# 线性代数（理科类）期中考试题（2017-11-5）

1(15). 设矩阵$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & a & 1 \\ 3 & -1 & 1 \end{pmatrix}$，已知$B$为3阶非零矩阵，满足$AB=0$，求$a$的值并写出$A$的相抵标准型.

answer: $a = -\frac{1}{5}$, $r(A) = 2$

2(15).设矩阵$A$的伴随矩阵$A^* = \begin{pmatrix} 4 & -2 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$,求$A$.

answer: $A = \frac{1}{2}\begin{pmatrix} 2 & 4 & 0 & 0 \\ 6 & 8 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 04 \end{pmatrix}$

3(15). 求下列矩阵方程：$\begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 2 \\ 1 & -1 & 0 \end{pmatrix} X = \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 3 & 1 \end{pmatrix}$

answer: $X = \begin{pmatrix} 1 & \frac{2}{5} & \frac{5}{6} \\ 0 & -\frac{1}{2} & -\frac{1}{6} \\ 0 & 0 & \frac{2}{3} \end{pmatrix}$

4(15).设$\alpha_1, \alpha_2, \alpha_3$是3维列向量，令$A = (\alpha_1, \alpha_2, \alpha_3)$, $B = (\alpha_1+\alpha_2+\alpha_3, \alpha_1 + 2\alpha_2 + 4\alpha_3, \alpha_1 + 3\alpha_2 + 9\alpha_3)$, 设$|B| = 1$, 求$|A|$.

answer: $1/2$.

5(15).设$A, B$是$n$阶方阵，$AB = kI_n, k \neq 0$. 证明：$b_{11} + b_{21}\lambda + \cdots + b_{n1}\lambda^{n-1} = \frac{k}{|A|}\begin{vmatrix} 1 & \lambda & \cdots & \lambda^{n-1} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & \cdots & \cdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$.

6(15).设$n$阶整数方阵$A$满足条件：对任意$n$维整数向量$\alpha \in \mathbb{Z}^n$, 线性方程组$Ax = \alpha$均有整数解. 矩阵$B$由矩阵$A$作以下变换得到：从第二列开始，每一列加上它的前一列，同时第一列加上原来的最后一列. 证明$B$的行列式$\det B$或为0, 或为$\pm 2$.

7(10）. 设$A$是数域$F$上$n$阶方阵，满足：对任意$n$维列向量$\alpha$,存在$\lambda_\alpha \in F$,使得$A\alpha = \lambda_\alpha \alpha$. 证明:$A$是纯量矩阵，即存在$c \in F$，使得$A = cI_n$.