电子电路与系统基础II

习题课第十一讲 习题讲解

二阶动态LTI电路时频分析（下半）
阻抗匹配与变换网络（上半）

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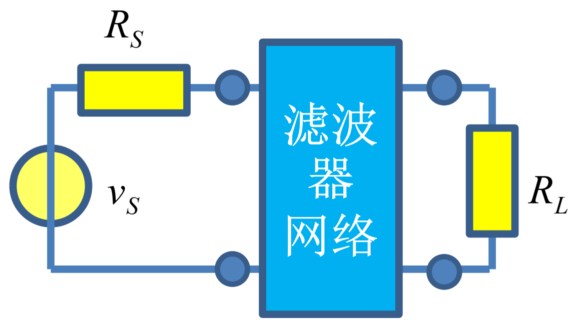
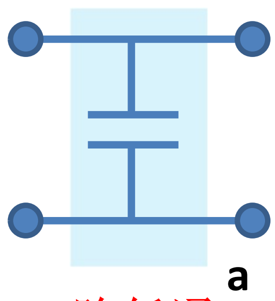
作业7：典型结构滤波器类型判断

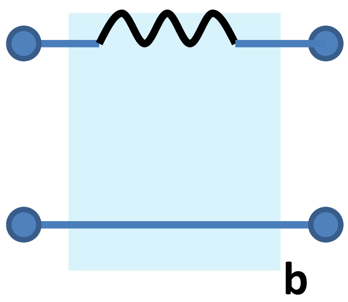
- 电容和电感的记忆能力或者积分效应，导致时域上的延时和频域上的选频特性

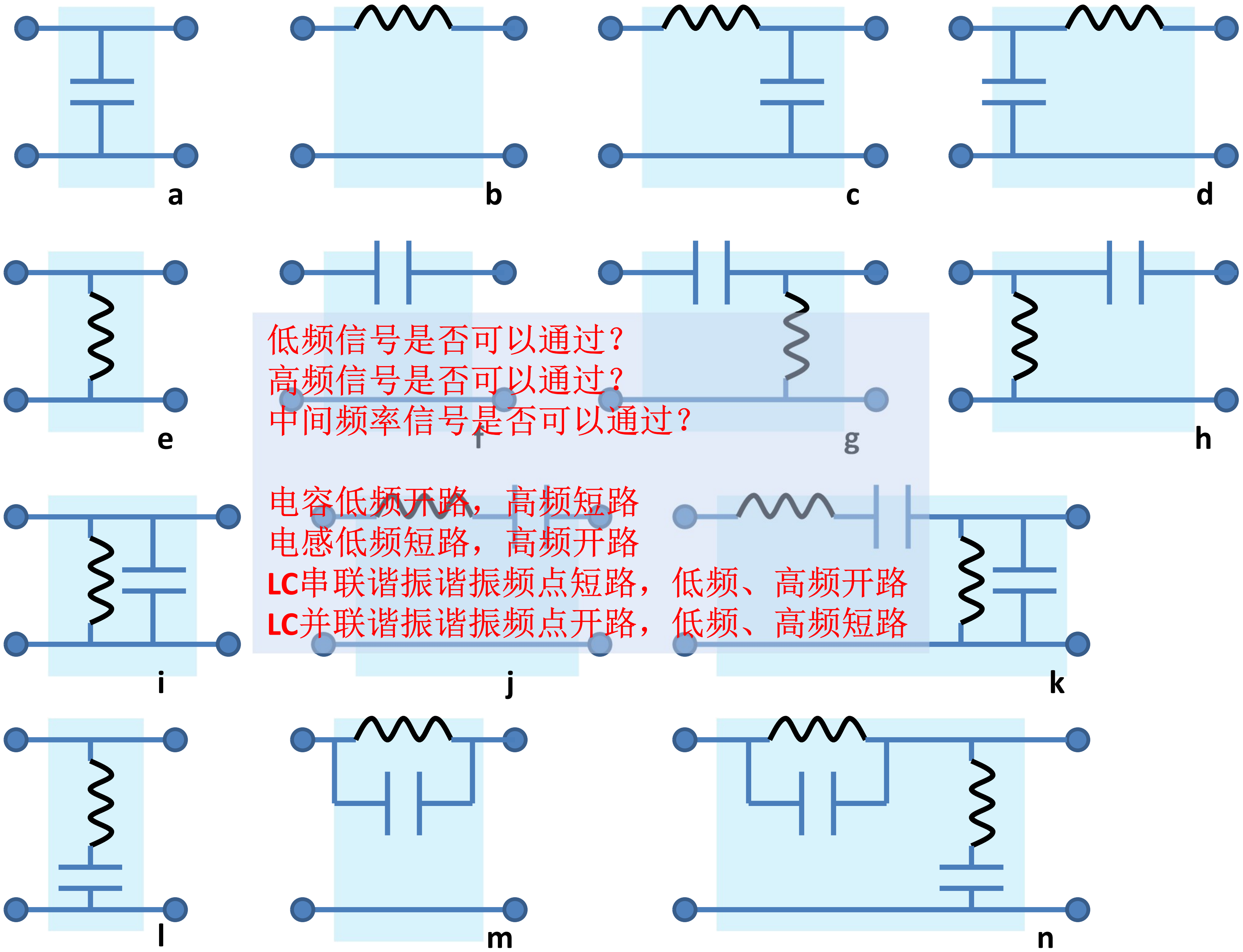
- 常见滤波器分类
  - 低通、高通、带通、带阻
  - 请给出正确的滤波器分类





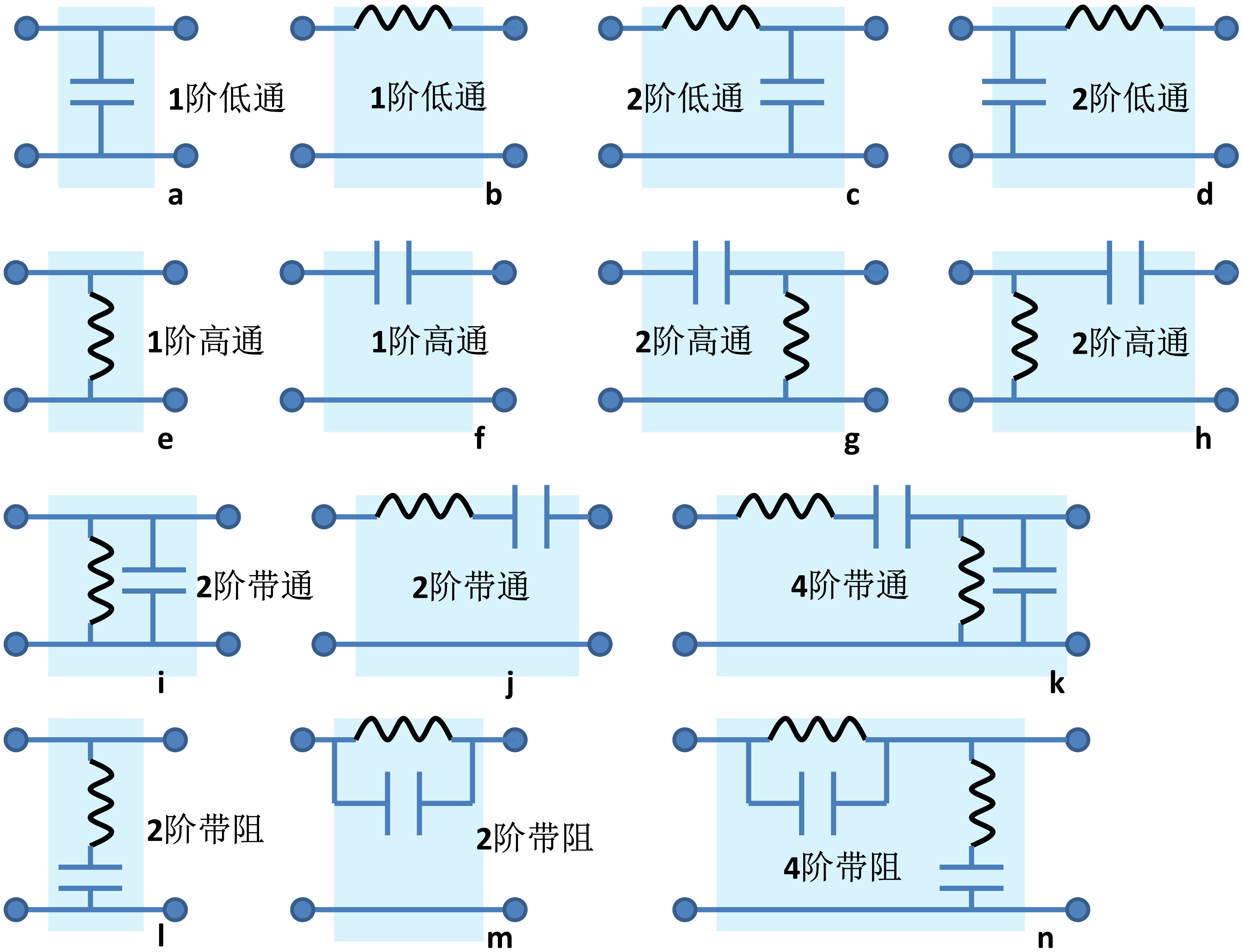
一阶低通





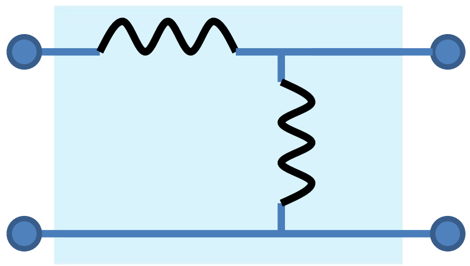
低频信号是否可以通过？
高频信号是否可以通过？
中间频率信号是否可以通过？

电容低频开路，高频短路
电感低频短路，高频开路
LC串联谐振谐振频点短路，低频、高频开路
LC并联谐振谐振频点开路，低频、高频短路

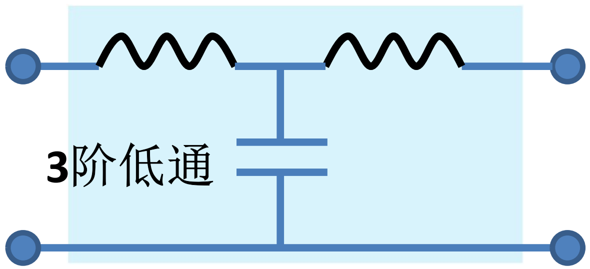

1阶低通
a
1阶低通
b
2阶低通
c
2阶低通
d
1阶高通
e
1阶高通
f
2阶高通
g
2阶高通
h
2阶带通
i
2阶带通
j
4阶带通
k
2阶带阻
l
2阶带阻
m
4阶带阻
n

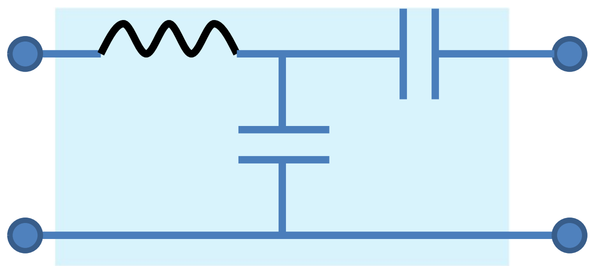
滤波器类型判定

- 电容低频开路，高频短路
- 电感低频短路，高频开路
- 中间频率？



二阶带通：无谐振，难匹配





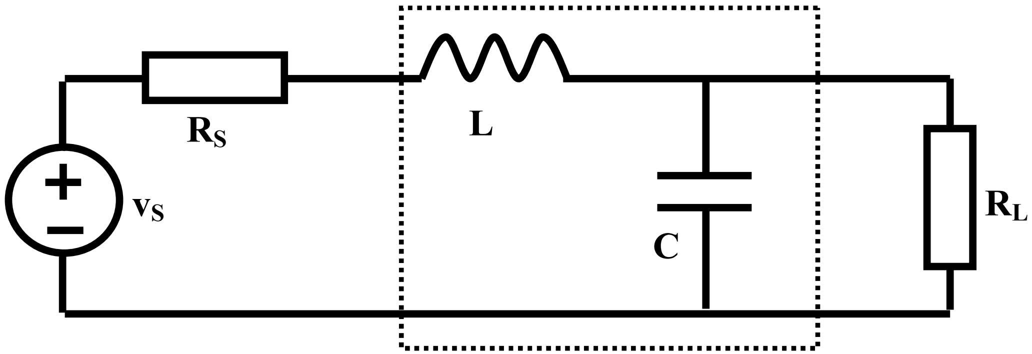
三阶带通：有谐振

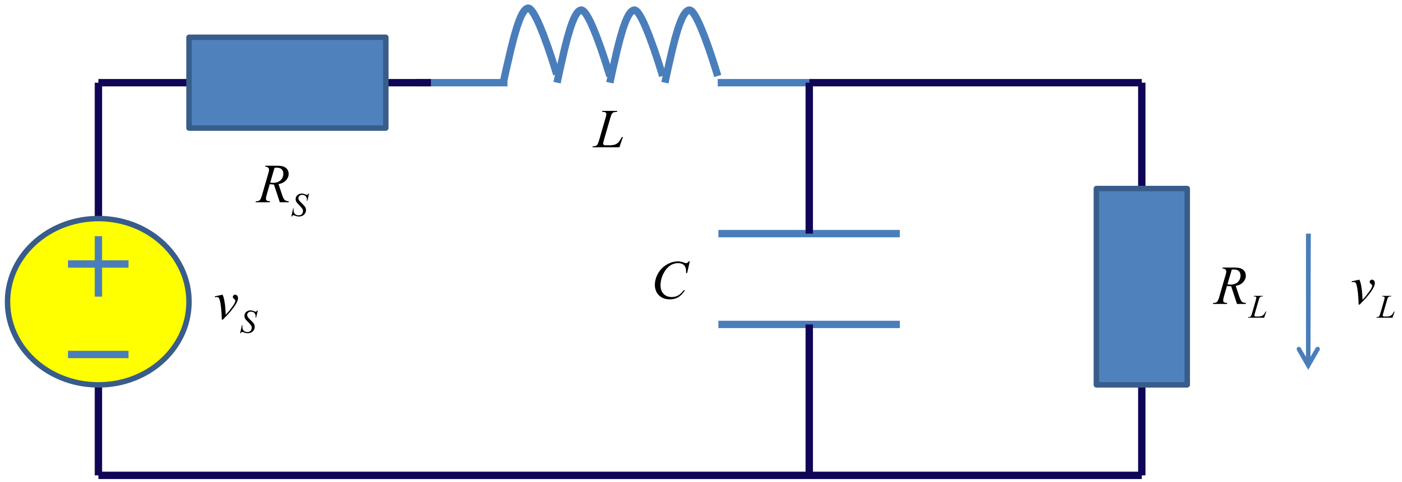
标准形态很容易判定

低频：电容开路，信号过不去
高频：电感开路信号过不去
中频：信号可以通过：带通匹配网络

作业8：低通滤波器设计

- 如图所示，已知信源内阻为**50Ω**，负载电阻也是**50Ω**，请设计一个具有群延时最大平坦特性的二阶低通**LC**滤波器，其**3dB**带宽为**1MHz**，请给出虚框表示的**LC**低通滤波器中电感和电容的具体数值。
  – 群延时最大平坦
  – 选作：幅度最大平坦





低通传输

$$H(s)=2\sqrt{\frac{R_S}{R_L}}\frac{V_L(s)}{V_S(s)}=2\sqrt{\frac{R_S}{R_L}}\frac{\dfrac{R_L}{1+sR_LC}}{R_S+sL+\dfrac{R_L}{1+sR_LC}}$$

$$=2\frac{\sqrt{R_SR_L}}{R_L+R_S}\frac{1}{s^2LC\dfrac{R_L}{R_L+R_S}+s\left(\dfrac{L}{R_L+R_S}+C\dfrac{R_SR_L}{R_L+R_S}\right)+1}$$

$$=H_0\frac{1}{\left(\dfrac{s}{\omega_n}\right)^2+2\xi\dfrac{s}{\omega_n}+1}=H_0\frac{\omega_n^2}{s^2+2\xi\omega_ns+\omega_n^2}$$

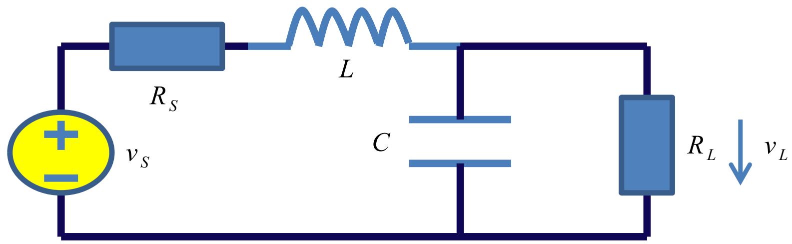
ω_n：系统的自由振荡频率

二阶低通

$$H(s)=2\frac{\sqrt{R_S R_L}}{R_L+R_S}\frac{1}{s^2LC\dfrac{R_L}{R_L+R_S}+s\left(\dfrac{L}{R_L+R_S}+C\dfrac{R_S R_L}{R_L+R_S}\right)+1}$$

$$=H_0\frac{1}{\left(\dfrac{s}{\omega_n}\right)^2+2\xi\dfrac{s}{\omega_n}+1}$$

二阶低通传函的典型形态



$$H_0=2\frac{\sqrt{R_S R_L}}{R_L+R_S}=1$$

$$\omega_0=\frac{1}{\sqrt{LC}}$$

$$\omega_n=\omega_0\sqrt{\frac{R_S+R_L}{R_L}}=\sqrt{2}\omega_0$$

$$Z_0=\sqrt{\frac{L}{C}},Y_0=\sqrt{\frac{C}{L}}$$

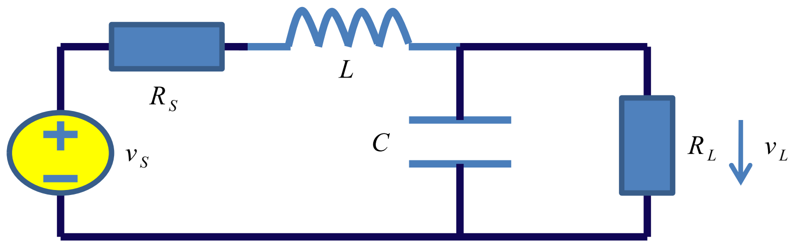
$$\xi=\frac{1}{2}\left(\frac{Z_0}{\sqrt{R_L(R_S+R_L)}}+\frac{Y_0}{\sqrt{G_S(G_S+G_L)}}\right)=\frac{1}{2\sqrt{2}}\left(\frac{Z_0}{R}+\frac{Y_0}{G}\right)$$

幅度最大平坦：ξ=0.707

$$H_0 = 1$$

$$\omega_n = \sqrt{2}\omega_0$$

$$\xi = \frac{1}{2\sqrt{2}}\left(\frac{Z_0}{R} + \frac{Y_0}{G}\right)$$



$$\xi = \frac{1}{2\sqrt{2}}\left(\frac{Z_0}{R} + \frac{Y_0}{G}\right) = \frac{1}{\sqrt{2}} \Rightarrow R = Z_0 = \sqrt{\frac{L}{C}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Z_0 = \sqrt{\frac{L}{C}}, Y_0 = \sqrt{\frac{C}{L}}$$

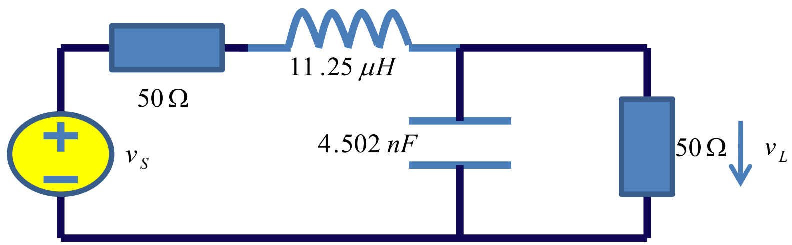
$$BW_{3dB} = \frac{\omega_n}{2\pi} = \frac{\sqrt{2}}{2\pi\sqrt{LC}} \Rightarrow$$

$$L = \frac{\sqrt{2}R}{2\pi BW_{3dB}} = \frac{\sqrt{2}\times 50}{2\pi\times 1\times 10^6} = 11.25\mu H$$

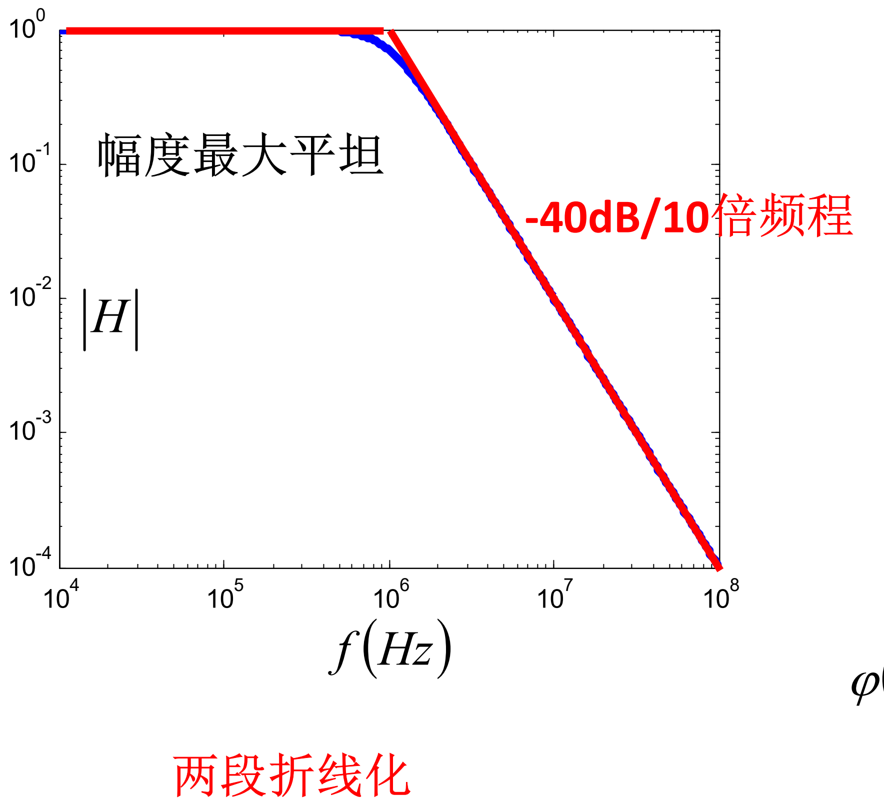
$$C = \frac{\sqrt{2}}{2\pi BW_{3dB}R} = \frac{\sqrt{2}}{2\pi\times 1\times 10^6\times 50} = 4.502nF$$

幅度最大平坦二阶低通的特点

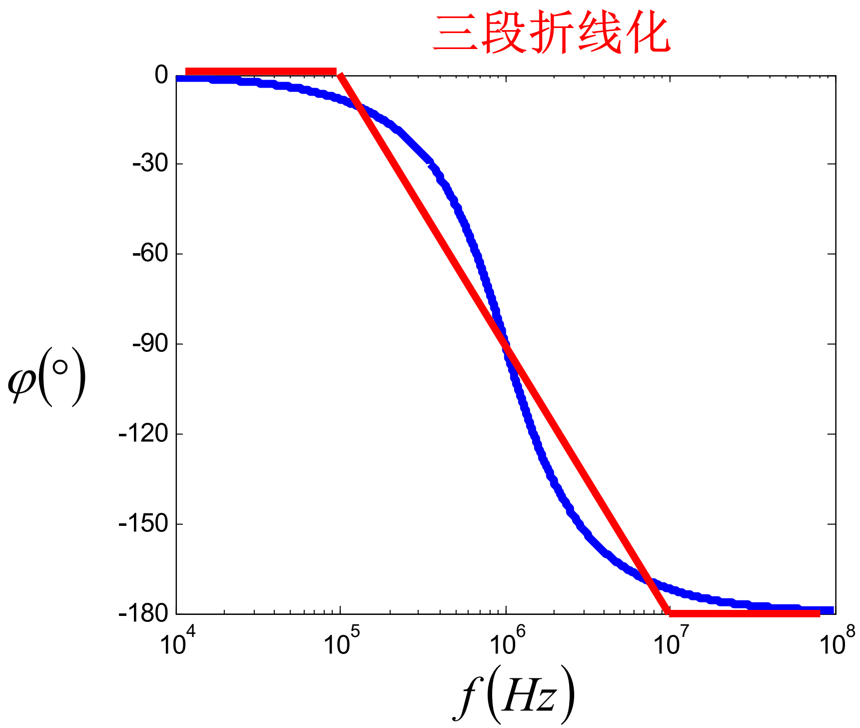
频率特性
伯特图



50 Ω
11.25 μH
4.502 nF
50 Ω
v_S
v_L



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^4
10^5
10^6
10^7
10^8
幅度最大平坦
-40dB/10倍频程
$|H|$
$f(Hz)$
两段折线化



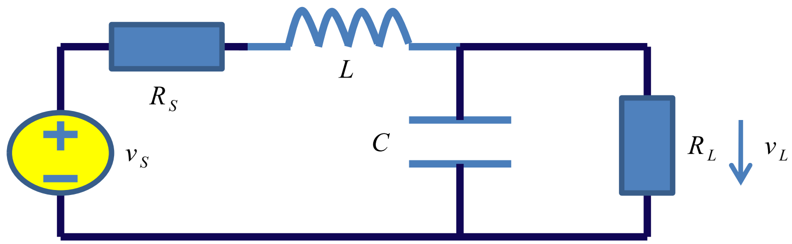
三段折线化
0
-30
-60
-90
-120
-150
-180
$\varphi(°)$
10^4
10^5
10^6
10^7
10^8
$f(Hz)$

群延时最大平坦：ξ=0.866

$$H_0 = 1$$

$$\omega_n = \sqrt{2}\omega_0$$

$$\xi = \frac{1}{2\sqrt{2}}\left(\frac{Z_0}{R} + \frac{Y_0}{G}\right)$$



$$\xi = \frac{1}{2\sqrt{2}}\left(\frac{Z_0}{R} + \frac{R}{Z_0}\right) = \frac{\sqrt{3}}{2}$$

⇒

$$\frac{Z_0}{R} + \frac{R}{Z_0} = \sqrt{6}$$

⇒

$$\frac{Z_0}{R} = \frac{\sqrt{6} \pm \sqrt{2}}{2} = 1.932, 0.5176$$

$$Z_0 = \sqrt{\frac{L}{C}} = 96.59\Omega, 25.88\Omega$$

$$BW_{3dB} = \frac{\omega_{3dB}}{2\pi} = \frac{\omega_n}{2\pi}\sqrt{-2\xi^2 + 1 + \sqrt{\left(2\xi^2 - 1\right)^2 + 1}} = \frac{\sqrt{2}\omega_0}{2\pi} 0.7862 = \frac{0.1769}{\sqrt{LC}}$$

$$\sqrt{LC} = \frac{0.1769}{BW_{3dB}} = 0.1769 \times 10^{-6} s$$

两个设计结果

$$Z_0 = \sqrt{\frac{L}{C}} = 96.59\Omega$$

$$\sqrt{LC} = \frac{0.1769}{BW_{3dB}} = 0.1769 \times 10^{-6} s$$

$$L = 0.1769 \times 10^{-6} \times 96.59 = 17.09 \mu H$$

$$C = 0.1769 \times 10^{-6} / 96.59 = 1.832 nF$$

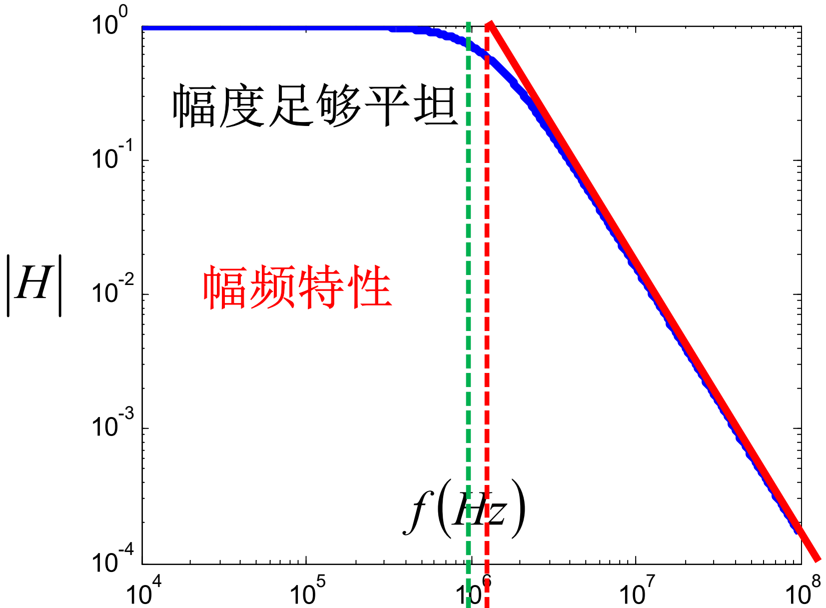
$$Z_0 = \sqrt{\frac{L}{C}} = 25.88\Omega$$

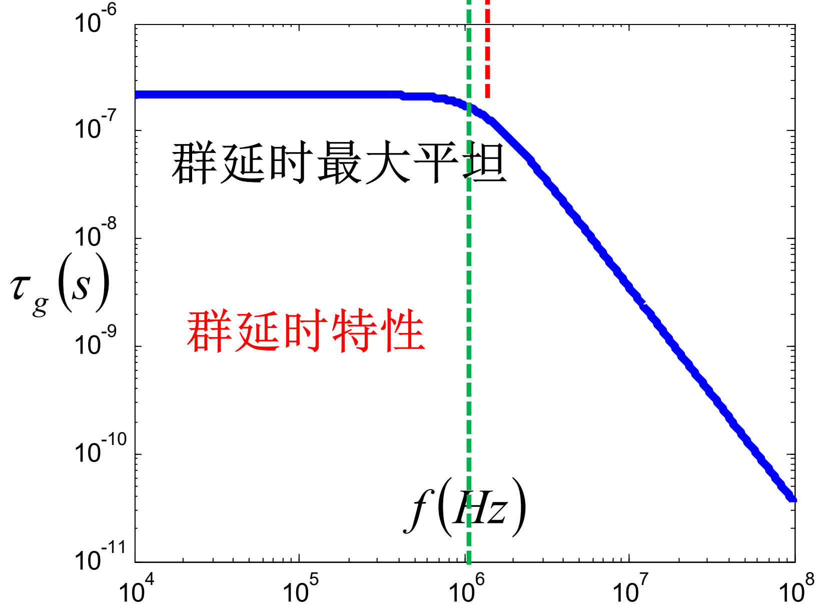
$$\sqrt{LC} = \frac{0.1769}{BW_{3dB}} = 0.1769 \times 10^{-6} s$$

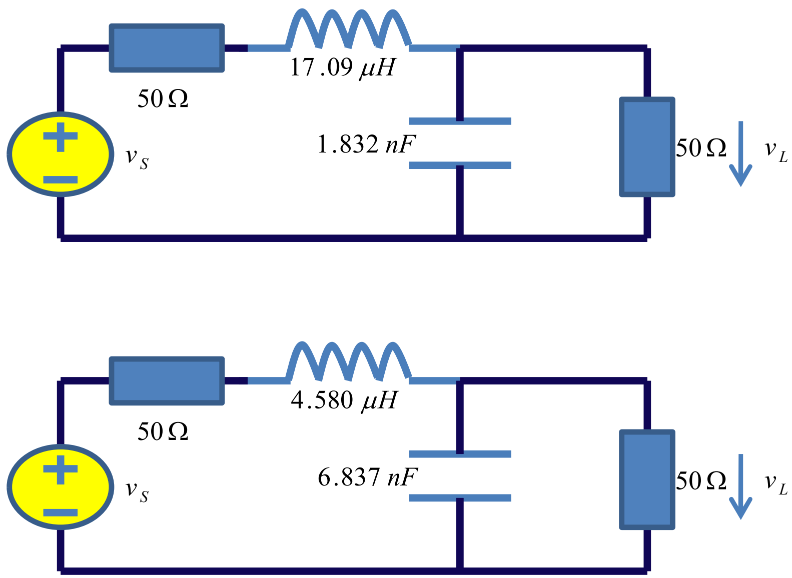
$$L = 0.1769 \times 10^{-6} \times 25.88 = 4.580 \mu H$$

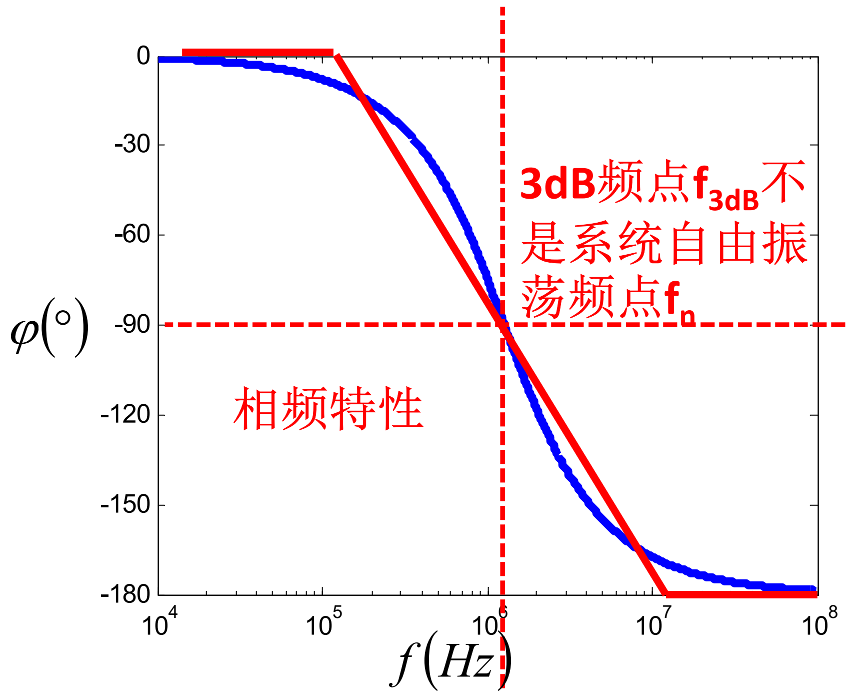
$$C = 0.1769 \times 10^{-6} / 25.88 = 6.837 nF$$

频率特性

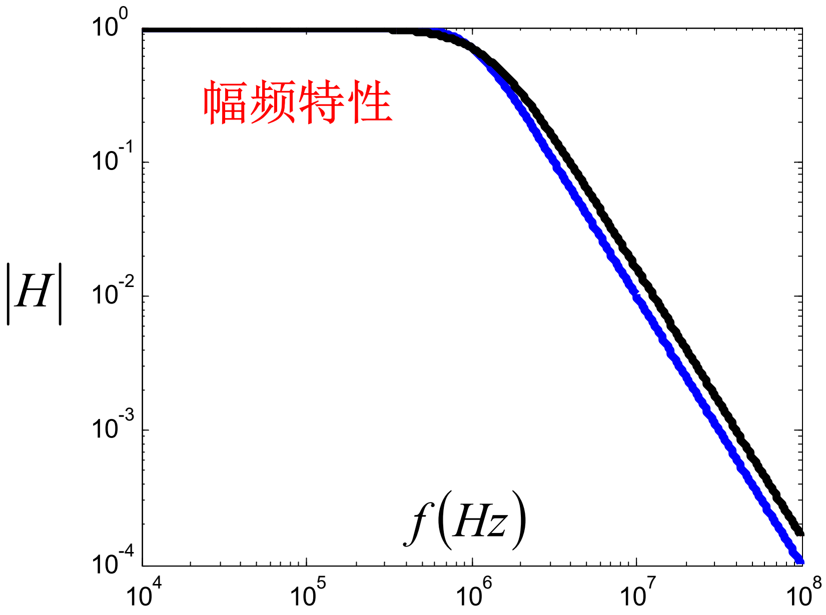

10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$|H|$
幅度足够平坦
幅频特性
$f(Hz)$
10^4
10^5
10^6
10^7
10^8

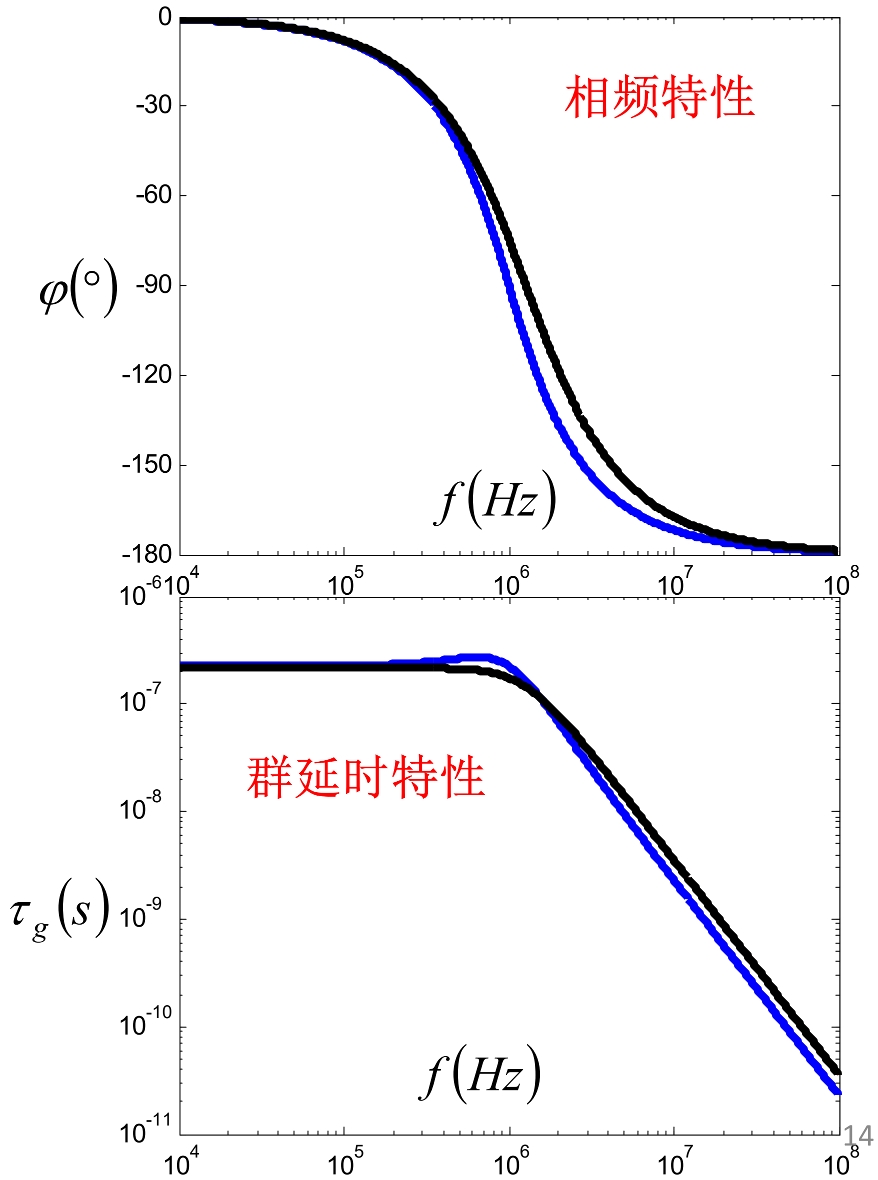

10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
$\tau_g(s)$
群延时最大平坦
群延时特性
$f(Hz)$
10^4
10^5
10^6
10^7
10^8


$50\,\Omega$
$17.09\ \mu H$
$1.832\ nF$
$50\,\Omega$
v_S
v_L
$50\,\Omega$
$4.580\ \mu H$
$6.837\ nF$
$50\,\Omega$
v_S
v_L


0
-30
-60
-90
-120
-150
-180
$\varphi(^\circ)$
3dB频点f_{3dB}不是系统自由振荡频点f_0
相频特性
$f(Hz)$
10^4
10^5
10^6
10^7
10^8

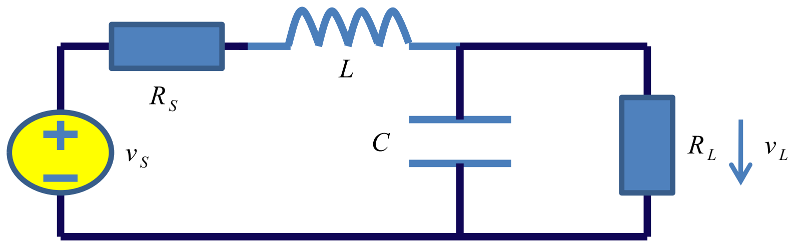
幅度最大平坦和群延时最大平坦





幅度最大平坦：蓝线
群延时最大平坦：黑线

matlab



- R=50;
- RS=R;
- RL=R;
- BW3=1E6;
-
- kesai=sqrt(2)/2;
- L1=sqrt(2)*R/2/pi/BW3;
- C1=sqrt(2)/2/pi/BW3/R;
-
- kesai=sqrt(3)/2;
- Z02=(sqrt(6)+sqrt(2))/2*R;
- Z03=(sqrt(6)-sqrt(2))/2*R;
-
- w3=BW3/(sqrt(-2*kesai^2+1+sqrt((2*kesai^2-1)^2+1))*sqrt(2)/2/pi);
- C2=1/(Z02*w3);
- L2=Z02/w3;
-
- C3=1/(Z03*w3);
- L3=Z03/w3;

已知参量

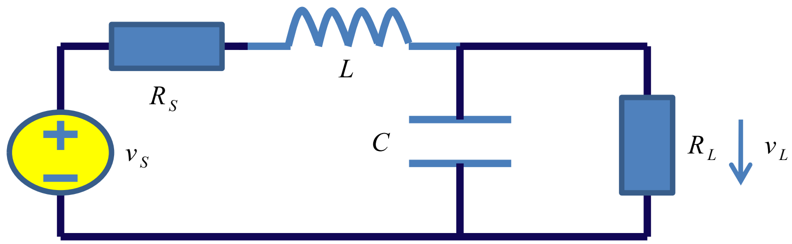
幅度最大平坦设计

群延时最大平坦设计1

群延时最大平坦设计2

```
freqstart=1E4;
freqstop=1E8;
freqnum=1000;
freqstep=10^(log10(freqstop/freqstart)/freqnum);

freq=freqstart/freqstep;
```

考察频段设置




```
for k=1:freqnum
   freq=freq*freqstep;

   f(k)=freq;
   w=2*pi*freq;
   s=i*w;

   ABCD=[1 RS+s*L1; 0 1]*[1 0; s*C1+1/RL 1];
```

传输参量矩阵计算

```
   H1=2*sqrt(RS/RL)*1/ABCD(1,1);
   absH1(k)=abs(H1);
   angleH1(k)=angle(H1)/pi*180;
```

幅频特性

相频特性

幅度最大平坦

```
   if k>1
      tgH1(k)=-(angleH1(k)-angleH1(k-1))/180*pi/(2*pi*(f(k)-f(k-1)));
   end
```

群延时特性

```
   ABCD=[1 RS+s*L2; 0 1]*[1 0; s*C2+1/RL 1];

   H2=2*sqrt(RS/RL)*1/ABCD(1,1);
   absH2(k)=abs(H2);
   angleH2(k)=angle(H2)/pi*180;

   if k>1
      tgH2(k)=-(angleH2(k)-angleH2(k-1))/180*pi/(2*pi*(f(k)-f(k-1)));
   end
```

群延时最大平坦设计1

```
   ABCD=[1 RS+s*L3; 0 1]*[1 0; s*C3+1/RL 1];

   H3=2*sqrt(RS/RL)*1/ABCD(1,1);
   absH3(k)=abs(H3);
   angleH3(k)=angle(H3)/pi*180;

   if k>1
      tgH3(k)=-(angleH3(k)-angleH3(k-1))/180*pi/(2*pi*(f(k)-f(k-1)));
   end

end
```

群延时最大平坦设计2

Matlab作图输出

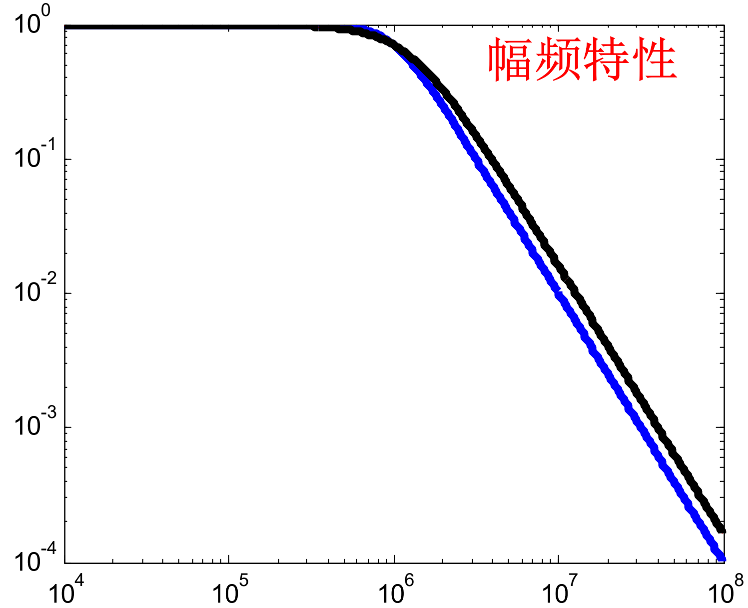
```
tgH1(1)=tgH1(2);
tgH2(1)=tgH2(2);
tgH3(1)=tgH3(2);

figure(1)
hold on
plot(f,absH1,'b')
plot(f,absH2,'r')
plot(f,absH3,'k')

figure(2)
hold on
plot(f,angleH1,'b')
plot(f,angleH2,'r')
plot(f,angleH3,'k')

figure(3)
hold on
plot(f,tgH1,'b')
plot(f,tgH2,'r')
plot(f,tgH3,'k')
```

群延时计算：差分计算，补点

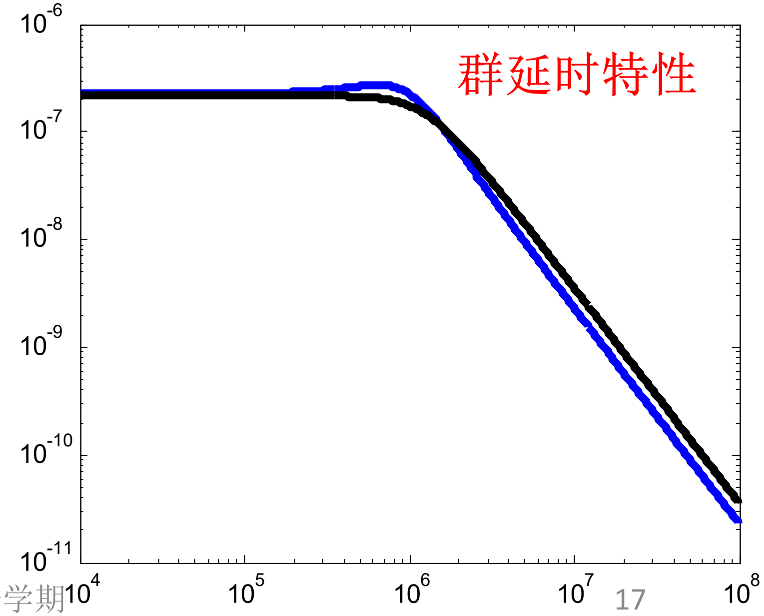
幅频特性

相频特性

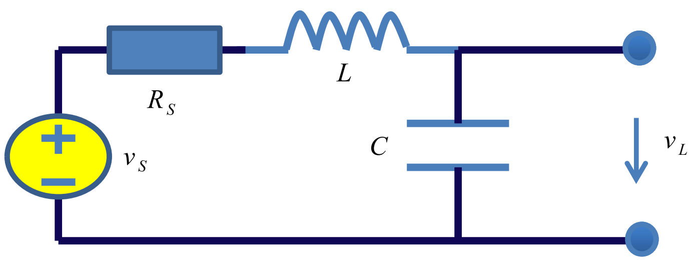
群延时特性



群延时最大平坦两个设计的曲线完全覆盖



最优特性如何体现？



- 如图所示的低通滤波器，设计$\xi =$ **0.1，0.866，10** 三种情况下的低通滤波器，说明$\xi =$ **0.866**的阶跃响应最优
  - **Matlab**代码，说明滤波效果

```
RS=50;     %信源内阻为50欧姆
f3dB=1E6;  %3dB带宽为1MHz

kesai=[0.1 sqrt(3)/2 10];  %三种阻尼系数情况

Dt=1E-9;
timenum=20000;
f0=f3dB/5;
w0=2*pi*f0;
T=1/f0;
for j=1:timenum
   t(j)=(j-1000)*Dt;
   if j<1000
      vs1(j)=0;
      vs2(j)=0;
   else
      vs1(j)=1;
      vs2(j)=0.5+2/pi*cos(w0*(t(j)-0.3*T))-2/3/pi*cos(3*w0*(t(j)-0.3*T))+2/5/pi*cos(5*w0*(t(j)-0.3*T));
         vs2n(j)=vs2(j)+0.3*sin(50*w0*t(j)+0.5*randn)+0.6*sin(52*w0*t(j)+0.5*randn)+0.3*sin(54*w0*t(j)+0.5*randn)+0.5*randn;

   end
end
```

方波的基波频率为**f0**

于是方波的**5**次谐波分量前位于通带之内

以此作为数字信号的抽象

信号**1**为阶跃信号用于考察阶跃响应

信号**2**为数字信号的抽象：幅度为**1**的方波信号的傅立叶展开前**4**项

滤波器输入为数字信号**+**噪声：位于通带外**10**倍**3dB**频点位置噪声**+**随机噪声

```
figure(1)
hold on
plot(t,vs1,'k')

figure(2)
hold on
plot(t,vs2,'k')
plot(t,vs2n,'b')
```

```
for k=1:3
   %串联RLC取值
   L(k)=RS*sqrt(-2*kesai(k)^2+1+sqrt((1-2*kesai(k)^2)^2+1))/(2*kesai(k)*2*pi*f3dB);
   C(k)=2*kesai(k)*sqrt(-2*kesai(k)^2+1+sqrt((1-2*kesai(k)^2)^2+1))/(RS*2*pi*f3dB);

   RCD=Dt/C(k);
   GLD=Dt/L(k);

   A=[1 -RCD; GLD 1+GLD*RS];
   invA=inv(A);
```

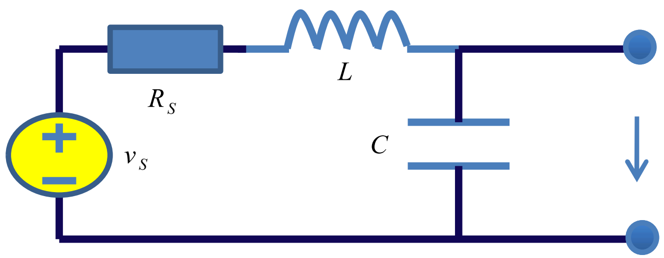
自行验证：根据**3dB**带宽、阻尼系数、信源内阻计算获得电容、电感大小

$$L = \frac{R_S\sqrt{-2\xi^2+1+\sqrt{(-2\xi^2+1)^2+1}}}{2\xi\omega_{3dB}}$$

$$C = \frac{2\xi\sqrt{-2\xi^2+1+\sqrt{(-2\xi^2+1)^2+1}}}{R_S\omega_{3dB}}$$

$$\begin{bmatrix} v_C(t_{k+1}) \\ i_L(t_{k+1}) \end{bmatrix} = \begin{bmatrix} 1 & -R_{C\Delta} \\ G_{L\Delta} & 1+G_{L\Delta}R_S \end{bmatrix}^{-1} \left(\begin{bmatrix} v_C(t_k) \\ i_L(t_k) \end{bmatrix} + \begin{bmatrix} 0 \\ G_{L\Delta} \end{bmatrix} v_S(t_{k+1}) \right)$$

后向欧拉法



$$v_s = v_R + v_L + v_C = i_L R_S + L\frac{di_L}{dt} + v_C$$

$$i_L = i_C = C\frac{dv_C}{dt} \qquad \frac{dv_C}{dt} = \frac{1}{C} i_L$$

$$\frac{di_L}{dt} = \frac{1}{L}v_s - \frac{1}{L}v_C - \frac{R_S}{L}i_L$$

$$\frac{d}{dt}\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_S}{L} \end{bmatrix}\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v_s(t)$$

状态方程

$$\frac{d}{dt}\boldsymbol{x}(t) = \mathbf{f}(\boldsymbol{x}(t), t)$$

$$\boldsymbol{x}(t_{k+1}) - \boldsymbol{x}(t_k) = \int_{t_k}^{t_{k+1}} \mathbf{f}(\boldsymbol{x}(t), t)dt \approx \mathbf{f}(\boldsymbol{x}(t_{k+1}), t_{k+1})\Delta t$$

$$\begin{bmatrix} v_C(t_{k+1}) \\ i_L(t_{k+1}) \end{bmatrix} - \begin{bmatrix} v_C(t_k) \\ i_L(t_k) \end{bmatrix} \approx \left(\begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_S}{L} \end{bmatrix}\begin{bmatrix} v_C(t_{k+1}) \\ i_L(t_{k+1}) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v_s(t_{k+1})\right)\Delta t$$

(t_k, t_{k+1})区间内积分

后向欧拉法

$$\left(I - \begin{bmatrix} 0 & \frac{\Delta t}{C} \\ -\frac{\Delta t}{L} & -\frac{\Delta t R_S}{L} \end{bmatrix}\right)\begin{bmatrix} v_C(t_{k+1}) \\ i_L(t_{k+1}) \end{bmatrix} \approx \begin{bmatrix} v_C(t_k) \\ i_L(t_k) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\Delta t}{L} \end{bmatrix} v_s(t_{k+1})$$

前一状态和当前激励决定当前状态

```
%时域特性1：阶跃响应
  vC(1)=0;
  iL(1)=0;

  x=[vC(1);iL(1)];

  for j=2:timenum
     x=invA*(x+[0; GLD*vs1(j)]);
     vC(j)=x(1);
     iL(j)=x(2);
  end

  figure(1)
  hold on
  plot(t,vC)
```

阶跃激励产生的阶跃响应

```
%时域特性2：噪声滤波
  vC(1)=0;
  iL(1)=0;

  x=[vC(1);iL(1)];

  for j=2:timenum
     x=invA*(x+[0; GLD*vs2n(j)]);
     vC(j)=x(1);
     iL(j)=x(2);
  end

  figure(5+k)
  hold on
  plot(t,vs2,'k')
  plot(t,vC,'b')
```

数字信号带噪声，滤波器应当将噪声滤除

$$\begin{bmatrix} v_C(t_{k+1}) \\ i_L(t_{k+1}) \end{bmatrix} = \begin{bmatrix} 1 & -R_{C\Delta} \\ G_{L\Delta} & 1+G_{L\Delta}R_S \end{bmatrix}^{-1} \left(\begin{bmatrix} v_C(t_k) \\ i_L(t_k) \end{bmatrix} + \begin{bmatrix} 0 \\ G_{L\Delta} \end{bmatrix} v_s(t_{k+1}) \right)$$

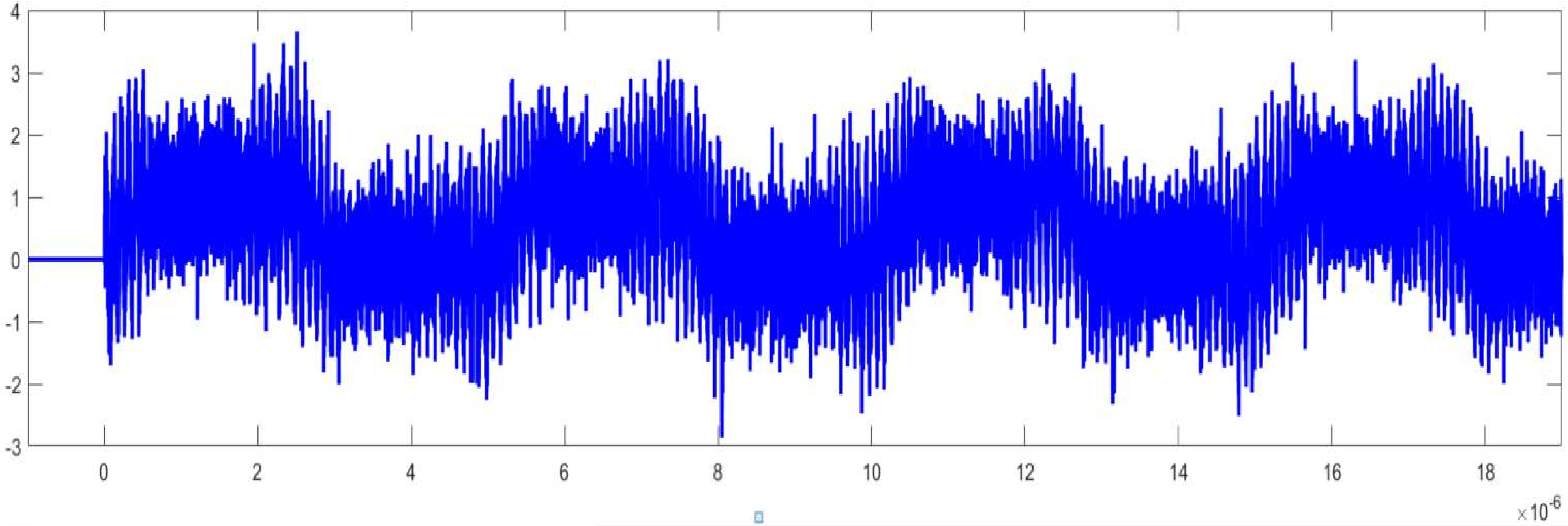
```
%频率特性
   freqstart=f3dB/100;
   freqstop=f3dB*10000;
   freqnum=10000;
   freqstep=10^(log10(freqstop/freqstart)/(freqnum-1));
   freq=freqstart/freqstep;
   taog(1)=0;
   for j=1:freqnum
      freq=freq*freqstep;
      f(j)=freq;
      s=i*2*pi*freq;
      ks=0.5*RS*sqrt(C(k)/L(k));
      w0=1/sqrt(L(k)*C(k));
      H=w0^2/(s^2+2*ks*w0*s+w0^2);
      absH(j)=20*log10(abs(H));
      angH(j)=angle(H)/pi*180;
      if j>1
         taog(j)=-(angH(j)-angH(j-1))/(f(j)-f(j-1))/360;
      end
   end
   taog(1)=taog(2);
end
```

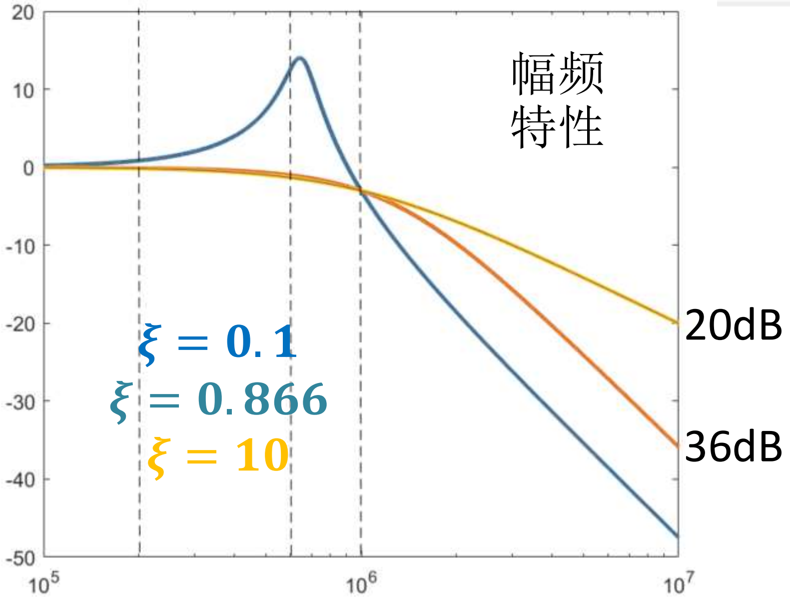
```
figure(3)
hold on
plot(f,absH)      %幅频特性

figure(4)
hold on
plot(f,angH)      %相频特性

figure(5)
hold on
plot(f,taog)      %群延时特性
```

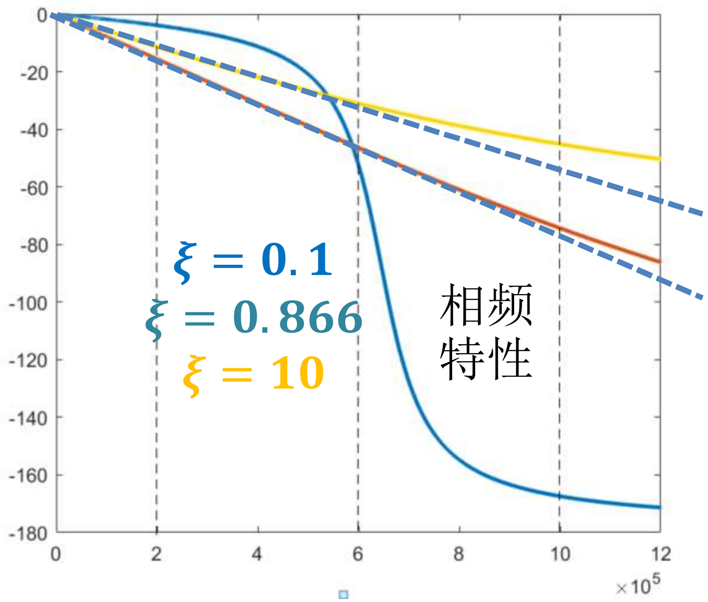
$$H(s)=\frac{\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}$$

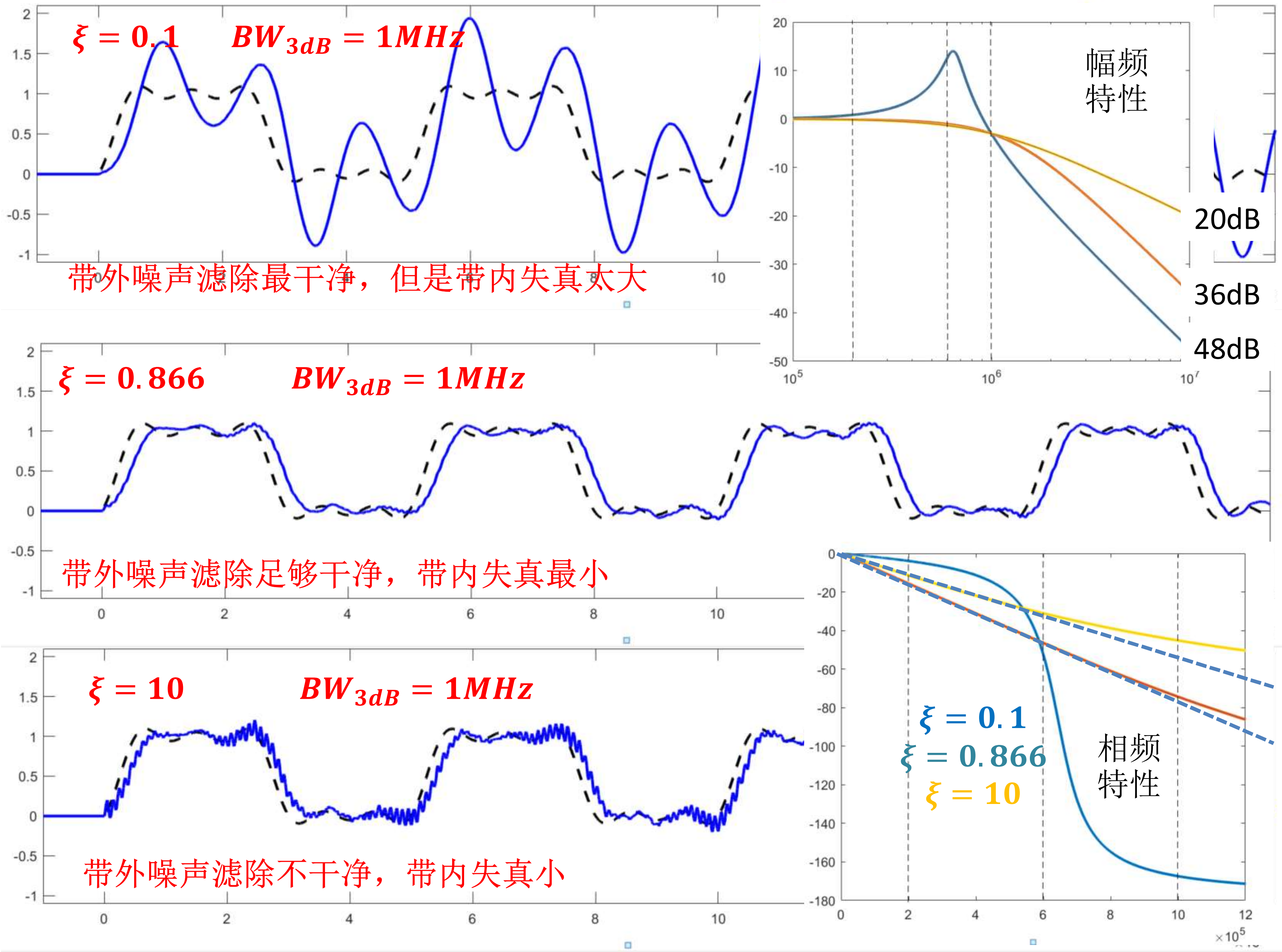
$$\xi=\frac{R_S}{2}\sqrt{\frac{C}{L}}\qquad \omega_0=\frac{1}{\sqrt{LC}}$$

被噪声淹没的信号，哪种滤波器好？

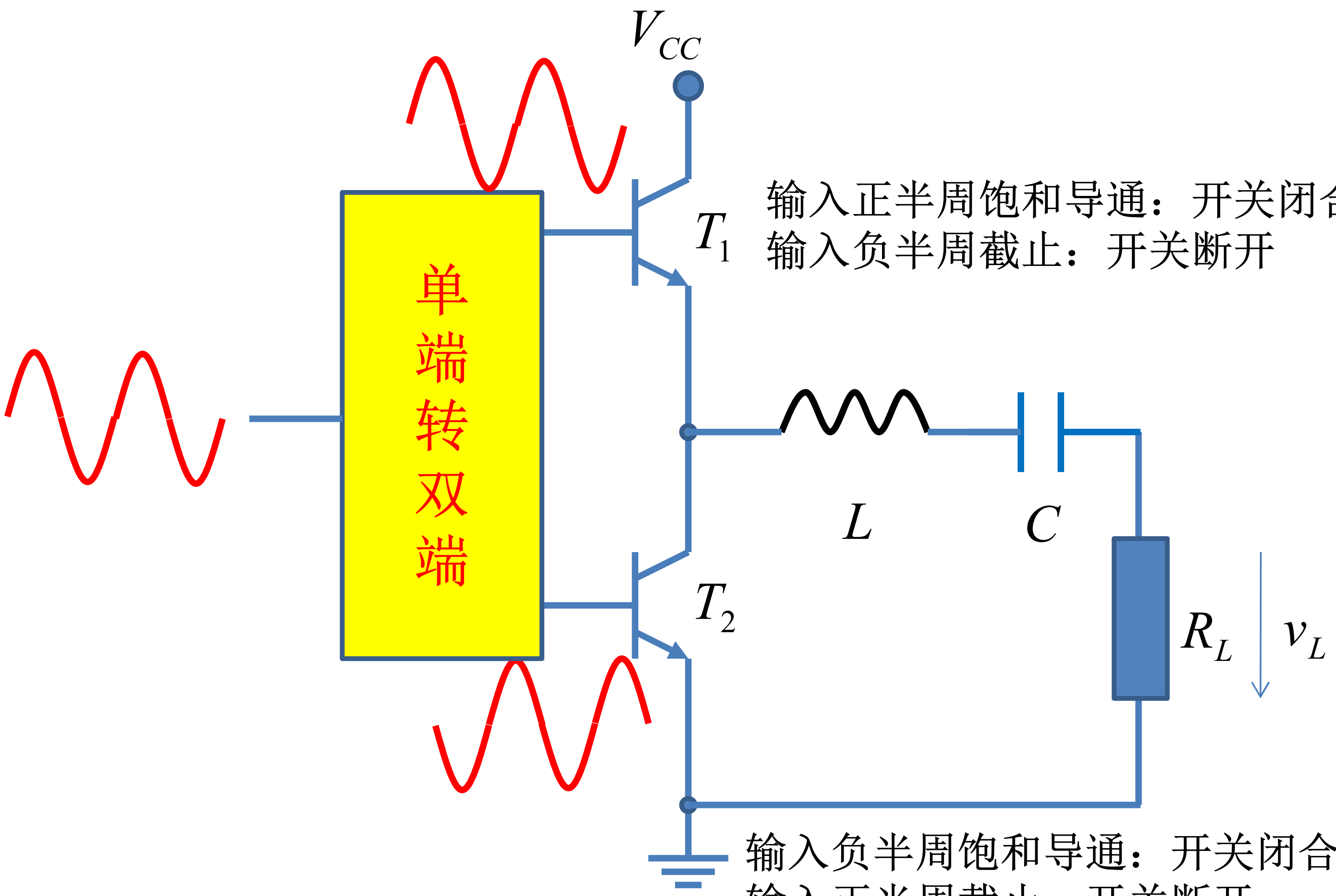







ξ = 0.1 BW3dB = 1MHz
带外噪声滤除最干净，但是带内失真太大
ξ = 0.866 BW3dB = 1MHz
带外噪声滤除足够干净，带内失真最小
ξ = 10 BW3dB = 1MHz
带外噪声滤除不干净，带内失真小
幅频
特性
20dB
36dB
48dB
ξ = 0.1
ξ = 0.866
ξ = 10
相频
特性
×10^5

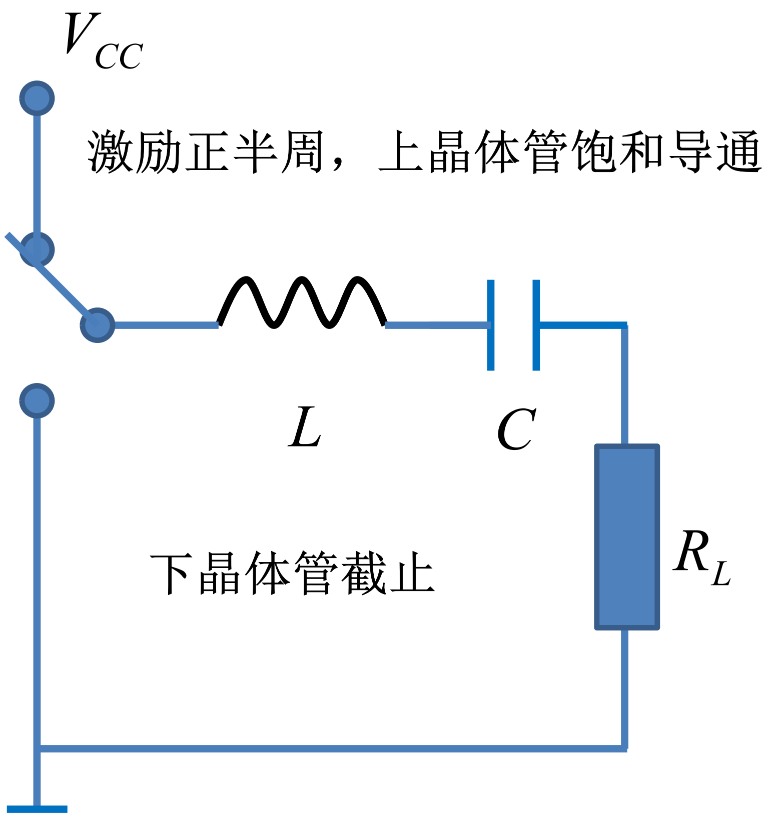
习题6 D类放大器



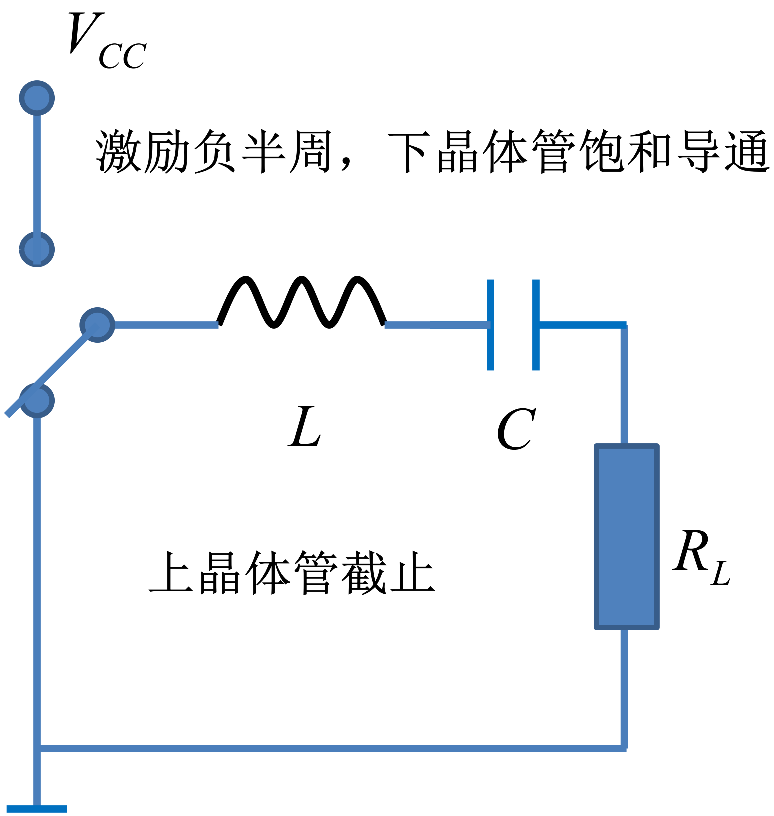
输入正半周饱和导通：开关闭合
输入负半周截止：开关断开

输入负半周饱和导通：开关闭合
输入正半周截止：开关断开

D类放大器等效电路

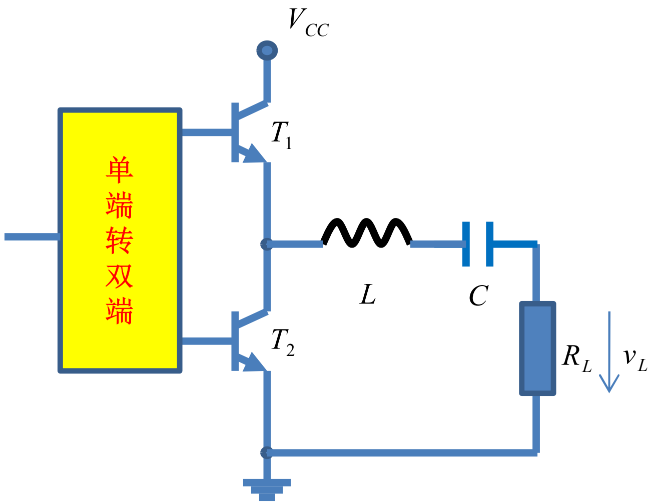


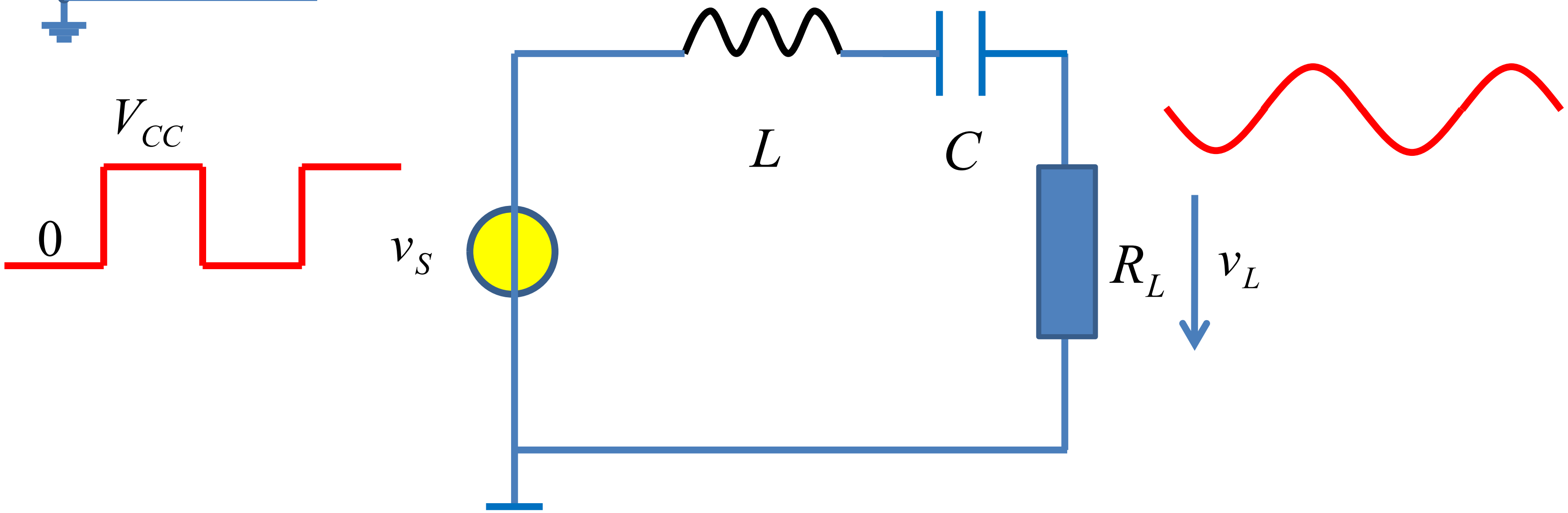
V_{CC}
激励正半周，上晶体管饱和导通
L
C
下晶体管截止
R_L



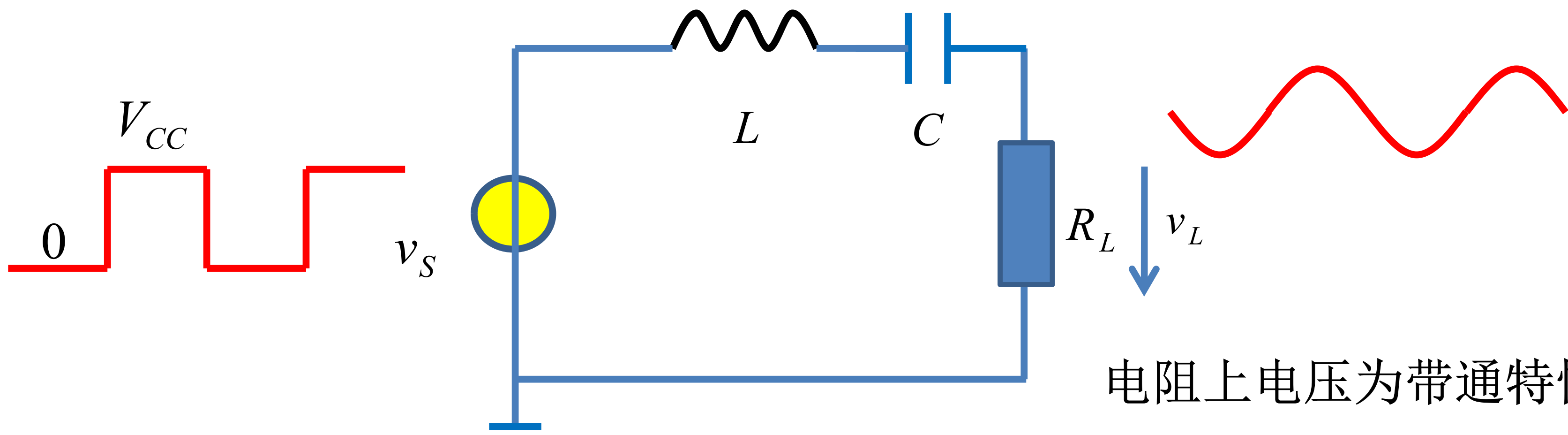
V_{CC}
激励负半周，下晶体管饱和导通
L
C
上晶体管截止
R_L

D类放大
电压源驱动RLC串联谐振回路





- 要想三次谐波分量低于基波分量**40dB**以上，谐振回路的**Q**值应取多大？



电阻上电压为带通特性

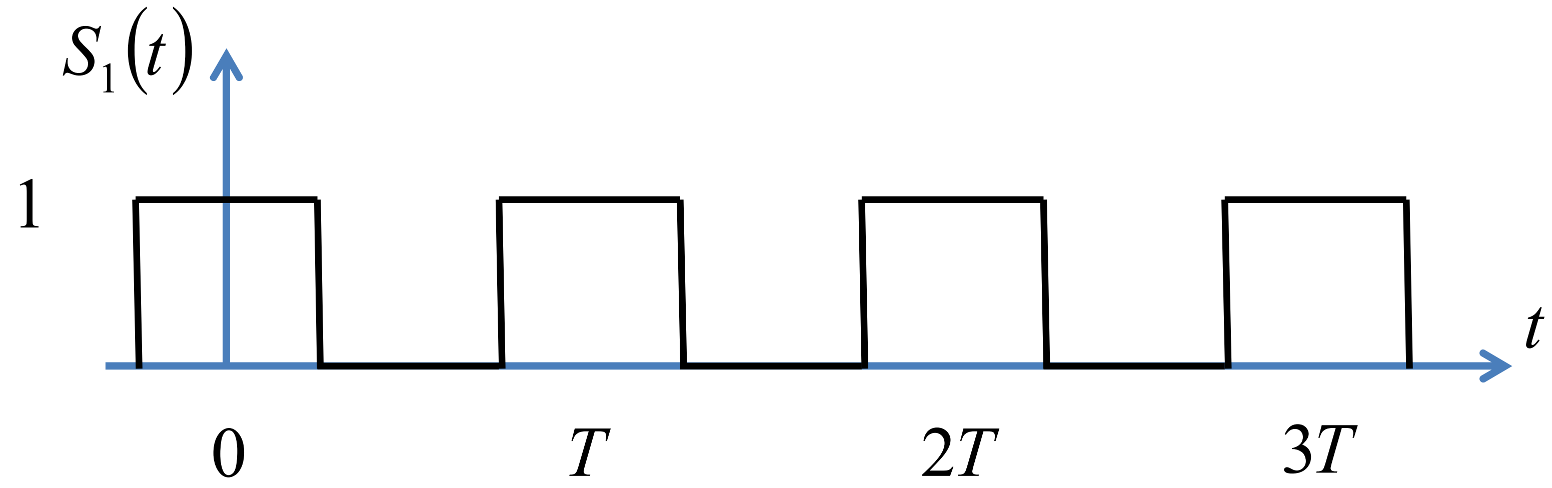
$$\dot{V}_{out}(j\omega)=H(j\omega)\dot{V}_{in}(j\omega)=\frac{R}{j\omega L+\frac{1}{j\omega C}+R}\dot{V}_{in}(j\omega)$$

$$=\frac{\dot{V}_{in}(j\omega)}{1+jQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}=\frac{\dot{V}_{in}(j\omega)}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}e^{-j\arctan Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}$$

$$\omega_0=\frac{1}{\sqrt{LC}}\qquad Q=\frac{1}{R}\sqrt{\frac{L}{C}}$$

串联谐振

方波信号



$$S_1(t)=\frac{1}{2}+\frac{2}{\pi}\cos\omega_0 t-\frac{2}{3\pi}\cos3\omega_0 t+\frac{2}{5\pi}\cos5\omega_0 t-\ldots$$

0/1方波信号中包含直流分量，基波分量，奇次谐波分量（三次、五次、七次、**…**）

$$\left|\dot{V}_{out}(j\omega_0)\right| = \left|H(j\omega_0)\dot{V}_{in}(j\omega_0)\right| = 1 \times a_0 = \frac{2}{\pi}V_{CC}$$

基波分量

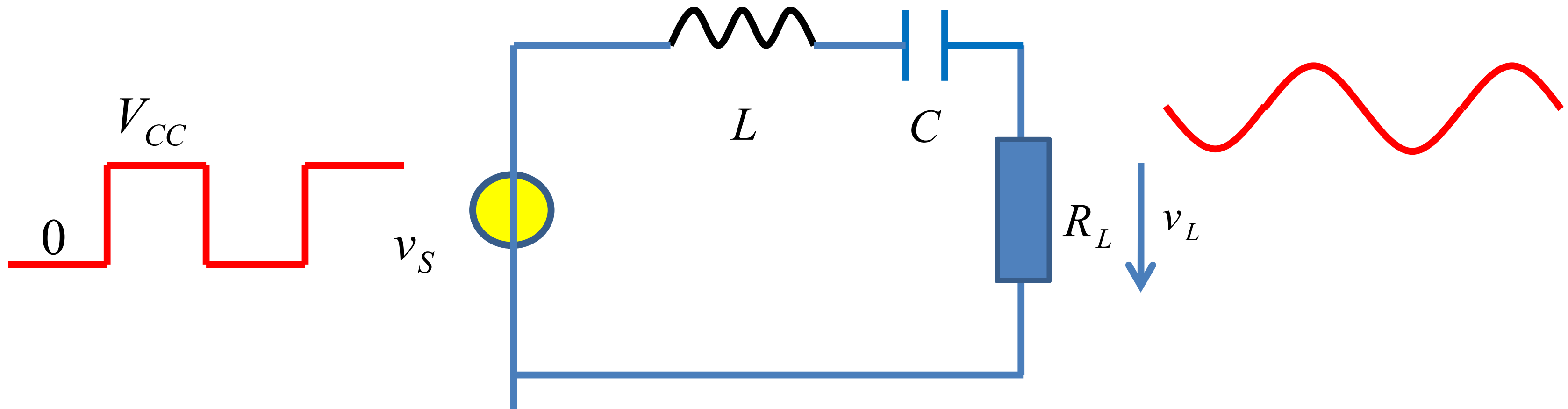
$$\left|\dot{V}_{out}(j3\omega_0)\right| = \left|H(j3\omega_0)\dot{V}_{in}(j3\omega_0)\right| = \frac{1}{\sqrt{1+Q^2\left(\frac{8}{3}\right)^2}} \times \frac{a_0}{3}$$

三次谐波分量

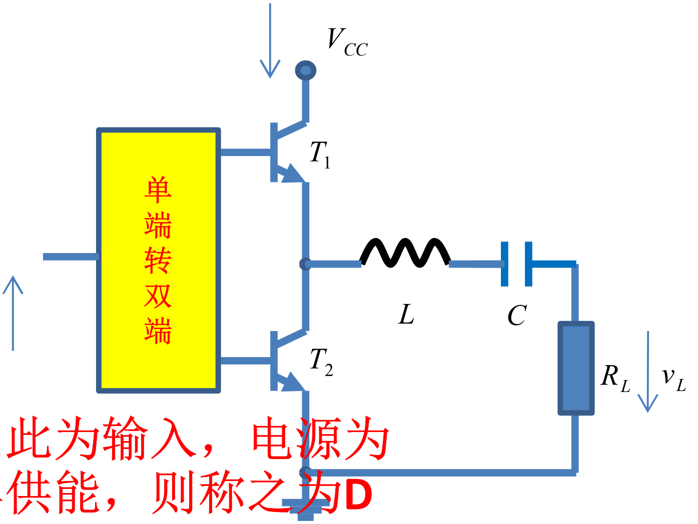
$$10\log\frac{P(3\omega_0)}{P(\omega_0)} = 20\log\frac{\left|\dot{V}_{out}(j3\omega_0)\right|}{\left|\dot{V}_{out}(j\omega_0)\right|} = 20\log\frac{\dfrac{a_0/3}{\sqrt{1+\left(\frac{8}{3}Q\right)^2}}}{a_0} \le -40$$

输出三次谐波功率比基波低**40dB**

$$\frac{1}{3\sqrt{1+\left(\frac{8}{3}Q\right)^2}} \le \frac{1}{100} \quad \Rightarrow \quad Q \ge \frac{3}{8}\sqrt{\frac{10^4}{9}-1} = 12.49$$



以此为输入，控制端控制开关状态，则称之为D类逆变器：将直流能量转换为交流能量



以此为输入，电源为其供能，则称之为D类放大器：对输入正弦信号进行非线性的功率放大

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} > 12.5$$

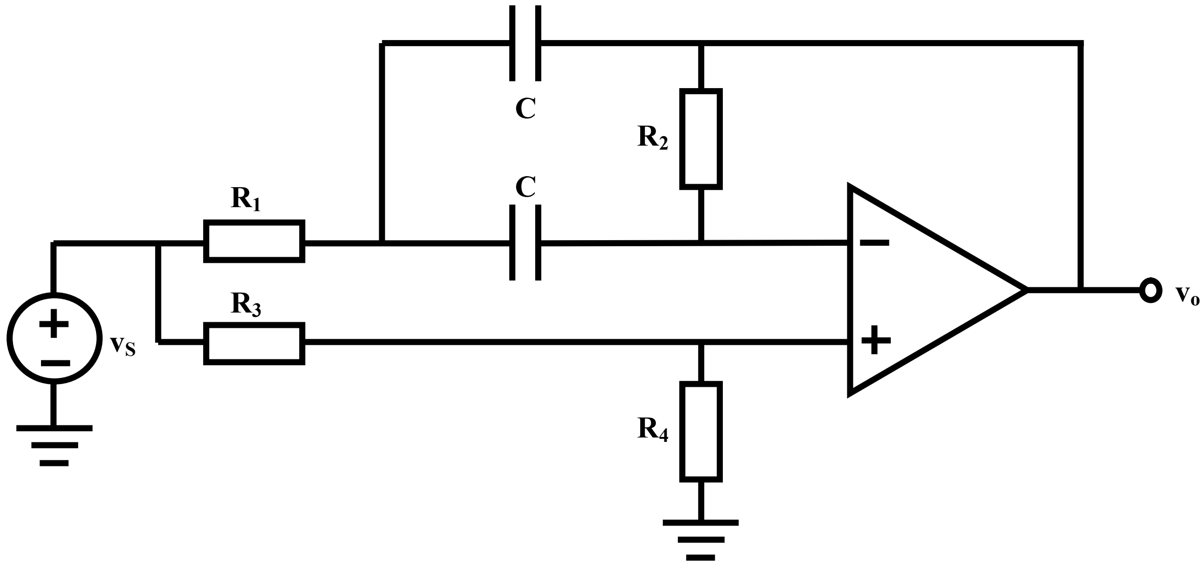
$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0 = 2\pi \frac{1}{T}$$

由此可以设计**L**、**C**值的大小?

用简单模型做原理性理解
更细致的分析见后续专业课程

作业9：全通滤波器

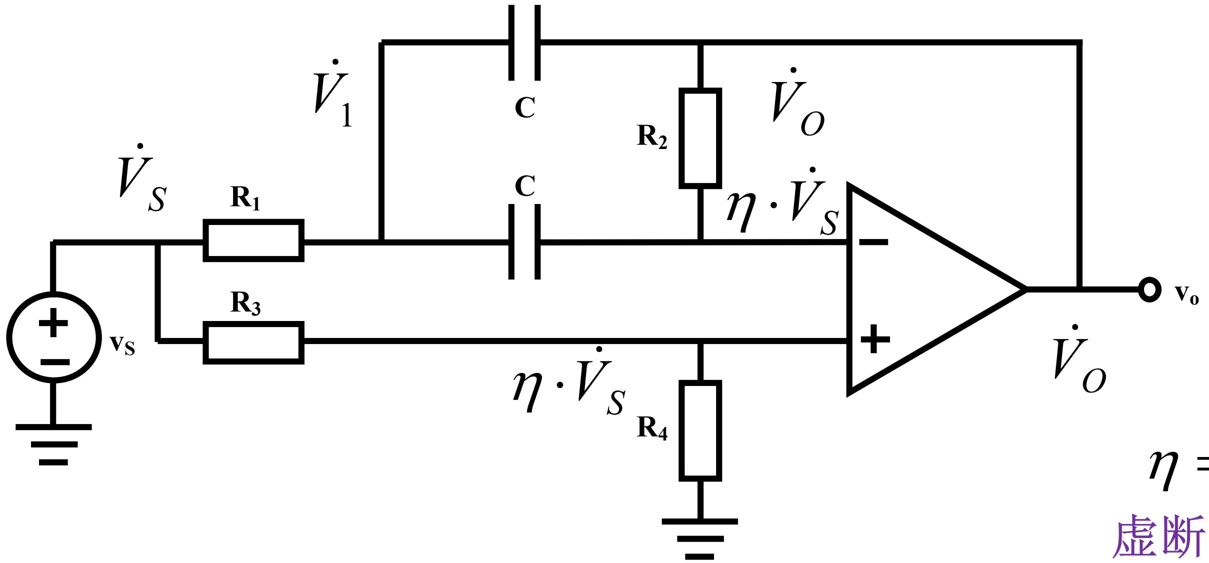
- 请分析图示电路，电阻R_1、R_2、R_3、R_4之间满足什么关系时，该电路可构成一个二阶全通滤波器。给出该全通滤波器的关键参量：H_0,ω_0,ξ。



$$H(s)=H_0\frac{s^2-2\xi\omega_0 s+\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}$$

$$H(j\omega)=\frac{\dot{V}_o}{\dot{V}_S}$$

结点电压方程



$$\eta=\frac{R_4}{R_3+R_4}$$

虚断

$$\frac{\dot V_S-\dot V_1}{R_1}=j\omega C\left(\dot V_1-\dot V_O\right)+j\omega C\left(\dot V_1-\eta\dot V_S\right)$$

虚短

$$\left(\frac{1}{R_1}+j\omega C\eta\right)\dot V_S+j\omega C\dot V_O=\left(2j\omega C+\frac{1}{R_1}\right)\dot V_1$$

$$j\omega C\left(\dot V_1-\eta\dot V_S\right)=\frac{\eta\dot V_S-\dot V_O}{R_2}$$

虚断

$$\dot V_1=\frac{1+j\omega R_1C\eta}{1+2j\omega R_1C}\dot V_S+\frac{j\omega R_1C}{1+2j\omega R_1C}\dot V_O$$

$$\dot V_1=\frac{\eta\dot V_S-\dot V_O}{j\omega CR_2}+\eta\dot V_S=\eta\dot V_S\left(1+\frac{1}{j\omega CR_2}\right)-\frac{\dot V_O}{j\omega CR_2}$$

传递函数

$$\dot{V}_1=\frac{1+j\omega R_1C\eta}{1+2j\omega R_1C}\dot{V}_S+\frac{j\omega R_1C}{1+2j\omega R_1C}\dot{V}_O \qquad \dot{V}_1=\eta\dot{V}_S\left(1+\frac{1}{j\omega CR_2}\right)-\frac{\dot{V}_O}{j\omega CR_2}$$

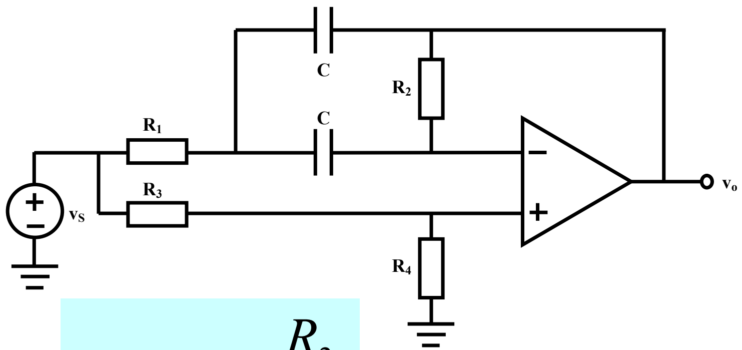
$$\frac{1+j\omega R_1C\eta}{1+2j\omega R_1C}\dot{V}_S+\frac{j\omega R_1C}{1+2j\omega R_1C}\dot{V}_O=\eta\dot{V}_S\left(1+\frac{1}{j\omega CR_2}\right)-\frac{\dot{V}_o}{j\omega CR_2}$$

$$\frac{\dot{V}_O}{\dot{V}_S}=\frac{\eta\left(1+\dfrac{1}{j\omega CR_2}\right)-\dfrac{1+j\omega R_1C\eta}{1+2j\omega R_1C}}{\dfrac{j\omega R_1C}{1+2j\omega R_1C}+\dfrac{1}{j\omega CR_2}}=\frac{\eta(1+j\omega R_2C)(1+2j\omega R_1C)-(1+j\omega R_1C\eta)j\omega R_2C}{j\omega R_1Cj\omega R_2C+1+2j\omega R_1C}$$

$$=\eta\frac{s^2R_1CR_2C+sC\left(R_2+2R_1-\dfrac{R_2}{\eta}\right)+1}{s^2R_1CR_2C+2sR_1C+1} \qquad 2R_1=-\left(R_2+2R_1-\frac{R_2}{\eta}\right)$$

$$4R_1=R_2\left(\frac{1}{\eta}-1\right)=R_2\left(\frac{R_3+R_4}{R_4}-1\right)=R_2\frac{R_3}{R_4}$$

二阶全通滤波器



$$4R_1 = R_2 \frac{R_3}{R_4}$$

$$\frac{\dot{V}_O}{\dot{V}_S} = \eta \frac{s^2 R_1 C R_2 C + sC\left(R_2 + 2R_1 - \frac{R_2}{\eta}\right) + 1}{s^2 R_1 C R_2 C + 2sR_1 C + 1}$$

$$= \eta \frac{s^2 R_1 C R_2 C - 2sR_1 C + 1}{s^2 R_1 C R_2 C + 2sR_1 C + 1}$$

$$= H_0 \frac{s^2 - \frac{2s}{R_2 C} + \frac{1}{R_1 C R_2 C}}{s^2 + \frac{2s}{R_2 C} + \frac{1}{R_1 C R_2 C}}$$

$$= H_0 \frac{s^2 - 2\xi\omega_0 s + \omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

1、改变电阻比改变阻尼系数

$$\xi = \sqrt{\frac{R_1}{R_2}}$$

2、改变C改变自由振荡频率

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2} C}$$

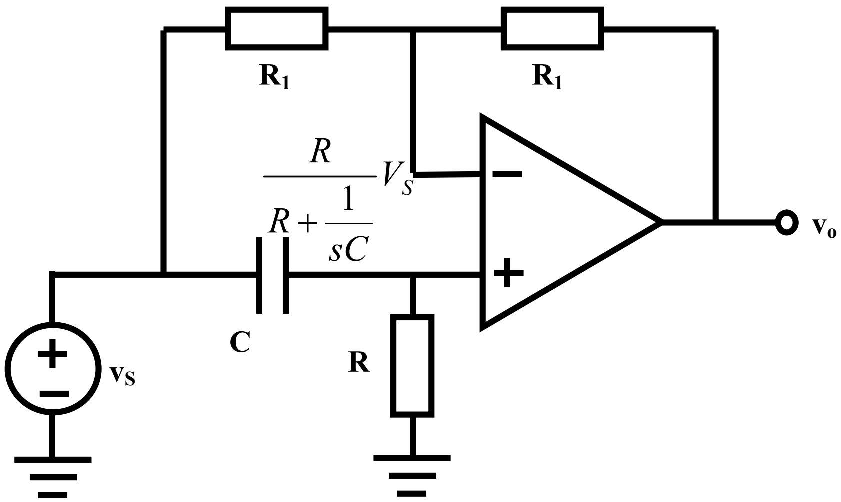
3、增益由阻尼系数决定，阻尼系数调整时，增益自动调整，不能单独调整增益

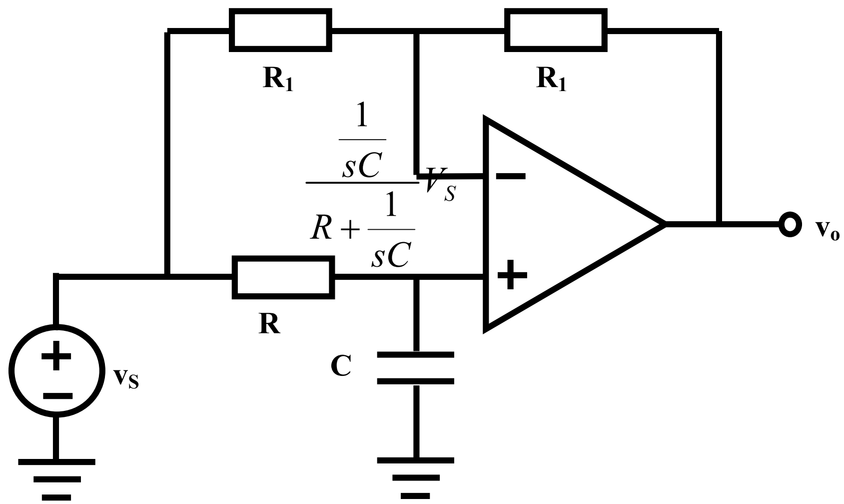
$$H_0 = \frac{R_4}{R_3 + R_4} = \frac{1}{\frac{R_3}{R_4} + 1}$$

$$= \frac{1}{4\frac{R_1}{R_2} + 1} = \frac{1}{4\xi^2 + 1}$$

一阶有源RC全通滤波器

$$\frac{V_S - \dfrac{R}{R+\dfrac{1}{sC}}V_S}{R_1} = \frac{\dfrac{R}{R+\dfrac{1}{sC}}V_S - V_O}{R_1}$$





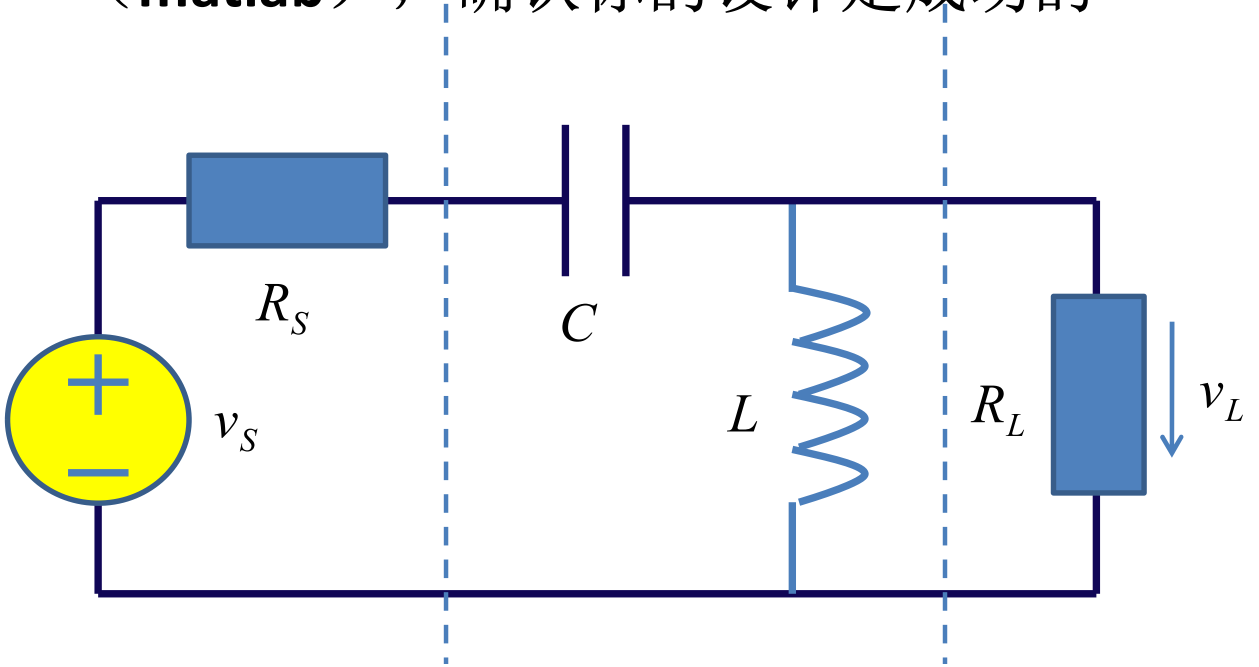
$$\frac{\dot{V}_O}{\dot{V}_S} = H_0 \frac{s-\omega_0}{s+\omega_0}$$

全通滤波器：所有频率分量均可通过，
但不同频率有不同的相位偏移

第10讲 阻抗变换与匹配网络

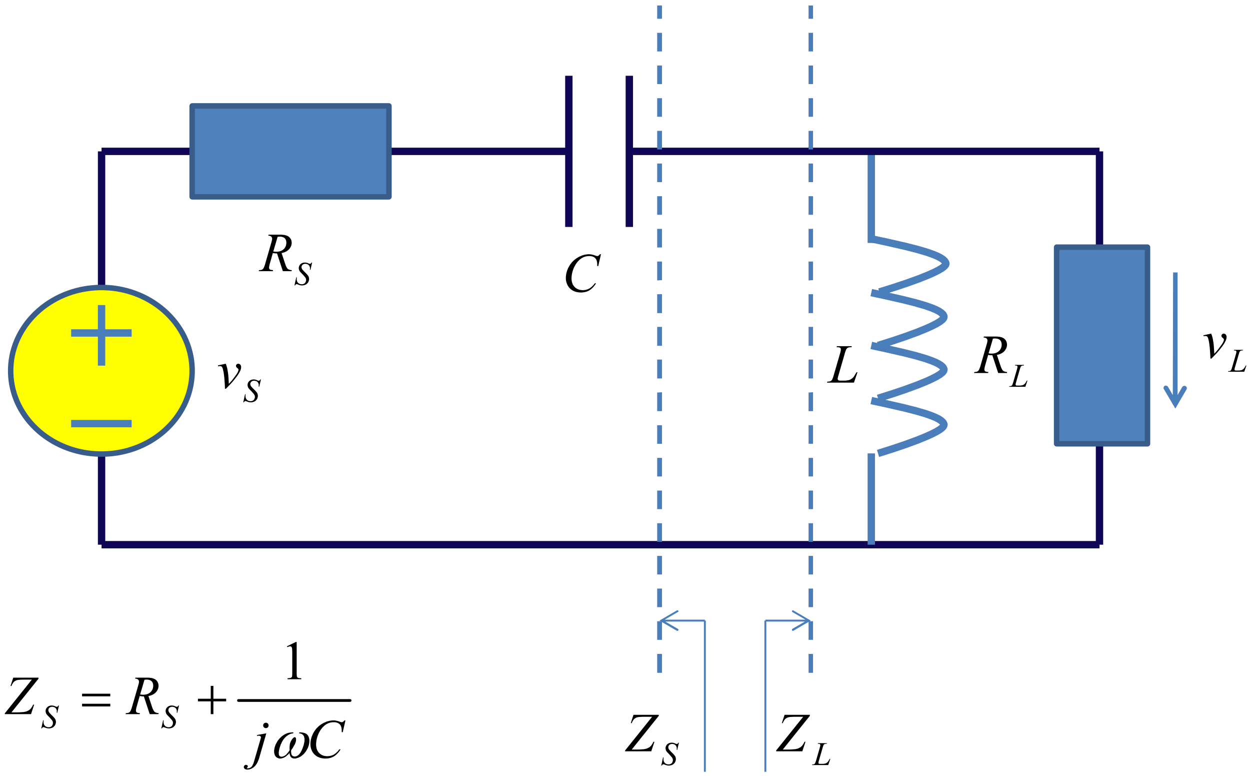
作业1 LC高通型匹配网络

- 推导传递函数，如果希望在**10MHz**频点上实现最大功率传输：负载获得信源的额定输出功率，实现**200Ω**和**50Ω**阻抗之间的匹配
  - 电感**L**、电容**C**如何取值？
  - 画出基于功率传输的传递函数的幅频特性和相频特性（**matlab**），确认你的设计是成功的



互易元件形成的网络是互易网络，设计反了，掉个个就可以完成最终设计了

直接方法共轭匹配



$$Z_S = R_S + \frac{1}{j\omega C}$$

$$Z_L = \frac{1}{\dfrac{1}{R_L} + \dfrac{1}{j\omega L}} = \frac{j\omega L R_L}{R_L + j\omega L}$$

$$= \frac{j\omega L R_L\left(R_L - j\omega L\right)}{R_L{}^2 + (\omega L)^2} = \frac{(\omega L)^2 R_L}{R_L{}^2 + (\omega L)^2} + \frac{j\omega L R_L{}^2}{R_L{}^2 + (\omega L)^2}$$

共轭匹配

$$Z_S = R_S + \frac{1}{j\omega C} \qquad Z_L = \frac{(\omega L)^2 R_L}{R_L{}^2 + (\omega L)^2} + \frac{j\omega L R_L{}^2}{R_L{}^2 + (\omega L)^2}$$

$$R_S - j\frac{1}{\omega_r C} = Z_S(j\omega_r) = Z_L^*(j\omega_r) = \frac{(\omega_r L)^2 R_L}{R_L{}^2 + (\omega_r L)^2} - j\frac{\omega_r L R_L{}^2}{R_L{}^2 + (\omega_r L)^2}$$

在特定频点可共轭匹配

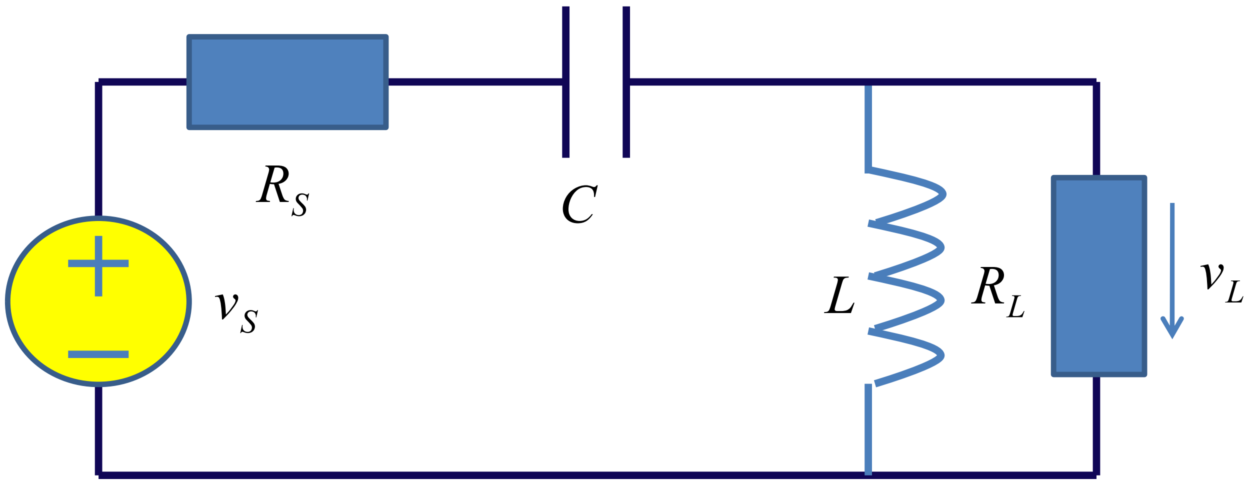
$$R_S = \frac{(\omega_r L)^2 R_L}{R_L{}^2 + (\omega_r L)^2}$$

$$\frac{1}{\omega_r C} = \frac{\omega_r L R_L{}^2}{R_L{}^2 + (\omega_r L)^2}$$

$$L = \frac{1}{\omega_r}\frac{R_L}{\sqrt{\frac{R_L}{R_S} - 1}}$$

$$C = \frac{1}{\omega_r}\frac{1}{R_S\sqrt{\frac{R_L}{R_S} - 1}}$$

用口诀设计



10MHz频点上实现**50Ω**和**200Ω**阻抗之间的匹配

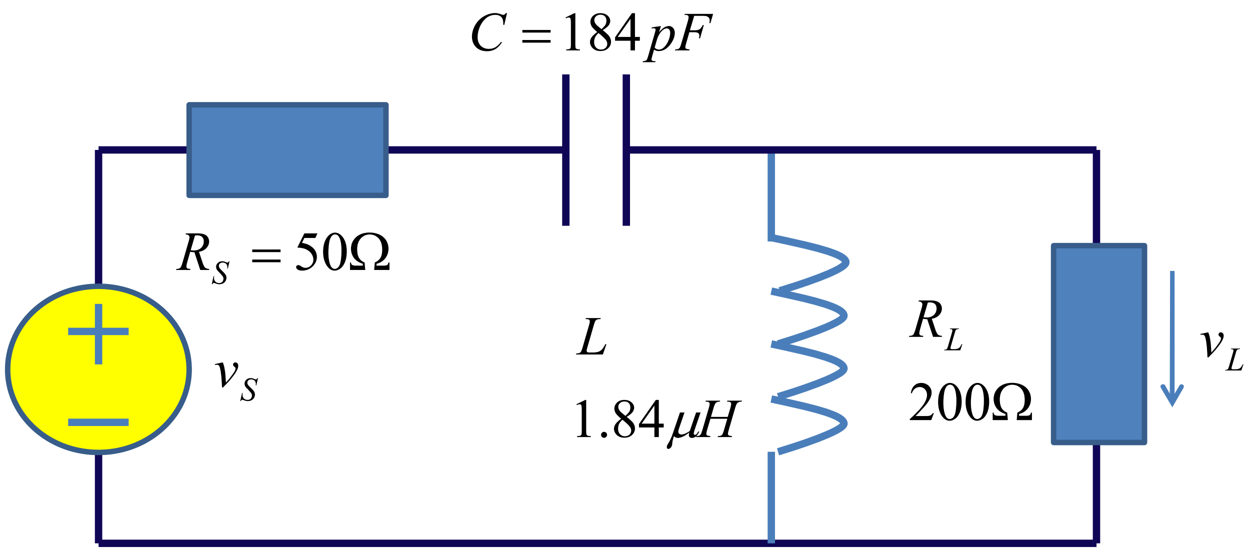
$$Q=\sqrt{\frac{R_L}{R_S}-1}=\sqrt{\frac{200}{50}-1}=\sqrt{3}$$

$$L=\frac{1}{\omega_r}\frac{R_L}{Q}=\frac{1}{2\pi\times 10\times 10^6}\frac{200}{\sqrt{3}}=1.84\mu H$$

$$C=\frac{1}{\omega_r}\frac{1}{R_S Q}=\frac{1}{2\pi\times 10\times 10^6}\frac{1}{50\times\sqrt{3}}=184pF$$

并大串小Q相等

确认



10MHz频点上实现**50Ω**和**200Ω**阻抗之间的匹配

$$\mathbf{ABCD}=\begin{bmatrix}1 & R_S+\frac{1}{j\omega C}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{R_L}+\frac{1}{j\omega L} & 1\end{bmatrix}=\begin{bmatrix}1+\left(R_S+\frac{1}{j\omega C}\right)\left(\frac{1}{R_L}+\frac{1}{j\omega L}\right) & *\\ * & *\end{bmatrix}$$

$$H(s)=2\sqrt{\frac{R_S}{R_L}}\frac{V_L}{V_S}=2\sqrt{\frac{R_S}{R_L}}\frac{1}{A}=2\sqrt{\frac{R_S}{R_L}}\frac{1}{1+\left(R_S+\frac{1}{sC}\right)\left(\frac{1}{R_L}+\frac{1}{sL}\right)}=2\sqrt{\frac{R_S}{R_L}}\frac{s^2LCR_L}{s^2LC(R_L+R_S)+s(L+CR_SR_L)+R_L}$$

$$=2\sqrt{\frac{R_S}{R_L}}\frac{R_L}{R_L+R_S}\frac{s^2}{s^2+s\left(\frac{1}{(R_L+R_S)C}+\frac{R_S\parallel R_L}{L}\right)+\frac{R_L}{(R_L+R_S)LC}}=2\frac{\sqrt{R_SR_L}}{R_L+R_S}\frac{s^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Matlab数值确认

```
RS=50;
RL=200;
fr=10E6;
```

已知条件

```
wr=2*pi*fr;
Q=sqrt(RL/RS-1);
L=RL/wr/Q;
C=1/wr/RS/Q;
```

匹配网络设计

```
freqstart=fr/100;;
freqstop=fr*100;
freqnum=100;
freqstep=10^(log10(freqstop/freqstart)/freqnum);
```

考察频率范围

```
freq=freqstart/freqstep;
for k=1:freqnum
   freq=freq*freqstep;

   f(k)=freq;
   w=2*pi*freq;
   s=i*w;

   ABCD=[1 RS+1/s/C; 0 1]*[1 0; 1/RL+1/s/L 1];

   H=2*sqrt(RS/RL)/ABCD(1,1);

   absH(k)=abs(H);
   angleH(k)=angle(H)/pi*180;
end
```

ABCD矩阵计算

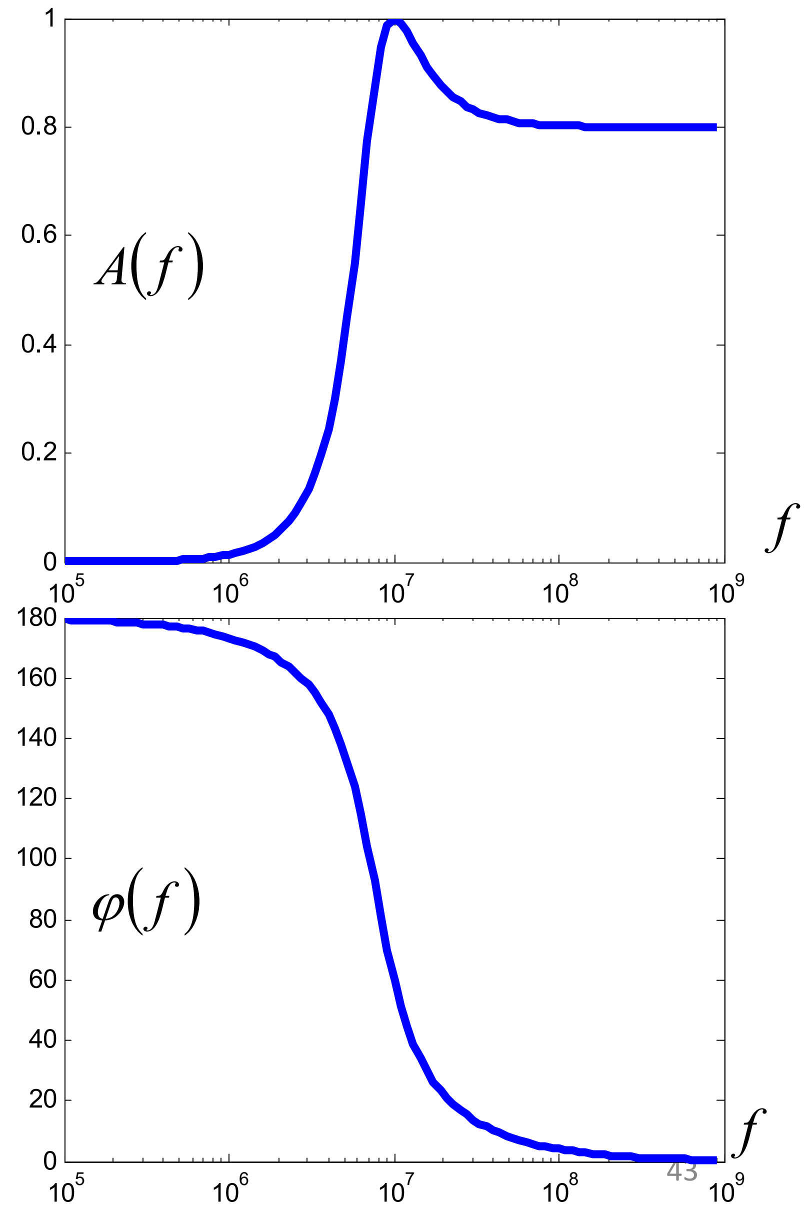
传递函数计算

幅频特性

相频特性

```
figure(1)
plot(f,absH)
```

```
figure(2)
plot(f,angleH)
```

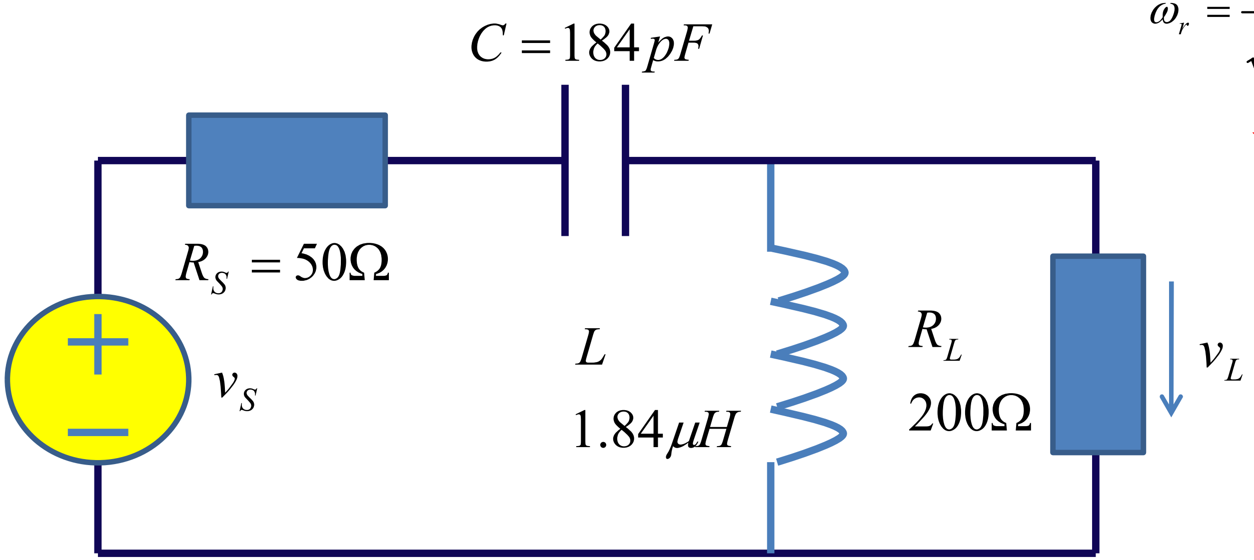


几个谐振频率

$$H(s)=2\frac{\sqrt{R_S R_L}}{R_L+R_S}\frac{s^2}{s^2+s\left(\frac{1}{(R_L+R_S)C}+\frac{R_S \parallel R_L}{L}\right)+\frac{R_L}{(R_L+R_S)LC}}=2\frac{\sqrt{R_S R_L}}{R_L+R_S}\frac{s^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\omega_r=\frac{\omega_n}{\sqrt{1-2\xi^2}}$$

谐振峰频点



$$f_0=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\times3.14\times\sqrt{1.84\mu\times184p}}=8.66MHz$$

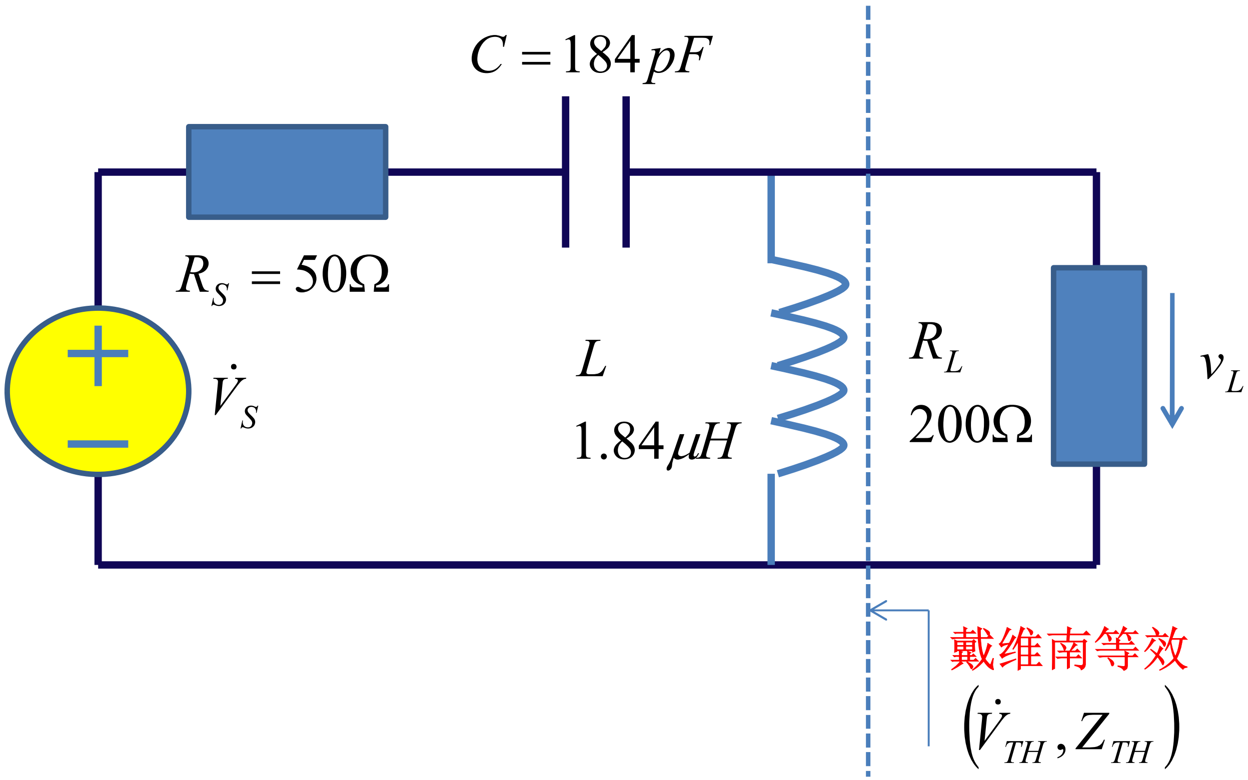
在$R_L=\infty$或$R_S=0$时的LC谐振频率

$$f_n=\frac{1}{2\pi\sqrt{\frac{R_L}{R_S+R_L}LC}}=\frac{1}{2\times3.14\times\sqrt{\frac{200}{200+50}\times1.84\mu\times184p}}=9.68MHz$$

二阶系统的自由振荡频率

$$f_r=\frac{f_n}{\sqrt{1-2\xi^2}}=\sqrt{\frac{R_L}{R_L-R_S}}f_0=10MHz$$

二阶高通系统的谐振峰频点：幅频特性的极大值频点

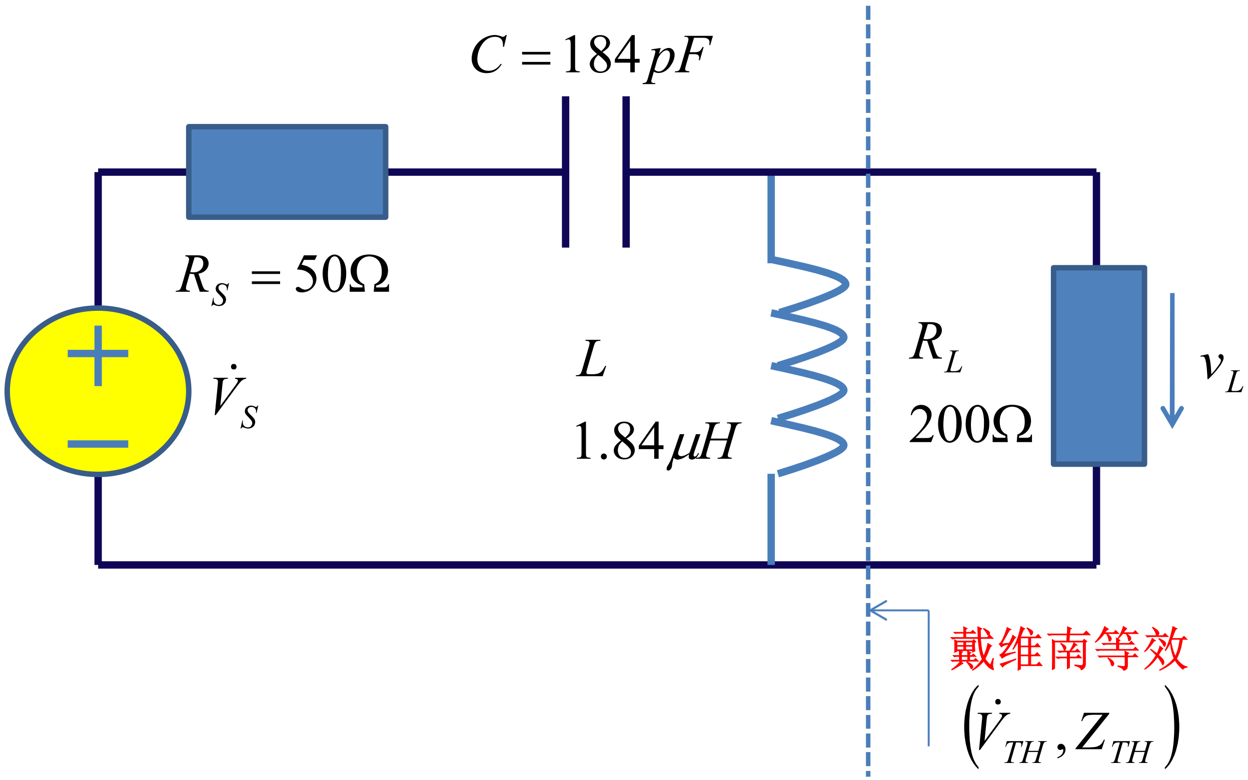


$$L=\frac{1}{\omega_r}\frac{R_L}{\sqrt{\frac{R_L}{R_S}-1}}$$

$$C=\frac{1}{\omega_r}\frac{1}{R_S\sqrt{\frac{R_L}{R_S}-1}}$$

$$\dot{V}_{TH}=\frac{j\omega L}{j\omega L+\frac{1}{j\omega C}+R_S}\dot{V}_S=\frac{-\omega^2 LC}{\left(1-\omega^2 LC\right)+j\omega R_S C}\dot{V}_S=\frac{-\frac{\omega^2}{\omega_r^2}\frac{R_L}{R_L-R_S}}{1-\frac{\omega^2}{\omega_r^2}\frac{R_L}{R_L-R_S}+j\frac{\omega}{\omega_r}\sqrt{\frac{R_S}{R_L-R_S}}}\dot{V}_S$$

$$Z_{TH}=\frac{j\omega L\left(\frac{1}{j\omega C}+R_S\right)}{j\omega L+\frac{1}{j\omega C}+R_S}=\frac{-\omega^2 LCR_S+j\omega L}{\left(1-\omega^2 LC\right)+j\omega R_S C}=R_L\frac{-\frac{\omega^2}{\omega_r^2}\frac{R_S}{R_L-R_S}+j\frac{\omega}{\omega_r}\sqrt{\frac{R_S}{R_L-R_S}}}{1-\frac{\omega^2}{\omega_r^2}\frac{R_L}{R_L-R_S}+j\frac{\omega}{\omega_r}\sqrt{\frac{R_S}{R_L-R_S}}}$$



$$P_L=\frac{|\dot{V}_L|^2}{R_L}=\frac{\left|\frac{1}{2}\dot{V}_{TH}\right|^2}{R_L}=\frac{1}{4}\frac{|\dot{V}_S|^2}{R_S}=P_{S,\max}$$

$$\dot{V}_{TH}=\frac{-\frac{\omega^2}{\omega_r^2}\frac{R_L}{R_L-R_S}}{1-\frac{\omega^2}{\omega_r^2}\frac{R_L}{R_L-R_S}+j\frac{\omega}{\omega_r}\sqrt{\frac{R_S}{R_L-R_S}}}\dot{V}_S\overset{\omega=\omega_r}{=}\frac{\frac{R_L}{R_L-R_S}}{\frac{R_S}{R_L-R_S}-j\sqrt{\frac{R_S}{R_L-R_S}}}\dot{V}_S=\sqrt{\frac{R_L}{R_S}}e^{j\arctan\sqrt{\frac{R_L-R_S}{R_S}}}\dot{V}_S$$

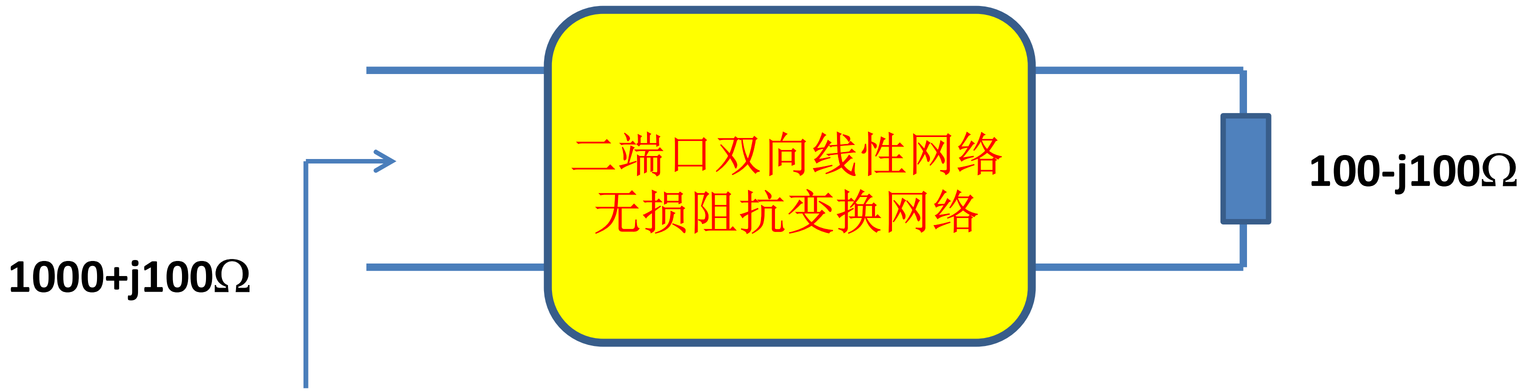
$$Z_{TH}=R_L\frac{-\frac{\omega^2}{\omega_r^2}\frac{R_S}{R_L-R_S}+j\frac{\omega}{\omega_r}\sqrt{\frac{R_S}{R_L-R_S}}}{1-\frac{\omega^2}{\omega_r^2}\frac{R_L}{R_L-R_S}+j\frac{\omega}{\omega_r}\sqrt{\frac{R_S}{R_L-R_S}}}\overset{\omega=\omega_r}{=}R_L$$

在ω_r频点，戴维南电压是激励电压的**sqrt(R_L/R_S)=2**倍，戴维南内阻恰好等于负载电阻，可实现最大功率传输匹配

作业2：
阻抗变换网络

请设计一个阻抗变换网络，在频点10MHz上，将阻抗100+j100Ω变换为1000-j100Ω。

请设计一个阻抗变换网络，在频点10MHz上，将阻抗100-j100Ω变换为1000+j100Ω



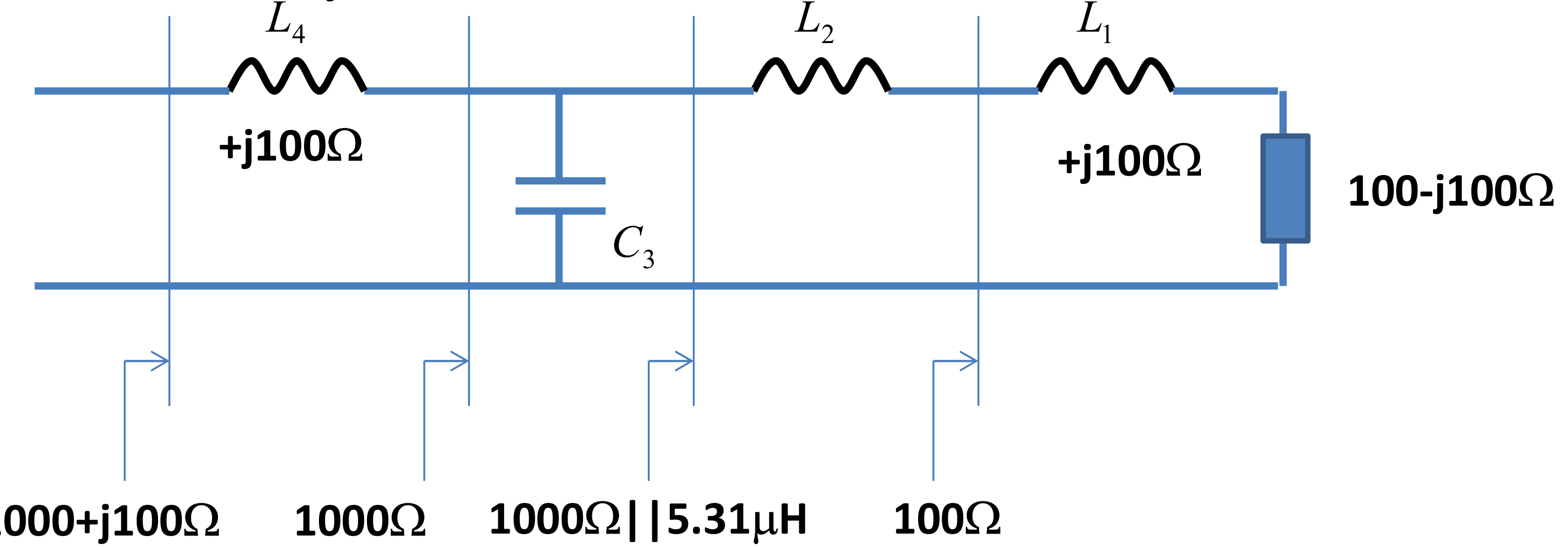
最简单的阻抗变换思路：
1、首先用正电抗（电感）抵偿负电抗
2、之后用串转并，将100Ω转化为1000Ω
3、最后通过补偿方法获得j100Ω正电抗

最简单的阻抗变换思路：

1、首先用正电抗（电感）抵偿负电抗

2、之后用串转并，将100Ω转化为1000Ω

3、最后通过补偿方法获得j100Ω正电抗



$$\omega_0 L_1 = 100\Omega$$

$$L_1 = \frac{100}{2\times 3.14\times 10\times 10^6} = 1.59\mu H$$

$$L_4 = 1.59\mu H$$

$$Q = \sqrt{\frac{R'}{R} - 1} = \sqrt{\frac{1000}{100} - 1} = 3$$

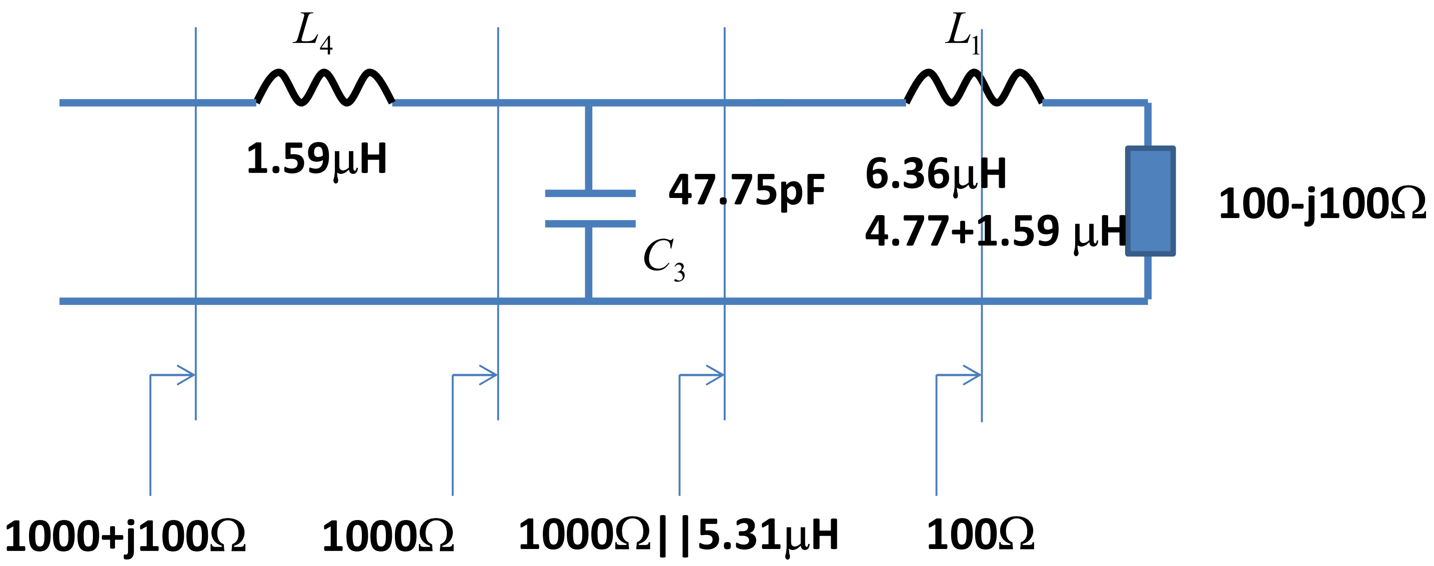
$$C_3 = \frac{1}{\omega_0^2 L'} = \frac{1}{\left(2\times 3.14\times 10\times 10^6\right)^2 \times 5.31\times 10^{-6}} = 47.75pF$$

$$L_2 = \frac{QR}{\omega_0} = \frac{3\times 100}{2\pi\times 10^7} = 4.77\mu H$$

$$L' = \frac{R'}{Q\omega_0} = \frac{1000}{3\times 2\pi\times 10^7} = 5.31\mu H$$

并大串小Q相等

阻抗变换网络1



答案不唯一，具体电路和实际应用环境有关，和实际负载有关
和是否需要进行带宽优化有关？

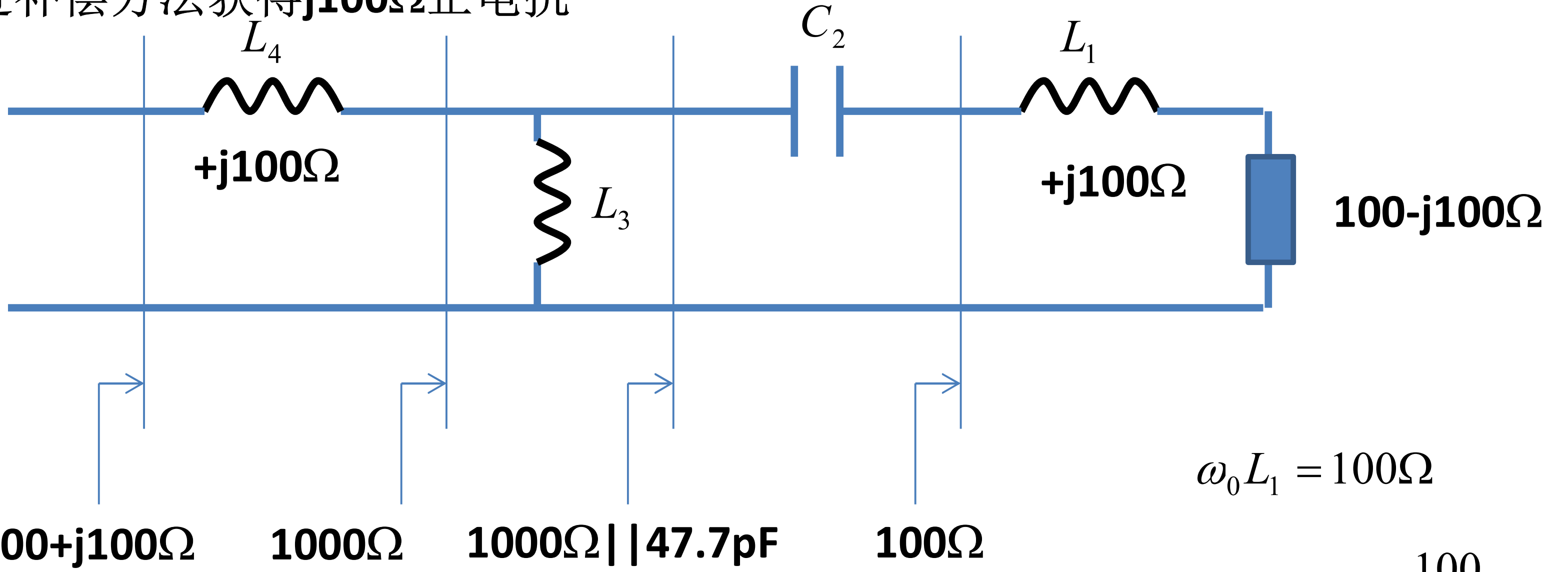
最简单的阻抗变换思路：

1、首先用正电抗（电感）抵偿负电抗

2、之后用串转并，将100Ω转化为1000Ω

3、最后通过补偿方法获得j100Ω正电抗

$C_2 = 53.1pF = 79.6pF$串$159pF$



$\omega_0 L_1 = 100\Omega$

$$L_1 = \frac{100}{2\times3.14\times10\times10^6} = 1.59\mu H$$

$L_4 = 1.59\mu H$

$$Q = \sqrt{\frac{R'}{R} - 1} = \sqrt{\frac{1000}{100} - 1} = 3$$

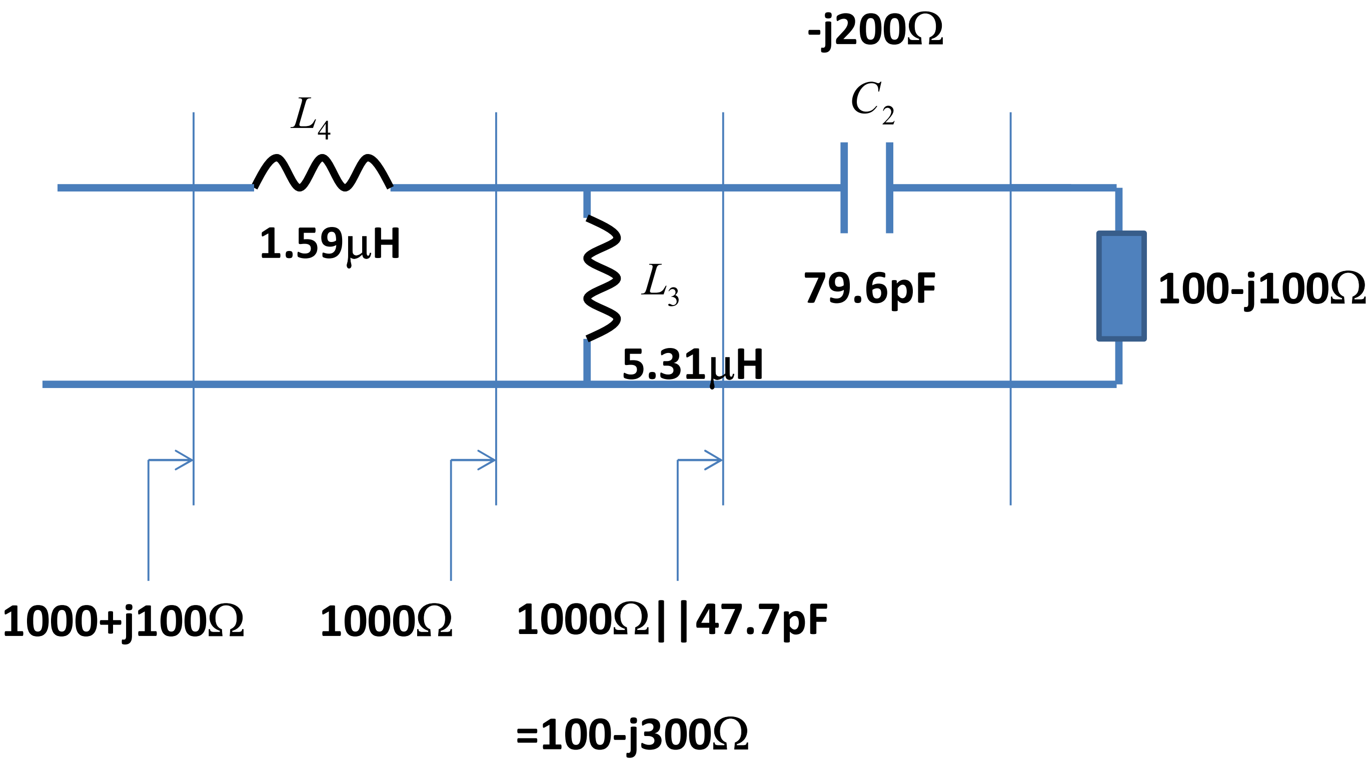
$$L_3 = \frac{1}{\omega_0^2 C'} = \frac{1}{(2\times3.14\times10\times10^6)^2 \times 47.7\times10^{-12}} = 5.31\mu H$$

$$C_2 = \frac{1}{\omega_0 R Q} = \frac{1}{2\pi\times10^7\times100\times3} = 53.1pF$$

$$C' = \frac{Q}{\omega_0 R'} = \frac{3}{2\pi\times10^7\times1000} = 47.7pF$$

等效并联电容和等效并联电阻的Q值不会发生变化

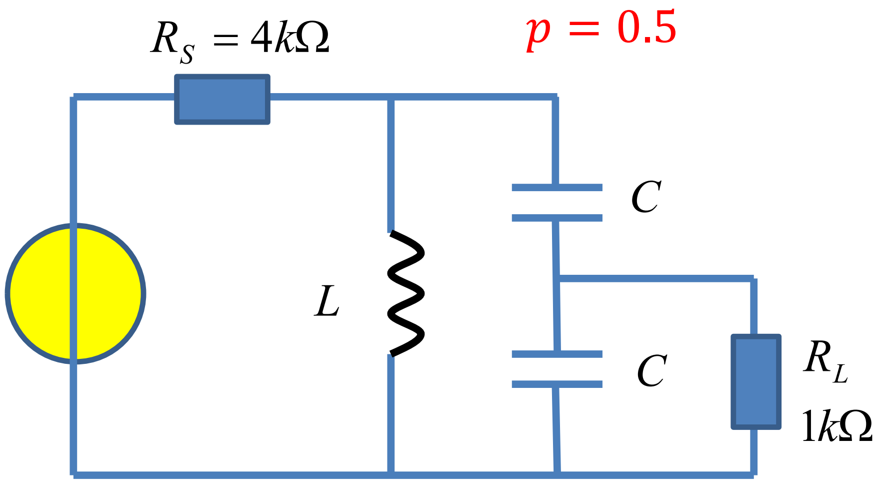
第二种方案

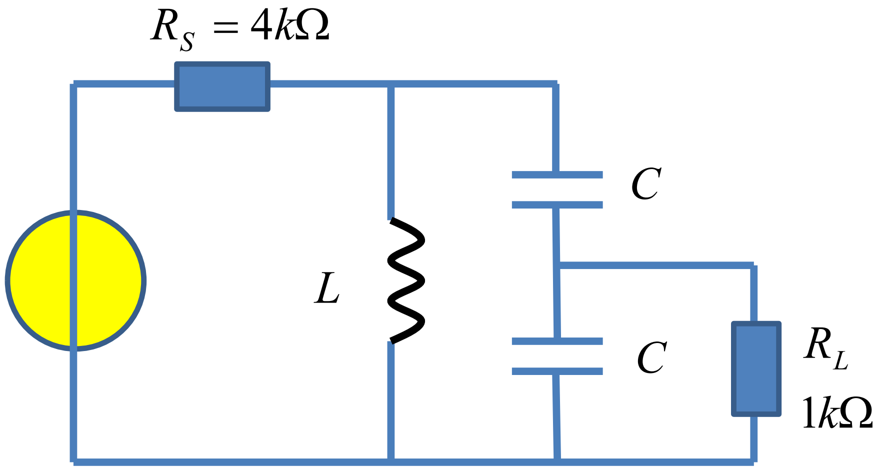


方案无数多种，实际采用哪种方案，由设计者根据系统需求而定

作业3 部分接入

- **3**、用部分接入方法，设计一个谐振频率为**2MHz**，电容接入系数为**0.5**，负载电阻为**1kΩ**，**3dB**带宽为**200kHz**的无损**LC**并联谐振匹配网络

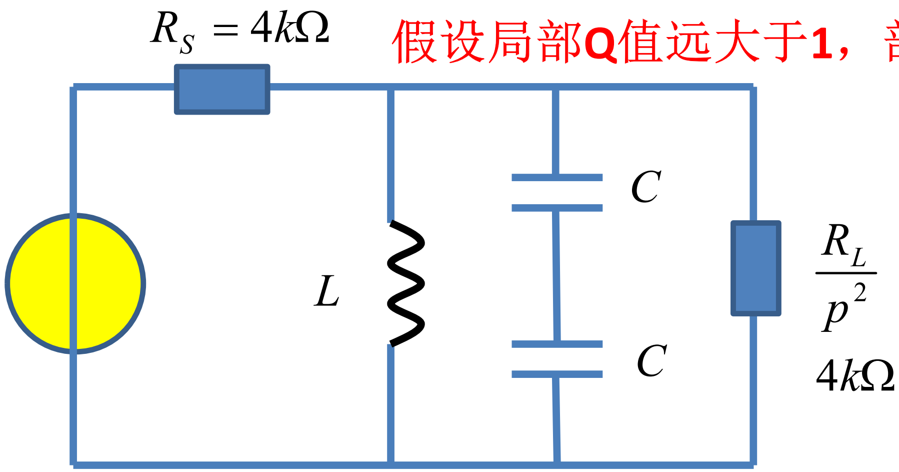




$$f_0 = \frac{1}{2\pi\sqrt{L \times 0.5C}} = 2MHz$$

$$Q = 0.5R_S\sqrt{\frac{0.5C}{L}} = 2\times10^3\sqrt{\frac{0.5C}{L}} = \frac{f_0}{BW_{3dB}} = \frac{2MHz}{200kHz} = 10$$

假设局部Q值远大于1，部分接入等效

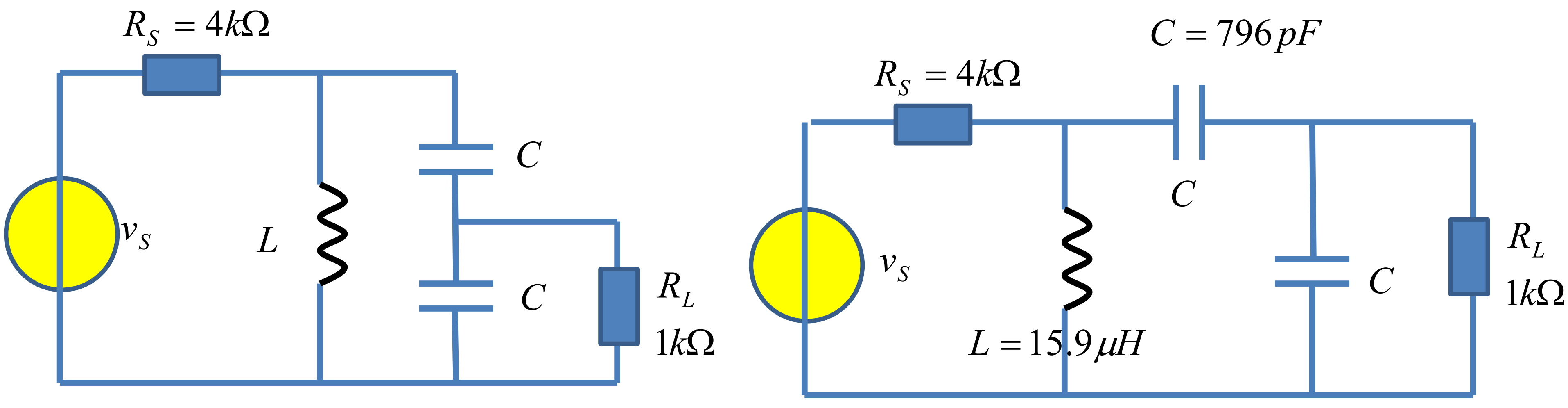


$$L = 15.9\mu H$$

$$C = 796pF$$

验证确认 $Q_{局部} = \omega C R_L = 2\pi \times 2M \times 796p \times 1k = 10 \gg 1$

验证设计



$$\mathbf{ABCD} = \begin{bmatrix} 1 & R_S \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{sL} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{sC} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ sC + G_L & 1 \end{bmatrix}$$

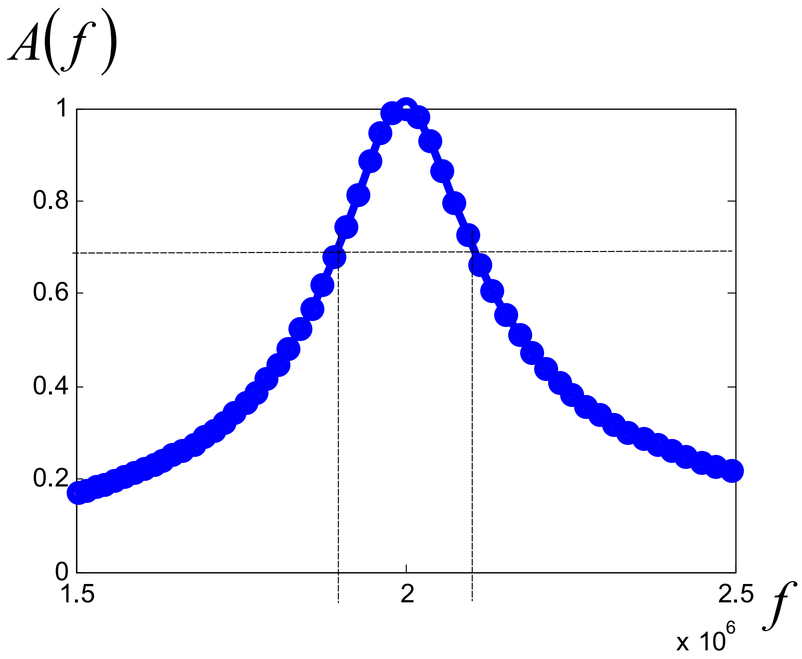
$$H(s) = 2\sqrt{\frac{R_S}{R_L}}\frac{1}{A}$$

系数确保其幅度平方为功率增益

Matlab代码

```
clear all

f0=2E6;
BW=200E3;
Q=f0/BW;
RS=4E3;
RL=1E3;
pp=Q/(0.5*RS);
L=1/(2*pi*f0*pp);
C=2*pp^2*L;

freqstart=f0/100;
freqstop=f0*100;
freqnum=1000;
freqstep=10^(log10(freqstop/freqstart)/freqnum);

freq=freqstart/freqstep;
for k=1:freqnum
   freq=freq*freqstep;
   f(k)=freq;
   w=2*pi*freq;
   s=i*w;

   abcd=[1 RS;0 1]*[1 0;1/(s*L) 1]*[1 1/(s*C);0 1]*[1
0; s*C+1/(RL) 1];

   H=2*sqrt(RS/RL)/abcd(1,1);

   absH(k)=abs(H);
   angleH(k)=angle(H)/pi*180;
end
```

```
figure(1)
plot(f,absH)

figure(2)
plot(f,angleH)
```



匹配频点2MHz
匹配带宽200kHz

部分接入

- 只要局部Q值足够高，部分接入简化分析就没有问题
  - 部分接入失效的频段是Q值小于1的频段，而我们感兴趣的带通中心频点附近的Q值必须足够的高