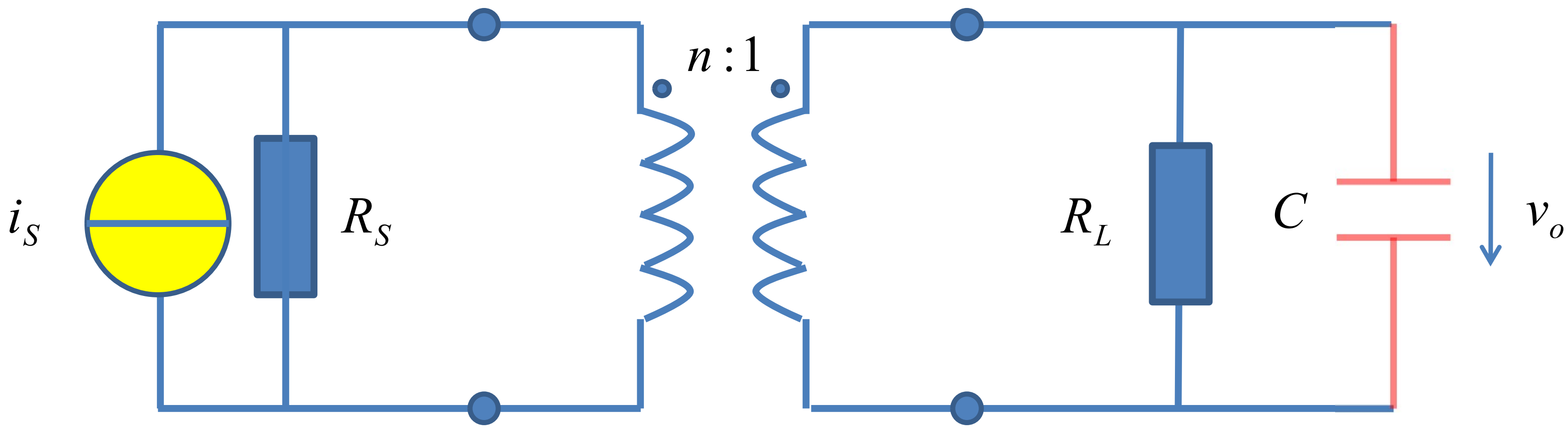
电子电路与系统基础II

习题课第十二讲 习题讲解

阻抗匹配与变换网络（下半）
晶体管电路（上半）

李国林
清华大学电子工程系

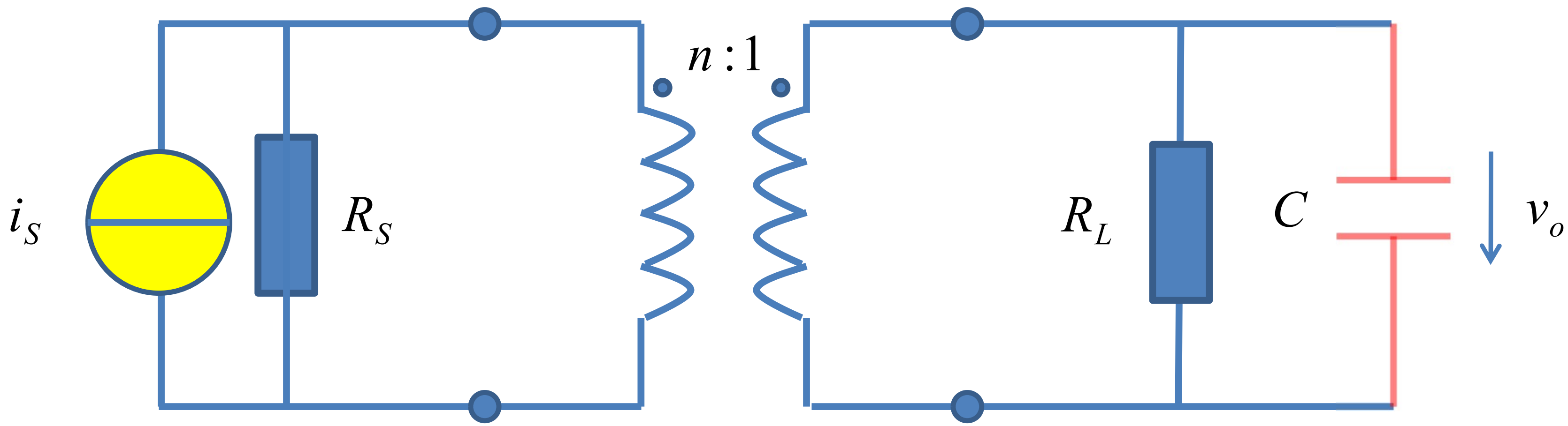
作业4 理想变压？



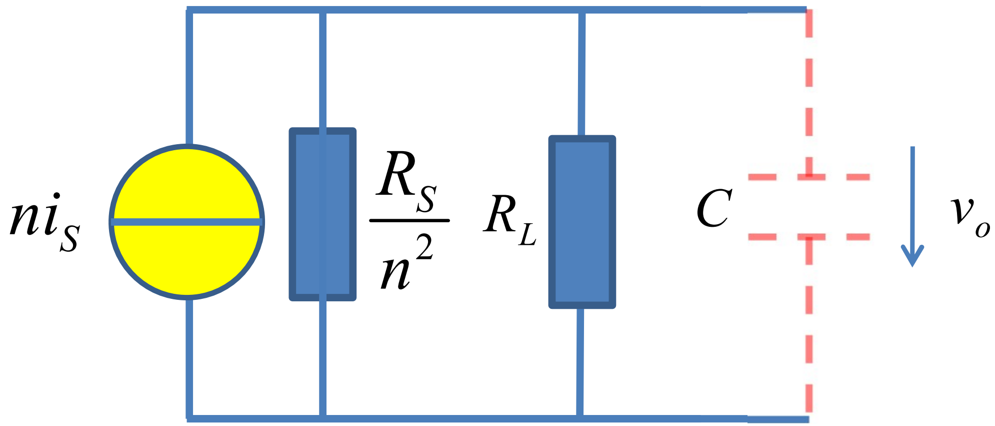
a、已知R_S=50Ω，R_L=200 Ω，若希望实现最大功率传输匹配，理想变压器变压比为多少？

b、负载端存在寄生电容效应，由于寄生电容C=200pF的影响，当输入电流为阶跃电流$I_{S0}U(t)$时，I_{S0}=1mA，负载电压变化情况如何？

c、（选作）如果耦合采用的全耦合互感变压器，输入是1kHz的方波信号，那么电感L_1，L_2至少取多大值时，该互感变压器可近似被视为理想变压器？



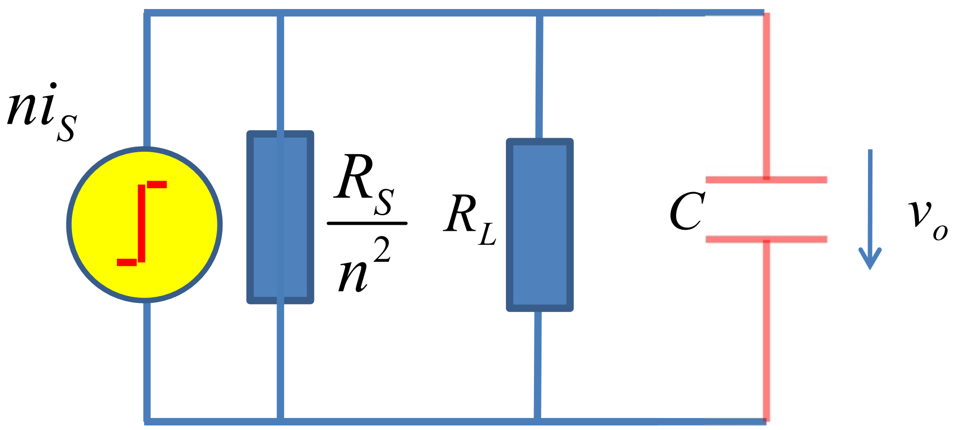
将输入回路的源和内阻等效到输出回路



最大功率传输匹配条件

$$\frac{R_S}{n^2} = R_L$$

$$n = \sqrt{\frac{R_S}{R_L}} = \sqrt{\frac{50}{200}} = \frac{1}{2}$$



三要素法求解

$$v_o\left(0^+\right)=v_C\left(0^-\right)=0$$

$$v_{o\infty}\left(t\right)=nI_{S0}\times 0.5R_L=0.5nI_{S0}R_L$$
$$=0.5\times 0.5\times 1mA\times 200\Omega=0.05V$$

$$\tau=RC=0.5R_LC=0.5\times 200\times 200\times 10^{-12}=20ns$$

$$v_o\left(t\right)=v_{o\infty}\left(t\right)+\left(v_o\left(0^+\right)-v_{o\infty}\left(0^+\right)\right)e^{-\frac{t}{\tau}}=0.5nI_{S0}R_L\left(1-e^{-\frac{t}{0.5R_LC}}\right)_{t\geq 0}$$

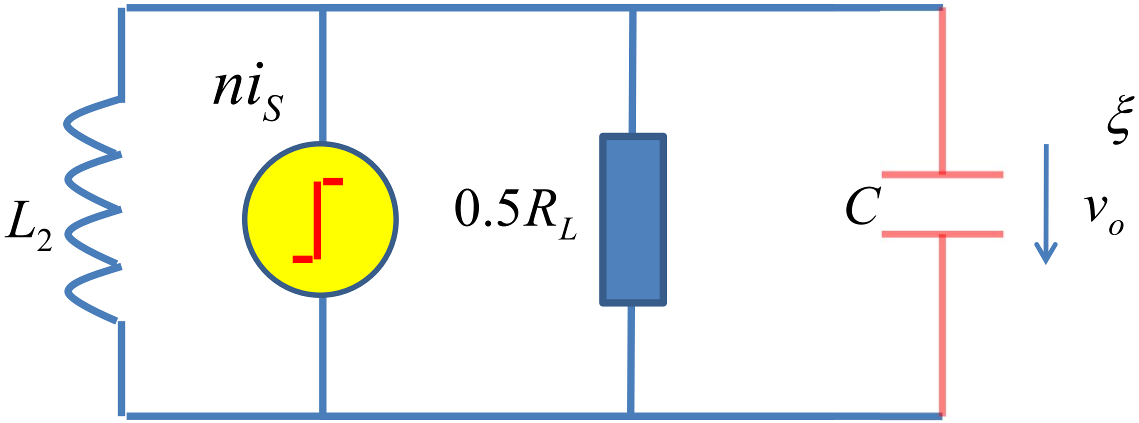
$$=0.05\left(1-e^{-\frac{t}{20ns}}\right)U\left(t\right)$$

阶跃信号是否是直流？

考虑非无穷大电感：全耦合

很大的电感

$$\omega_0=\frac{1}{\sqrt{L_2C}}$$



$$\xi=\frac{G}{2Y_0}=\frac{1}{2R}\sqrt{\frac{L_2}{C}}=\frac{1}{R_L}\sqrt{\frac{L_2}{C}}>>1$$

$$v_{o\infty}(t)=0$$

$$v_o(0^+)=0$$

$$\frac{d}{dt}v_o(0^+)=\frac{i_C(0^+)}{C}=\frac{nI_{S0}}{C}$$

$$=\frac{0.5\times 1mA}{200pF}=2.5\times10^6\ V/s$$

$$=nI_{S0}Z_0\omega_0$$

$$v_o(t)=v_{o\infty}(t)+(V_{o0}-V_{o\infty0})e^{-\xi\omega_0t}\cos\sqrt{1-\xi^2}\omega_0t$$

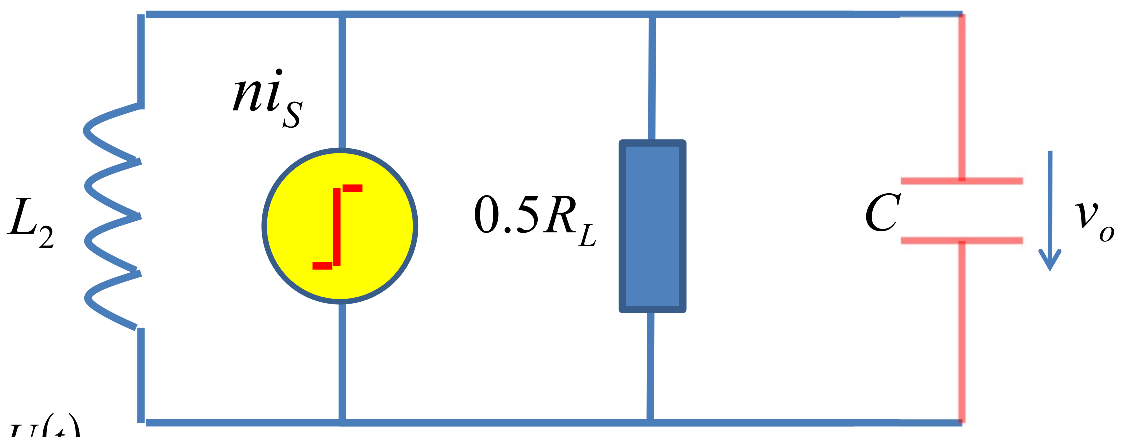
$$+\left(\frac{\dot V_{o0}-\dot V_{o\infty0}}{\xi\omega_0}+V_{o0}-V_{o\infty0}\right)\frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_0t}\sin\sqrt{1-\xi^2}\omega_0t$$

$$=0+(0-0)e^{-\xi\omega_0t}\cos\sqrt{1-\xi^2}\omega_0t$$

$$+\left(\frac{nI_{S0}Z_0\omega_0-0}{\xi\omega_0}+0-0\right)\frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_0t}\sin\sqrt{1-\xi^2}\omega_0t$$

$$=nI_{S0}Z_0\frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_0t}\sin\sqrt{1-\xi^2}\omega_0t$$

阶跃响应



$$i_S(t) = I_{S0}U(t)$$

属过阻尼情况

$$v_o(t) = nI_{S0}Z_0 \frac{1}{\sqrt{\xi^2-1}} e^{-\xi\omega_0 t} \sinh\sqrt{\xi^2-1}\omega_0 t \cdot U(t)$$

$$= nI_{S0}Z_0 \frac{1}{\sqrt{\xi^2-1}} e^{-\xi\omega_0 t} \frac{e^{+\sqrt{\xi^2-1}\omega_0 t} - e^{-\sqrt{\xi^2-1}\omega_0 t}}{2} \cdot U(t)$$

$$= nI_{S0}Z_0 \frac{1}{\sqrt{\xi^2-1}} \frac{e^{\left(-\xi+\sqrt{\xi^2-1}\right)\omega_0 t} - e^{\left(-\xi-\sqrt{\xi^2-1}\right)\omega_0 t}}{2} \cdot U(t)$$

$$= nI_{S0}Z_0 \frac{1}{2\sqrt{\xi^2-1}} \left(e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}}\right) \cdot U(t)$$

$$\tau_1 = \frac{1}{\left(\xi-\sqrt{\xi^2-1}\right)\omega_0} = \frac{\xi+\sqrt{\xi^2-1}}{\omega_0} \approx \frac{2\xi}{\omega_0} = \frac{L_2}{0.5R_L} = \tau_L$$

长寿命项

$$\tau_1 = \frac{1}{\left(\xi+\sqrt{\xi^2-1}\right)\omega_0} = \frac{\xi-\sqrt{\xi^2-1}}{\omega_0} \approx \frac{1}{2\xi\omega_0} = 0.5R_L C = \tau_C$$

短寿命项

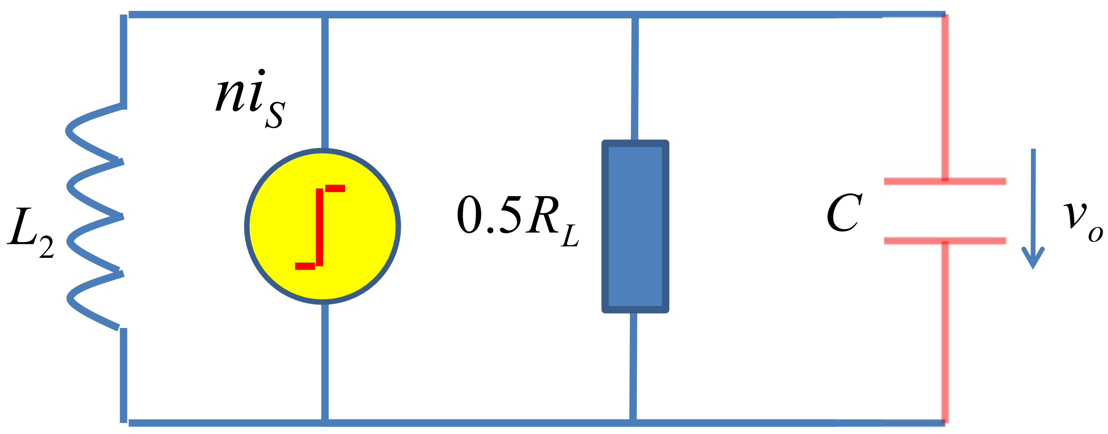
$$\xi = \frac{1}{R_L}\sqrt{\frac{L_2}{C}} >> 1 \qquad L_2 >> R_L^2 C$$

$$L_2 >> R_L^2 C = 200^2 \times 200 \times 10^{-12} = 8\mu H$$

1、电感足够大，过阻尼情况，二阶带通系统在很宽频率范围内才能足够接近一阶低通系统

2、在很小的观测时间尺度内（超过**1kHz**方波，观察时间小于**1ms**），其实是短寿命项在起主导作用

阶跃响应
从瞬态分析



考虑有限电感影响的二阶系统

$$v_{o2}(t) = nI_{S0}Z_0\frac{1}{2\sqrt{\xi^2-1}}\left(e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}}\right)\cdot U(t)$$

$$\overset{\xi>>1}{\approx} nI_{S0}Z_0\frac{1}{2\xi}\left(e^{-\frac{t}{\tau_L}} - e^{-\frac{t}{\tau_C}}\right)\cdot U(t)$$

$$\approx 0.5nI_{S0}R_L\left(1-e^{-\frac{t}{\tau_C}}\right)\cdot U(t) = v_{o1}(t)$$

$$L_2 >> R_L^2C = 200^2\times 200\times 10^{-12} = 8\mu H$$

确保是过阻尼系统

$$v_{o1}(t) = 0.5nI_{S0}R_L\left(1-e^{-\frac{t}{\tau_C}}\right)U(t)$$

电感无穷大时的一阶系统

在观测时间尺度内，二阶和一阶足够近似：

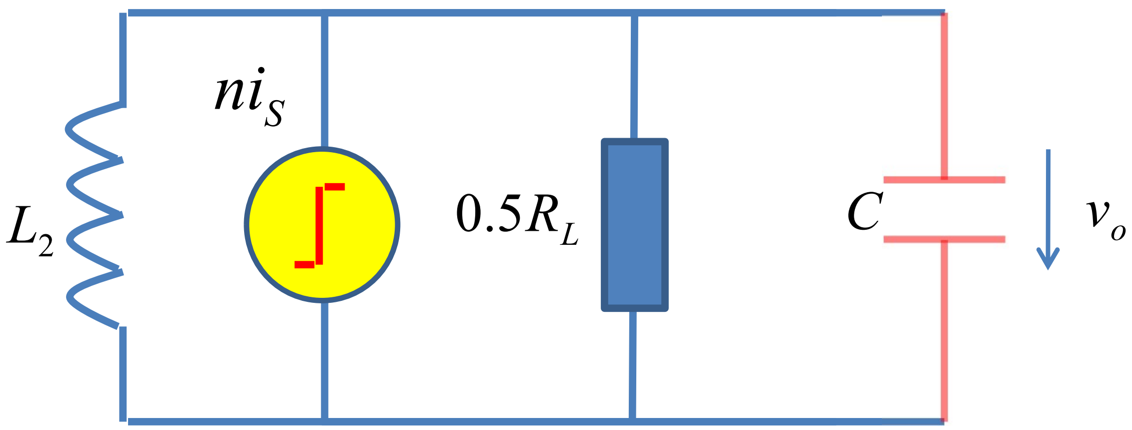
$$\frac{T}{\tau_L} < 0.01 \qquad T < 0.01L_2G = \frac{0.01L_2}{0.5R_L}$$

<1%的误差

$$L_2 > 50R_LT = 50\times 200\Omega\times 1ms = 10H$$

确保在观测时间尺度**1ms**内，
二阶系统行为犹如一阶系统

频域分析



$$H(s)=\frac{\dot{V}_o}{\dot{I}_S}=n\frac{\dot{V}_o}{n\dot{I}_S}=n\left(0.5R_L \parallel L_2 \parallel C\right)$$

$$=n\frac{0.5R_L}{1+jQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}=0.5nR_L\frac{2\xi\omega_0 s}{s^2+2\xi\omega_0 s+\omega_0^2}$$

$$=0.5nR_L\frac{2\xi\omega_0 s}{\left(s+\omega_{p1}\right)\left(s+\omega_{p2}\right)}\overset{\omega>>\omega_{p1}}{\approx}0.5nR_L\frac{2\xi\omega_0 s}{s\left(s+\omega_{p2}\right)}$$

$$\approx 0.5nR_L\frac{\frac{1}{0.5R_L C}}{s+\frac{1}{0.5R_L C}}=0.5nR_L\frac{\omega_{p2}}{s+\omega_{p2}}$$

$$\omega >> \omega_{p1}$$

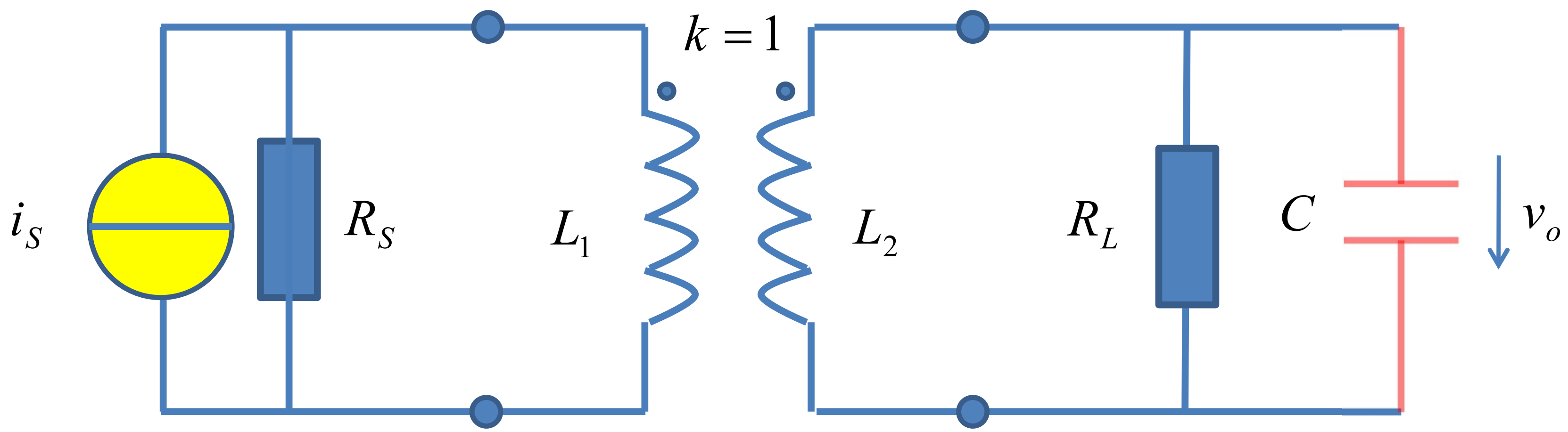
$$\frac{2\pi}{T}>100\frac{0.5R_L}{L_2}$$

$$L_2>\frac{50R_L T}{2\pi}=1.59H$$

无论如何，对于本例数值，如果期望互感变压器在**1kHz**方波激励下可被视为理想变压器，电感感值需要在**1H**量级

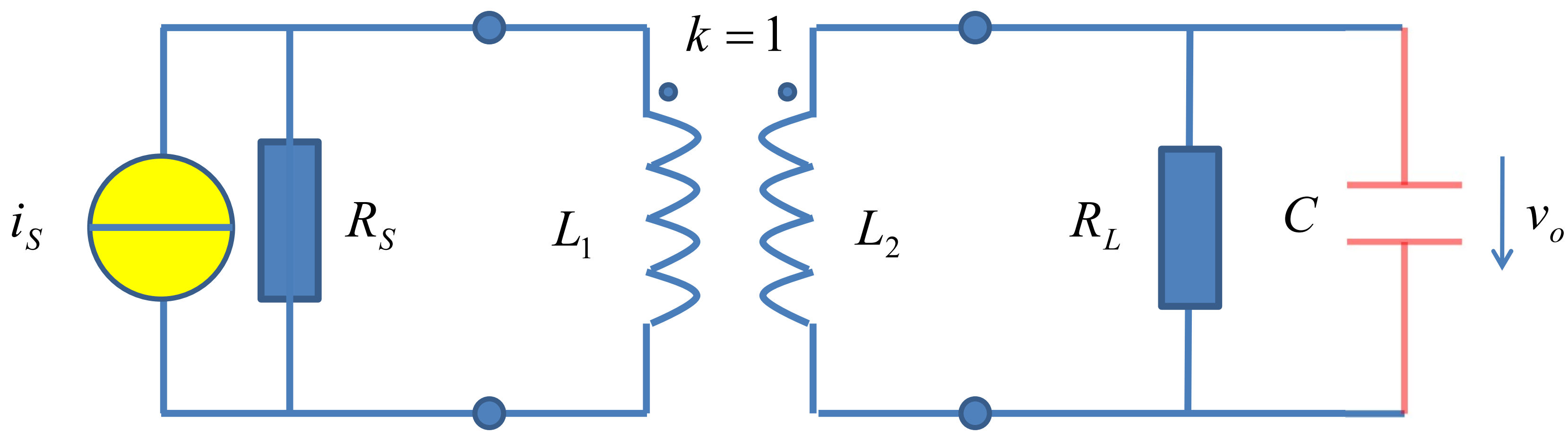
作业5

带通选频



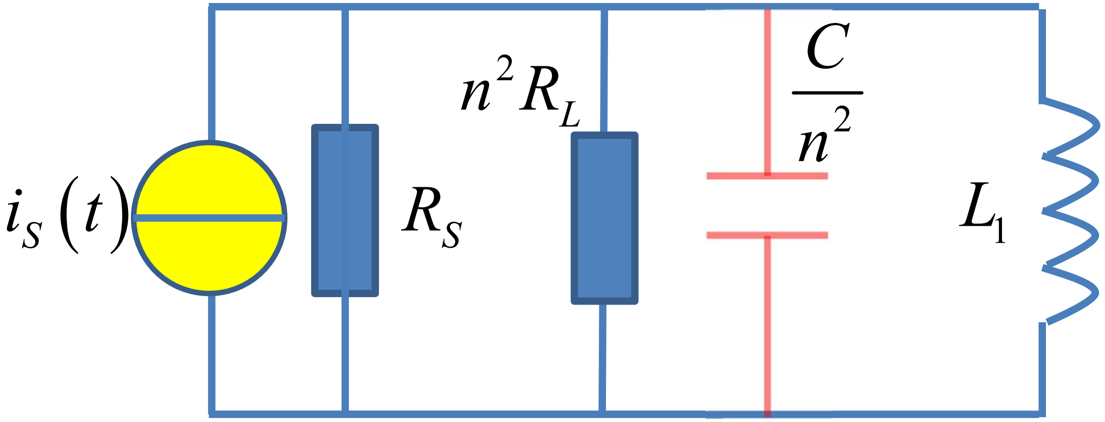
- 已知R_S=1kΩ，R_L=50Ω，现希望在1MHz频点上实现10kHz带宽的最大功率传输匹配
- 采用全耦合互感变压器，请设计全耦合变压器参数，并给出谐振电容的取值大小
  - L_1，L_2，C=?
  - 请画出有谐振电容C和无谐振电容C时的传递函数幅频特性
- （选作）如果选用的全耦合互感变压器不理想，其耦合系数只有0.90，请画出有谐振电容C和无谐振电容C时的传递函数幅频特性

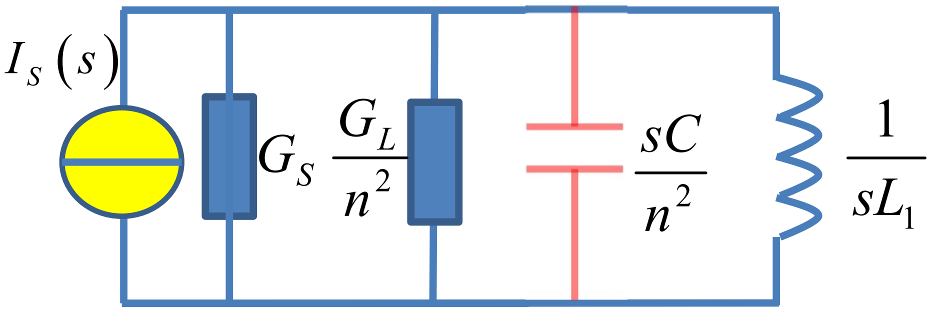
$$H(j\omega)=\frac{\dot{V}_o}{\dot{I}_S}$$



将输出回路等效到输入回路

$$\sqrt{\frac{L_1}{L_2}}=n$$





$$R = R_S \parallel n^2 R_L = 500\Omega$$

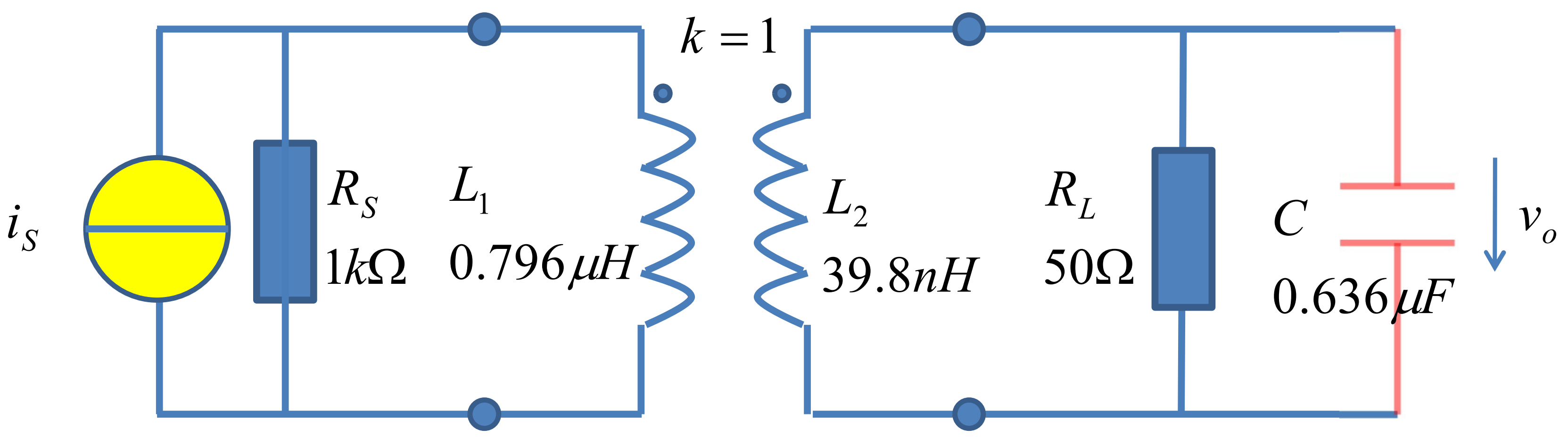
最大功率传输匹配 $G_S = \frac{G_L}{n^2} \Rightarrow n^2 = \frac{G_L}{G_S} = \frac{R_S}{R_L} = \frac{1k\Omega}{50\Omega} = 20 \Rightarrow n = 4.47$

$$\left.\begin{aligned} f_0 &= 1MHz \\ \Delta f &= 10kHz \end{aligned}\right\} \Rightarrow Q = \frac{f_0}{\Delta f} = 100$$

$$\begin{aligned} f_0 &= \frac{1}{2\pi\sqrt{L_1 C'}} \\ Q &= R\sqrt{\frac{C'}{L_1}} \end{aligned} \Rightarrow \begin{aligned} L_1 &= \frac{R}{Q 2\pi f_0} \\ C' &= \frac{Q}{2\pi f_0 R} \end{aligned} \Rightarrow \begin{aligned} L_1 &= \frac{500}{100 \times 2\pi \times 1M} = 0.796\mu H \\ C' &= \frac{100}{2\pi \times 1M \times 500} = 31.8nF \end{aligned}$$

$$L_2 = \frac{L_1}{n^2} = \frac{0.796\mu H}{20} = 39.8nH$$

$$C = n^2 C' = 20 \times 31.8nF = 0.636\mu F$$



$$G_p=\frac{P_L}{P_{S,\max}}=\frac{\frac{V_{L,rms}^2}{R_L}}{\frac{1}{4}I_{S,rms}^2R_S}=\frac{4}{R_SR_L}\left(\frac{V_{L,rms}}{I_{S,rms}}\right)^2$$

$$H(j\omega)=\frac{2}{\sqrt{R_SR_L}}\frac{\dot{V}_L}{\dot{I}_S}=\frac{2}{\sqrt{R_SR_L}}\frac{1}{C(j\omega)}$$

C参量
不是电容C

$$\mathbf{ABCD}=\begin{bmatrix}1 & 0\\ \frac{1}{R_S}+\frac{1}{sL} & 1\end{bmatrix}\begin{bmatrix}n & 0\\ 0 & \frac{1}{n}\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{R_L}+sC & 1\end{bmatrix}=\begin{bmatrix}n & 0\\ \frac{n}{R_S}+\frac{n}{sL}+\frac{1}{nR_L}+\frac{sC}{n} & \frac{1}{n}\end{bmatrix}$$

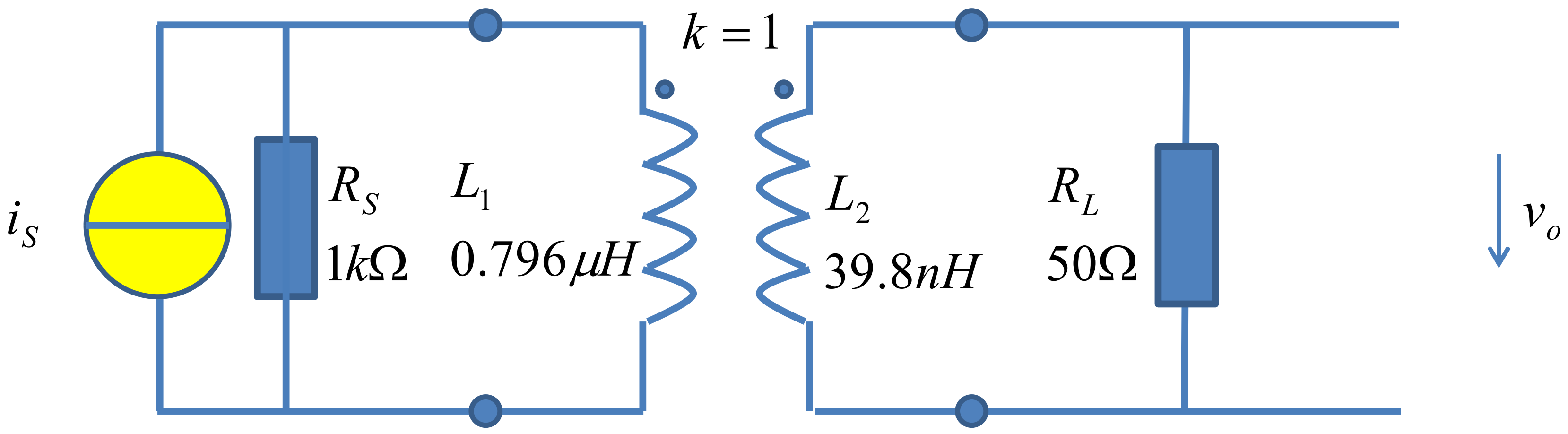
$$H(j\omega)=\frac{2}{\sqrt{R_SR_L}}\frac{\dot{V}_L}{\dot{I}_S}=\frac{2}{\sqrt{R_SR_L}}\frac{1}{C(j\omega)}=\frac{2}{\sqrt{R_SR_L}}\frac{1}{\frac{n}{R_S}+\frac{n}{j\omega L}+\frac{1}{nR_L}+\frac{j\omega C}{n}}=\frac{1}{1+jQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}$$

典型的带通传输

$$\omega_0=\frac{1}{\sqrt{LC'}},Q=0.5R_S\sqrt{\frac{C'}{L}},C'=\frac{C}{n^2}$$

电容C

$f_0=1MHz, BW_{3dB}=10kHz$



$$H(j\omega)=\frac{2}{\sqrt{R_SR_L}}\frac{\dot{V}_L}{\dot{I}_S}=\frac{2}{\sqrt{R_SR_L}}\frac{1}{C(j\omega)}=\frac{2}{\sqrt{R_SR_L}}\frac{1}{\frac{n}{R_S}+\frac{n}{j\omega L}+\frac{1}{nR_L}+\frac{j\omega C}{n}}$$

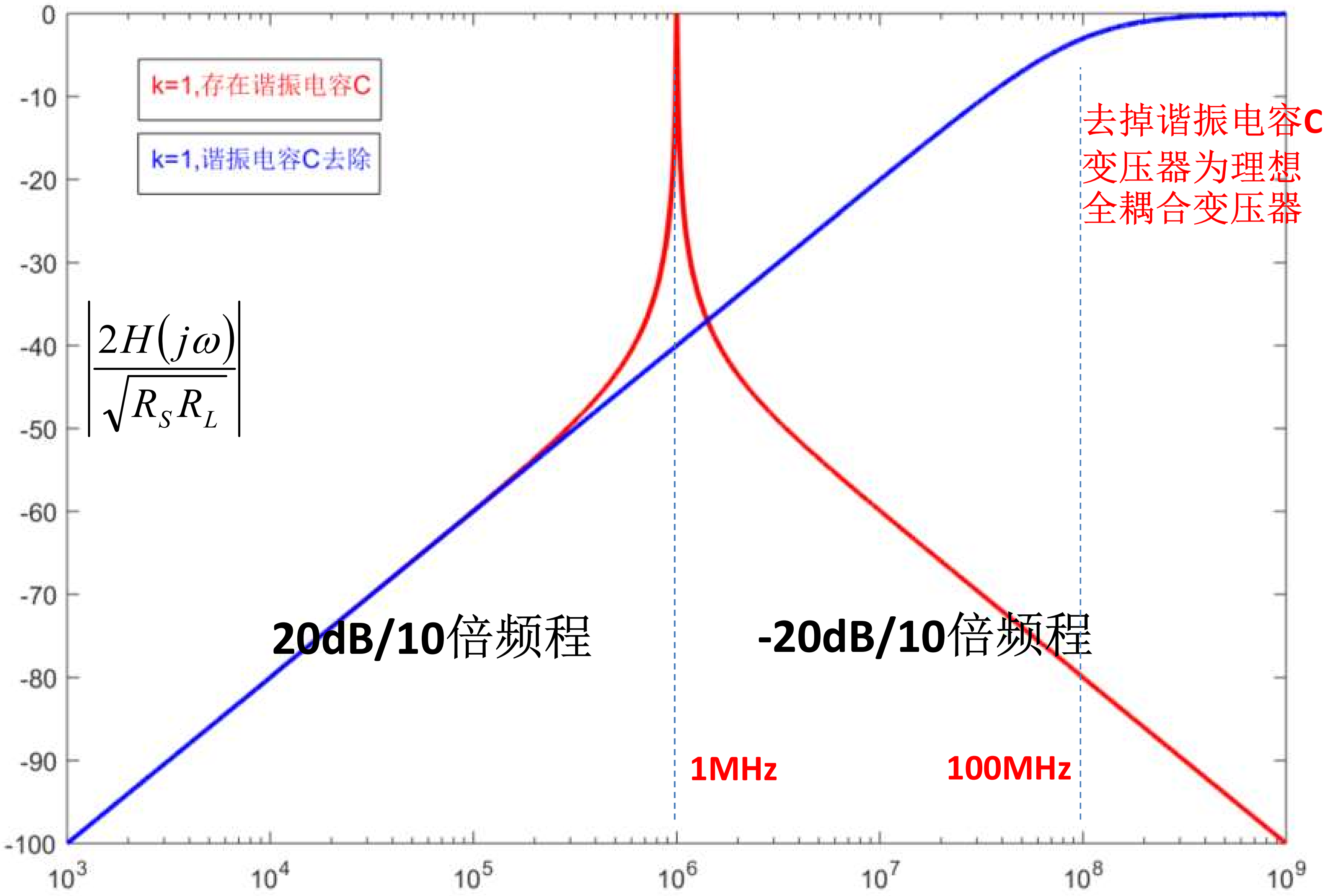
$$\overset{C=0}{=}\frac{2}{\sqrt{R_SR_L}}\frac{1}{\frac{n}{R_S}+\frac{n}{j\omega L}+\frac{1}{nR_L}}=\frac{2}{\sqrt{R_SR_L}\,n}\frac{1}{\frac{1}{R_S}+\frac{1}{j\omega L}+\frac{1}{n^2R_L}}$$

$$=\frac{2}{R_S}\frac{1}{\frac{2}{R_S}+\frac{1}{j\omega L}}=\frac{1}{1+\frac{0.5R_S}{j\omega L}}=\frac{1}{1+\frac{1}{j\omega\tau}}=\frac{j\omega\tau}{1+j\omega\tau}$$

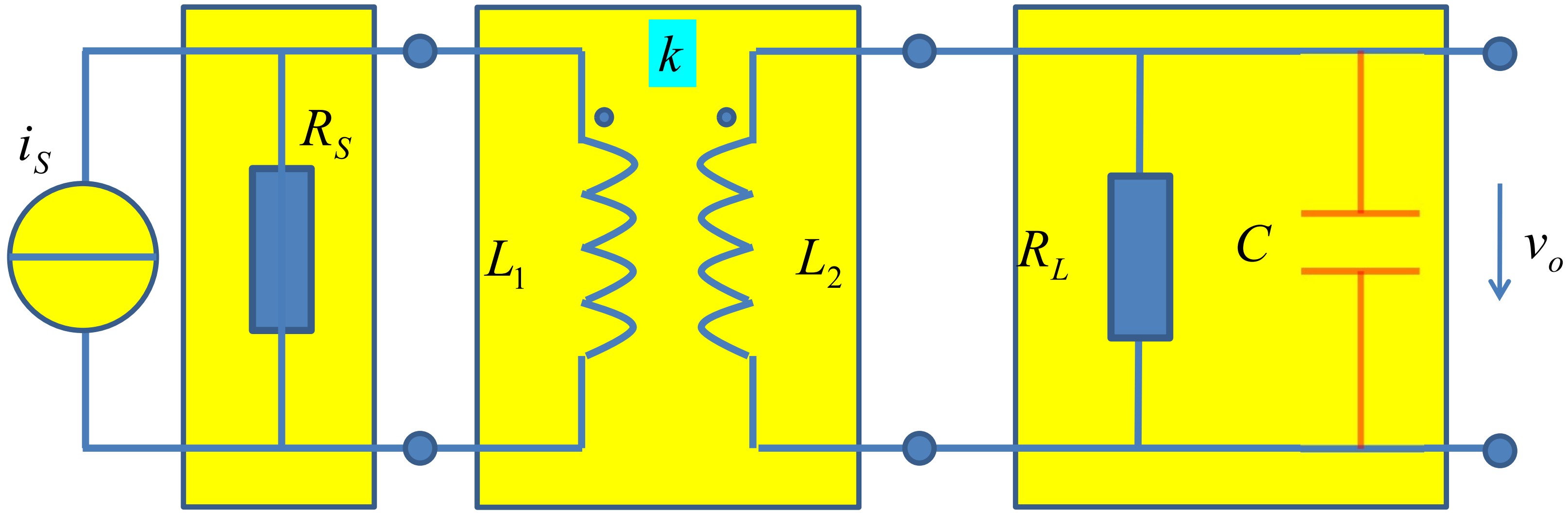
典型的一阶高通传输

$$\tau=GL=\frac{L}{0.5R_S}=\frac{0.796\mu H}{0.5\times1000\Omega}=1.592ns \qquad f_{3dB}=\frac{1}{2\pi\tau}=100MHz$$

有无谐振电容时幅频特性曲线比较



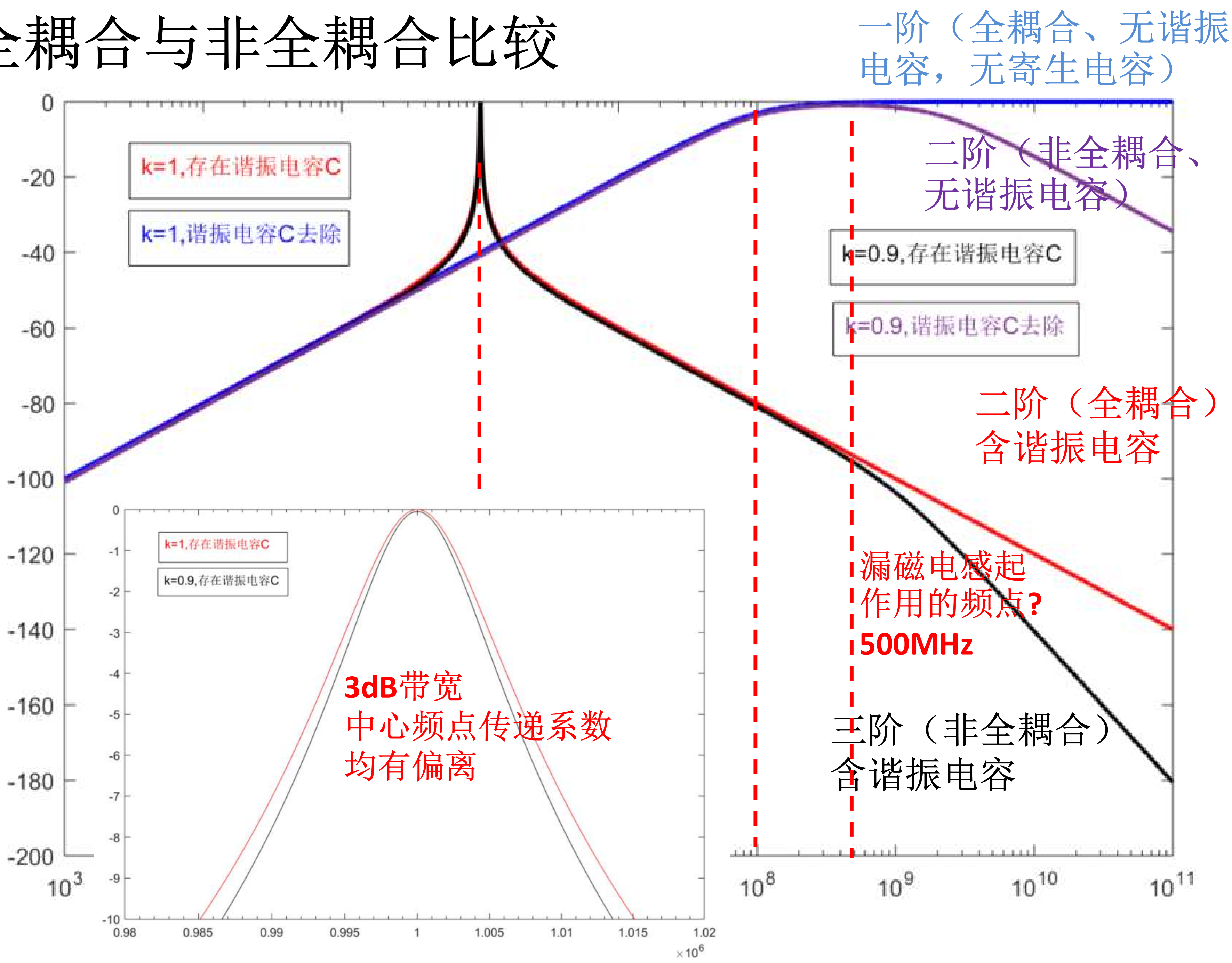
传输矩阵求解



$$\mathbf{ABCD}=\begin{bmatrix}1 & 0\\ \frac{1}{R_S} & 1\end{bmatrix}\begin{bmatrix}\frac{n}{k} & sM_0\frac{1-k^2}{k}\\ \frac{1}{skM_0} & \frac{1}{kn}\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{R_L}+sC & 1\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}=\begin{bmatrix}\frac{1}{A_{v0}} & \frac{1}{G_{m0}}\\ \frac{1}{R_{m0}} & \frac{1}{A_{i0}}\end{bmatrix}$$

$$H(j\omega)=\frac{2}{\sqrt{R_SR_L}}\frac{\dot{V}_L}{\dot{I}_S}=\frac{2R_{m0}(j\omega)}{\sqrt{R_SR_L}}=\frac{2}{\sqrt{R_SR_L}}\frac{1}{C(j\omega)}$$

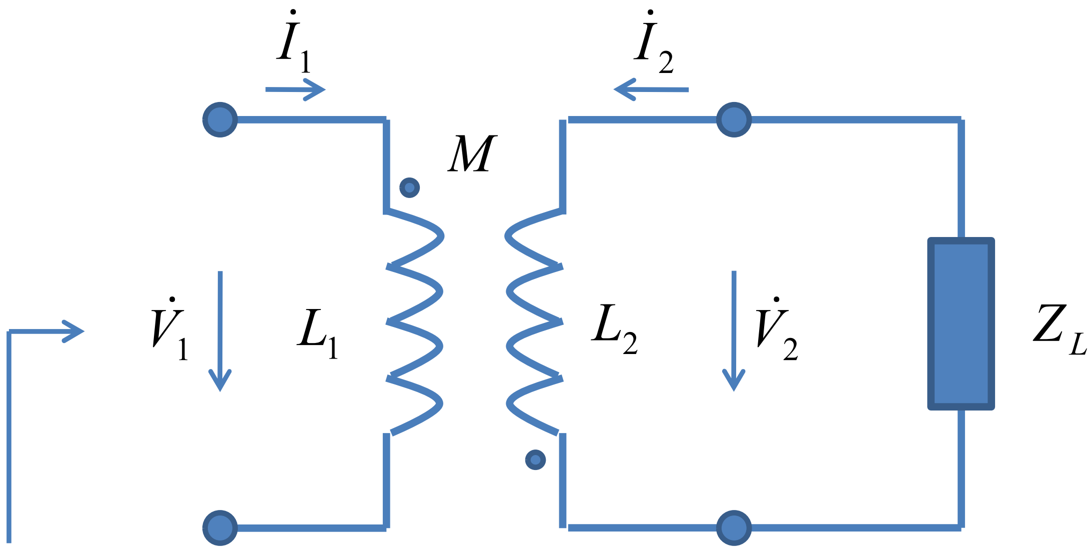
全耦合与非全耦合比较



一阶（全耦合、无谐振电容，无寄生电容）
k=1,存在谐振电容C
k=1,谐振电容C去除
二阶（非全耦合、无谐振电容）
k=0.9,存在谐振电容C
k=0.9,谐振电容C去除
二阶（全耦合）含谐振电容
漏磁电感起作用的频点?
500MHz
3dB带宽
中心频点传递系数
均有偏离
三阶（非全耦合）含谐振电容
k=1,存在谐振电容C
k=0.9,存在谐振电容C

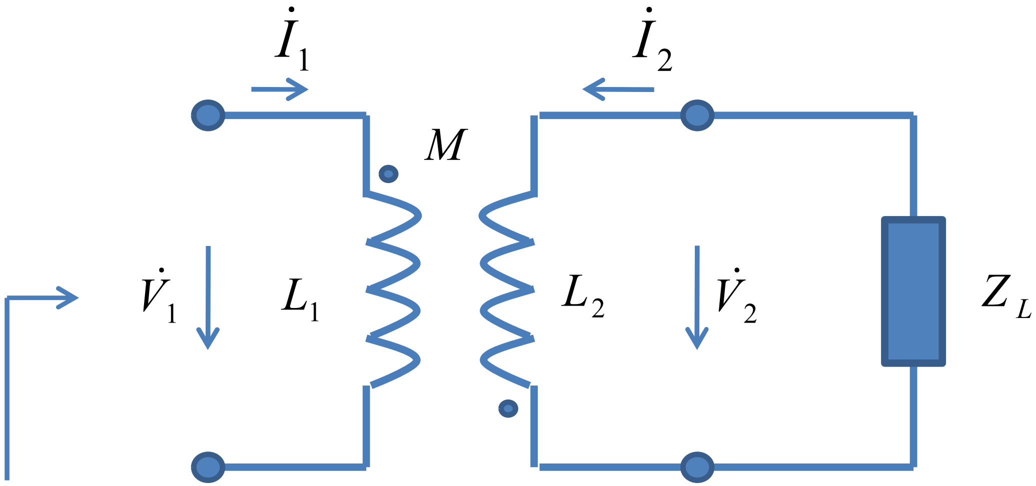
作业6 阻抗变换

- 在相量域（频域）分析互感变压器的阻抗变换关系



$$Z_{in} = \frac{\dot{V}_1}{\dot{I}_1} = j\omega L_1 + ...$$

阻抗变换能力和同名端有无关系？



$$\dot{V}_1 = j\omega L_1\dot{I}_1 + j\omega M\dot{I}_2$$
$$\dot{V}_2 = j\omega M\dot{I}_1 + j\omega L_2 I_2$$

M可正可负

$$Z_{in} = \frac{\dot{V}_1}{\dot{I}_1} = j\omega L_1 + j\omega M\frac{\dot{I}_2}{\dot{I}_1}$$

$$Z_L = \frac{\dot{V}_2}{-I_2} = -j\omega M\frac{\dot{I}_1}{\dot{I}_2} - j\omega L_2$$

$$\frac{\dot{I}_1}{\dot{I}_2} = \frac{Z_L + j\omega L_2}{-j\omega M}$$

$$Z_{in} = \frac{\dot{V}_1}{\dot{I}_1} = j\omega L_1 + j\omega M\frac{\dot{I}_2}{\dot{I}_1}$$
$$= j\omega L_1 + \frac{(\omega M)^2}{Z_L + j\omega L_2}$$

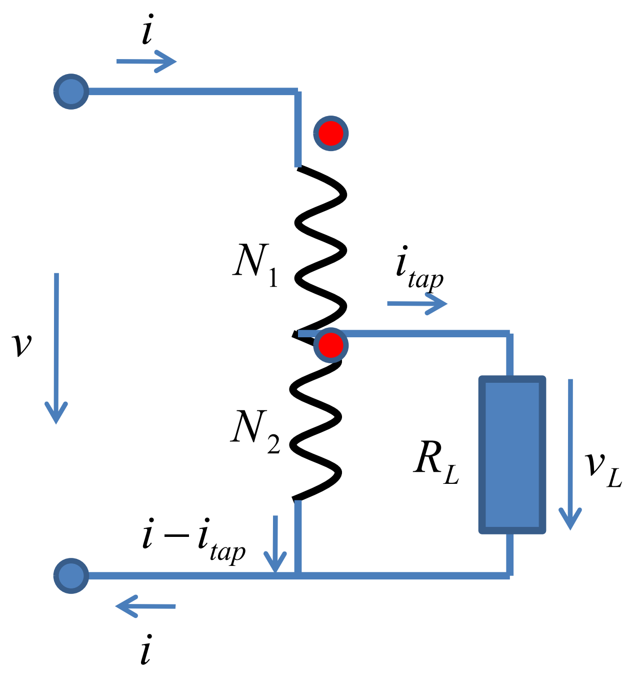
阻抗变换与同名端无关

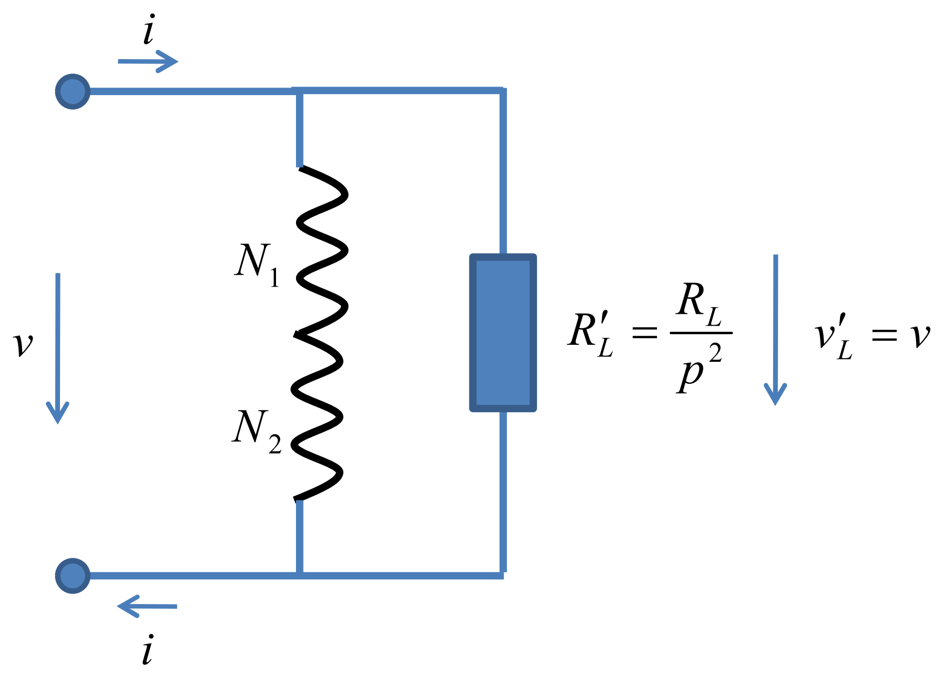
同名端位置同时影响电压和电流的方向，
但它们的比值，即阻抗保持不变

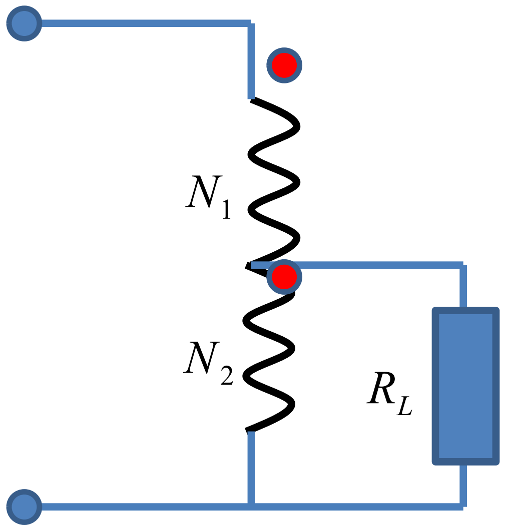
也可直接用Z参量矩阵进行输入阻抗运算，结果一致

作业7 全耦合变压器的部分接入

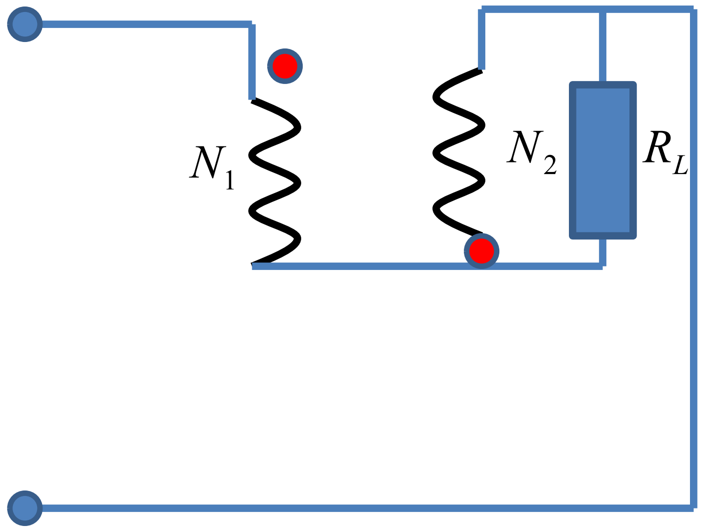
证明：全耦合变压器部分接入公式无需近似，给出部分接入系数



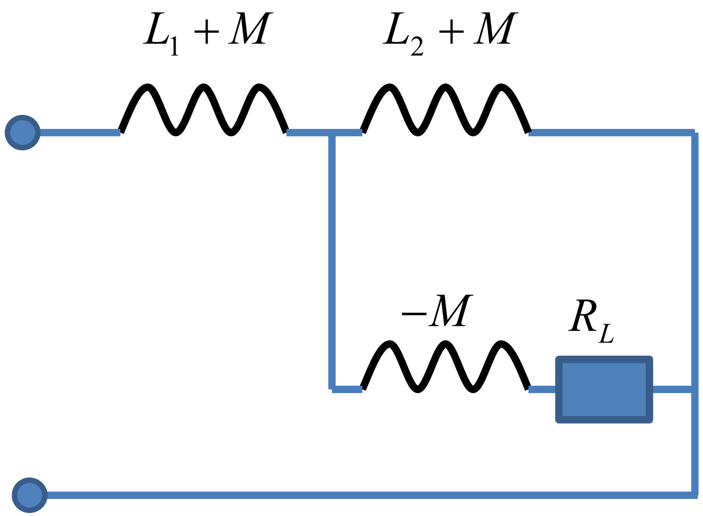




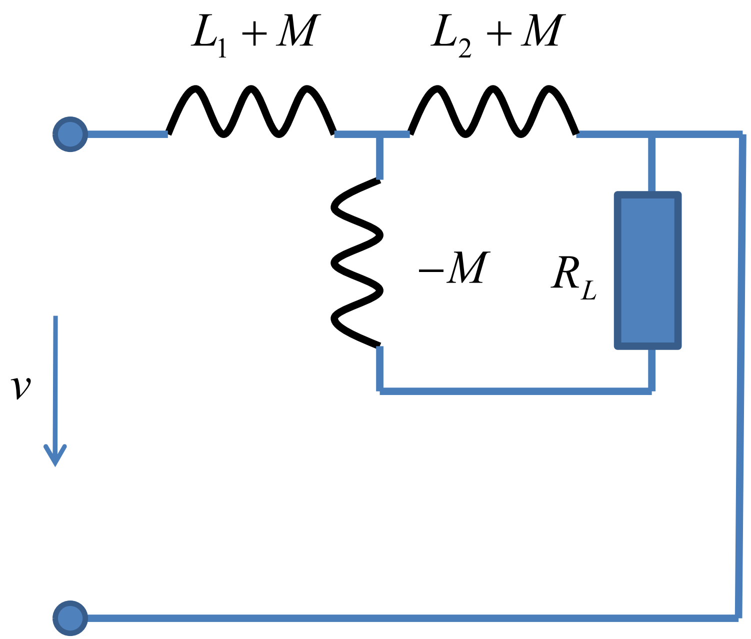
N_1
N_2
R_L



N_1
N_2
R_L



L_1+M
L_2+M
$-M$
R_L



L_1+M
L_2+M
$-M$
R_L
v

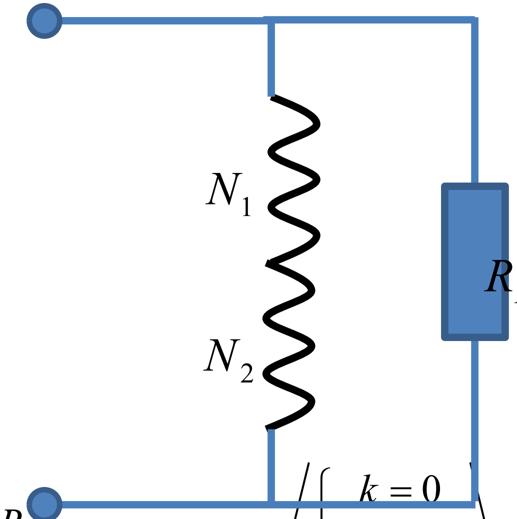
$$Z_{in}=(L_1+M)\text{串}\left((L_2+M)\text{并}(-M\text{串}R_L)\right)$$

$$=j\omega(L_1+M)+\frac{j\omega(L_2+M)\times(-j\omega M+R_L)}{j\omega(L_2+M)+(-j\omega M+R_L)}$$

$$=j\omega(L_1+M)+\frac{\omega^2 M(L_2+M)+j\omega(L_2+M)R_L}{j\omega L_2+R_L}$$

$$=\frac{-\omega^2(L_1+M)L_2+j\omega(L_1+M)R_L+\omega^2 M(L_2+M)+j\omega(L_2+M)R_L}{j\omega L_2+R_L}$$

$$=\frac{-\omega^2\left(L_1L_2-M^2\right)+j\omega(L_1+L_2+2M)R_L}{j\omega L_2+R_L}$$



$$R_L'=\frac{R_L}{p^2}$$

$$Y_{in}=Z_{in}^{-1}$$

$$=\frac{j\omega L_2+R_L}{-\omega^2\left(L_1L_2-M^2\right)+j\omega(L_1+L_2+2M)R_L}\left\langle\begin{cases}k=0\\k=1\\0<k<1\end{cases}\right\rangle$$

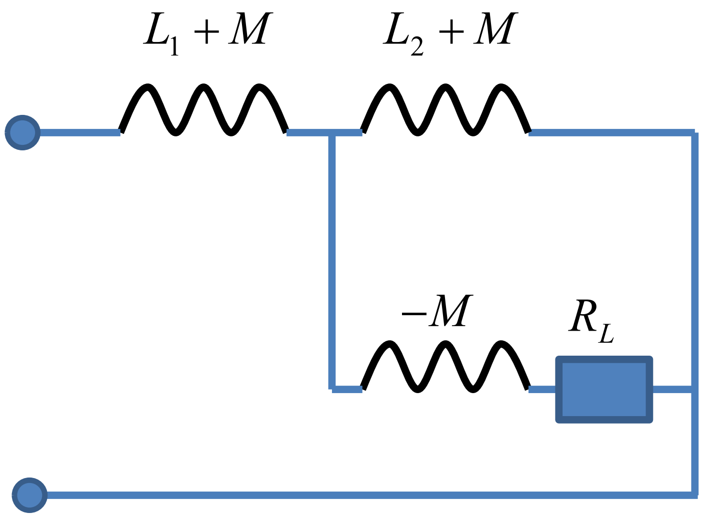
$$\overset{\text{全耦合}}{\underset{k=1}{=}}\frac{j\omega L_2+R_L}{j\omega(L_1+L_2+2M)R_L}$$

$$=\frac{L_2}{(L_1+L_2+2M)R_L}+\frac{1}{j\omega(L_1+L_2+2M)}$$

$$=\frac{1}{\left(\dfrac{L_1+L_2+2\sqrt{L_1L_2}}{L_2}\right)R_L}+\frac{1}{j\omega(L_1+L_2+2M)}$$

$$=\frac{1}{\left(\dfrac{\sqrt{L_1}+\sqrt{L_2}}{\sqrt{L_2}}\right)^2 R_L}+\frac{1}{j\omega(L_1+L_2+2M)}$$

$$=\frac{R_L}{p^2}\text{并}(L_1+L_2+2M)$$

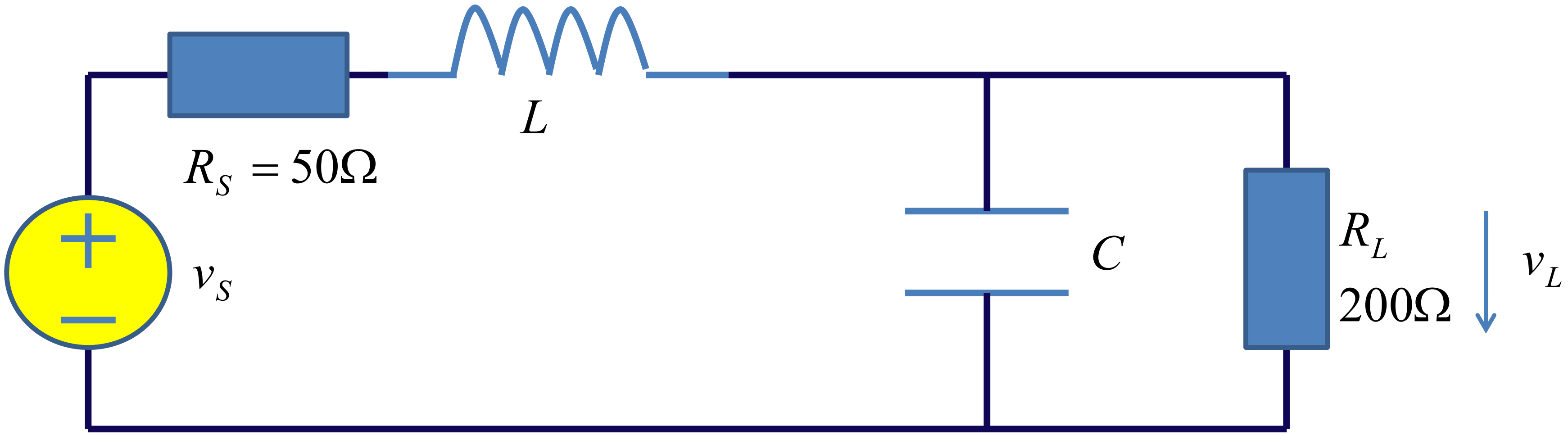


$$p=\frac{\sqrt{L_2}}{\sqrt{L_1}+\sqrt{L_2}}=\frac{N_2}{N_1+N_2}$$

作业8 匹配带宽（选作）

- 设计一个在**10MHz**频点上最大功率传输的**50Ω**到**200Ω**的匹配网络。
  - （**1**）设计一个低通型的**L**型匹配网络，通过数值计算获得幅频特性曲线，在曲线上确认**1dB**匹配带宽；
  - （**2**）先设计一个可将**50Ω**变换为**100Ω**的低通型**L**型匹配网络，再设计一个可将**100Ω**变换为**200Ω**的高通型**L**型匹配网络，将这两个匹配网络级联，用数值方法考察总网络传递函数确认匹配网络设计成功。通过幅频特性曲线，确认**1dB**匹配带宽，和低通**L**型匹配网络比，带宽是变宽了还是变窄了？
  - （**3**）设计一个可将**50Ω**变换为**1kΩ**的低通型**L**型匹配网络，再设计一个可将**1kΩ**变换为**200Ω**的高通型**L**型匹配网络，将这两个匹配网络级联，用数值方法考察总网络传递函数确认匹配网络设计成功。通过幅频特性曲线确认**1dB**匹配带宽，和低通**L**型匹配网络比，带宽是变宽了还是变窄了？
  - （**4**）通过上述问题的解决，分析是什么因素决定了匹配网络的带宽？

设计一个低通型的**L**型匹配网络，通过数值计算获得幅频特性曲线，在曲线上确认**1dB**匹配带宽；



$$Q = \sqrt{\frac{R_L}{R_S} - 1} = \sqrt{\frac{200}{50} - 1} = \sqrt{3} = 1.732$$

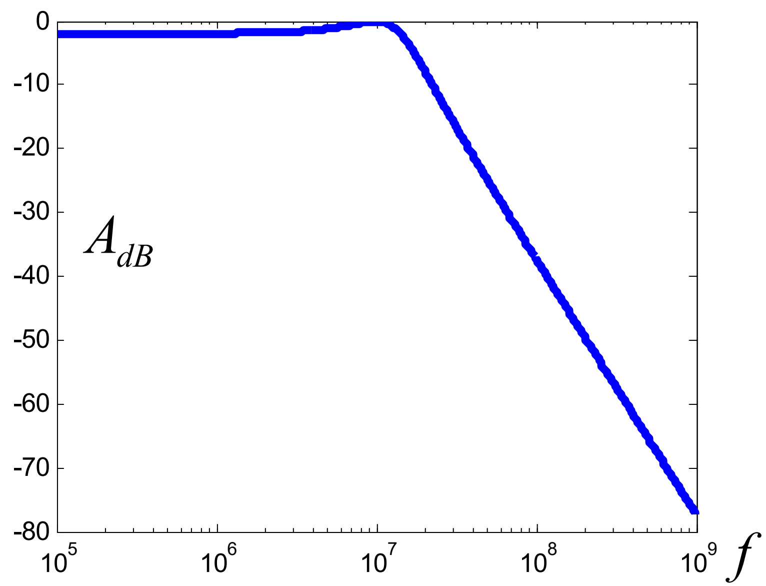
并大串小**Q**相等

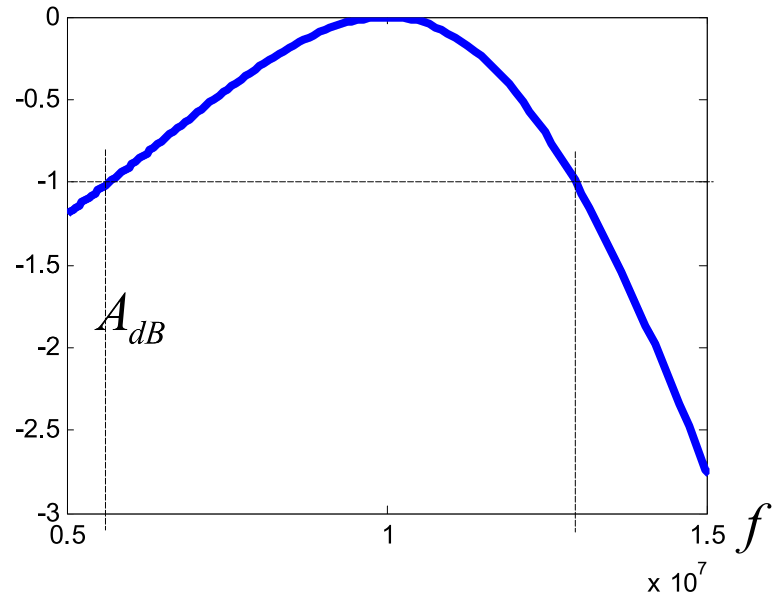
$$L = \frac{R_S}{\omega_r} Q = \frac{50}{2 \times 3.14 \times 10 \times 10^6} \sqrt{3} = 1.378 \mu H$$

$$C = \frac{1}{\omega_r R_L} Q = \frac{1}{2 \times 3.14 \times 10 \times 10^6 \times 200} \sqrt{3} = 137.8 pF$$

1dB带宽

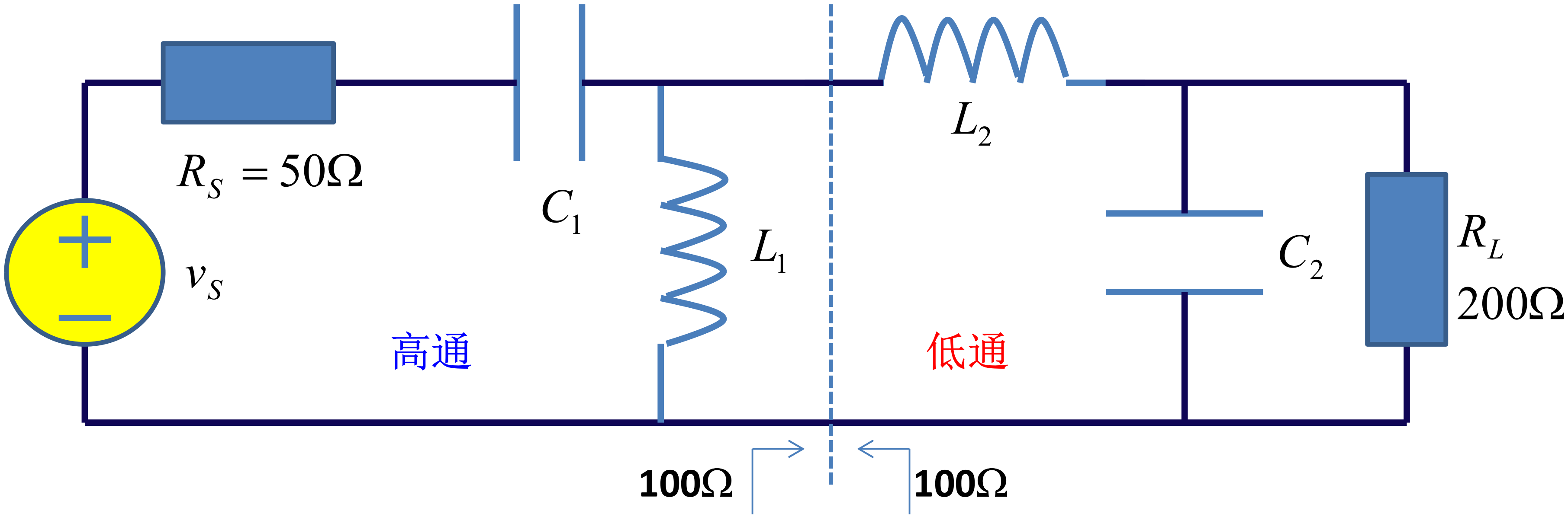
```
RS=50;
RL=200;

f0=10E6;

L=RS/(2*pi*f0)*sqrt(RL/RS-1);
C=1/(2*pi*f0*RL)*sqrt(RL/RS-1);

freqstart=f0/100;
freqstop=f0*100;
freqnum=1000;
freqstep=10^(log10(freqstop/freqstart)/freqnum);

freq=freqstart/freqstep;

for k=1:freqnum
   freq=freq*freqstep;
   f(k)=freq;

   w=2*pi*freq;
   s=i*w;
   ABCD=[1 RS+s*L;0 1]*[1 0; s*C+1/RL 1];

   H=2*sqrt(RS/RL)/ABCD(1,1);
   absH(k)=20*log10(abs(H));
end

figure(1)
plot(f,absH)
```





$$BW_{1dB} = 12.96MHz - 5.64MHz$$
$$= 7.32MHz$$

先设计一个可将50Ω变换为100Ω的低通型L型匹配网络，再设计一个可将100Ω变换为200Ω的高通型L型匹配网络，将这两个匹配网络级联，用数值方法考察总网络传递函数确认匹配网络设计成功。通过幅频特性曲线，确认1dB匹配带宽，和低通L型匹配网络比，带宽是变宽了还是变窄了？



$$Q_1 = \sqrt{\frac{R_r}{R_S} - 1} = \sqrt{\frac{100}{50} - 1} = 1$$

$$L_1 = \frac{R_r}{\omega_r Q_1} = \frac{100}{2\times 3.14\times 10\times 10^6 \times 1} = 1.592\mu H$$

$$C_1 = \frac{1}{\omega_r R_S Q_1} = \frac{1}{2\times 3.14\times 10\times 10^6 \times 50\times 1} = 318.3pF$$

$$Q_2 = \sqrt{\frac{R_L}{R_r} - 1} = \sqrt{\frac{200}{100} - 1} = 1$$

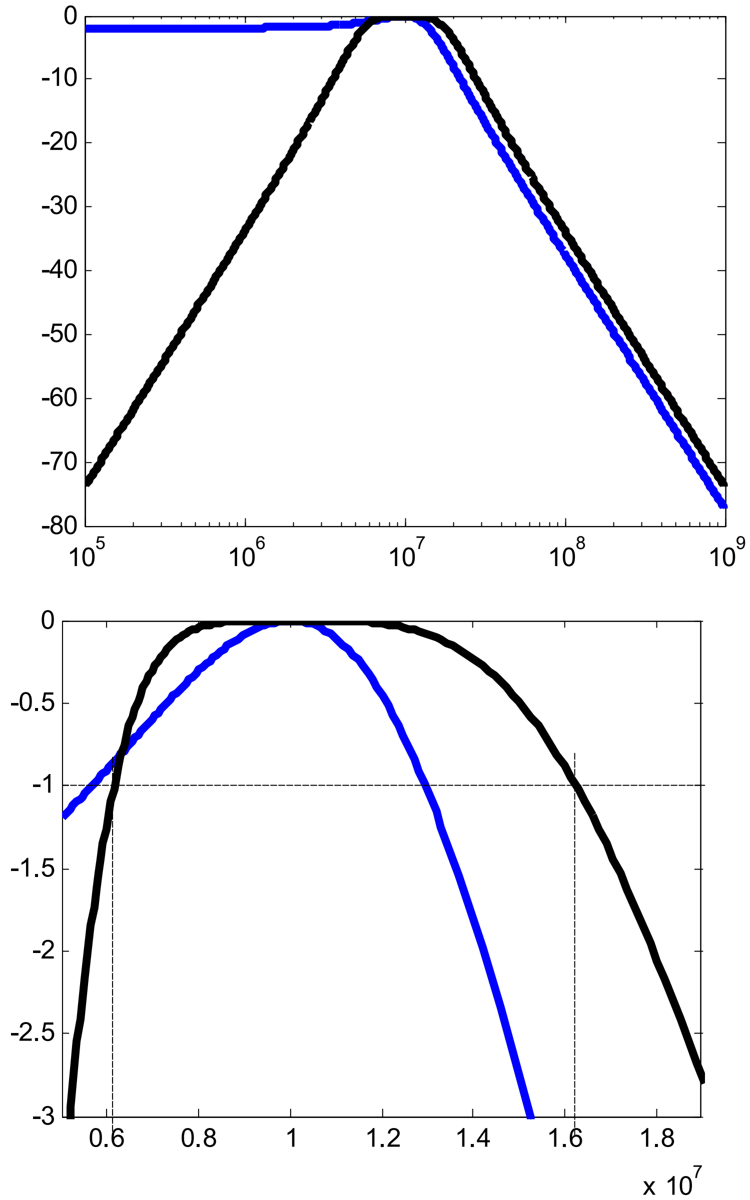
$$L_2 = \frac{R_r}{\omega_r} Q_2 = \frac{100}{2\times 3.14\times 10\times 10^6} \times 1 = 1.592\mu H$$

$$C_2 = \frac{1}{\omega_r R_L} Q_2 = \frac{1}{2\times 3.14\times 10\times 10^6 \times 200} \times 1 = 79.58pF$$

1dB带宽

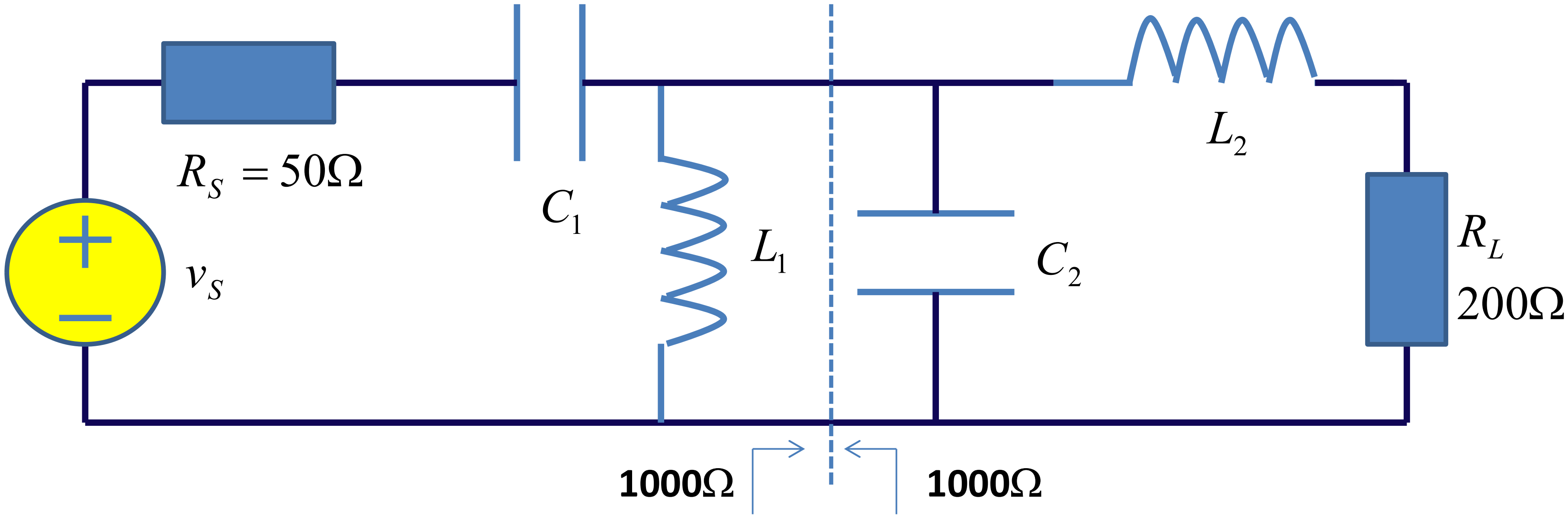
```
RS=50;
RL=200;
Rr=100;
 f0=10E6;

L=RS/(2*pi*f0)*sqrt(RL/RS-1);
C=1/(2*pi*f0*RL)*sqrt(RL/RS-1);

L2=Rr/(2*pi*f0)*sqrt(RL/Rr-1);
C2=1/(2*pi*f0*RL)*sqrt(RL/Rr-1);
 L1=Rr/(2*pi*f0*sqrt(Rr/RS-1));
C1=1/(2*pi*f0*RS*sqrt(Rr/RS-1));

freqstart=f0/100;
freqstop=f0*100;
freqnum=1000;
freqstep=10^(log10(freqstop/freqstart)/freqnum);
 freq=freqstart/freqstep;

for k=1:freqnum
   freq=freq*freqstep;
   f(k)=freq;

   w=2*pi*freq;
   s=i*w;
   ABCD=[1 RS+s*L;0 1]*[1 0; s*C+1/RL 1];
   H=2*sqrt(RS/RL)/ABCD(1,1);
   absH(k)=20*log10(abs(H));

   ABCD=[1 RS+1/(s*C1);0 1]*[1 0; 1/(s*L1) 1]*[1 s*L2;0 1]*[1 0; s*C2+1/RL,1];
    H2=2*sqrt(RS/RL)/ABCD(1,1);
   absH2(k)=20*log10(abs(H2));
end
```

```
figure(1)
hold on
plot(f,absH)
plot(f,absH2)
```



$$BW_{1dB} = 16.25MHz - 6.15MHz$$
$$= 10.10MHz$$

（3）设计一个可将50Ω变换为1kΩ的低通型L型匹配网络，再设计一个可将1kΩ变换为200Ω的高通型L型匹配网络，将这两个匹配网络级联，用数值方法考察总网络传递函数确认匹配网络设计成功。通过幅频特性曲线确认1dB匹配带宽，和低通L型匹配网络比，带宽是变宽了还是变窄了？



$$Q_1=\sqrt{\frac{R_r}{R_S}-1}=\sqrt{\frac{1000}{50}-1}=\sqrt{19}=4.359$$

$$L_1=\frac{R_r}{\omega_r Q_1}=\frac{1000}{2\times3.14\times10\times10^6\times4.359}=3.651\mu H$$

$$C_1=\frac{1}{\omega_r R_S Q_1}=\frac{1}{2\times3.14\times10\times10^6\times50\times4.359}=73.03pF$$

$$Q_2=\sqrt{\frac{R_r}{R_L}-1}=\sqrt{\frac{1000}{200}-1}=2$$

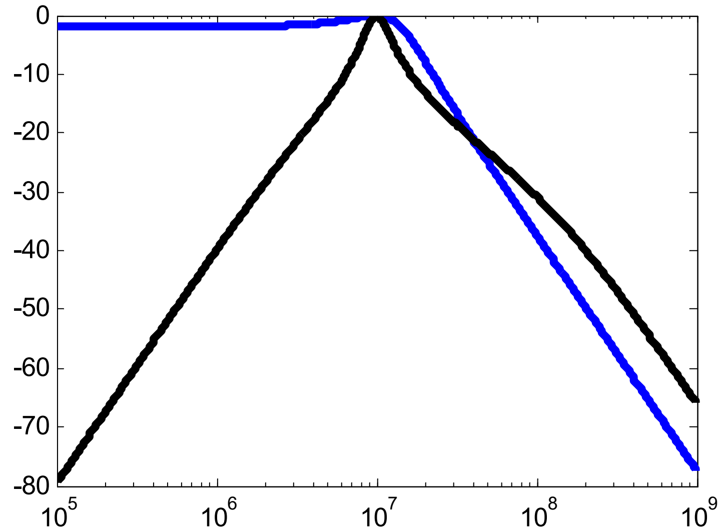
$$L_2=\frac{R_L}{\omega_r}Q_2=\frac{200}{2\times3.14\times10\times10^6}\times2=6.366\mu H$$

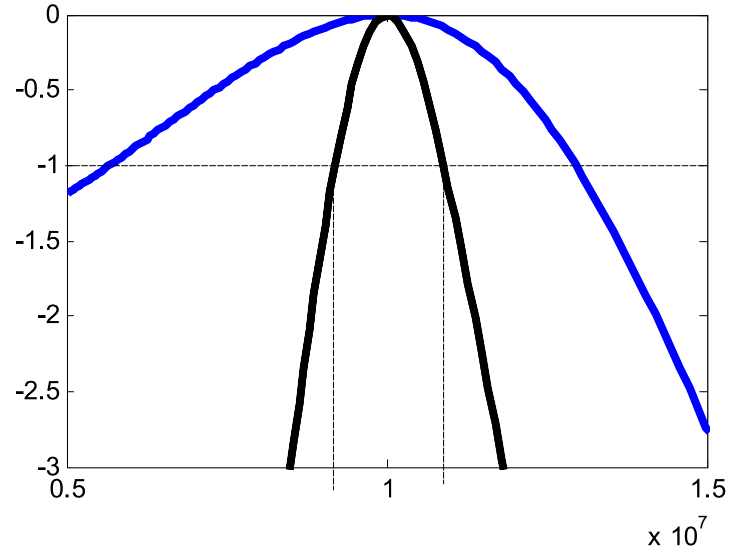
$$C_2=\frac{1}{\omega_r R_r}Q_2=\frac{1}{2\times3.14\times10\times10^6\times1000}\times2=31.83pF$$

1dB带宽

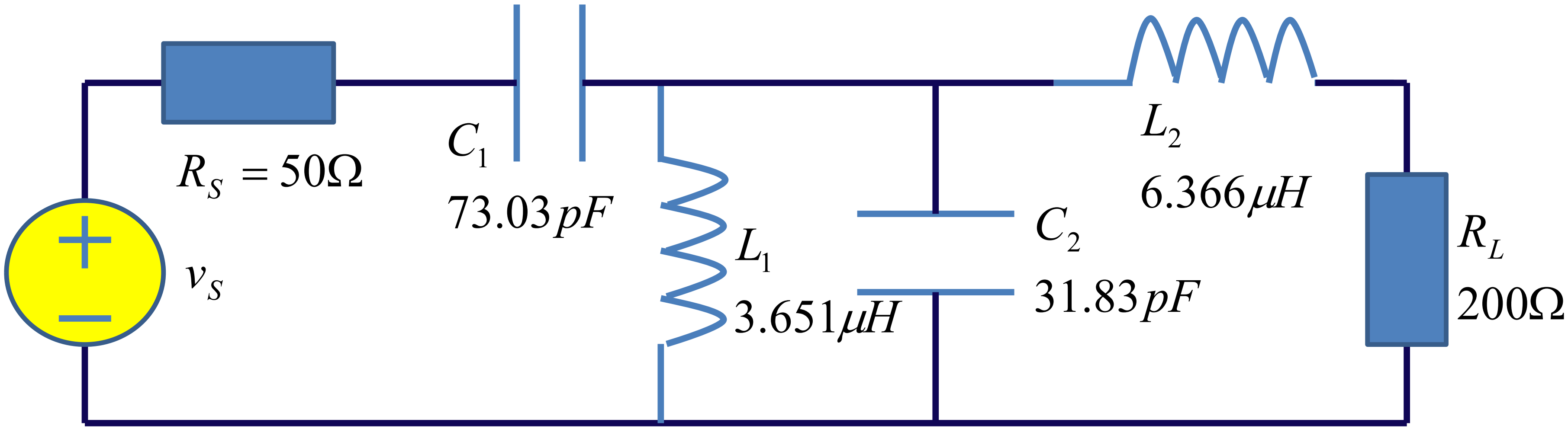
```
• RS=50;
• RL=200;
• Rr=1000;
•  f0=10E6;
•
• L=RS/(2*pi*f0)*sqrt(RL/RS-1);
• C=1/(2*pi*f0*RL)*sqrt(RL/RS-1);
•
• L2=RL/(2*pi*f0)*sqrt(Rr/RL-1);
• C2=1/(2*pi*f0*Rr)*sqrt(Rr/RL-1);
•  L1=Rr/(2*pi*f0*sqrt(Rr/RS-1));
• C1=1/(2*pi*f0*RS*sqrt(Rr/RS-1));
•
• freqstart=f0/100;
• freqstop=f0*100;
• freqnum=1000;
• freqstep=10^(log10(freqstop/freqstart)/freqnum);
•  freq=freqstart/freqstep;
•
• for k=1:freqnum
•    freq=freq*freqstep;
•    f(k)=freq;
•
•    w=2*pi*freq;
•    s=i*w;
•    ABCD=[1 RS+s*L;0 1]*[1 0; s*C+1/RL 1];
•    H=2*sqrt(RS/RL)/ABCD(1,1);
•    absH(k)=20*log10(abs(H));
•
•    ABCD=[1 RS+1/(s*C1);0 1]*[1 0; 1/(s*L1)+s*C2 1]*[1
  s*L2;0 1]*[1 0; 1/RL,1];
•    H2=2*sqrt(RS/RL)/ABCD(1,1);
•    absH2(k)=20*log10(abs(H2));
• end
```

```
figure(2)
hold on
plot(f,absH)
plot(f,absH2)
```



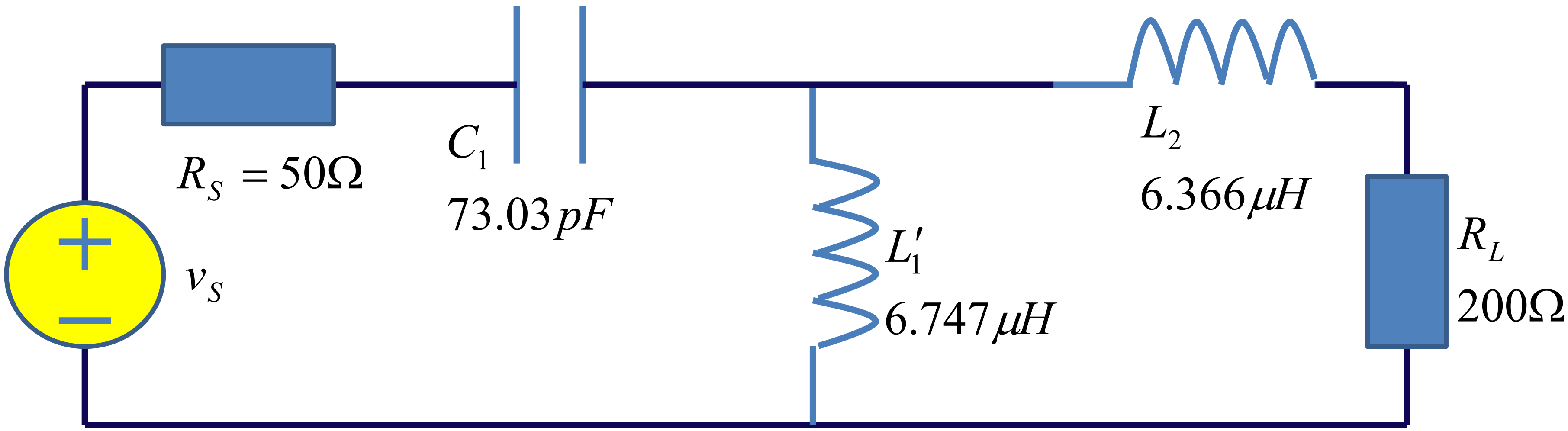


$$BW_{1dB} = 10.89MHz - 9.19MHz$$
$$= 1.70MHz$$



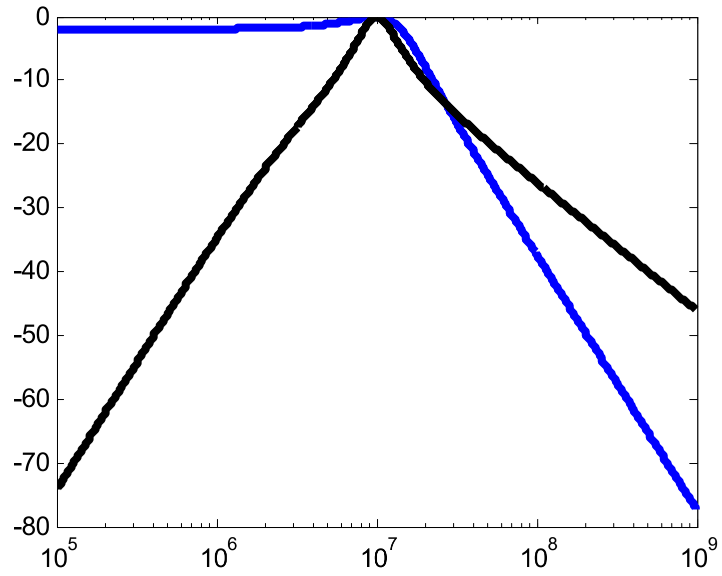
$$3.651\mu H = 6.747\mu H \parallel 7.958\mu H$$

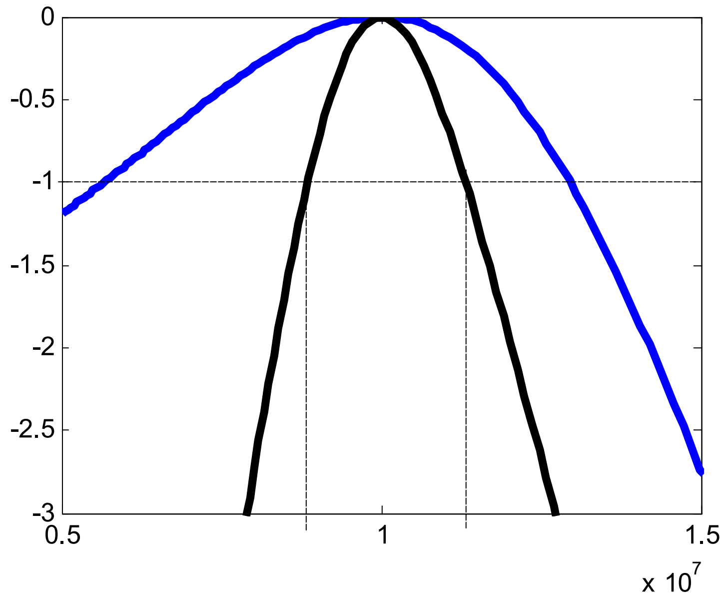
T型匹配网络



1dB带宽

```
RS=50;
RL=200;
Rr=1000;

f0=10E6;

L=RS/(2*pi*f0)*sqrt(RL/RS-1);
C=1/(2*pi*f0*RL)*sqrt(RL/RS-1);

L2=RL/(2*pi*f0)*sqrt(Rr/RL-1);
C2=1/(2*pi*f0*Rr)*sqrt(Rr/RL-1);
L1=Rr/(2*pi*f0*sqrt(Rr/RS-1));
C1=1/(2*pi*f0*RS*sqrt(Rr/RS-1));

L3=1/(2*pi*f0)^2/C2;
L1=1/(1/L1-1/L3);

freqstart=f0/100;
freqstop=f0*100;
freqnum=1000;
freqstep=10^(log10(freqstop/freqstart)/freqnum);

freq=freqstart/freqstep;

for k=1:freqnum
   freq=freq*freqstep;
   f(k)=freq;

   w=2*pi*freq;
   s=i*w;
   ABCD=[1 RS+s*L;0 1]*[1 0; s*C+1/RL 1];
   H=2*sqrt(RS/RL)/ABCD(1,1);
   absH(k)=20*log10(abs(H));

   ABCD=[1 RS+1/(s*C1);0 1]*[1 0; 1/(s*L1) 1]*[1 s*L2;0 1]*[1 0;
1/RL,1];
   H2=2*sqrt(RS/RL)/ABCD(1,1);
   absH2(k)=20*log10(abs(H2));
end
```





$$BW_{1dB} = 11.32MHz - 8.85MHz$$
$$= 2.47MHz$$

更复杂的匹配网络

- **L**型匹配网络是最简单的匹配网络，其匹配带宽由$\mathbf{R_S/R_L}$决定
  - 两个电阻相差越大，匹配带宽越窄
    - **Q**值越大，带宽越窄

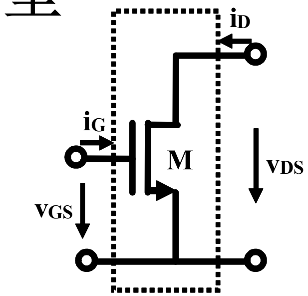
$$Q=\sqrt{\frac{R_{大}}{R_{小}}-1}$$

- 可以通过更复杂的匹配网络调整匹配带宽
  - 匹配带宽可以调得更宽
    - 中间阻抗位于$\mathbf{R_S}$、$\mathbf{R_L}$之间：降低**Q**值
  - 匹配带宽可以调得更窄
    - 中间阻抗位于$\mathbf{R_S}$、$\mathbf{R_L}$范围之外：提高**Q**值

第11讲 晶体管电路的回顾与拓展

作业1 为何晶体管需要工作在恒流区

- 当晶体管做放大管使用时，需要将其偏置在恒流导通区，此区晶体管具有最大增益
  - 分析**CS**组态晶体管在不同工作区的跨导增益和电压增益
    - 交流小信号分析，分析其**y**参量矩阵及其模型

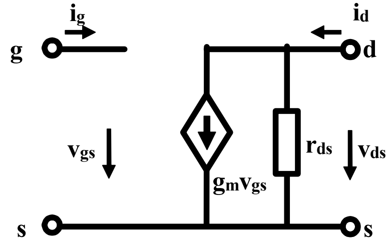


$$i_G = 0$$

$$i_D = \begin{cases} 0 & v_{GS} < V_{TH} \\ 2\beta_n\left(\left(v_{GS} - V_{TH}\right)v_{DS} - 0.5v_{DS}^2\right) & v_{GS} > V_{TH}, v_{GD} > V_{TH} \\ \beta_n\left(v_{GS} - V_{TH}\right)^2\left(1 + \lambda v_{DS}\right) & v_{GS} > V_{TH}, v_{GD} < V_{TH} \end{cases}$$

$$\beta_n = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}$$

有源区交流小信号模型建立



$$i_G = f_G(v_{GS}, v_{DS}) = 0$$

$$\mathbf{y}_{MOSFET} = \begin{bmatrix} \frac{\partial f_G}{\partial v_{GS}} & \frac{\partial f_G}{\partial v_{DS}} \\ \frac{\partial f_D}{\partial v_{GS}} & \frac{\partial f_D}{\partial v_{DS}} \end{bmatrix}_{v_{GS}=V_{GS0}, v_{DS}=V_{DS0}} = \begin{bmatrix} 0 & 0 \\ g_m & g_{ds} \end{bmatrix}$$

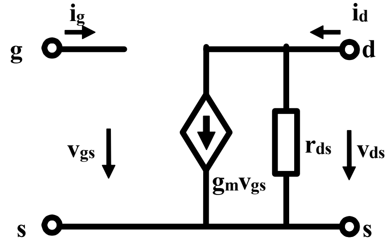
$$i_D = f_D(v_{GS}, v_{DS}) = \beta_n (v_{GS} - V_{TH})^2 \left(1 + \frac{v_{DS}}{V_A}\right)$$

$$I_{D0} = f_D(V_{GS0}, V_{DS0}) = \beta_n (V_{GS0} - V_{TH})^2 \left(1 + \frac{V_{DS0}}{V_A}\right) \approx \beta_n (V_{GS0} - V_{TH})^2$$

$$g_m = \frac{\partial f_D}{\partial v_{GS}}\bigg|_{v_{GS}=V_{GS0}, v_{DS}=V_{DS0}} = 2\beta_n (V_{GS0} - V_{TH})\left(1 + \frac{V_{DS0}}{V_A}\right) = \frac{2I_{D0}}{V_{GS0} - V_{TH}} = \frac{2I_{D0}}{V_{od}}$$

$$g_{ds} = \frac{\partial f_D}{\partial v_{DS}}\bigg|_{v_{GS}=V_{GS0}, v_{DS}=V_{DS0}} = \beta_n (V_{GS0} - V_{TH})^2 \frac{1}{V_A} \approx \frac{I_{D0}}{V_A}$$

欧姆区交流小信号电路模型



$$i_G = f_G(v_{GS}, v_{DS}) = 0$$

$$\mathbf{y}_{MOSFET} = \begin{bmatrix} \frac{\partial f_G}{\partial v_{GS}} & \frac{\partial f_G}{\partial v_{DS}} \\ \frac{\partial f_D}{\partial v_{GS}} & \frac{\partial f_D}{\partial v_{DS}} \end{bmatrix}_{v_{GS}=V_{GS0}, v_{DS}=V_{DS0}} = \begin{bmatrix} 0 & 0 \\ g_m & g_{ds} \end{bmatrix}$$

$$i_D = f_D(v_{GS}, v_{DS}) = 2\beta_n\left((v_{GS} - V_{TH})v_{DS} - 0.5v_{DS}^2\right)$$

$$g_m = \frac{\partial f_D}{\partial v_{GS}}\bigg|_{v_{GS}=V_{GS0}, v_{DS}=V_{DS0}} = 2\beta_n V_{DS0}$$

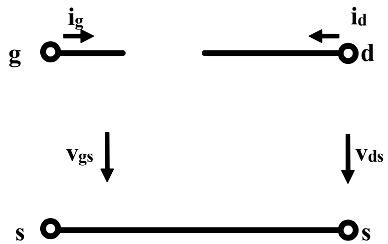
$$g_{ds} = \frac{\partial f_D}{\partial v_{DS}}\bigg|_{v_{GS}=V_{GS0}, v_{DS}=V_{DS0}} = 2\beta_n\left(V_{GS0} - V_{TH} - V_{DS0}\right)$$

截止区交流小信号电路模型

$$i_G = f_G(v_{GS}, v_{DS}) = 0$$

$$i_D = f_D(v_{GS}, v_{DS}) = 0$$

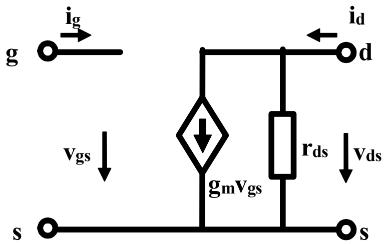
$$y_{MOSFET} = \begin{bmatrix} \dfrac{\partial f_G}{\partial v_{GS}} & \dfrac{\partial f_G}{\partial v_{DS}} \\ \dfrac{\partial f_D}{\partial v_{GS}} & \dfrac{\partial f_D}{\partial v_{DS}} \end{bmatrix}_{v_{GS}=V_{GS0},\, v_{DS}=V_{DS0}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

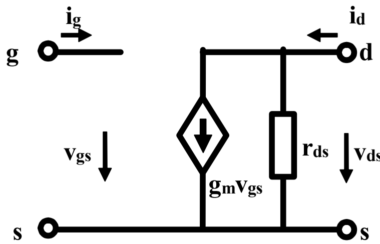


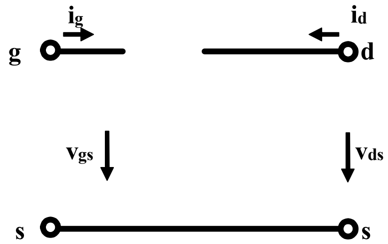
电路模型

做放大器使用时，绝大部分情况下晶体管偏置在恒流区，极个别应用晶体管偏置在欧姆区

欧姆导通区 $V_{DS0} < V_{GS0} - V_{TH}$　　恒流导通区 $V_{DS0} > V_{GS0} - V_{TH}$　　截止区







$$i_D = f_D(v_{GS}, v_{DS}) = 2\beta_n\left((v_{GS} - V_{TH})v_{DS} - 0.5v_{DS}^2\right)$$

未做任何修正

$$i_D = f_D(v_{GS}, v_{DS}) = \beta_n(v_{GS} - V_{TH})^2\left(1 + \frac{v_{DS}}{V_A}\right)$$

厄利效应修正

两个模型在分界点$V_{DS0}=V_{GS0}-V_{TH}$不匹配，有断点，实际晶体管不会有断点

$$g_m = 2\beta_n V_{DS0}$$

跨导增益小

$$g_m \approx 2\beta_n(V_{GS0} - V_{TH})$$

$$g_{ds} = 2\beta_n(V_{GS0} - V_{TH} - V_{DS0})$$

输出电导大

$$g_{ds} = \beta_n(V_{GS0} - V_{TH})^2 \frac{1}{V_A}$$

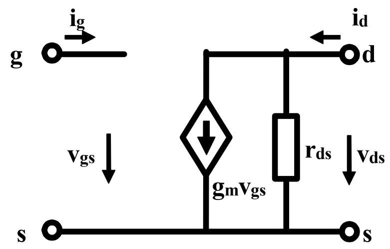
$$A_{v0} = -\frac{g_m}{g_{ds}} = -\frac{V_{DS0}}{V_{GS0} - V_{TH} - V_{DS0}}$$

电压增益小

$$A_{v0} = -\frac{g_m}{g_{ds}} = -\frac{2V_A}{V_{GS0} - V_{TH}}$$

模型不充分，在$V_{DS0}=V_{GS0}-V_{TH}$饱和电压位置，饱和区电压增益高于恒流区电压增益。事实上，假设偏置电压$V_{GS0}>V_{TH}$不变，随着输出端口直流工作点电压V_{DS0}的增加，从欧姆区到恒流区，电压增益越来越大，直至工作点进入恒流区，电压增益几乎保持不变。

欧姆区模型修正



$$i_G = f_G(v_{GS}, v_{DS}) = 0$$

$$\mathbf{y}_{MOSFET} = \begin{bmatrix} \frac{\partial f_G}{\partial v_{GS}} & \frac{\partial f_G}{\partial v_{DS}} \\ \frac{\partial f_D}{\partial v_{GS}} & \frac{\partial f_D}{\partial v_{DS}} \end{bmatrix}_{v_{GS}=V_{GS0}, v_{DS}=V_{DS0}} = \begin{bmatrix} 0 & 0 \\ g_m & g_{ds} \end{bmatrix}$$

$$i_D = f_D(v_{GS}, v_{DS}) = 2\beta_n\left((v_{GS} - V_{TH})v_{DS} - 0.5v_{DS}^2\right)(1 + \lambda v_{DS})$$

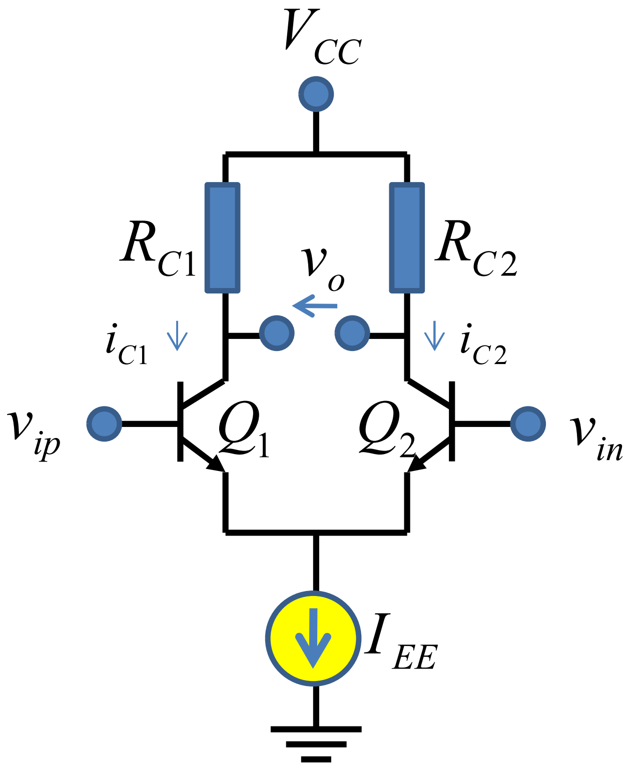
欧姆区v_{DS}本来就很小，这个修正几乎不影响原欧姆区电流大小，但是可确保欧姆区和恒流区是连续的

$$g_m = \frac{\partial f_D}{\partial v_{GS}}\bigg|_{v_{GS} = V_{GS0}, v_{DS} = V_{DS0}} = 2\beta_n V_{DS0}(1 + \lambda V_{DS0})$$

$$g_{ds} = \frac{\partial f_D}{\partial v_{DS}}\bigg|_{v_{GS} = V_{GS0}, v_{DS} = V_{DS0}} = 2\beta_n(V_{GS0} - V_{TH} - V_{DS0})$$

$$+2\beta_n\lambda(2(V_{GS0} - V_{TH})V_{DS0} - 1.5V_{DS0}^2)$$

作业2：BJT差分对跨导转移特性



证明**BJT**差分对跨导控制关系：

$$i_d = i_{C1} - i_{C2} = f(v_{id}) = I_{EE}\tanh\frac{v_{id}}{2v_T}$$

已知**BJT**跨导控制关系

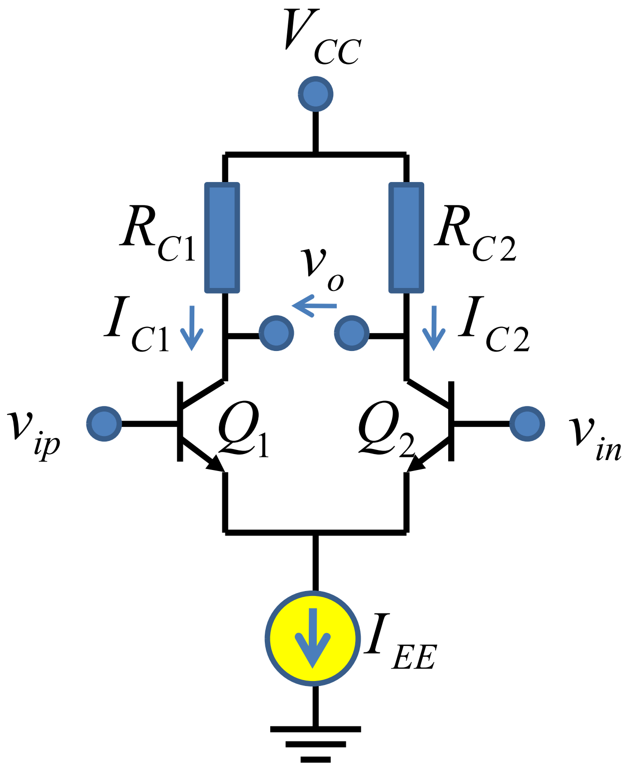
$$i_b \approx 0$$

$$i_c \approx I_{CS0} e^{\frac{v_{BE}}{v_T}}$$

忽略β、V_A的影响

$\beta\to\infty$, $V_A\to\infty$

BJT差分对跨导转移特性



$$v_o = (V_{CC} - I_{C2}R_{C2}) - (V_{CC} - I_{C1}R_{C1})$$
$$= (I_{C1} - I_{C2})R_C = i_d R_C$$

输出差分电流和差分电压的线性关系

$$I_{C1} \approx I_{CS1} e^{\frac{V_{BE1}}{v_T}} = I_{CS0} e^{\frac{V_{BE1}}{v_T}}$$

忽略厄利效应
$V_A \to \infty$

$$I_{C2} \approx I_{CS2} e^{\frac{V_{BE2}}{v_T}} = I_{CS0} e^{\frac{V_{BE2}}{v_T}}$$

$$I_{EE} = I_{E1} + I_{E2} = \frac{1}{\alpha}(I_{C1} + I_{C2}) \approx I_{C1} + I_{C2}$$

电流增益$\beta \to \infty$

$$v_{id} = V_{BE1} - V_{BE2}$$

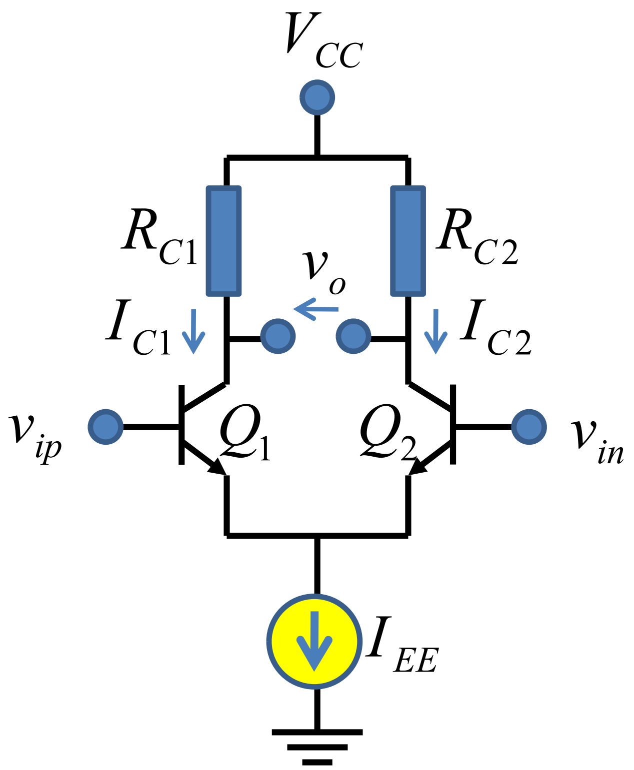
$$I_{C1} = I_{CS0} e^{\frac{V_{BE1}}{v_T}}$$

$$V_{BE1} = v_T \ln \frac{I_{C1}}{I_{CS0}}$$

$$I_{C2} = I_{CS0} e^{\frac{V_{BE2}}{v_T}}$$

$$V_{BE2} = v_T \ln \frac{I_{C2}}{I_{CS0}}$$

$$v_{id} = V_{BE1} - V_{BE2} = v_T \ln \frac{I_{C1}}{I_{C2}}$$



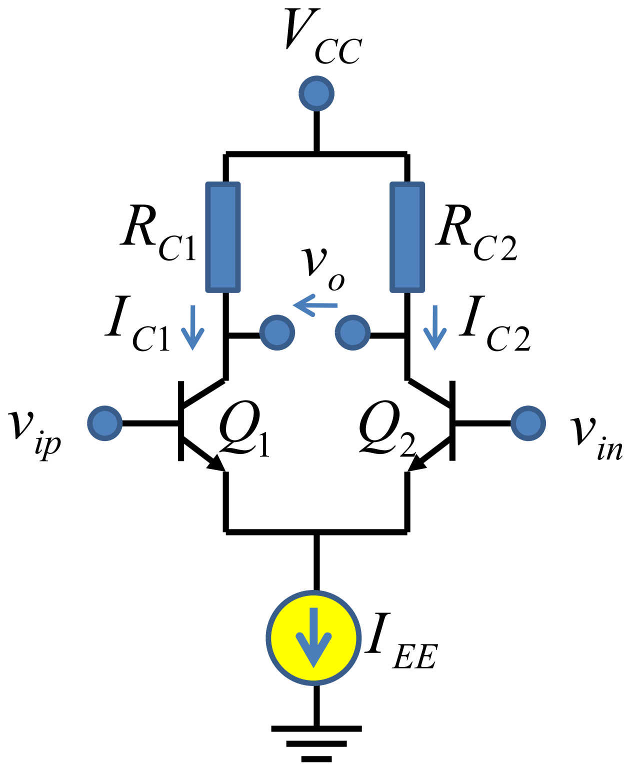
$$\frac{I_{C1}}{I_{C2}} = e^{\frac{v_{id}}{v_T}}$$

$$I_{C2} = \frac{I_{EE}}{1 + e^{+\frac{v_{id}}{v_T}}}$$

$$I_{C1} + I_{C2} = I_{EE}$$

$$I_{C1} = \frac{I_{EE}}{1 + e^{-\frac{v_{id}}{v_T}}}$$

$$I_{C1}=\frac{I_{EE}}{1+e^{-\frac{v_{id}}{v_T}}}\qquad I_{C2}=\frac{I_{EE}}{1+e^{+\frac{v_{id}}{v_T}}}$$



$$i_d=I_{C1}-I_{C2}=\frac{e^{\frac{v_{id}}{v_T}}-1}{e^{\frac{v_{id}}{v_T}}+1}I_{EE}$$

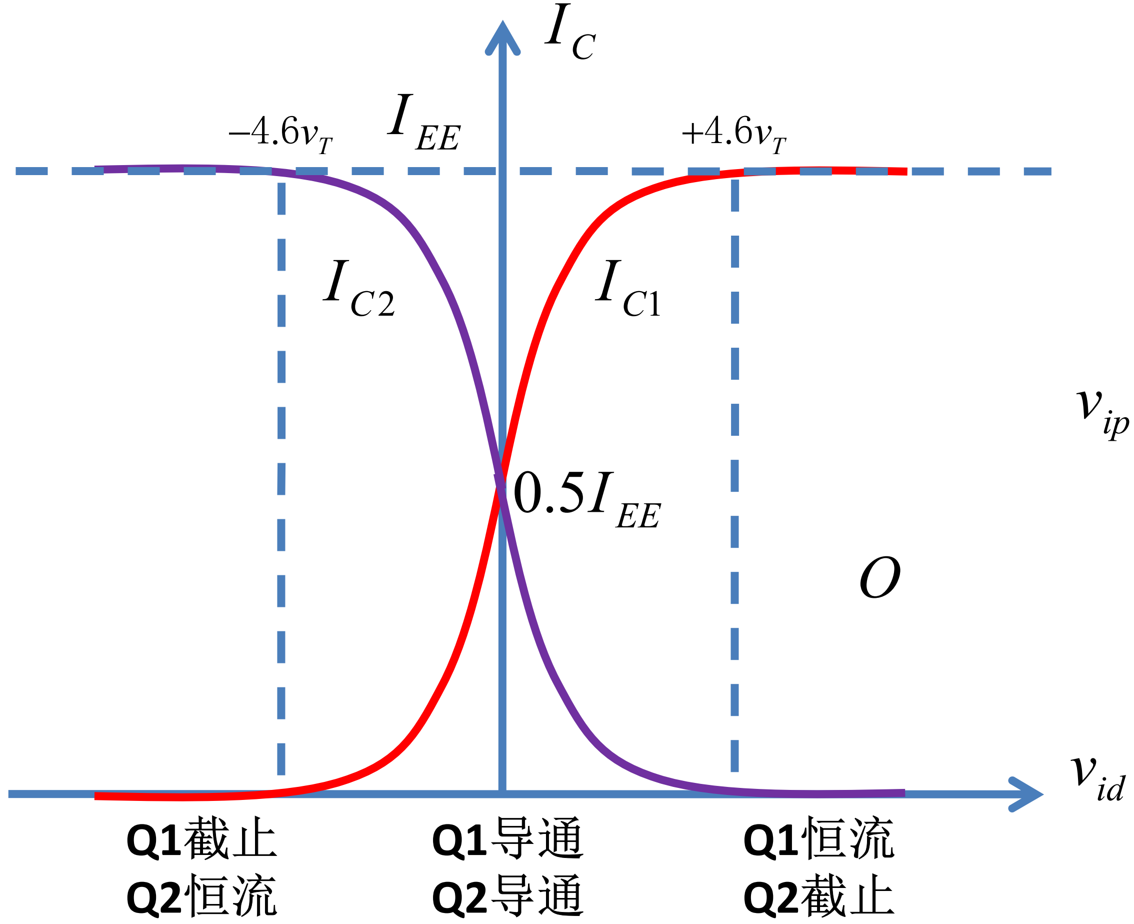
$$=\frac{e^{\frac{v_{id}}{2v_T}}-e^{-\frac{v_{id}}{2v_T}}}{e^{\frac{v_{id}}{2v_T}}+e^{-\frac{v_{id}}{2v_T}}}I_{EE}=I_{EE}\tanh\frac{v_{id}}{2v_T}$$

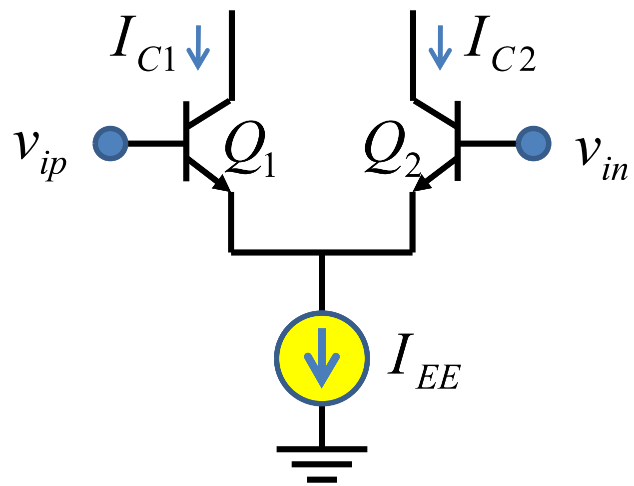
$$I_{C1}=0.5I_{EE}+0.5i_d$$
$$=0.5I_{EE}\left(1+\tanh\frac{v_{id}}{2v_T}\right)$$

$$I_{C2}=0.5I_{EE}-0.5i_d$$
$$=0.5I_{EE}\left(1-\tanh\frac{v_{id}}{2v_T}\right)$$

差分对差分电压—电流转移特性

$$I_{C1} = 0.5 I_{EE}\left(1 + \tanh\frac{v_{id}}{2v_T}\right) \qquad I_{C2} = 0.5 I_{EE}\left(1 - \tanh\frac{v_{id}}{2v_T}\right)$$





作业4：1dB线性范围

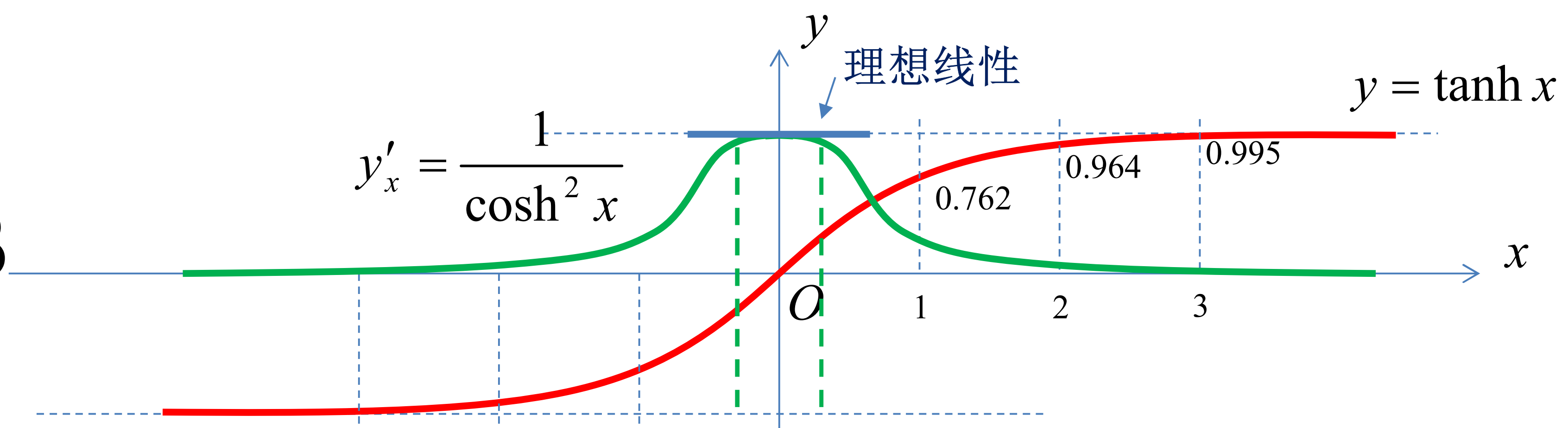
- 求差分对管的1dB线性范围

$$i_d = \begin{cases} +I_{SS} & v_{id} \ge +\sqrt{2}V_{od0} \\ I_{SS}\dfrac{v_{id}}{V_{od0}}\sqrt{1-\dfrac{1}{4}\left(\dfrac{v_{id}}{V_{od0}}\right)^2} & |v_{id}| \le \sqrt{2}V_{od0} \\ -I_{SS} & v_{id} \le -\sqrt{2}V_{od0} \end{cases}$$

$$\overset{1dB线性范围内}{\triangleq} I_{SS}\frac{v_{id}}{V_{od0}} = \frac{I_{SS}}{V_{od0}}v_{id} = g_{m0}v_{id}$$

$$i_d = I_{EE}tanh\frac{v_{id}}{2v_T} \overset{1dB线性范围内}{\triangleq} I_{EE}\frac{v_{id}}{2v_T} = \frac{I_{EE}}{2v_T}v_{id} = g_{m0}v_{id}$$

BJT差分对管



最大的线性区在**x=0**位置

$$i_d = I_{EE}\tanh\frac{v_{id}}{2v_T}$$

$$g_m = \frac{di_d}{dv_{id}} = \frac{I_{EE}}{2v_T}\frac{1}{\cosh^2\frac{v_{id}}{2v_T}}$$

$$g_{m0} = \frac{di_d}{dv_{id}}|(v_{id}=0) = \frac{I_{EE}}{2v_T}$$

$$\frac{g_{m0}}{g_m} = \cosh^2\frac{v_{id}}{2v_T}$$

$$1dB = 20\log\left(\frac{g_{m0}}{g_m}\right) = 20\log\left(\cosh^2\frac{v_{id,1dB}}{2v_T}\right)$$

$$\cosh^2\frac{v_{id,1dB}}{2v_T} = 10^{\frac{1}{20}}$$

$$\cosh\frac{v_{id,1dB}}{2v_T} = 10^{\frac{1}{40}} = 1.059$$

$$v_{id,1dB} = 2v_T\cdot\cosh^{-1}1.059$$
$$= \pm 0.685v_T = \pm 17.8mV \approx \pm 18mV$$

$$i_d = I_{EE}\tanh\frac{v_{id}}{2v_T} \overset{|v_{id}|\le 18mV}{\approx} I_{EE}\frac{v_{id}}{2v_T} = \frac{I_{EE}}{2v_T}v_{id} = g_{m0}v_{id}$$

MOS差分对管

$$i_d = \frac{I_{SS}}{V_{od0}} v_{id} \sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{od0}}\right)^2}$$

$$g_m = \frac{di_d}{dv_{id}} = \frac{I_{SS}}{V_{od0}} \frac{1 - \frac{1}{2}\left(\frac{v_{id}}{V_{od}}\right)^2}{\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{od0}}\right)^2}}$$

$$g_{m0} = \frac{di_d}{dv_{id}} |(v_{id} = 0) = \frac{I_{SS}}{V_{od0}}$$

$$\frac{g_{m0}}{g_m} = \frac{\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{od0}}\right)^2}}{1 - \frac{1}{2}\left(\frac{v_{id}}{V_{od0}}\right)^2}$$

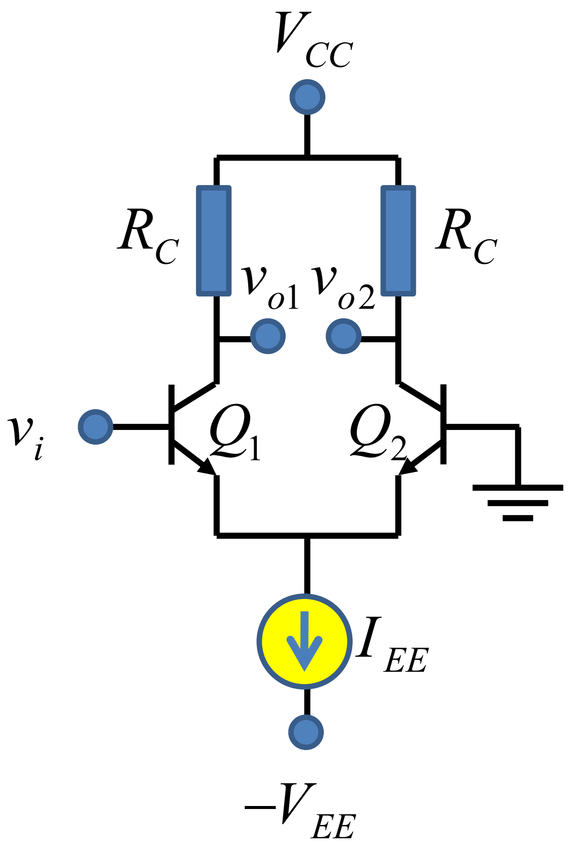
$$1dB = 20\log\left(\frac{g_{m0}}{g_m}\right) = 20\log\left(\frac{\sqrt{1 - \frac{1}{4}\left(\frac{v_{id,1dB}}{V_{od0}}\right)^2}}{1 - \frac{1}{2}\left(\frac{v_{id,1dB}}{V_{od0}}\right)^2}\right)$$

$$\frac{\sqrt{1 - \frac{1}{4}\left(\frac{v_{id,1dB}}{V_{od0}}\right)^2}}{1 - \frac{1}{2}\left(\frac{v_{id,1dB}}{V_{od0}}\right)^2} = 10^{\frac{1}{20}}$$

$$v_{id,1dB} = 0.53 V_{od0}$$

$$i_d = \frac{I_{SS}}{V_{od0}} v_{id} \sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{od0}}\right)^2} \overset{|v_{id}| \le 0.53\ od0}{\approx} \frac{I_{SS}}{V_{od0}} v_{id} = g_{m0} v_{id}$$

作业3：差分放大器单端转双端



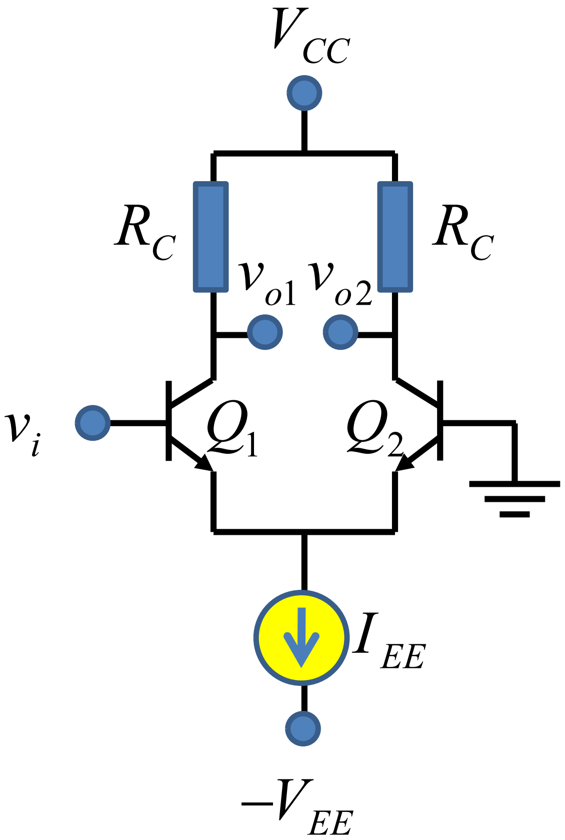
$I_{EE} = 1mA$

- 电源电压为±**10V**，差分对管参数一致，**R_C=3kΩ**，画出如下三种输入情况下的两个输出电压**v_{o1},v_{o2}**的波形示意图

$$v_i = 10\sin\left(2\pi\times10^3 t\right)(mV)$$

$$v_i = 0.5\sin\left(2\pi\times10^3 t\right)(V)$$

$$v_i = 50 + 100\sin\left(2\pi\times10^3 t\right)(mV)$$



$I_{EE} = 1mA$

$$v_{ip} = v_i \qquad v_{ic} = \frac{v_{ip} + v_{in}}{2} = \frac{v_i}{2}$$

$$v_{in} = 0 \qquad v_{id} = v_{ip} - v_{in} = v_i$$

$$i_d = I_{EE} \tanh\frac{v_{id}}{2v_T} = I_{EE} \tanh\frac{v_i}{2v_T}$$

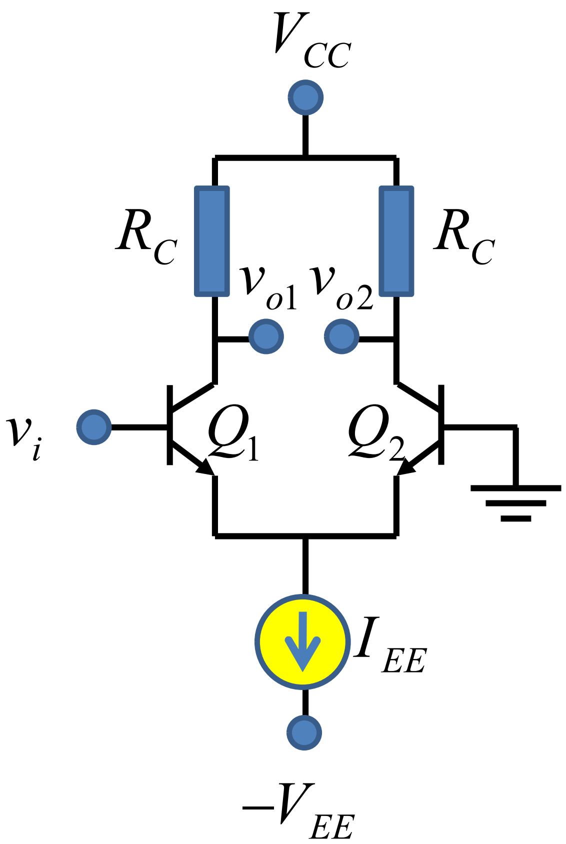
$$I_{C1} = 0.5I_{EE}\left(1 + \tanh\frac{v_{id}}{2v_T}\right) \qquad I_{C2} = 0.5I_{EE}\left(1 - \tanh\frac{v_{id}}{2v_T}\right)$$

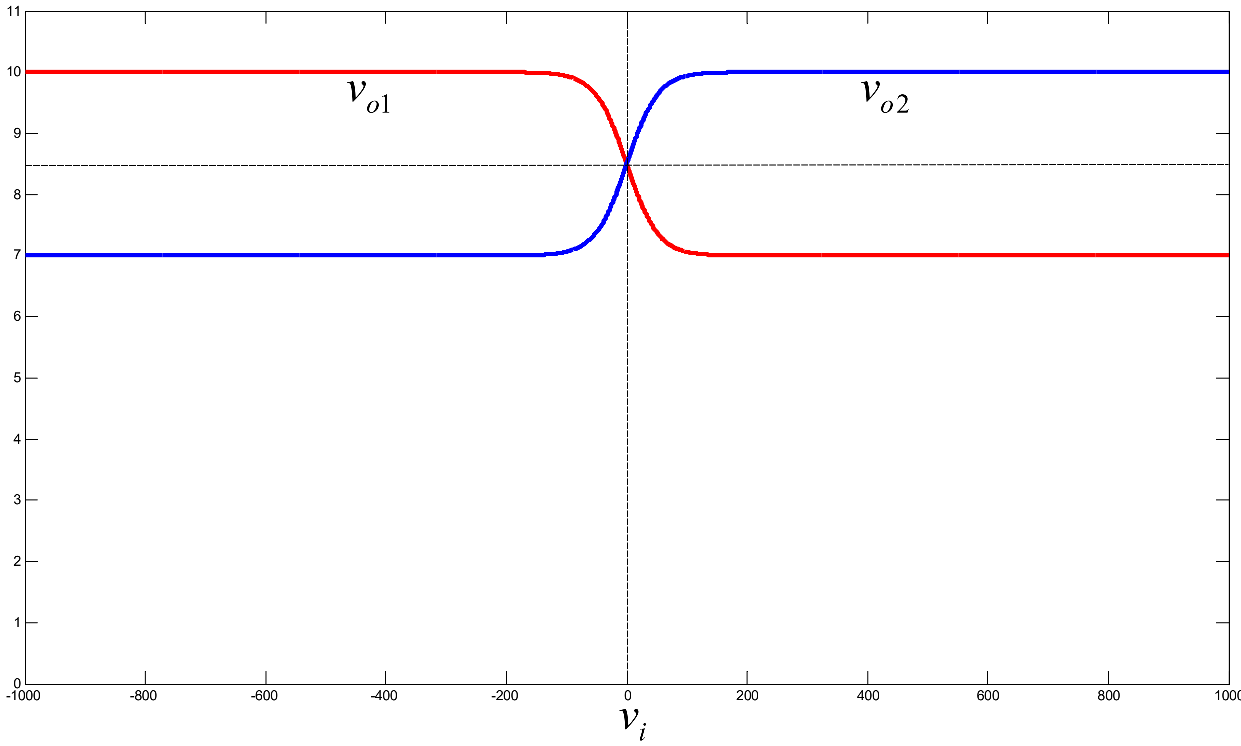
$$v_{o1} = V_{CC} - I_{C1}R_C = V_{CC} - 0.5I_{EE}R_C - 0.5I_{EE}R_C \tanh\frac{v_i}{2v_T}$$

$$= 10 - 0.5 \times 1m \times 3k - 0.5 \times 1m \times 3k \times \tanh\frac{v_i}{2v_T} = 8.5 - 1.5\tanh\frac{v_i}{2v_T}$$

$$v_{o2} = V_{CC} - I_{C2}R_C = 8.5 + 1.5\tanh\frac{v_i}{2v_T}$$

$$v_{o1} = 8.5 - 1.5\tanh\frac{v_i}{2v_T} \qquad v_{o2} = 8.5 + 1.5\tanh\frac{v_i}{2v_T}$$





$$I_{EE} = 1mA$$

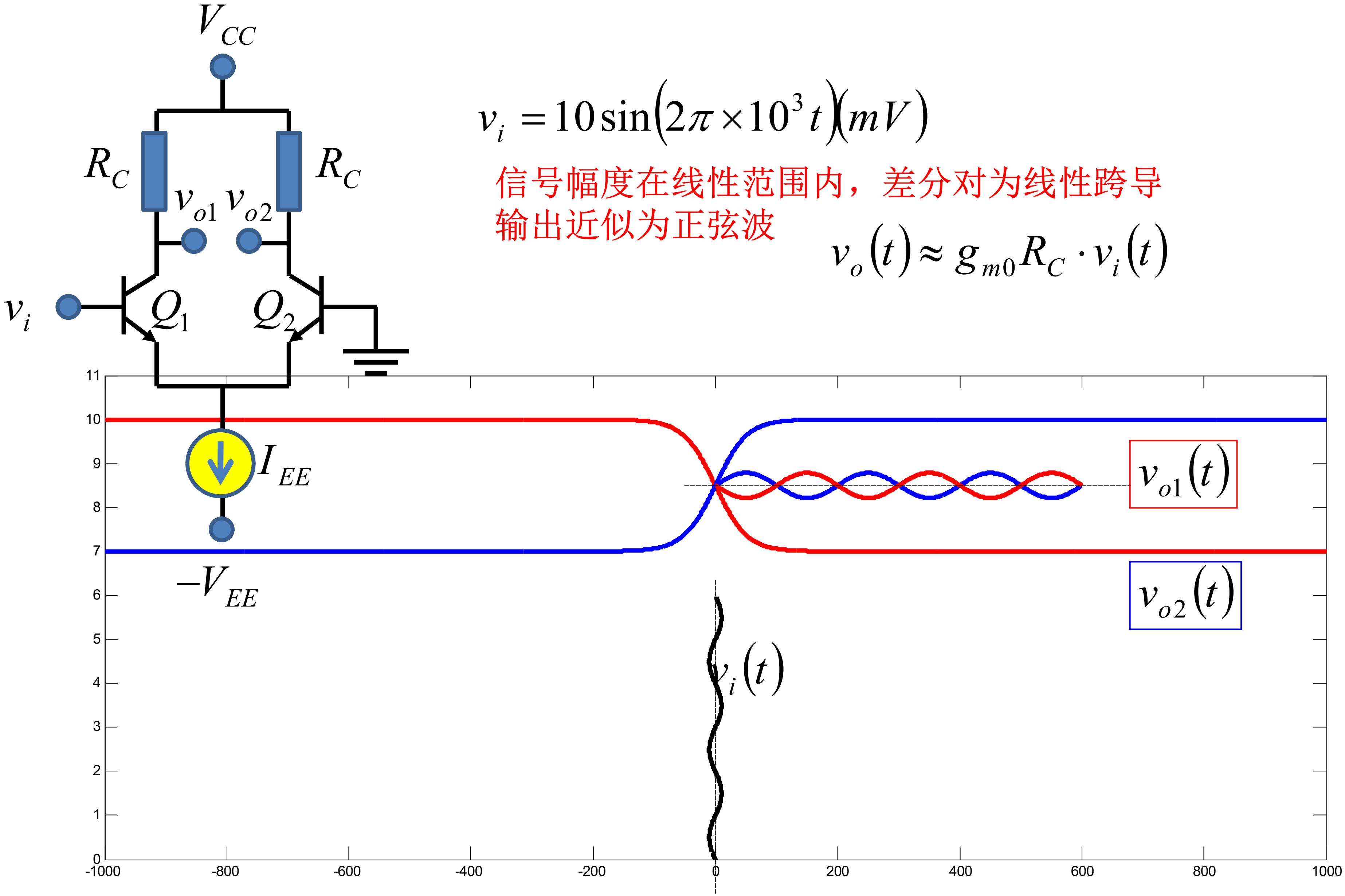
$$v_i = 10\sin\left(2\pi\times10^3 t\right)\left(mV\right) \qquad v_i = 500\sin\left(2\pi\times10^3 t\right)\left(mV\right)$$

$$v_i = 50 + 100\sin\left(2\pi\times10^3 t\right)\left(mV\right)$$

$$v_i = 10\sin\left(2\pi \times 10^3 t\right)(mV)$$

信号幅度在线性范围内，差分对为线性跨导输出近似为正弦波

$$v_o(t) \approx g_{m0} R_C \cdot v_i(t)$$



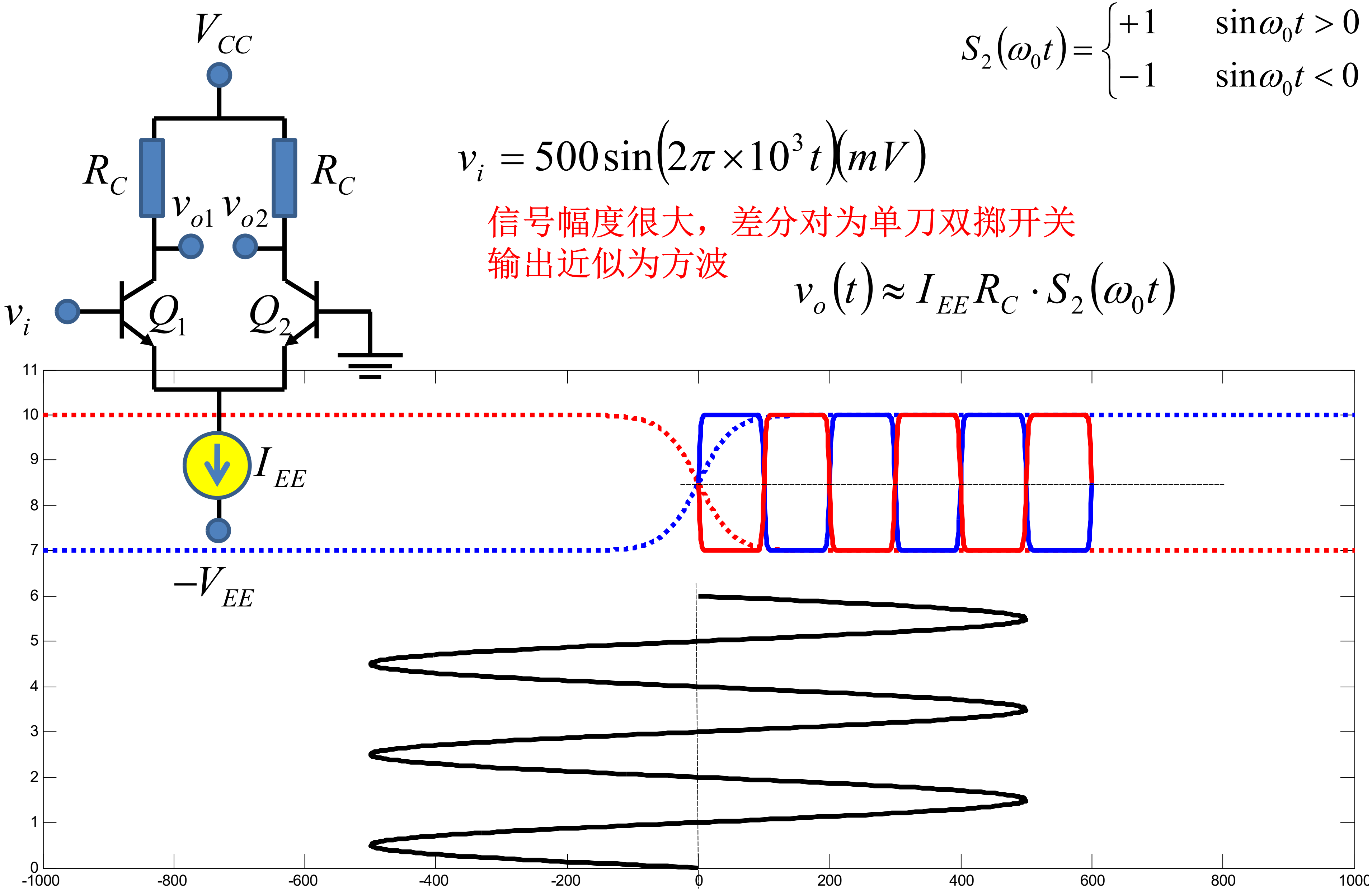
$$S_2(\omega_0 t)=\begin{cases}+1 & \sin\omega_0 t>0\\ -1 & \sin\omega_0 t<0\end{cases}$$

$$v_i=500\sin\left(2\pi\times10^3 t\right)(mV)$$

信号幅度很大，差分对为单刀双掷开关

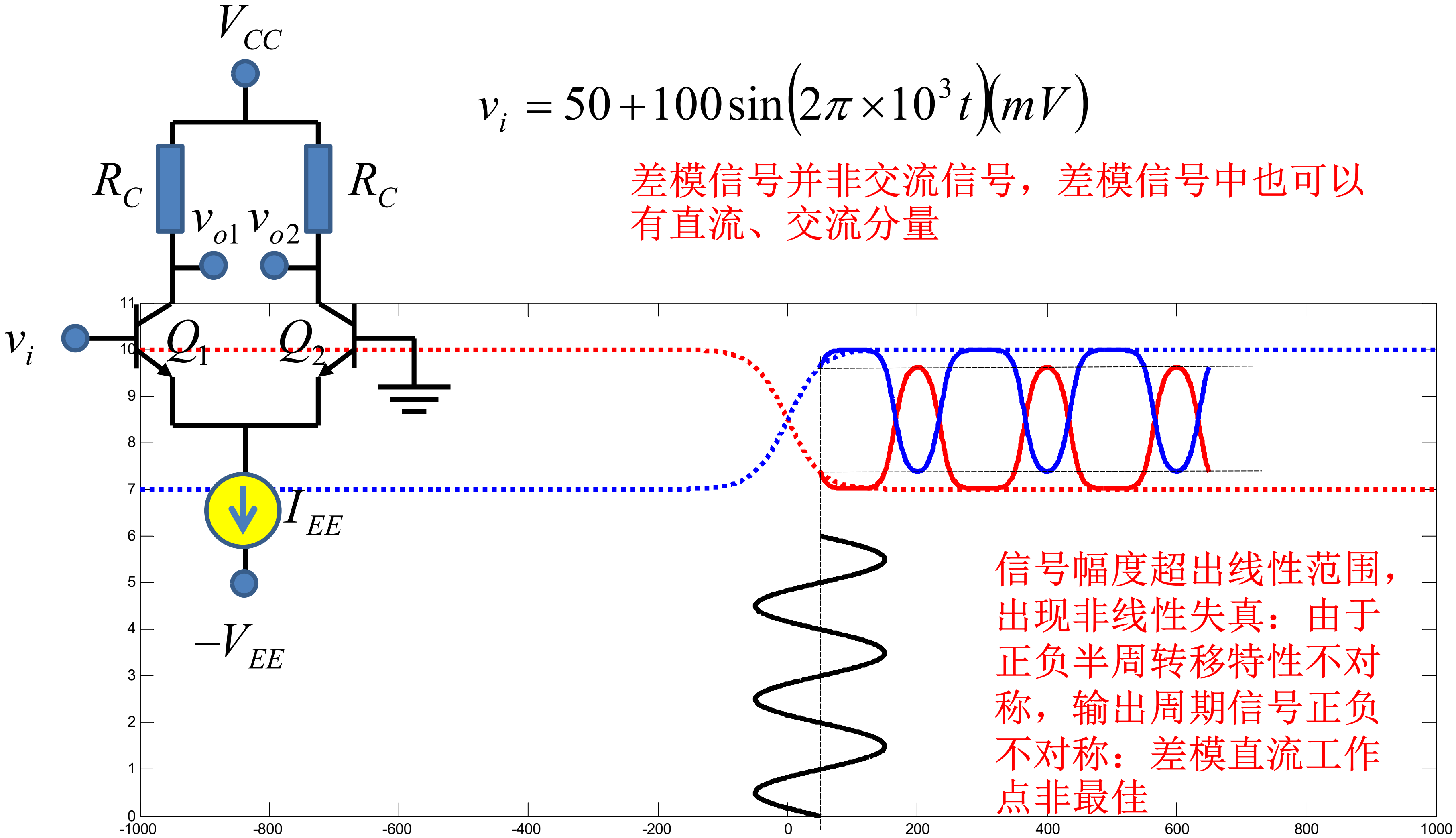
输出近似为方波

$$v_o(t)\approx I_{EE}R_C\cdot S_2(\omega_0 t)$$



$$v_i = 50 + 100\sin\left(2\pi\times10^3 t\right)(mV)$$

差模信号并非交流信号，差模信号中也可以有直流、交流分量

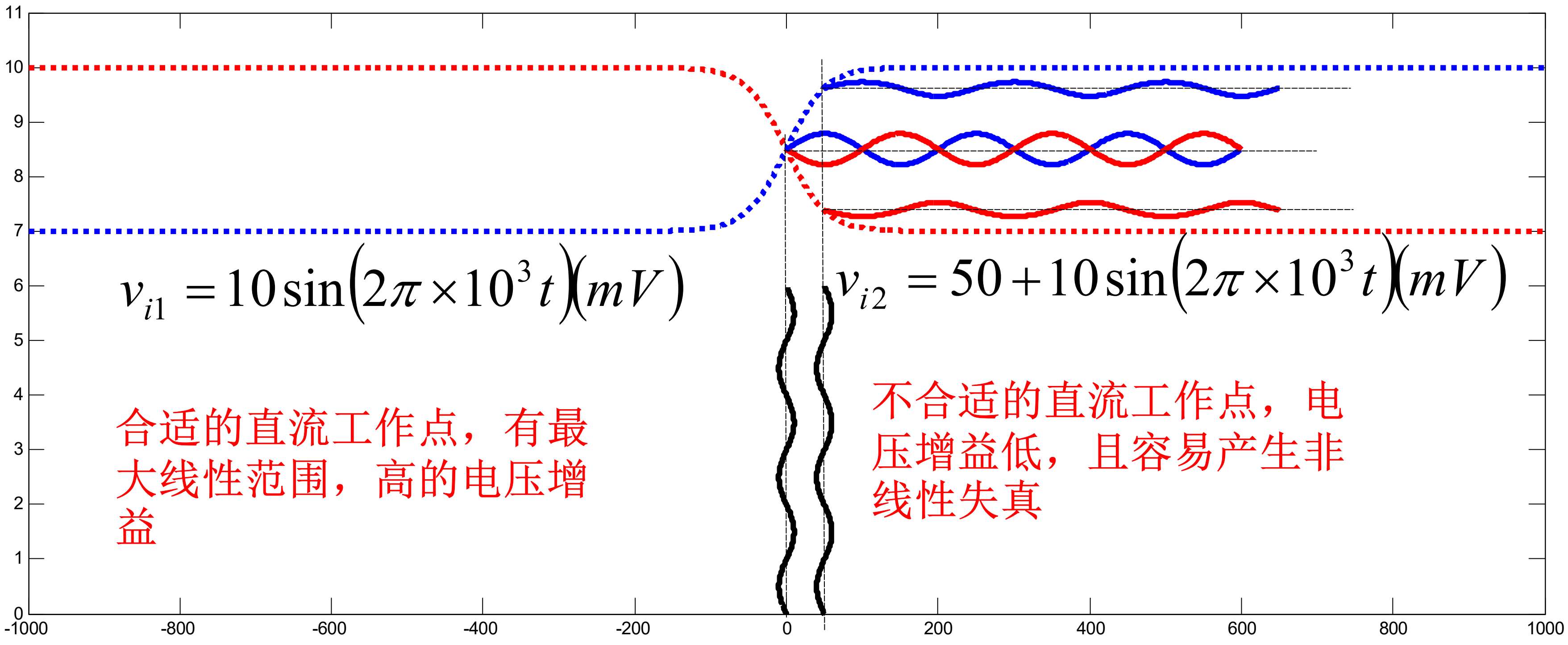


信号幅度超出线性范围，出现非线性失真：由于正负半周转移特性不对称，输出周期信号正负不对称：差模直流工作点非最佳

$$v_{ic} = 25 + 50\sin\left(2\pi\times10^3 t\right)(mV)$$

$$v_{id} = 50 + 100\sin\left(2\pi\times10^3 t\right)(mV)$$

差模、共模信号中可以有直流，可以有交流



11
10
9
8
7
6
5
4
3
2
1
0
-1000
-800
-600
-400
-200
0
200
400
600
800
1000
$v_{i1}=10\sin\left(2\pi\times10^{3}t\right)\left(mV\right)$
$v_{i2}=50+10\sin\left(2\pi\times10^{3}t\right)\left(mV\right)$
合适的直流工作点，有最大线性范围，高的电压增益
不合适的直流工作点，电压增益低，且容易产生非线性失真

差模信号与共模信号
直流信号与交流信号

$$v_{ip}=\frac{v_{ip}+v_{in}}{2}+\frac{v_{ip}-v_{in}}{2}=v_{ic}+0.5v_{id}$$

$$v_{ip}=V_{IP0}+\Delta v_{ip}$$

$$v_{in}=\frac{v_{ip}+v_{in}}{2}-\frac{v_{ip}-v_{in}}{2}=v_{ic}-0.5v_{id}$$

$$v_{in}=V_{IN0}+\Delta v_{in}$$

$$v_{ic}=\frac{v_{ip}+v_{in}}{2}=\frac{V_{IP0}+V_{IN0}}{2}+\frac{\Delta v_{ip}+\Delta v_{in}}{2}=V_{IC0}+\Delta v_{ic}$$

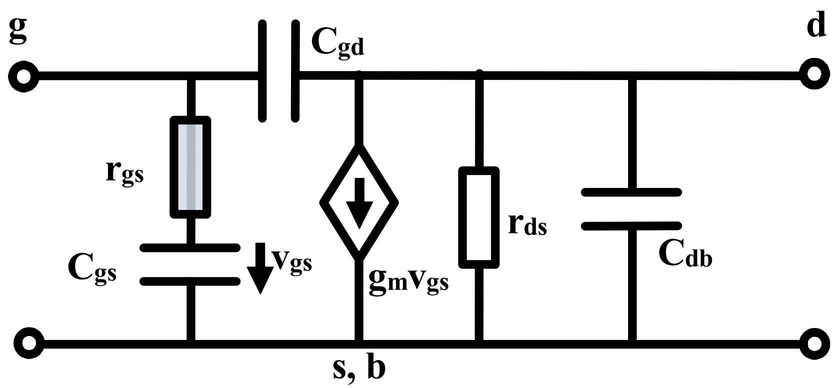
共模可以存在直流和交流
无论如何，差分对电桥平衡，可抑制共模信号

$$v_{id}=v_{ip}-v_{in}=V_{IP0}-V_{IN0}+\Delta v_{ip}-\Delta v_{in}=V_{ID0}+\Delta v_{id}$$

差模可以存在直流和交流
不平衡电桥导致输出不为零
直流导致工作点不同位置
交流小信号则线性放大，大信号有非线性失真

作业5：晶体管的有源性

- 请分析如下网络的有源性条件
  - 列写其**y**参量矩阵
  - 由有源性定义证明有源性条件为**$f<f_{max}$**



$$f_{\max} = \frac{g_m}{4\pi C_{gs}}\sqrt{\frac{r_{ds}}{r_{gs}}}$$

8.3.2节：相量域的有源和无源定义

- 线性时不变网络在相量域的有源性描述为：端口描述方程为线性代数方程的线性时不变网络，如果其端口总吸收实功恒不小于零，

$$P=\sum_{k=1}^{n}P_k=\frac{1}{2}\mathrm{Re}\sum_{k=1}^{n}\dot{V}_k\dot{I}_k^*=\frac{1}{2}\mathrm{Re}\,\dot{\mathbf{V}}^T\dot{\mathbf{I}}^*\geq 0 \qquad \left(\forall\dot{\mathbf{V}},\dot{\mathbf{I}},\mathbf{f}\left(\dot{\mathbf{V}},\dot{\mathbf{I}}\right)=0\right)$$

- 该网络就是无源网络。如果存在某种负载条件，使得端口总吸收实功小于**0**的情况可以出现，该网络则是有源的

$$P=\sum_{k=1}^{n}P_k=\frac{1}{2}\mathrm{Re}\sum_{k=1}^{n}\dot{V}_k\dot{I}_k^*=\frac{1}{2}\mathrm{Re}\,\dot{\mathbf{V}}^T\dot{\mathbf{I}}^*<0 \qquad \left(\exists\dot{\mathbf{V}},\dot{\mathbf{I}},\mathbf{f}\left(\dot{\mathbf{V}},\dot{\mathbf{I}}\right)=0\right)$$

-
- $\dot{\mathbf{V}}$ $\dot{\mathbf{I}}$是联参考方向定义的端口电压和端口电流列向量，
- $\mathbf{f}\left(\dot{\mathbf{V}},\dot{\mathbf{I}}\right)=0$ 则是该线性时不变网相量域的端口描述线性代数方程。

$$\mathbf{Y}=\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}=\begin{bmatrix} G_{11}+jB_{11} & G_{12}+jB_{12} \\ G_{21}+jB_{21} & G_{22}+jB_{22} \end{bmatrix}$$

有源性条件

无源性定义要求任意满足元件约束方程的端口电压电流均有

$$\mathrm{Re}\,\dot{\mathbf{V}}^T\dot{\mathbf{I}}^* \geq 0 \qquad\qquad \dot{\mathbf{V}}^T\dot{\mathbf{I}}^* + \dot{\mathbf{I}}^T\dot{\mathbf{V}}^* \geq 0$$

$$\dot{\mathbf{V}}^T\mathbf{Y}^*\dot{\mathbf{V}}^* + \dot{\mathbf{V}}^T\mathbf{Y}^T\dot{\mathbf{V}}^* = \dot{\mathbf{V}}^T\left(\mathbf{Y}^* + \mathbf{Y}^T\right)\dot{\mathbf{V}}^* \geq 0$$

故而只要 $\mathbf{Y}^* + \mathbf{Y}^T$ 是半正定矩阵（positive semidefinite matrix）即可

$$\mathbf{Y}^* + \mathbf{Y}^T = \begin{bmatrix} G_{11}-jB_{11} & G_{12}-jB_{12} \\ G_{21}-jB_{21} & G_{22}-jB_{22} \end{bmatrix} + \begin{bmatrix} G_{11}+jB_{11} & G_{21}+jB_{21} \\ G_{12}+jB_{12} & G_{22}+jB_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 2G_{11} & G_{12}+G_{21}-j\left(B_{12}-B_{21}\right) \\ G_{12}+G_{21}+j\left(B_{12}-B_{21}\right) & 2G_{22} \end{bmatrix}$$

半正定条件

$$\mathbf{Y}^* + \mathbf{Y}^T = \begin{bmatrix} G_{11} - jB_{11} & G_{12} - jB_{12} \\ G_{21} - jB_{21} & G_{22} - jB_{22} \end{bmatrix} + \begin{bmatrix} G_{11} + jB_{11} & G_{21} + jB_{21} \\ G_{12} + jB_{12} & G_{22} + jB_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 2G_{11} & G_{12} + G_{21} - j(B_{12} - B_{21}) \\ G_{12} + G_{21} + j(B_{12} - B_{21}) & 2G_{22} \end{bmatrix}$$

$$\Delta_{11} = 2G_{22} \geq 0$$

$$\Delta_{22} = 2G_{11} \geq 0$$

这三个条件同时满足则无源。反之，三个条件有一个不满足，二端口网络就是有源的

$$\Delta = 2G_{11} \cdot 2G_{22} - (G_{12} + G_{21} - j(B_{12} - B_{21}))(G_{12} + G_{21} + j(B_{12} - B_{21})) \geq 0$$

有源性条件

$$\mathbf{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} G_{11} + jB_{11} & G_{12} + jB_{12} \\ G_{21} + jB_{21} & G_{22} + jB_{22} \end{bmatrix}$$

$$\mathrm{y} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

$G_{11} < 0$ 端口1看入导纳出现负电导可向外输出电能 $G_{11} < 0$

$G_{22} < 0$ 端口2看入导纳出现负电导可向外输出电能 $G_{22} < 0$

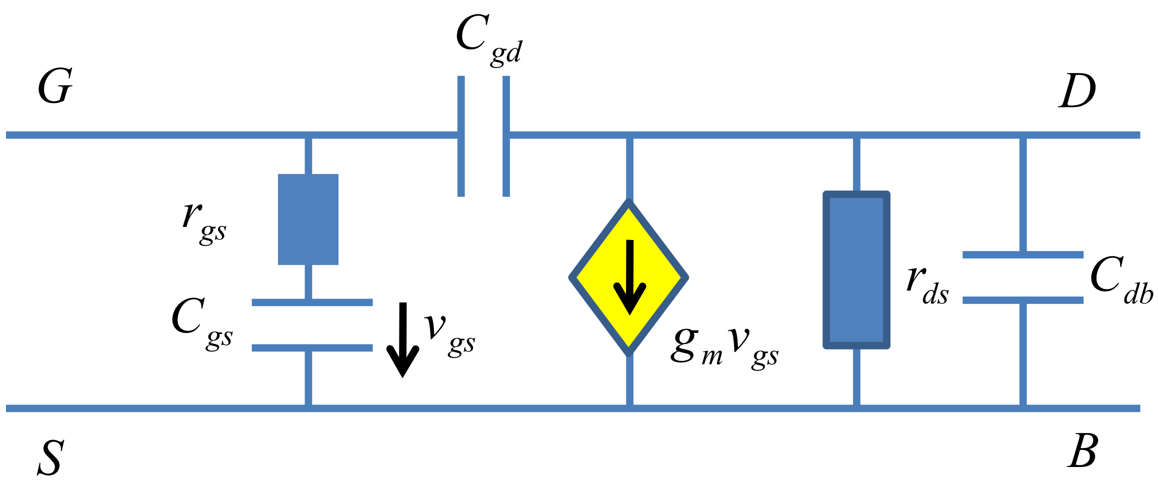
$$(G_{12} + G_{21})^2 + (B_{12} - B_{21})^2 > 4G_{11}G_{22}$$

$$(G_{12} + G_{21})^2 > 4G_{11}G_{22}$$

跨导增益足够高，除了抵偿内部电导损耗外，还可向外输出额外能量

三者满足其一，二端口网络即是有源网络

导纳参量矩阵



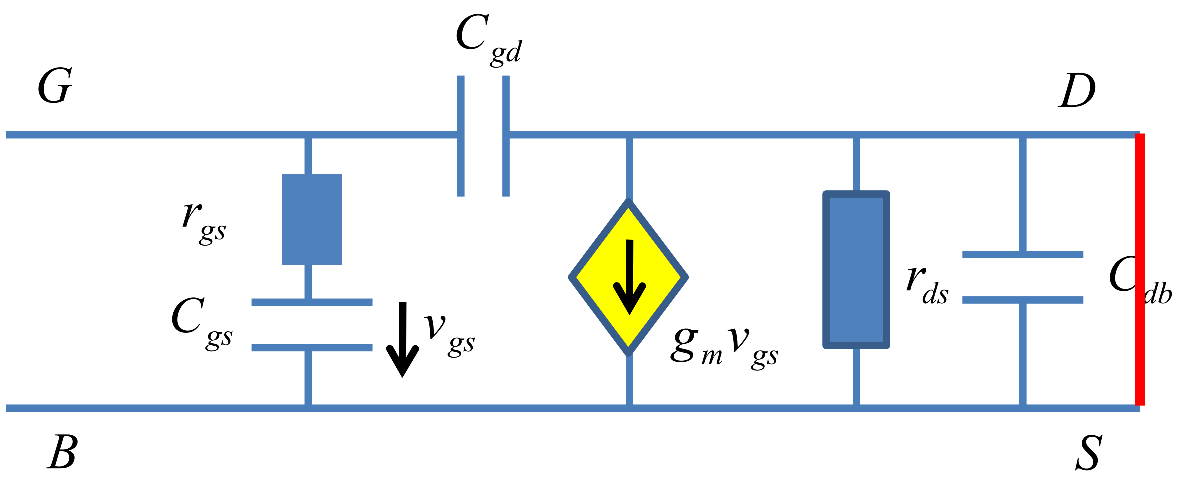
$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \end{bmatrix}$$

$$Y_{11} = \frac{\dot{I}_1}{\dot{V}_1}\bigg|_{\dot{V}_2=0}$$

$$Y_{12} = \frac{\dot{I}_1}{\dot{V}_2}\bigg|_{\dot{V}_1=0}$$

$$Y_{21} = \frac{\dot{I}_2}{\dot{V}_1}\bigg|_{\dot{V}_2=0}$$

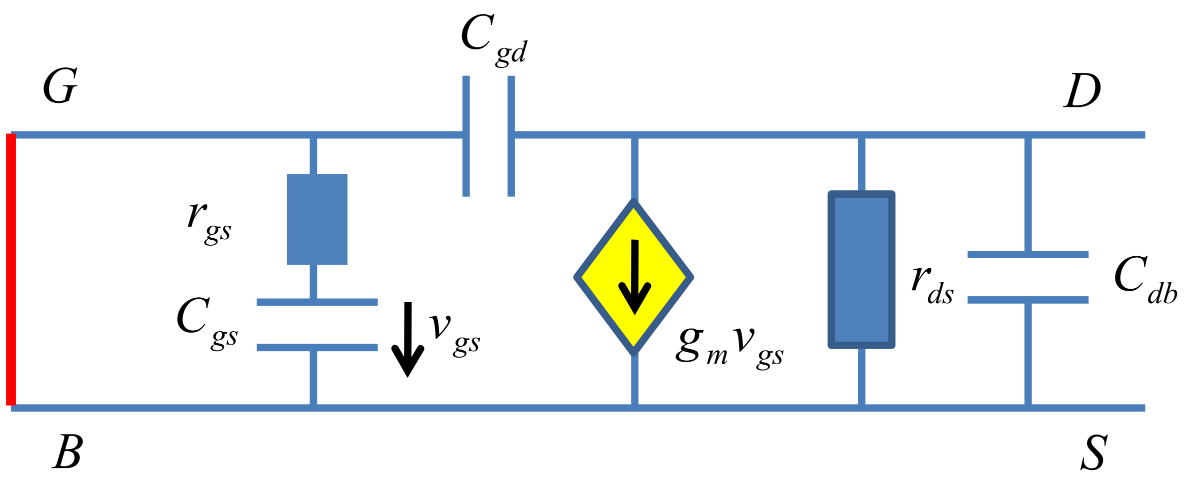
$$Y_{22} = \frac{\dot{I}_2}{\dot{V}_2}\bigg|_{\dot{V}_1=0}$$



$$Y_{11}=\frac{\dot{I}_1}{\dot{V}_1}\bigg|_{\dot{V}_2=0}=\left(r_{gs}\text{串}C_{gs}\right)\text{并}C_{gd}=\frac{sC_{gs}}{1+sC_{gs}r_{gs}}+sC_{gd}$$

$$s=j\omega$$

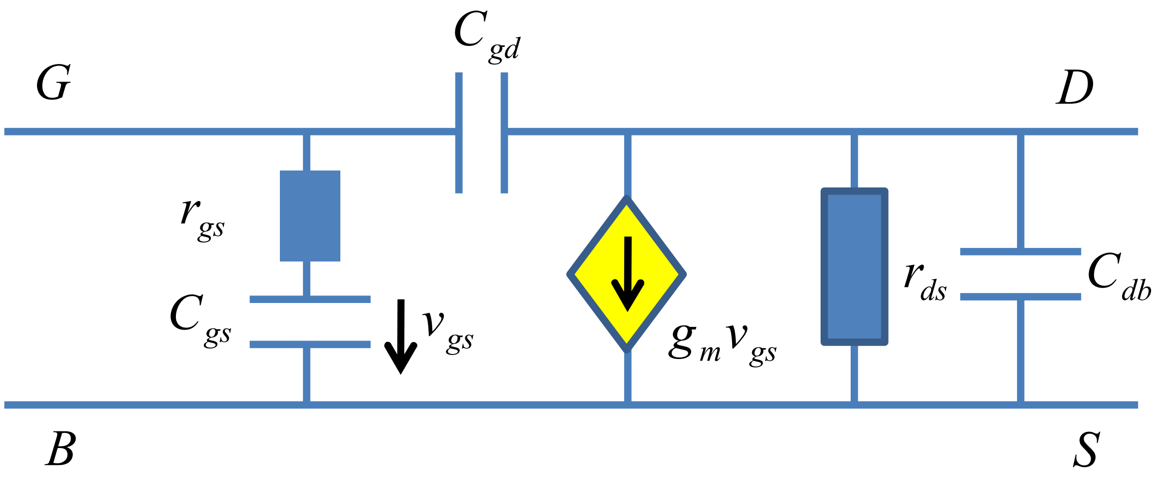
$$Y_{21}=\frac{\dot{I}_2}{\dot{V}_1}\bigg|_{\dot{V}_2=0}=\frac{g_m\dot{V}_{gs}-\dot{I}_{gd}}{\dot{V}_g}=g_m\frac{1}{1+sC_{gs}r_{gs}}-sC_{gd}$$



$$Y_{22}=\frac{\dot{I}_2}{\dot{V}_2}\Big|_{\dot{V}_1=0}=r_{ds}并C_{db}并C_{gd}=\frac{1}{r_{ds}}+sC_{db}+sC_{gd}$$

$$Y_{12}=\frac{\dot{I}_1}{\dot{V}_2}\Big|_{\dot{V}_1=0}=-sC_{gd}$$

导纳参量矩阵



$$\mathbf{Y}=\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}=\begin{bmatrix} \dfrac{sC_{gs}}{1+sC_{gs}r_{gs}}+sC_{gd} & -sC_{gd} \\ g_m\dfrac{1}{1+sC_{gs}r_{gs}}-sC_{gd} & \dfrac{1}{r_{ds}}+sC_{db}+sC_{gd} \end{bmatrix}$$

$$s=j\omega$$

$$\left(G_{12}+G_{21}\right)^2+\left(B_{12}-B_{21}\right)^2>4G_{11}G_{22}$$

$$\mathbf{Y}=\begin{bmatrix} \dfrac{sC_{gs}}{1+sC_{gs}r_{gs}}+sC_{gd} & -sC_{gd} \\ g_m\dfrac{1}{1+sC_{gs}r_{gs}}-sC_{gd} & \dfrac{1}{r_{ds}}+sC_{db}+sC_{gd} \end{bmatrix}=\begin{bmatrix} \dfrac{j\omega C_{gs}}{1+j\omega C_{gs}r_{gs}}+j\omega C_{gd} & -j\omega C_{gd} \\ g_m\dfrac{1}{1+j\omega C_{gs}r_{gs}}-j\omega C_{gd} & \dfrac{1}{r_{ds}}+j\omega C_{db}+j\omega C_{gd} \end{bmatrix}$$

$$=\begin{bmatrix} \dfrac{\left(\omega C_{gs}\right)^2 r_{gs}}{1+\left(\omega C_{gs}r_{gs}\right)^2}+\dfrac{j\omega C_{gs}}{1+\left(\omega C_{gs}r_{gs}\right)^2}+j\omega C_{gd} & -j\omega C_{gd} \\ \dfrac{g_m}{1+\left(\omega C_{gs}r_{gs}\right)^2}-j\omega\left(\dfrac{g_m r_{gs}C_{gs}}{1+\left(\omega C_{gs}r_{gs}\right)^2}+C_{gd}\right) & \dfrac{1}{r_{ds}}+j\omega C_{db}+j\omega C_{gd} \end{bmatrix}$$

$$\left(\frac{g_m}{1+\left(\omega C_{gs}r_{gs}\right)^2}+0\right)^2+\left(\omega\left(\frac{g_m r_{gs}C_{gs}}{1+\left(\omega C_{gs}r_{gs}\right)^2}+C_{gd}\right)-\omega C_{gd}\right)^2>4\left(\frac{\left(\omega C_{gs}\right)^2 r_{gs}}{1+\left(\omega C_{gs}r_{gs}\right)^2}\right)\frac{1}{r_{ds}}$$

最高振荡频率f_{max}

$$\left(\frac{g_m}{1+(\omega C_{gs} r_{gs})^2}+0\right)^2+\left(\omega\left(\frac{g_m r_{gs} C_{gs}}{1+(\omega C_{gs} r_{gs})^2}+C_{gd}\right)-\omega C_{gd}\right)^2>4\left(\frac{(\omega C_{gs})^2 r_{gs}}{1+(\omega C_{gs} r_{gs})^2}\right)\frac{1}{r_{ds}}$$

$$g_m^2+(\omega C_{gs} g_m r_{gs})^2>4(\omega C_{gs})^2\frac{r_{gs}}{r_{ds}}\left(1+(\omega C_{gs} r_{gs})^2\right)$$

$$g_m^2\left(1+(\omega C_{gs} r_{gs})^2\right)>4(\omega C_{gs})^2\frac{r_{gs}}{r_{ds}}\left(1+(\omega C_{gs} r_{gs})^2\right)$$

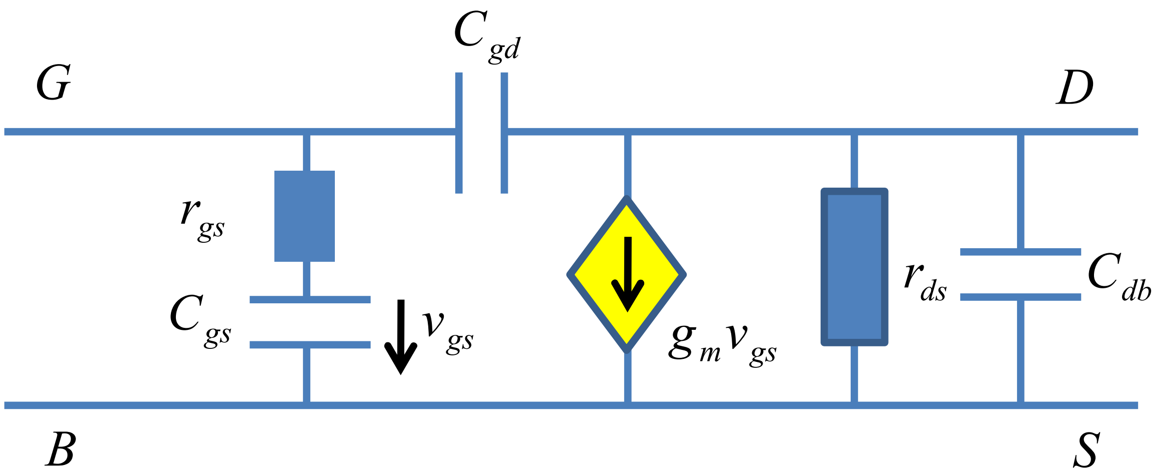
$$g_m^2>4(\omega C_{gs})^2\frac{r_{gs}}{r_{ds}} \qquad (\omega C_{gs})^2<\frac{g_m^2 r_{ds}}{4r_{gs}}$$

$$\omega<\frac{g_m}{2C_{gs}}\sqrt{\frac{r_{ds}}{r_{gs}}}=\omega_{\max} \qquad f_{\max}=\frac{g_m}{4\pi C_{gs}}\sqrt{\frac{r_{ds}}{r_{gs}}}$$

$$f<f_{\max}$$

晶体管的有源性条件

如何理解f_{max}表达式



$$f_{\max} = \frac{g_m}{4\pi C_{gs}}\sqrt{\frac{r_{ds}}{r_{gs}}}$$

$f < f_{\max}$

晶体管的有源性条件

网络内部损耗体现在r_{gs}和r_{ds}上

r_{gs}越小，输入损耗越小；r_{ds}越大，输出损耗越小

因而当r_{gs}很小或 r_{ds}极大，都可以做到扩大有源区

g_m是跨导增益，其值越大，有源性越强

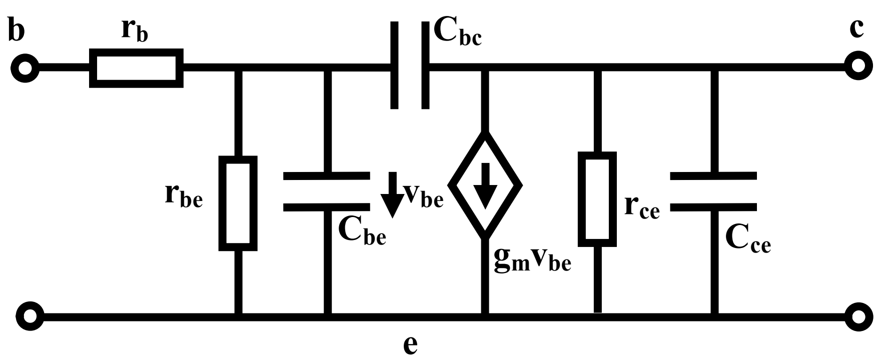
寄生电容C_{gs}越大，其上分压越小，导致跨导控制作用消失

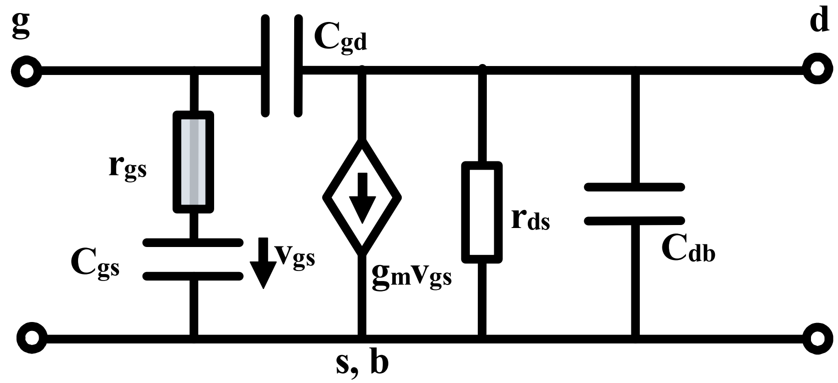
$g_m^2 > 4g_{be}g_{ce}(1+g_{be}r_b)$

不考虑寄生电容的有源性条件

BJT的f_{max}

不考虑寄生电容：该模型绝对有源





$$f_{\max} = \frac{1}{4\pi}\sqrt{\frac{g_m^2 - 4g_{be}g_{ce}(1+g_{be}r_b)}{\left(g_m C_{bc}(C_{be}+C_{bc}) + g_{be}C_{bc}^2 + g_{ce}(C_{be}+C_{bc})^2\right)r_b}}$$

$$\approx \frac{1}{4\pi}\sqrt{\frac{g_m}{C_{bc}(C_{be}+C_{bc})r_b}} = \frac{1}{4\pi}\frac{g_m}{(C_{be}+C_{bc})}\sqrt{\frac{C_{be}+C_{bc}}{C_{bc}}\frac{1}{g_m r_b}}$$

$$f_{\max} = \frac{g_m}{4\pi C_{gs}}\sqrt{\frac{r_{ds}}{r_{gs}}}$$

$$f < f_{\max}$$

考虑了寄生电容效应后的晶体管的有源性条件

最大功率增益和f_{max}

$$G_{pmax}(f) \sim \left(\frac{f_{max}}{f}\right)^2$$

最大功率增益在高频端的估算公式

$$G_{pmax}\left(f=\frac{1}{4}f_{max}\right)=\left(\frac{f_{max}}{0.25f_{max}}\right)^2=16=12dB$$

放大器和振荡器工作频率范围不应超过$f_{max}/4$