电子电路与系统基础II

习题课第十三讲

晶体管电路回顾与拓展（下半）
负阻LC正弦波振荡器

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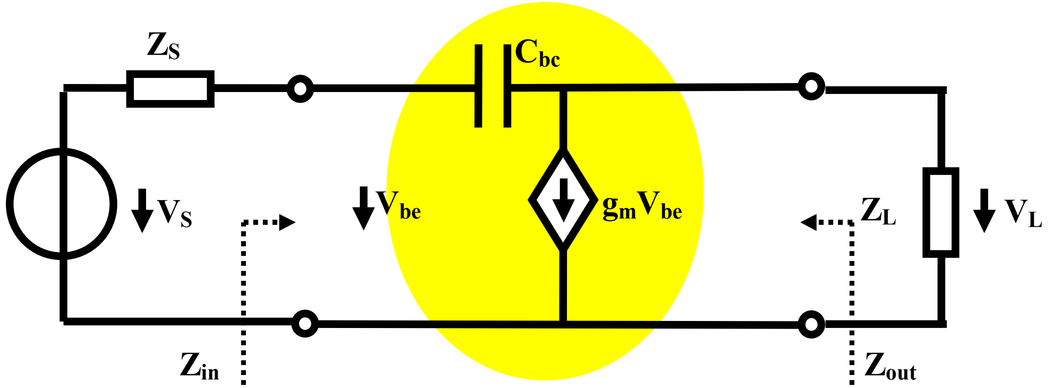
作业6：晶体管放大器不稳定的原因

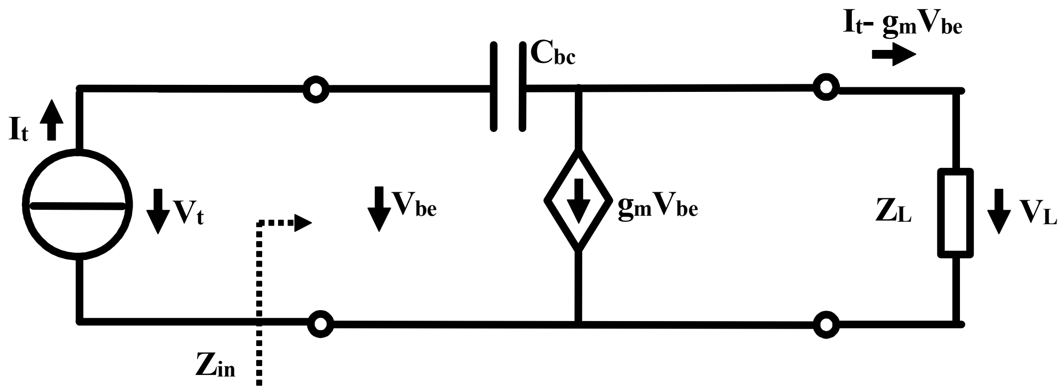
- 练习10.4.10：图E10.4.6是用来考察CE组态晶体管C_{bc}对输入阻抗和输出阻抗影响的原理性电路，其中只剩下晶体管原本设计的压控流源和跨接在压控流源输出和输入之间的寄生电容C_{bc}，考察当$Z_L=R_L$，$j\omega L_2$两种负载情况下，输入阻抗Z_{in}的性质；考察当$Z_S=R_S$，$j\omega L_1$两种负载情况下，输出阻抗Z_{out}的性质。



只要是三点式连接关系，在GS、DS、GD端口都能看到等效负阻

方法1：先求二端口网络的y参量，再由y参量求输入输出阻抗
方法2：加压求流

输入端口加流求压



$$\dot{V}_t = \dot{I}_t \frac{1}{j\omega C_{bc}} + (\dot{I}_t - g_m \dot{V}_t) Z_L = \dot{I}_t \left(\frac{1}{j\omega C_{bc}} + Z_L \right) - g_m \dot{V}_t Z_L$$

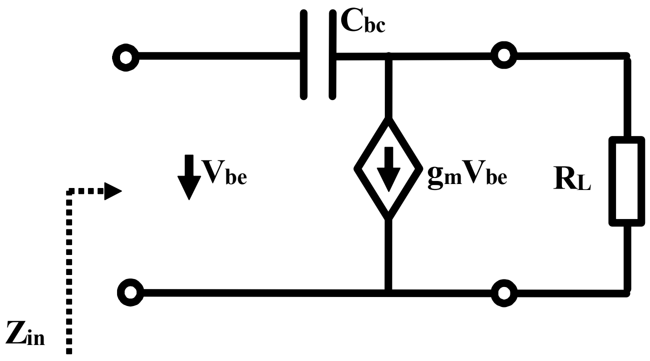
$$Z_{in} = \frac{\dot{V}_t}{\dot{I}_t} = \frac{\frac{1}{j\omega C_{bc}} + Z_L}{1 + g_m Z_L} = \frac{1}{j\omega(1 + g_m Z_L) C_{bc}} + \frac{Z_L}{1 + g_m Z_L}$$

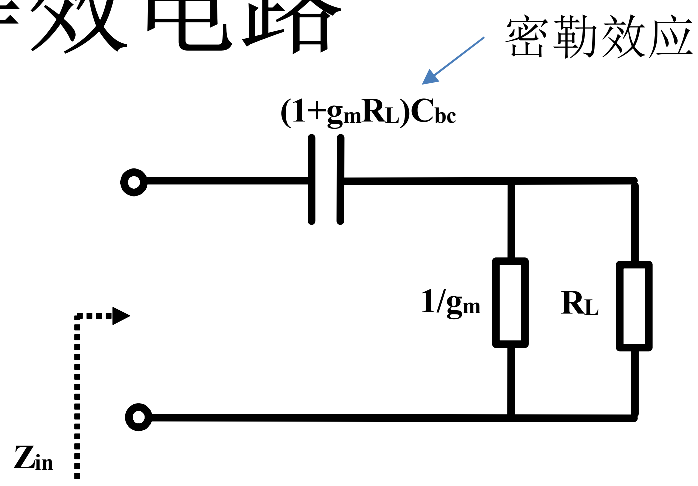
$$Z_L \,||\, \frac{1}{g_m}$$

$$Z_{in}(Z_L = R_L) = \frac{1}{j\omega(1 + g_m R_L) C_{bc}} + R_L \,||\, \frac{1}{g_m}$$

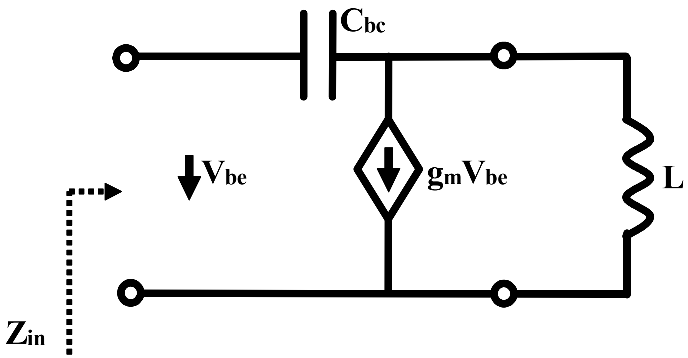
$$Z_{in}(Z_L = j\omega L) = \frac{1}{j\omega C_{bc} - \omega^2 L C_{bc} g_m} + j\omega L \,||\, \frac{1}{g_m}$$

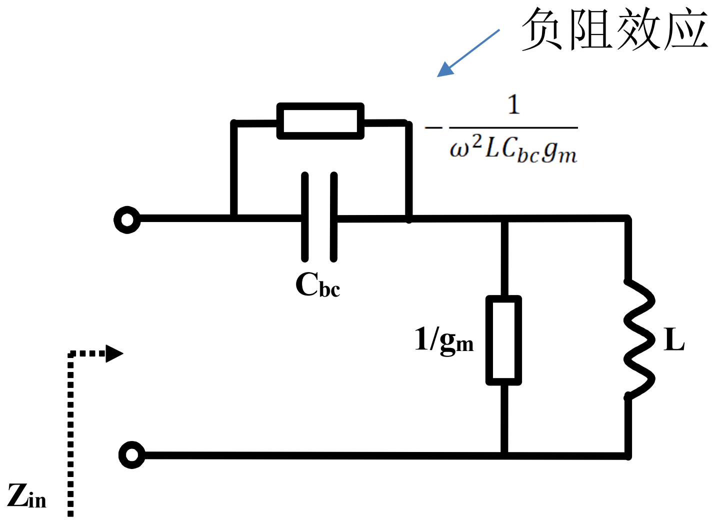
输入阻抗等效电路





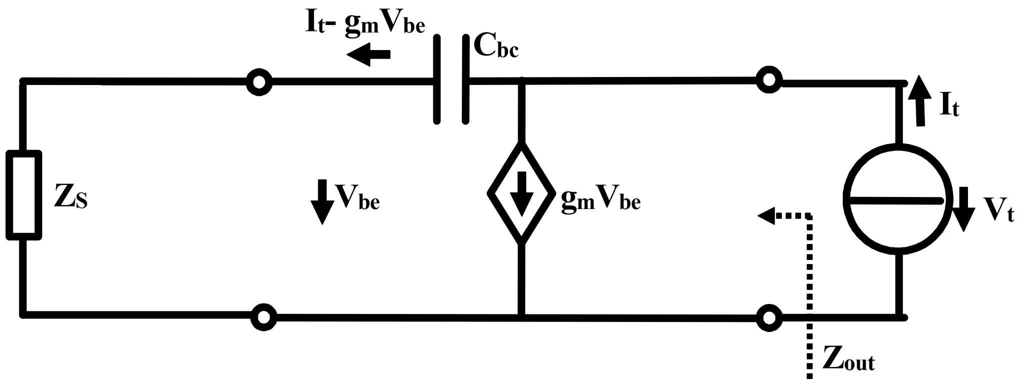
$$Z_{in}(Z_L = R_L) = \frac{1}{j\omega(1+g_m R_L)C_{bc}} + R_L \,||\, \frac{1}{g_m}$$





$$Z_{in}(Z_L = j\omega L) = \frac{1}{j\omega C_{bc} - \omega^2 L C_{bc} g_m} + j\omega L || \frac{1}{g_m}$$

输出端口加流求压



$$\dot{V}_{be}=(\dot{I}_t-g_m\dot{V}_{be})Z_S$$

$$\dot{V}_{be}=\frac{Z_S}{1+g_mZ_S}\dot{I}_t$$

$$\dot{V}_t=(\dot{I}_t-g_m\dot{V}_{be})\left(\frac{1}{j\omega C_{bc}}+Z_S\right)=\left(\dot{I}_t-g_m\frac{Z_S}{1+g_mZ_S}\dot{I}_t\right)\left(\frac{1}{j\omega C_{bc}}+Z_S\right)$$

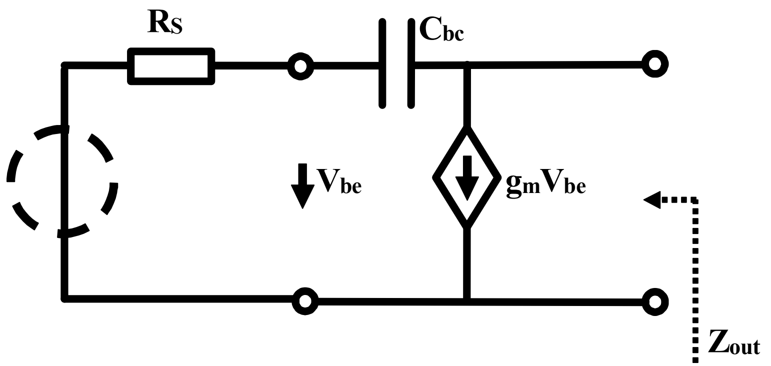
$$Z_{out}=\frac{\dot{V}_t}{\dot{I}_t}=\frac{\frac{1}{j\omega C_{bc}}+Z_S}{1+g_mZ_S}=\frac{1}{j\omega(1+g_mZ_S)C_{bc}}+\frac{Z_S}{1+g_mZ_S}$$

$$Z_S\,||\,\frac{1}{g_m}$$

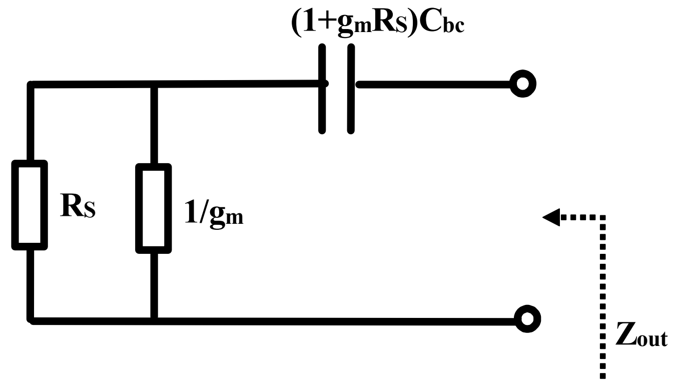
$$Z_{out}(Z_S=R_S)=\frac{1}{j\omega(1+g_mR_S)C_{bc}}+R_S\,||\,\frac{1}{g_m}$$

$$Z_{out}(Z_S=j\omega L)=\frac{1}{j\omega C_{bc}-\omega^2LC_{bc}g_m}+j\omega L||\frac{1}{g_m}$$

输出阻抗等效电路

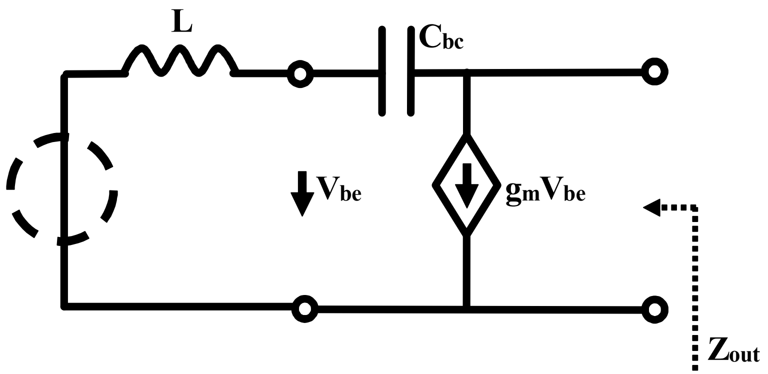


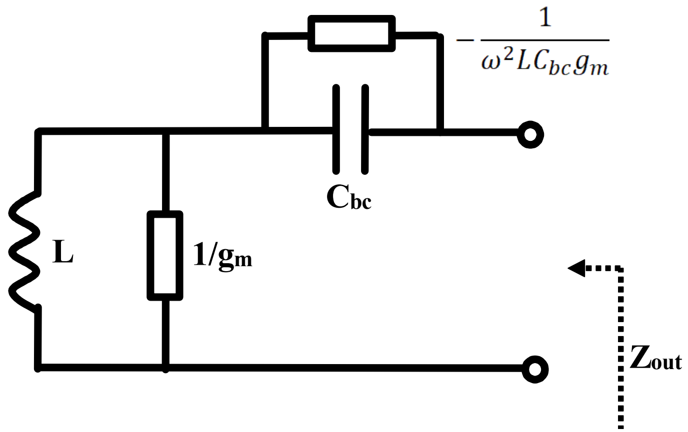
密勒效应



$$Z_{out}(Z_S = R_S) = \frac{1}{j\omega(1 + g_m R_S)C_{bc}} + R_S \,||\, \frac{1}{g_m}$$

负阻效应





$$Z_{out}(Z_S = j\omega L) = \frac{1}{j\omega C_{bc} - \omega^2 L C_{bc} g_m} + j\omega L \,||\, \frac{1}{g_m}$$

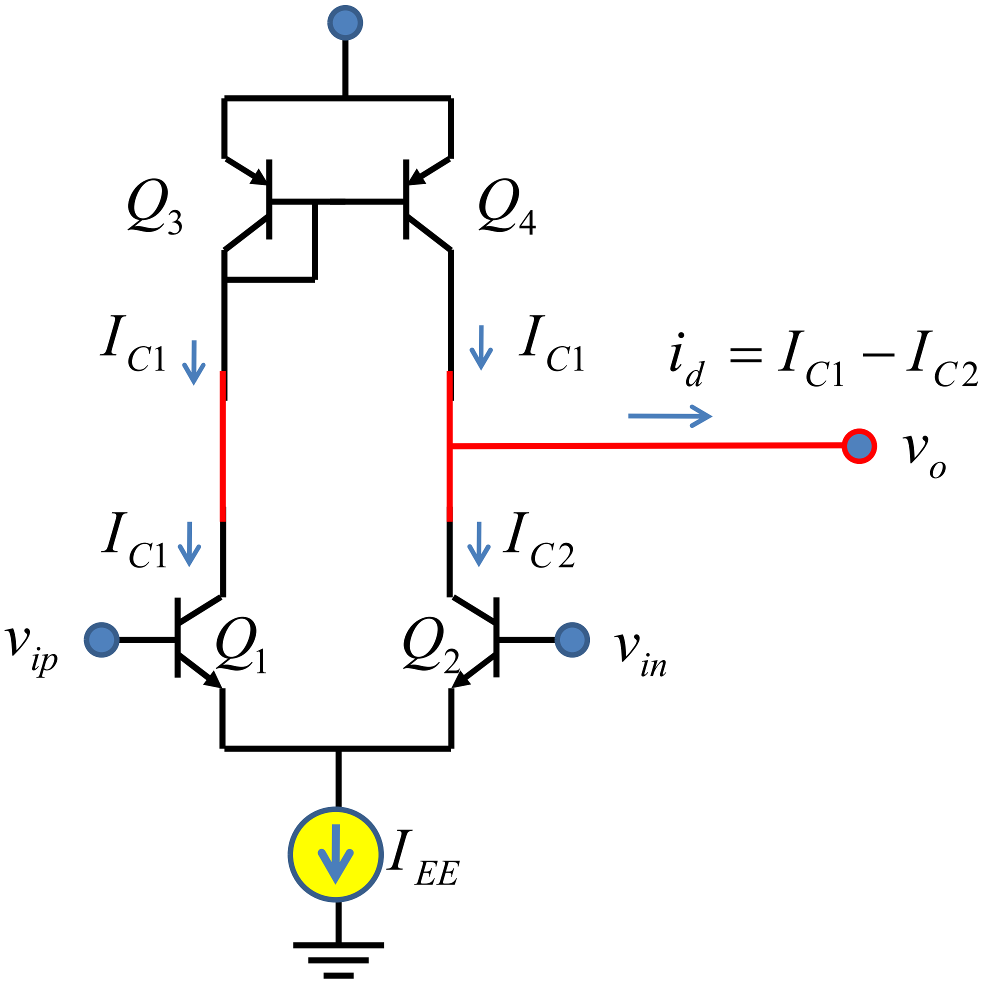
寄生电容效应

- **C_{bc}**是跨接在**CE**组态晶体管输入端和输出端的跨接电容
  - 密勒效应：当一个端口接电阻负载时，另一个端口看入阻抗中有一个等效大电容，从而**C_{bc}**很容易呈现高频短路效应，从而高频增益严重下降

$$C_{in} = (1 + g_m R_L) C_{bc} \qquad C_{out} = (1 + g_m R_S) C_{bc}$$

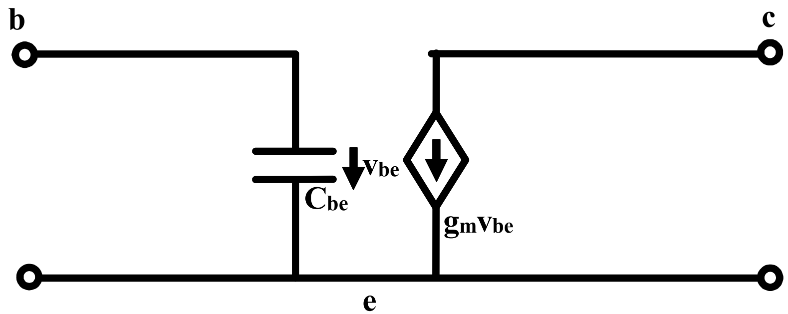
  - 负阻效应：当一个端口接电感负载时，另一个端口看入阻抗中有一个等效负阻，当端口正阻无法抵偿等效负阻时，放大器将自激振荡，**C_{bc}**是**CE**组态晶体管放大器的不稳定来源
    - 哈特莱三点式结构，从任何一个端口看，都会看到等效负阻，其中**C_{bc}**是寄生的，而两个电感则是用来做共轭匹配的，从而放大器调试时出现自激振荡

作业7：寄生电容产生的零点

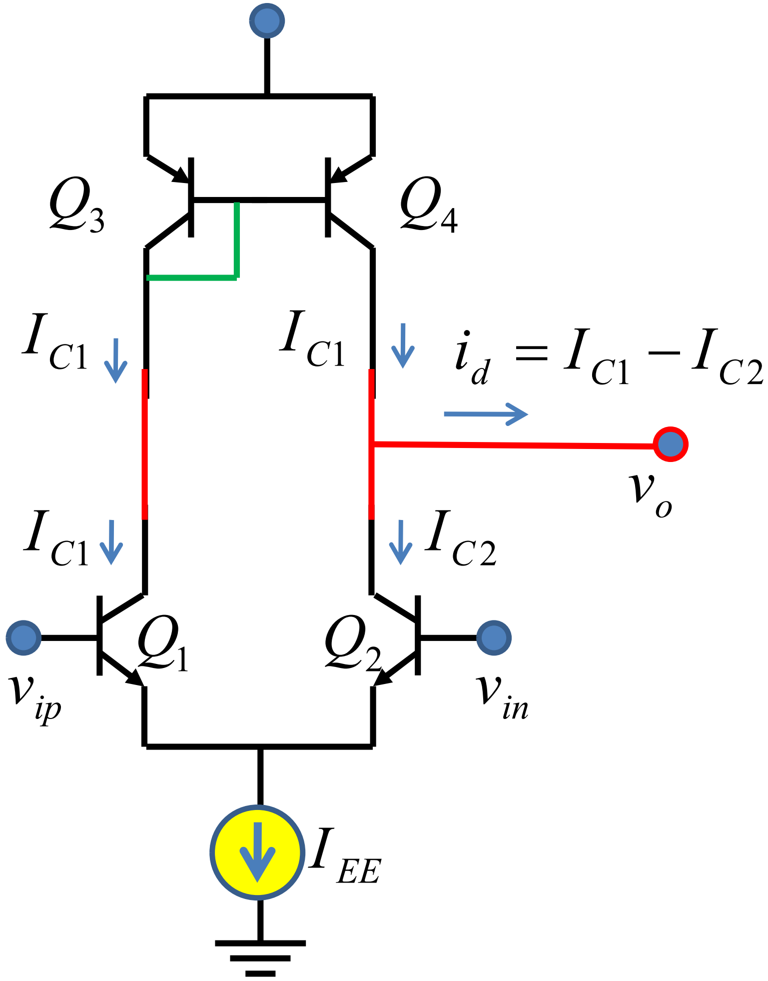


- 仅考虑寄生电容C_{be}影响，求传递函数表达式

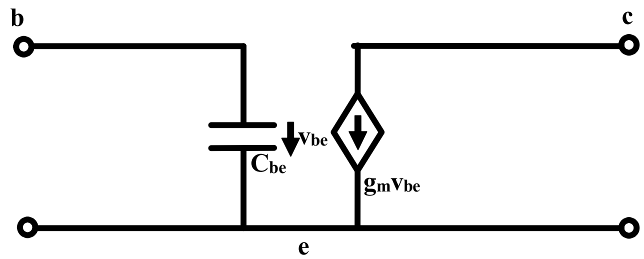
$$H = \frac{i_d}{v_{id}}$$



晶体管简化模型

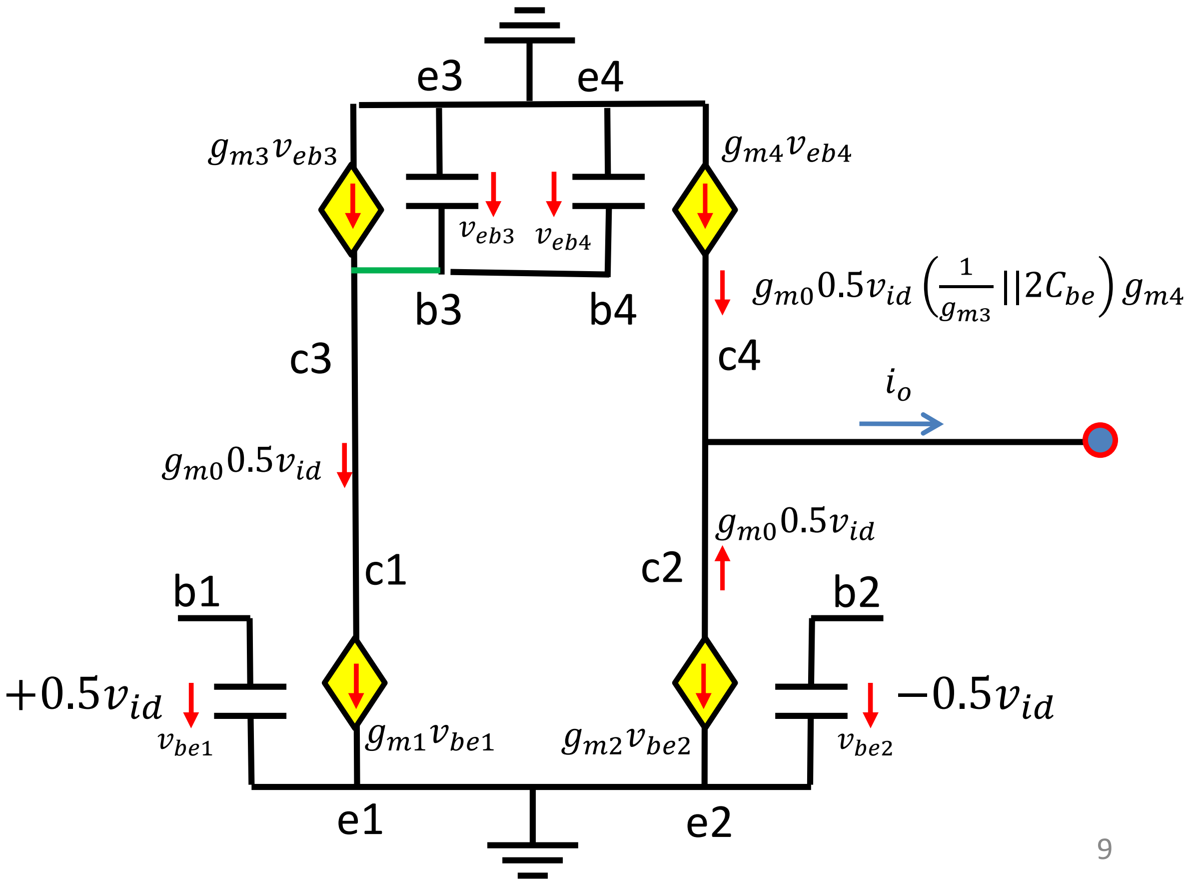


Q_3
Q_4
I_{C1}
I_{C1}
$i_d = I_{C1} - I_{C2}$
v_o
I_{C1}
I_{C2}
Q_1
Q_2
v_{ip}
v_{in}
I_{EE}

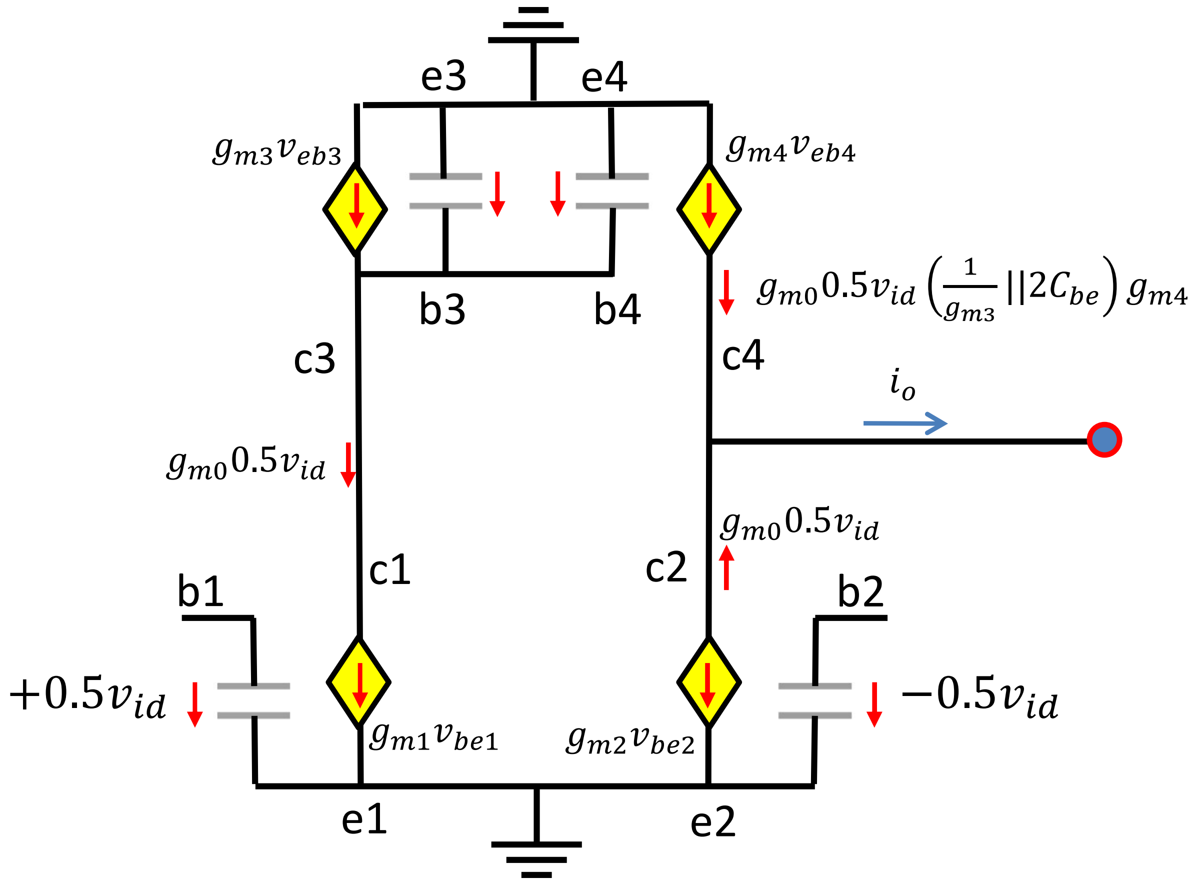


b
c
v_{be}
C_{be}
$g_m v_{be}$
e

等效
电路



e3
e4
$g_{m3} v_{eb3}$
$g_{m4} v_{eb4}$
v_{eb3}
v_{eb4}
$g_{m0} 0.5 v_{id} \left(\frac{1}{g_{m3}} \| 2C_{be}\right) g_{m4}$
b3
b4
c3
c4
i_o
$g_{m0} 0.5 v_{id}$
$g_{m0} 0.5 v_{id}$
c1
c2
b1
b2
$+0.5 v_{id}$
v_{be1}
$g_{m1} v_{be1}$
$g_{m2} v_{be2}$
v_{be2}
$-0.5 v_{id}$
e1
e2

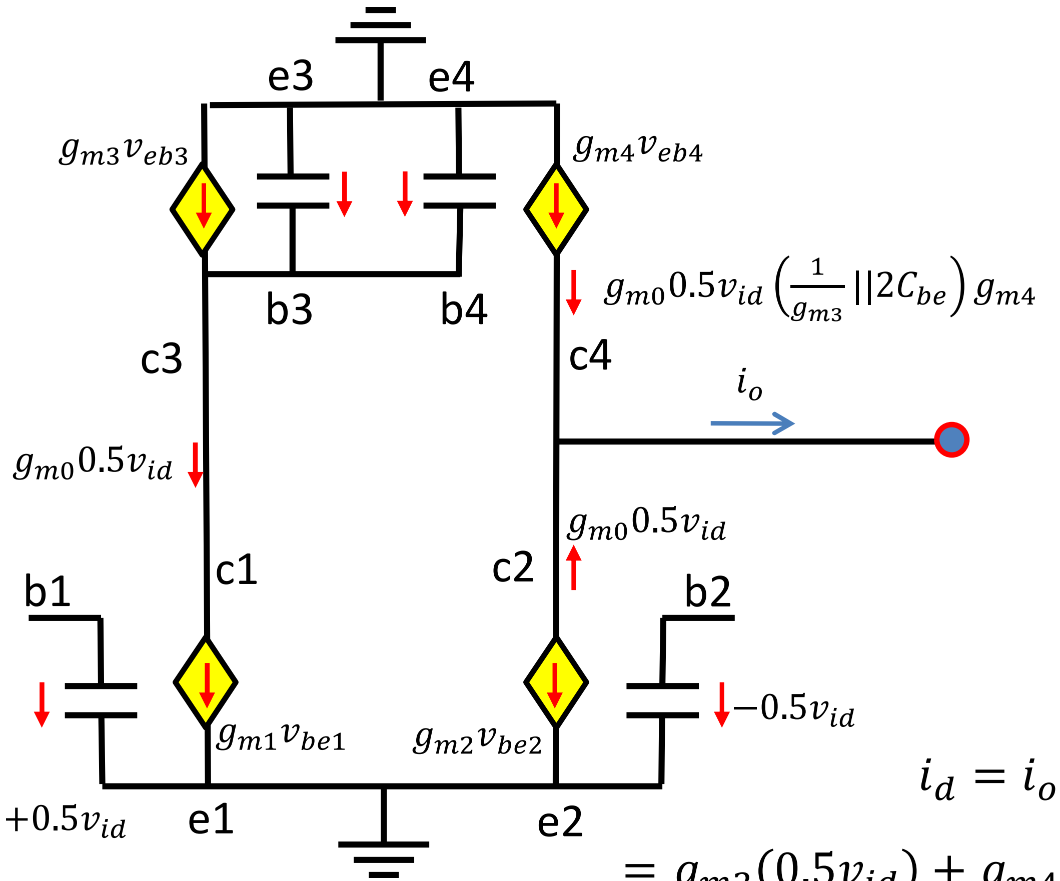


不考虑寄生电容

$$
\begin{aligned}
i_d = i_o &= g_{m2}(0.5v_{id}) + g_{m4}v_{eb4} \\
&= g_{m0}(0.5v_{id}) + g_{m4}\left(g_{m0}(0.5v_{id})\frac{1}{g_{m3}}\right) \\
&= g_{m0}(0.5v_{id}) + g_{m0}(0.5v_{id}) \\
&= g_{m0}\ v_{id}
\end{aligned}
$$

电阻电路分析结果
电流镜完成双端转单端功能
实现差分电流的合并

考虑寄生电容影响



$$i_d = i_o = g_{m2}(0.5v_{id}) + g_{m4}v_{eb4}$$

$$= g_{m2}(0.5v_{id}) + g_{m4}\left(g_{m1}(0.5v_{id}) \times \frac{1}{g_{m3} + j\omega(C_{be3} + C_{be4})}\right)$$

$$= \left(g_{m0} + \frac{g_{m4}g_{m0}}{g_{m3} + j\omega(C_{be4} + C_{be4})}\right)(0.5v_{id})$$

$$= \left(\frac{g_{m4}g_{m0} + g_{m0}\, g_{m4} + g_{m0}j\omega(2C_{be4})}{g_{m4} + j\omega(2C_{be4})}\right)(0.5v_{id})$$

$$= g_{m0}\left(\frac{1 + j\omega\frac{C_{be4}}{g_{m4}}}{1 + j\omega\frac{2C_{be4}}{g_{m4}}}\right)v_{id}$$

传递函数

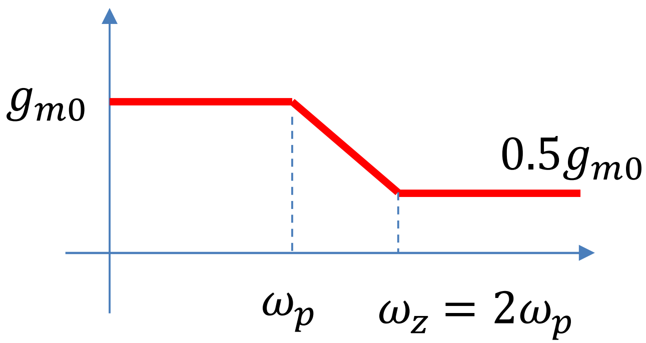
$$i_d = g_{m0}\left(\frac{1 + j\omega\dfrac{C_{be4}}{g_{m4}}}{1 + j\omega\dfrac{2C_{be}}{g_{m4}}}\right)v_{id}$$

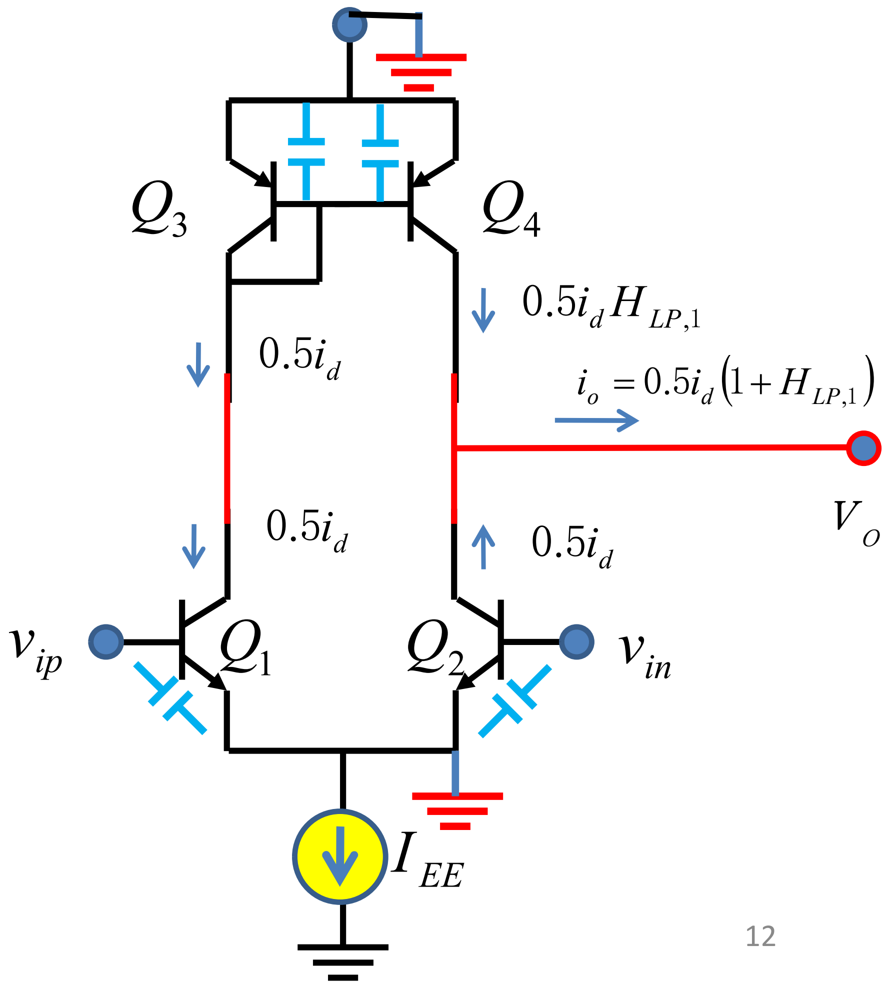
$$H = \frac{i_d}{v_{id}} = H_0\left(\frac{1 + \dfrac{j\omega}{\omega_z}}{1 + \dfrac{j\omega}{\omega_p}}\right)$$

$$H_0 = g_{m0}$$

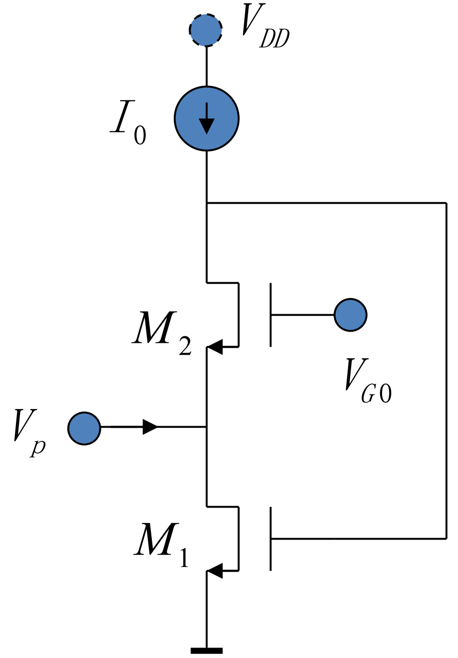
$$\omega_p = \frac{g_{m4}}{2C_{be4}}$$

$$\omega_z = \frac{g_{m4}}{C_{be4}} = 2\omega_p$$

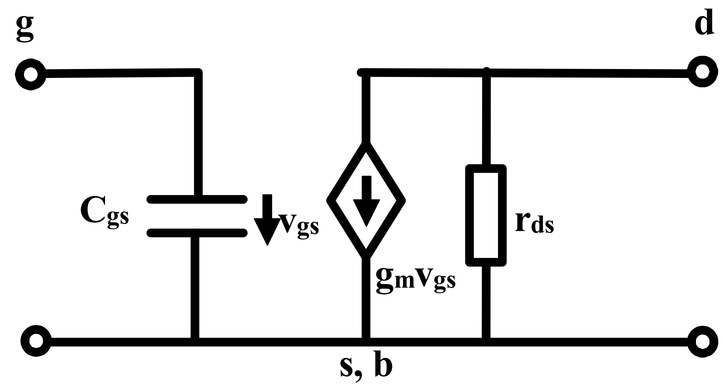




作业8：寄生电容的回旋对偶变换导致阻容电路出现谐振现象

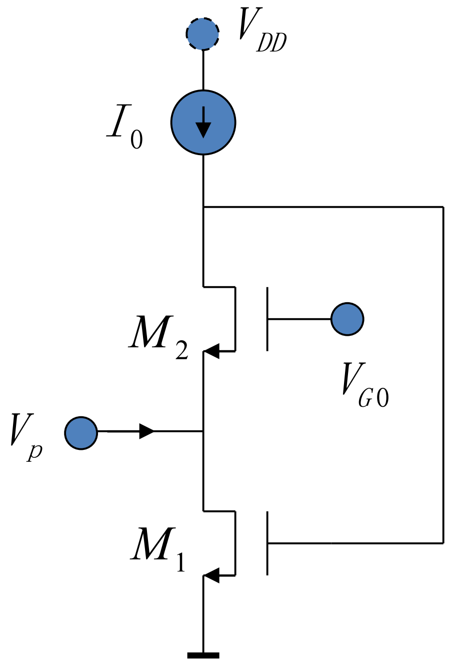


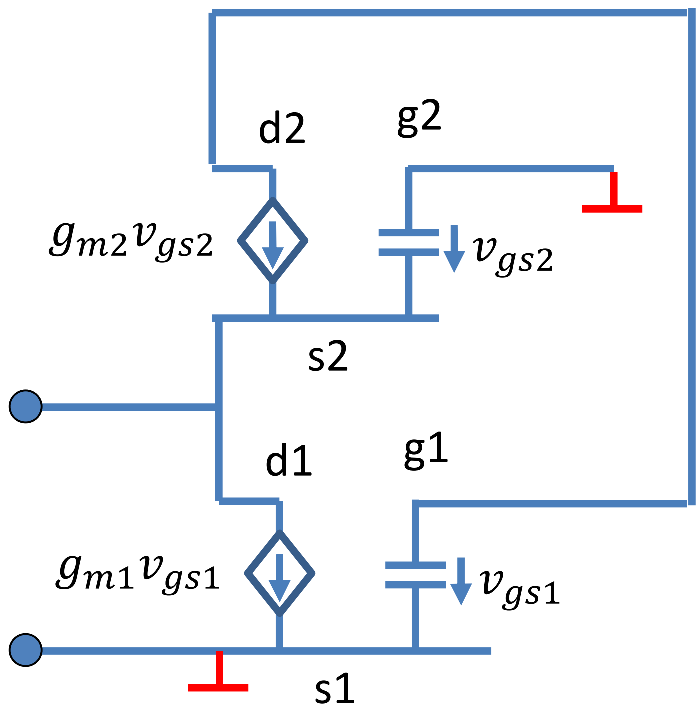
- 证明：考虑了晶体管的寄生电容效应后，从V_p端口看入，其等效电路为RLC并联谐振回路
  - 给出等价RLC



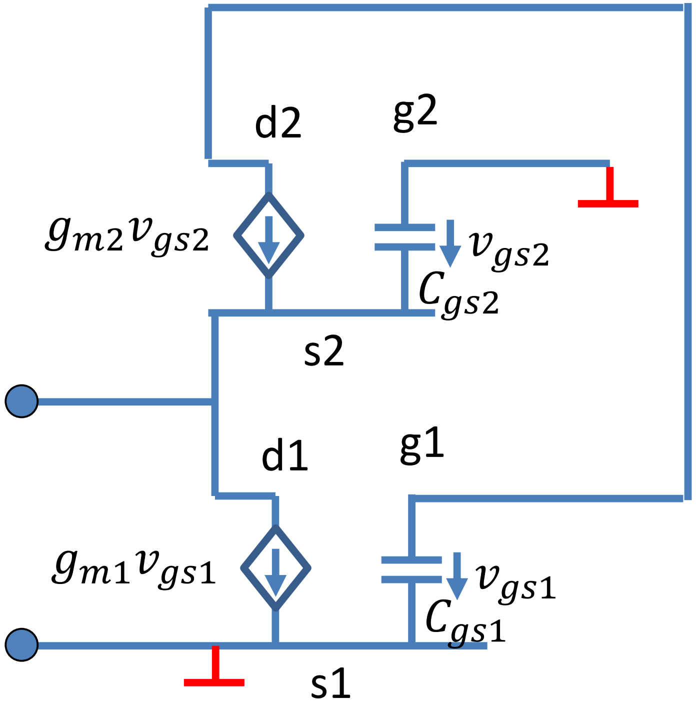
MOS简化模型

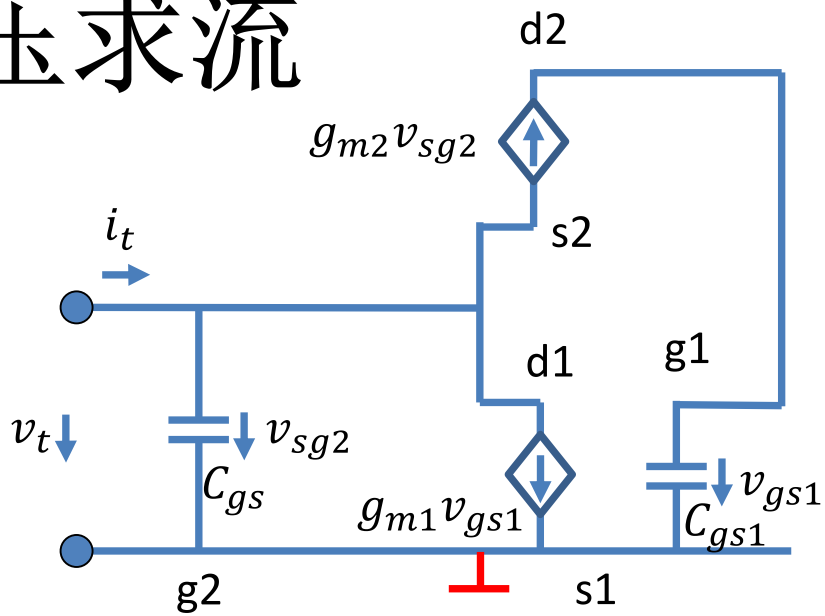
简化模型





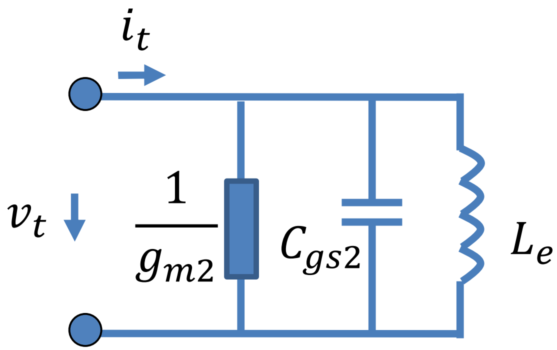
加压求流





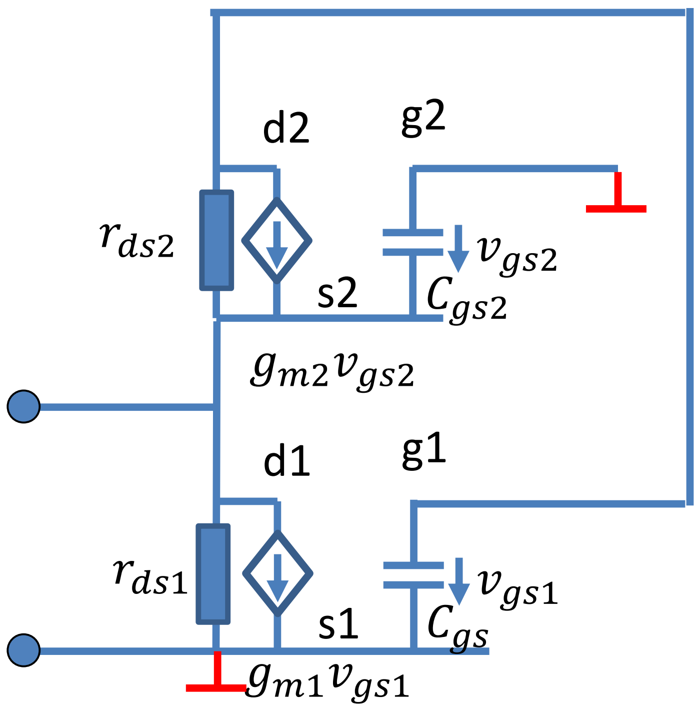
$$I_t = sC_{gs2}V_t + g_{m2}V_t + g_{m1}\left(\frac{g_{m2}V_t}{sC_{gs1}}\right)$$

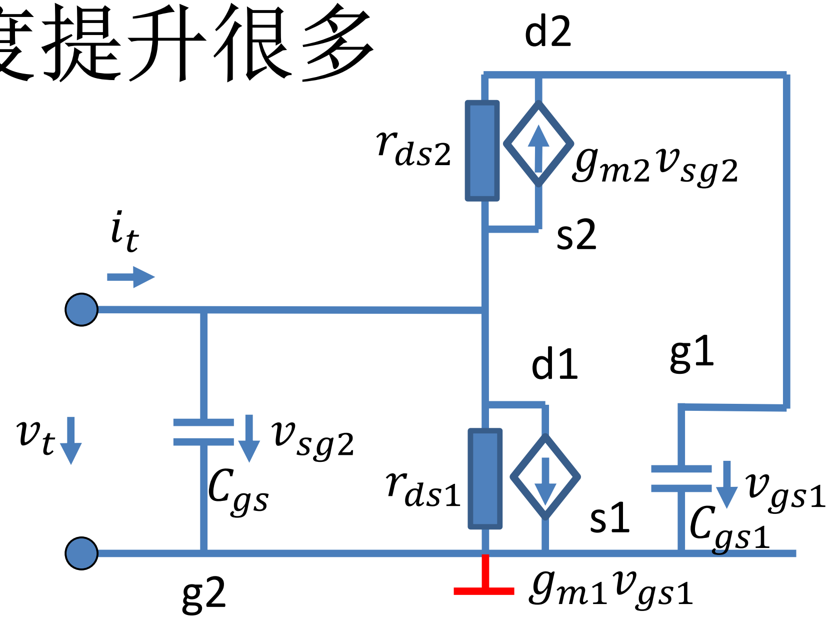
$$Y_{in} = \frac{I_t}{V_t} = sC_{gs2} + g_{m2} + \frac{g_{m1}g_{m2}}{sC_{gs1}} = g_{m2} + sC_{gs2} + \frac{1}{sL_e}$$



$$L_e = \frac{C_{gs1}}{g_{m1}g_{m2}}$$

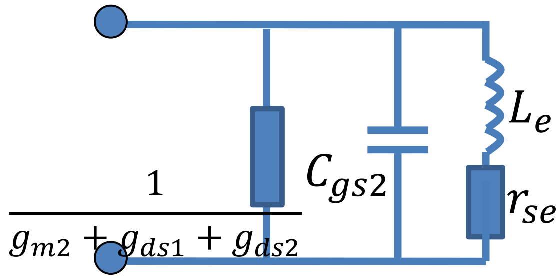
多一个参量
复杂度提升很多





$$I_t = V_t(sC_{gs2} + g_{ds1}) + g_{m1}V_{gs1} + g_{m2}V_t + (V_t - V_{gs1})g_{ds2}$$

$$sC_{gs1}V_{gs1} = g_{m2}V_t + (V_t - V_{gs1})g_{ds2} \qquad V_{gs1} = V_t\frac{g_{m2} + g_{ds2}}{sC_{gs1} + g_{ds2}}$$



$$Y_{in} = \frac{I_t}{V_t} = g_{m2} + g_{ds1} + g_{ds} + sC_{gs2} + \frac{(g_{m1} - g_{ds2})(g_{m2} + g_{ds2})}{sC_{gs1} + g_{ds2}}$$

$$= g_{m2} + g_{ds1} + g_{ds2} + sC_{gs2} + \frac{1}{sL_e + r_{se}}$$

$$L_e = \frac{C_{gs1}}{(g_{m1} - g_{ds2})(g_{m2} + g_{ds2})}$$

$$r_{se} = \frac{g_{ds2}}{(g_{m1} - g_{ds2})(g_{m2} + g_{ds2})}$$

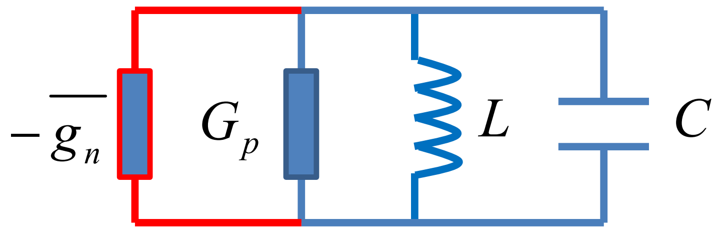
第12讲 负阻正弦振荡原理

作业1：求振荡幅度和振荡频率

- 已知某负阻元件的准线性负导大小和振荡幅度的关系为

$$\bar{g}_n = \frac{0.01}{V_m}$$

电压单位：V
跨导单位：S



- 已知电感为0.1μH，电容为200pF，电感无载Q值为Q_0=100

$$Q_0 = \frac{Y_0}{G_{p0}} = \frac{1}{G_{p0}\omega_0 L}$$

- 负载电阻为1kΩ

$$G_{p0} = \frac{1}{Q_0 \omega_0 L}$$

- 求输出正弦振荡信号的频率和幅度

并联型负阻振荡器

$$\sum B(\omega_{osc}) = 0$$

虚部（频率）平衡条件
决定振荡频率

$$\omega_{osc} C - \frac{1}{\omega_{osc} L} = 0$$

$$f_{osc} = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

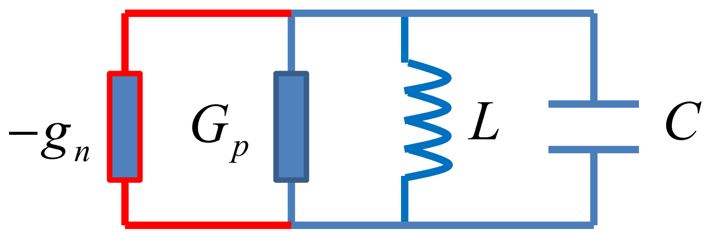
$$= \frac{1}{2\times 3.14\times\sqrt{0.1\times 10^{-6}\times 200\times 10^{-12}}}$$

$$= 35.6MHz$$

$$G_p = G_L + G_{p,L} = \frac{1}{R_L} + \frac{1}{Q_0\omega_0 L}$$

$$= \frac{1}{1000} + \frac{1}{100\times 2\times 3.14\times 35.6\times 10^6\times 0.1\times 10^{-6}}$$

$$= \frac{1}{1000} + \frac{1}{2236} = 1.45mS$$



$$\sum Y = 0 \qquad \sum G + j\sum B = 0$$

实部条件　虚部条件

$$\overline{g_n} = G_p$$

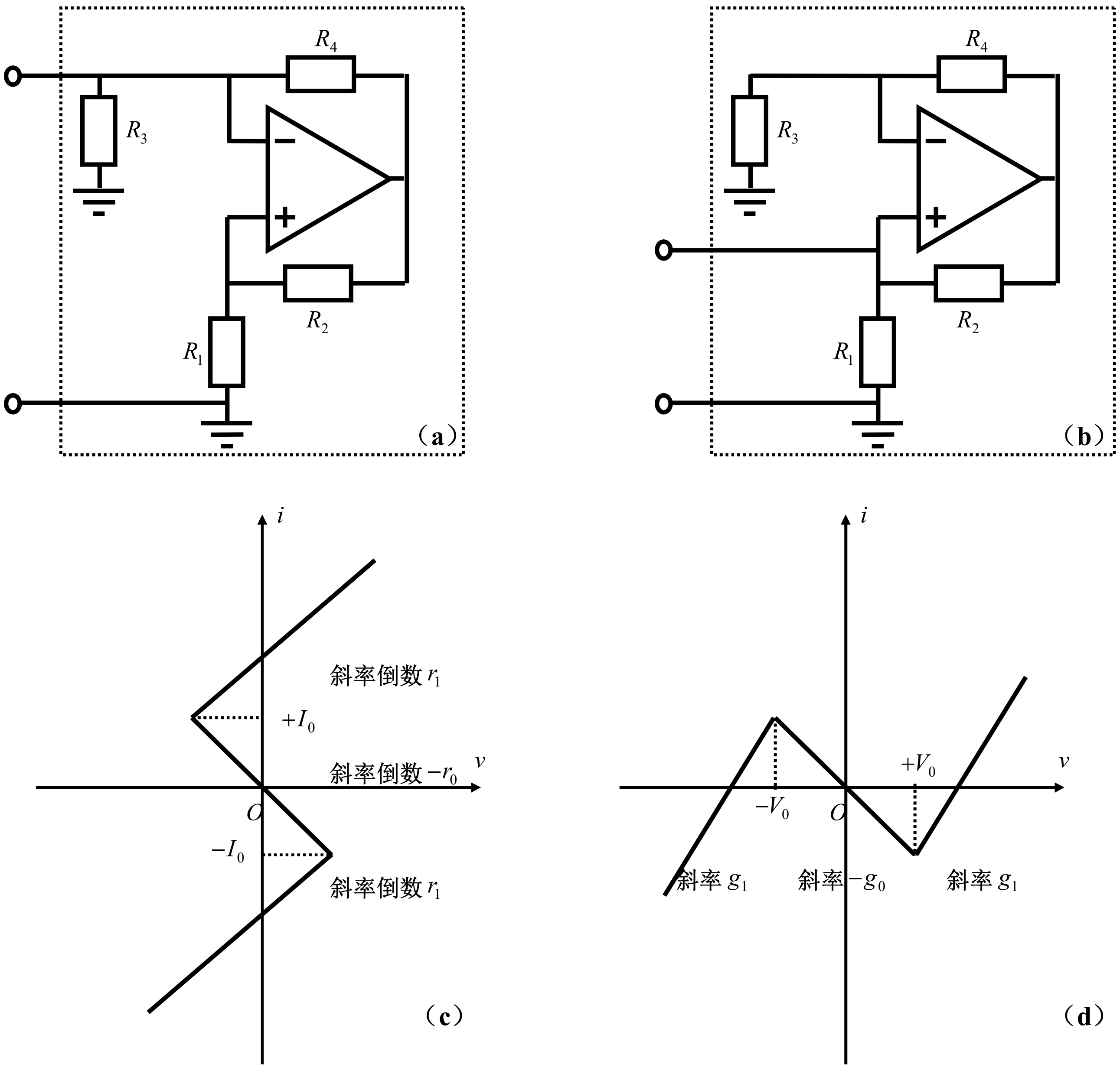
实部（幅度）平衡条件
决定振荡幅度

$$\overline{g_n}(V_{m\infty}) = \frac{0.01}{V_{m\infty}} = G_p = 1.45mS$$

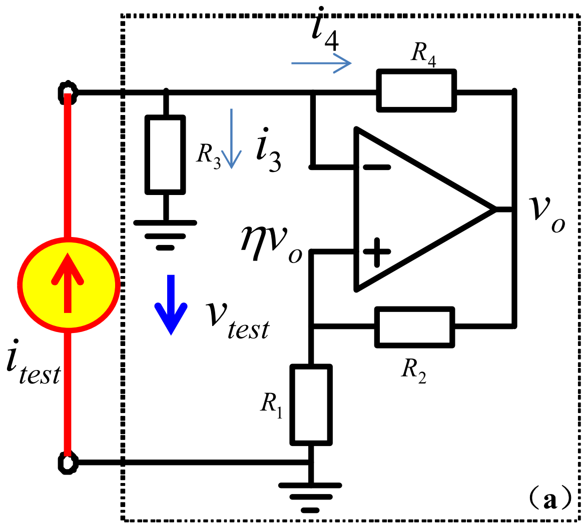
$$V_{m\infty} = \frac{0.01}{1.45\times 10^{-3}} = 6.91V$$

作业2
等效负阻

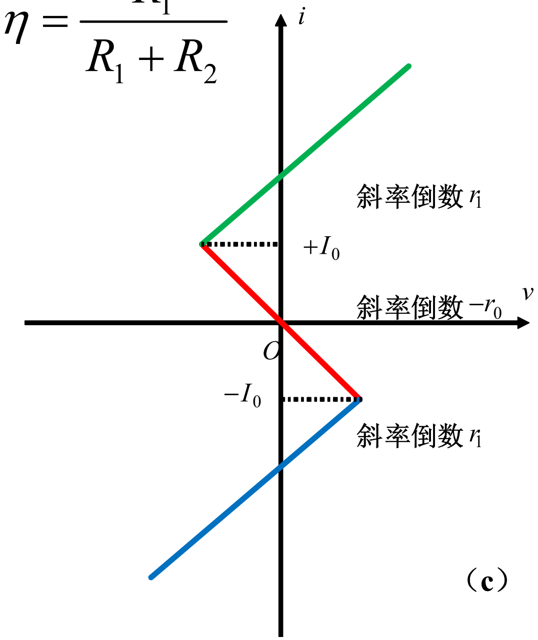
- 图**E10.4.1a**可用来实现**S**型负阻，图**E10.4.1b**可用来实现**N**型负阻。请设计运放外围电路器件大小，用来分别实现图**E10.4.1c**所示的**S**型负阻特性和图**E10.4.1d**所示的**N**型负阻特性。分析中假设运放为理想运放，其正负饱和电压为$\pm \mathbf{V_{sat}}$。



S型负阻



$$\eta = \frac{R_1}{R_1+R_2}$$



$$i_{test} = i_3 + i_4 = \frac{v_{test}}{R_3} + \frac{v_{test} - v_o}{R_4}$$

$$v_{test} = \frac{i_{test}}{G_3+G_4} + \frac{G_4}{G_3+G_4} v_o$$

情形1：运放工作在正饱和区： $v_o = +V_{sat}$

$$v_p > v_n \Rightarrow v_{test} < \eta V_{sat}$$

$$v_{test} = \frac{i_{test}}{G_3+G_4} + \frac{G_4}{G_3+G_4} V_{sat}$$

$$i_{test} < \left(\eta\left(G_3+G_4\right) - G_4\right) V_{sat} = -I_0$$

情形3：运放工作在负饱和区： $v_o = -V_{sat}$

$$v_p < v_n \Rightarrow v_{test} > -\eta V_{sat}$$

$$v_{test} = \frac{i_{test}}{G_3+G_4} - \frac{G_4}{G_3+G_4} V_{sat}$$

$$i_{test} > \left(G_4 - \eta\left(G_3+G_4\right)\right) V_{sat} = I_0$$

情形2：运放工作在线性区 $-V_{sat} < v_o < +V_{sat}$

$$v_p = v_n \Rightarrow v_{test} = \eta v_o$$

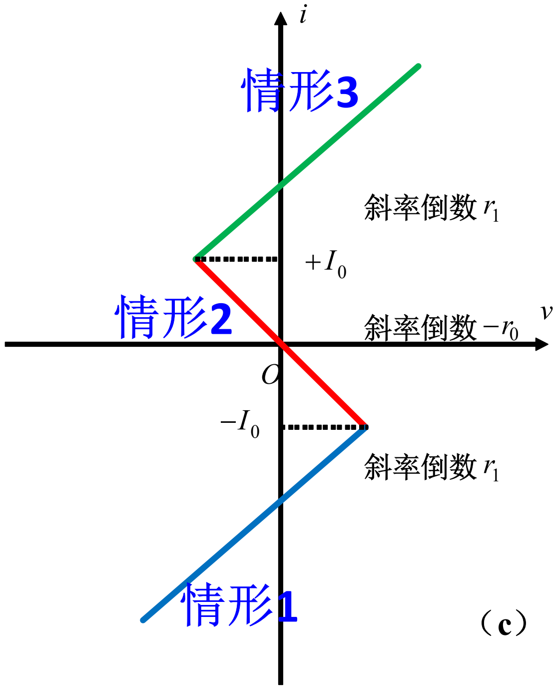
$$v_{test} = \frac{i_{test}}{G_3+G_4} + \frac{G_4}{G_3+G_4} v_o$$

$$= \frac{i_{test}}{G_3+G_4} + \frac{G_4}{G_3+G_4} \frac{v_{test}}{\eta}$$

$$v_{test} = \frac{i_{test}}{G_3+G_4-\dfrac{G_4}{\eta}}$$

$$-I_0 < i_{test} < I_0$$

一一对应



$$r_1=\frac{1}{G_3+G_4}$$

$$-r_0=\frac{1}{G_3+G_4-\frac{G_4}{\eta}}$$

$$I_0=\left(G_4-\eta\left(G_3+G_4\right)\right)V_{sat}$$

情形1：运放工作在正饱和区：

$$v_{test}=\frac{i_{test}}{G_3+G_4}+\frac{G_4}{G_3+G_4}V_{sat}$$

$$i_{test}<-I_0$$

$$I_0=\left(G_4-\eta\left(G_3+G_4\right)\right)V_{sat}$$

情形2：运放工作在线性区

$$v_{test}=\frac{i_{test}}{G_3+G_4-\frac{G_4}{\eta}}$$

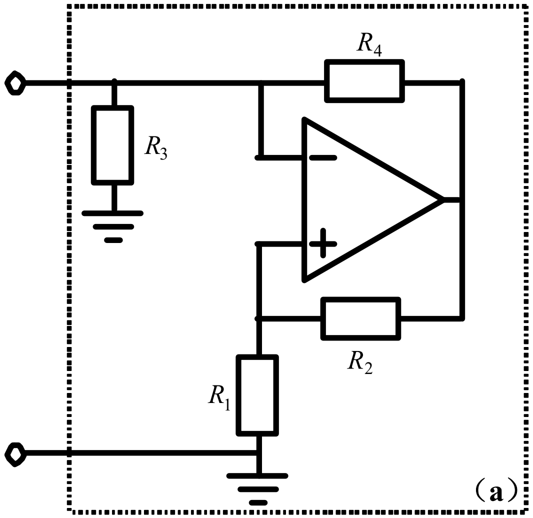
$$-I_0<i_{test}<+I_0$$

情形3：运放工作在负饱和区：

$$v_{test}=\frac{i_{test}}{G_3+G_4}-\frac{G_4}{G_3+G_4}V_{sat}$$

$$i_{test}>+I_0$$

电路设计



$$r_1=\frac{1}{G_3+G_4}$$

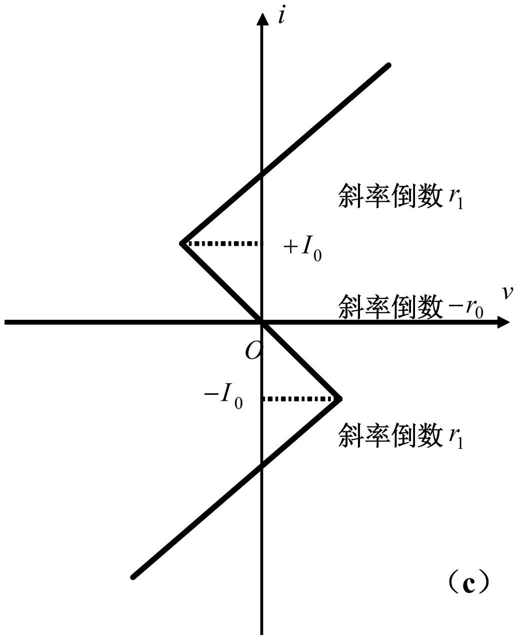
$$-r_0=\frac{1}{G_3+G_4-\dfrac{G_4}{\eta}}$$

$$I_0=\left(G_4-\eta\left(G_3+G_4\right)\right)V_{sat}$$

$$G_3+G_4=\frac{1}{r_1}$$

$$\frac{G_4}{\eta}=\frac{1}{r_1}+\frac{1}{r_0}$$

$$I_0=\eta\frac{V_{sat}}{r_0}$$



$$\eta=\frac{I_0r_0}{V_{sat}}=\frac{R_1}{R_1+R_2}$$

$$\eta<1\Rightarrow V_{sat}>r_0I_0$$

$$G_4=\eta\left(\frac{1}{r_1}+\frac{1}{r_0}\right)=\frac{I_0}{V_{sat}}\left(\frac{r_0}{r_1}+1\right)$$

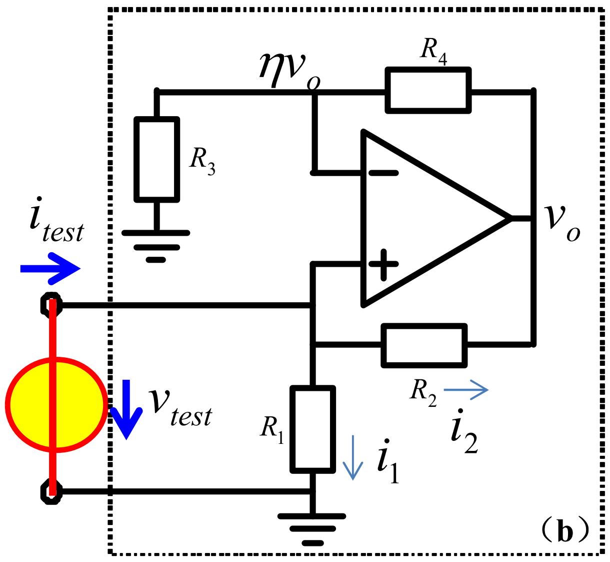
对运放提出的要求

$$G_3=\frac{1}{r_1}-G_4=\frac{1}{r_1}-\frac{I_0}{V_{sat}}\left(\frac{r_0}{r_1}+1\right)$$

$$G_3>0\Rightarrow V_{sat}>I_0\left(r_1+r_0\right)$$

不是所有负阻均可实现，对运放有要求：饱和电压，输出电流

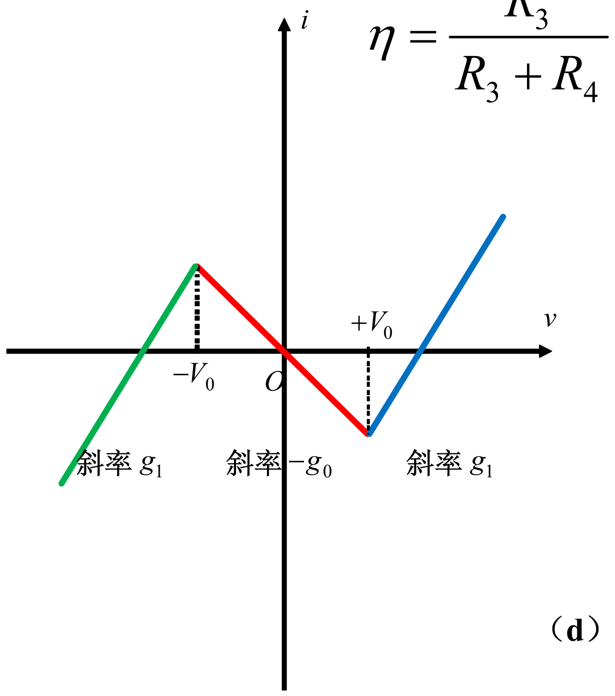
N型负阻



$$i_{test} = i_1 + i_2 = \frac{v_{test}}{R_1} + \frac{v_{test} - v_o}{R_2}$$

$$i_{test} = (G_1 + G_2)v_{test} - G_2 v_o$$

$$\eta = \frac{R_3}{R_3 + R_4}$$



情形1：运放工作在正饱和区： $v_o = +V_{sat}$

$$i_{test} = (G_1 + G_2)v_{test} - G_2 V_{sat} \qquad v_p > v_n \Rightarrow v_{test} > \eta V_{sat} = V_0$$

情形3：运放工作在负饱和区： $v_o = -V_{sat}$

$$i_{test} = (G_1 + G_2)v_{test} + G_2 V_{sat} \qquad v_p < v_n \Rightarrow v_{test} < -\eta V_{sat} = -V_0$$

情形2：运放工作在线性区

$$-V_{sat} < v_o < +V_{sat}$$

$$i_{test} = (G_1 + G_2)v_{test} - G_2 v_o \qquad v_p = v_n \Rightarrow v_{test} = \eta v_o$$

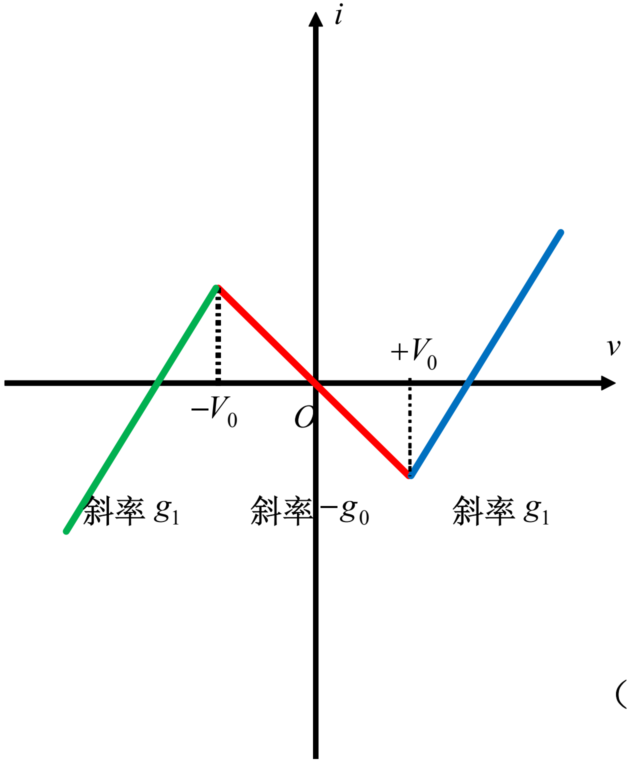
$$= (G_1 + G_2)v_{test} - G_2 \frac{v_{test}}{\eta}$$

$$-\eta V_{sat} < v_{test} < \eta V_{sat}$$

$$= \left(G_1 + G_2 - \frac{G_2}{\eta}\right)v_{test}$$

$$-V_0 < v_{test} < +V_0$$

一一对应



（d）

$$g_1 = G_1 + G_2$$

$$-g_0 = G_1 + G_2 - \frac{G_2}{\eta}$$

$$\eta V_{sat} = V_0$$

情形1：运放工作在正饱和区：

$$i_{test} = (G_1 + G_2)v_{test} - G_2 V_{sat}$$

$$v_{test} > \eta V_{sat}$$

情形2：运放工作在线性区

$$i_{test} = \left(G_1 + G_2 - \frac{G_2}{\eta}\right)v_{test}$$

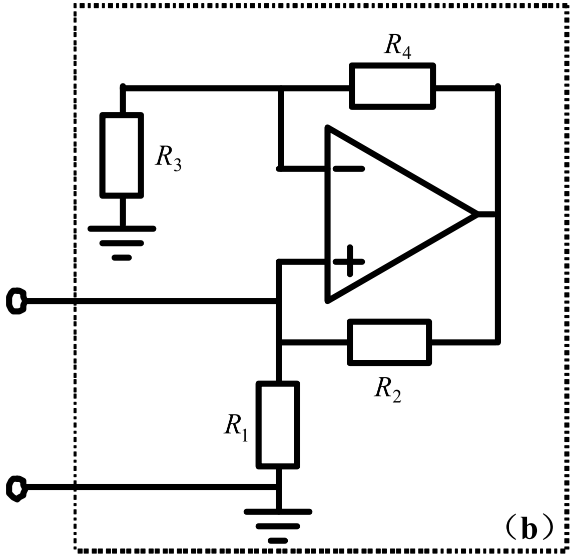
$$-\eta V_{sat} < v_{test} < \eta V_{sat}$$

情形3：运放工作在负饱和区：

$$i_{test} = (G_1 + G_2)v_{test} + G_2 V_{sat}$$

$$v_{test} < -\eta V_{sat}$$

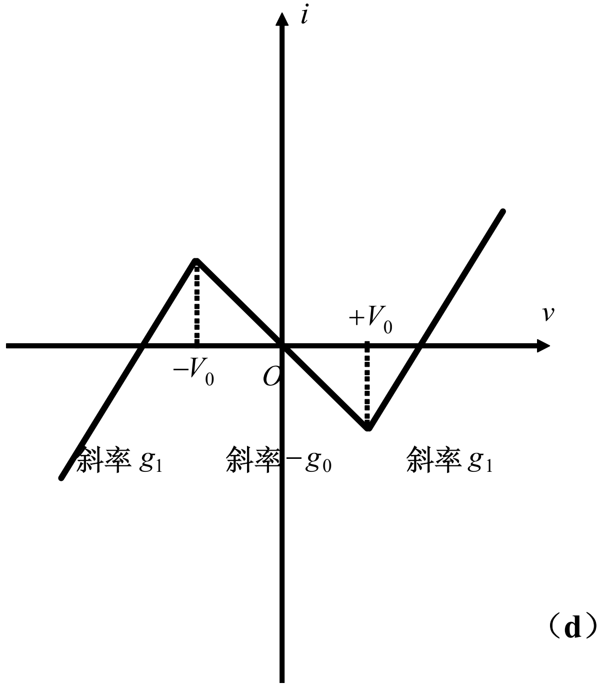
电路设计



$$g_1 = G_1 + G_2$$

$$-g_0 = G_1 + G_2 - \frac{G_2}{\eta}$$

$$\eta V_{sat} = V_0$$



❸确定**R₃R₄**电阻

$$\eta = \frac{V_0}{V_{sat}} = \frac{R_3}{R_3 + R_4}$$

$$\eta = \frac{V_0}{V_{sat}} < 1 \Rightarrow V_{sat} > V_0$$

❷确定分压系数

对运放提出的要求

$$G_2 = \eta(g_0 + g_1)$$

❹确定**R₂ R₁**电阻

$$G_1 = g_1 - G_2$$
$$= g_1 - \eta(g_0 + g_1)$$

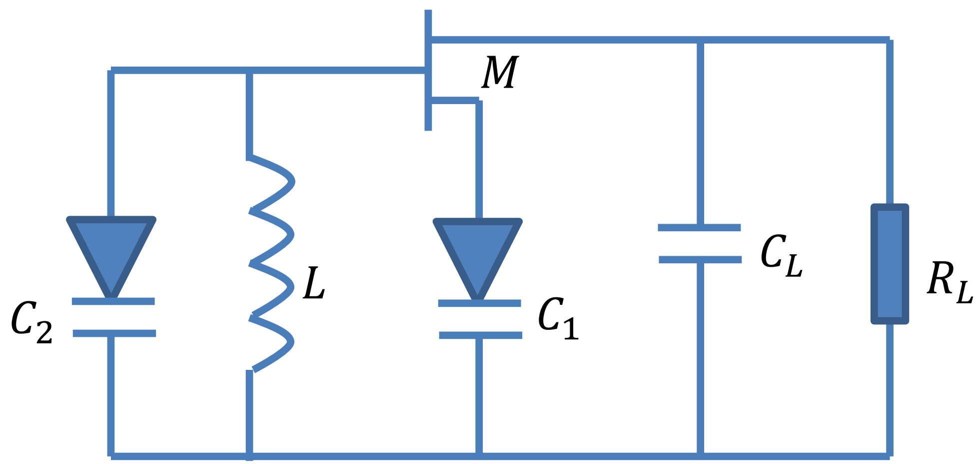
$$G_1 > 0 \Rightarrow V_{sat} > \left(\frac{g_0}{g_1} + 1\right) V_0$$

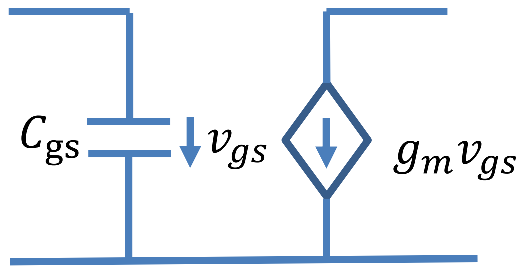
❶选运放

❺验证运放输出电流在工作范围内满足要求（可提供如此大电流）

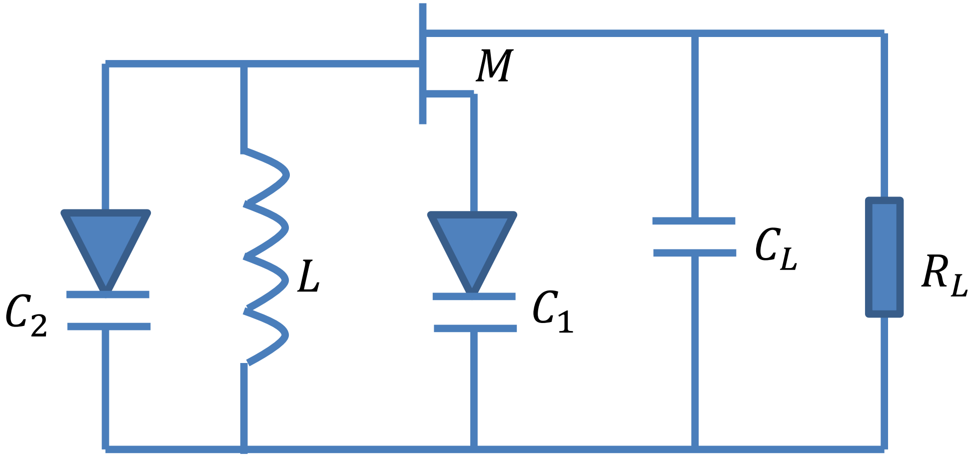
作业3 等效负阻

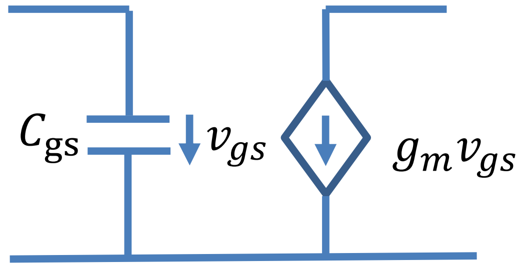
- 图示为微波频段的变容管调谐的正弦波振荡器，请用负阻振荡原理说明它是振荡器



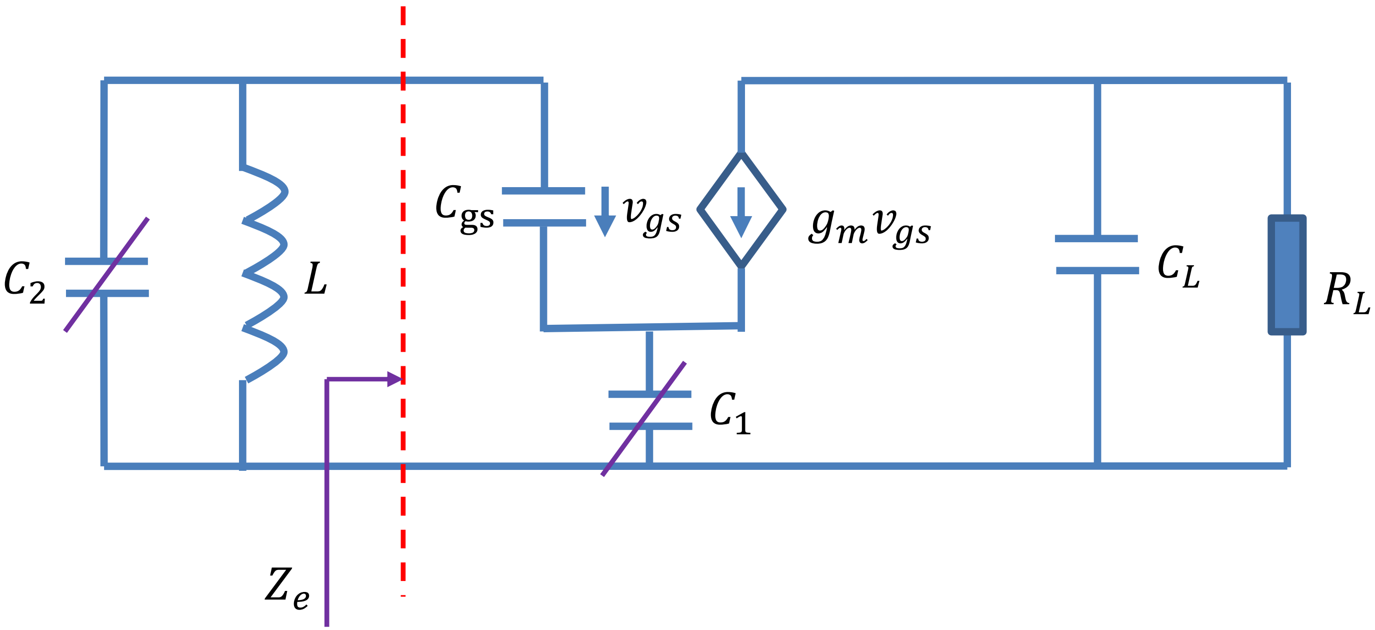


GaAs MESFET简化分析模型

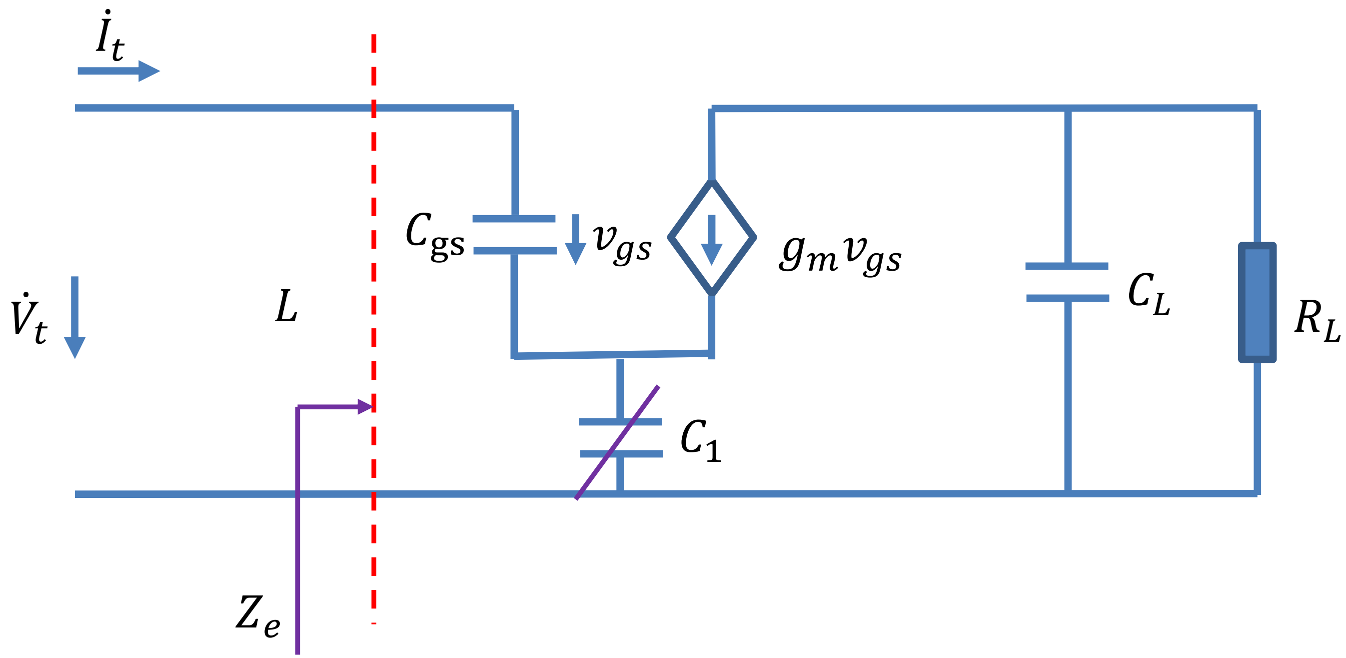




GaAs MESFET简化分析模型

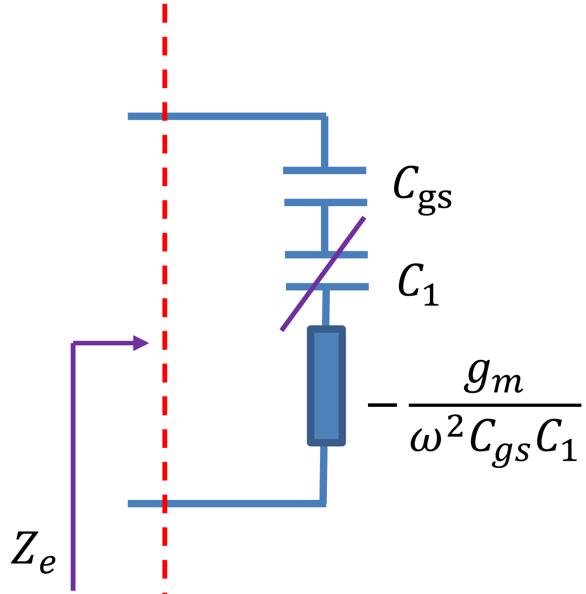


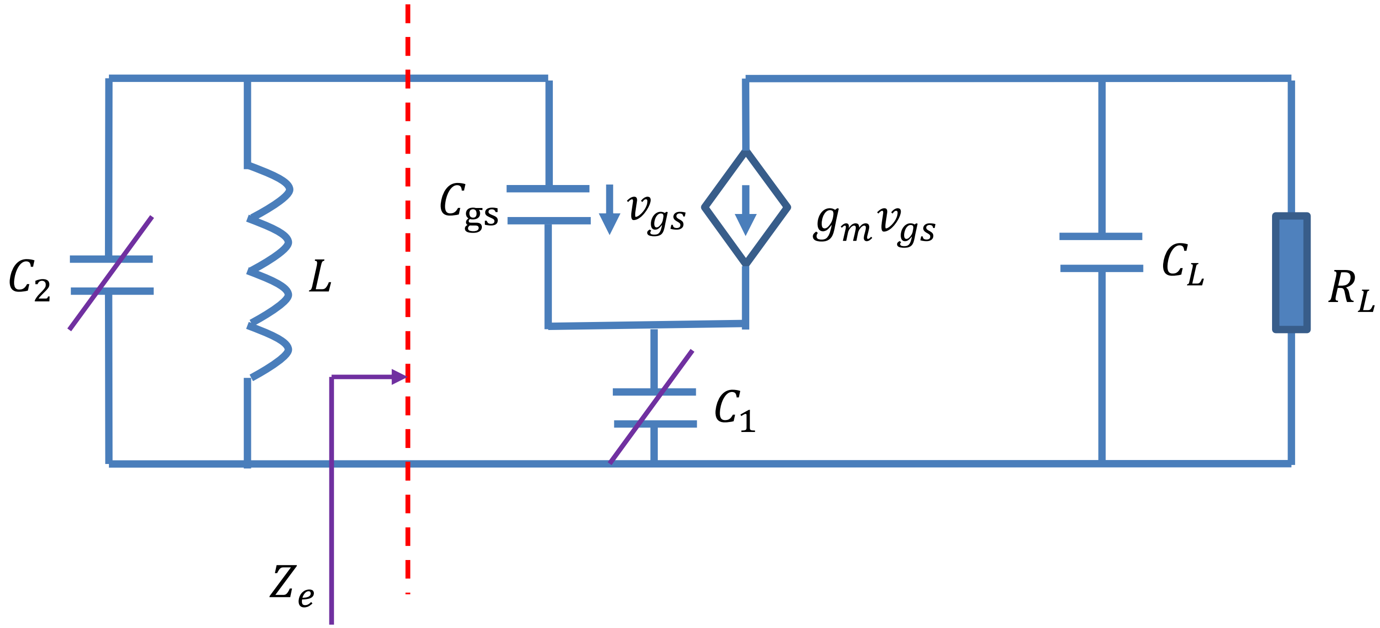
基本思路：首先找到谐振腔，证明谐振腔外并联负导或谐振腔内串联负阻



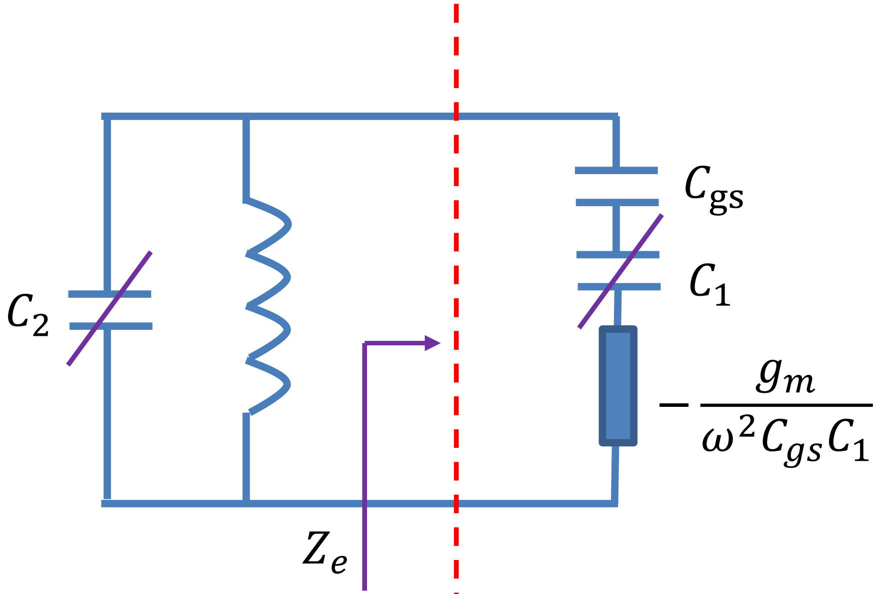
$$\dot{V}_t=\dot{I}_t\frac{1}{j\omega C_{gs}}+\left(\dot{I}_t+g_m\dot{I}_t\frac{1}{j\omega C_{gs}}\right)\frac{1}{j\omega C_1}$$

$$Z_e=\frac{\dot{V}_t}{\dot{I}_t}=\frac{1}{j\omega C_{gs}}+\left(1+g_m\frac{1}{j\omega C_{gs}}\right)\frac{1}{j\omega C_1}$$
$$=\frac{1}{j\omega C_{gs}}+\frac{1}{j\omega C_1}+g_m\frac{1}{j\omega C_{gs}}\frac{1}{j\omega C_1}$$
$$=\frac{1}{j\omega C_{gs}}+\frac{1}{j\omega C_1}-\frac{g_m}{\omega^2 C_{gs}C_1}$$

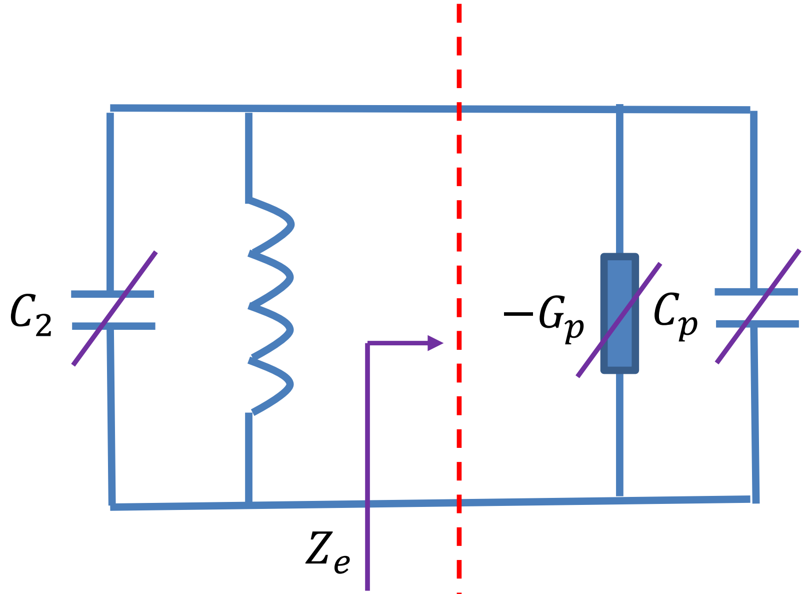




C_2
L
C_{gs}
v_{gs}
$g_m v_{gs}$
C_L
R_L
C_1
Z_e

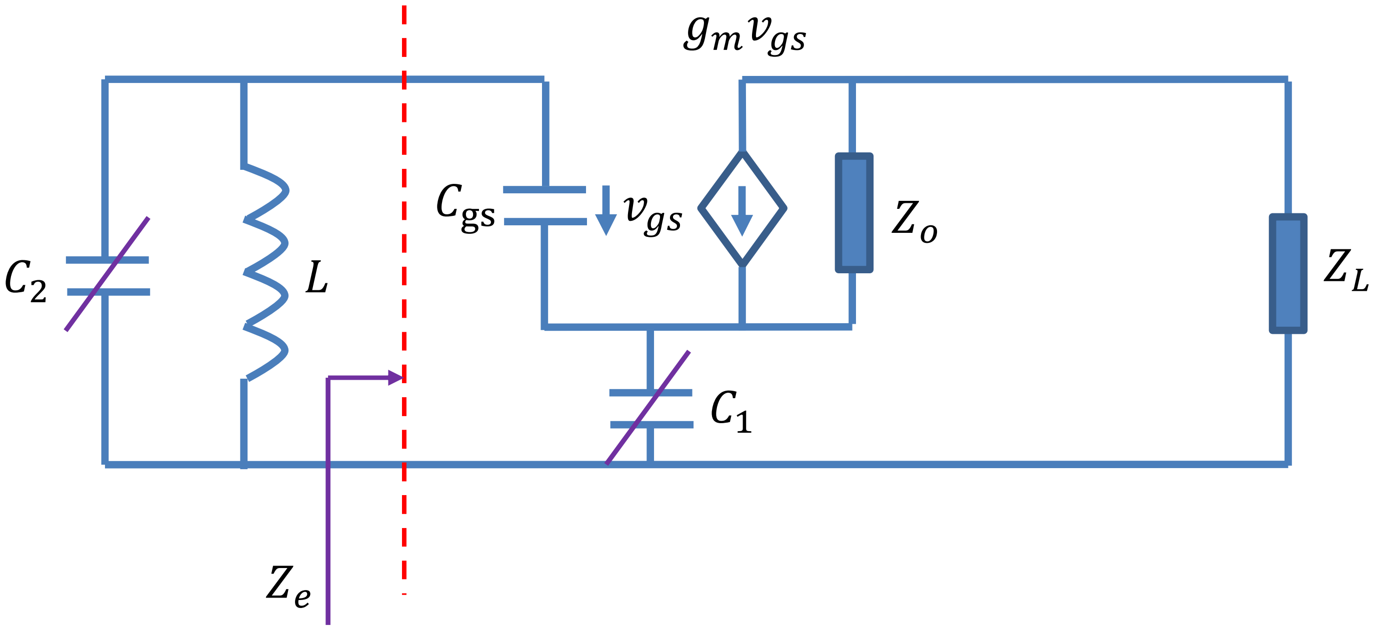


C_2
C_{gs}
C_1
$-\frac{g_m}{\omega^2 C_{gs} C_1}$
Z_e



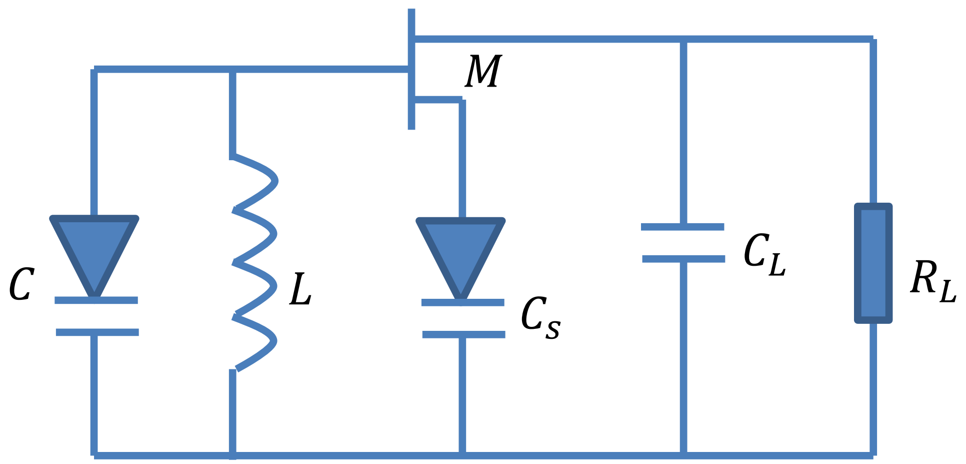
C_2
$-G_p$
C_p
Z_e

三点式的变种



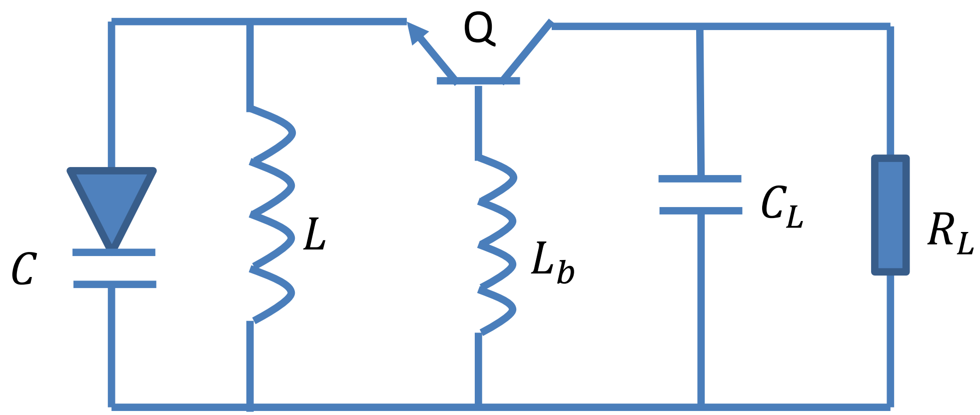
$Z_o \to \infty$时， Z_L和恒流源串联，可视为短路，构成三点式结构

微波频段常见三点式振荡器变种

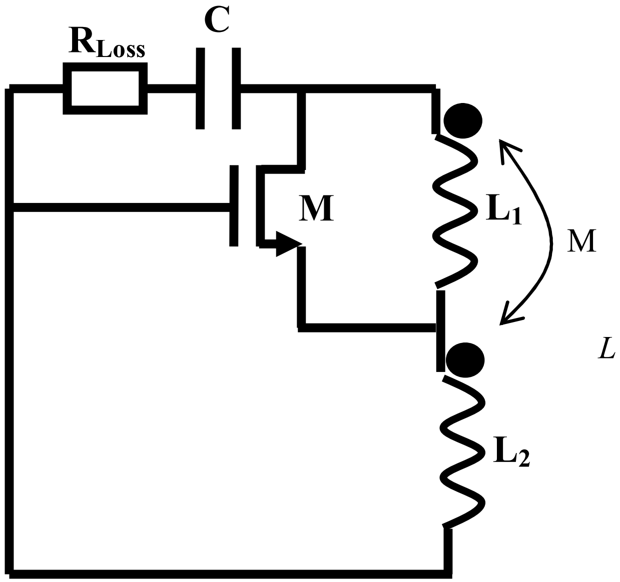


微波频段大多不用正反馈原理，多用负阻原理进行分析；微波频段大多采用由测量获得的二端口网络参量电路模型进行振荡器设计

微波频段考虑的寄生效应太多，基于器件结构的电路模型过于复杂，因而一般用二端口网络参量这种二端口戴维南或诺顿等效电路模型进行分析和设计



基本思路：首先找到谐振腔，证明谐振腔外并联负导或谐振腔内串联负阻



$$L = L_1 + 2M + L_2 = (N_1 + N_2)^2 \Xi$$

作业4 起振条件分析

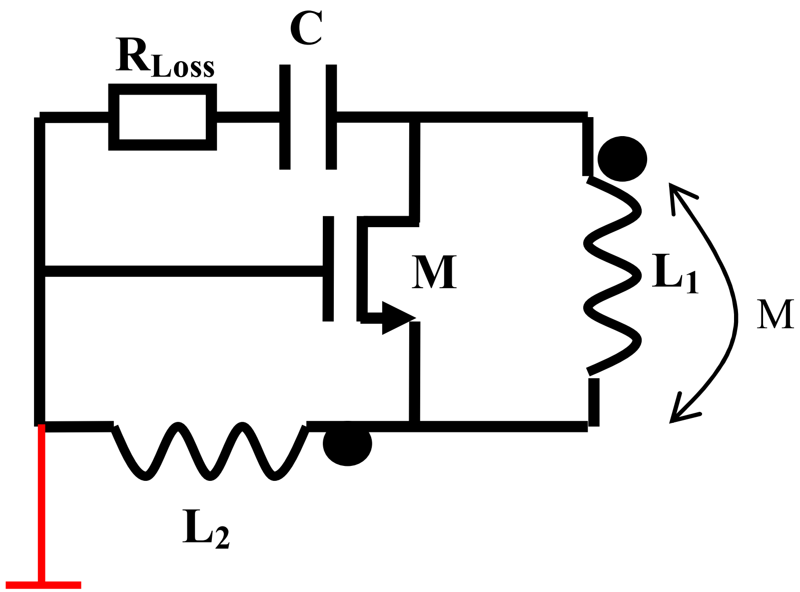
- 某同学在设计哈特莱正弦波振荡器时，首先将一个在磁环上绕了**N**圈制成的电感**L**（**=N²**Ξ，Ξ为磁环磁导）中间引出一个抽头，接到晶体管源极上，电感的两端则分别接在晶体管的漏极和栅极，如图所示。由于一分为二的两个电感绕在同一个磁环上，它们之间具有全耦合关系，即 $M=\sqrt{L_1L_2}$，其中 $L_1 = N_1^2\Xi$，$L_2 = N_2^2\Xi$，这里$\mathbf{N_1}$,$\mathbf{N_2}$和**N**为电感在磁环上的绕线匝数，$\mathbf{N=N_1+N_2}$。假设电路中的所有能量损耗全部折合等效为和电容串联的电阻$\mathbf{R_{Loss}}$，且 $Q=\frac{1}{R_{Loss}}\sqrt{\frac{L}{C}} >> 1$。此时图中晶体管可以建模为理想压控流源，其跨导增益为$\mathbf{g_m}$。

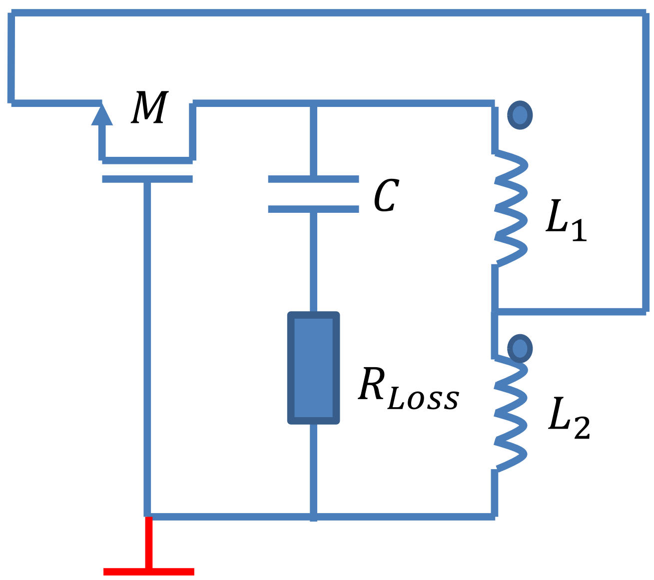
（**1**）请分析该振荡器，用图示的已知电路元件参量**L**、**M**、**C**、$\mathbf{R_{Loss}}$、$\mathbf{g_m}$表述该正弦波振荡器的振荡频率和起振条件。

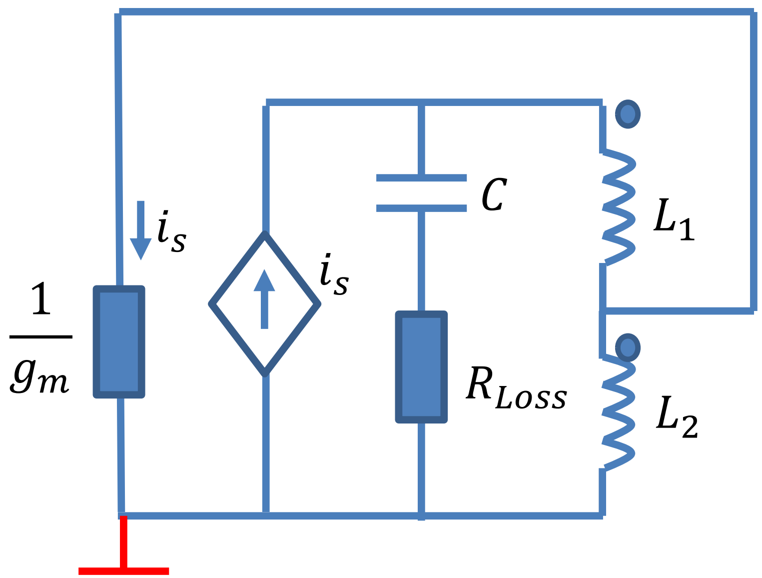
（**2**）在实际电路设计中，我们往往期望低功耗设计，因而希望直流偏置电流足够的小，换句话说，希望和直流偏置电流成正比关系的跨导$\mathbf{g_m}$应足够的小，该振荡器仍然可以起振。请分析图示振荡电路的电感中间抽头如何引出（即接入系数$\mathbf{p=N_2/N}$ 如何取值），该电路可以在较小的$\mathbf{g_m}$（对应较小的直流偏置电流）条件下就可以起振。

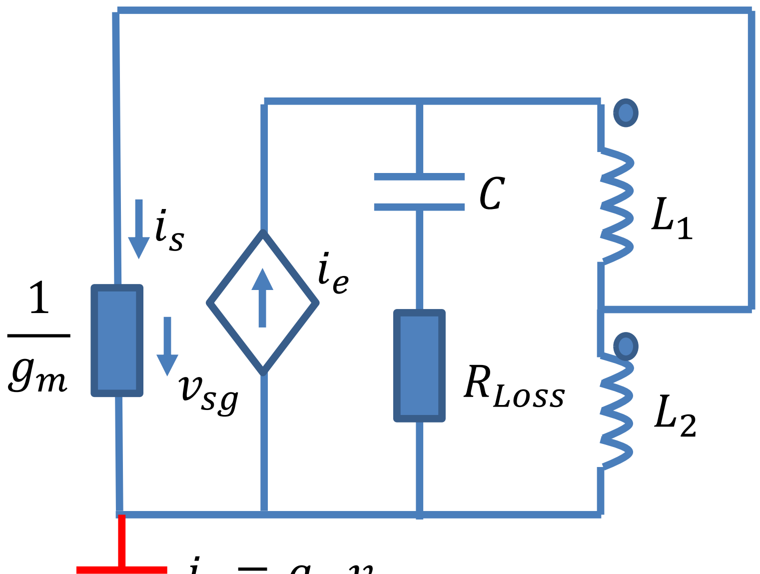
正反馈原理

把晶体管建模为放大器
需要考虑晶体管组态
三点式振荡结构和地无关
人为添加地（任意位置均可）

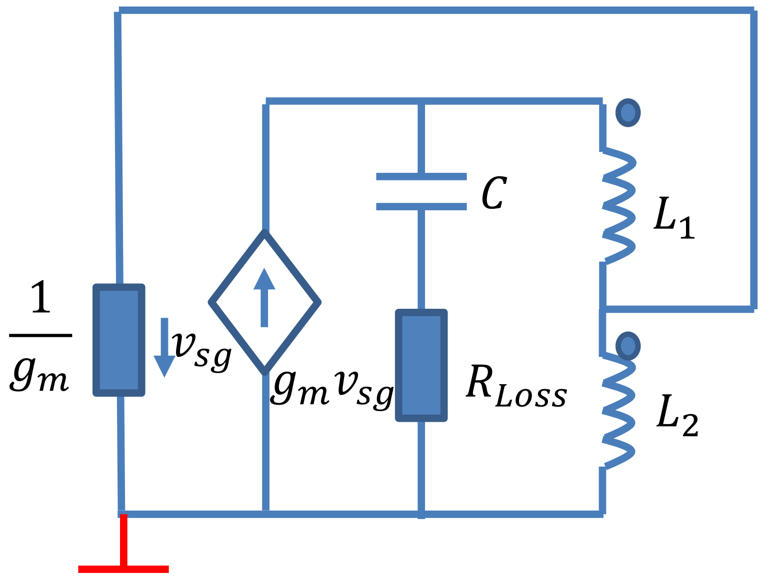


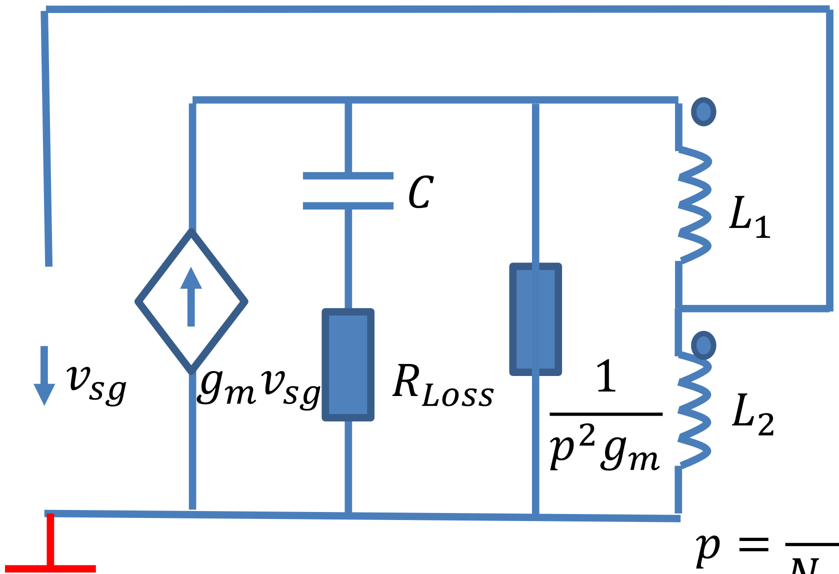




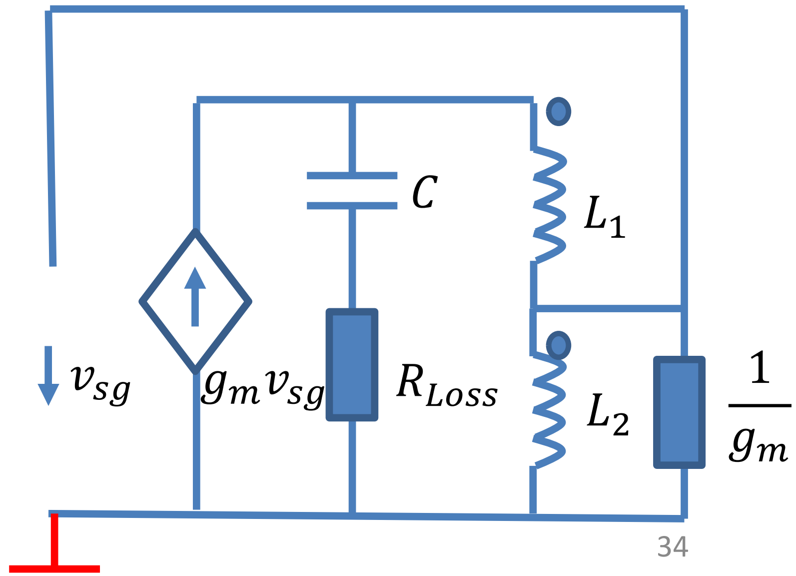


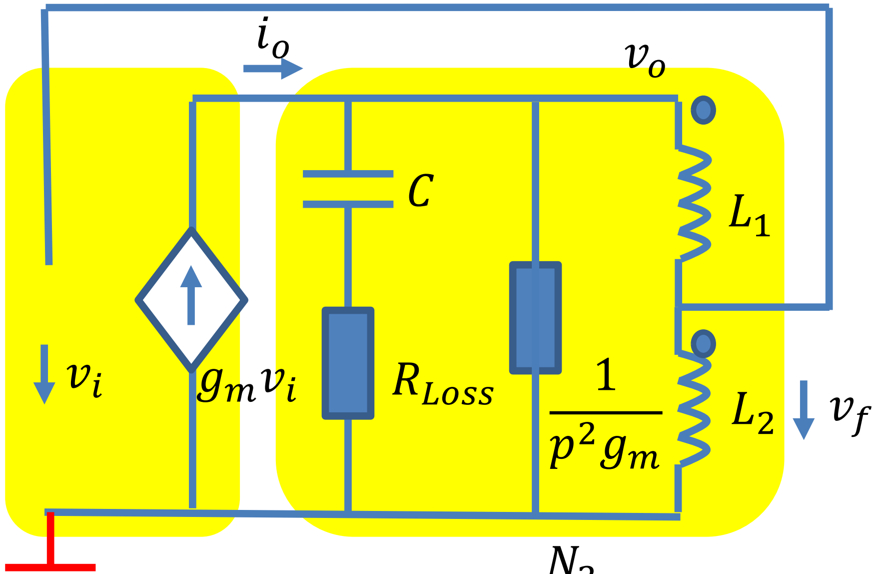
$i_s = g_m v_{sg}$





$p = \frac{N_2}{N_1 + N_2}$





$$p = \frac{N_2}{N_1 + N_2}$$

$$A = \frac{\dot{I}_o}{\dot{V}_i} = g_m$$

$$F = \frac{\dot{V}_f}{\dot{I}_o} = \frac{\dot{V}_f}{\dot{V}_o}\frac{\dot{V}_o}{\dot{I}_o} = p\left(L\|\frac{1}{p^2 g_m}\|(R_{Loss}\text{串}C)\right)$$

$$= p\left(\frac{1}{p^2 g_m + \frac{1}{j\omega L} + \frac{1}{R_{Loss} + \frac{1}{j\omega C}}}\right)$$

$$= p\left(\frac{1}{p^2 g_m + \frac{1}{j\omega L} + \frac{j\omega C}{1 + j\omega C R_{Loss}}}\right)$$

$$AF > 1$$

$$p g_m\left(\frac{1}{p^2 g_m + \frac{1}{j\omega L} + \frac{j\omega C}{1 + j\omega C R_{Loss}}}\right) > 1$$

$$p g_m\left(\frac{1}{p^2 g_m + \frac{(\omega C)^2 R_{Loss}}{1 + (\omega C R_{Loss})^2} + \frac{1}{j\omega L} + \frac{j\omega C}{1 + (\omega C R_{Loss})^2}}\right) > 1$$

起振条件

$$pg_m\left(\frac{1}{p^2g_m+\frac{(\omega C)^2R_{Loss}}{1+(\omega CR_{Loss})^2}+\frac{1}{j\omega L}+\frac{j\omega C}{1+(\omega CR_{Loss})^2}}\right)>1$$

$$p=\frac{N_2}{N_1+N_2}$$

相位条件（虚部条件）决定振荡频率

$$\frac{1}{j\omega L}+\frac{j\omega C}{1+(\omega CR_{Loss})^2}=0\qquad \omega_0^2=\frac{1}{LC-R_{Loss}^2C^2}\qquad \omega_0=\frac{1}{\sqrt{LC-R_{Loss}^2C^2}}$$

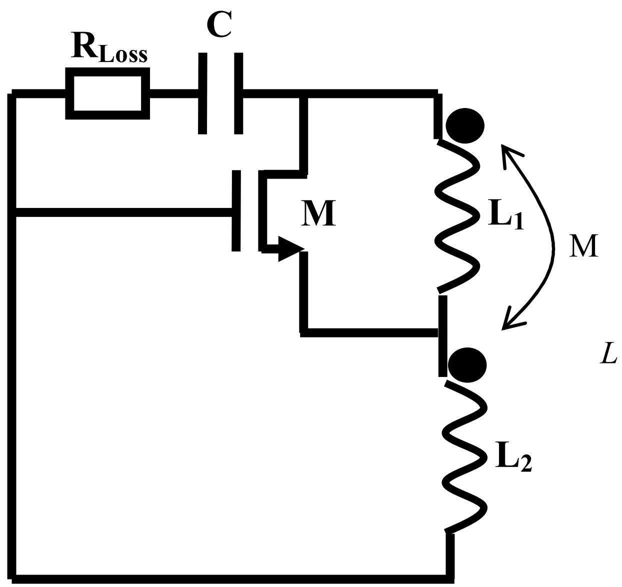
幅度条件（实部条件）决定振荡幅度（是否起振）

$$pg_m\left(\frac{1}{p^2g_m+\frac{(\omega_0 C)^2R_{Loss}}{1+(\omega_0 CR_{Loss})^2}}\right)>1\qquad pg_m>p^2g_m+\frac{(\omega_0 C)^2R_{Loss}}{1+(\omega_0 CR_{Loss})^2}$$

$$g_m>\frac{1}{p(1-p)}\frac{(\omega_0 C)^2R_{Loss}}{1+(\omega_0 CR_{Loss})^2}=\frac{1}{\frac{N_2}{N_1+N_2}\frac{N_1}{N_1+N_2}}\frac{\frac{C^2}{LC-R_{Loss}^2C^2}R_{Loss}}{1+\frac{R_{Loss}^2C^2}{LC-R_{Loss}^2C^2}}$$

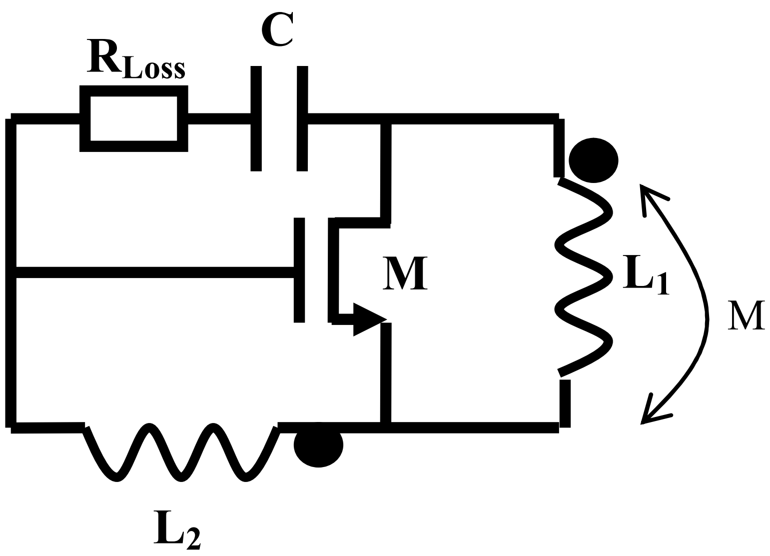
$$=\frac{1}{\frac{N_2}{N_1+N_2}\frac{N_1}{N_1+N_2}}\frac{R_{Loss}C^2}{LC}=\frac{1}{\frac{N_2}{N_1+N_2}\frac{N_1}{N_1+N_2}}\frac{R_{Loss}C}{L}=\frac{R_{Loss}C}{M}$$

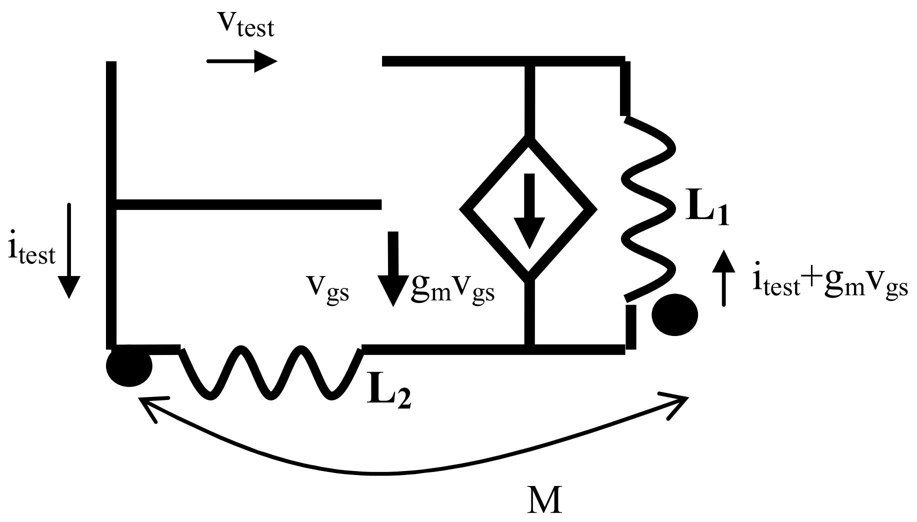
负阻原理

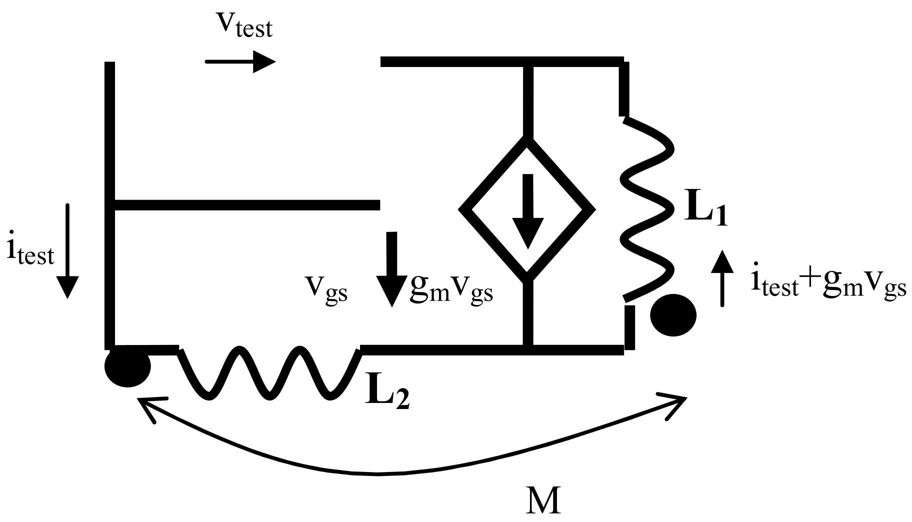


$$L = L_1 + 2M + L_2 = (N_1 + N_2)^2 \Xi$$

加流求压







$$\dot{V}_{gs} = j\omega L_2 \dot{I}_{test} + j\omega M\left(\dot{I}_{test} + g_m \dot{V}_{gs}\right)$$

$$\dot{V}_{gs} = \frac{j\omega\left(L_2 + M\right)}{1 - j\omega M g_m}\dot{I}_{test}$$

$$\dot{V}_1 = j\omega L_1\left(\dot{I}_{test} + g_m \dot{V}_{gs}\right) + j\omega M \dot{I}_{test}$$

$$\dot{V}_1 = j\omega\left(L_1 + M\right)\dot{I}_{test} + j\omega L_1 g_m \frac{j\omega\left(L_2 + M\right)}{1 - j\omega M g_m}\dot{I}_{test}$$

$$\dot{V}_{test} = \dot{V}_{gs} + \dot{V}_1 = \frac{j\omega\left(L_2 + M\right)}{1 - j\omega M g_m}\dot{I}_{test} + j\omega\left(L_1 + M\right)\dot{I}_{test} + j\omega L_1 g_m \frac{j\omega\left(L_2 + M\right)}{1 - j\omega M g_m}\dot{I}_{test}$$

$$Z_{in} = \frac{\dot{V}_{test}}{\dot{I}_{test}} = j\omega\left(L_1 + M\right) + \left(1 + j\omega L_1 g_m\right)\frac{j\omega\left(L_2 + M\right)}{1 - j\omega M g_m}$$

$$= j\omega\left(L_1 + 2M + L_2\right) - \frac{\omega^2\left(L_1 + M\right)\left(L_2 + M\right)g_m}{1 - j\omega M g_m}$$

$$Z_{in} = j\omega(L_1 + 2M + L_2) - \frac{\omega^2 (L_1 + M)(L_2 + M) g_m}{1 - j\omega M g_m}$$

$$= j\omega(N_1^2\Xi + 2N_1N_2\Xi + N_2^2\Xi) - \frac{\omega^2 (N_1^2\Xi + N_1N_2\Xi)(N_2^2\Xi + N_1N_2\Xi) g_m}{1 - j\omega M g_m}$$

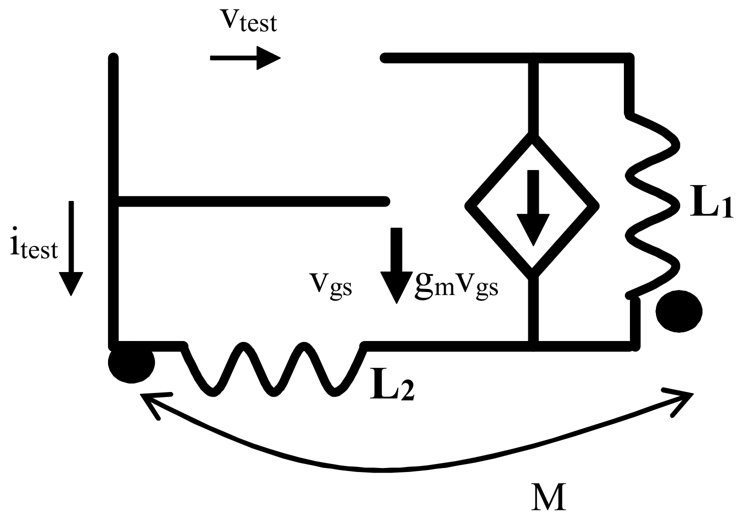
$$= j\omega(N_1 + N_2)^2 \Xi - \frac{\omega^2 N_1N_2\Xi(N_1 + N_2)^2 \Xi g_m}{1 - j\omega M g_m}$$

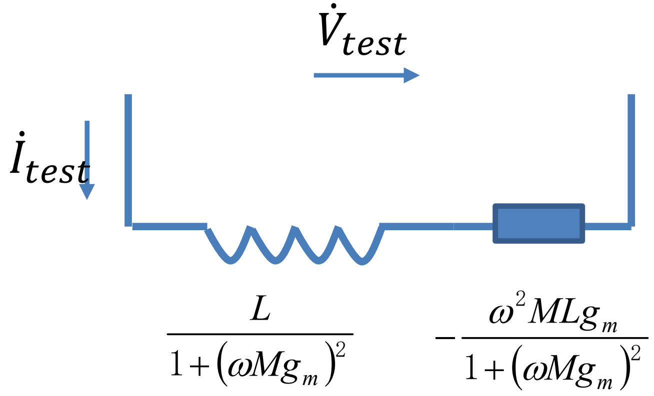
$$= j\omega L - \frac{\omega^2 M L g_m}{1 - j\omega M g_m}$$

$$= j\omega L - \frac{\omega^2 M L g_m (1 + j\omega M g_m)}{1 + (\omega M g_m)^2}$$

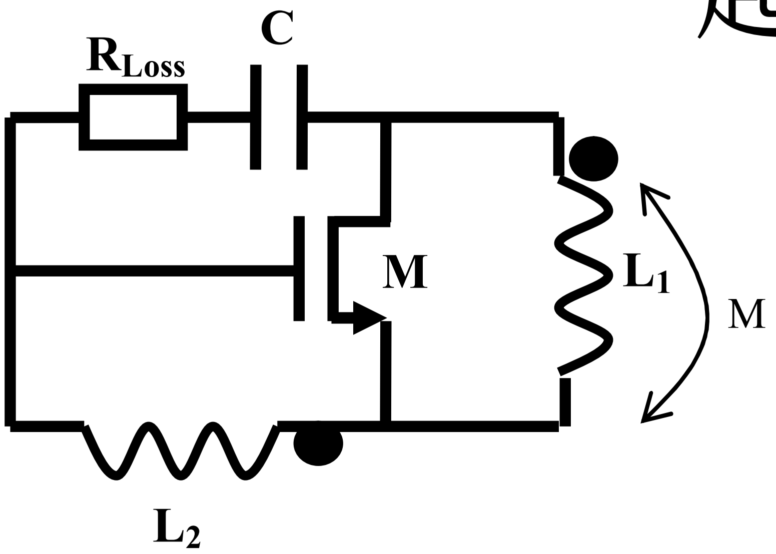
$$= j\omega L \frac{1}{1 + (\omega M g_m)^2} - \frac{\omega^2 M L g_m}{1 + (\omega M g_m)^2}$$

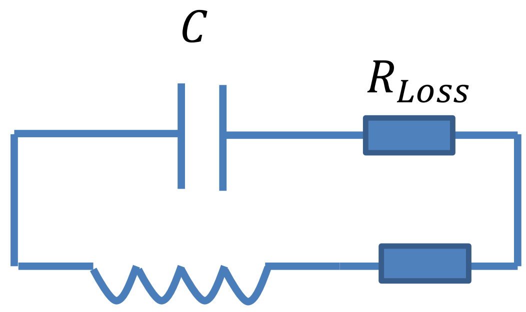
等效负阻





起振条件





$$\frac{L}{1+(\omega M g_m)^2} \quad - \frac{\omega^2 M L g_m}{1+(\omega M g_m)^2}$$

起振条件 $$\frac{\omega^2 M L g_m}{1+\left(\omega M g_m\right)^2} > R_{Loss}$$

振荡频率 $$\frac{1}{\omega_0^2} = \frac{LC}{1+\left(\omega_0 M g_m\right)^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC - g_m^2 M^2}}$$

$$\frac{\dfrac{1}{LC - g_m^2 M^2} M L g_m}{1+\dfrac{g_m^2 M^2}{LC - g_m^2 M^2}} > R_{Loss}$$

振荡频率和增益有关，增益代入满足平衡条件的准线性增益

$$g_m > \frac{R_{Loss} C}{M}$$

$$\omega_0 \overset{g_m = \frac{R_{Loss} C}{M}}{\triangleq} \frac{1}{\sqrt{LC - R_{Loss}^2 C^2}}$$

在实际电路设计中，我们往往期望低功耗设计，因而希望直流偏置电流足够的小，换句话说，希望和直流偏置电流成正比关系的跨导$\mathbf{g_m}$应足够的小，该振荡器仍然可以起振。请分析图示振荡电路的电感中间抽头如何引出（即接入系数$\mathbf{p=N_2/N}$ 如何取值），该电路可以在较小的$\mathbf{g_m}$（对应较小的直流偏置电流）条件下就可以起振。

$$g_m > \frac{R_{Loss}C}{M} = \frac{R_{Loss}C}{p(1-p)L} = \frac{R_{Loss}}{p(1-p)Z_0^2} = \frac{G_p}{p(1-p)}$$

恢复为三点式振荡器起振条件的一般形式

$G_p = \frac{R_{Loss}}{Z_0^2}$是电路中的损耗折合到bc（gd）端口的等效导纳

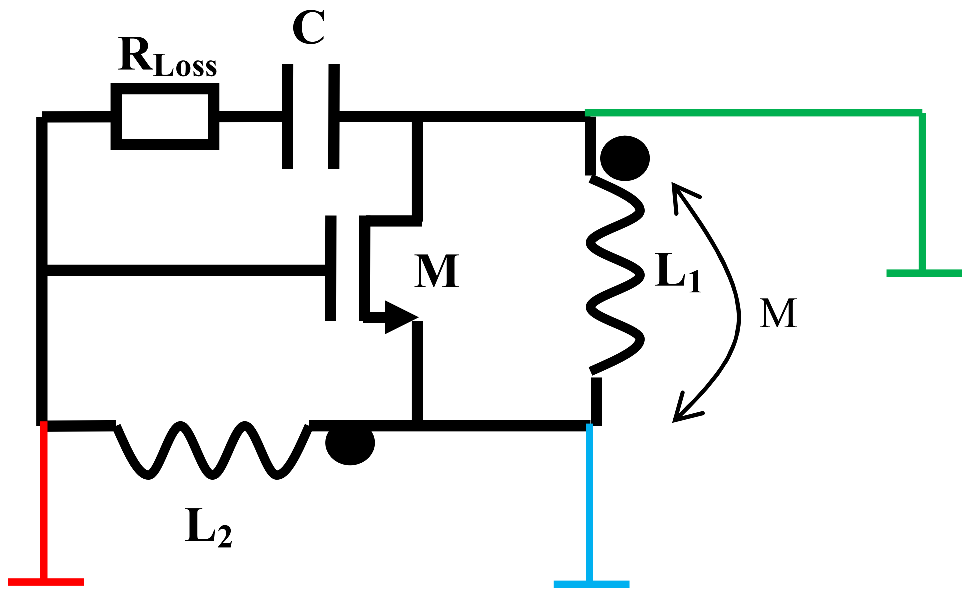
$p = \frac{N_2}{N_1+N_2}$是三点式（电容或）电感接入系数

$$p(1-p) \le \frac{1}{4} \qquad g_m > \frac{G_p}{p(1-p)} \ge 4G_p$$

显然接入系数$p = \frac{N_2}{N_1+N_2} = 0.5$可使得跨导增益不必很大即可起振，属低功耗设计方案

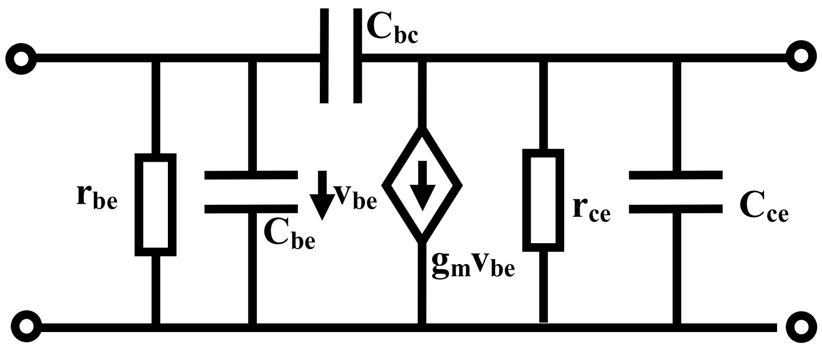
这是三点式振荡器设计的一般初始设计方案：取部分介入系数为0.5

自行练习
地的选择不同，不影响最终结论

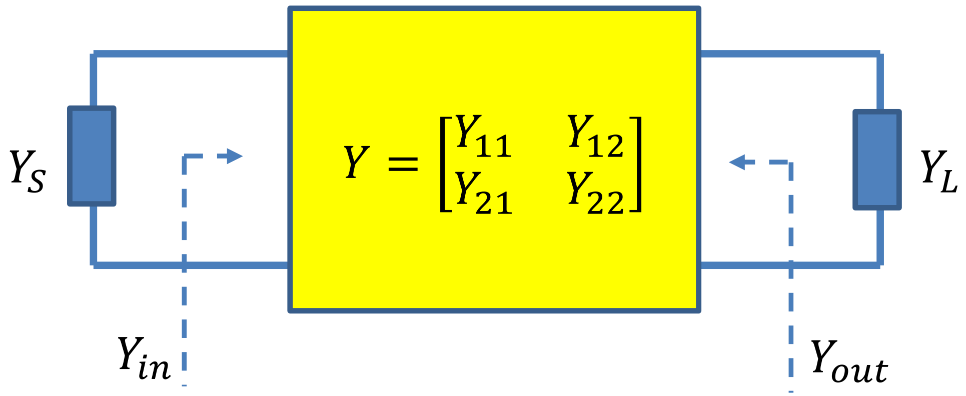


选作1：绝对稳定区

- 二端口网络所谓绝对稳定，指的是其两个端口端接无源负载时，另一个端口的看入阻抗也是无源的。绝对稳定的二端口网络，在端接任意无源负载时，不会出现振荡现象。求图示晶体管二端口网络的绝对稳定区（不会自激振荡的工作频率范围）。



绝对稳定

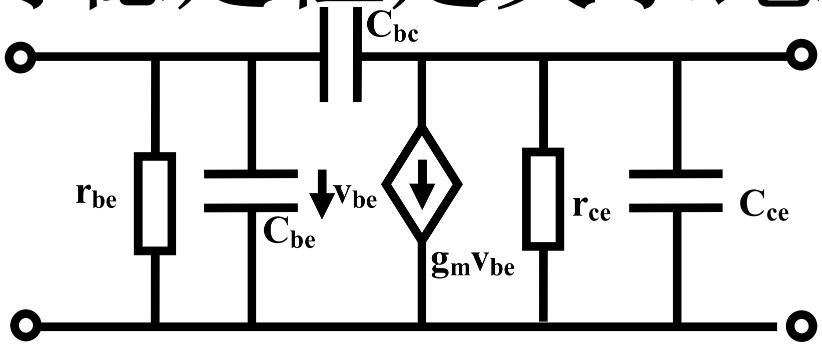


如果$\mathrm{Re}Y_L \geq 0$，则必有$\mathrm{Re}Y_{in} \geq 0$

如果$\mathrm{Re}Y_S \geq 0$，则必有$\mathrm{Re}Y_{out} \geq 0$

绝对稳定：无源负载情况下不会出现等效负阻（负导），于是在进行放大器共轭匹配调试时，绝对不会出现振荡不稳定现象，则称之为绝对稳定

根据绝对稳定性定义求绝对稳定区



$$\mathbf{Y}=\begin{bmatrix} g_{be} & 0 \\ g_m & g_{ce} \end{bmatrix}+\begin{bmatrix} j\omega C_{be}+j\omega C_{bc} & -j\omega C_{bc} \\ -j\omega C_{bc} & j\omega C_{ce}+j\omega C_{bc} \end{bmatrix}$$

$$\mathbf{Y}=\begin{bmatrix} g_{be}+j\omega C_{be}+j\omega C_{bc} & -j\omega C_{bc} \\ g_m-j\omega C_{bc} & g_{ce}+j\omega C_{ce}+j\omega C_{bc} \end{bmatrix}$$

$$Y_{in}=Y_{11}-\frac{Y_{12}Y_{21}}{Y_{22}+Y_L}=g_{be}+j\omega C_{be}+j\omega C_{bc}+\frac{j\omega C_{bc}(g_m-j\omega C_{bc})}{g_{ce}+j\omega C_{ce}+j\omega C_{bc}+G_L+jB_L}$$

$$=g_{be}+j\omega C_{be}+j\omega C_{bc}+\frac{(\omega C_{bc})^2+j\omega C_{bc}g_m}{g_{ce}+G_L+j(\omega C_{ce}+\omega C_{bc}+B_L)}$$

$$\forall B_L \qquad G_L\ge 0 \qquad \Longrightarrow \qquad \mathrm{Re}Y_{in}\ge 0$$

$$Y_{in} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22}+Y_L} = g_{be} + j\omega C_{be} + j\omega C_{bc} + \frac{(\omega C_{bc})^2 + j\omega C_{bc} g_m}{g_{ce} + G_L + j(\omega C_{ce} + \omega C_{bc} + B_L)}$$

$$\mathrm{Re} Y_{in} = g_{be} + \frac{(\omega C_{bc})^2 (g_{ce} + G_L) + \omega C_{bc} g_m (\omega C_{ce} + \omega C_{bc} + B_L)}{(g_{ce} + G_L)^2 + (\omega C_{ce} + \omega C_{bc} + B_L)^2}$$

$$= \frac{g_{be}(\omega C_{ce} + \omega C_{bc} + B_L)^2 + \omega C_{bc} g_m (\omega C_{ce} + \omega C_{bc} + B_L) + (\omega C_{bc})^2 (g_{ce} + G_L) + g_{be}(g_{ce} + G_L)^2}{(g_{ce} + G_L)^2 + (\omega C_{ce} + \omega C_{bc} + B_L)^2}$$

$$\forall B_L \qquad G_L \geq 0 \qquad \Longrightarrow \qquad \mathrm{Re} Y_{in} \geq 0$$

$$\Delta = (\omega C_{bc} g_m)^2 - 4 g_{be}\left((\omega C_{bc})^2 (g_{ce} + G_L) + g_{be}(g_{ce} + G_L)^2\right) \leq 0$$

$$\Delta = (\omega C_{bc})^2 \left(g_m^2 - 4 g_{be}(g_{ce} + G_L)\right) - 4 g_{be}^2 (g_{ce} + G_L)^2 \leq 0$$

$$(\omega C_{bc})^2 \left(g_m^2 - 4 g_{be}(g_{ce} + G_L)\right) \leq 4 g_{be}^2 (g_{ce} + G_L)^2$$

$$(\omega C_{bc})^2 \leq \frac{4 g_{be}^2 (g_{ce} + G_L)^2}{g_m^2 - 4 g_{be}(g_{ce} + G_L)}$$

$$\forall B_L \qquad G_L \geq 0 \qquad \Longrightarrow \qquad \mathrm{Re}Y_{in} \geq 0$$

绝对稳定区

$$(\omega C_{bc})^2 \leq \frac{4g_{be}^2(g_{ce}+G_L)^2}{g_m^2 - 4g_{be}(g_{ce}+G_L)}$$

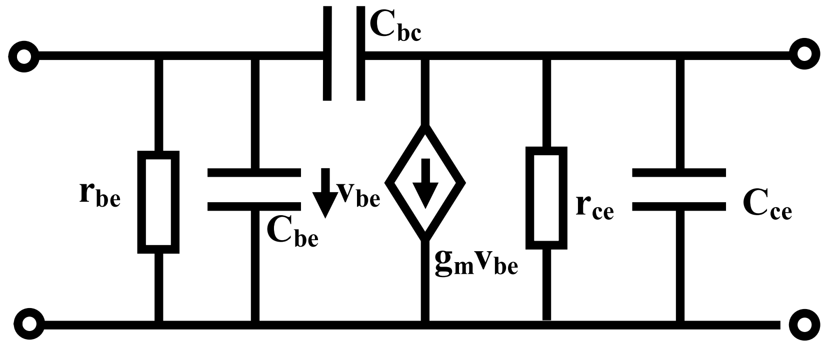
$$\omega \leq \frac{2g_{be}(g_{ce}+G_L)}{C_{bc}\sqrt{g_m^2 - 4g_{be}(g_{ce}+G_L)}}$$

$$\frac{2g_{be}(g_{ce}+G_L)}{C_{bc}\sqrt{g_m^2 - 4g_{be}(g_{ce}+G_L)}} \geq \frac{2g_{be}g_{ce}}{C_{bc}\sqrt{g_m^2 - 4g_{be}g_{ce}}} = \omega_{us}$$

只要$\boldsymbol{\omega} \leq \boldsymbol{\omega_{us}}$，则 $\boldsymbol{\omega} \leq \frac{2g_{be}(g_{ce}+G_L)}{C_{bc}\sqrt{g_m^2-4g_{be}(g_{ce}+G_L)}}$恒成立

于是 $\boldsymbol{\omega} \leq \boldsymbol{\omega_{us}}$ 为绝对稳定区

BJT晶体管核心模型的绝对稳定区



$$\omega \le \omega_{us} = \frac{2g_{be}g_{ce}}{C_{bc}\sqrt{g_m^2 - 4g_{be}g_{ce}}}$$

CE组态不稳定的来源是C_{bc}

- 晶体管核心模型
  - 绝对稳定区范围和C_{bc}成反比关系，C_{bc}=0时，绝对稳定区为全频带：没有输出到输入的反馈，则没有产生负阻的可能性
  - 如果网络本身无源，则全频带绝对稳定
    - 不稳定是因为有源，但有源未必不稳定 $g_m^2 \le 4g_{be}g_{ce}$

习题10.9给出的一般性结论
和前述推导相同的过程，非特定网络

- 满足如下条件的网络称为绝对稳定网络

网络参量矩阵$\boldsymbol{P} = \mathbf{Z}, \mathbf{Y}, \mathbf{h}, \mathbf{g}$

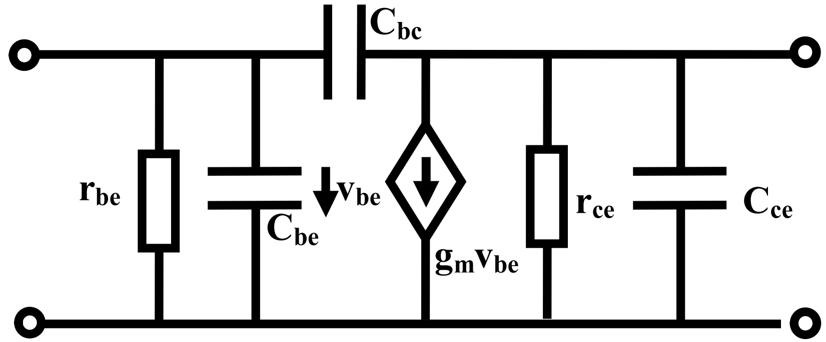
$\mathrm{Re}P_{11} \geq 0$　　输出开路或短路时，输入阻抗或导纳必须无源

$\mathrm{Re}P_{22} \geq 0$　　输入开路或短路时，输出阻抗或导纳必须无源

$$k = \frac{2\mathrm{Re}P_{11}\mathrm{Re}P_{22} - \mathrm{Re}(P_{12}P_{21})}{|P_{12}P_{21}|} \geq 1$$

罗莱特稳定性系数：确定二端口网络稳定性的一个参量

Y参量
稳定性系数
绝对稳定区



$$\mathrm{Y}=\begin{bmatrix} g_{be}+j\omega C_{be}+j\omega C_{bc} & -j\omega C_{bc} \\ g_m-j\omega C_{bc} & g_{ce}+j\omega C_{ce}+j\omega C_{bc} \end{bmatrix}$$

$$\mathrm{Re}Y_{11}=g_{be}>0 \qquad \mathrm{Re}Y_{22}=g_{ce}>0$$

$$k=\frac{2\mathrm{Re}Y_{11}\mathrm{Re}Y_{22}-\mathrm{Re}(Y_{12}Y_{21})}{|Y_{12}Y_{21}|}=\frac{2g_{be}g_{ce}+\omega^2C_{bc}^2}{\omega C_{bc}\sqrt{g_m^2+\omega^2C_{bc}^2}}\geq 1$$

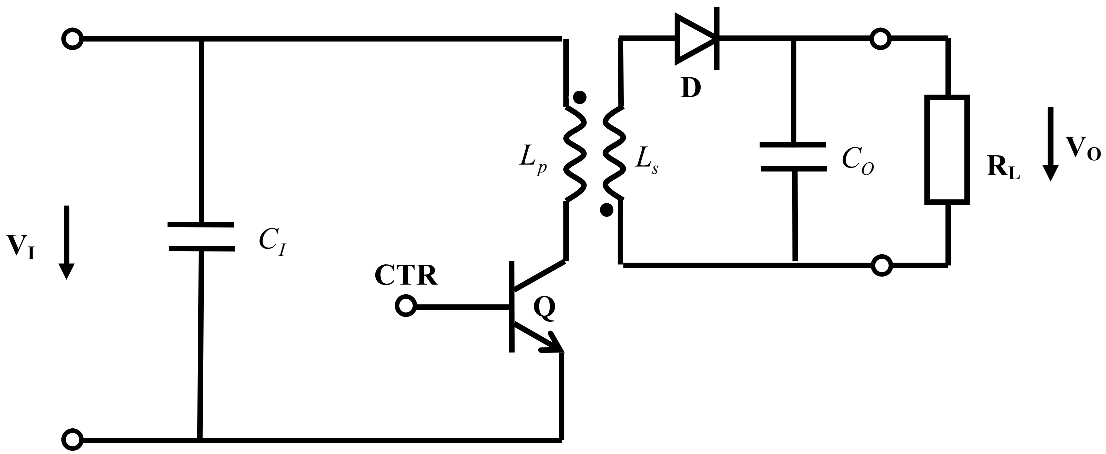
$$\frac{2g_{be}g_{ce}+\omega^2C_{bc}^2}{\omega C_{bc}}\geq\sqrt{g_m^2+\omega^2C_{bc}^2}$$

$$\omega\leq\frac{2g_{be}g_{ce}}{C_{bc}\sqrt{g_m^2-4g_{be}g_{ce}}}=\omega_{us}$$

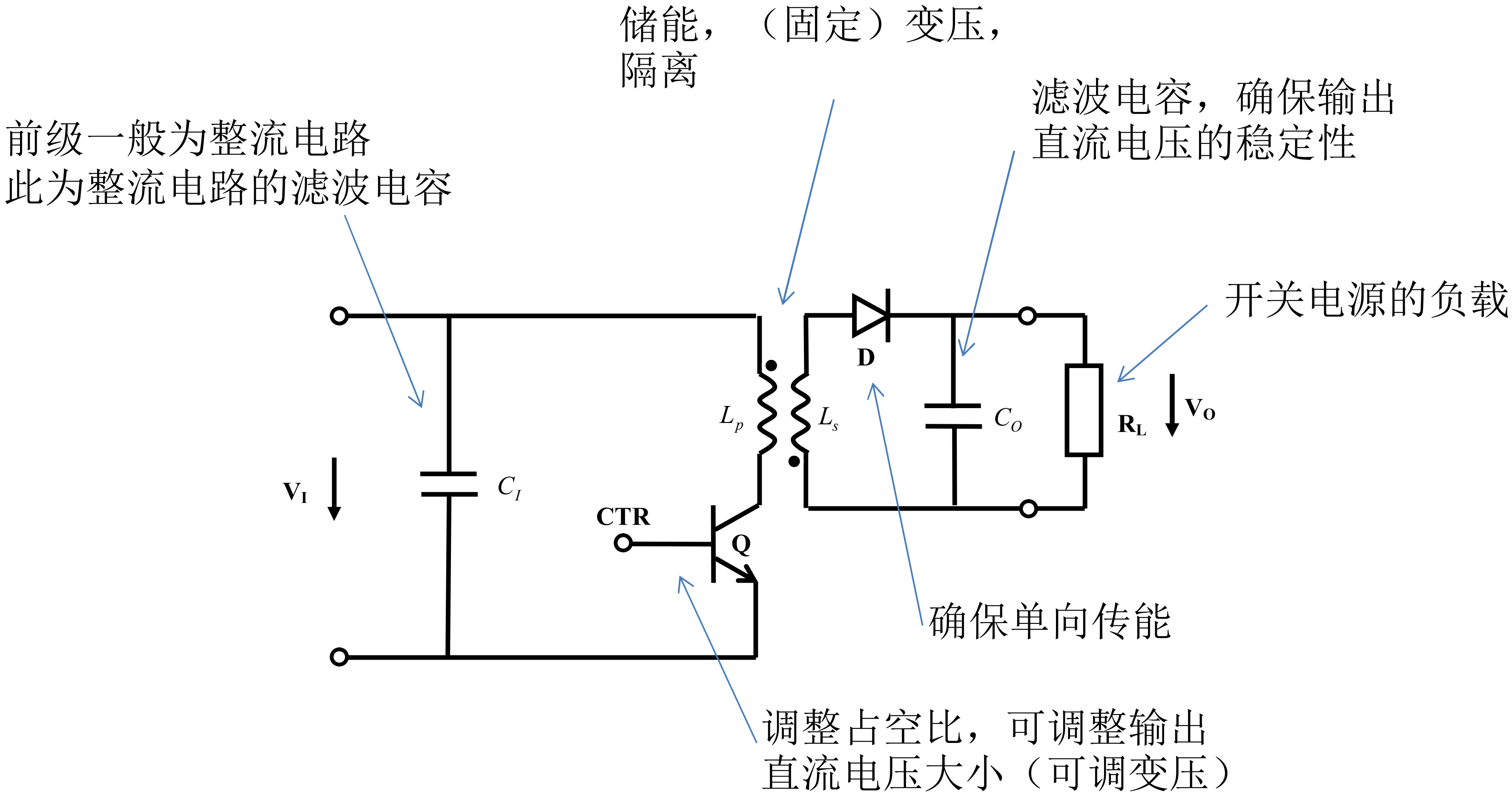
绝对稳定工作区

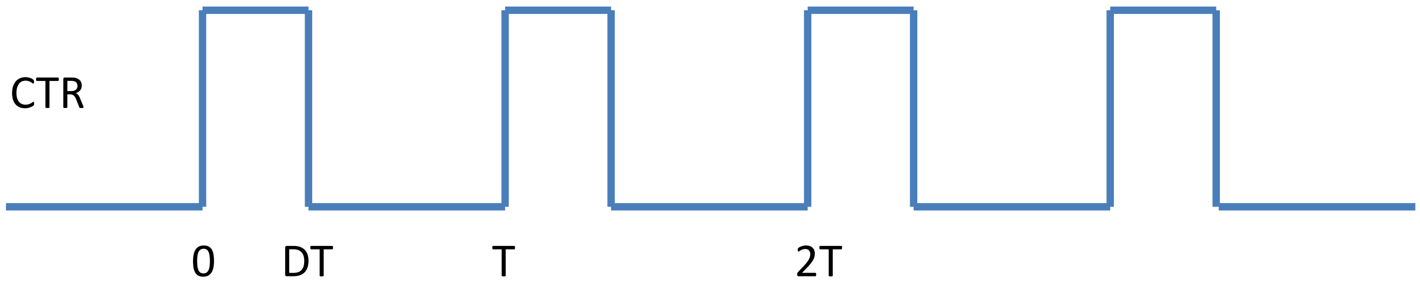
$$\left(\frac{2g_{be}g_{ce}}{\omega C_{bc}}\right)^2+\frac{4g_{be}g_{ce}}{\omega C_{bc}}\omega C_{bc}+(\omega C_{bc})^2\geq g_m^2+\omega^2C_{bc}^2$$

选作2：Flyback converter

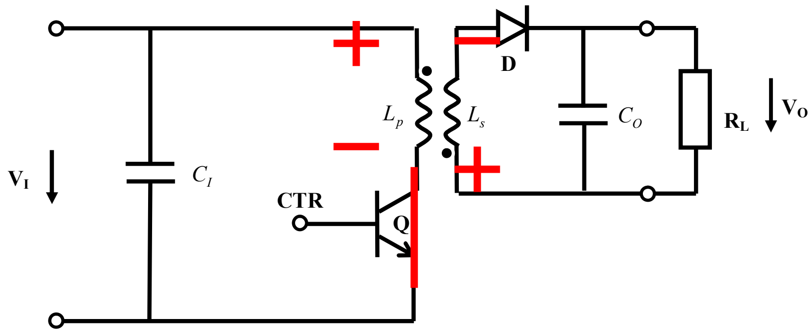


- 反激DC-DC转换器：**CTR=1**，晶体管**Q**饱和导通，输入直流电压V_I为初级线圈L_p充磁，变压器同名端接法使得次级线圈电压反相从而二极管**D**反偏截止，故而V_I对初级线圈的充磁能量全部存储于变压器结构中。此时，负载电阻自大电容C_O获得电能。当**CTR=0**使得晶体管**Q**截止，晶体管集电极电压将急剧上升，次级线圈电压反相导致二极管**D**正偏导通，抽象为开关闭合，假设变压器是全耦合变压器，存储于变压器中的能量将全部自次级线圈L_s对外释放，它对C_O电容充电补充**CTR=1**阶段释放的电能，同时也为R_L提供直流电能。假设滤波电容C_O极大，导致V_O直流输出电压在充放磁的完整周期内几乎保持不变，**CTR**方波占空比为**D**，请分析输出直流电压V_O和输入直流电压V_I之间的关系


储能，（固定）变压，
隔离
滤波电容，确保输出
直流电压的稳定性
前级一般为整流电路
此为整流电路的滤波电容
开关电源的负载
V_I
C_I
L_p
L_s
D
C_O
R_L
V_O
CTR
Q
确保单向传能
调整占空比，可调整输出
直流电压大小（可调变压）

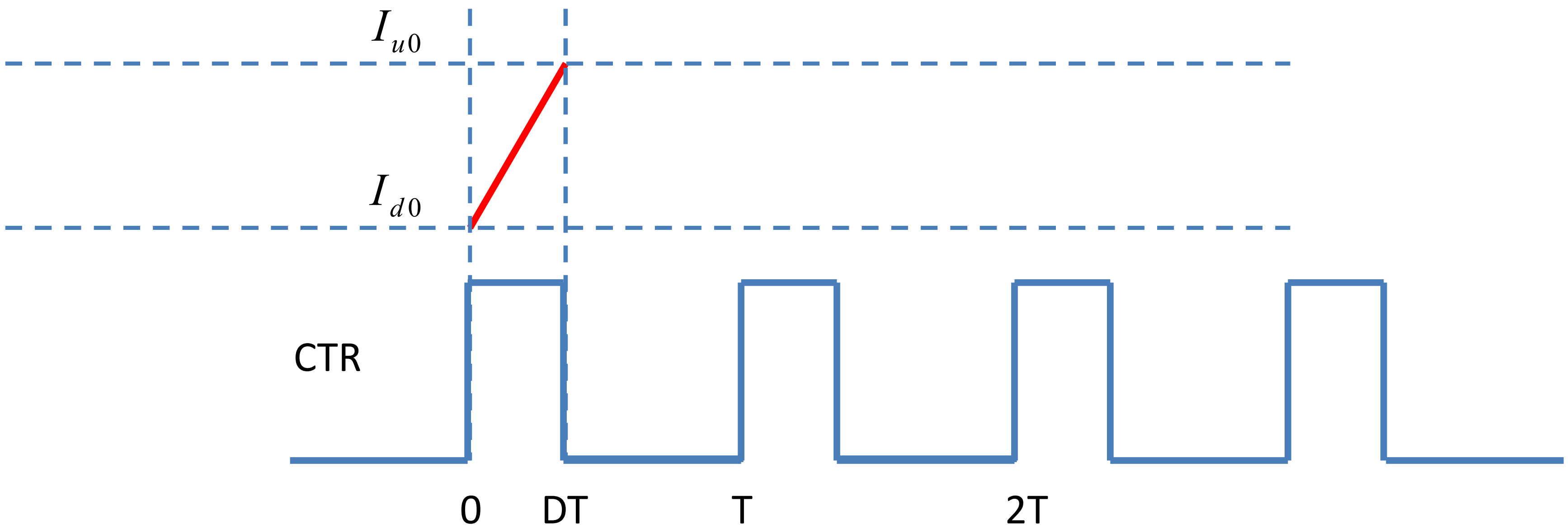

CTR
0
DT
T
2T

CTR=1
开关闭合

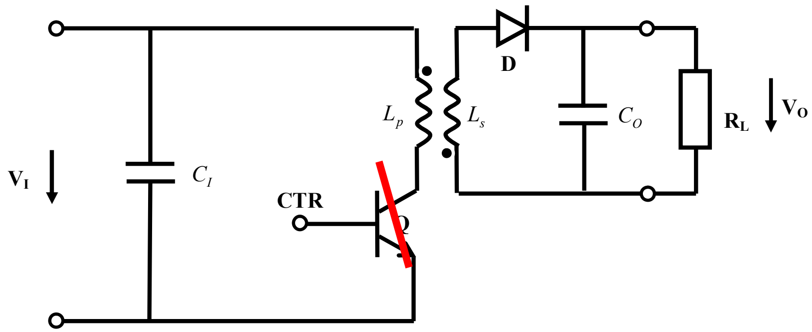


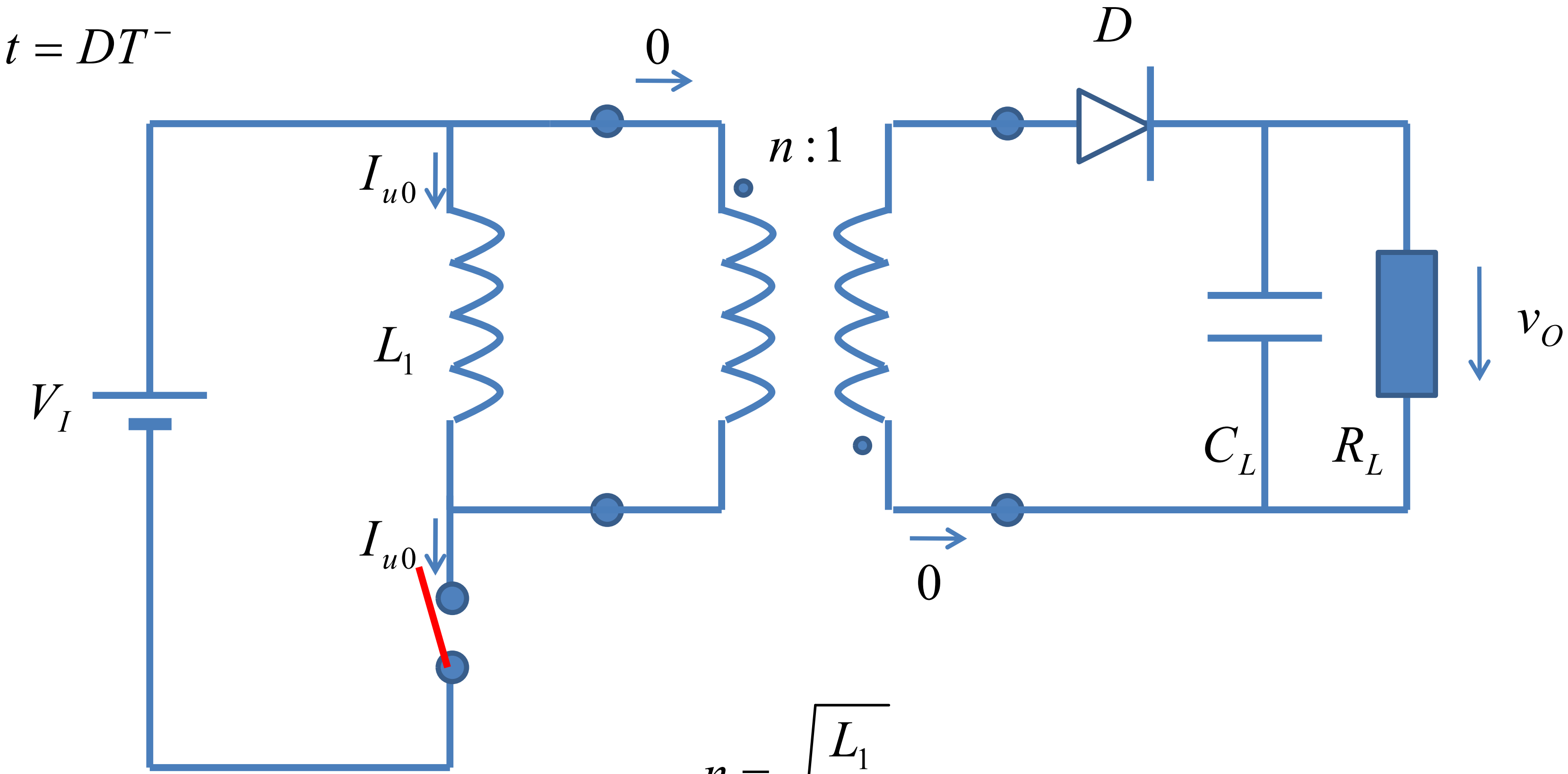
$$i_1(t)=I_{d0}+\frac{1}{L_1}\int_0^t V_I dt=I_{d0}+\frac{V_I}{L_1}t$$

$$I_{u0}=i_1(DT)=I_{d0}+\frac{V_I}{L_1}DT$$



CTR=0
开关断开前瞬间

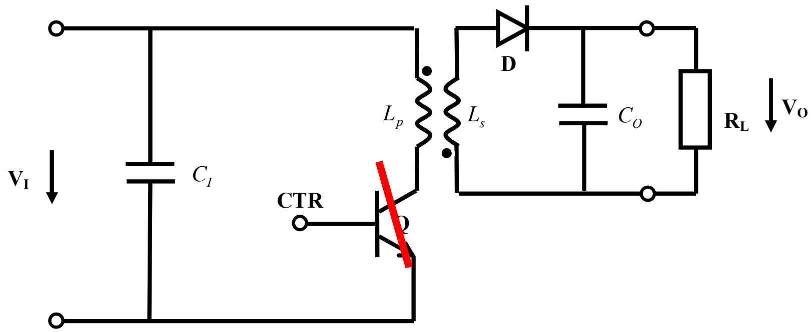




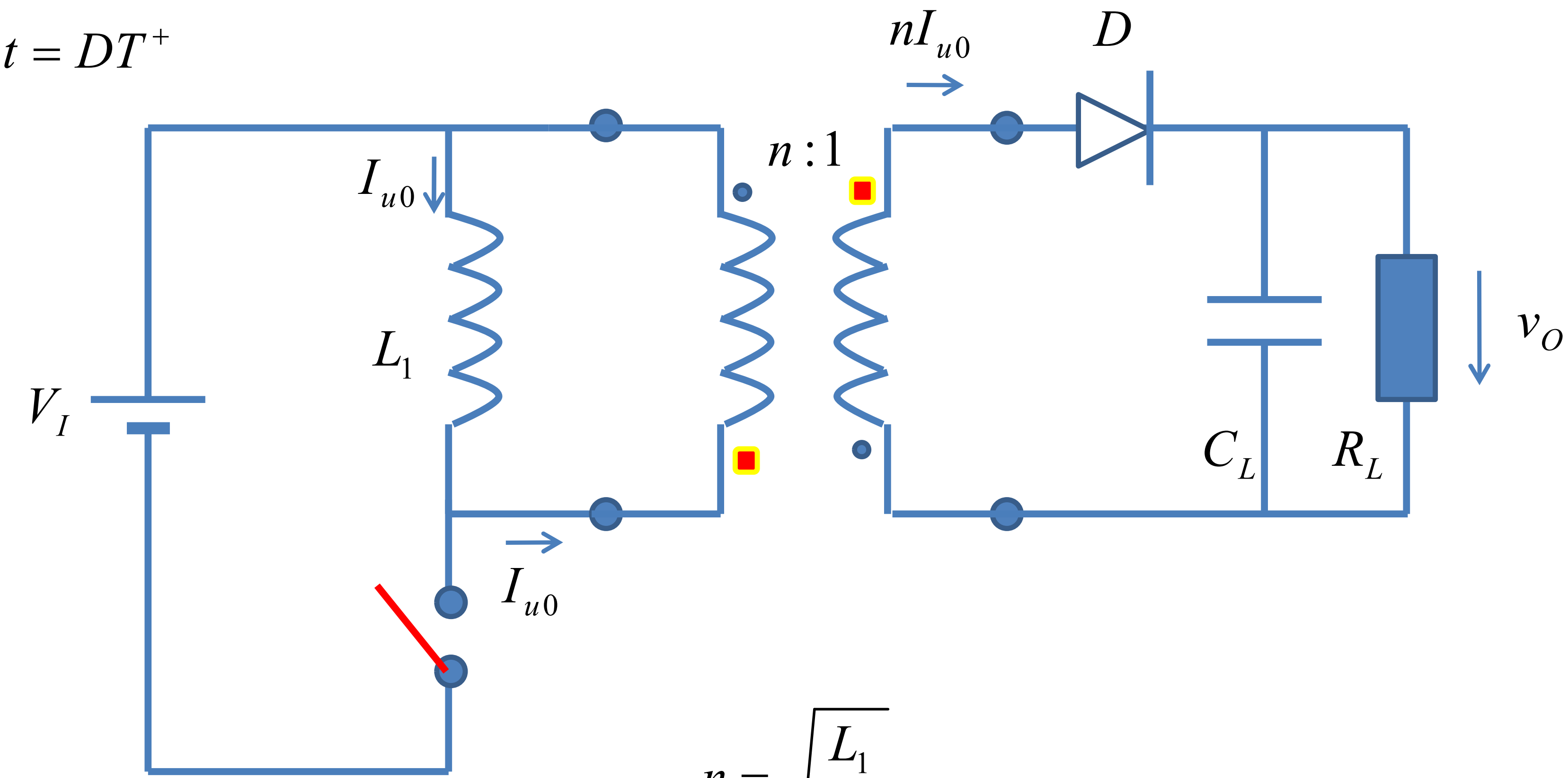
$$n = \sqrt{\frac{L_1}{L_2}}$$

$$k = 1$$

CTR=0
开关断开后瞬间

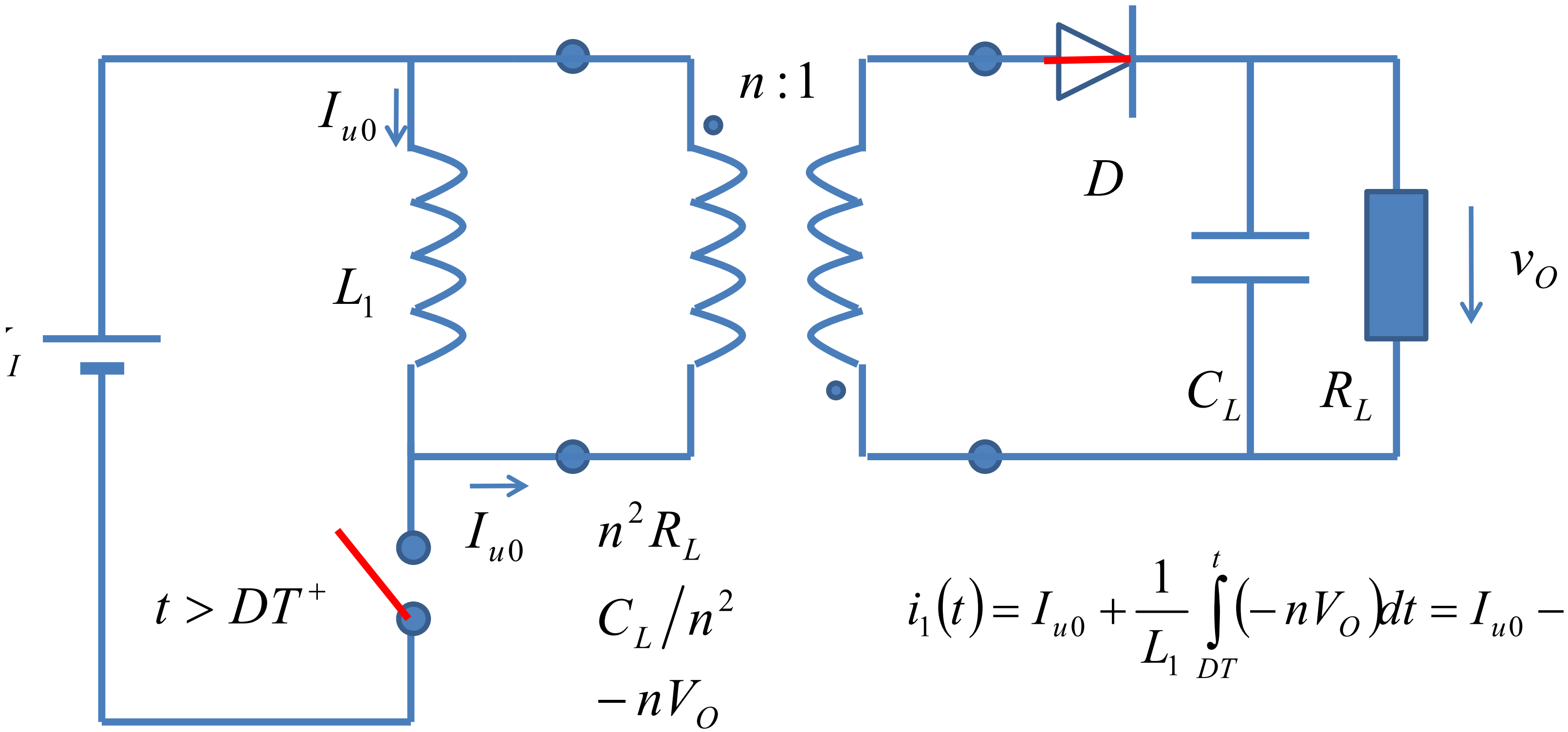


$t = DT^+$



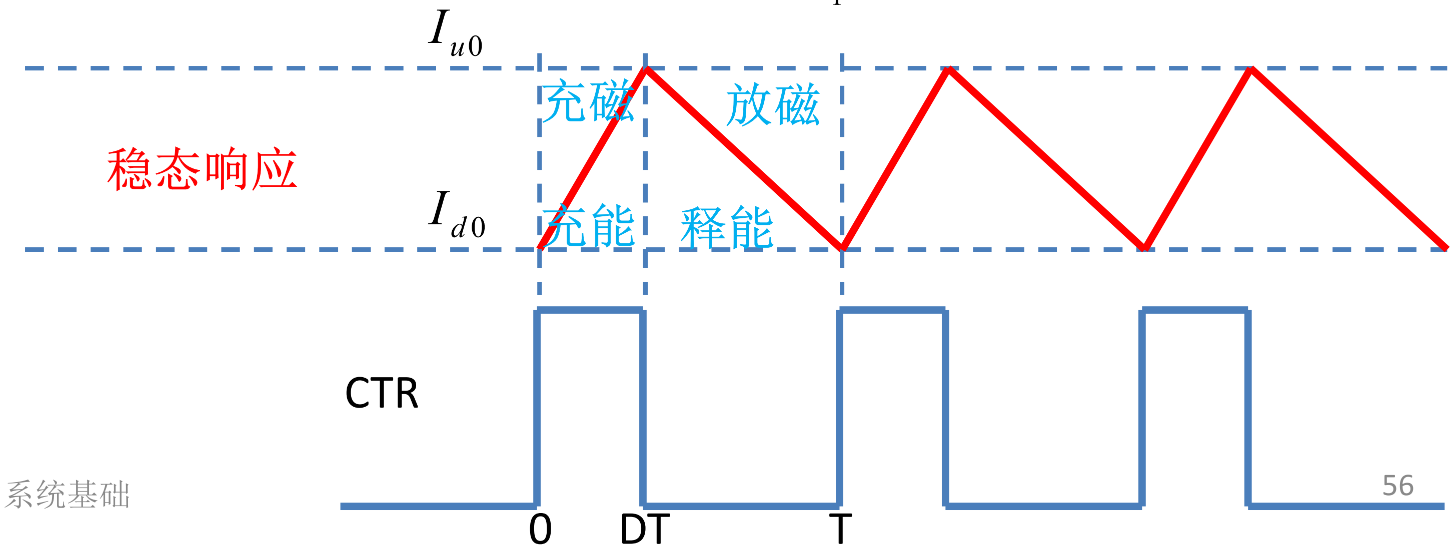
$$n = \sqrt{\frac{L_1}{L_2}}$$

$$k = 1$$



$$i_1(t) = I_{u0} + \frac{1}{L_1}\int_{DT}^{t}(-nV_O)dt = I_{u0} - \frac{nV_O}{L_1}(t - DT)$$

$$i_1(T) = I_{u0} - \frac{nV_O}{L_1}(1 - D)T = I_{d0}$$



$$I_{u0} = i_1(DT) = I_{d0} + \frac{V_I}{L_1} DT$$

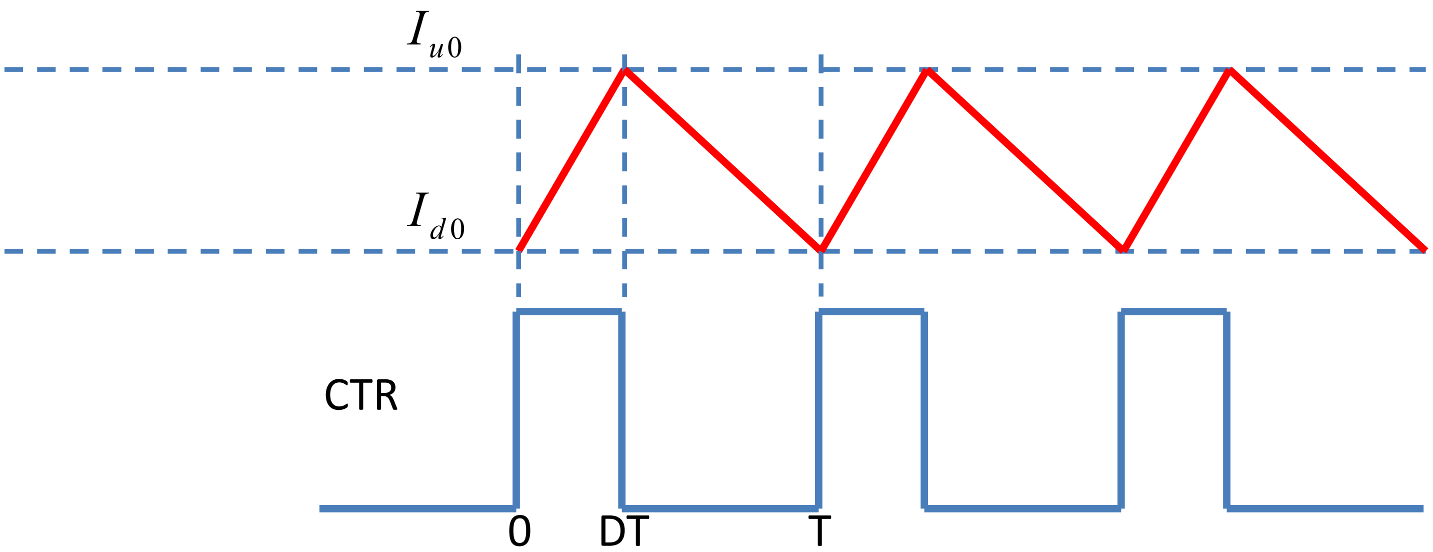
$$I_{d0} = i_1(T) = I_{u0} - \frac{nV_O}{L_1}(1-D)T$$

$$\frac{V_I}{L_1} DT = \frac{nV_O}{L_1}(1-D)T$$

磁通守恒：变压器次级释放的磁通等于初级吸收的磁通

$$V_O = \frac{D}{1-D}\frac{V_I}{n}$$

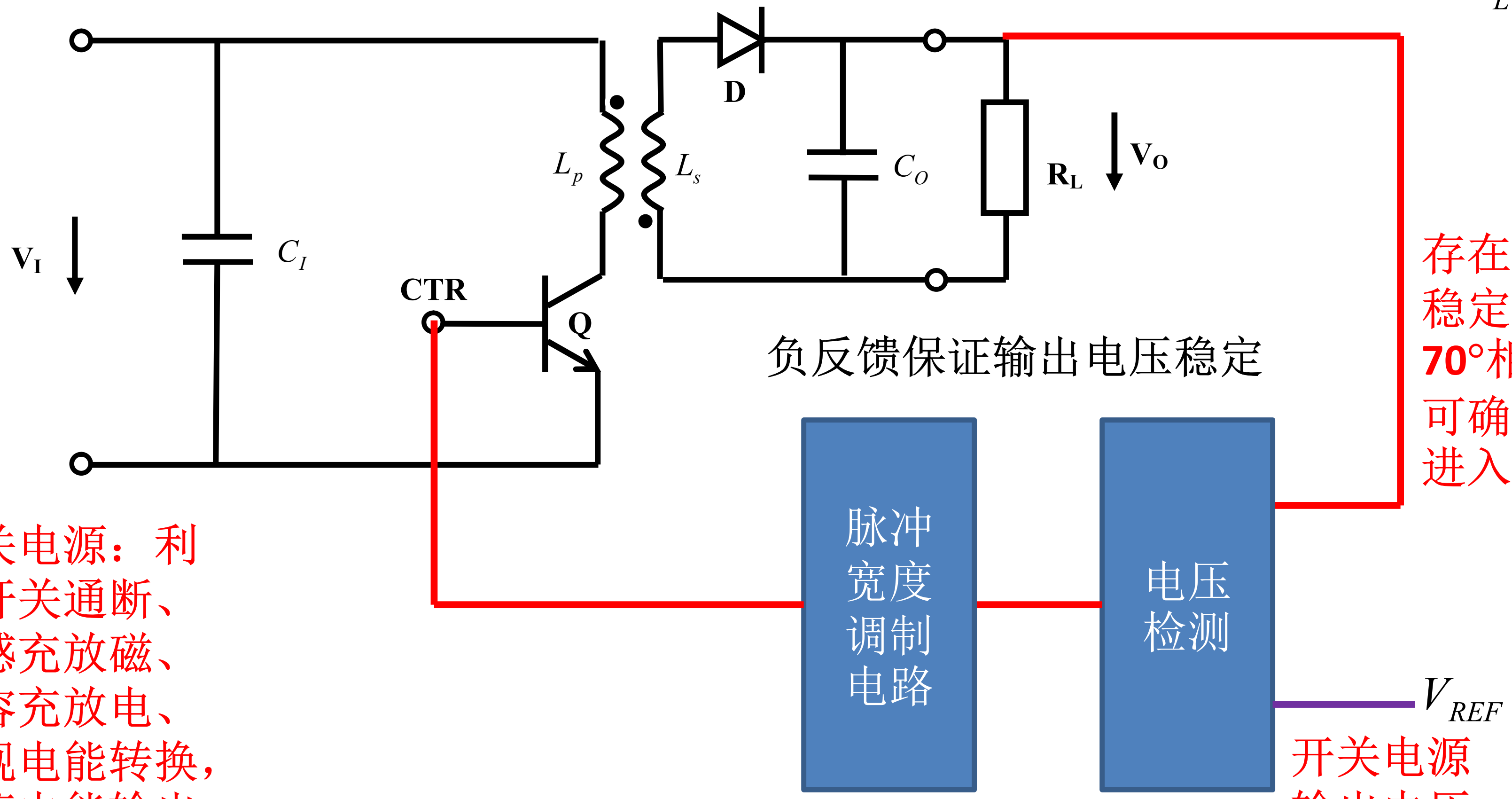
输出直流电压**Vo**可通过调节占空比**D**调整



可用于实现开关电源

$$V_O = \frac{D}{1-D}\frac{V_I}{n}$$

$$\Delta V = \pm\frac{T}{R_L C_L} D V_O$$



负反馈保证输出电压稳定

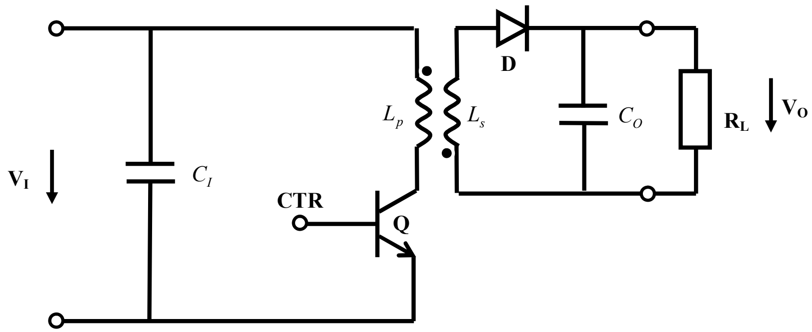
存在负反馈稳定性问题：**70°相位裕度**可确保最快进入稳态

开关电源：利用开关通断、电感充放磁、电容充放电、实现电能转换，直流电能输出

开关电源输出电压控制旋钮

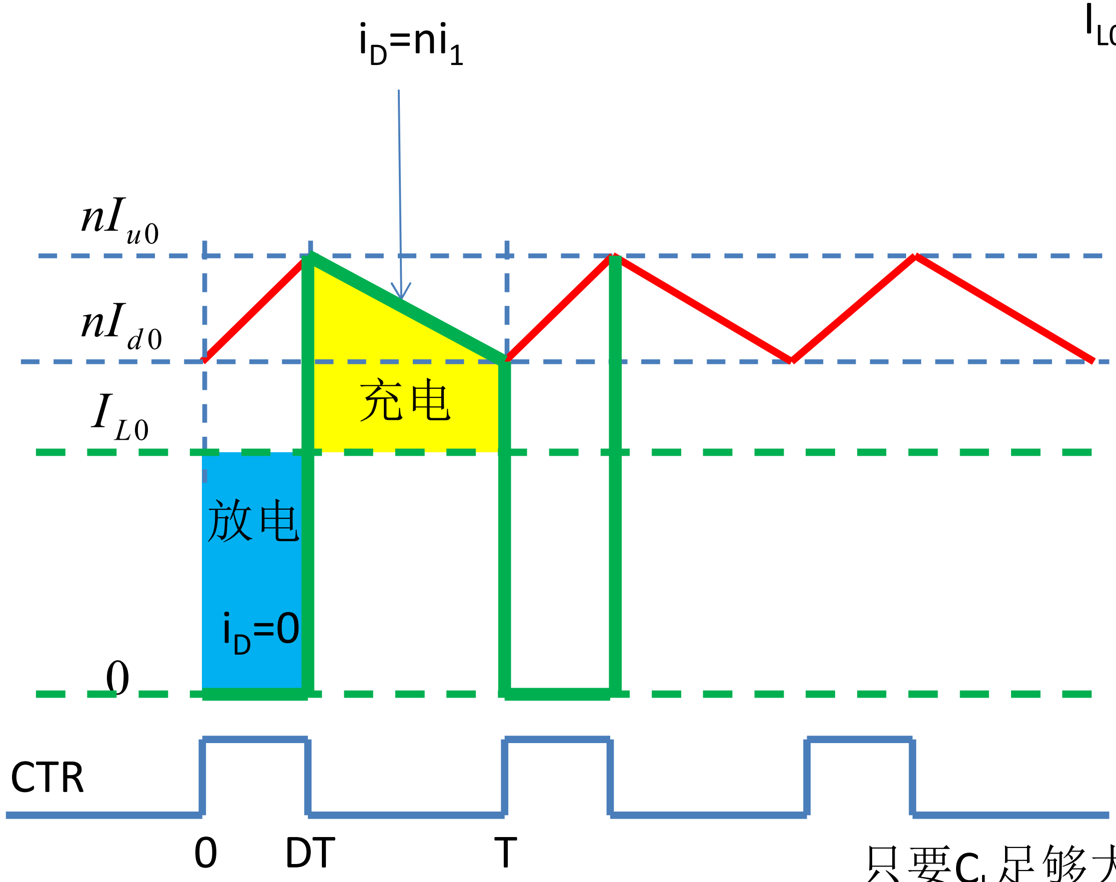
纹波电压

假设Vo不变，其实Vo在变
但可以使得其变化很小



对输出R_LC_L而言
R_L一直在消耗电流
I_{L0}为i_D的直流分量

$$I_{L0}=\frac{V_O}{R_L}$$



CTR=1:电容放电

$$\Delta Q=I_{L0}DT$$

CTR=0:电容充电ΔQ补偿放电

$$\Delta V=\pm\frac{\Delta Q}{C_L}$$

$$=\pm\frac{V_0}{R_LC_L}DT$$

$$=\pm\frac{T}{R_LC_L}DV_0$$

只要C_L足够大，则可认为Vo几乎不变