电子电路与系统基础II

习题课第十四讲

正反馈正弦波振荡器　习题讲解

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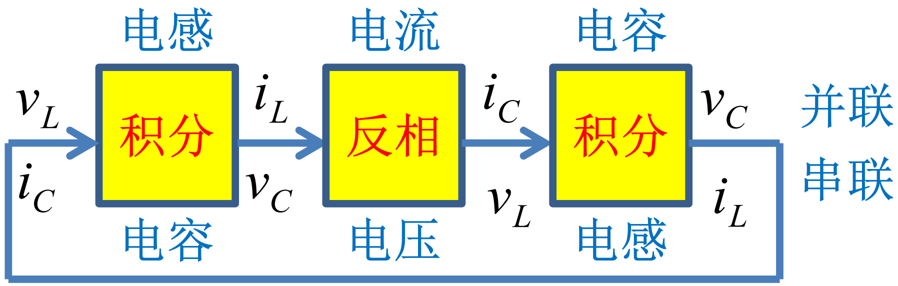
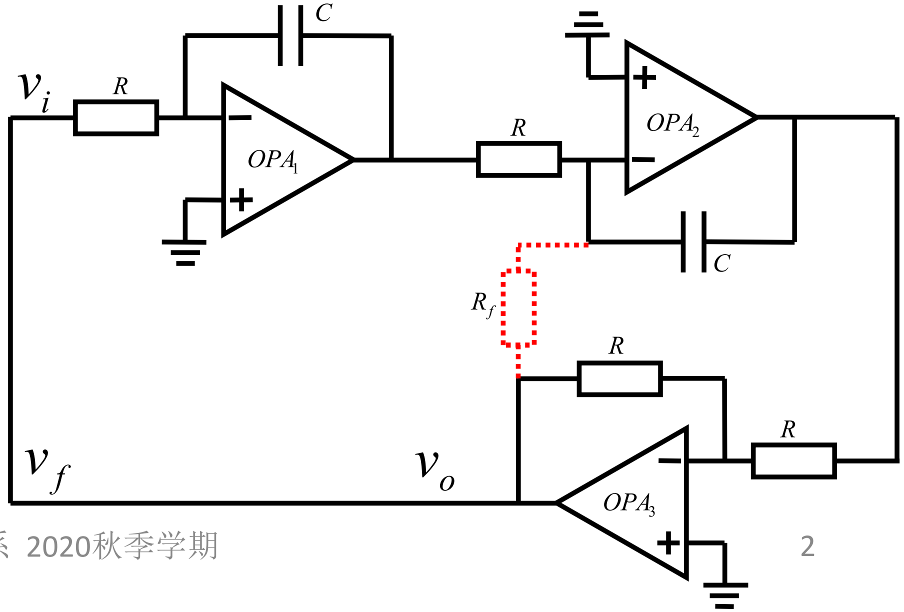
作业1 双积分正弦波振荡器

- 如图E10.4.11所示，OPA_1及其周边RC构成第一个RC反相积分器，OPA_2及其周边RC构成第二个RC反相积分器，OPA_3及其周边R构成一个反相器，原则上说，构成的‘积分器－积分器－反相器’闭环可以形成正弦波振荡器。电路调试中还需要外加一个R_f电阻，该电路才能自激振荡，考察原因，并说明该正弦波振荡器的振荡频率为多少？



正弦振荡器平衡原理模型

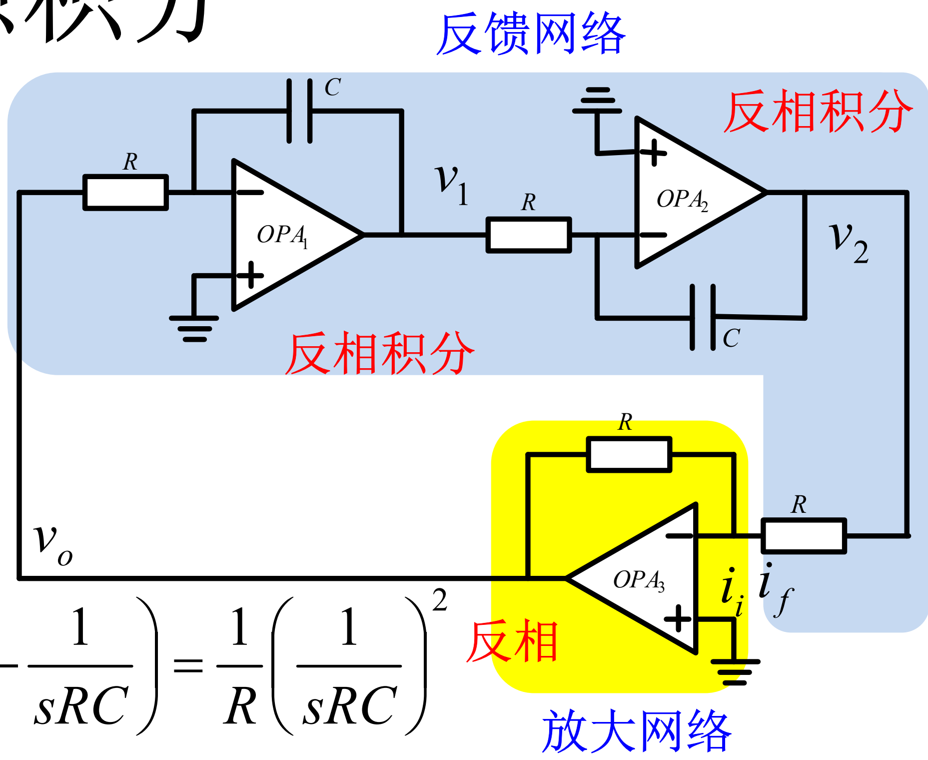


正弦振荡原理

- 负阻原理和正反馈原理，都是力图从电路角度考察自激振荡正弦波振荡器
  - 要求振荡器是自激振荡（**AF>1**，**r>R**）
    - 无需任何外加激励，靠电路内部噪声激励，形成从增幅振荡到等幅的稳定振荡
  - 用电路语言加以描述
    - 三个振荡条件：起振，平衡，稳定

- 对于难以用负阻原理和正反馈原理分析的电路，可以回到最原始的电路方程，从二阶系统的阻尼系数入手，考察其是否属于负阻尼情况，以此说明自激振荡的可能性
  - 零阻尼$\xi=0$：等幅正弦振荡（两种能量转换，维持总能量不变）
  - 负欠阻尼$-1<\xi<0$：增幅正弦振荡（可自激形成正弦振荡）
  - 正欠阻尼$0<\xi<1$：减幅正弦振荡（无法自激振荡）

理想运放实现的理想积分正反馈分析？



理想受控源：跨阻放大网络

$$A_0=\frac{\dot V_o}{\dot I_i}=-R=R_{m0}$$

跨导反馈网络：视运放网络为单向网络

$$F=\frac{\dot I_f}{\dot V_o}=\frac{\dot I_f}{\dot V_2}\cdot\frac{\dot V_2}{\dot V_1}\cdot\frac{\dot V_1}{\dot V_o}=\left(\frac{1}{R}\right)\cdot\left(-\frac{1}{sRC}\right)\cdot\left(-\frac{1}{sRC}\right)=\frac{1}{R}\left(\frac{1}{sRC}\right)^2$$

$$F(j\omega)=\frac{1}{R}\left(\frac{1}{j\omega RC}\right)^2=-\frac{1}{R}\frac{1}{\omega^2R^2C^2}$$

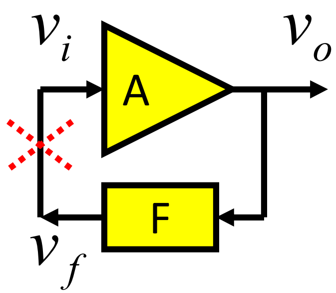
$$T_0=A_0F(j\omega)=(-R)\cdot\left(-\frac{1}{R}\frac{1}{\omega^2R^2C^2}\right)=\frac{1}{\omega^2R^2C^2}=\left(\frac{\omega_0}{\omega}\right)^2>1$$

$$\omega<\omega_0=\frac{1}{RC}$$

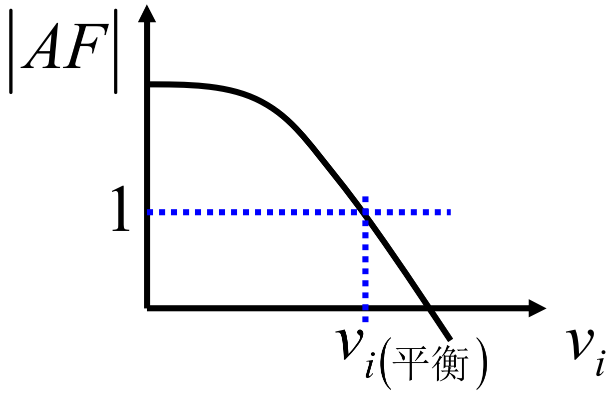
只要频率低于ω_0，
都满足起振条件？

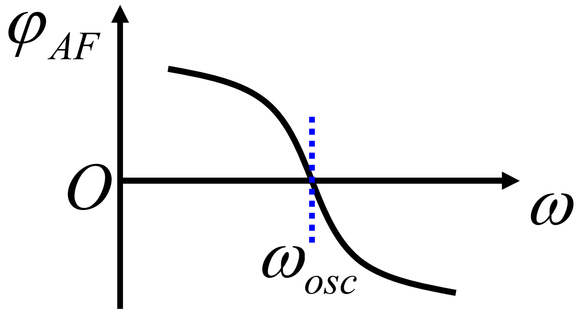
正反馈原理

环路增益和幅度无关？
环路增益相位和频率无关？



$AF = \left|AF\right| e^{j\varphi_{AF}}$



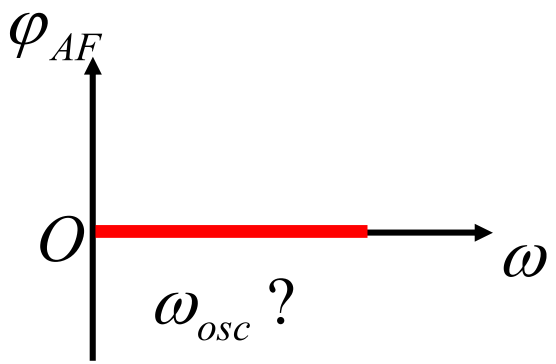


	幅度条件	相位（频率）条件
起振条件	$\left\|A_0 F\right\| > 1$	$\varphi_{A_0 F}(\omega_{osc}) = 0$（正反馈条件）
平衡条件	$\left\|\overline{A}F\right\| = 1$	$\varphi_{\overline{A}F}(\omega_{osc}) = 0$（正反馈条件）
	$V_{im} = V_{im\infty}$ （平衡点 ）	$\omega = \omega_{osc}$ （平衡点 ）
稳定条件	$\left.\frac{\partial\left\|\overline{A}F\right\|}{\partial V_{im}}\right\|_{V_{im}=V_{im\infty}} < 0$	$\left.\frac{\partial\varphi_{\overline{A}F}}{\partial\omega}\right\|_{\omega=\omega_{osc}} < 0$
$T = \overline{A}F = \left\|\overline{A}F\right\| e^{j\varphi_{\overline{A}F}} = \left\|T\right\| e^{j\varphi_T}$		

错在哪里？临界系统情况
根本就不是负阻尼自激振荡！

$$T_0 = A_0 F(j\omega) = (-R)\cdot\left(-\frac{1}{R}\frac{1}{\omega^2 R^2 C^2}\right) = \frac{1}{\omega^2 R^2 C^2} = \left(\frac{\omega_0}{\omega}\right)^2 > 0$$

环路增益和幅度无关？幅度平衡条件如何满足？



在所有频点上，$\varphi_{AF} = 0$ 任意频点均正反馈？

$$\left.\frac{\partial \varphi_{AF}}{\partial \omega}\right|_{\omega=\omega_{osc}} < 0$$

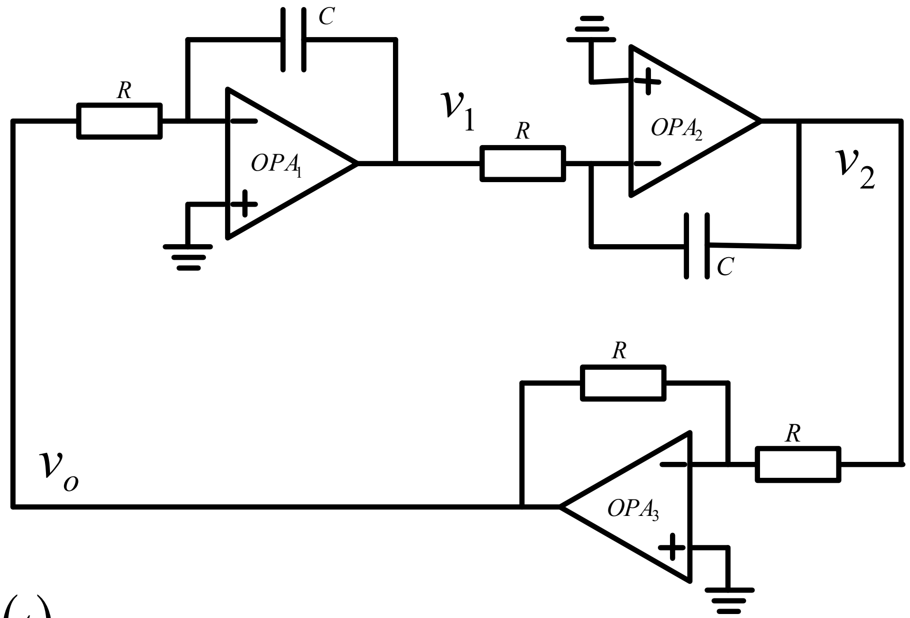
哪个频点上满足相位（频率）稳定条件？

属临界稳定情况：仅在 $T_0 = 1$ 频点上，可正弦振荡

$$\omega_{osc} = \omega_0 = \frac{1}{RC}$$

否则电路方程无法满足

回到电路方程
正弦振荡



$$v_1(t) = -\frac{1}{RC}\int v_o(t)dt$$

$$v_2(t) = -\frac{1}{RC}\int v_1(t)dt \qquad v_o(t) = -v_2(t)$$

$$v_o(t) = -v_2(t) = \frac{1}{RC}\int v_1(t)dt = -\frac{1}{(RC)^2}\iint v_o(t)dt \qquad \frac{d^2}{dt^2}v_o(t) + \omega_0^2 v_o(t) = 0$$

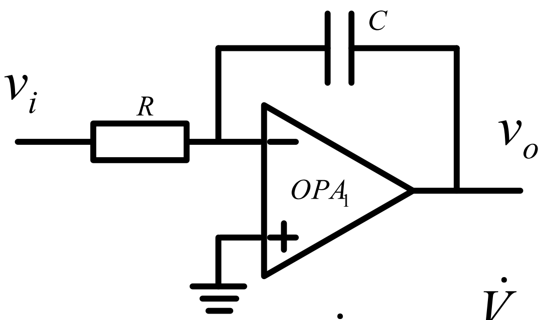
$$v_o(t) = V_{op}\cos(\omega_0 t + \varphi_0)$$

两个参量由初始条件决定
幅度一开始就定了，既不增加，也不降低
这是数学上的理想情况，物理上不可能发生

由电容存储初始能量决定

由观测时间起点决定（两个电容的初始储能）

实际器件非理想
只考虑运放有限增益的影响



$$\dot{V}_n = -\frac{\dot{V}_o}{A_{v0}}$$

有限增益：虚短不再满足

假设零输出电阻：输出端理想压控压源模型

增益足够大，输出电阻足够小，单向化条件很容易满足，传递函数很容易获得

$$\frac{\dot{V}_i - \dot{V}_n}{R} = \left(\dot{V}_n - \dot{V}_o\right)sC$$

假设无限大输入电阻，虚断仍然满足

$$\dot{V}_i = -\frac{\dot{V}_o}{A_{v0}} - \left(\frac{\dot{V}_o}{A_{v0}} + \dot{V}_o\right)sRC$$

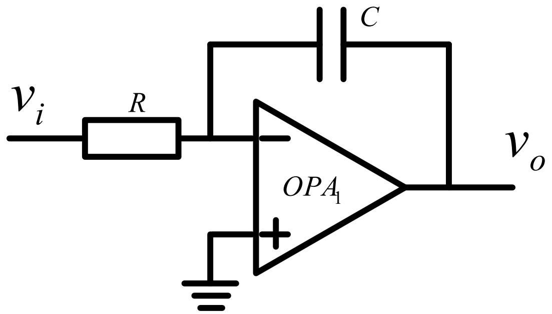
$$\frac{\dot{V}_o}{\dot{V}_i} = -\frac{1}{\frac{1}{A_{v0}} + \left(\frac{1}{A_{v0}} + 1\right)sRC}$$

$$= -\frac{A_{v0}}{1 + \left(1 + A_{v0}\right)sRC} \approx -\frac{A_{v0}}{1 + sA_{v0}RC}$$

有限增益导致非理想积分

在原理性分析时，往往只考虑一个非理想因素就够了，过多考虑将导致问题过度复杂

理想积分和非理想积分



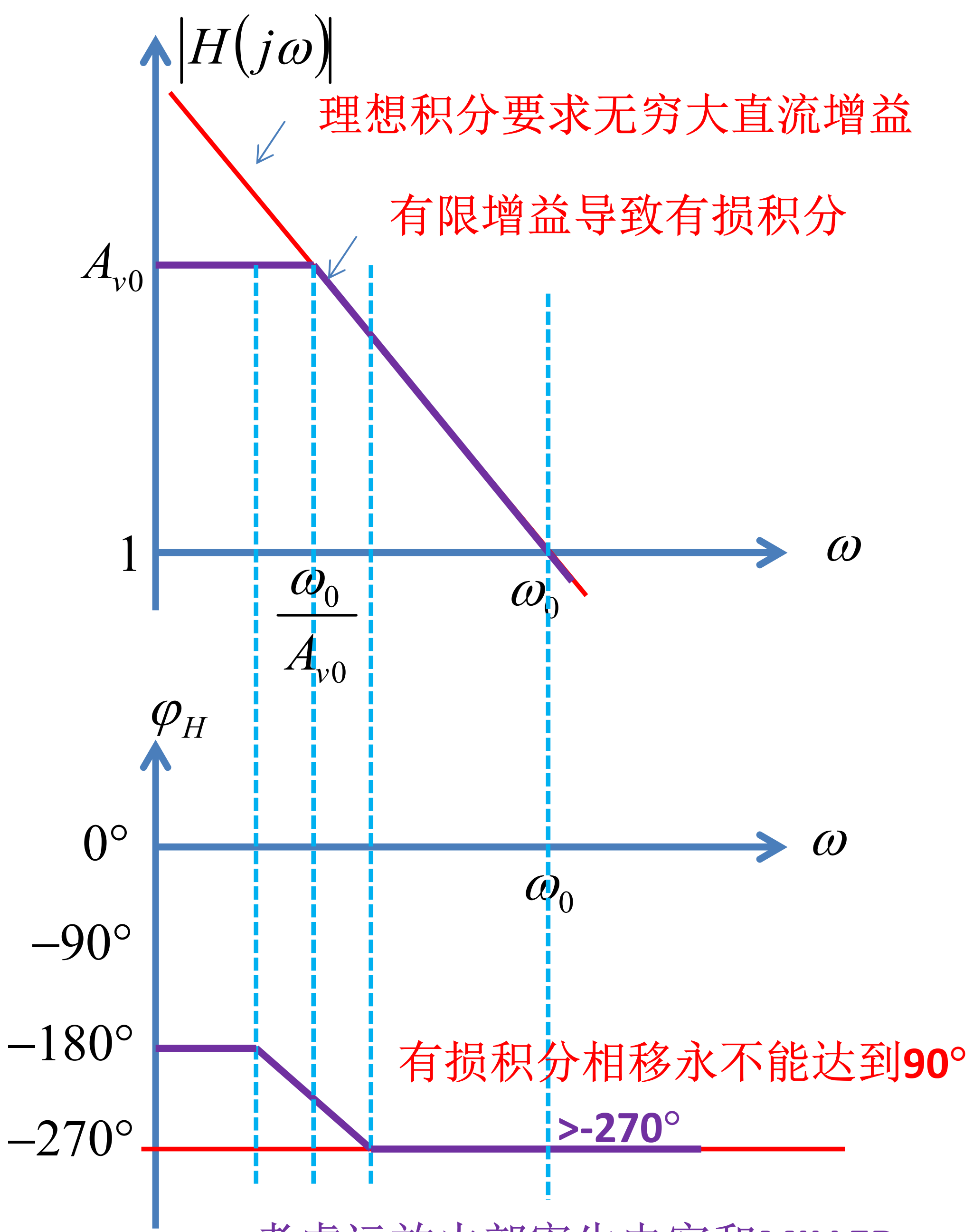
$$H(s)=\frac{\dot{V}_o}{\dot{V}_i}$$

$$=-\frac{A_{v0}}{1+sA_{v0}RC}$$

非理想反相积分

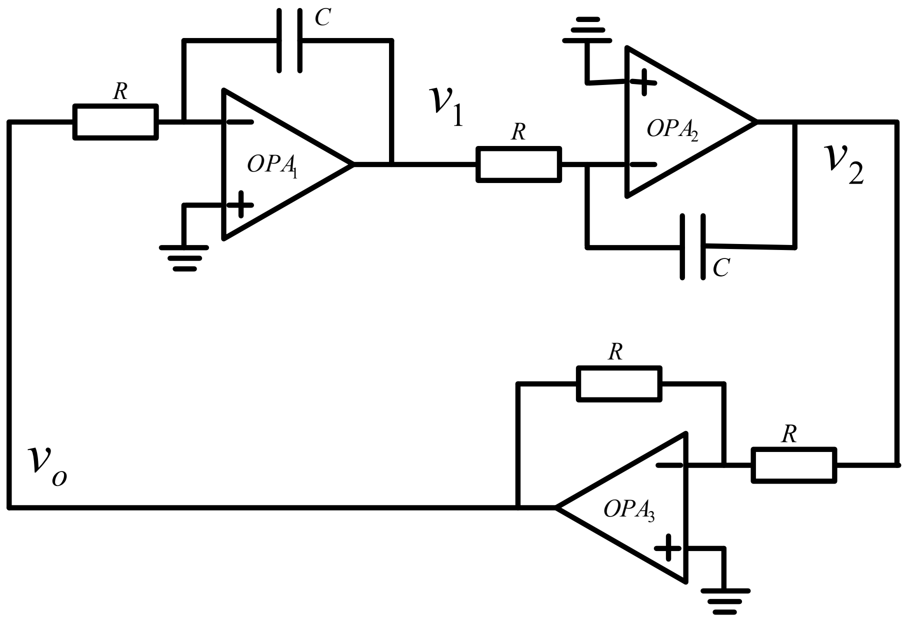
$$\xrightarrow{A_{v0}\to\infty}-\frac{1}{sRC}$$

理想反相积分



考虑运放内部寄生电容和**MILLER**补偿电容作用，相移可超**90°**

有损双积分
正阻尼



$$V_1(s)=-\frac{A_{v0}}{1+sA_{v0}RC}V_o(s)$$

$$V_2(s)=-\frac{A_{v0}}{1+sA_{v0}RC}V_1(s)$$

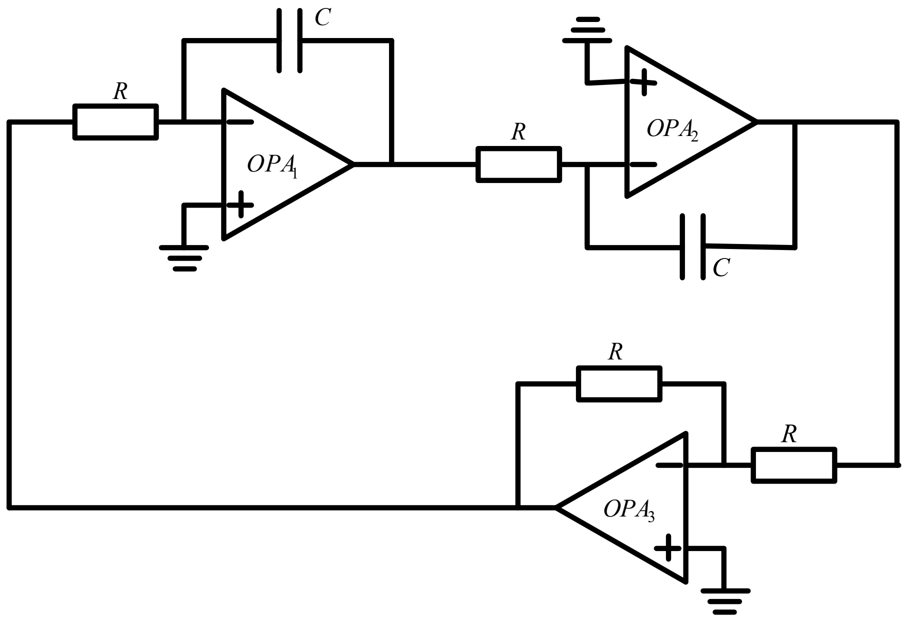
$$V_o(s)=-V_2(s)=\frac{A_{v0}}{1+sA_{v0}RC}V_1(s)=-\left(\frac{A_{v0}}{1+sA_{v0}RC}\right)^2V_o(s)$$

$$(1+sA_{v0}RC)^2V_o(s)+A_{v0}{}^2V_o(s)=0$$

$$s^2(A_{v0}RC)^2V_o(s)+2sA_{v0}RCV_o(s)+\left(1+A_{v0}{}^2\right)V_o(s)=0$$

$$s^2V_o(s)+\frac{2}{A_{v0}RC}sV_o(s)+\frac{A_{v0}{}^2+1}{A_{v0}{}^2(RC)^2}V_o(s)=0$$

有限增益正阻尼减幅正弦振荡



$$s^2V_o(s)+\frac{2}{A_{v0}RC}sV_o(s)+\frac{A_{v0}{}^2+1}{A_{v0}{}^2(RC)^2}V_o(s)=0$$

$$s^2V_o(s)+\frac{2}{A_{v0}RC}sV_o(s)+\frac{1}{(RC)^2}V_o(s)=0$$

$$\frac{d^2}{dt^2}v_o(t)+\frac{2}{A_{v0}RC}\frac{d}{dt}v_o(t)+\frac{1}{(RC)^2}v_o(t)=0$$

$$s\to\frac{d}{dt}$$

$$V_o(s)\to v_o(t)$$

$$\frac{d^2}{dt^2}v_o(t)+2\xi\omega_n\frac{d}{dt}v_o(t)+\omega_n^2v_o(t)=0$$

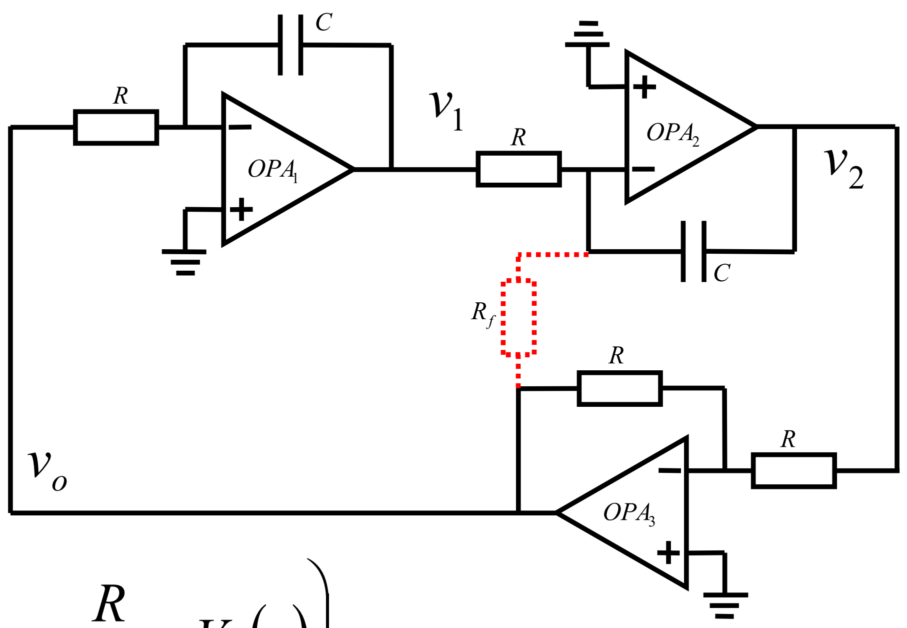
$$\omega_n=\frac{1}{RC}$$ 自由振荡频率

$$\xi=\frac{1}{A_{v0}}$$ 阻尼系数由增益决定

无穷增益则零阻尼

阻尼系数很小，初始能量导致严重振铃，但最终会消停

真正的正反馈



$$V_1(s) = -\frac{A_{v0}}{1+sA_{v0}RC}V_o(s)$$

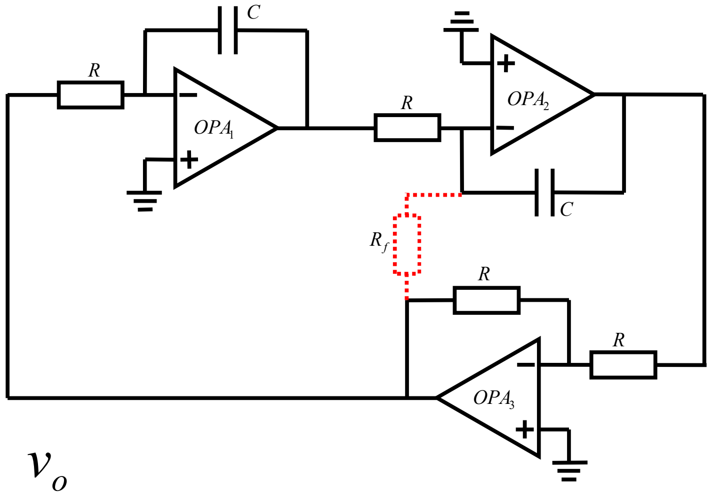
$$V_2(s) = -\frac{A_{v0}}{1+sA_{v0}(R_f \parallel R)C}\left(\frac{R_f}{R_f+R}V_1(s)+\frac{R}{R_f+R}V_o(s)\right)$$

$$V_o(s) = -V_2(s) = \frac{A_{v0}}{1+sA_{v0}(R_f \parallel R)C}\left(\frac{R_f}{R_f+R}V_1(s)+\frac{R}{R_f+R}V_o(s)\right)$$

$$= \frac{A_{v0}}{1+sA_{v0}(R_f \parallel R)C}\left(-\frac{R_f}{R_f+R}\frac{A_{v0}}{1+sA_{v0}RC}V_o(s)+\frac{R}{R_f+R}V_o(s)\right)$$

$$(1+sA_{v0}(R_f \parallel R)C)(1+sA_{v0}RC)V_o(s)-\frac{R}{R_f+R}A_{v0}(1+sA_{v0}RC)V_o(s)+\frac{R_f}{R_f+R}A_{v0}^2V_o(s)=0$$

电路方程
负阻尼增幅振荡



随振荡幅度增加，进入运放饱和区，准线性增益下降，当增益**Av**下降到使得$\mathbf{R_f=0.5A_vR}$时，则零阻尼正弦振荡

$$\left(1+sA_{v0}\left(R_f \parallel R\right)C\right)\left(1+sA_{v0}RC\right)V_o(s)-\frac{R}{R_f+R}A_{v0}\left(1+sA_{v0}RC\right)V_o(s)+\frac{R_f}{R_f+R}A_{v0}^2V_o(s)=0$$

$$\left(s^2+s\frac{\left(2R_f+R-RA_{v0}\right)}{A_{v0}RCR_f}+\frac{\left(R_f+R_fA_{v0}^2+R-RA_{v0}\right)}{A_{v0}^2(RC)^2R_f}\right)V_o(s)=0$$

$$\left(s^2+s\frac{\left(2R_f-RA_{v0}\right)}{A_{v0}RCR_f}+\frac{\left(R_fA_{v0}^2-RA_{v0}\right)}{A_{v0}^2(RC)^2R_f}\right)V_o(s)=0$$

$$R_f<\frac{A_{v0}}{2}R$$

正反馈电阻，可导致负阻尼

$$R_f\to\infty:\left(s^2+\frac{2}{A_{v0}RC}s+\frac{1}{(RC)^2}\right)V_o(s)=0$$

中间验证：和前面无$\mathbf{R_f}$结果一致

$$0.5R<<R_f<0.5A_{v0}R$$

$$A_{v0}\to\infty:\left(s^2-\frac{1}{R_fC}s+\frac{1}{(RC)^2}\right)V_o(s)=0$$

理想运放：一定是负阻尼

$$-1<\xi<0$$

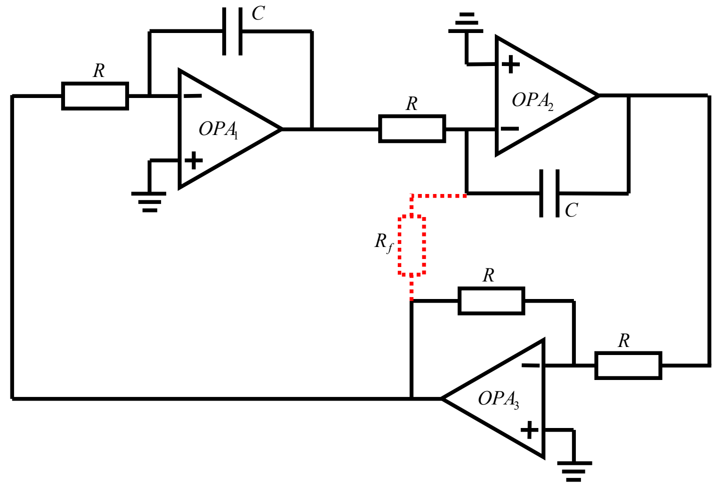
$$\xi=-\frac{R}{2R_f}>-1 \qquad R_f>>0.5R$$

确保高**Q**值

阻尼系数和自由振荡频率

此方程由运放在线性区工作假设推导而来，
属起振阶段小信号分析

$$\left(s^2+s\frac{\left(2R_f-RA_{v0}\right)}{A_{v0}RCR_f}+\frac{\left(R_fA_{v0}^2-RA_{v0}\right)}{A_{v0}^2\left(RC\right)^2R_f}\right)V_o\left(s\right)=0$$



$$\omega_n=\frac{1}{RC}\sqrt{1-\frac{1}{A_{v0}}\frac{R}{R_f}}\approx\frac{1}{RC}$$

二阶系统自由振荡频率

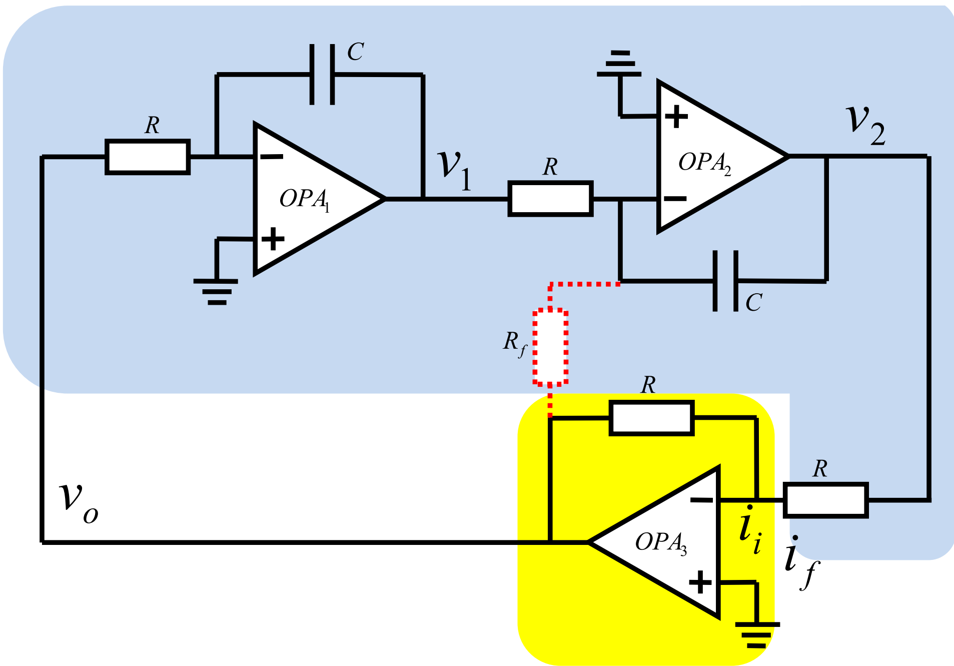
$$\xi=\frac{\frac{1}{A_{v0}}-0.5\frac{R}{R_f}}{\sqrt{1-\frac{1}{A_{v0}}\frac{R}{R_f}}}\approx\frac{1}{A_{v0}}-0.5\frac{R}{R_f}$$

二阶系统阻尼系数

$$0.5R<<R_f<0.5A_{v0}R$$

确保负阻尼且振荡形态

负阻尼就可以用正反馈原理？



跨阻放大网络

$$A_0=\frac{\dot{V}_o}{\dot{I}_i}=-R=R_{m0}$$

跨导反馈网络

$$F=\frac{\dot{I}_f}{\dot{V}_o}=\frac{\dot{I}_f}{\dot{V}_2}\frac{\dot{V}_2}{\dot{V}_o}=\left(\frac{1}{R}\right)\cdot\left(-\frac{A_{v0}}{1+sA_{v0}\left(R\parallel R_f\right)C}\right)\left(-\frac{A_{v0}}{1+sA_{v0}RC}\frac{R_f}{R_f+R}+\frac{R}{R_f+R}\right)$$

$$\overset{s=j\omega}{=}\frac{1}{R}\frac{\left(A_{v0}^2R_f-A_{v0}R\right)-j\omega A_{v0}^2R^2C}{\left(R_f+R-\omega^2A_{v0}^2R^2C^2R_f\right)+j\omega A_{v0}RC\left(2R_f+R\right)}$$

$$T_0=A_0F\left(j\omega\right)=\frac{\left(A_{v0}R-A_{v0}^2R_f\right)+j\omega A_{v0}^2R^2C}{\left(R_f+R-\omega^2A_{v0}^2R^2C^2R_f\right)+j\omega A_{v0}RC\left(2R_f+R\right)}$$

正反馈条件决定频率

$$-1 < \xi < 0$$

$$0.5R << R_f < 0.5A_{v0}R$$

$$T_0 = A_0 F(j\omega) = \frac{\left(A_{v0}R - A_{v0}^2 R_f\right) + j\omega A_{v0}^2 R^2 C}{\left(R_f + R - \omega^2 A_{v0}^2 R^2 C^2 R_f\right) + j\omega A_{v0} RC\left(2R_f + R\right)}$$

$$\varphi_{T_0}(\omega_{osc}) = 0 \Rightarrow \frac{\left(A_{v0}R - A_{v0}^2 R_f\right)}{\left(R_f + R - \omega_{osc}^2 A_{v0}^2 R^2 C^2 R_f\right)} = \frac{j\omega_{osc} A_{v0}^2 R^2 C}{j\omega_{osc} A_{v0} RC\left(2R_f + R\right)} = A_{v0}\frac{R}{\left(2R_f + R\right)}$$

相位平衡条件（虚部条件）决定振荡频率，振荡频率和增益相关

$$\omega_{osc,0} = \frac{1}{RC}\sqrt{\frac{A_{v0} - 1 + 2\frac{R_f}{R}A_{v0}}{A_{v0}^2}} \approx \frac{1}{RC}\sqrt{\frac{1 + 2\frac{R_f}{R}}{A_{v0}}}$$

实部条件决定起振条件具体要求：

$$T_0\left(j\omega_{osc,0}\right) = A_{v0}\frac{R}{\left(2R_f + R\right)} > 1$$

$$R_f < \frac{\left(A_{v0} - 1\right)R}{2}$$

起振条件就是负阻尼系数条件

数值例

A_{v0}=200000
R=10kΩ
C=0.01μF

$$0.5R << R_f < 0.5A_{v0}R$$

$$5k\Omega << R_f < 1G\Omega$$

$$\omega_0 = \frac{1}{RC} = \frac{1}{10\times10^3\times0.01\times10^{-6}} = 10000 rad/s = 2\pi\times1.59kHz$$

$$\omega_{osc,0} = \frac{1}{RC}\sqrt{\frac{A_{v0}-1+2\frac{R_f}{R}A_{v0}}{A_{v0}^2}}$$

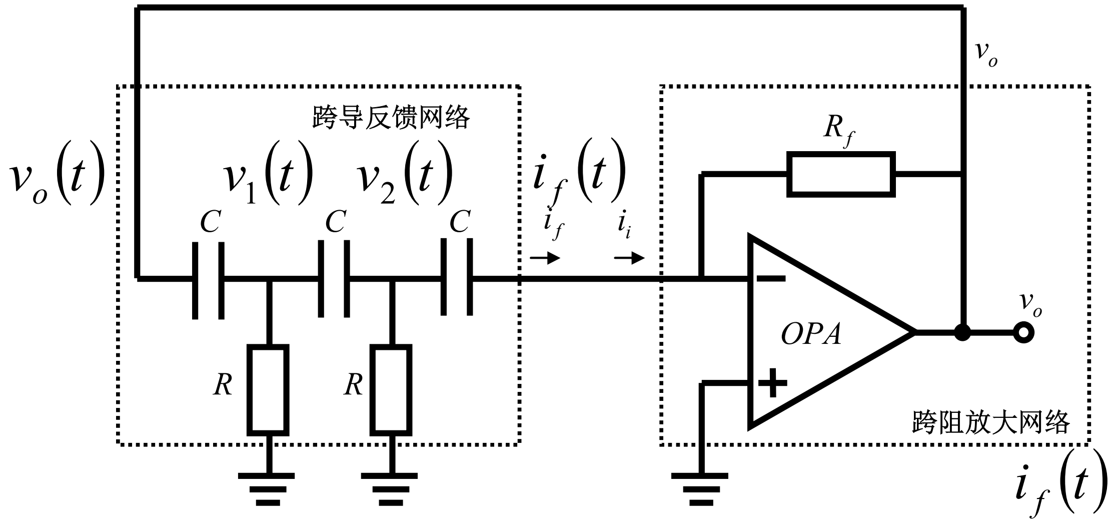
$$R_f < \frac{(A_{v0}-1)R}{2}$$

$$\approx \omega_0\sqrt{\frac{1+2\frac{R_f}{R}}{A_{v0}}} \quad \overset{\overline{A_{v0}}=\frac{2R_f}{R}+1}{=} \quad \omega_0$$

$$\overline{A_{v0}} = \frac{2R_f}{R} + 1$$ 平衡条件

考虑运放寄生电容效应：可能不用加正反馈电阻，就可以振荡，但振荡频率将偏离理论频率很远

作业2 超前RC移相正弦波振荡器



$$v_o(t) = V_{om}\cos\omega_{osc}t$$

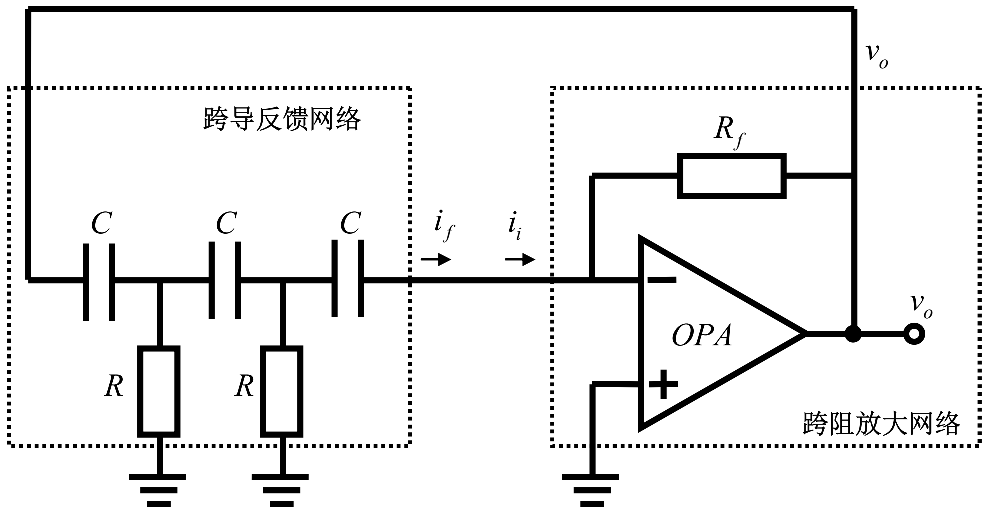
$$v_1(t) = V_{1m}\cos(\omega_{osc}t + \Delta\varphi_1)$$

$$v_2(t) = V_{2m}\cos(\omega_{osc}t + \Delta\varphi_1 + \Delta\varphi_2)$$

$$i_f(t) = I_{fm}\cos(\omega_{osc}t + \Delta\varphi_1 + \Delta\varphi_2 + \Delta\varphi_3)$$

- 请分析确认输出**v_o**电压经第一级**RC**高通后相位超前多少度？经第二级**RC**高通后相位又超前多少度？经第三级单电容理想微分网络（电容电压转化为电容电流），相位又超前多少度？验证相位总共超前度数为**180°**，和后面的反相跨阻放大的**180°**抵偿，恰好形成正反馈。

正反馈分析



$$T_0 = A_0 F = -R_f \cdot \frac{sC \cdot (sRC)^2}{3s^2R^2C^2 + 4sRC + 1}\bigg|_{s=j\omega} = R_f \cdot \frac{j\omega C \cdot (\omega RC)^2}{\left(1 - 3\omega^2R^2C^2\right) + 4j\omega RC}\bigg|_{s=j\omega}$$

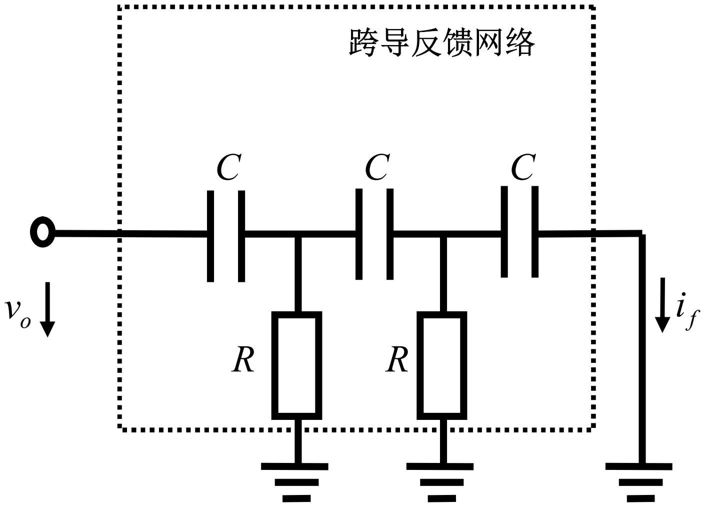
$$\varphi_T(\omega)_{\omega=\omega_{osc}} = 0 \Rightarrow 1 - 3\omega_{osc}^2 R^2C^2 = 0 \Rightarrow \omega_{osc} = \frac{1}{\sqrt{3}RC}$$

正反馈条件（相位平衡条件）决定振荡频率

$$T_0(j\omega_{osc}) = \frac{R_f}{12R} > 1$$

起振条件

跨导反馈网络



$$\mathbf{ABCD}=\begin{bmatrix}1 & \frac{1}{sC}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{R} & 1\end{bmatrix}\begin{bmatrix}1 & \frac{1}{sC}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{R} & 1\end{bmatrix}\begin{bmatrix}1 & \frac{1}{sC}\\ 0 & 1\end{bmatrix}$$

$$=\ldots=\begin{bmatrix}* & \left(\left(\frac{1}{sRC}\right)^2+\frac{4}{sRC}+3\right)\frac{1}{sC}\\ * & *\end{bmatrix}$$

$$F(j\omega)=G_f=\frac{1}{B}=\frac{\dot{I}_f}{\dot{V}_o}=-\frac{j\omega C\cdot(\omega RC)^2}{1-3\omega^2R^2C^2+4j\omega RC}$$

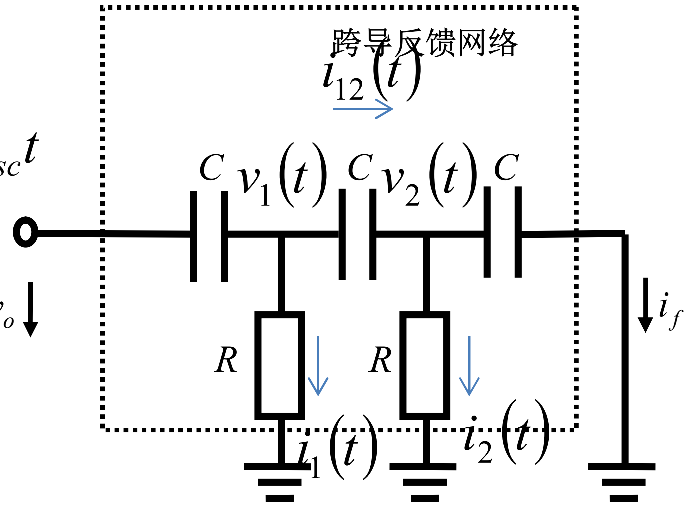
$$F(j\omega_{osc})=\frac{\dot{I}_f}{\dot{V}_o}(j\omega_{osc})=-\frac{1}{12R}$$

$$v_o(t)=V_{om}\cos\omega_{osc}t$$

$$i_f(t)=-\frac{V_{om}}{12R}\cos\omega_{osc}t=\frac{V_{om}}{12R}(\cos\omega_{osc}t+\pi)$$

反馈网络总共移相180°

$$v_o(t)=V_{om}\cos\omega_{osc}t$$



RC均为双向/互易元件，分析移相时必须考虑前后级影响

倒推法获得各结点电压和支路电流

$$i_f(t)=\frac{V_{om}}{12R}\cos(\omega_{osc}t+\pi)$$

$$\omega_{osc}=\frac{1}{\sqrt{3}RC}$$

$$\frac{\dot{V}_2}{\dot{I}_f}=\frac{1}{j\omega C}$$

$$v_2(t)=\frac{V_{om}}{12R}\frac{1}{\omega_{osc}C}\cos\left(\omega_{osc}t+\pi-\frac{\pi}{2}\right)=\frac{\sqrt{3}}{12}V_{om}\cos\left(\omega_{osc}t+\frac{\pi}{2}\right)$$

$$i_2(t)=\frac{v_2(t)}{R}=\frac{\sqrt{3}}{12}\frac{V_{om}}{R}\cos\left(\omega_{osc}t+\frac{\pi}{2}\right)$$

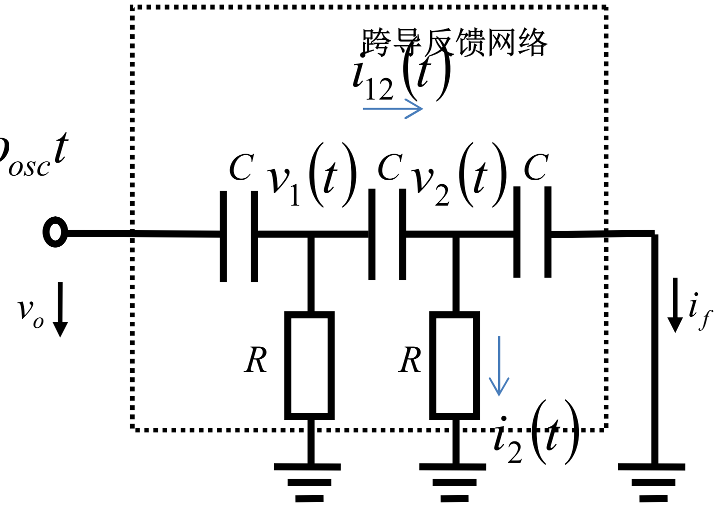
$$\dot{I}_{12}=\dot{I}_2+\dot{I}_f=\frac{\sqrt{3}}{12}\frac{V_{om}}{R}\angle 90^\circ+\frac{1}{12}\frac{V_{om}}{R}\angle 180^\circ$$

$$=-\frac{1}{12}\frac{V_{om}}{R}+j\frac{\sqrt{3}}{12}\frac{V_{om}}{R}=\frac{1}{6}\frac{V_{om}}{R}\angle 120^\circ$$

$$i_{12}(t)=\frac{1}{6}\frac{V_{om}}{R}\cos\left(\omega_{osc}t+\frac{2}{3}\pi\right)$$

$$v_o(t)=V_{om}\cos\omega_{osc}t$$

$$\omega_{osc}=\frac{1}{\sqrt{3}RC}$$



$$i_f(t)=\frac{V_{om}}{12R}\cos(\omega_{osc}t+\pi)$$

0.083

$$v_2(t)=\frac{\sqrt{3}}{12}V_{om}\cos\left(\omega_{osc}t+\frac{\pi}{2}\right)$$

0.144

$$i_{12}(t)=\frac{1}{6}\frac{V_{om}}{R}\cos\left(\omega_{osc}t+\frac{2}{3}\pi\right)$$

$$\dot{V}_1=\dot{I}_{12}\frac{1}{j\omega_{osc}C}+\dot{V}_2$$

$$=\left(\frac{1}{6}\frac{V_{om}}{R}\angle 120^\circ\right)\left(\frac{\sqrt{3}RC}{C}\angle -90^\circ\right)+\frac{\sqrt{3}}{12}V_{om}\angle 90^\circ$$

$$=\frac{\sqrt{3}}{6}V_{om}\angle 30^\circ+\frac{\sqrt{3}}{12}V_{om}\angle 90^\circ=\frac{\sqrt{3}}{6}V_{om}\left(\frac{\sqrt{3}}{2}+j\frac{1}{2}\right)+j\frac{\sqrt{3}}{12}V_{om}$$

$$=\frac{1}{4}V_{om}+j\frac{\sqrt{3}}{6}V_{om}=\frac{3}{12}V_{om}+j\frac{2\sqrt{3}}{12}V_{om}=\frac{\sqrt{9+12}}{12}V_{om}\angle\arctan\frac{2\sqrt{3}}{3}$$

$$=\frac{\sqrt{21}}{12}V_{om}\angle 49.1^\circ$$

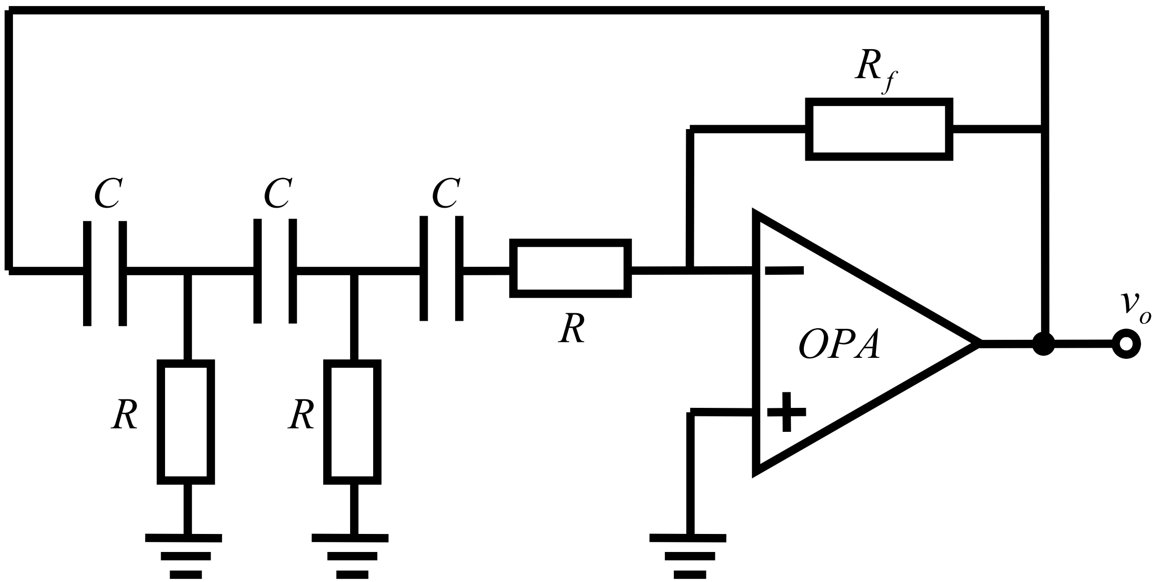
$$v_1(t)=\frac{\sqrt{21}}{12}V_{om}\cos(\omega_{osc}t+0.273\pi)$$

0.382

在ω_{osc}频点上，第一级**CR**网络相位超前**49.1°**；第二级**CR**网络相位超前**40.9°**；第三级**C**微分网络相位超前**90°**；反馈网络总共超前移相**180°**；放大网络滞后移相**180°**，闭环形成正反馈。

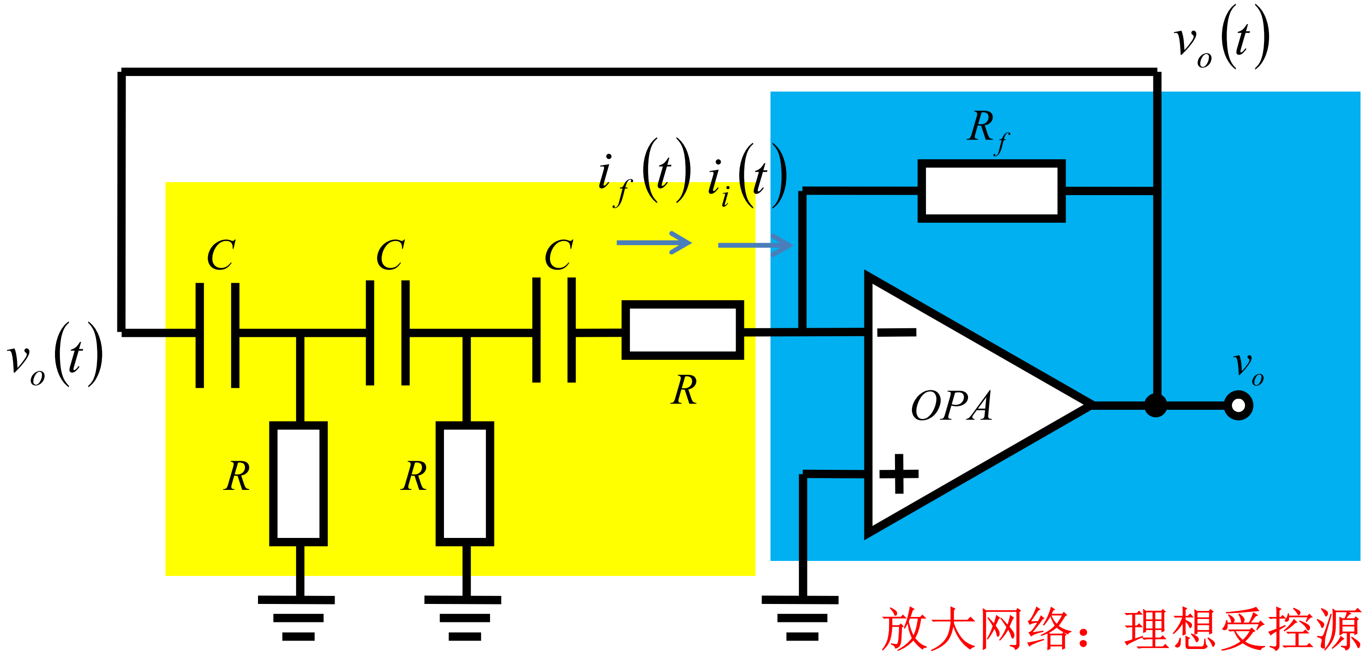
作业3 超前RC移相正弦波振荡器

- 证明图E10.4.19所示RC移相正弦波振荡器的起振条件为 $R_f > 29R$ ，振荡频率为 $f_0 = \frac{1}{2\pi\sqrt{6}RC}$ 。



提示：放大网络和反馈网络划分时，应确保放大网络具有理想受控源特性，于是则可将等效受控源之外的所有其他元件全部划分到反馈网络中，如是分析可有效去除负载效应，获得正确的分析结果。

放大网络和反馈网络分解



放大网络：理想受控源
理想受控源没有负载效应

理想流控压源

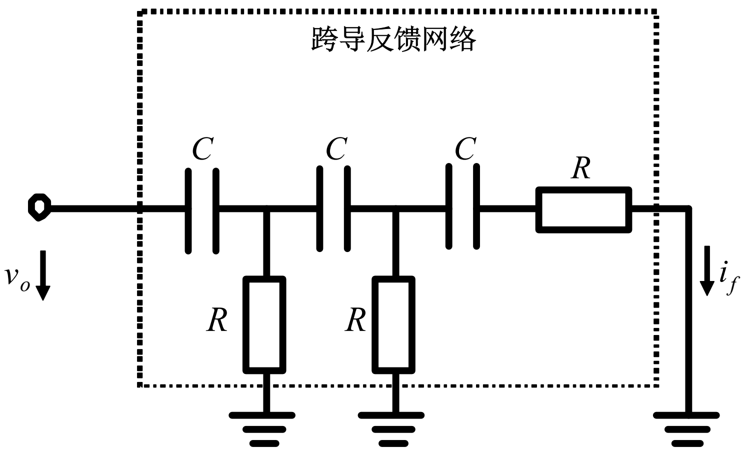
$$F = \frac{\dot{I}_i}{\dot{V}_o} = G_F = \ldots$$

$$A_0 = \frac{\dot{V}_o}{\dot{I}_i} = R_{m0} = -R_f$$

跨导反馈网络

$$\mathbf{ABCD}=\begin{bmatrix}1 & \frac{1}{sC}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{R} & 1\end{bmatrix}\begin{bmatrix}1 & \frac{1}{sC}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{R} & 1\end{bmatrix}\begin{bmatrix}1 & \frac{1}{sC}+R\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+\frac{1}{sRC} & \frac{1}{sC}\\ \frac{1}{R} & 1\end{bmatrix}\begin{bmatrix}1+\frac{1}{sRC} & \frac{1}{sC}\\ \frac{1}{R} & 1\end{bmatrix}\begin{bmatrix}1 & \frac{1}{sC}+R\\ 0 & 1\end{bmatrix}$$



$$=\begin{bmatrix}\left(1+\frac{1}{sRC}\right)^2+\frac{1}{sRC} & \frac{1}{sC}\left(2+\frac{1}{sRC}\right)\\ \frac{1}{R}\left(2+\frac{1}{sRC}\right) & 1+\frac{1}{sRC}\end{bmatrix}\begin{bmatrix}1 & \frac{1}{sC}+R\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}* & \left(\left(1+\frac{1}{sRC}\right)^2+\frac{1}{sRC}\right)\left(\frac{1}{sC}+R\right)+\frac{1}{sC}\left(2+\frac{1}{sRC}\right)\\ * & *\end{bmatrix}$$

$$F=\frac{\dot{I}_i}{\dot{V}_o}=G_F=\frac{1}{B}$$

$$=\begin{bmatrix}* & R\left(\frac{1}{(sRC)^3}+\frac{5}{(sRC)^2}+\frac{6}{sRC}+1\right)\\ * & *\end{bmatrix}$$

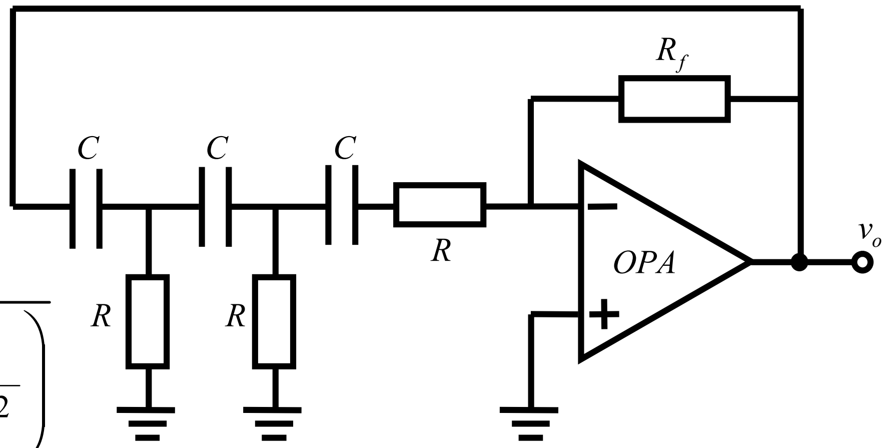
$$=\frac{1}{R}\frac{1}{\frac{1}{(sRC)^3}+\frac{5}{(sRC)^2}+\frac{6}{sRC}+1}$$

$$\overset{s=j\omega}{=}\frac{1}{R}\frac{1}{\frac{1}{j\omega RC}\left(6-\frac{1}{(\omega RC)^2}\right)+\left(1-\frac{5}{(\omega RC)^2}\right)}$$

相位平衡条件（正反馈条件）决定振荡频率

$$A_0 = \frac{\dot{V}_o}{\dot{I}_i} = R_{m0} = -R_f$$

$$F = \frac{\dot{I}_i}{\dot{V}_o} = \frac{1}{R}\frac{1}{\frac{1}{j\omega RC}\left(6-\frac{1}{(\omega RC)^2}\right)+\left(1-\frac{5}{(\omega RC)^2}\right)}$$



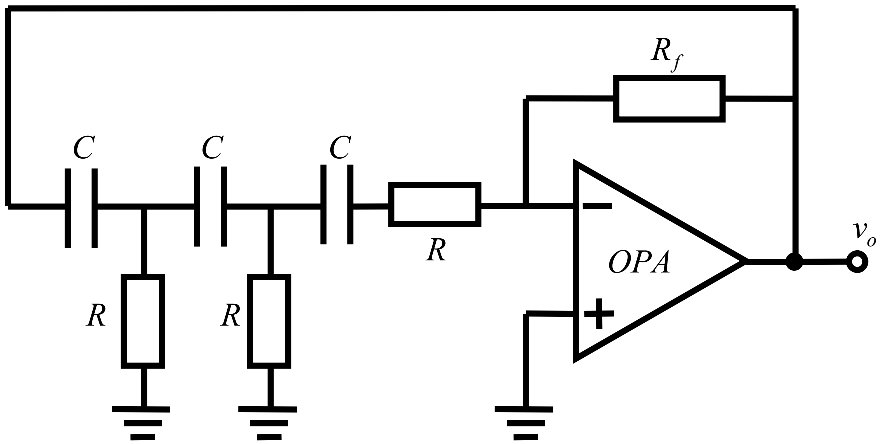
$$T_0 = A_0F = -\frac{R_f}{R}\frac{1}{\frac{1}{j\omega RC}\left(6-\frac{1}{(\omega RC)^2}\right)+\left(1-\frac{5}{(\omega RC)^2}\right)}$$

$$\varphi_{T_0}(\omega_{osc}) = 0 \Rightarrow 6-\frac{1}{(\omega_{osc}RC)^2} = 0 \Rightarrow \omega_{osc} = \frac{1}{\sqrt{6}RC}$$

正反馈条件决定振荡频率

起振条件

$$T_0 = A_0 F = -\frac{R_f}{R}\frac{1}{\frac{1}{j\omega RC}\left(6-\frac{1}{(\omega RC)^2}\right)+\left(1-\frac{5}{(\omega RC)^2}\right)}$$



$$\varphi_{T_0}(\omega_{osc}) = 0 \Rightarrow 6-\frac{1}{(\omega_{osc}RC)^2}=0$$

$$\Rightarrow \omega_{osc} = \frac{1}{\sqrt{6}RC}$$

$$T_0(j\omega_{osc}) = -\frac{R_f}{R}\frac{1}{j0+(1-5\times 6)} = \frac{R_f}{29R} > 1$$

$$\Rightarrow R_f > 29R$$

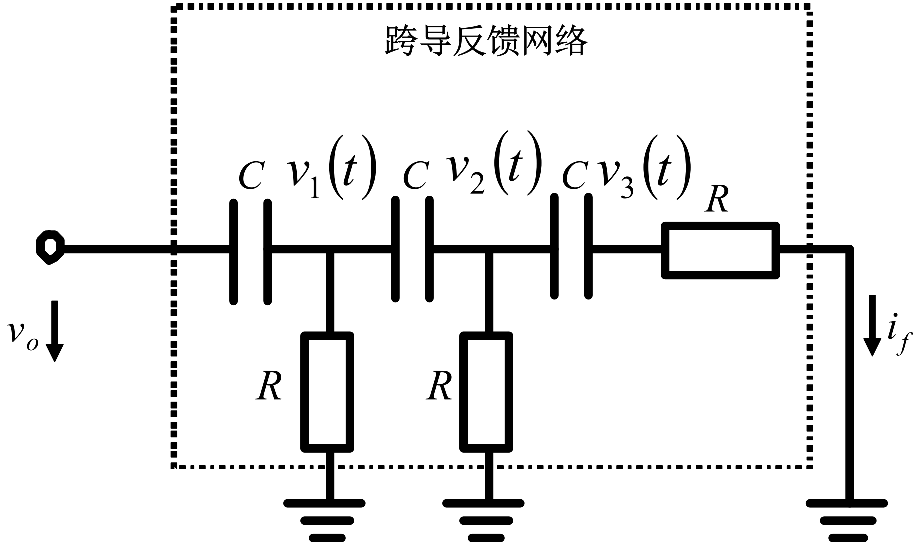
R_f越大，负反馈越小，只有正反馈大于负反馈，才能起振

三级移相

$$F(j\omega_{osc})=\frac{\dot{I}_i}{\dot{V}_o}(j\omega_{osc})$$

$$=\frac{1}{R}\frac{1}{\frac{1}{j\omega RC}\left(6-\frac{1}{(\omega RC)^2}\right)+\left(1-\frac{5}{(\omega RC)^2}\right)}\Bigg|_{\omega=\omega_{osc}}$$

$$=\frac{1}{-29R}$$



$$v_o(t)=V_{om}\cos\omega_{osc}t$$

$$i_f(t)=-\frac{V_{om}}{29R}\cos\omega_{osc}t=\frac{V_{om}}{29R}\cos(\omega_{osc}t+\pi)$$

$$\dot{V}_2=\dot{I}_f\left(R+\frac{1}{j\omega_{osc}C}\right)=\dot{I}_fR\left(1+\frac{1}{j\omega_{osc}RC}\right)$$

$$=\dot{I}_fR\left(1-j\sqrt{6}\right)=\frac{V_{om}}{29}\angle 180°\cdot\sqrt{7}\angle-\arctan\sqrt{6}$$

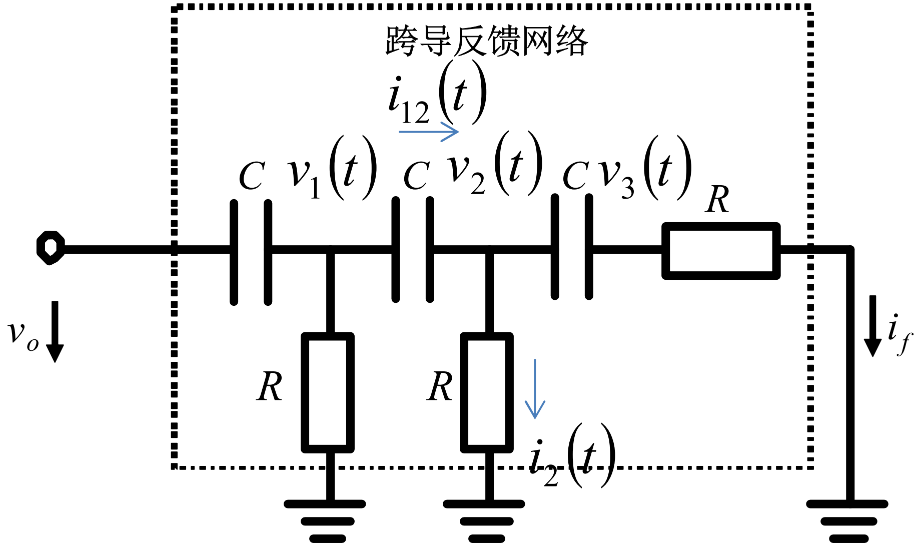
$$=\frac{\sqrt{7}}{29}V_{om}\angle 112.2°$$

$$v_3(t)=i_fR=\frac{V_{om}}{29}\cos(\omega_{osc}t+\pi)$$

$$v_2(t)=\frac{\sqrt{7}}{29}V_{om}\cos(\omega_{osc}t+0.623\pi)$$

$$\omega_{osc}=\frac{1}{\sqrt{6}RC}$$

三阶移相



$$v_o(t)=V_{om}\cos\omega_{osc}t$$

$$i_f(t)=-\frac{V_{om}}{29R}\cos\omega_{osc}t=\frac{V_{om}}{29R}\cos(\omega_{osc}t+\pi)$$

$$v_3(t)=i_fR=\frac{V_{om}}{29}\cos(\omega_{osc}t+\pi)$$

$$v_2(t)=\frac{\sqrt{7}}{29}V_{om}\cos(\omega_{osc}t+0.623\pi)$$

$$i_2(t)=\frac{\sqrt{7}}{29}\frac{V_{om}}{R}\cos(\omega_{osc}t+0.623\pi)$$

$$\dot{I}_{12}=\dot{I}_2+\dot{I}_f=\frac{\sqrt{7}}{29}\frac{V_{om}}{R}\angle 112.2^\circ+\frac{1}{29}\frac{V_{om}}{R}\angle 180^\circ$$

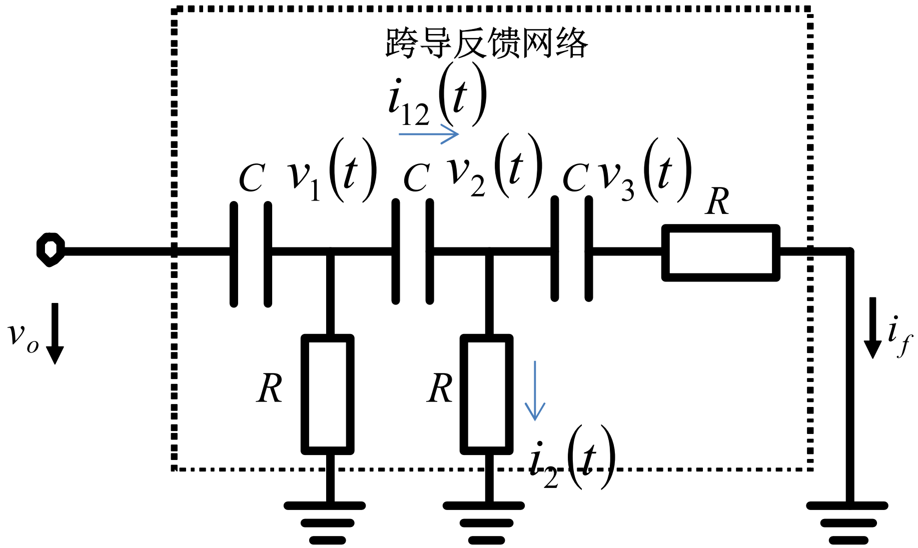
$$=\frac{1}{29}\frac{V_{om}}{R}\left(-1+j\sqrt{6}\right)-\frac{1}{29}\frac{V_{om}}{R}$$

$$=\frac{1}{29}\frac{V_{om}}{R}\left(-2+j\sqrt{6}\right)=\frac{\sqrt{10}}{29}\frac{V_{om}}{R}\angle 129.2^\circ$$

$$i_{12}(t)=i_2(t)+i_f(t)$$

$$=\frac{\sqrt{10}}{29}\frac{V_{om}}{R}\cos(\omega_{osc}t+0.718\pi)$$

三级移相



$$v_o(t) = V_{om}\cos\omega_{osc}t$$

$$i_f(t) = -\frac{V_{om}}{29R}\cos\omega_{osc}t = \frac{V_{om}}{29R}\cos(\omega_{osc}t + \pi)$$

$$v_3(t) = i_f R = \frac{V_{om}}{29}\cos(\omega_{osc}t + \pi)$$

$$v_2(t) = \frac{\sqrt{7}}{29}V_{om}\cos(\omega_{osc}t + 0.623\pi)$$

$$i_2(t) = \frac{\sqrt{7}}{29}\frac{V_{om}}{R}\cos(\omega_{osc}t + 0.623\pi)$$

$$i_{12}(t) = i_2(t) + i_f(t)$$

$$= \frac{\sqrt{10}}{29}\frac{V_{om}}{R}\cos(\omega_{osc}t + 0.718\pi)$$

$$\dot{V}_1 = \frac{1}{j\omega_{osc}C}\dot{I}_{12} + \dot{V}_2$$

$$= \frac{\sqrt{10}}{29}\frac{V_{om}}{\omega_{osc}RC}\angle 39.2^\circ + \frac{\sqrt{7}}{29}V_{om}\angle 112.2^\circ$$

$$= \frac{\sqrt{60}}{29}V_{om}\angle 39.2^\circ + \frac{\sqrt{7}}{29}V_{om}\angle 112.2^\circ$$

$$= \frac{V_{om}}{29}\left(6 + j\sqrt{24} - 1 + j\sqrt{6}\right) = \frac{\sqrt{79}}{29}V_{om}\angle 55.8^\circ$$

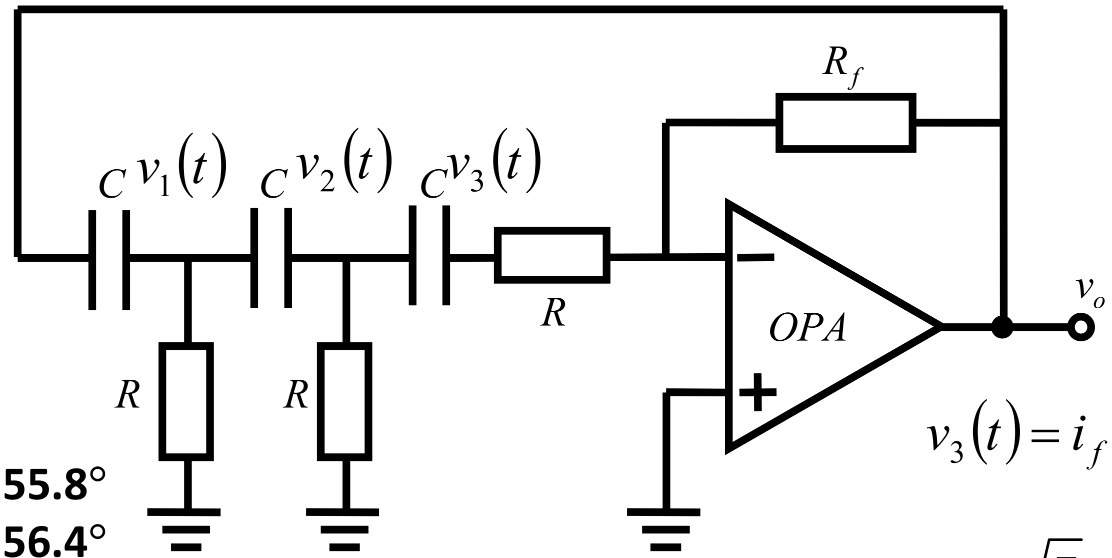
$$v_1(t) = \frac{\sqrt{79}}{29}V_{om}\cos(\omega_{osc}t + 0.310\pi)$$

三级移相

理论推导时，一定是抽取出最核心最简单模型进行分析
电路抽象
实际电路不符合理论分析，是由于分析模型过于简单
但电路设计必须会用简单模型

$$R_f > 29R$$

$$f_{osc} = \frac{1}{2\pi\sqrt{6}RC}$$



$$v_o(t) = V_{om}\cos\omega_{osc}t$$

$$v_3(t) = i_f R = \frac{V_{om}}{29}\cos(\omega_{osc}t + \pi)$$

0.034

$$v_2(t) = \frac{\sqrt{7}}{29}V_{om}\cos(\omega_{osc}t + 0.623\pi)$$

0.091

$$v_1(t) = \frac{\sqrt{79}}{29}V_{om}\cos(\omega_{osc}t + 0.310\pi)$$

0.306

第一级超前**55.8°**
第二级超前**56.4°**
第三级超前**67.8°**

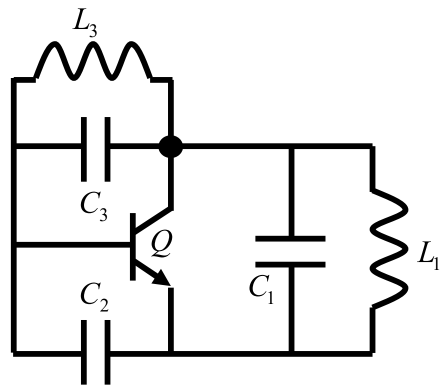
共超前移相**180°**，反相放大器移相**180°**，
形成正反馈回路。

作业4 三点式LC振荡

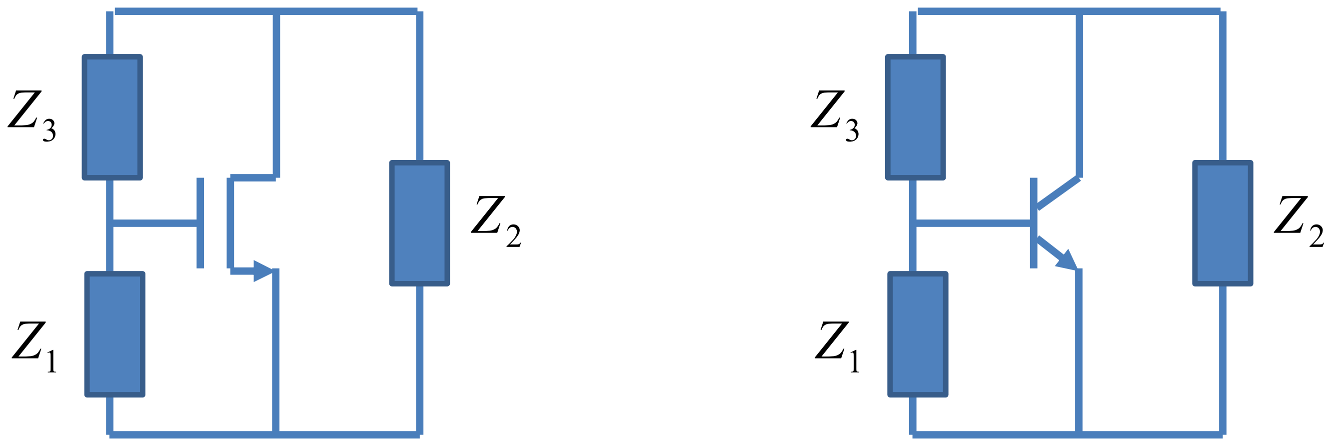
- 图**E10.4.27**是某振荡电路去除电阻影响后剩下的纯电抗元件和晶体管的三点式连接关系图，已知

$$f_{01}=\frac{1}{2\pi\sqrt{L_1C_1}}=1MHz \qquad f_{03}=\frac{1}{2\pi\sqrt{L_3C_3}}=1.1MHz$$

  – 请问该振荡器有无可能振荡？如果可能振荡，振荡频率大约为多少？



三点式振荡器



三点式：三个电抗元件分别接在晶体管的三个极间

只要和源极（发射极）相连的两个电抗元件是同属性的，同为容性或同为感性，第三个电抗元件反属性的，则满足正反馈条件：和参考地无关，只和结构有关

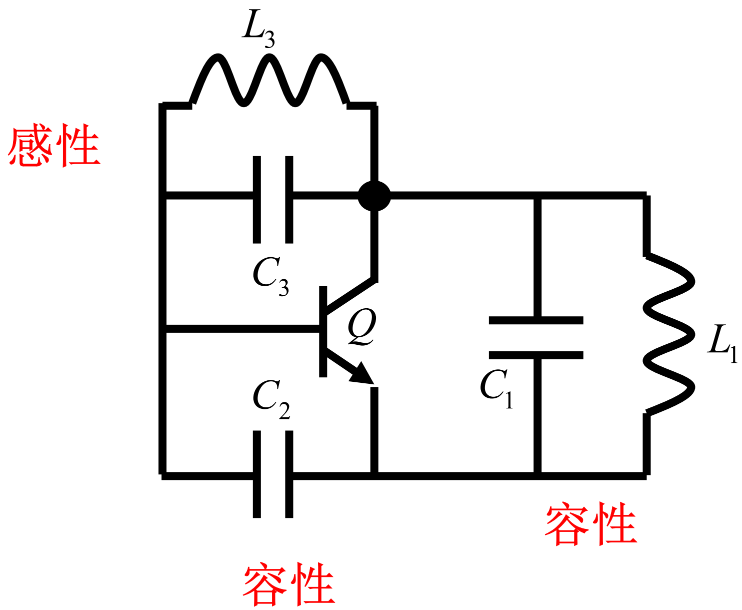
如果满足起振条件且**Q>>1**，则正弦振荡

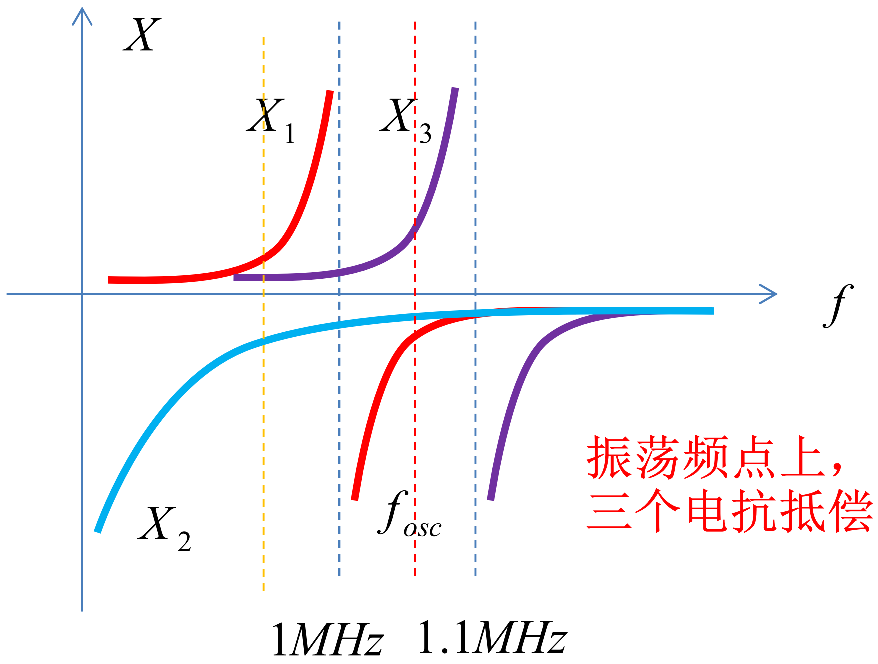
正弦振荡频率完全由三个电抗元件决定：晶体管作用为提供抵偿正阻的负阻，相互抵偿则去除

$$0=\sum\dot{V}_i=\dot{I}\sum Z_i=j\dot{I}\sum X_i$$ 满足虚部平衡条件（相位平衡条件）

$$X_1(\omega_{osc})+X_2(\omega_{osc})+X_3(\omega_{osc})=0$$

三点式的
正反馈条件要求





$$f_{01}=\frac{1}{2\pi\sqrt{L_1C_1}}=1MHz$$

$$f_{03}=\frac{1}{2\pi\sqrt{L_3C_3}}=1.1MHz$$

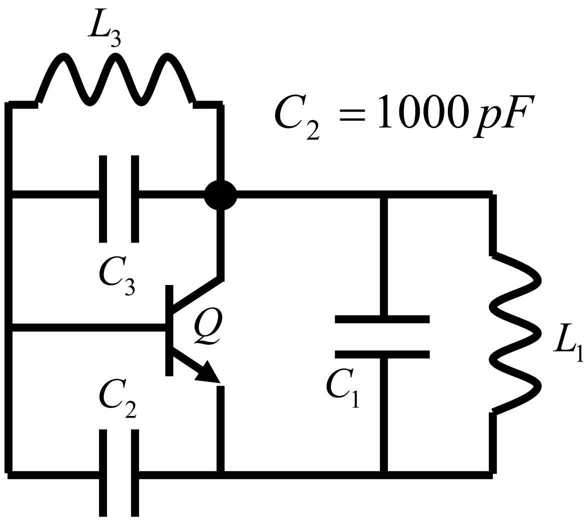
$$Z_2=\frac{1}{j\omega C_2}=jX_2 \qquad X_2=-\frac{1}{\omega C_2}$$

$$Z_1=\frac{1}{j\omega C_1+\frac{1}{j\omega L_1}}=jX_1$$

$$X_1=\frac{1}{\frac{1}{\omega L_1}-\omega C_1}=\frac{Z_{01}}{\frac{\omega_{01}}{\omega}-\frac{\omega}{\omega_{01}}}$$

$$X_3=\frac{1}{\frac{1}{\omega L_3}-\omega C_3}=\frac{Z_{03}}{\frac{\omega_{03}}{\omega}-\frac{\omega}{\omega_{03}}}$$

数値例



$$C_2 = 1000pF$$

$$f_{01} = 1MHz = \frac{1}{2\pi\sqrt{L_1 C_1}} = \frac{1}{2\pi\sqrt{50\mu H \times 507pF}}$$

$$f_{03} = 1.1MHz = \frac{1}{2\pi\sqrt{L_3 C_3}} = \frac{1}{2\pi\sqrt{50\mu H \times 419pF}}$$

$$0 = X_1 + X_2 + X_3 = \frac{Z_{01}}{\frac{\omega_{01}}{\omega_{osc}} - \frac{\omega_{osc}}{\omega_{01}}} - \frac{1}{\omega_{osc} C_2} + \frac{Z_{03}}{\frac{\omega_{03}}{\omega_{osc}} - \frac{\omega_{osc}}{\omega_{03}}}$$

$$= \frac{Z_{01}}{\frac{f_{01}}{f_{osc}} - \frac{f_{osc}}{f_{01}}} - \frac{1}{2\pi f_{osc} C_2} + \frac{Z_{03}}{\frac{f_{03}}{f_{osc}} - \frac{f_{osc}}{f_{03}}} = \frac{314}{\frac{1}{f_{osc}} - \frac{f_{osc}}{1}} - \frac{159}{f_{osc}} + \frac{345}{\frac{1.1}{f_{osc}} - \frac{f_{osc}}{1.1}}$$

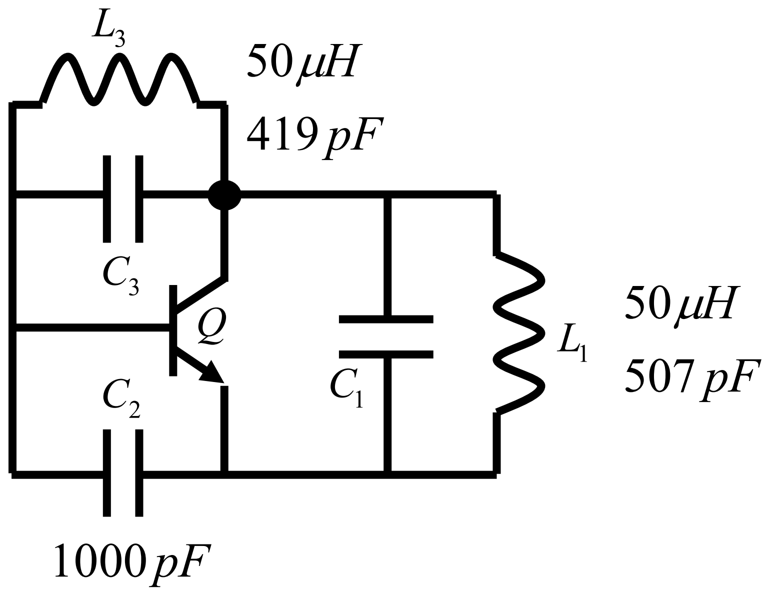
$$= \frac{314 f_{osc}\left(\frac{1.1}{f_{osc}} - \frac{f_{osc}}{1.1}\right) - 159\left(\frac{1}{f_{osc}} - \frac{f_{osc}}{1}\right)\left(\frac{1.1}{f_{osc}} - \frac{f_{osc}}{1.1}\right) + 345\left(\frac{1}{f_{osc}} - \frac{f_{osc}}{1}\right) f_{osc}}{\left(\frac{1}{f_{osc}} - \frac{f_{osc}}{1}\right) f_{osc} \left(\frac{1.1}{f_{osc}} - \frac{f_{osc}}{1.1}\right)}$$

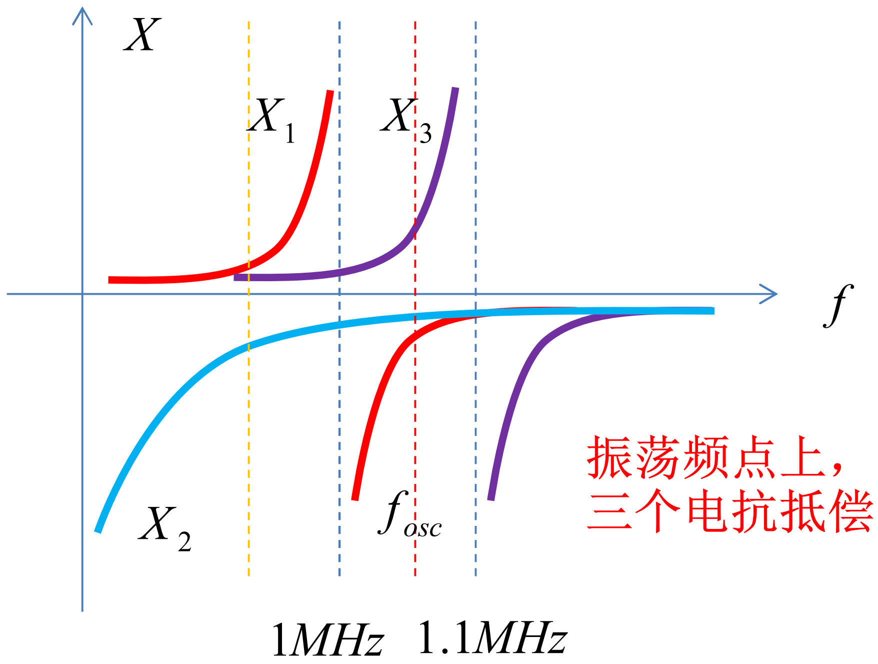
$$= \frac{314\left(1.1 - \frac{f_{osc}^2}{1.1}\right) - 159\left(\frac{1.1}{f_{osc}^2} - 1.1 - \frac{1}{1.1} + \frac{f_{osc}^2}{1.1}\right) + 345\left(1 - f_{osc}^2\right)}{\left(\frac{1}{f_{osc}} - \frac{f_{osc}}{1}\right) f_{osc} \left(\frac{1.1}{f_{osc}} - \frac{f_{osc}}{1.1}\right)}$$

$$= \frac{345 - 285 f_{osc}^2 - \left(\frac{175}{f_{osc}^2} - 319 + 145 f_{osc}^2\right) + 345 - 345 f_{osc}^2}{\left(\frac{1}{f_{osc}} - \frac{f_{osc}}{1}\right) f_{osc} \left(\frac{1.1}{f_{osc}} - \frac{f_{osc}}{1.1}\right)}$$

$$= \frac{1009 - 775 f_{osc}^2 - \frac{175}{f_{osc}^2}}{\left(\frac{1}{f_{osc}} - \frac{f_{osc}}{1}\right) f_{osc} \left(\frac{1.1}{f_{osc}} - \frac{f_{osc}}{1.1}\right)}$$

$$f_{osc} = 1.047MHz, 0.454MHz$$

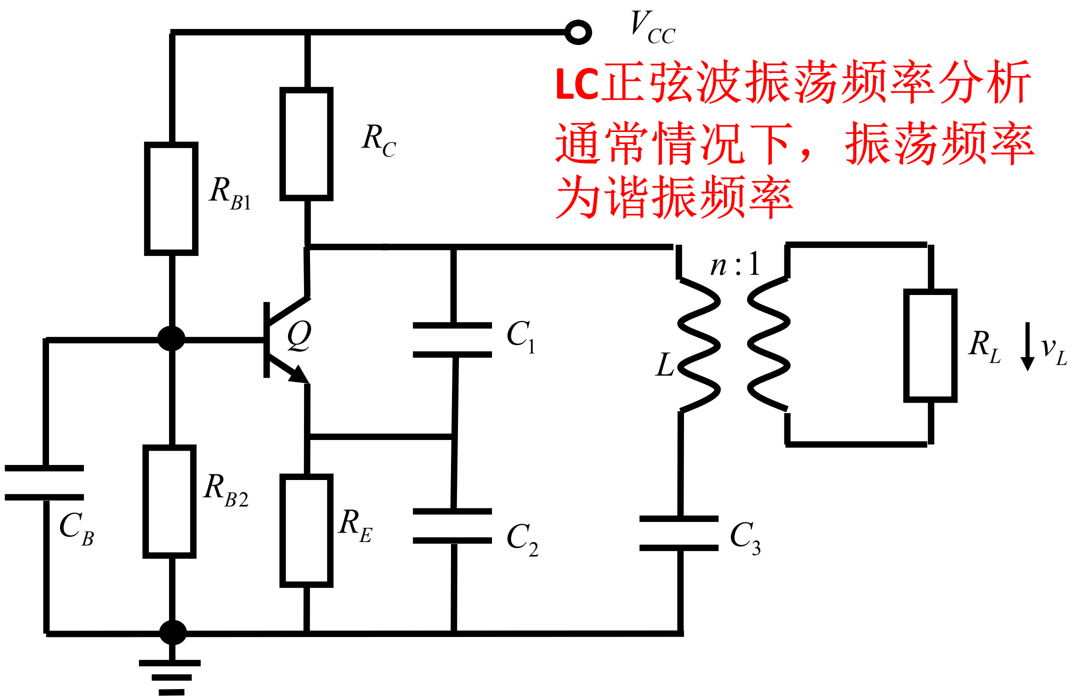




$$f_{osc} = 1.047\,MHz$$

只有满足三点式结构（正反馈条件）的才是振荡频率

作业5 克拉泼振荡器



LC正弦波振荡频率分析

通常情况下，振荡频率为谐振频率

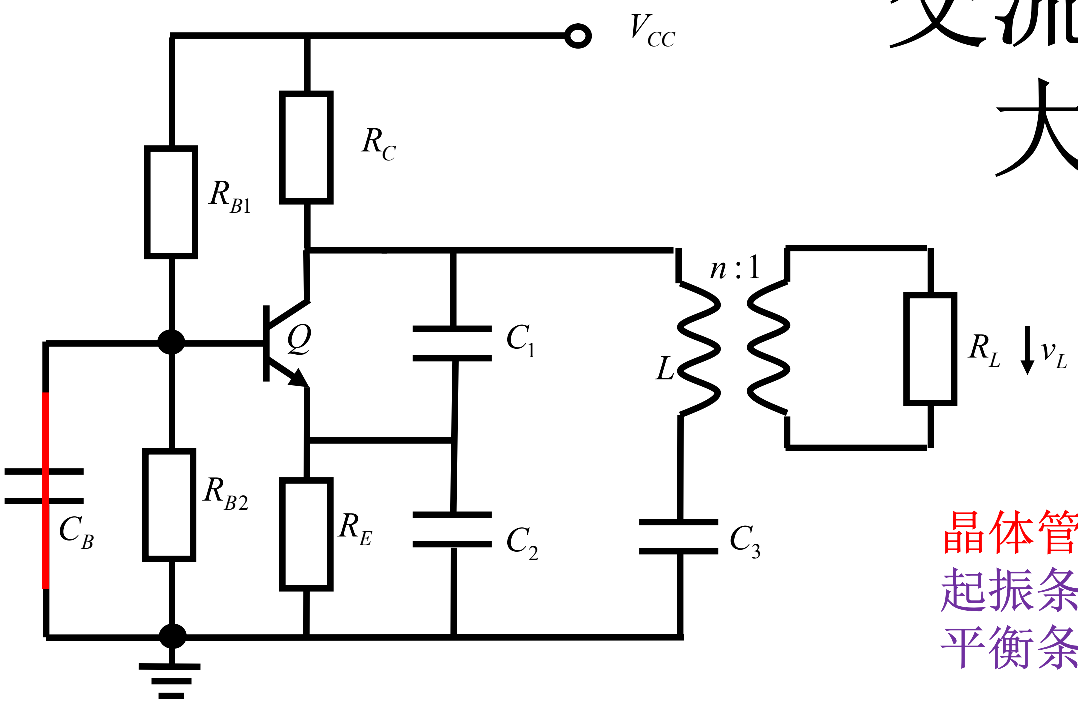
想方设法地提高频率稳定度

提高谐振腔Q值

降低谐振腔外影响

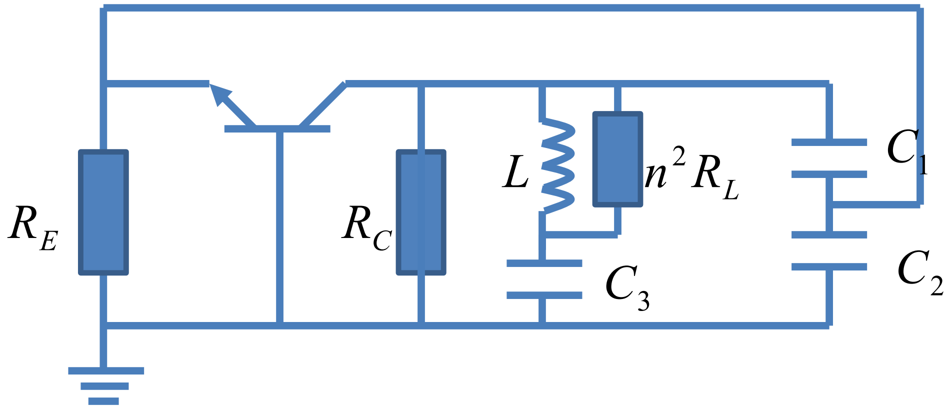
- 克拉泼电路，通过加入一个和电感串联的小电容C_3，使得谐振腔外元件（包括晶体管）对谐振腔的影响减弱
  - 三点式谐振电容C_1=200pF，C_2=200pF，克拉泼电容C_3=20pF
  - 谐振电感L=10μH
  - 晶体管寄生电容C_{be}=30pF
  - 振荡频率为多少？
- 如果没有C_3电容，C_1=C_2=24pF，振荡频率为多少？
- 晶体管寄生电容C_{be}随温度变化有±10%的变化，有克拉泼电容和无克拉泼电容，两种情况下，振荡频率随C_{be}变化分别为多少？

交流小信号线性分析
大信号准线性分析



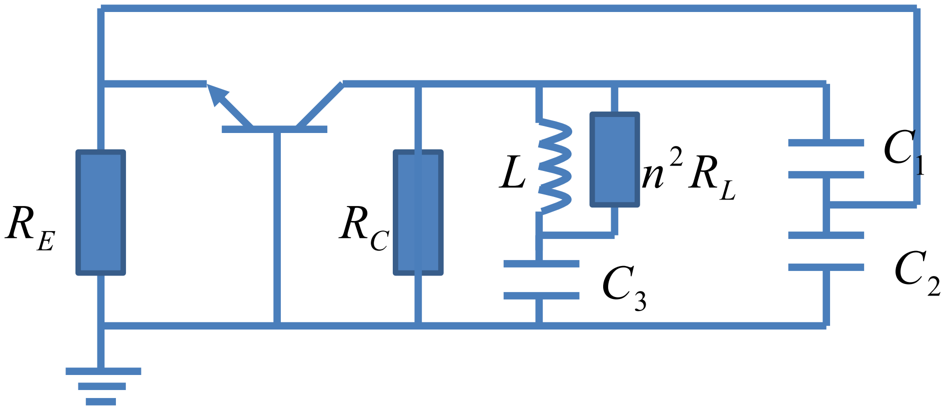
晶体管模型：
起振条件：小信号，微分线性放大元件
平衡条件：大信号，准线性放大元件

起振分析：必须将所有电阻考虑在内
起振条件和这些电阻密切相关



假设起振，振荡频率分析
假设高Q值，振荡频率则完全由谐振频率决定

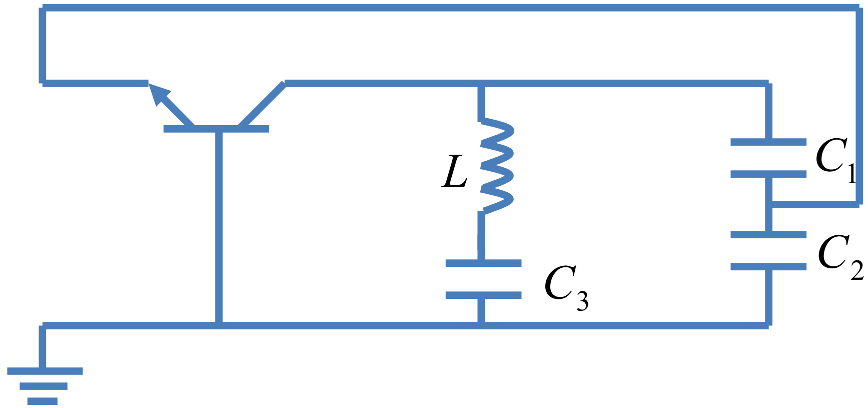
共基组态：电容三点式正弦波振荡器



LC振荡频率估算

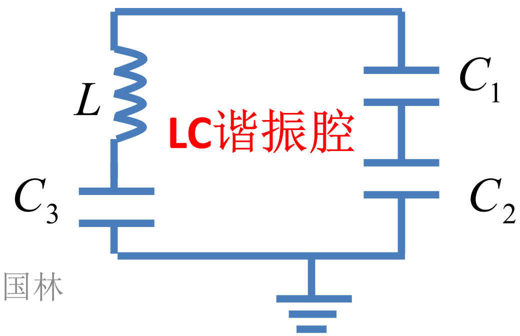
非起振分析，仅做振荡频率估算：：

（1）先把电阻全部去除（谐振腔内并联大电阻开路，串联小电阻短路），高Q值假设下，电阻对LC谐振腔振荡频率的影响可以忽略不计



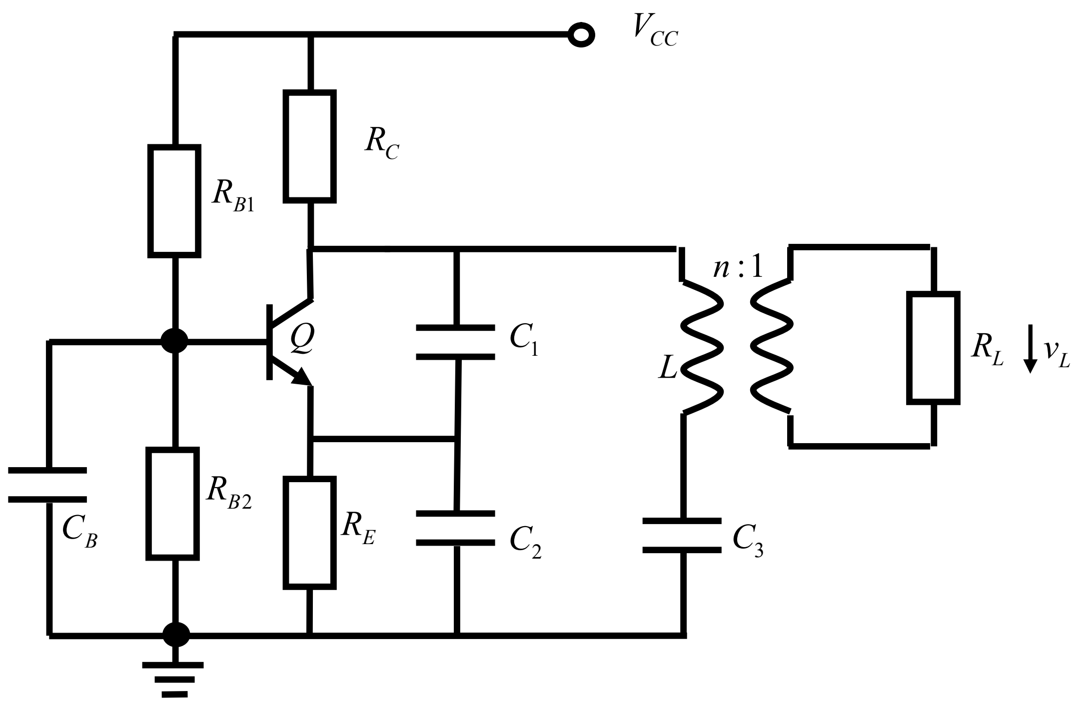
（2）再次检查正反馈条件（如果是三点式，则必须满足三点式结构）

（3）去除晶体管，晶体管等效负阻和正阻抵偿，不影响振荡频率



（4）在高Q值假设下，正弦振荡频率就是LC谐振腔谐振频率

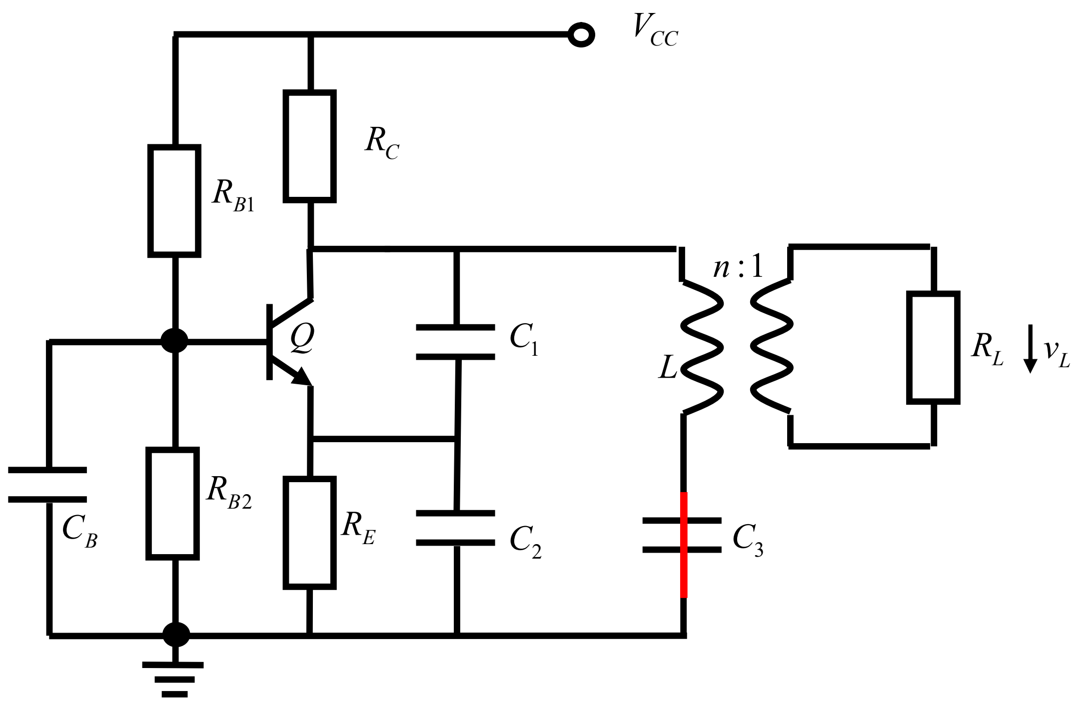
$$\sum X_i(\omega_{osc})=0 \quad \Rightarrow \quad f_{osc}\approx f_0=\frac{1}{2\pi\sqrt{LC_{123}}}$$



- 克拉泼电路，通过加入一个和电感串联的小电容**C$_3$**，使得谐振腔外元件（包括晶体管）对谐振腔的影响减弱
  - 三点式谐振电容**C$_1$=200pF**，**C$_2$=200pF**，克拉泼电容**C$_3$=20pF**
  - 谐振电感**L=10μH**
  - 晶体管寄生电容**C$_{be}$=30pF**
  - 振荡频率为多少？

$$C_{123}=\frac{1}{\frac{1}{C_1}+\frac{1}{C_2'}+\frac{1}{C_3}}=\frac{1}{\frac{1}{200p}+\frac{1}{230p}+\frac{1}{20p}}=16.85pF$$

$$f_{osc}\approx f_0=\frac{1}{2\pi\sqrt{LC_{123}}}=\frac{1}{2\pi\times\sqrt{10\times10^{-6}\times16.85\times10^{-12}}}=12.26MHz$$

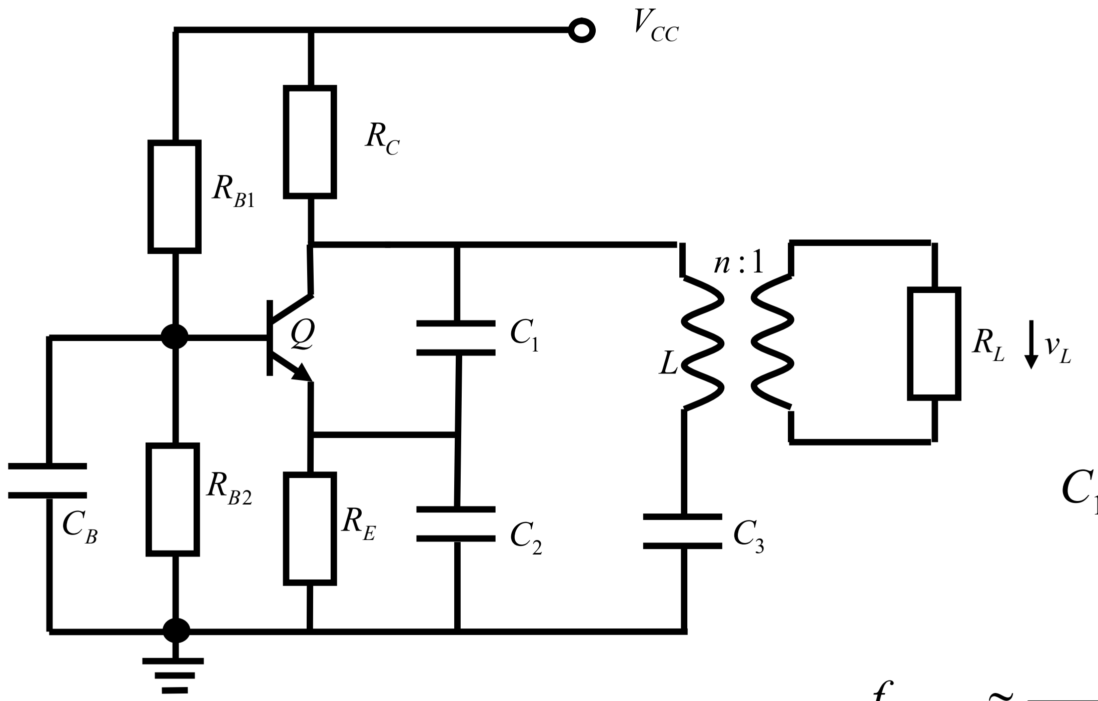


- 克拉泼电路，通过加入一个和电感串联的小电容**C$_3$**，使得谐振腔外元件（包括晶体管）对谐振腔的影响减弱
  - 三点式谐振电容**C$_1$=200pF**，**C$_2$=200pF**，克拉泼电容**C$_3$=20pF**
  - 谐振电感**L=10μH**
  - 晶体管寄生电容**C$_{be}$=30pF**
  - 振荡频率为多少？

$$C_{123}=\frac{1}{\frac{1}{C_1}+\frac{1}{C_2'}}=\frac{1}{\frac{1}{24p}+\frac{1}{54p}}=16.62\,pF$$

- 如果没有**C$_3$**电容，**C$_1$=C$_2$=24pF**，振荡频率为多少？

$$f_{osc}\approx f_0=\frac{1}{2\pi\sqrt{LC_{123}}}=\frac{1}{2\pi\times\sqrt{10\times10^{-6}\times16.62\times10^{-12}}}=12.35MHz$$



– 晶体管寄生电容C_{be}随温度变化有±10%的变化，有克拉泼电容和无克拉泼电容，两种情况下，振荡频率随C_{be}变化分别为多少？

$$C_{123} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2'} + \frac{1}{C_3}}$$

$$= \frac{1}{\frac{1}{200p} + \frac{1}{230p} + \frac{1}{20p}} = 16.85pF$$

$$f_{osc} \approx f_0 = \frac{1}{2\pi\sqrt{LC_{123}}}$$

$$= \frac{1}{2\pi \times \sqrt{10\times10^{-6} \times 16.85\times10^{-12}}} = 12.26MHz$$

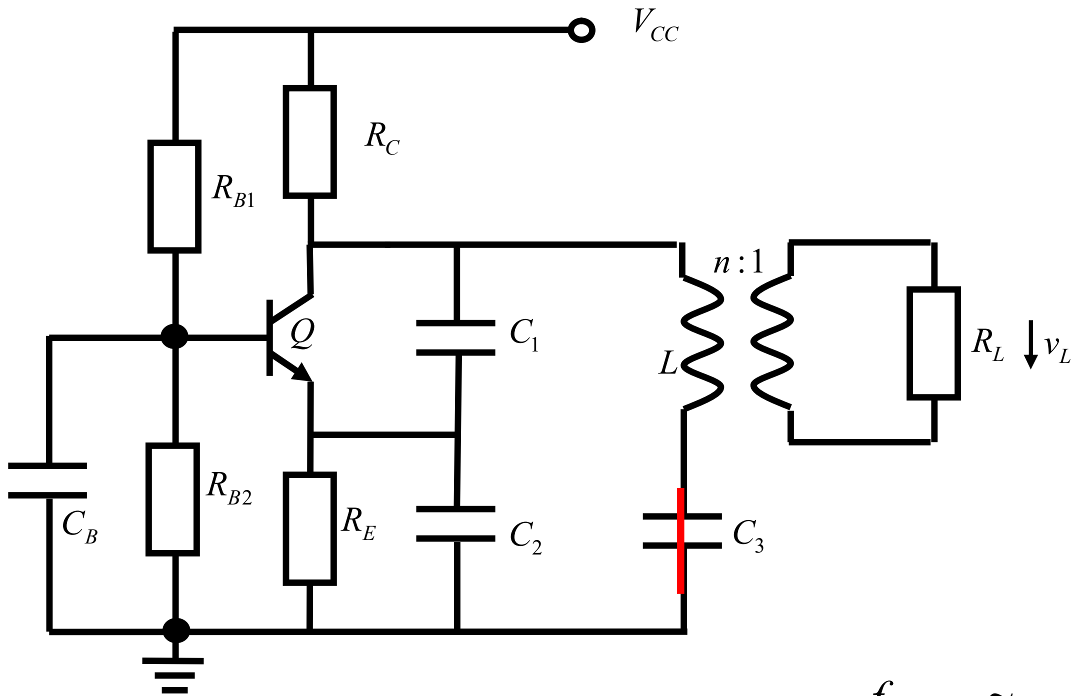
$$C_{123,\Delta 1} = \frac{1}{\frac{1}{200p} + \frac{1}{227p} + \frac{1}{20p}} = 16.83pF$$

$$f_{osc,\Delta 1} \approx \frac{1}{2\pi \times \sqrt{10\times10^{-6} \times 16.83\times10^{-12}}} = 12.27MHz$$

$$C_{123,\Delta 2} = \frac{1}{\frac{1}{200p} + \frac{1}{233p} + \frac{1}{20p}} = 16.87pF$$

$$f_{osc,\Delta 2} \approx \frac{1}{2\pi \times \sqrt{10\times10^{-6} \times 16.83\times10^{-12}}} = 12.26MHz$$

$$\Delta f = f_{osc,\Delta 1} - f_{osc,\Delta 2} \approx 11.72kHz \sim 0.1\% f_0$$



– 晶体管寄生电容C_{be}随温度变化有±10%的变化，有克拉泼电容和无克拉泼电容，两种情况下，振荡频率随C_{be}变化分别为多少？

$$C_{123} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2'}}$$

$$= \frac{1}{\frac{1}{24p} + \frac{1}{54p}} = 16.62\,pF$$

$$f_{osc} \approx f_0 = \frac{1}{2\pi\sqrt{LC_{123}}}$$

$$= \frac{1}{2\pi \times \sqrt{10\times10^{-6} \times 16.62\times10^{-12}}} = 12.35MHz$$

$$C_{123,\Delta1} = \frac{1}{\frac{1}{24p} + \frac{1}{51p}} = 16.32\,pF$$

$$f_{osc,\Delta1} \approx \frac{1}{2\pi \times \sqrt{10\times10^{-6} \times 16.32\times10^{-12}}} = 12.46MHz$$

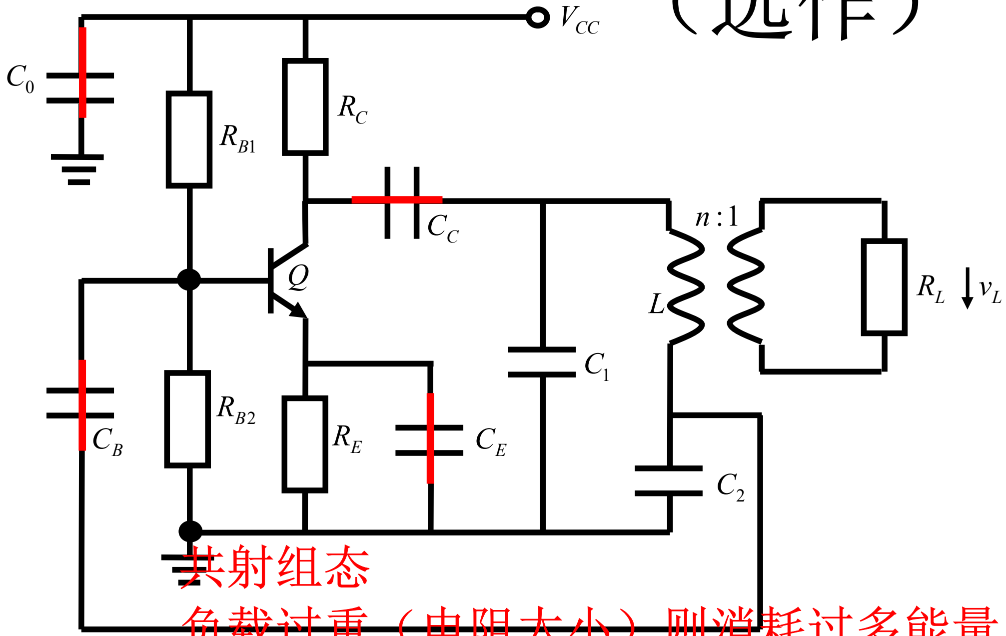
$$C_{123,\Delta2} = \frac{1}{\frac{1}{24p} + \frac{1}{57p}} = 16.89\,pF$$

$$f_{osc,\Delta2} \approx \frac{1}{2\pi \times \sqrt{10\times10^{-6} \times 16.89\times10^{-12}}} = 12.25MHz$$

$$\Delta f = f_{osc,\Delta1} - f_{osc,\Delta2} \approx 211.6kHz \sim 1.7\% f_0$$

作业6 起振条件（选作）

- 图E10.4.26是由共射组态BJT构成的电容三点式LC振荡器。其中偏置电阻R_{B1}=68kΩ，R_{B2}=5.6kΩ，R_E=1kΩ，R_C=3.3kΩ；耦合电容、旁路电容及电源滤波电容 C_B、C_C、C_E、C_0 均为0.1μF大电容；三点式谐振电容C_1=300pF，C_2=1000pF；变压器为2:1全耦合变压器，电感L可调谐，使得振荡频率为1MHz，电感的无载Q值为100；电源电压V_{CC}=12V，负载电阻R_L=1kΩ；晶体管电流增益β=400，厄利电压V_A=100V。分析该电路的起振条件，如果能够起振，负载电阻变化到多少则不能振荡？如果不能起振，负载电阻变化到多少则可起振？



共射组态

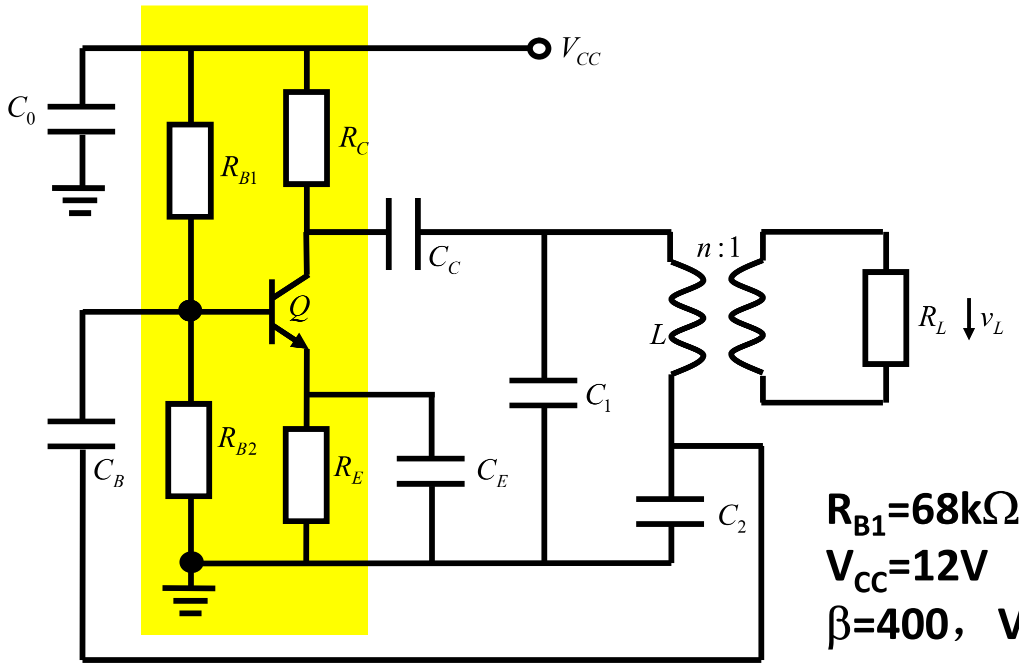
负载过重（电阻太小）则消耗过多能量，晶体管等效负阻无法提供足够补充能量，不满足起振条件

$$g_m > \frac{G_{eL}}{p(1-p)} \approx \frac{G'_L}{p(1-p)}$$

$$R_L \approx > \frac{1}{n^2 g_m p(1-p)} \approx \frac{1}{n^2 g_m p(1-p)}$$

$$= \frac{1}{4\times 8m\times 0.23\times 0.77} = 176\Omega$$

用共基结果
只考虑负载损耗的粗略估算



直流分析
获得微分模型

起振阶段可交流小信号分析

R_{B1}=68kΩ，R_{B2}=5.6kΩ，R_E=1kΩ，R_C=3.3kΩ；
V_{CC}=12V
β=400，V_A=100V

$$V_{BB}=\frac{R_{B2}}{R_{B1}+R_{B2}}V_{CC}=\frac{5.6}{68+5.6}\times 12=0.913V$$

$$R_B=\frac{R_{B1}R_{B2}}{R_{B1}+R_{B2}}=\frac{68\times 5.6}{68+5.6}=5.174k\Omega$$

$$V_{BB}=I_BR_B+0.7+(\beta+1)I_BR_E$$

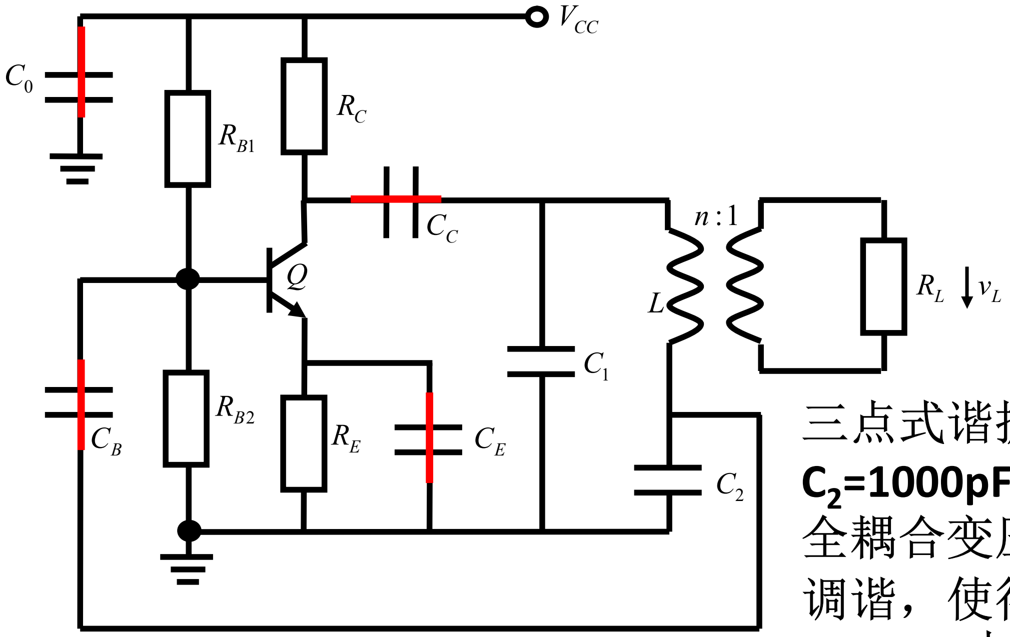
$$I_C=\beta I_B=\beta\frac{V_{BB}-0.7}{R_B+(\beta+1)R_E}=400\frac{0.913-0.7}{5.174k+401k}=0.21mA$$

$$g_m=\frac{I_C}{v_T}=\frac{0.21mA}{26mV}=8.07mS$$

$$r_{be}=\beta\frac{1}{g_m}=\frac{400}{8.07m}=49.6k\Omega$$

$$r_{ce}=\frac{V_A}{I_{C0}}=\frac{100}{0.21m}=477k\Omega$$

交流分析模型



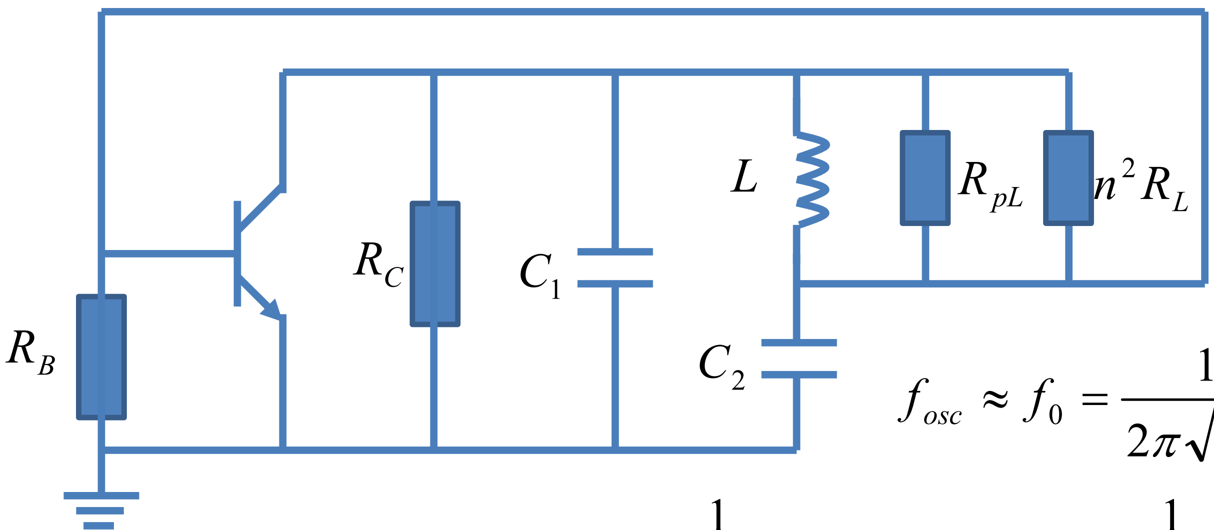
C_B、 C_C 、 C_E 、 C_0 均为0.1μF大电容

三点式谐振电容C_1=300pF，C_2=1000pF；变压器为2:1全耦合变压器，电感L可调谐，使得振荡频率为1MHz，电感的无载Q值为100；

$$R_B = 5.174k\Omega$$

$$R_C = 3.3k\Omega$$

$$n^2 R_L = 2^2 \times 1k\Omega = 4k\Omega$$

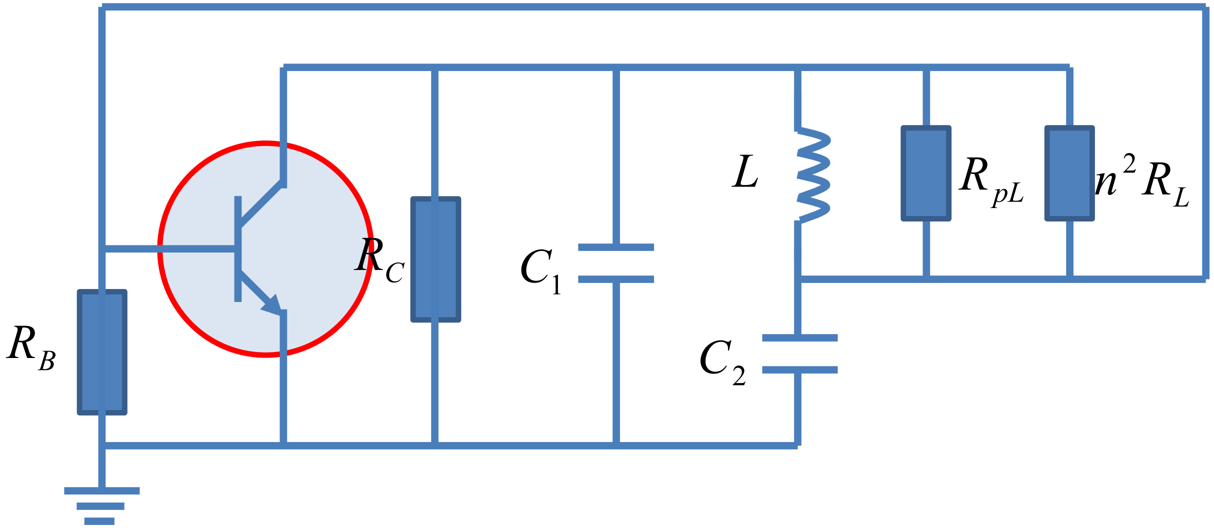


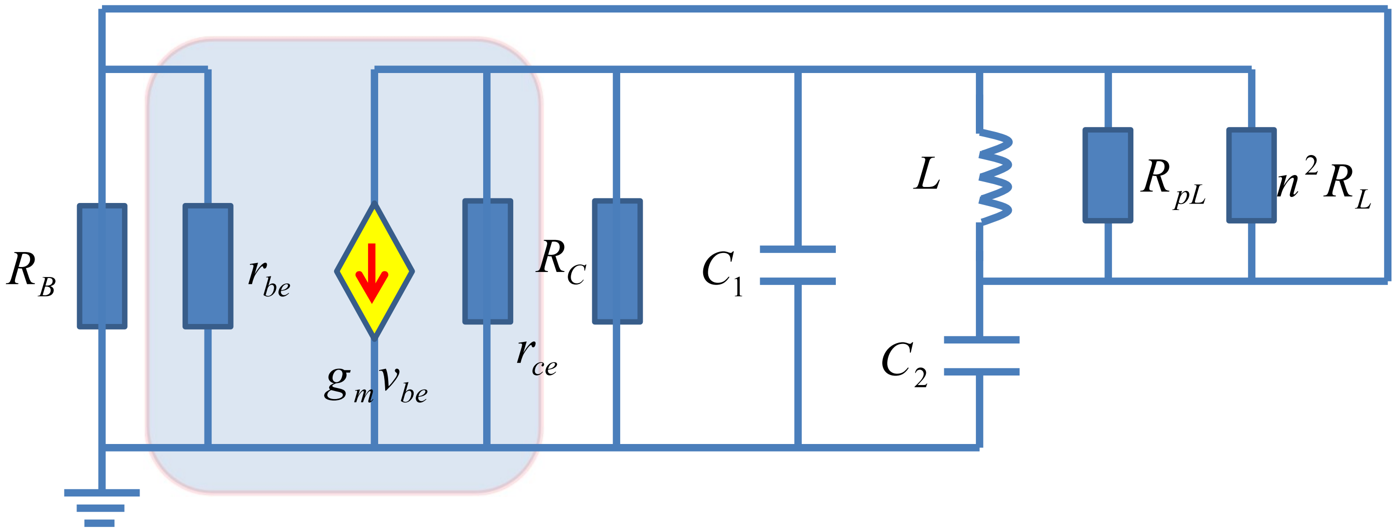
$$R_{pL} = Q_0 \omega_0 L$$
$$= 100 \times 2 \times 3.14 \times 1M \times 110\mu$$
$$= 69k\Omega$$

$$f_{osc} \approx f_0 = \frac{1}{2\pi\sqrt{LC}}$$

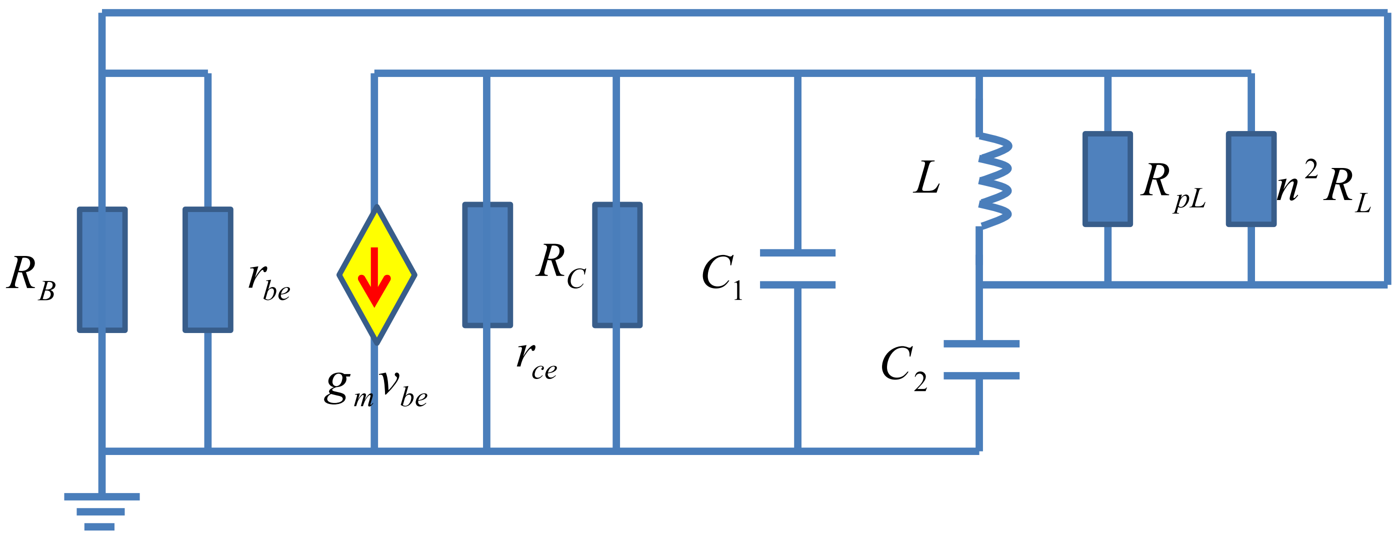
$$L \approx \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\times 3.14\times 1\times 10^6)^2 \times 231\times 10^{-12}} = 110\mu H$$

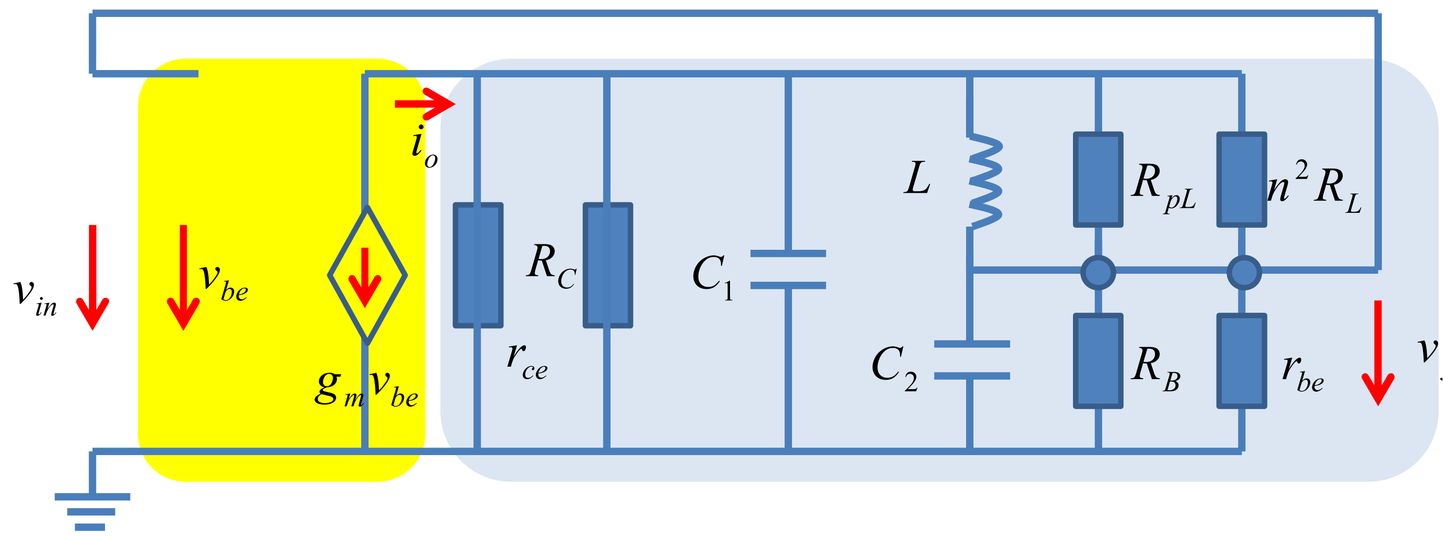
交流小信号电路模型





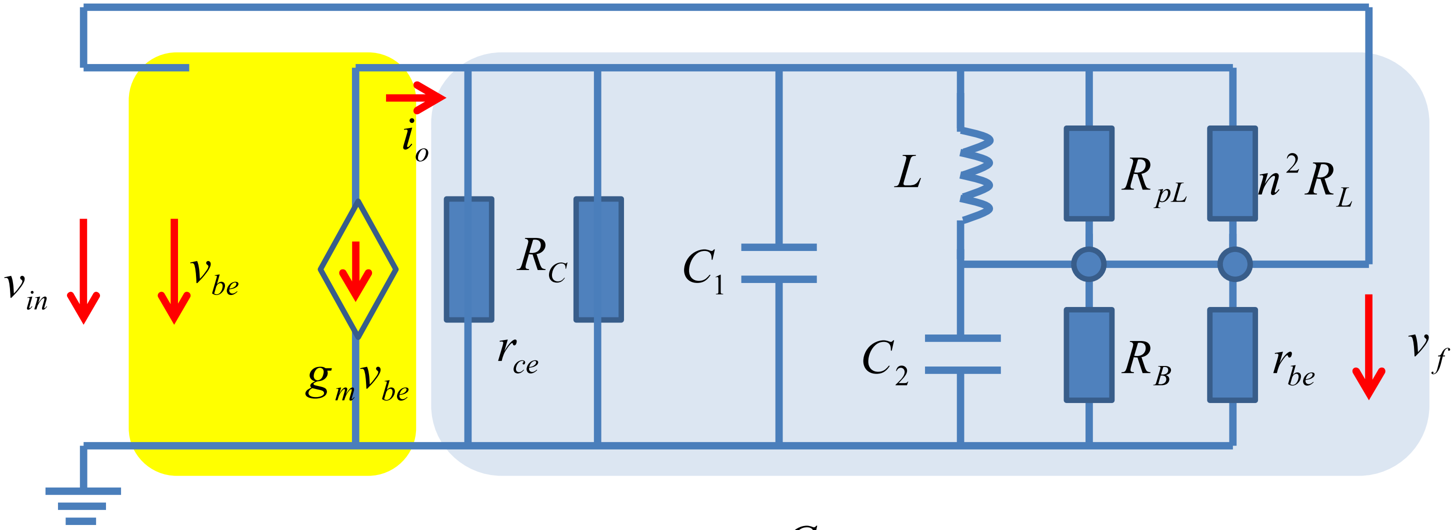
放大网络和反馈网络分离



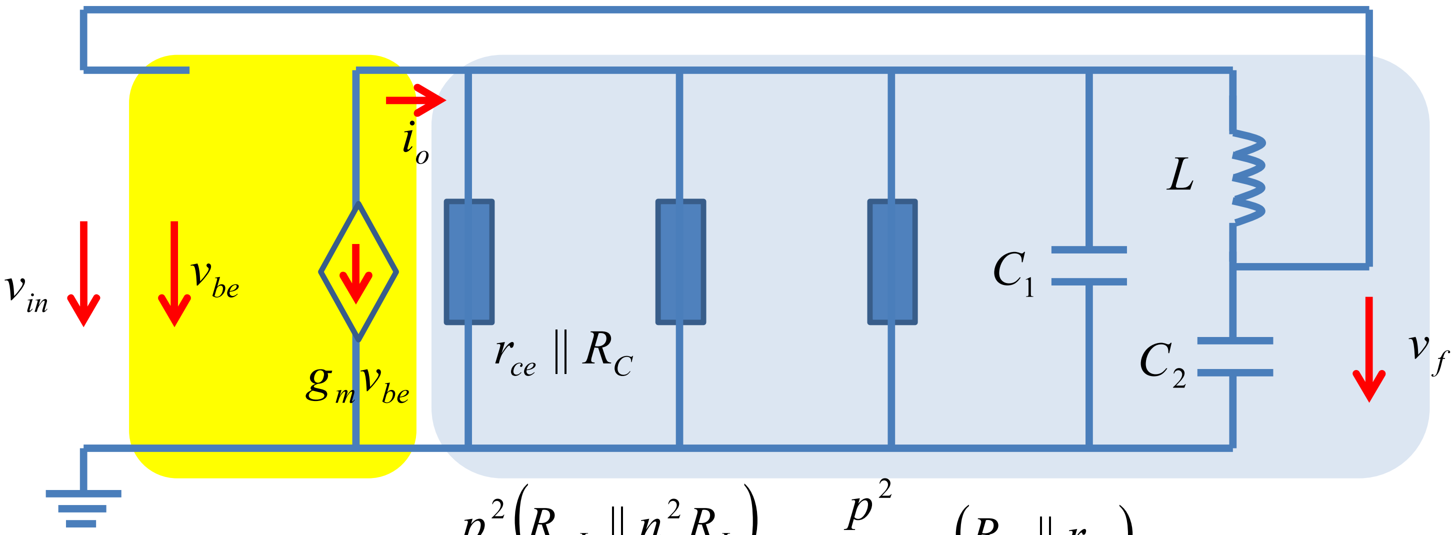


正确划分放大网络和反馈网络是正反馈振荡器起振分析的关键

部分接入是简化分析方法



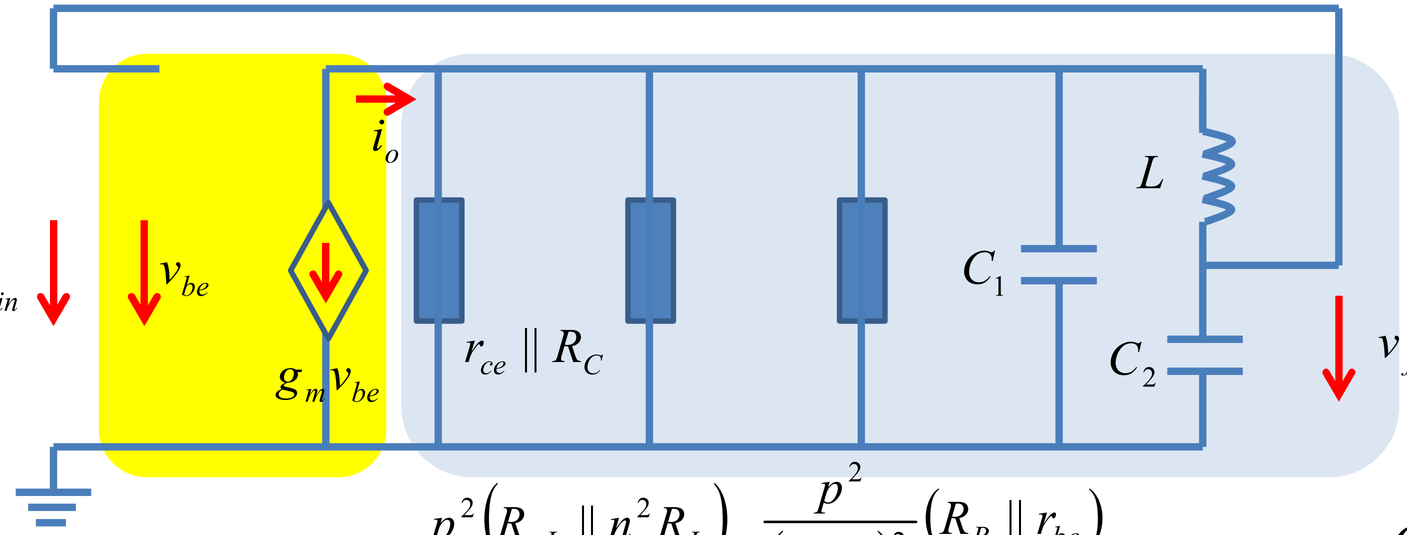
$$p = \frac{C_2}{C_1 + C_2}$$



$p^2\left(R_{pL} \parallel n^2 R_L\right)$ $\frac{p^2}{(1-p)^2}\left(R_B \parallel r_{be}\right)$

假设局部**Q**值足够高

放大倍数与反馈系数



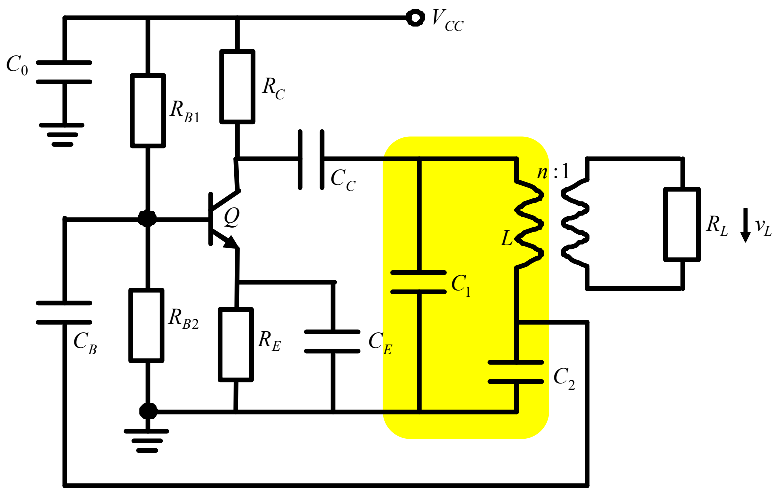
$$p^2\left(R_{pL} \parallel n^2 R_L\right) \qquad \frac{p^2}{(1-p)^2}\left(R_B \parallel r_{be}\right)$$

$$p = \frac{C_2}{C_1 + C_2}$$

$$A_0 = \frac{\dot{I}_o}{\dot{V}_i} = -g_m$$

$$F = \frac{\dot{V}_f}{\dot{I}_o} = \frac{\dot{V}_f}{\dot{V}_o}\frac{\dot{V}_o}{\dot{I}_o} = \left(C_2\text{分压系数}\right)\cdot\left(R \parallel C_1 \parallel \left(L\text{串}C_2\right)\right)$$

$$= \frac{\dfrac{1}{j\omega C_2}}{j\omega L + \dfrac{1}{j\omega C_2}} \cdot \frac{1}{\dfrac{1}{R} + j\omega C_1 + \dfrac{1}{j\omega L + \dfrac{1}{j\omega C_2}}}$$



环路增益

$$F=\frac{\dot V_f}{\dot I_o}=\frac{\dot V_f}{\dot V_o}\frac{\dot V_o}{\dot I_o}=\left(C_2\text{分压系数}\right)\cdot\left(R\,\|\,C_1\,\|\left(L\text{串}C_2\right)\right)$$

$$=\frac{\frac{1}{j\omega C_2}}{j\omega L+\frac{1}{j\omega C_2}}\cdot\frac{1}{\frac{1}{R}+j\omega C_1+\frac{1}{j\omega L+\frac{1}{j\omega C_2}}}$$

$$=\frac{1}{j\omega C_2}\cdot\frac{1}{\left(\frac{1}{R}+j\omega C_1\right)\left(j\omega L+\frac{1}{j\omega C_2}\right)+1}$$

$$=\frac{1}{\omega C_2}\cdot\frac{1}{\left(\frac{1}{\omega RC_2}-\frac{\omega L}{R}\right)+j\left(1+\frac{C_1}{C_2}-\omega^2LC_1\right)}$$

$$T=A_0F=-g_m\frac{1}{\omega C_2}\cdot\frac{1}{\left(\frac{1}{\omega RC_2}-\frac{\omega L}{R}\right)+j\left(1+\frac{C_1}{C_2}-\omega^2LC_1\right)}$$

$$\varphi_T\left(\omega_{osc}\right)=0\Rightarrow 1+\frac{C_1}{C_2}-\omega_{osc}{}^2LC_1=0\Rightarrow\omega_{osc}=\sqrt{\frac{C_2+C_1}{LC_1C_2}}=\frac{1}{\sqrt{L\left(C_1\text{串}C_2\right)}}$$

正好是我们期望的结果

起振条件

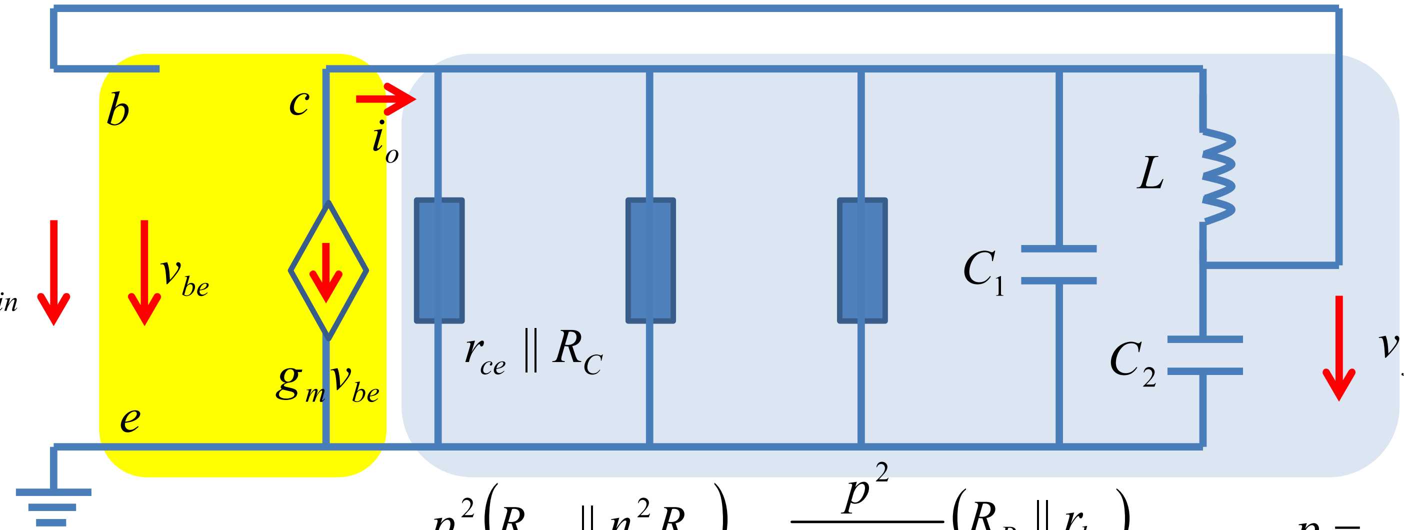
$$T = A_0 F = -g_m \frac{1}{\omega C_2} \cdot \frac{1}{\left(\frac{1}{\omega R C_2} - \frac{\omega L}{R}\right) + j\left(1 + \frac{C_1}{C_2} - \omega^2 L C_1\right)}$$

$$\varphi_T\left(\omega_{osc}\right) = 0 \Rightarrow 1 + \frac{C_1}{C_2} - \omega_{osc}{}^2 L C_1 = 0 \Rightarrow \omega_{osc} = \sqrt{\frac{C_2 + C_1}{L C_1 C_2}} = \frac{1}{\sqrt{L\left(C_1 串 C_2\right)}}$$

$$T\left(j\omega_{osc}\right) = -g_m R \frac{1}{1 - \omega_{osc}^2 L C_2} = -g_m R \frac{1}{1 - \frac{L C_2}{L \frac{C_1 C_2}{C_1 + C_2}}} = -g_m R \frac{1}{1 - \frac{C_1 + C_2}{C_1}} = g_m R \frac{C_1}{C_2} > 1$$

起振条件：跨导增益足够大

起振条件的进一步分析



$$p^2\left(R_{pL} \parallel n^2 R_L\right) \qquad \frac{p^2}{(1-p)^2}\left(R_B \parallel r_{be}\right) \qquad p = \frac{C_2}{C_1 + C_2}$$

$$T(j\omega_{osc}) = g_m R \frac{C_1}{C_2} = \frac{C_1}{C_2} \frac{g_m}{g_{ce} + G_c + \dfrac{G_{pL} + G_L/n^2}{p^2} + \left(\dfrac{1-p}{p}\right)^2 (G_B + g_{be})} > 1$$

折合到CE端口的总导纳

$$\frac{C_1}{C_2} \frac{p^2 g_m}{p^2(g_{ce} + G_c) + G_{pL} + G_L/n^2 + (1-p)^2 (G_B + g_{be})} > 1$$

折合到bc端口的总导纳

起振条件的一致性

$$\frac{C_1}{C_2}\frac{p^2 g_m}{p^2\left(g_{ce}+G_c\right)+G_{pL}+G_L/n^2+(1-p)^2\left(G_B+g_{be}\right)}>1$$

$$p=\frac{C_2}{C_1+C_2}$$

$$\frac{C_1C_2}{\left(C_1+C_2\right)^2}g_m>p^2\left(g_{ce}+G_c\right)+G_{pL}+G_L/n^2+(1-p)^2\left(G_B+g_{be}\right)=G_{eL}$$

$$g_m>\frac{G_{eL}}{p(1-p)}$$

折合到bc端口的总导纳

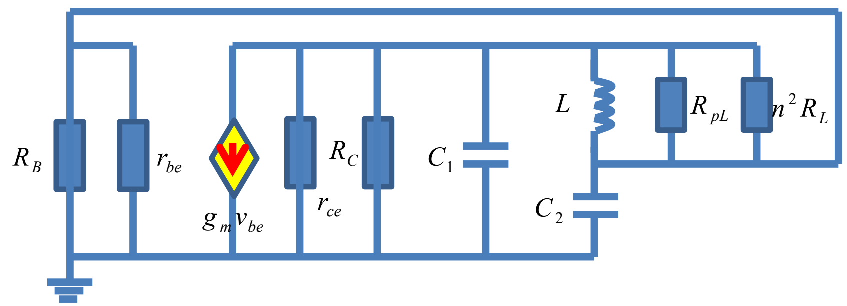
电容部分接入系数

这个公式在共基组态电容三点式正反馈分析中获得过，在并联**LC**负导等效中获得过，在串联**LC**负阻等效中获得过

三点式结构一旦确定，无论组态，将获得一致的起振条件

对负载电阻的要求估算

$$g_m > \frac{G_{eL}}{p(1-p)}$$



$$\frac{C_1 C_2}{(C_1+C_2)^2} g_m > G_{eL} = \left(\frac{C_2}{C_1+C_2}\right)^2 (g_{ce}+G_c) + G_{pL} + \frac{G_L}{n^2} + \left(\frac{C_1}{C_2+C_2}\right)^2 (G_B + g_{be})$$

折合到bc端口的总电导，代表LC并联谐振回路中的正阻损耗

$$G_L < n^2\left(p(1-p)g_m - p^2(g_{ce}+G_c) - G_{pL} - (1-p)^2(G_B+g_{be})\right)$$

$$= 2^2 \times \left(\frac{300\times1000}{1300^2}\times 8.07mS - \left(\frac{1000}{1300}\right)^2 \times \left(\frac{1}{477k}+\frac{1}{3.3k}\right) - \frac{1}{69k} - \left(\frac{300}{1300}\right)^2\left(\frac{1}{5.174k}+\frac{1}{49.6k}\right)\right)$$

$$= 4\times(1.433mS - 0.181mS - 0.014mS - 0.011mS)$$

$$= 4\times 1.227mS = 4.908mS$$

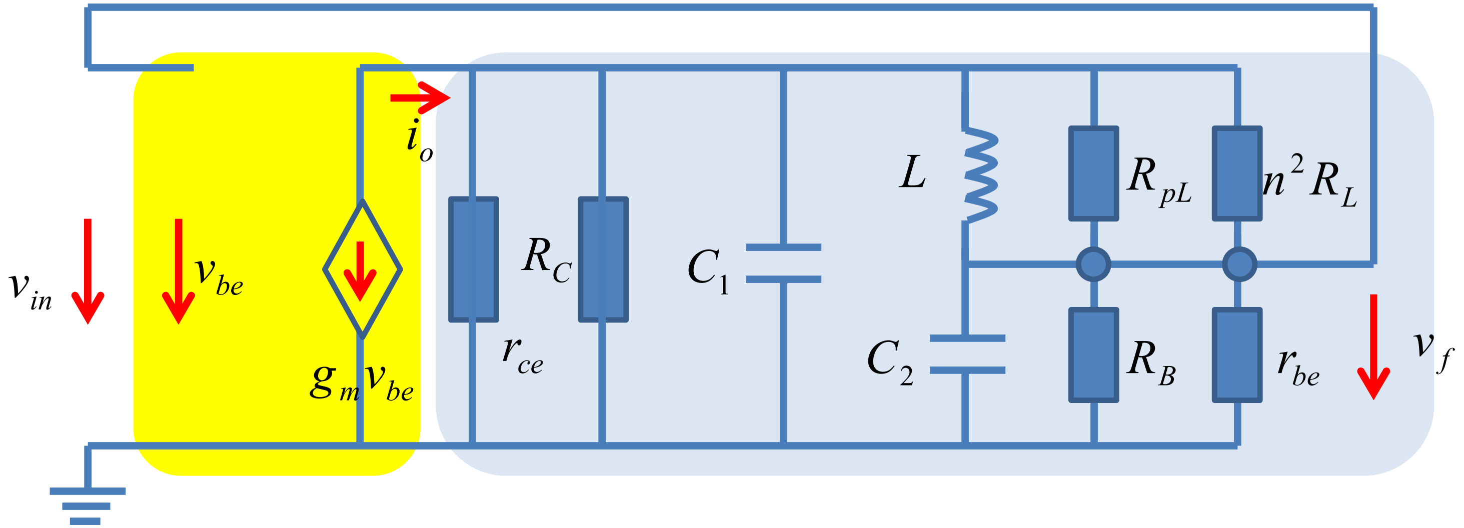
谐振腔中损耗最大来源为R_C电阻

可通过串联高频扼流圈消除其影响

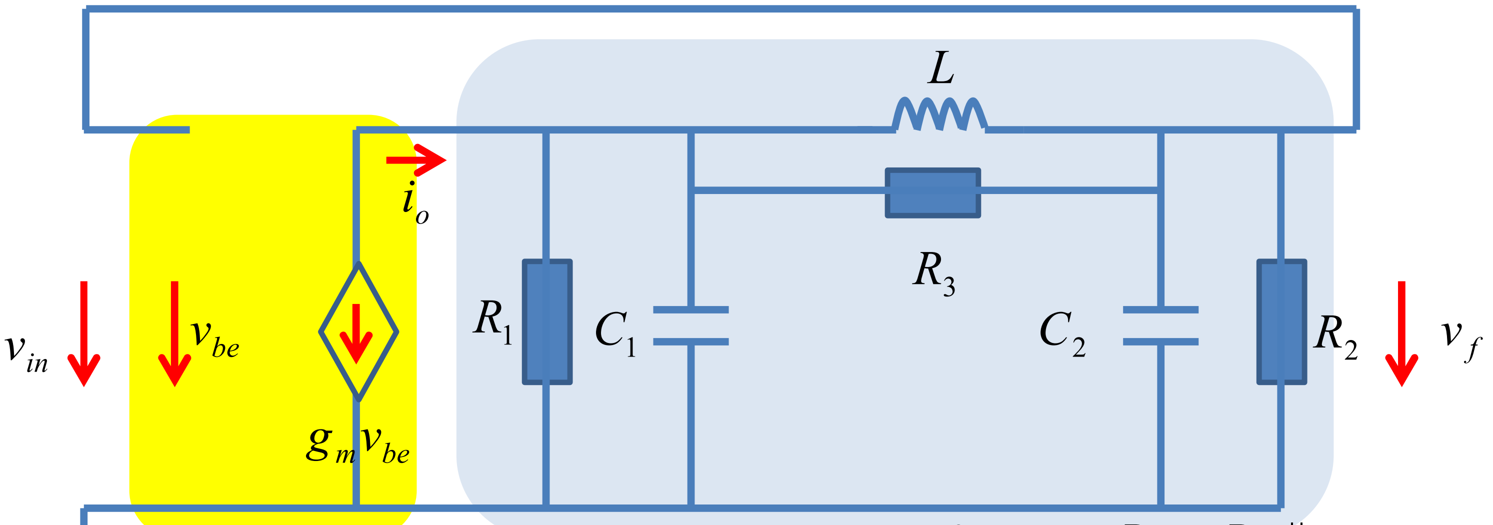
$$R_L > \frac{1}{4.908mS} = 204\Omega$$

部分接入方法：概念清晰，结果准确度是否有保障？Q>>1?

不做部分接入的分析



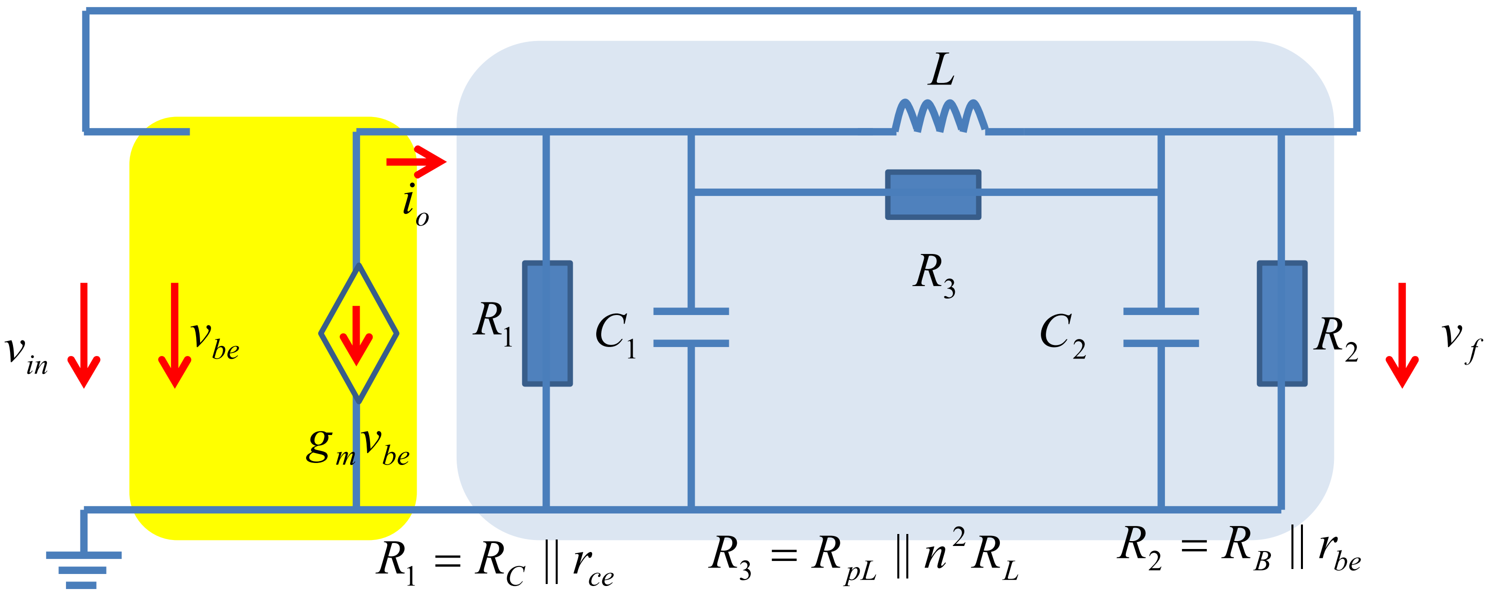
采用部分接入是高Q值近似分析



$R_1 = R_C \parallel r_{ce}$ $R_3 = R_{pL} \parallel n^2 R_L$ $R_2 = R_B \parallel r_{be}$

$A_0 = -g_m$

$F = \frac{1}{C}$ 高Q值低通：谐振频点附近近似带通



$$R_1 = R_C \parallel r_{ce} \qquad R_3 = R_{pL} \parallel n^2 R_L \qquad R_2 = R_B \parallel r_{be}$$

$$A_0 = -g_m$$

$$\mathbf{ABCD} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R_1} + sC_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{sLR_3}{sL+R_3} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2} + sC_2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cdots & \cdots \\ \frac{1}{R_1} + sC_1 + \frac{1}{R_2} + sC_2 + \left(\frac{1}{R_1} + sC_1\right)\left(\frac{1}{R_2} + sC_2\right)\frac{sLR_3}{sL+R_3} & \cdots \end{bmatrix}$$

$$F = \frac{1}{C} = \frac{1}{\frac{1}{R_1} + sC_1 + \frac{1}{R_2} + sC_2 + \left(\frac{1}{R_1} + sC_1\right)\left(\frac{1}{R_2} + sC_2\right)\frac{sLR_3}{sL+R_3}}$$

$$A_0 = -g_m$$

$$F = \frac{1}{C} = \frac{1}{\frac{1}{R_1} + sC_1 + \frac{1}{R_2} + sC_2 + \left(\frac{1}{R_1} + sC_1\right)\left(\frac{1}{R_2} + sC_2\right)\frac{sLR_3}{sL + R_3}}$$

$$= \frac{sL + R_3}{(G_1 + G_2 + sC_1 + sC_2)(sL + R_3) + (G_1 + sC_1)(G_2 + sC_2)sLR_3}$$

$$= \frac{sL + R_3}{\begin{array}{l} s^2 L(C_1 + C_2) + sR_3(C_1 + C_2) + sL(G_1 + G_2) + R_3(G_1 + G_2) \\ + \left(sLR_3G_1G_2 + s^2LR_3C_1G_2 + s^2LR_3C_2G_1 + s^3LR_3C_1C_2\right) \end{array}}$$

$$= \frac{sL + R_3}{\begin{array}{l} R_3(G_1 + G_2) + s\left(R_3(C_1 + C_2) + L(G_1 + G_2) + LR_3G_1G_2\right) \\ + s^2\left(L(C_1 + C_2) + LR_3G_2C_1 + LR_3G_1C_2\right) + s^3LR_3C_1C_2 \end{array}}$$

$$= \frac{j\omega L + R_3}{\begin{array}{l} j\omega\left(\left(R_3(C_1 + C_2) + L(G_1 + G_2) + LR_3G_1G_2\right) - \omega^2 LR_3C_1C_2\right) \\ + R_3(G_1 + G_2) - \omega^2\left(L(C_1 + C_2) + LR_3G_2C_1 + LR_3G_1C_2\right) \end{array}}$$

$$F=\frac{1}{C}=\frac{j\omega L+R_3}{j\omega\left(\left(R_3\left(C_1+C_2\right)+L\left(G_1+G_2\right)+LR_3G_1G_2\right)-\omega^2LR_3C_1C_2\right)+R_3\left(G_1+G_2\right)-\omega^2\left(L\left(C_1+C_2\right)+LR_3G_2C_1+LR_3G_1C_2\right)}$$

$$\varphi_{AF}\left(\omega_{osc}\right)=0$$

$$\frac{L}{\left(R_3\left(C_1+C_2\right)+L\left(G_1+G_2\right)+LR_3G_1G_2\right)-\omega_{osc}^2LR_3C_1C_2}=\frac{R_3}{R_3\left(G_1+G_2\right)-\omega_{osc}^2\left(L\left(C_1+C_2\right)+LR_3G_2C_1+LR_3G_1C_2\right)}$$

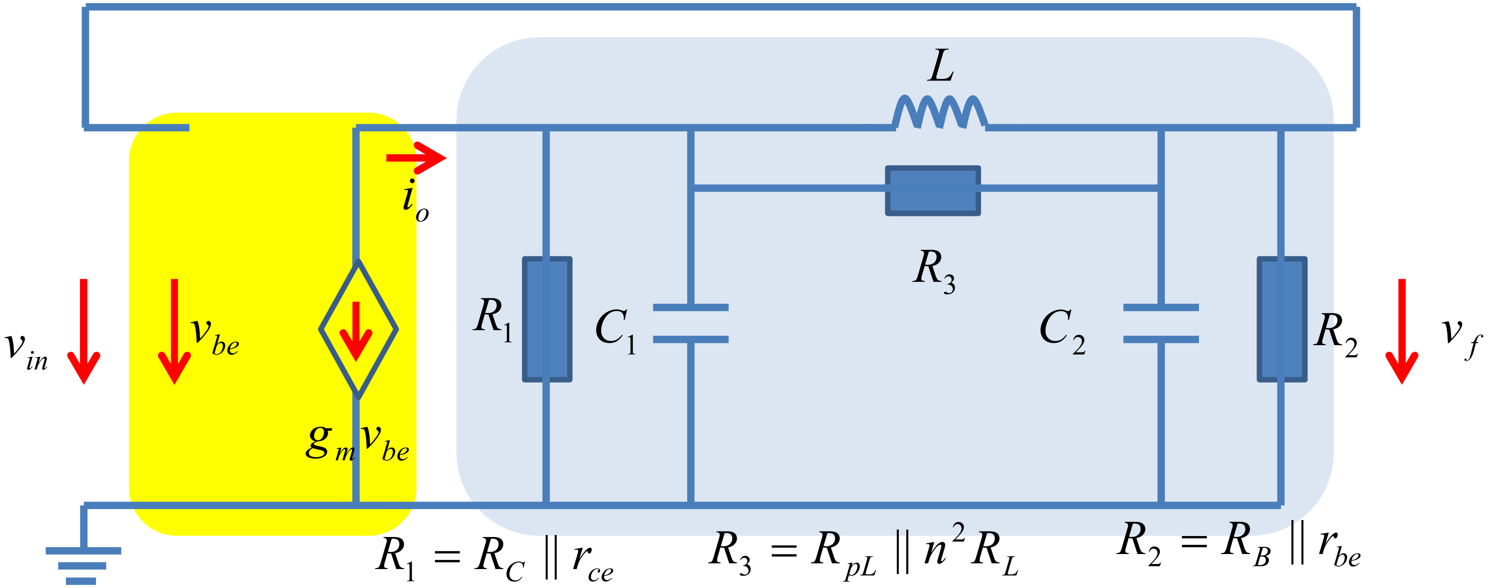
相位平衡条件获得振荡频率

$$\omega_{osc}^2\left[1-L\frac{1}{C_1C_2}\frac{R_1C_1+R_2C_2}{R_3R_2R_1}\right]=\left(\frac{C_1+C_2}{LC_1C_2}+\frac{1}{R_1C_1R_2C_2}\right)$$

$$\omega_{osc}=\sqrt{\frac{\frac{C_1+C_2}{LC_1C_2}+\frac{1}{R_1C_1R_2C_2}}{1-L\frac{1}{C_1C_2}\frac{R_1C_1+R_2C_2}{R_3R_2R_1}}}=\sqrt{\frac{1}{L\frac{C_1C_2}{C_1+C_2}}}\sqrt{\frac{1+\frac{L}{R_1R_2\left(C_1+C_2\right)}}{1-L\frac{1}{C_1C_2}\frac{R_1C_1+R_2C_2}{R_3R_2R_1}}}$$

$$=\omega_0\sqrt{\frac{1+\frac{L}{R_1R_2\left(C_1+C_2\right)}}{1-\frac{L}{C_1C_2}\frac{R_1C_1+R_2C_2}{R_3R_2R_1}}}$$

并联电阻**R1**，**R2**，**R3**越大，**Q**值越大，
电阻对振荡频率的影响就越小



$R_1 = R_C \parallel r_{ce}$ $R_3 = R_{pL} \parallel n^2 R_L$ $R_2 = R_B \parallel r_{be}$

$$R_1 = R_C \parallel r_{ce} = 3.3k \parallel 477k = 3.277k\Omega$$

$$R_2 = R_B \parallel r_{be} = 5.174k \parallel 49.6k = 4.685k\Omega$$

$$R_3 = n^2 R_L \parallel R_{pL} = 4k \parallel 69k = 3.781k\Omega$$

$C_1 = 300pF$ $C_2 = 1000pF$

$L = 110\mu H$

$$\omega_{osc} = \omega_0 \sqrt{\frac{1+\dfrac{L}{R_1R_2(C_1+C_2)}}{1-\dfrac{L}{C_1C_2}\dfrac{R_1C_1+R_2C_2}{R_3R_2R_1}}} = \omega_0 \sqrt{\frac{1+\dfrac{110\mu}{3.277k\times 4.685k\times 1300p}}{1-\dfrac{110\mu}{300p\times 1000p}\dfrac{3.277k\times 300p+4.685k\times 1000p}{3.781k\times 3.277k\times 4.685k}}}$$

$$= \omega_0\sqrt{\frac{1+0.00551}{1-0.0358}} = \omega_0\sqrt{1.0428} = 1.0212\omega_0$$

电阻对振荡频率影响有限，使得振荡频率略大于原理性分析结果：**2%**误差可以容忍

$$1 < A_0 F(\omega_{osc}) = -g_m \frac{R_3}{R_3(G_1+G_2) - \omega_{osc}^2 \left(L(C_1+C_2) + LR_3G_2C_1 + LR_3G_1C_2\right)}$$

$$= -g_m \frac{R_3}{R_3(G_1+G_2) - \dfrac{\dfrac{C_1+C_2}{LC_1C_2} + \dfrac{1}{R_1C_1R_2C_2}}{1 - L\dfrac{1}{C_1C_2}\dfrac{R_1C_1+R_2C_2}{R_3R_2R_1}} \left(L(C_1+C_2) + LR_3G_2C_1 + LR_3G_1C_2\right)}$$

按电容部分接入处理，起振条件和电感无关

$$R_3 > \frac{\dfrac{(C_1+C_2)^2}{C_1C_2} + \dfrac{L(C_1+C_2)}{R_1R_2C_1C_2} + (G_1+G_2+g_m)\dfrac{L(R_1C_1+R_2C_2)}{R_1R_2C_1C_2}}{g_m+G_1+G_2 - \left(\dfrac{L}{R_1R_2C_1C_2} + \dfrac{C_1+C_2}{C_1C_2}\right)\left(\dfrac{R_1C_1+R_2C_2}{R_1R_2}\right)}$$

$$R_3 > \ldots \xrightarrow{L\to 0} \frac{1}{p(1-p)g_m - p^2G_1 + (1-p)^2G_2}$$

$$= \frac{\dfrac{(1300p)^2}{300p\times 1000p} + \dfrac{110\mu\times 1300p}{3.277k\times 4.685k\times 300p\times 1000p} + (0.305m+0.213m+8.07m)\dfrac{110\mu\times(3.277k\times 300p+4.685k\times 1000p)}{3.277k\times 4.685k\times 300p\times 1000p}}{8.07m+0.305m+0.213m - \left(\dfrac{110\mu}{3.277k\times 4.685k\times 300p\times 1000p} + \dfrac{1300p}{300p\times 1000p}\right)\left(\dfrac{3.277k\times 300p+4.685k\times 1000p}{3.277k\times 4.685k}\right)}$$

0.0000239　　0.0043333

$$= \frac{5.633+0.031+1.159}{8.588m-1.609m} = 978\Omega$$

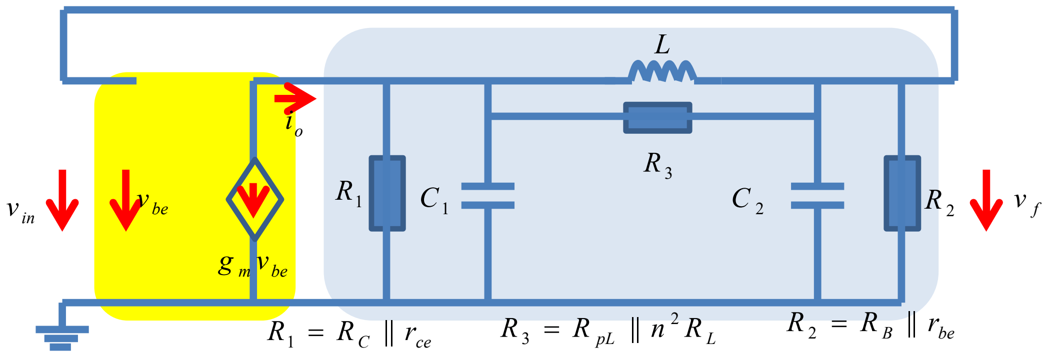
结论：用部分接入方法做近似分析足够用！

$$R_L > \frac{1}{n^2\left(\dfrac{1}{R_{3\min}} - \dfrac{1}{R_{pL}}\right)} = \frac{1}{4\times\left(\dfrac{1}{978} - \dfrac{1}{69000}\right)} = 248\Omega$$

部分接入没有考虑bc端口并联电感的作用，是高Q值假设下的分析结果

$R_L = 250\Omega$

负载太重，Q值下降，频率偏移



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$$R_3 = n^2 R_L \parallel R_{pL} = 4k \parallel 69k = 3.781k\Omega$$

$$R_3 = n^2 R_L \parallel R_{pL} = 1k \parallel 69k = 986\Omega$$

$$G_{bc} = G_3 + p^2 G_1 + (1-p)^2 G_2 = 1.014m + 0.181m + 0.011m = 1.207mS$$

$$Q = \frac{1}{\omega_0 L G_{bc}} = \frac{1}{2\pi \times 1M \times 110\mu \times 1.207m} = 1.2$$

自行分析对Q值影响最大的因素，电路如何调整使得频率稳定度提高，…

频率稳定度下降

$$\omega_{osc} = \omega_0 \sqrt{\frac{1 + \frac{L}{R_1 R_2 (C_1 + C_2)}}{1 - \frac{L}{C_1 C_2} \frac{R_1 C_1 + R_2 C_2}{R_1 R_2 R_3}}} = \omega_0 \sqrt{\frac{1 + \frac{110\mu}{3.277k \times 4.685k \times 1300p}}{1 - \frac{110\mu}{300p \times 1000p} \frac{3.277k \times 300p + 4.685k \times 1000p}{3.277k \times 4.685k \times 0.986k}}}$$

$$= \omega_0 \sqrt{\frac{1 + 0.00551}{1 - 0.137292}} = 1.08\omega_0$$

频率偏差加大