

电子电路与系统基础II

习题课第八讲 一阶非线性动态电路分析

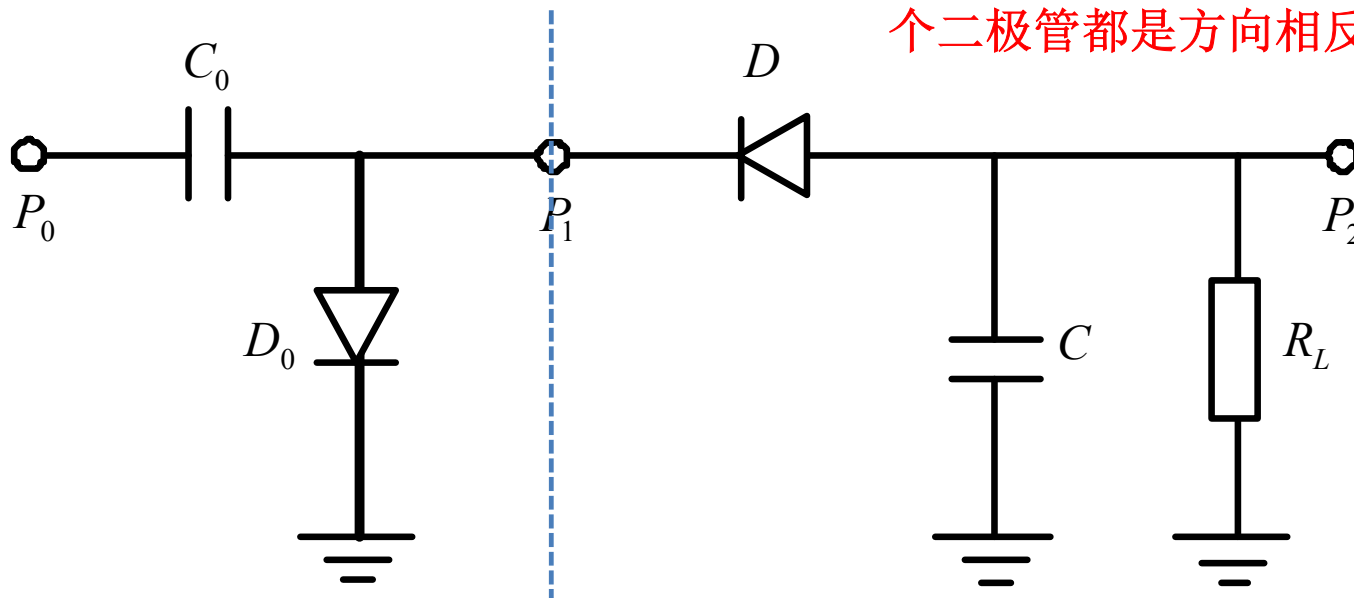
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作业1 整流器电路

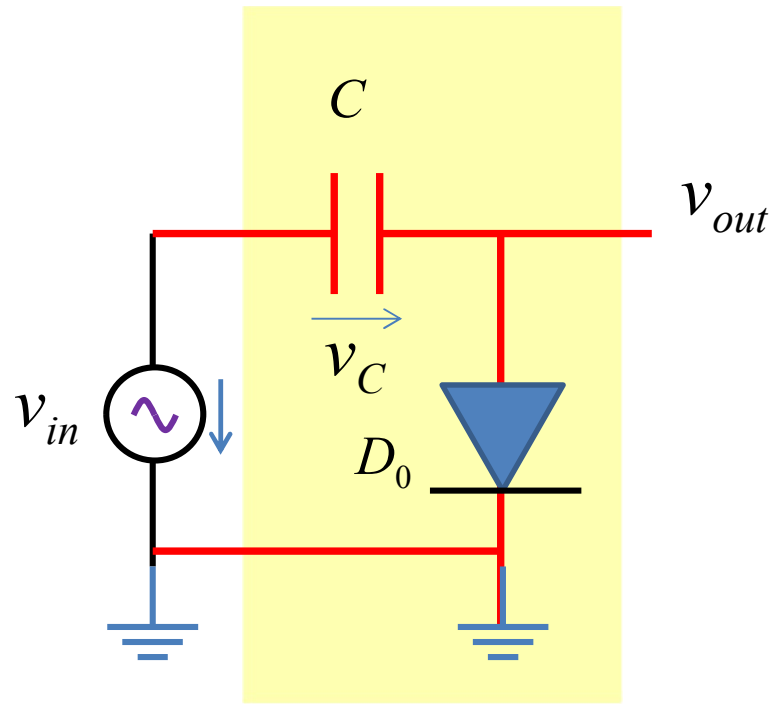
- 如图所示电路是否有错误，如果有，如何修正，修正后完成什么功能？如果没有错误，它可完成什么功能？

课堂上所讲倍压整流电路，两个二极管都是方向相反

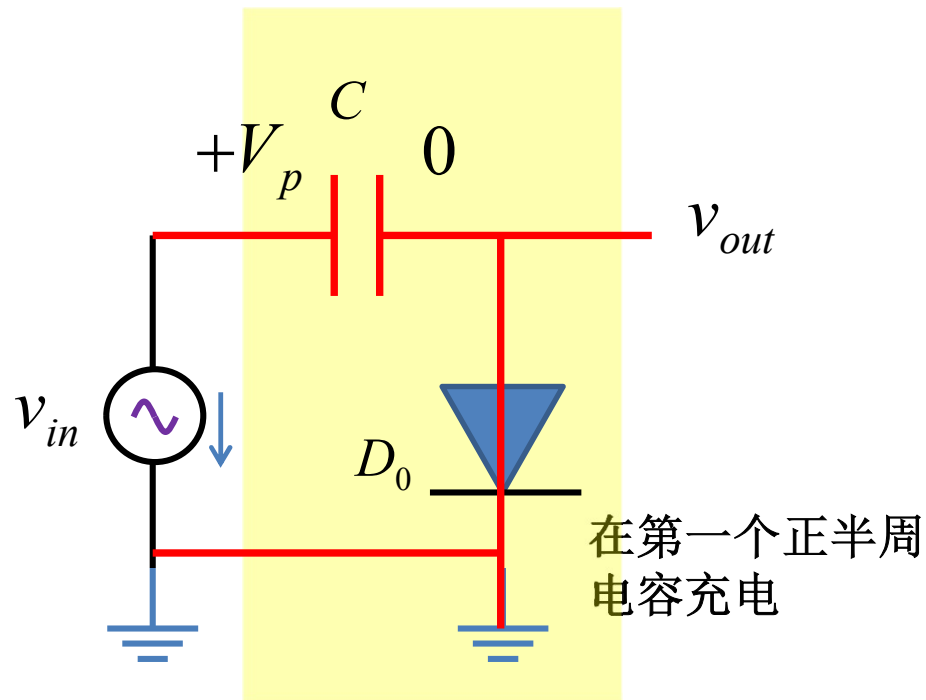
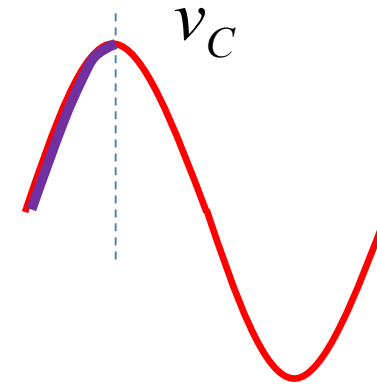


两个一阶系统的级联

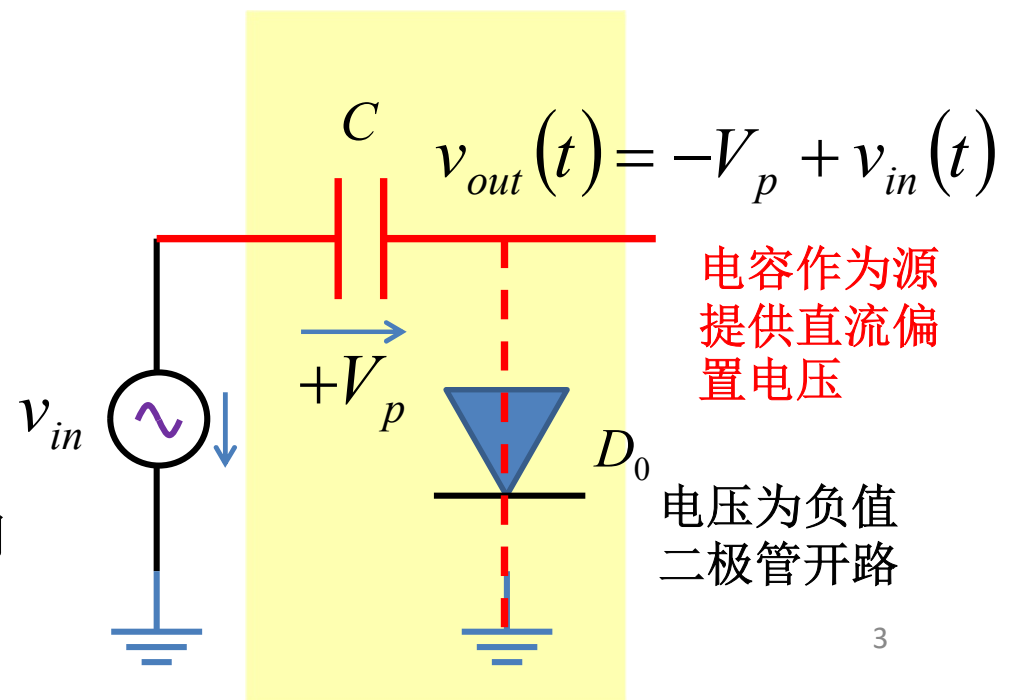
嵌位器电路



电容充电



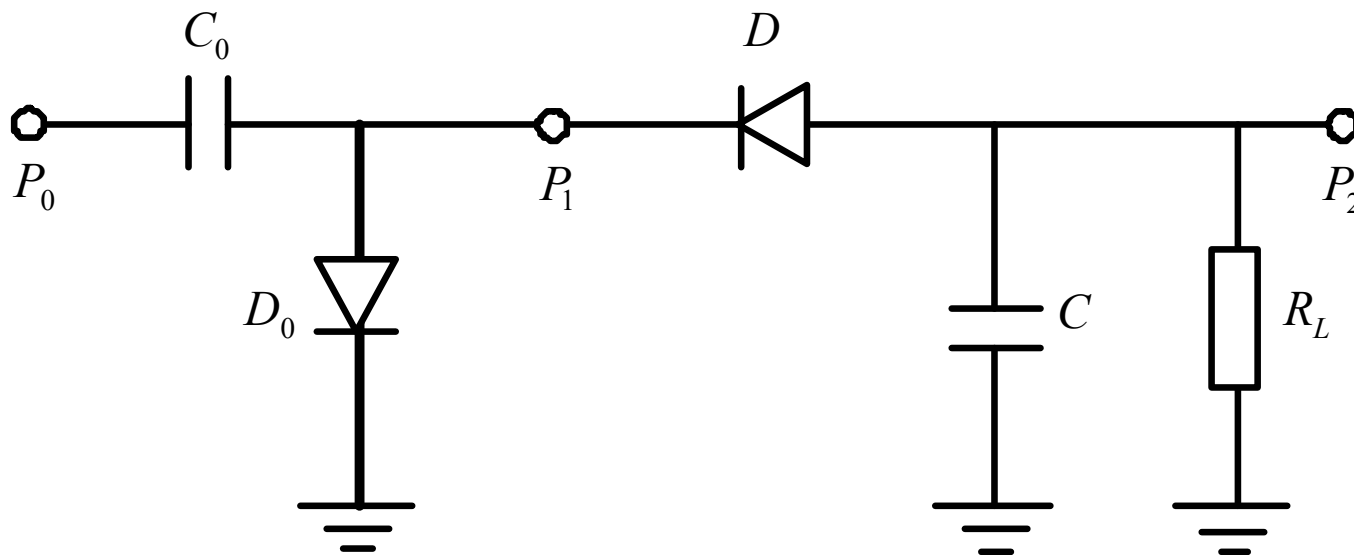
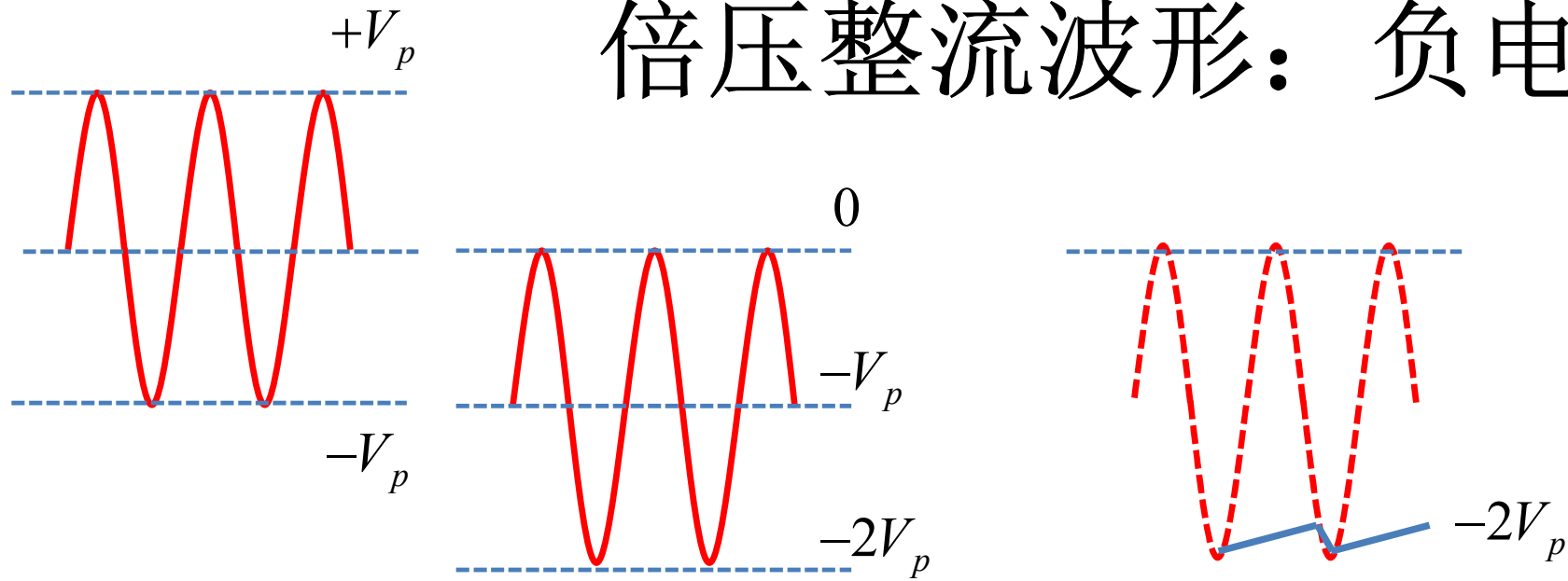
在第一个正半周
电容充电



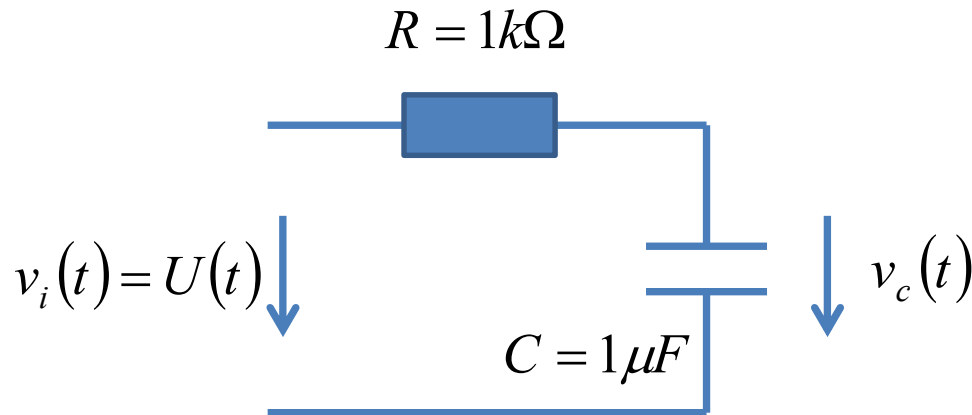
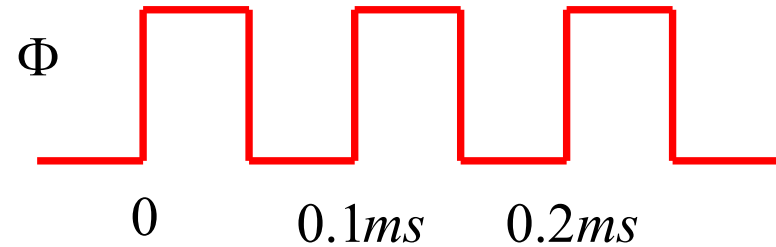
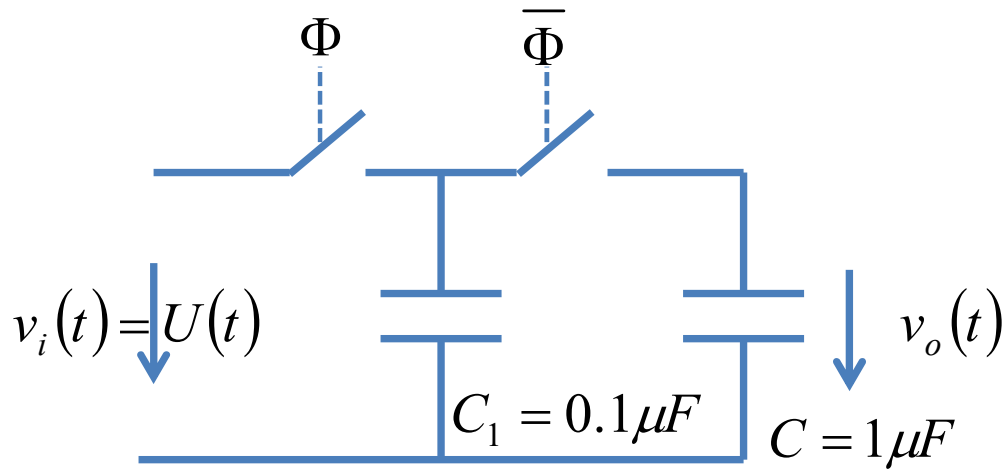
电容作为源
提供直流偏
置电压

电压为负值
二极管开路

倍压整流波形：负电压



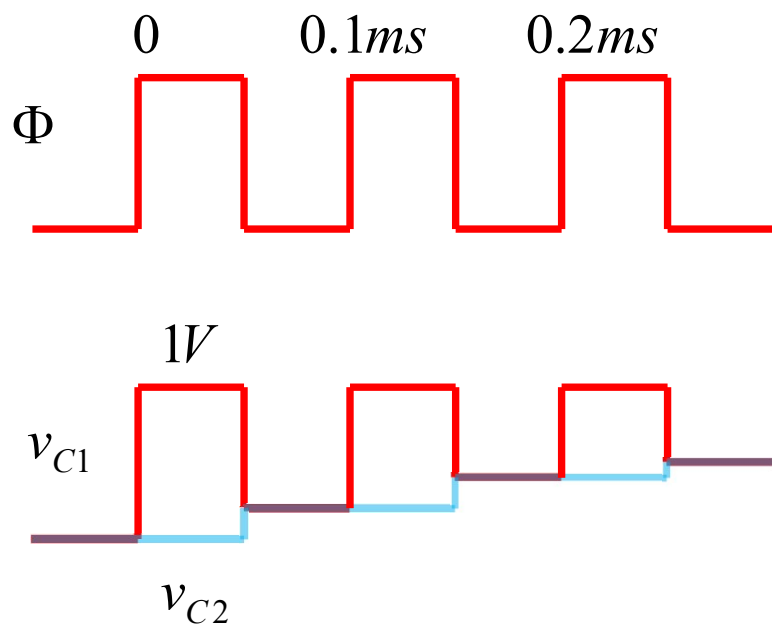
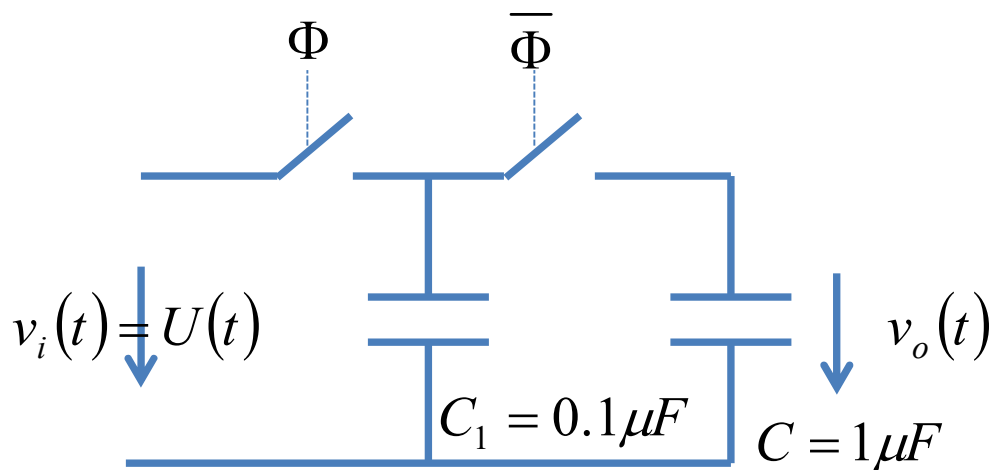
作业2 开关电容等效电阻



假设开关是理想开关
考察两个电路输出电压波形
是否一致？

研究开关电容对电阻的可替代性？

$$R_{eff} = \frac{T}{C}$$



$t = 0.05ms$

$$v_{C_2}(1) = \frac{C_1 \cdot 1 + C_2 \cdot 0}{C_1 + C_2} = \frac{1}{11} (V)$$

1/电荷守恒
2/源等效

$t = 0.15ms$

$$v_{C_2}(2) = \frac{C_1 \cdot 1 + C_2 \cdot v_{C_2}(1)}{C_1 + C_2}$$

$$= \frac{0.1 + \frac{1}{11}}{1.1} = \frac{1}{11} + \frac{10}{11^2} (V)$$

$t = 0.25ms$

$$v_{C_2}(3) = \frac{C_1 \cdot 1 + C_2 \cdot v_{C_2}(2)}{C_1 + C_2}$$

$$= \frac{1 + \frac{10}{11} + \frac{100}{11^2}}{11} = \frac{1}{11} + \frac{10}{11^2} + \frac{100}{11^3} (V)$$

$$t = (0.1n - 0.05)ms$$

电容电压变化规律

$$\begin{aligned}
 v_{C_2}(n) &= \frac{C_1 \cdot 1 + C_2 \cdot v_{C_2}(n-1)}{C_1 + C_2} = \frac{C_1 \cdot 1 + C_2 \cdot \frac{C_1 \cdot 1 + C_2 \cdot v_{C_2}(n-2)}{C_1 + C_2}}{C_1 + C_2} \\
 &= \frac{C_1 \cdot 1 + C_2 \cdot \frac{C_1 \cdot 1 + C_2 \cdot v_{C_2}(n-3)}{C_1 + C_2}}{C_1 + C_2} \\
 &= \frac{C_1 \cdot 1 + C_2 \cdot \frac{C_1 \cdot 1 + C_2 \cdot v_{C_2}(n-4)}{C_1 + C_2}}{C_1 + C_2} = \dots \\
 &= \frac{C_1}{C_1 + C_2} + \frac{C_2 C_1}{(C_1 + C_2)^2} + \frac{C_2^2 C_1}{(C_1 + C_2)^3} + \dots + \frac{C_2^{n-1} C_1}{(C_1 + C_2)^n} \\
 &= \frac{C_1}{C_1 + C_2} \left(1 + \frac{C_2}{C_1 + C_2} + \frac{C_2^2}{(C_1 + C_2)^2} + \dots + \frac{C_2^{n-1}}{(C_1 + C_2)^{n-1}} \right)
 \end{aligned}$$

性质

$$\begin{aligned}v_{C_2}(n) &= \frac{C_1}{C_1 + C_2} + \frac{C_2 C_1}{(C_1 + C_2)^2} + \frac{C_2^2 C_1}{(C_1 + C_2)^3} + \dots + \frac{C_2^{n-1} C_1}{(C_1 + C_2)^n} \\ &= \frac{C_1}{C_1 + C_2} \left(1 + \frac{C_2}{C_1 + C_2} + \frac{C_2^2}{(C_1 + C_2)^2} + \dots + \frac{C_2^{n-1}}{(C_1 + C_2)^{n-1}} \right) = 1 - \left(\frac{C_2}{C_1 + C_2} \right)^n\end{aligned}$$

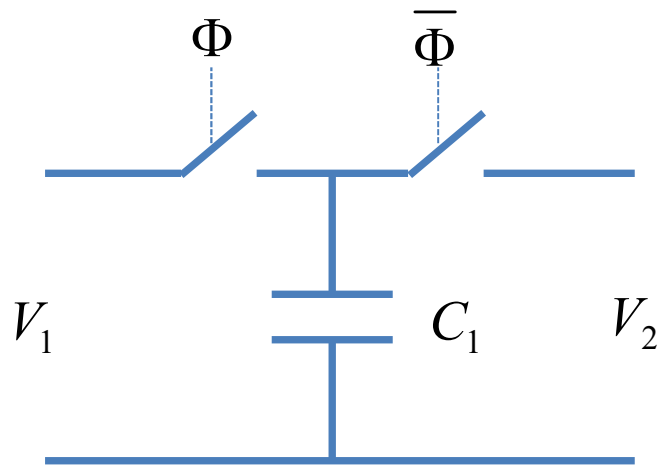
$$v_{C_2}(n) = v_{C_2}(n-1) + \left(\frac{C_2}{C_1 + C_2} \right)^{n-1} \frac{C_1}{C_1 + C_2}$$

后一个状态是前一个状态的增量
增量随时间增加是指数衰减的

$$v_{C_2}(\infty) = 1(V)$$

状态值在时间趋于无穷时趋于终值**1V**

开关电容等效为电阻



一个周期内，有 ΔQ 的电荷从端口**1**转移到端口**2**，相当于有电流从端口**1**流到端口**2**

$$\bar{I} = \frac{Q_{\Phi} - Q_{\bar{\Phi}}}{T} = \frac{C_1(V_1 - V_2)}{T}$$

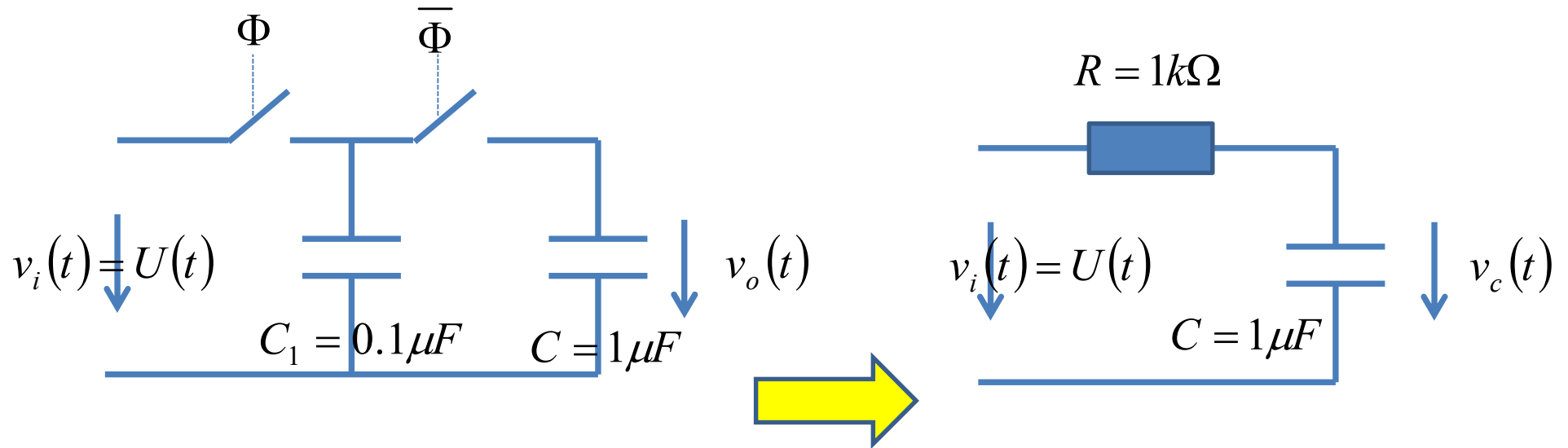
端口**1**流到端口**2**有电压差时，就有电流流过，因而可等效为一个电阻

$$\Phi \quad Q_{\Phi} = C_1 V_1$$

$$\bar{\Phi} \quad Q_{\bar{\Phi}} = C_1 V_2$$

$$R_{eq} = \frac{V_1 - V_2}{\bar{I}} = \frac{T}{C_1} = \frac{0.1ms}{0.1\mu F} = 1k\Omega$$

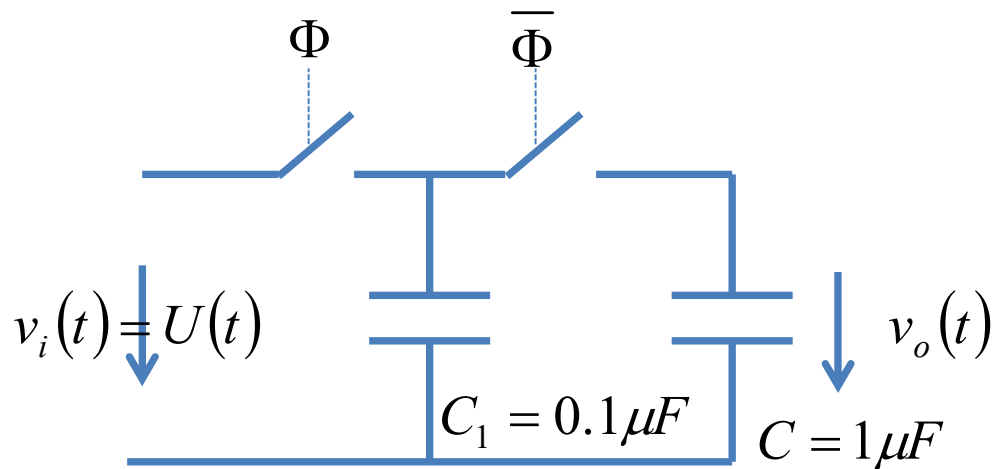
一阶RC的充电过程



$$v_{C2}(n) = \left(1 - \left(\frac{C_2}{C_1 + C_2} \right)^n \right) \cdot U(n)$$

$$v_c(t) = \left(1 - e^{-\frac{t}{\tau}} \right) U(t) = \left(1 - e^{-\frac{t}{RC}} \right) \cdot U(t)$$

两者对比 高度近似



$$v_{C2}(n) = 1 - \left(\frac{C_2}{C_1 + C_2} \right)^n$$

$$n \geq 0$$

离散时间的充电过程：一个时钟周期完成一次快速充电（瞬间充电）

$$v_c(t) = 1 - e^{-\frac{t}{RC}} \Big|_{t=nT} = 1 - e^{-\frac{nT}{RC}} = 1 - \left(e^{-\frac{T}{RC}} \right)^n \quad t \geq 0$$

连续时间的充电过程：时时刻刻在充电进行中

$$\frac{C_2}{C_1 + C_2} \Leftrightarrow e^{-\frac{T}{RC}} \Big|_{T \ll RC} \approx \frac{1}{1 + \frac{T}{RC}} = \frac{1}{1 + \frac{T}{RC_2}} = \frac{C_2}{C_2 + \frac{T}{R}} = \frac{C_2}{C_2 + C_1} \quad R = \frac{T}{C_1}$$

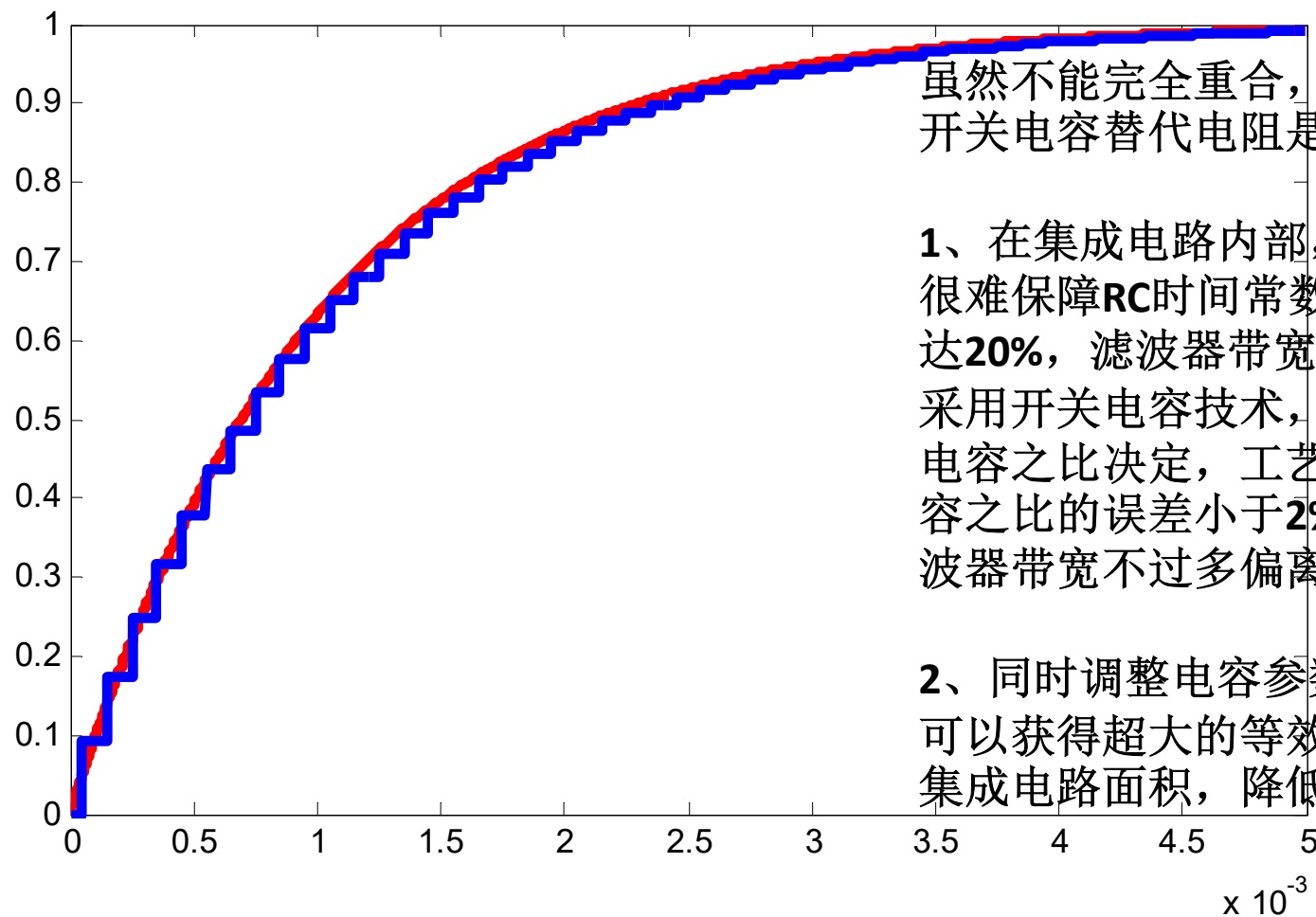
一个周期内的衰减量

$$R = \frac{T}{C_1} \ll \frac{RC}{C_1} \quad C \gg C_1$$

转移电荷的开关电容越小，用电阻等价的误差就越小

充电时域波形

$$R_{eq} = \frac{T}{C_1} \quad \tau = R_{eq} C_2 = T \frac{C_2}{C_1}$$



虽然不能完全重合，但足够接近，用开关电容替代电阻是一种合理的选择

1、在集成电路内部，由于工艺问题，很难保障RC时间常数的精度，误差可达**20%**，滤波器带宽无法保证，如果采用开关电容技术，时间常数由两个电容之比决定，工艺上可保证两个电容之比的误差小于**2%**，从而可确保滤波器带宽不过多偏离设计值

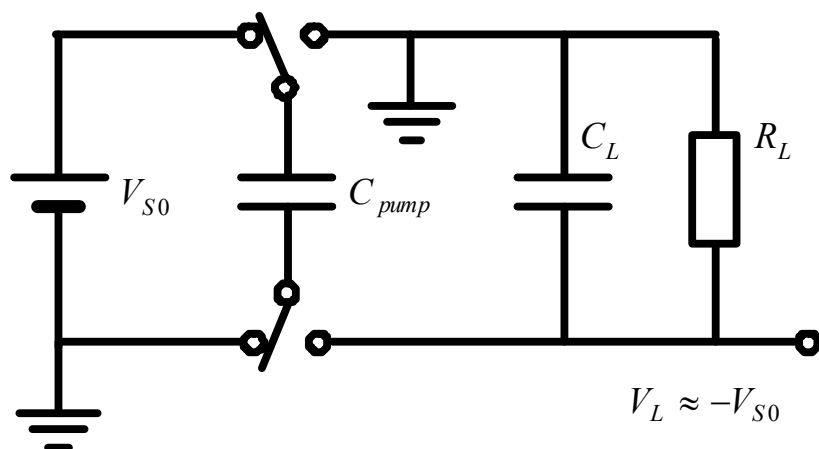
2、同时调整电容参数和时钟周期，可以获得超大的等效电阻，可以节约集成电路面积，降低系统成本

Matlab 作图程序

- clear all
-
- R=1E3;
- C2=1E-6;
- C1=0.1E-6;
-
- tao=R*C2;
- tstop=5*tao;
- tnum=10000;
- tstep=tstop/tnum;
- t=-tstep;
-
- for kk=1:tnum
- t=t+tstep;
-
- tt(kk)=t;
- v2(kk)=1-exp(-t/tao);
-
- n=floor(t/0.05E-3);
- if mod(n,2)==1
- v3(kk)=1-
- (C2/(C1+C2))^((n+1)/2);
- else
- if kk==1
- v3(kk)=0;
- else
- v3(kk)=v3(kk-1);
- end
- end
- end
-
-
- figure(2)
- hold on
- plot(tt,v2,'r')
- plot(tt,v3)

作业3：开关电容做DC-DC转换

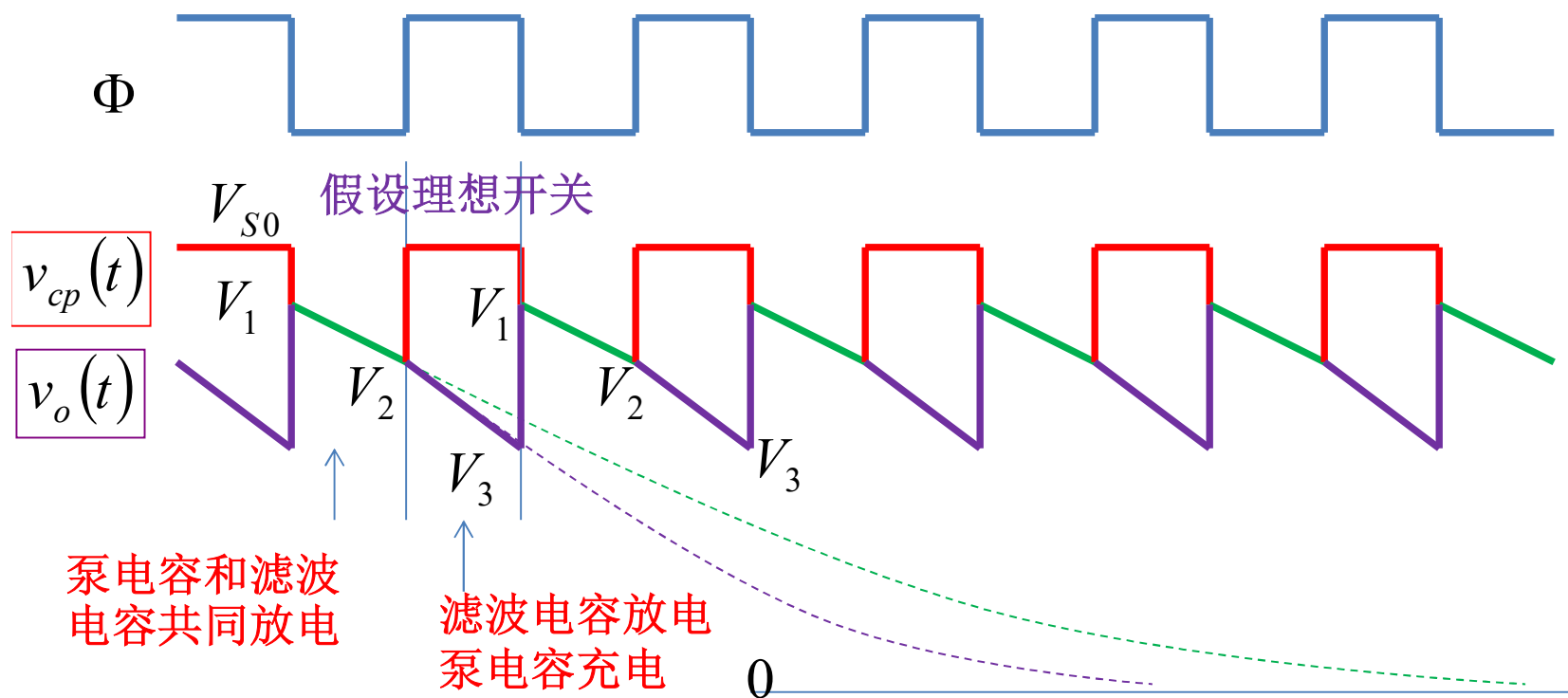
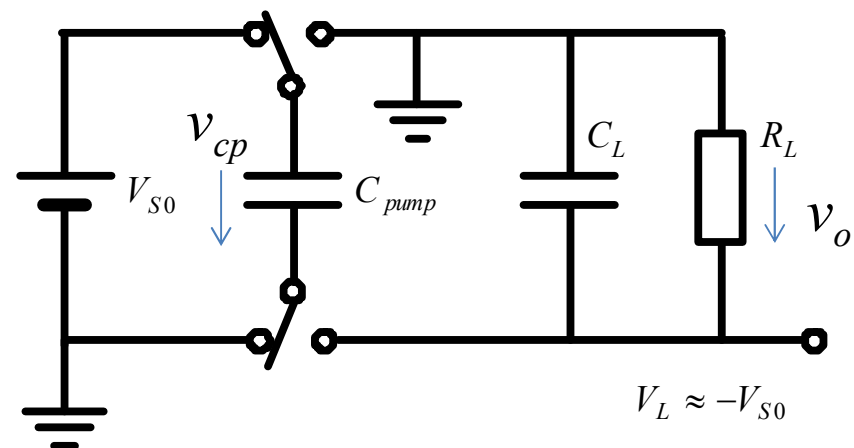
- **习题9.9 开关电容实现反压** 两个开关在占空比为50%的时钟控制下，在前50%方波周期内使得泵电容 C_{pump} 接到直流电压源 V_{S0} 上，从 V_{S0} 上获取电荷（电能），后50%方波周期内再接到负载电路上，泵电容将部分电荷转移到滤波电容 C_L 上，在泵电容接电源的50%周期内，滤波电容为负载提供电能。分析当电路进入稳态后，输出反相直流电压的纹波电压为多少？分析提高能量转换效率的措施？

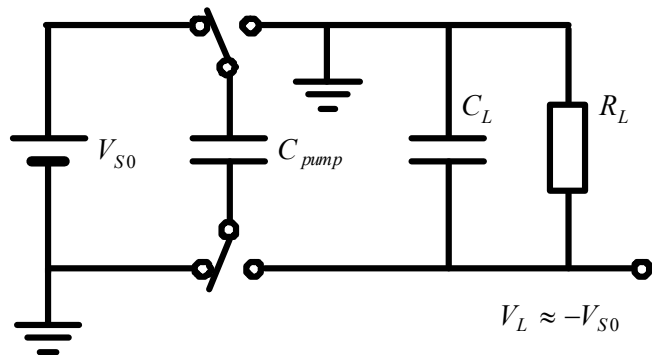


和前一个电路的区别：

- 1、双掷开关使得两个电路不共地（完全隔离）
- 2、有负载电阻耗能

不考虑瞬态过程
只考虑稳定后的稳态响应

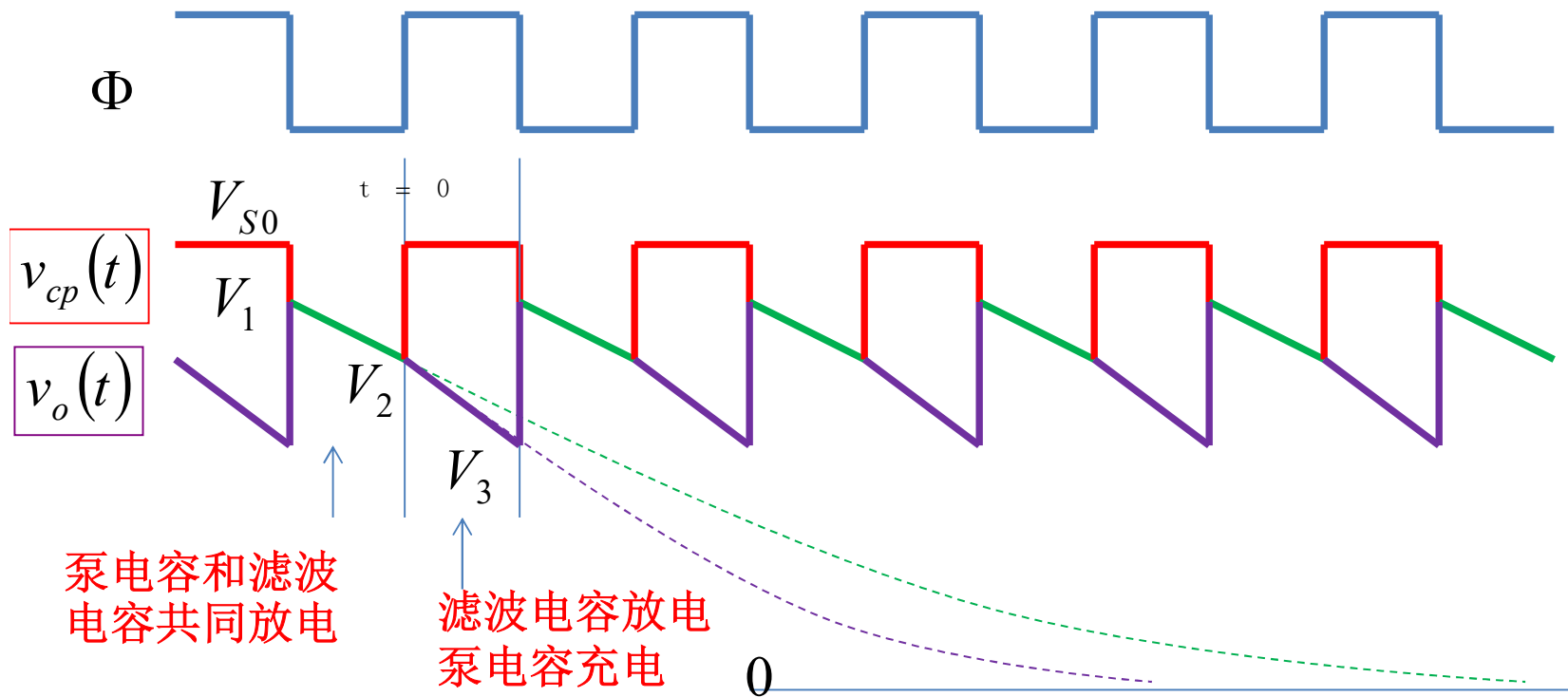


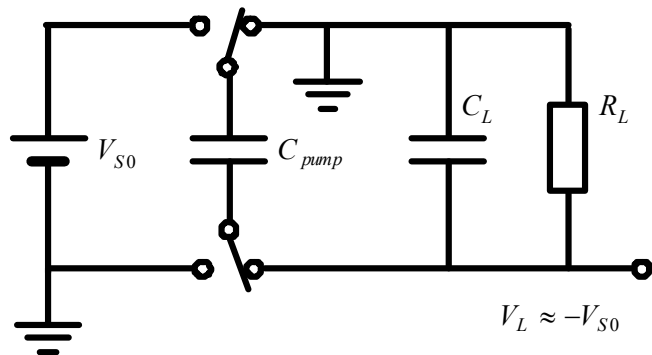


滤波电容单独放电，泵电容充电

$$v_o(t) = V_2 e^{-\frac{t}{R_L C_L}} \approx V_2 \left(1 - \frac{t}{R_L C_L} \right)$$

$$V_3 \approx V_2 \left(1 - \frac{0.5T}{R_L C_L} \right)$$





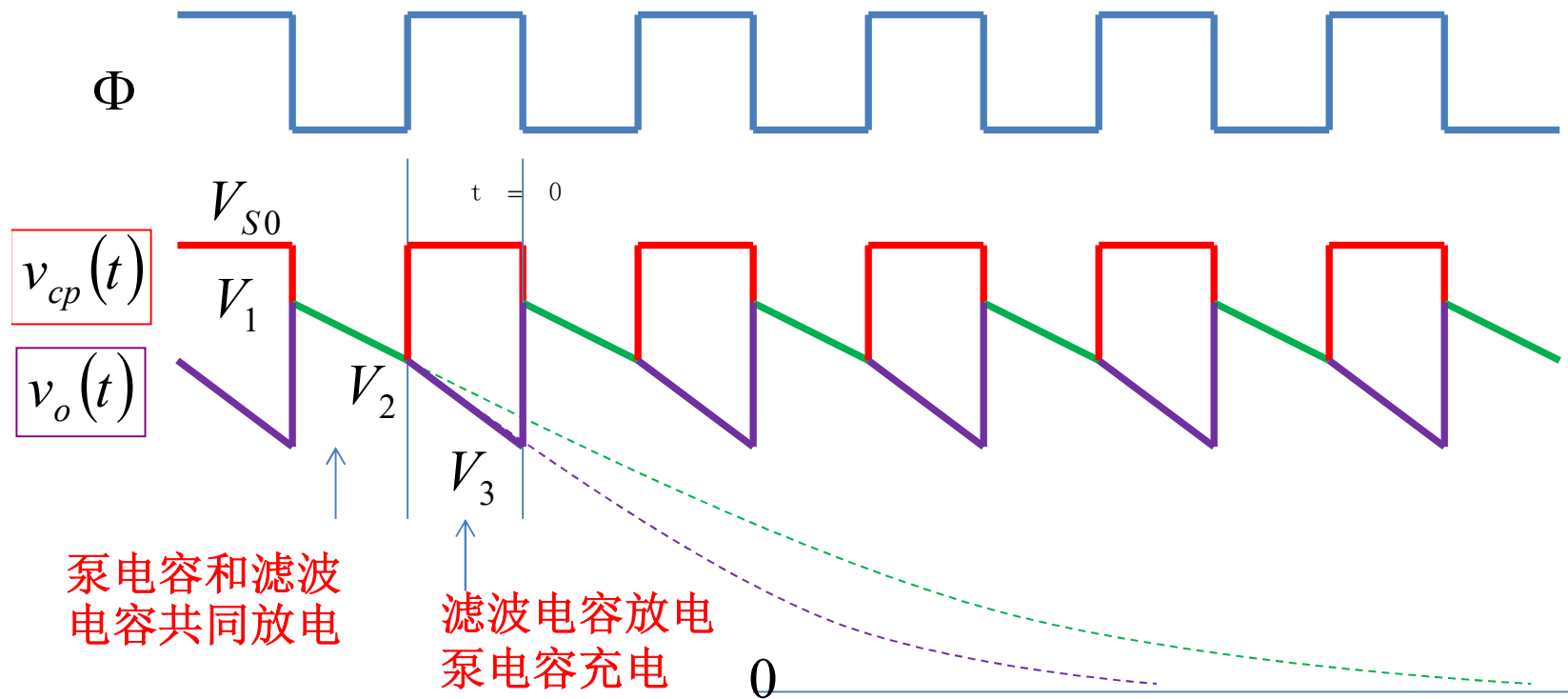
开关换向瞬间

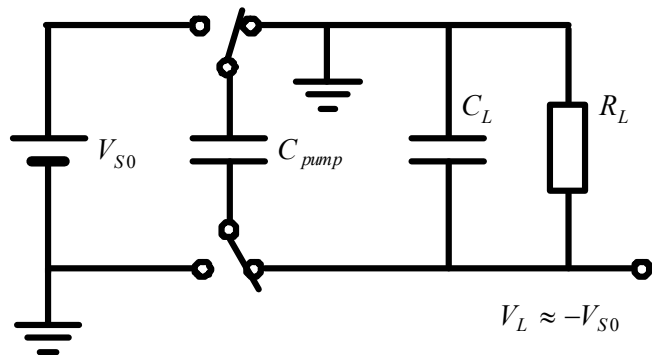
$$v_{cp}(0^-) = V_{S0} \quad v_o(0^-) = V_3$$

$$C_L v_o(0^-) + C_p v_{cp}(0^-) = (C_L + C_p) v_o(0^+)$$

电荷守恒/源等效

$$v_o(0^+) = \frac{C_L}{C_L + C_p} V_3 + \frac{C_p}{C_L + C_p} V_{S0} = V_1$$

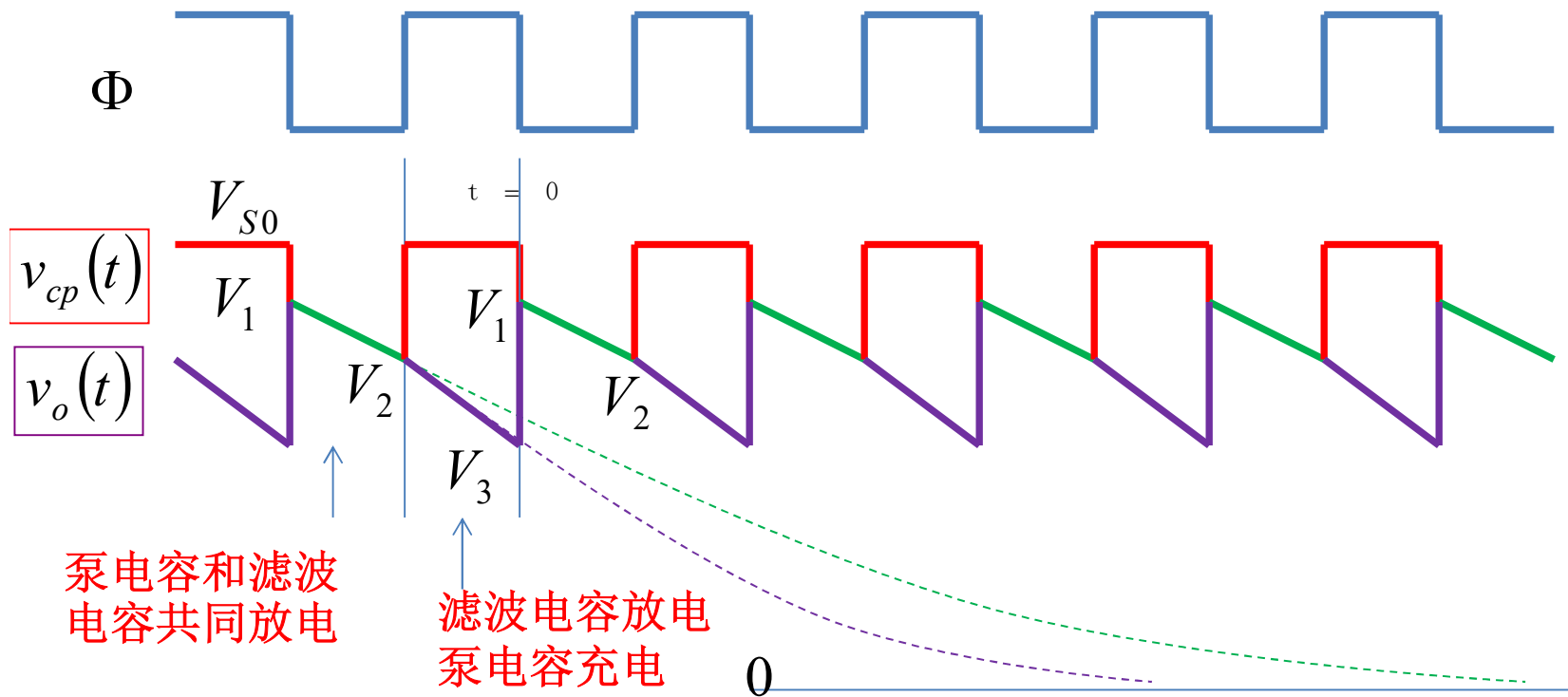


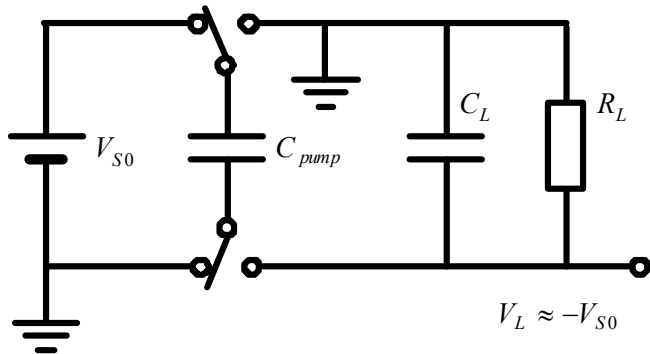


滤波电容和泵电容共同放电

$$v_o(t) = V_1 e^{-\frac{t}{R_L(C_L + C_p)}} \approx V_2 \left(1 - \frac{t}{R_L(C_L + C_p)} \right)$$

$$V_2 \approx V_1 \left(1 - \frac{0.5T}{R_L(C_L + C_p)} \right)$$

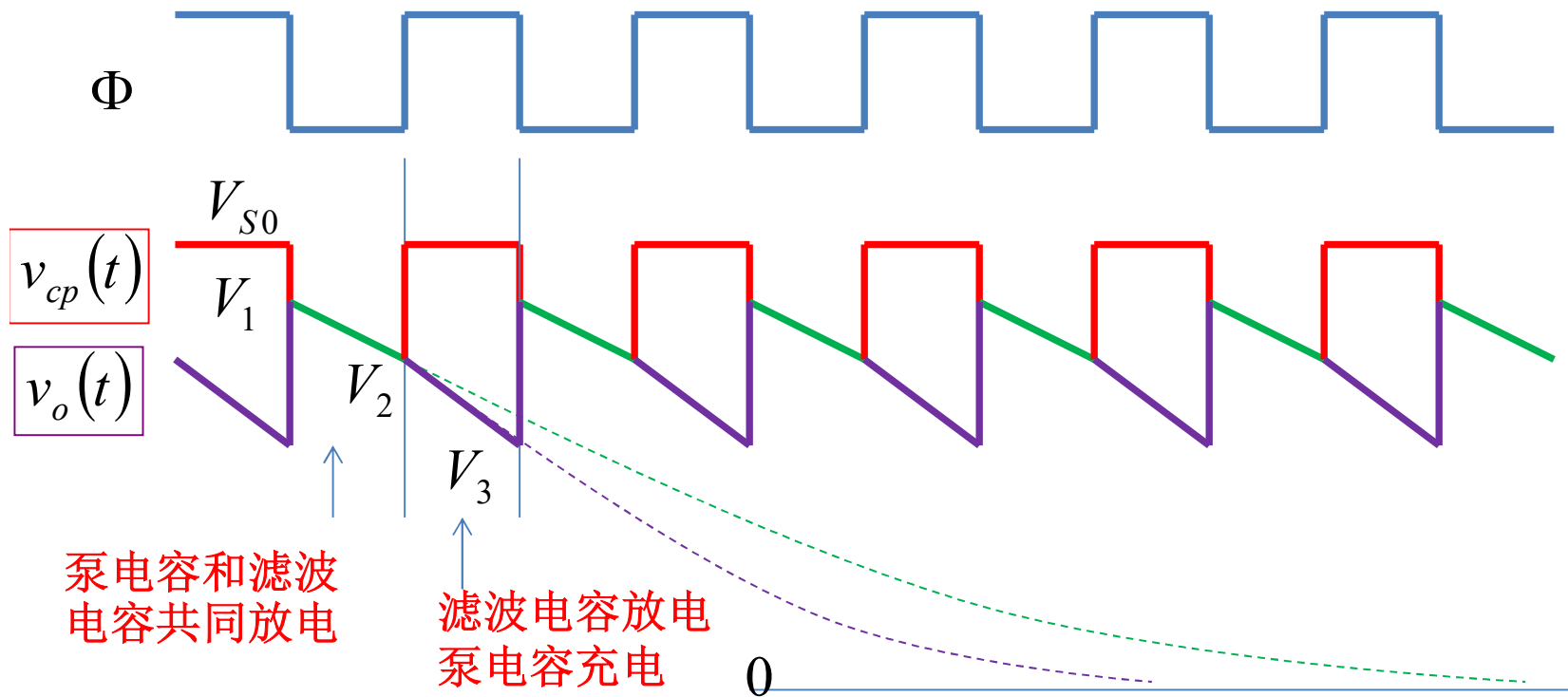




$$V_3 \approx V_2 \left(1 - \frac{0.5T}{R_L C_L} \right)$$

$$V_1 = \frac{C_L}{C_L + C_p} V_3 + \frac{C_p}{C_L + C_p} V_{S0}$$

$$V_2 \approx V_1 \left(1 - \frac{0.5T}{R_L (C_L + C_p)} \right)$$



$$V_2 \approx V_1 \left(1 - \frac{0.5T}{R_L(C_L + C_p)} \right)$$

$$V_1 \left(\frac{C_p}{C_L + C_p} + \frac{C_L}{C_L + C_p} \frac{0.5T}{R_L((C_L + C_p) \text{串} C_L)} \right) \approx \frac{C_p}{C_L + C_p} V_{S0}$$

$$V_3 \approx V_2 \left(1 - \frac{0.5T}{R_L C_L} \right)$$

$$V_1 \approx \frac{C_p}{C_p + C_L} \frac{0.5T}{R_L((C_L + C_p) \text{串} C_L)} V_{S0}$$

$$\approx V_1 \left(1 - \frac{0.5T}{R_L(C_L + C_p)} \right) \left(1 - \frac{0.5T}{R_L C_L} \right)$$

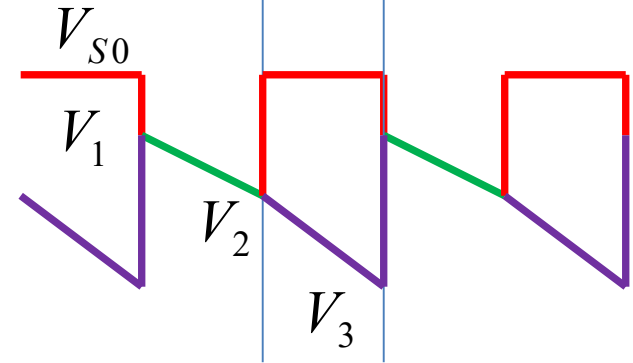
$$\approx V_1 \left(1 - \frac{0.5T}{R_L(C_L + C_p)} - \frac{0.5T}{R_L C_L} \right)$$

$$\approx V_1 \left(1 - \frac{0.5T}{R_L((C_L + C_p) \text{串} C_L)} \right)$$



$v_{cp}(t)$

$v_o(t)$



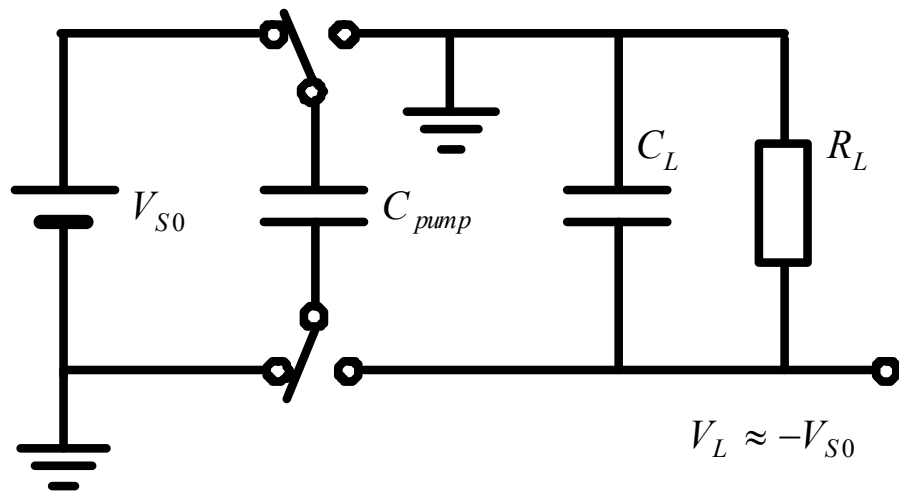
$$\Delta V = V_1 - V_3$$

$$\approx \frac{0.5T}{R_L((C_L + C_p) \text{串} C_L)} V_1$$

$$\approx \frac{C_p}{R_L((C_L + C_p) \text{串} C_L) C_p + C_L} V_{S0}$$

$$V_1 = \frac{C_L}{C_L + C_p} V_3 + \frac{C_p}{C_L + C_p} V_{S0}$$

$$\approx \frac{C_L}{C_L + C_p} V_1 \left(1 - \frac{0.5T}{R_L((C_L + C_p) \text{串} C_L)} \right) + \frac{C_p}{C_L + C_p} V_{S0}$$

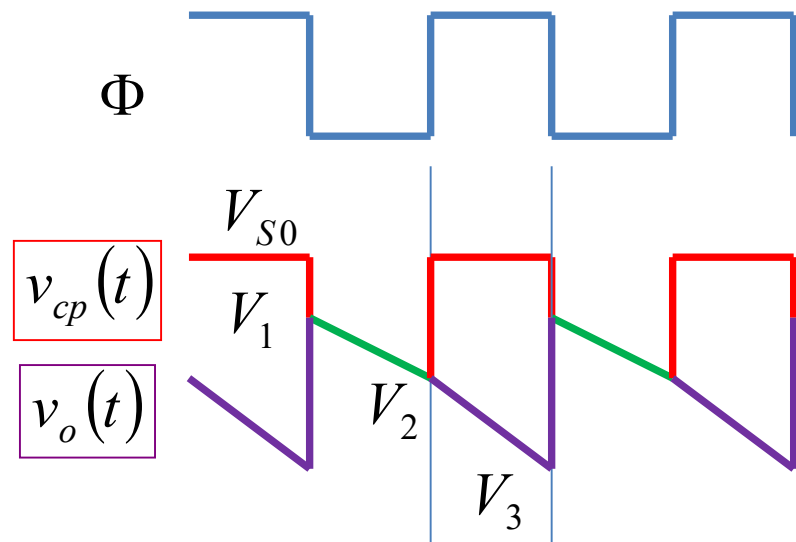


$$V_1 \approx \frac{C_p}{C_p + C_L} \frac{0.5T}{R_L((C_L + C_p) \text{串} C_L)} V_{S0}$$

$$\Delta V \approx \frac{C_p}{R_L((C_L + C_p) \text{串} C_L)} \frac{0.5T}{C_p + C_L} V_{S0}$$

$$C_L \rightarrow \infty \quad \Delta V \rightarrow 0$$

滤波电容越大，纹波电压越小，降低纹波的措施是提高滤波电容 C_L



$$V_1 \rightarrow \frac{C_p}{C_p + \frac{T}{R_L}} V_{S0} = \frac{R_L}{\frac{T}{C_p} + R_L} V_{S0} = \frac{R_L}{R_{pe} + R_L} V_{S0}$$

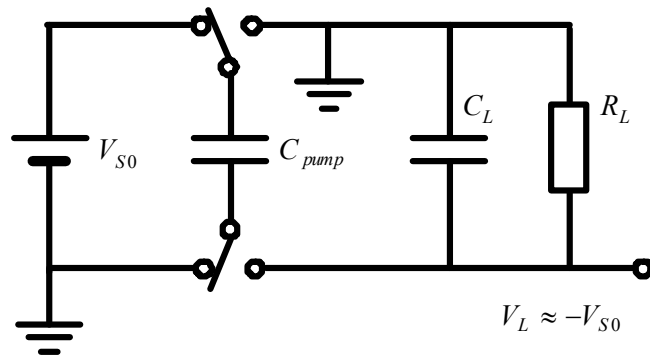
滤波电容很大时，输出电压近似为输入电压的等效分压
时钟频率越高，泵电容越大，等效电阻越小，分压越大

$$R_{pe} = \frac{T}{C_p}$$

$$C_L \gg C_p \gg \frac{T}{R_L}$$

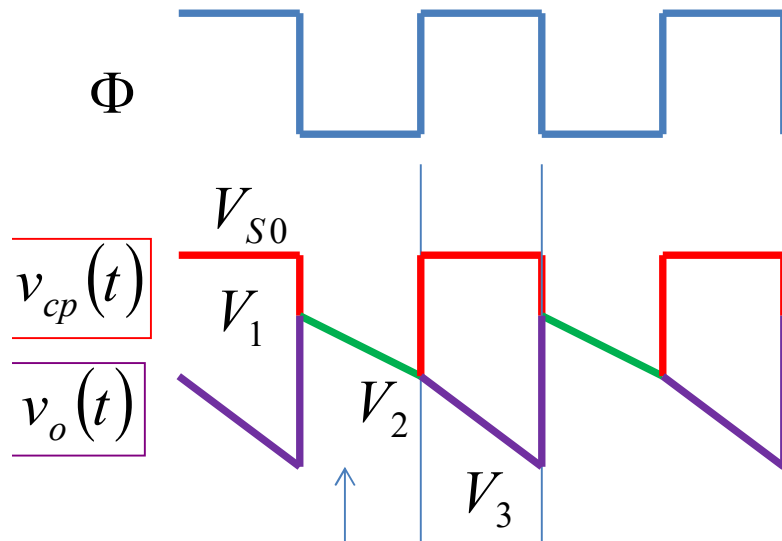
$$\Delta V \approx \frac{R_{pe}}{R_L + R_{pe}} \frac{C_p}{C_L} V_{S0}$$

非负载耗能的 额外耗能分析



C_p 拨向恒压源瞬间充电，冲激电流耗能

$$\Delta E_1 = \frac{1}{2} C_p (V_{S0} - V_2)^2$$



C_p 拨向 C_L 瞬间充放电，冲激电流耗能

$$\begin{aligned} \Delta E_2 &= \frac{1}{2} C_p V_{S0}^2 + \frac{1}{2} C_L V_3^2 - \frac{1}{2} (C_p + C_L) V_1^2 \\ &= \frac{1}{2} \frac{C_L C_p}{C_L + C_p} (V_{S0} - V_3)^2 \end{aligned}$$

均可理解为具有初始电压等效电容瞬间以冲激电流（电磁辐射）形式将其初始储能释放掉

$$\Delta E_1 = \frac{1}{2} C_p (V_{S0} - V_2)^2$$

$$\Delta E_2 = \frac{1}{2} \frac{C_L C_p}{C_L + C_p} (V_{S0} - V_3)^2$$

$$V_{S0} - V_2 \approx V_{S0} \left(\frac{R_{pe}}{R_L + R_{pe} \frac{C_L + 0.5C_p}{C_L + C_p}} \right) \stackrel{C_L \gg C_p}{\approx} V_{S0} \frac{R_{pe}}{R_L + R_{pe}}$$

$$\Delta E = \Delta E_1 + \Delta E_2$$

$$\stackrel{C_L \gg C_p}{\approx} C_p V_{S0}^2 \left(\frac{R_{pe}}{R_{pe} + R_L} \right)^2$$

$$R_{pe} = \frac{T}{C_p}$$

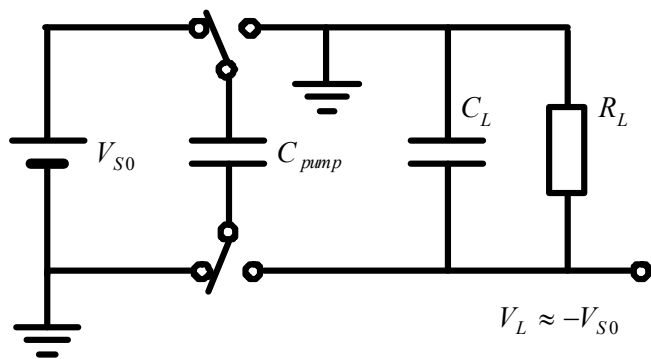
$$\stackrel{R_{pe} \ll R_L}{\approx} C_p V_{S0}^2 \left(\frac{R_{pe}}{R_L} \right)^2$$

$$V_{S0} - V_3 \approx V_{S0} \frac{R_{pe}}{R_L \frac{C_L}{C_L + 0.5C_p} + R_{pe} \frac{C_L}{C_L + C_p}}$$

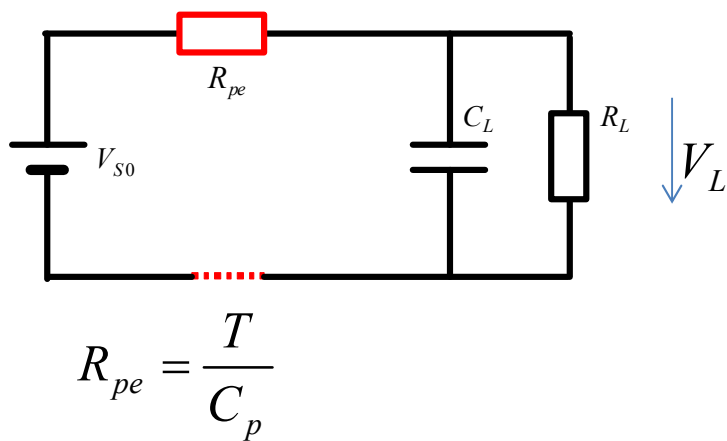
$$= \frac{V_{S0}^2}{R_L^2} \frac{T^2}{C_p}$$

$$\stackrel{C_L \gg C_p}{\approx} V_{S0} \frac{R_{pe}}{R_L + R_{pe}}$$

$$P_D = \frac{\Delta E}{T} \approx \frac{V_{S0}^2}{R_L^2} \frac{T}{C_p} \approx I_L^2 R_{pe}$$



$C_L \gg C_p$: 分析可以用如下等效电路简化分析



波纹由电荷转移导致:

$$C_L \gg C_p \gg \frac{T}{R_L}$$

输出直流电压: 为 R_L 分压

$$V_1 \approx \frac{C_p}{C_p + C_L} \frac{0.5T}{R_L((C_L + C_p) \text{串} C_L)} V_{S0}$$

$$\begin{aligned} C_L \gg C_p & \approx \frac{R_L}{R_L + R_{pe}} V_{S0} \\ R_{pe} \ll R_L & \approx V_{S0} \end{aligned}$$

$$P_D \approx I_L^2 R_{pe} \quad \begin{matrix} C_L \gg C_p \\ R_{pe} \ll R_L \end{matrix}$$

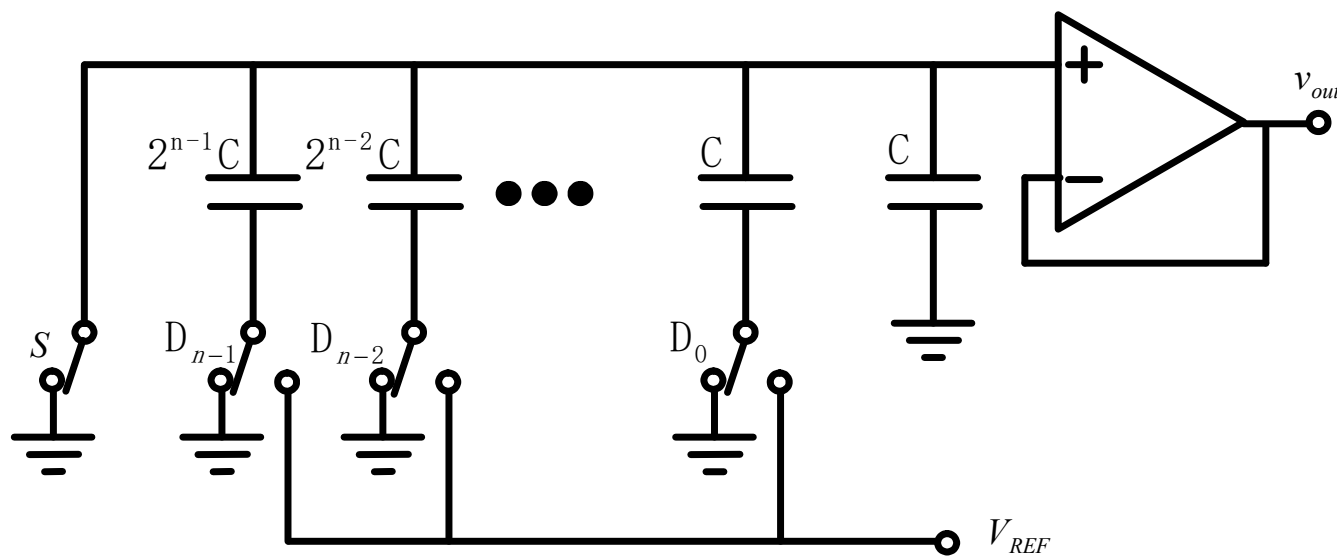
开关损耗为开关电容等效电阻消耗电能
 C_p 越大, 开关电容等效电阻越小, 等效电阻损耗越小, 效率越高

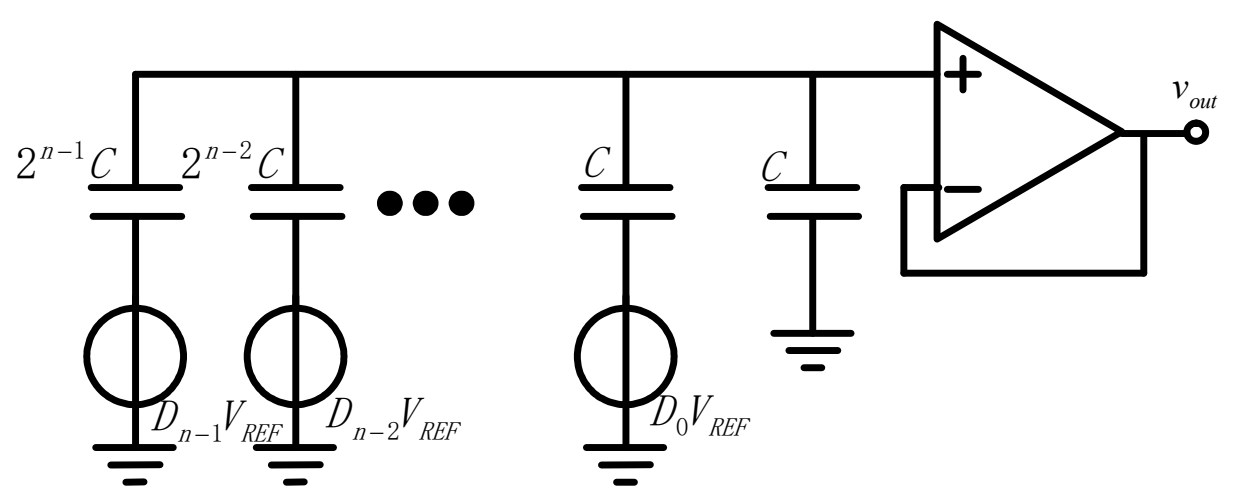
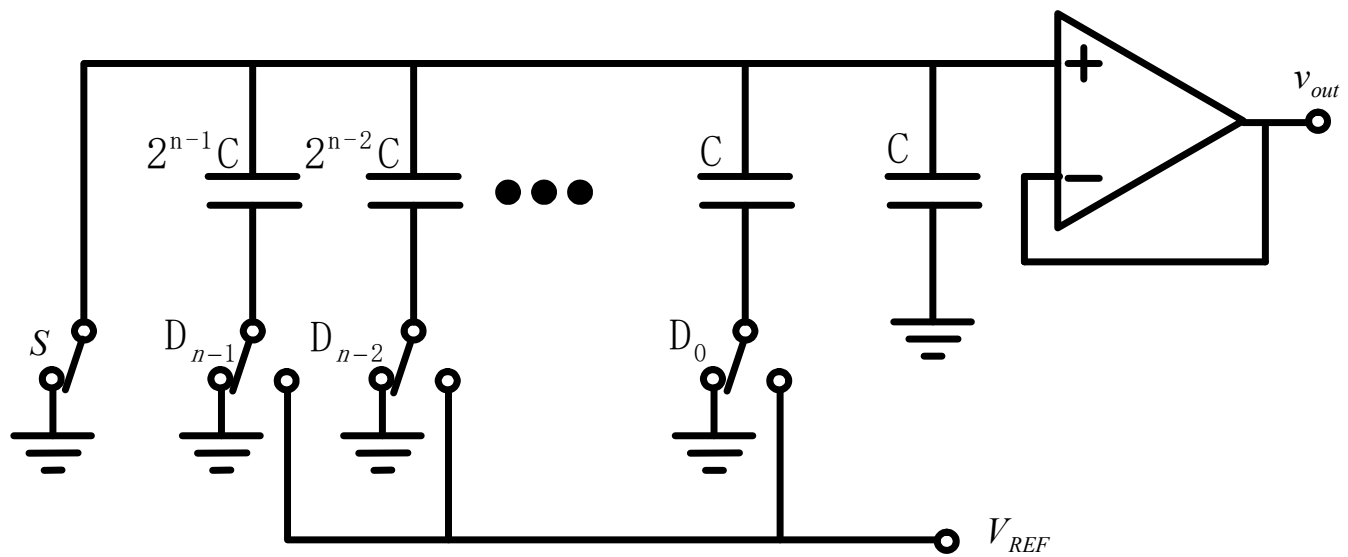
$$\Delta V \approx \frac{C_p}{C_L} \frac{R_{pe}}{R_{pe} + R_L} V_{S0}$$

$$\approx \frac{T}{R_L C_L} V_{S0} = \frac{I_L}{f C_L}$$

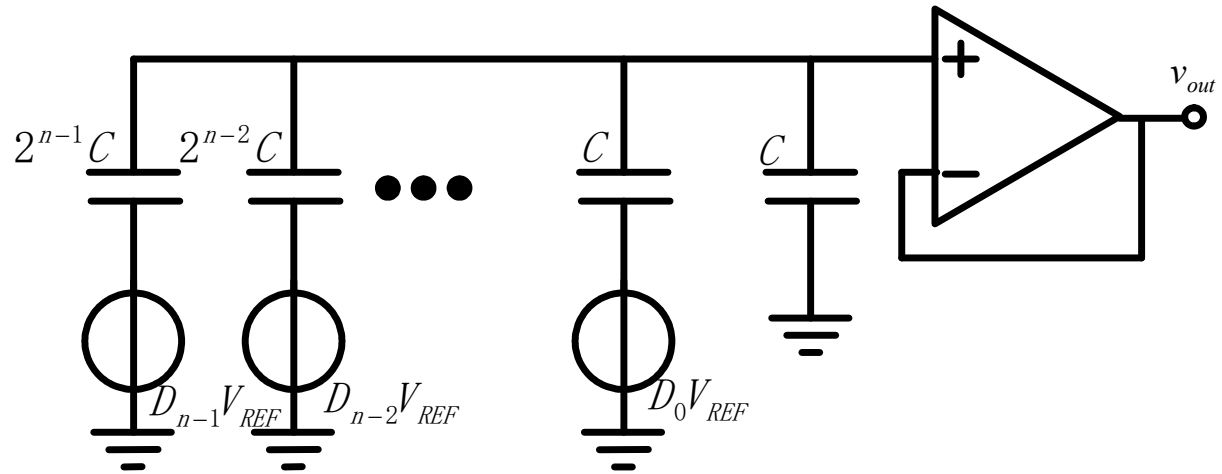
C_L 越大, 纹波越小, 输出越接近理想直流
 C_p 越大, 损耗越小, 输出电压越接近输入

- 习题9.6 加权电容DAC** 如图E9.4.14所示，这是加权电容DAC电路，请证明它完成了n-bit的DA转换。其工作顺序为：在复位相，所有开关全部接到地上，如图示。在采样相，开关S断开，开关 D_0 到 D_{n-1} 则依数字输入而定，如果输入 $D_i=1$ ，相应开关则拨向 V_{REF} ，如果 $D_i=0$ ，相应开关则仍然保持和地连通。

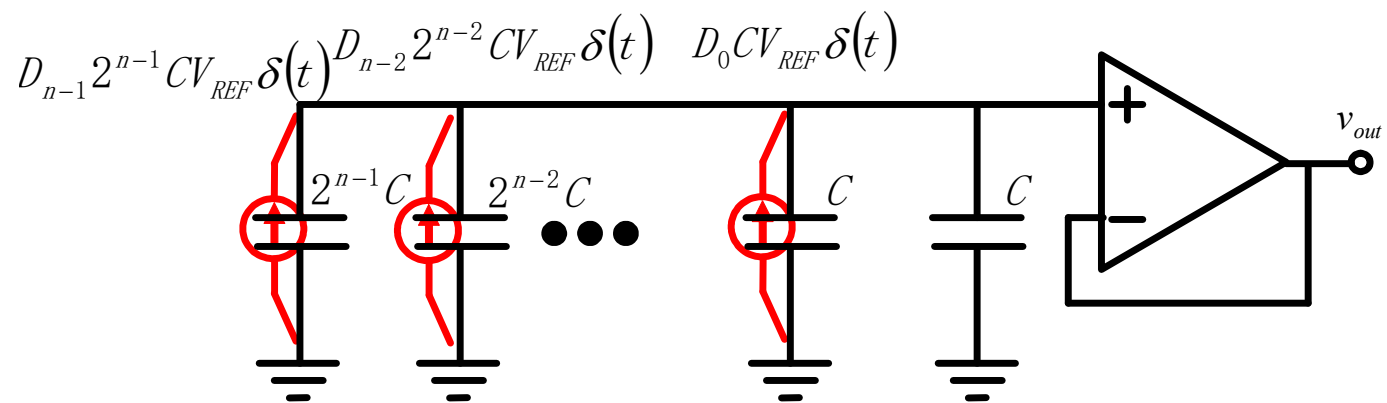


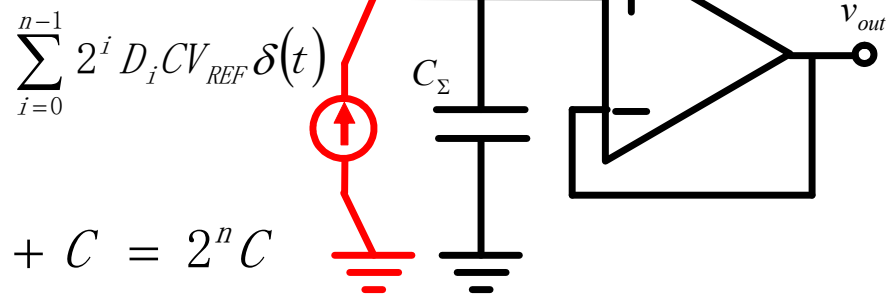
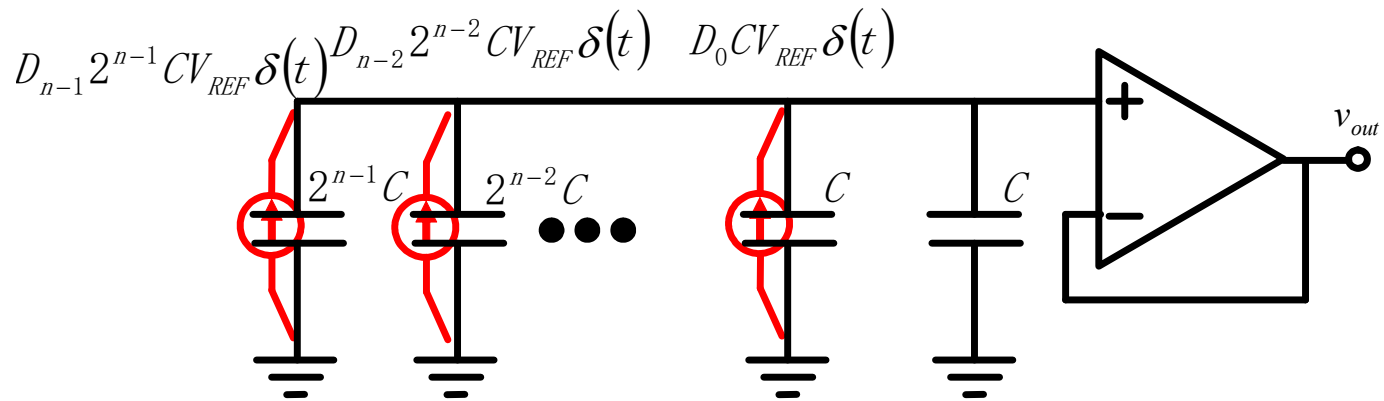


戴维南源表述
 也可用电容分压，电荷重分配分析，不如源等效简洁



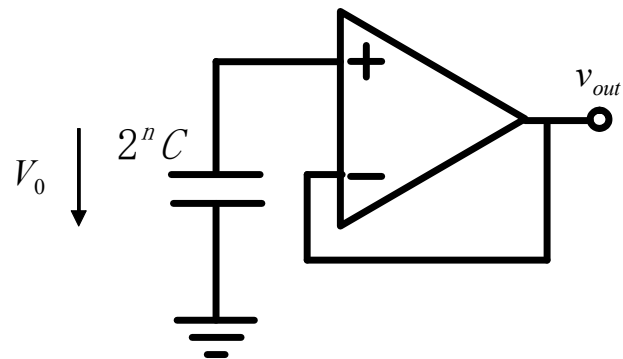
并联用诺顿等效表述最为简单

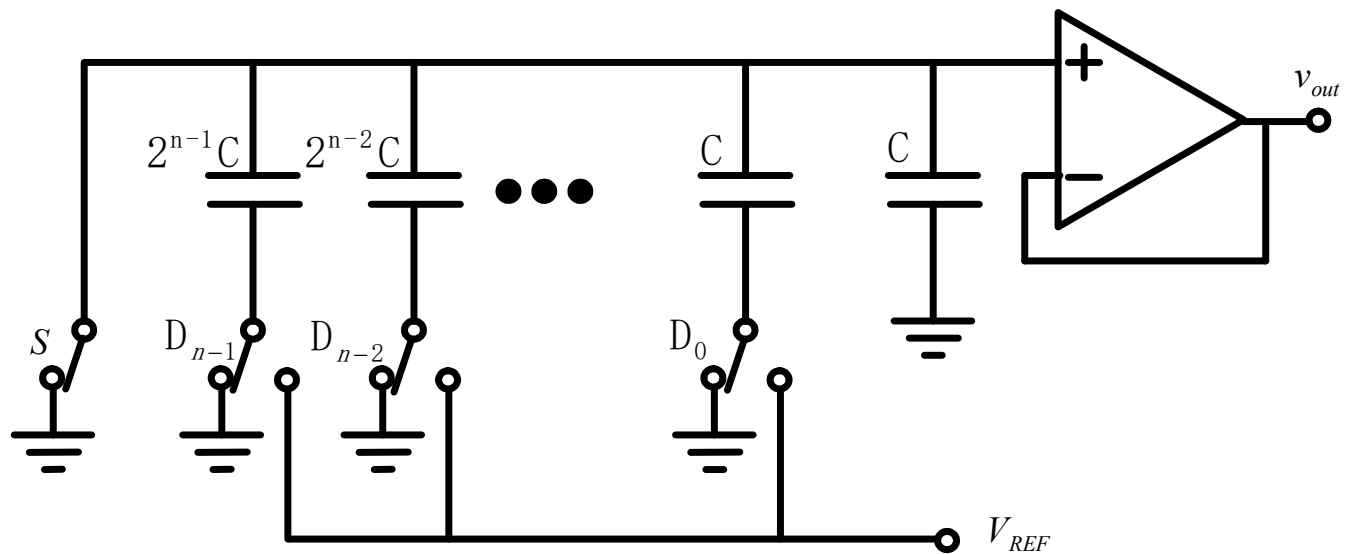




$$C_{\Sigma} = 2^{n-1} C + 2^{n-2} C + \dots + C + C = 2^n C$$

$$\begin{aligned}
 V_{out} = V_0 &= \frac{\sum_{i=0}^{n-1} 2^i D_i C V_{REF}}{2^n C} = \frac{1}{2^n} \sum_{i=0}^{n-1} 2^i D_i V_{REF} \\
 &= \left(2^{n-1} D_{n-1} + \dots + 2D_1 + D_0 \right) \frac{V_{REF}}{2^n}
 \end{aligned}$$





$$V_{out} = V_0 = \frac{\sum_{i=0}^{n-1} 2^i D_i C V_{REF}}{2^n C} = \frac{1}{2^n} \sum_{i=0}^{n-1} 2^i D_i V_{REF}$$

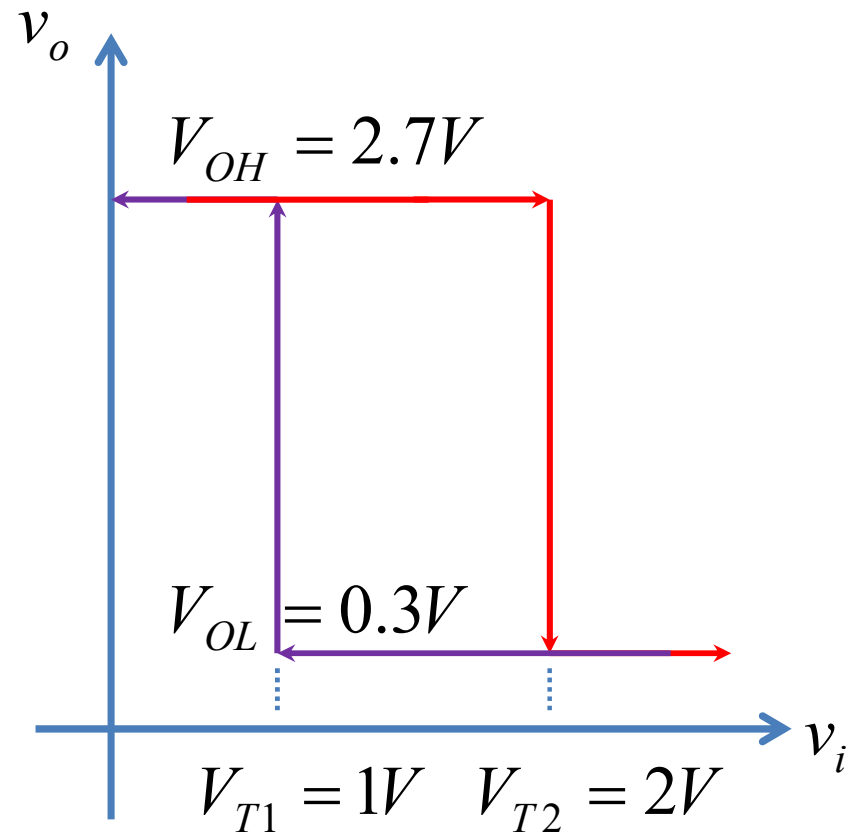
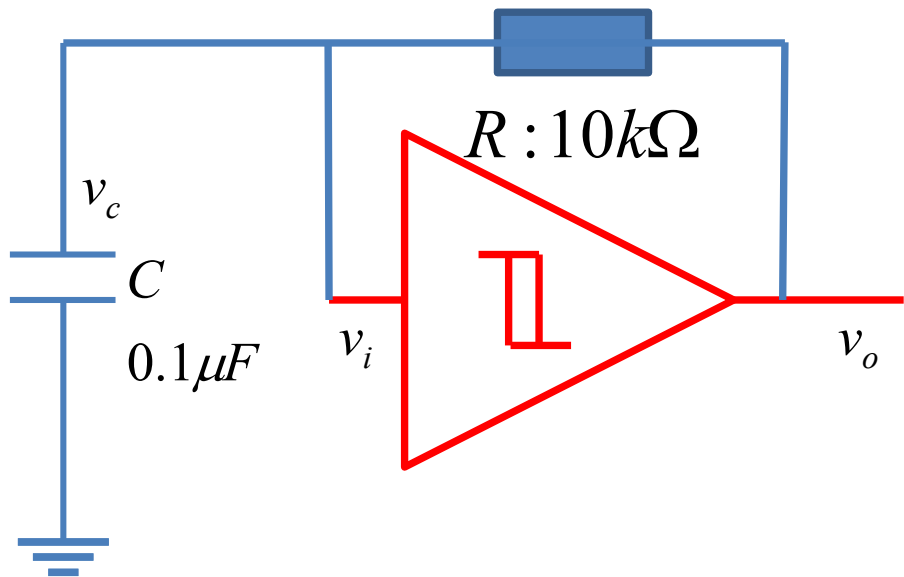
D₃D₂D₁D₀
1111

$$= \left(2^{n-1} D_{n-1} + \dots + 2D_1 + D_0 \right) \frac{V_{REF}}{2^n}$$

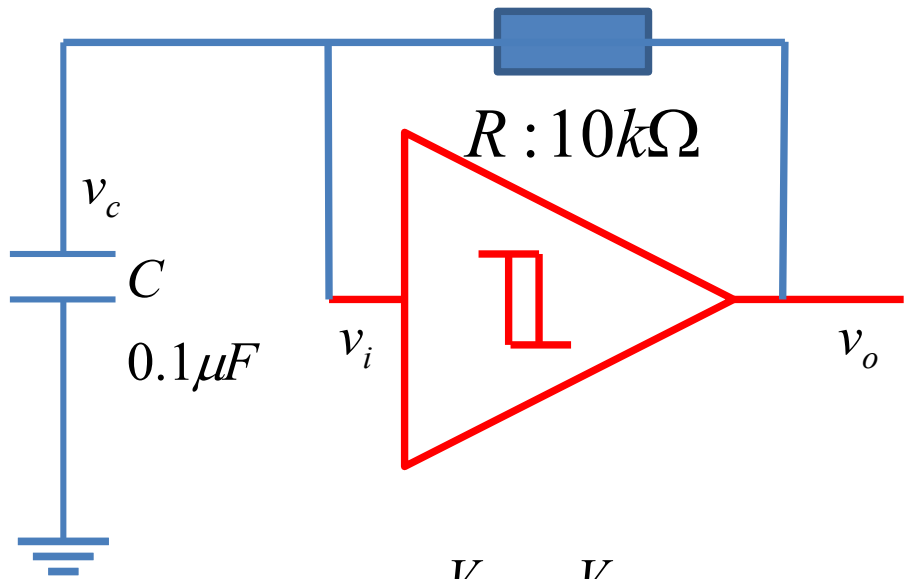
$$V_{out} = (8 + 4 + 2 + 1) \frac{V_{REF}}{16} = \frac{15}{16} V_{REF}$$

作业5

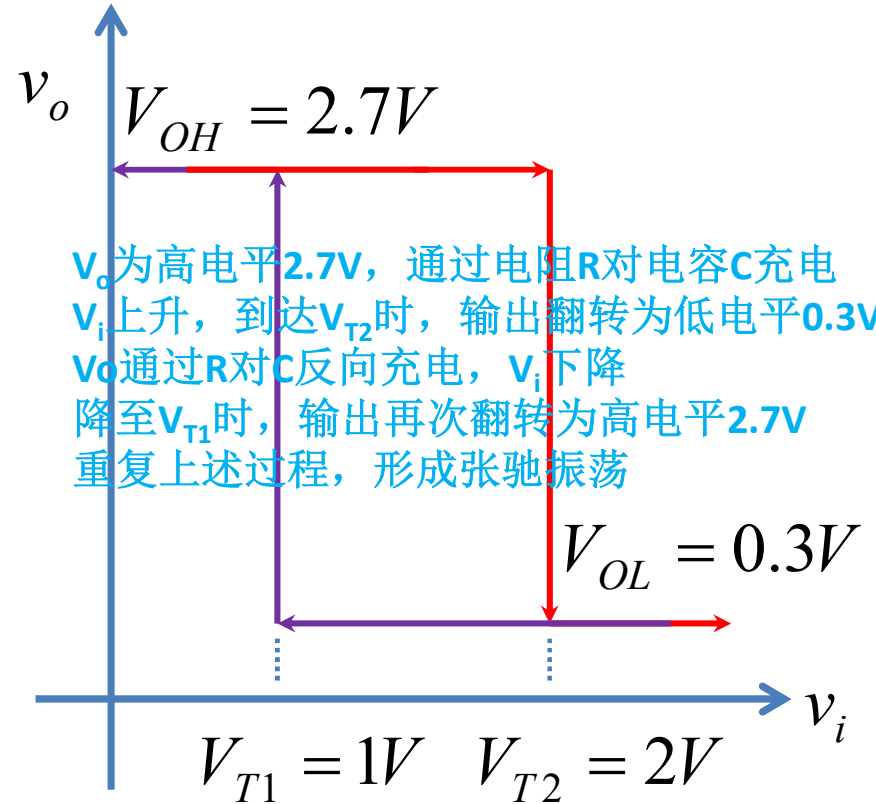
反相施密特触发器RC方波振荡器



- 已知施密特触发器的滞回曲线如图所示
- 画出电容电压和触发器输出电压波形，根据波形对其工作原理进行描述，并给出振荡频率，请用 $R, C, V_{OH}, V_{OL}, V_{T1}, V_{T2}$ 参量表述振荡频率



1. v_o 为高电平 $2.7V$ ，通过电阻 R 对电容 C 充电
2. v_i 上升，到达 V_{T2} 时，输出翻转为低电平 $0.3V$
3. v_o 通过 R 对 C 反向充电， v_i 下降
4. 降至 V_{T1} 时，输出再次翻转为高电平 $2.7V$
5. 重复上述过程，形成张弛振荡



$$T_1 = RC \ln \frac{V_{OH} - V_{T1}}{V_{OH} - V_{T2}}$$

$$= 10k \times 0.1\mu \times \ln \frac{1.7}{0.7} = 0.8873ms$$

$$T_2 = RC \ln \frac{V_{OL} - V_{T2}}{V_{OL} - V_{T1}}$$

$$= 0.8873ms$$

$$T = T_1 + T_2 = 1.7746ms$$

$$f = \frac{1}{T} = 564Hz$$

$V_{OH} = 2.7V$

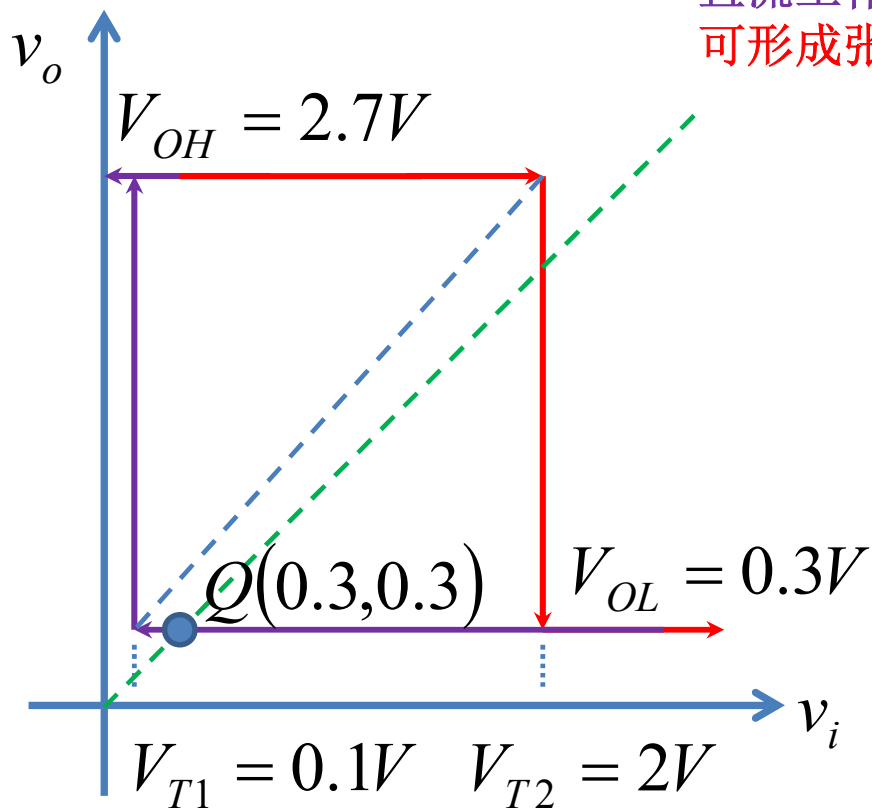
$V_{T2} = 2V$

$V_{T1} = 1V$

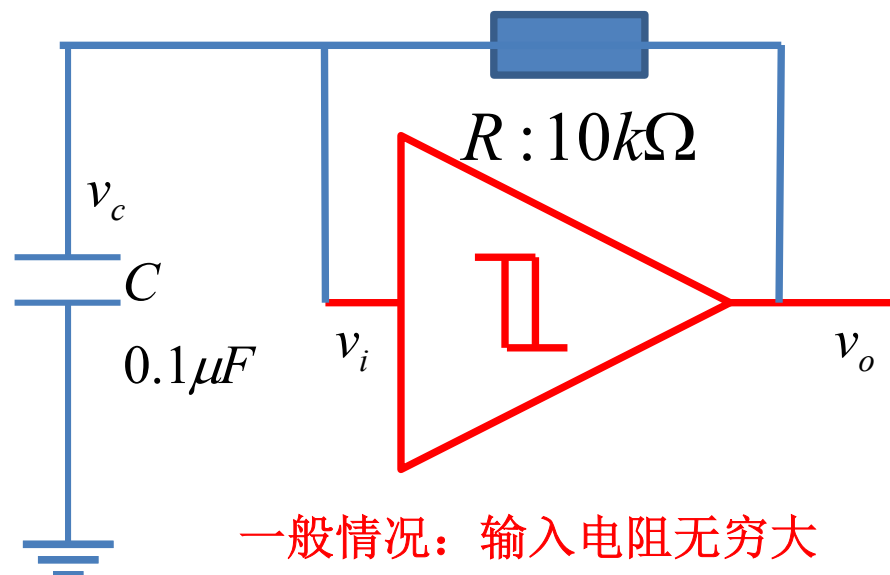
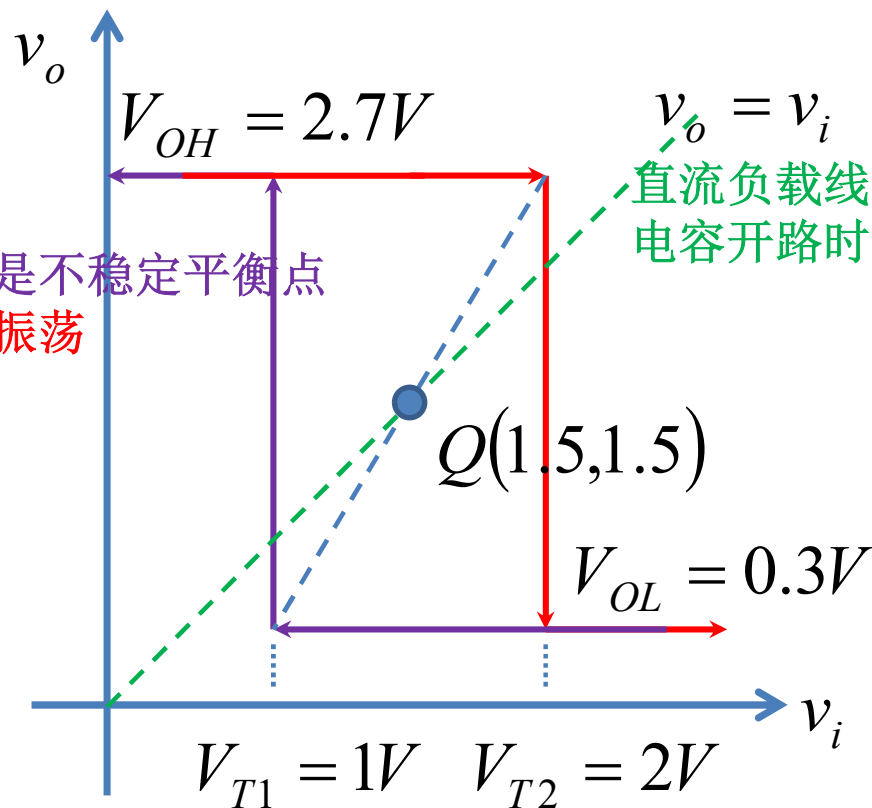
$V_{OL} = 0.3V$

形成张弛振荡？

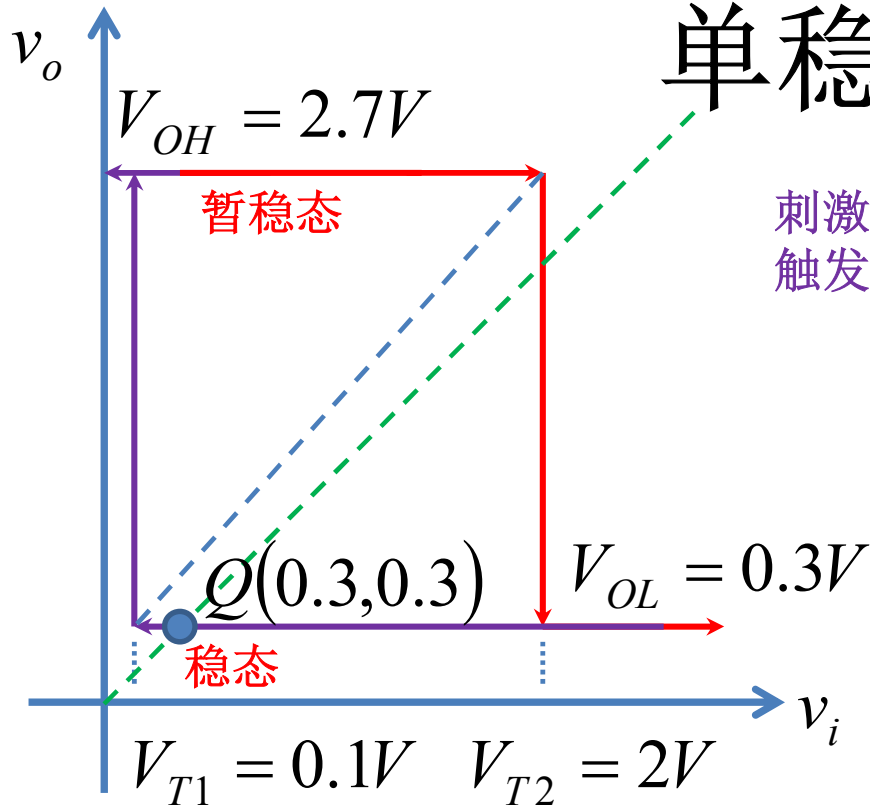
条件：直流工作点偏置在负阻区，偏置点为不稳定平衡点



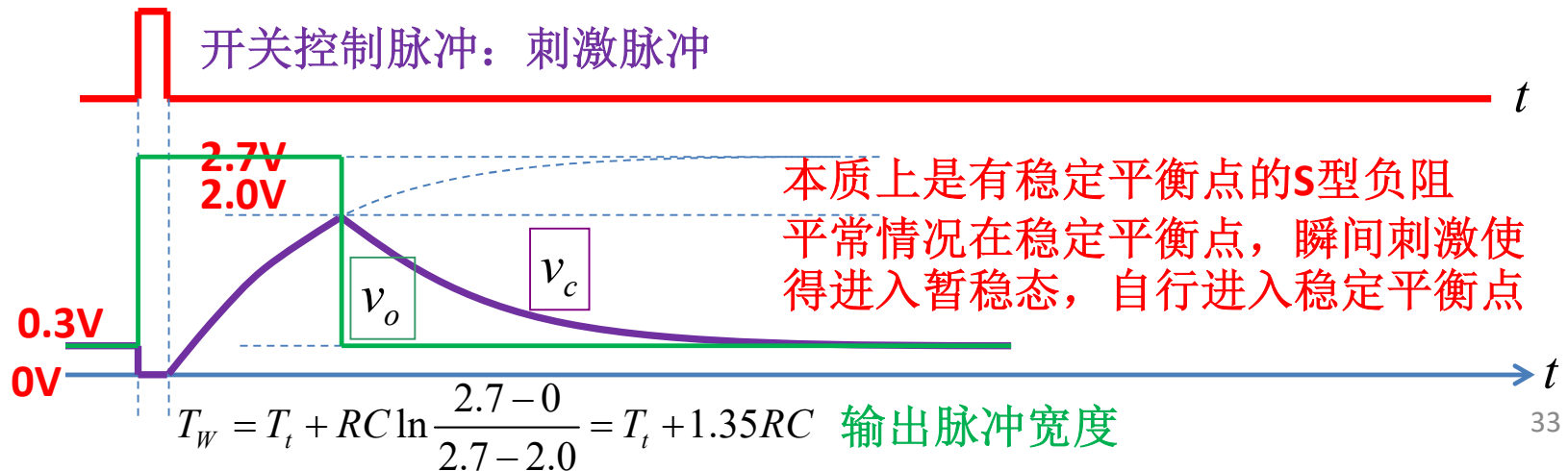
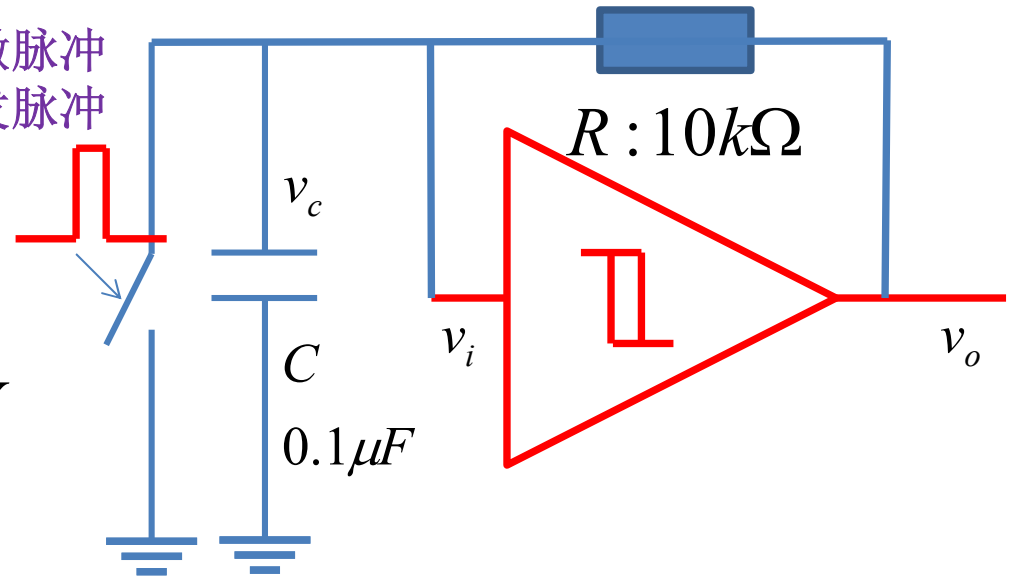
如果设计的振荡不振荡，原因？



单稳电路

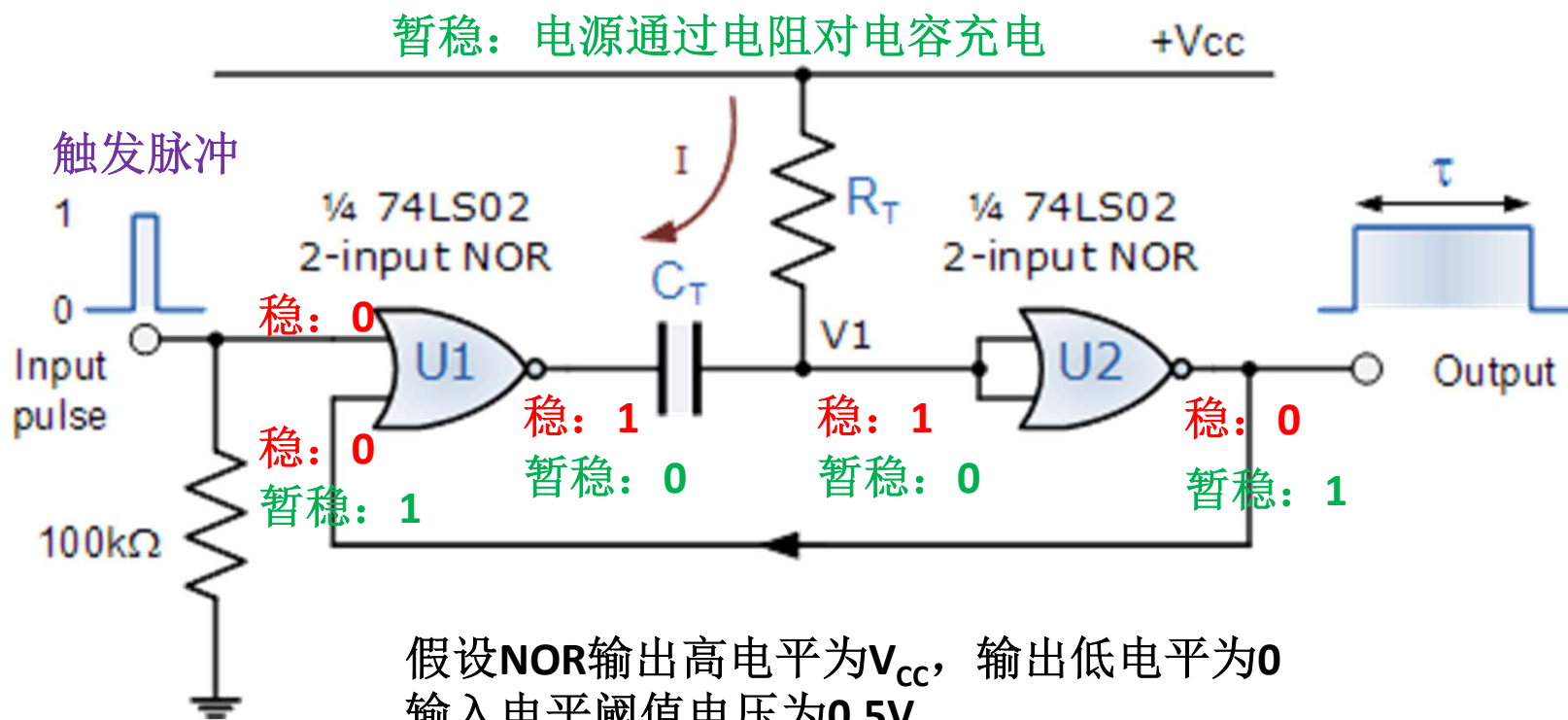


刺激脉冲
触发脉冲



作业6 NOR gate monostable multivibrator

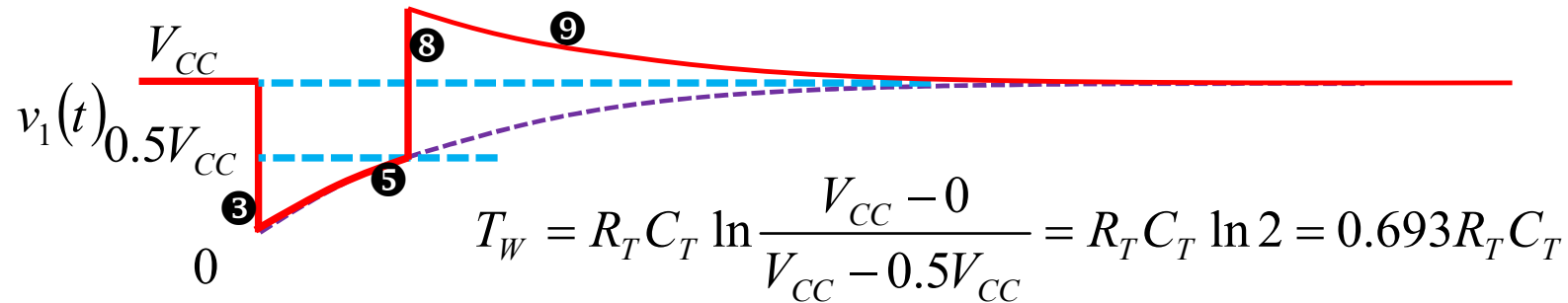
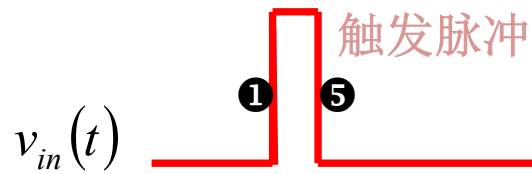
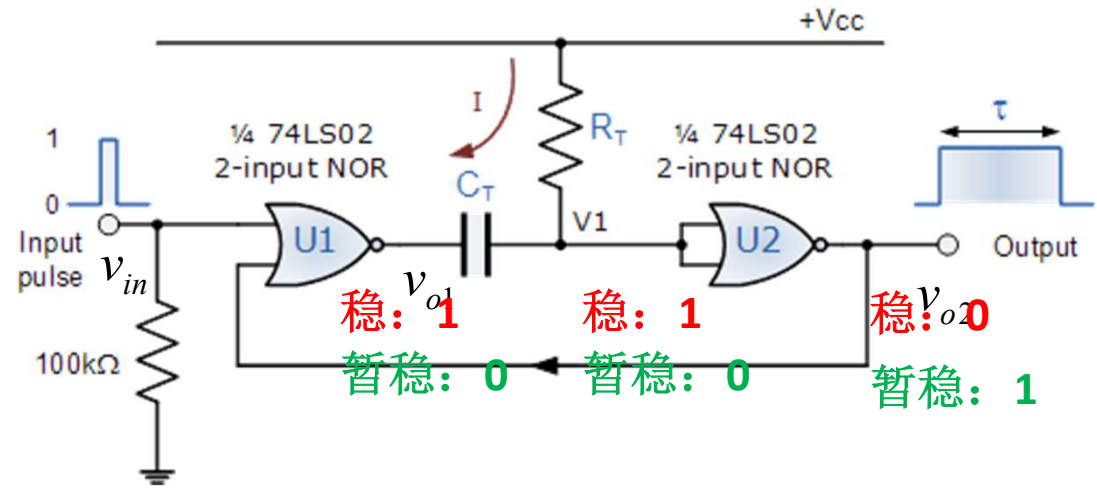
单稳态
单脉冲产生



假设NOR输出高电平为 V_{CC} ，输出低电平为0
输入电平阈值电压为 $0.5V_{CC}$

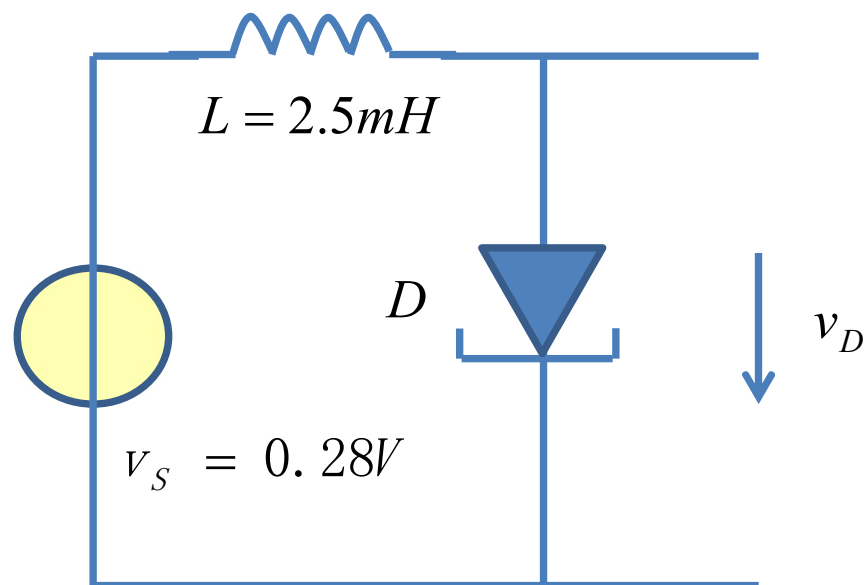
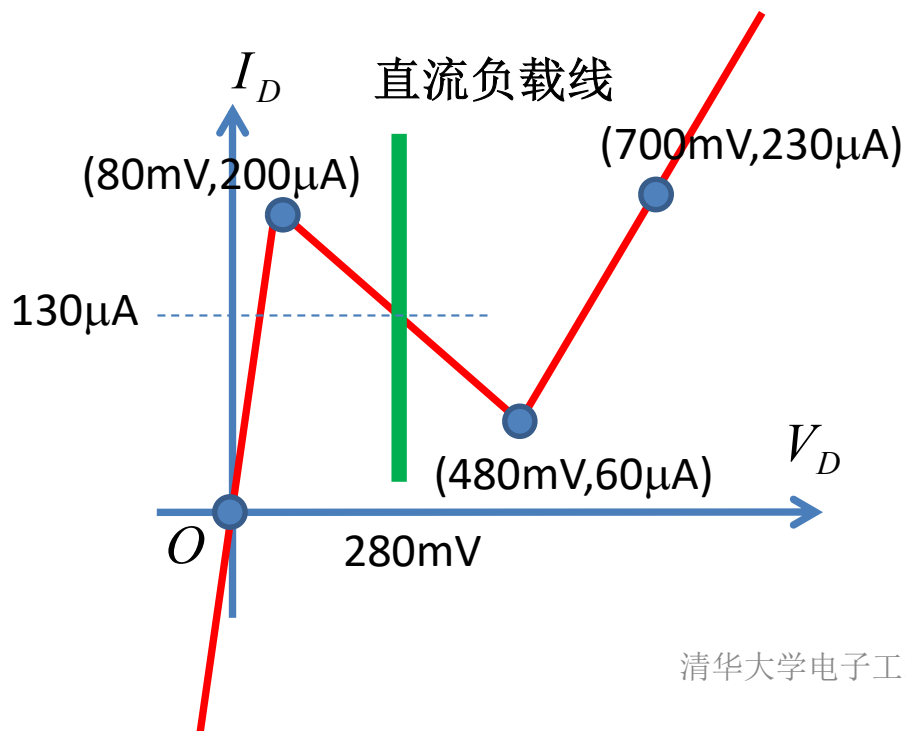
- 1、画出各个结点的波形
- 2、求单脉冲输出脉宽

本质上是有稳定平衡点的S型负阻
平常情况在稳定平衡点，瞬间刺激使得进入暂稳态，自行进入稳定平衡点



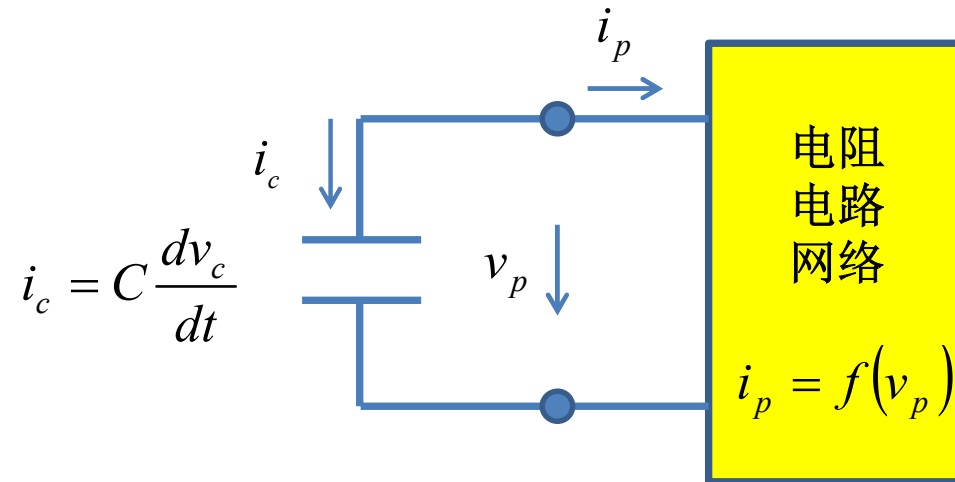
作业7 L和N型负阻：张弛振荡？

- 相图分析后，给出张弛振荡器振荡频率，画出 v_D 振荡时域波形图



- 1、如果分析不清楚，请先PSPICE仿真，根据仿真结果分析（选作）
- 2、用PSPICE仿真确认你的分析结果（选作）

课堂讨论内容：非线性加单电容



压控表述

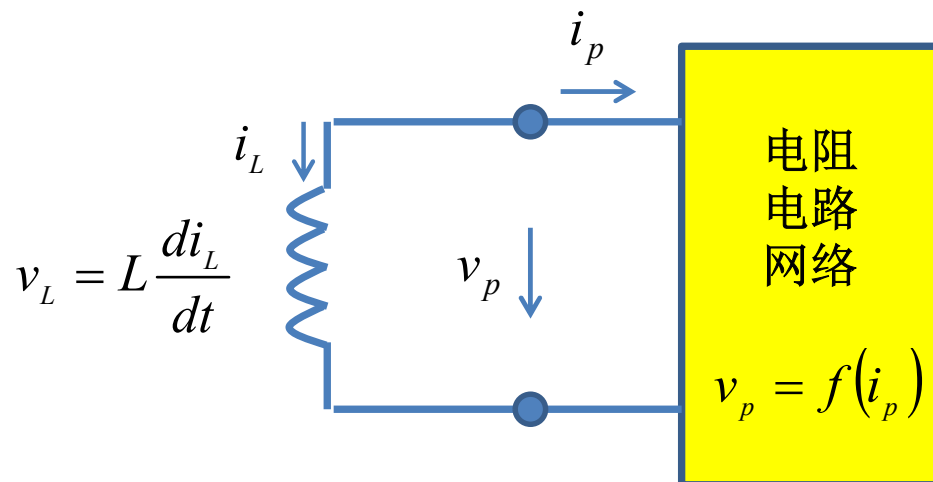
$$C \frac{dv_c}{dt} = i_c = -i_p = -f(v_p) = -f(v_c)$$

$$y = -\frac{1}{C} f(x)$$

$$x = v_c$$
$$y = \frac{dv_c}{dt} = \frac{dx}{dt}$$

相图研究

对偶研究：非线性加单电感



流控表述

$$L \frac{di_L}{dt} = v_L = v_p = f(i_p) = f(-i_L)$$

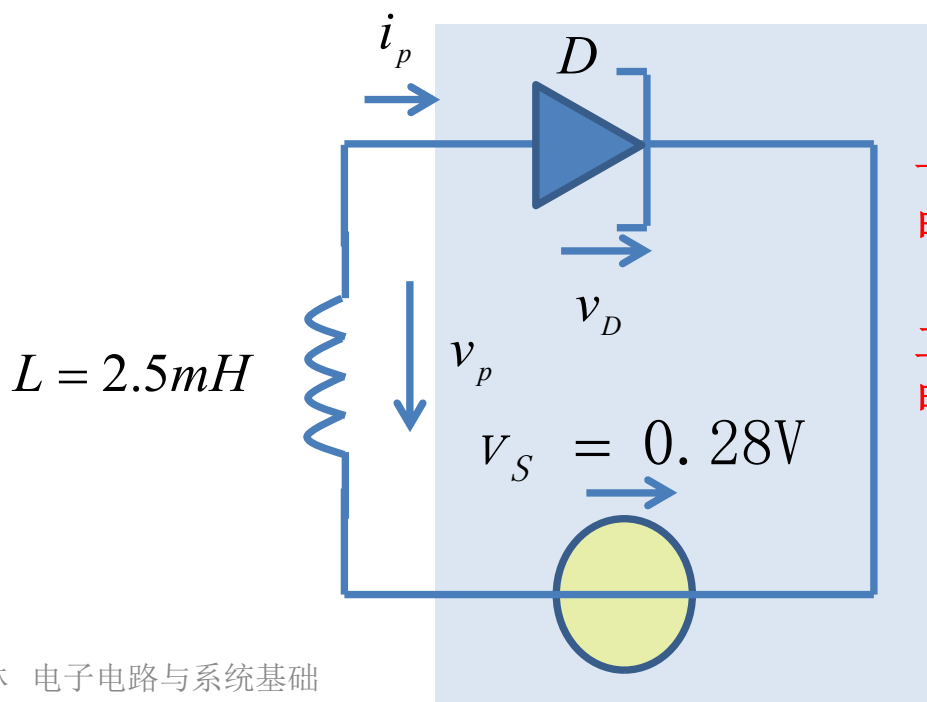
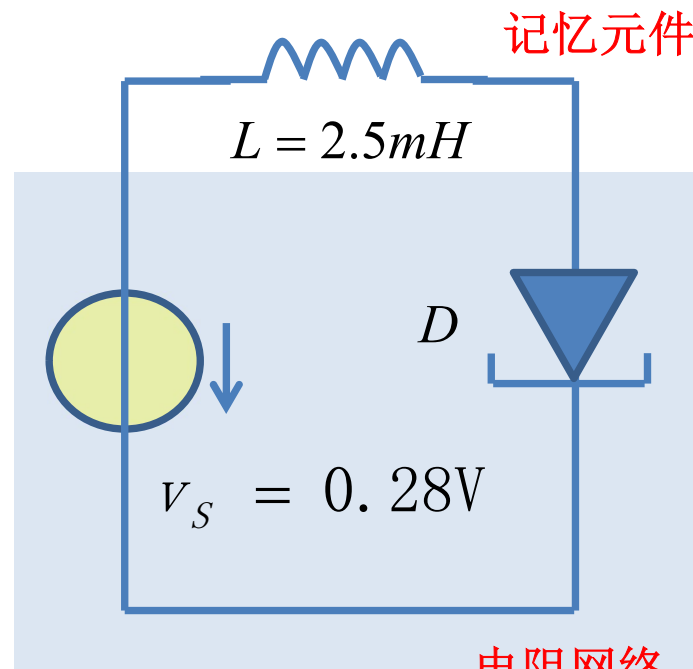
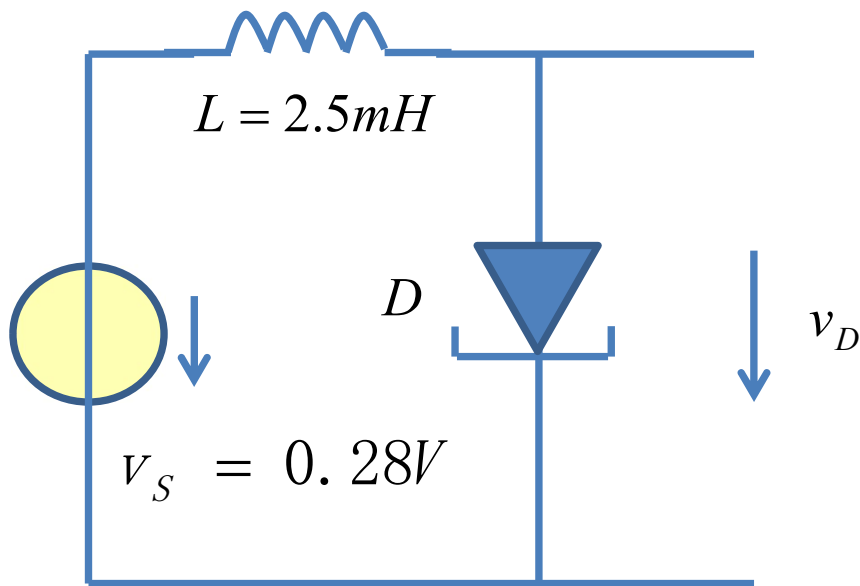
$$y = \frac{1}{L} f(-x)$$

$$x = i_L$$

$$y = \frac{di_L}{dt} = \frac{dx}{dt}$$

相图研究

电阻电路重新封装为单端口网络



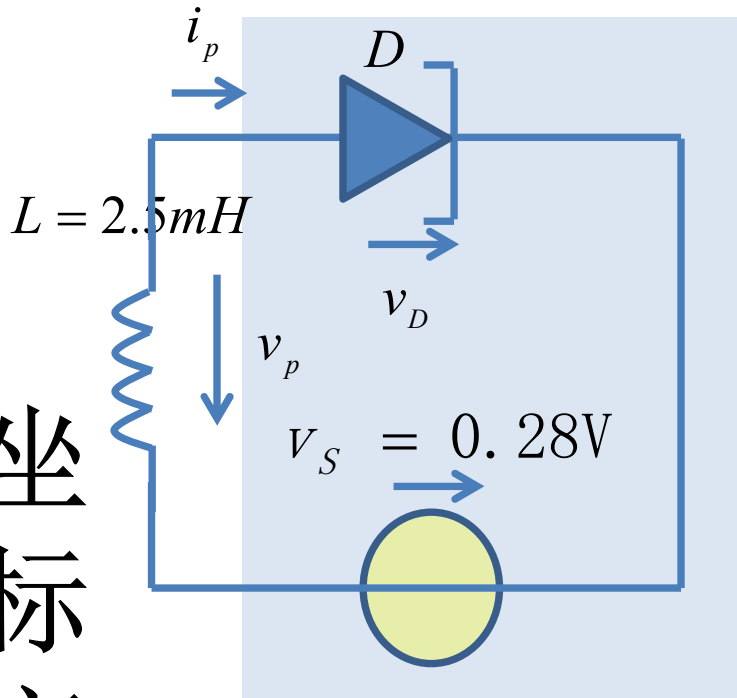
电阻网络分离

一阶系统
电阻网络抽象为单端口网络

二阶系统
电阻网络抽象为二端口网络

单端口网络元件约束
 $v_p = v_D - v_S = v_D - 0.28V$
 $i_p = i_D$ v_D, i_D 已知

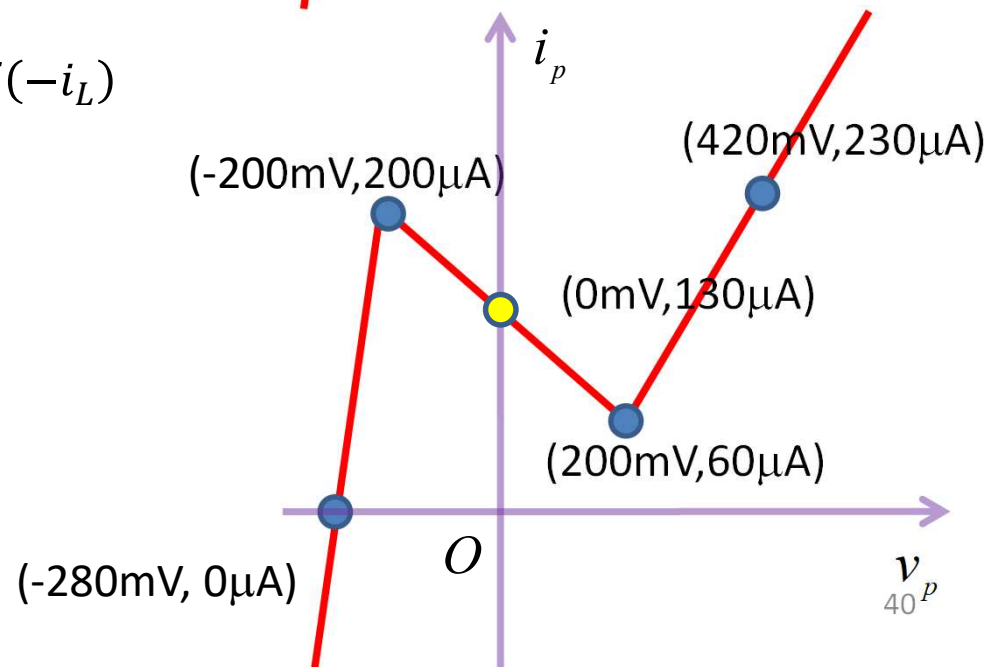
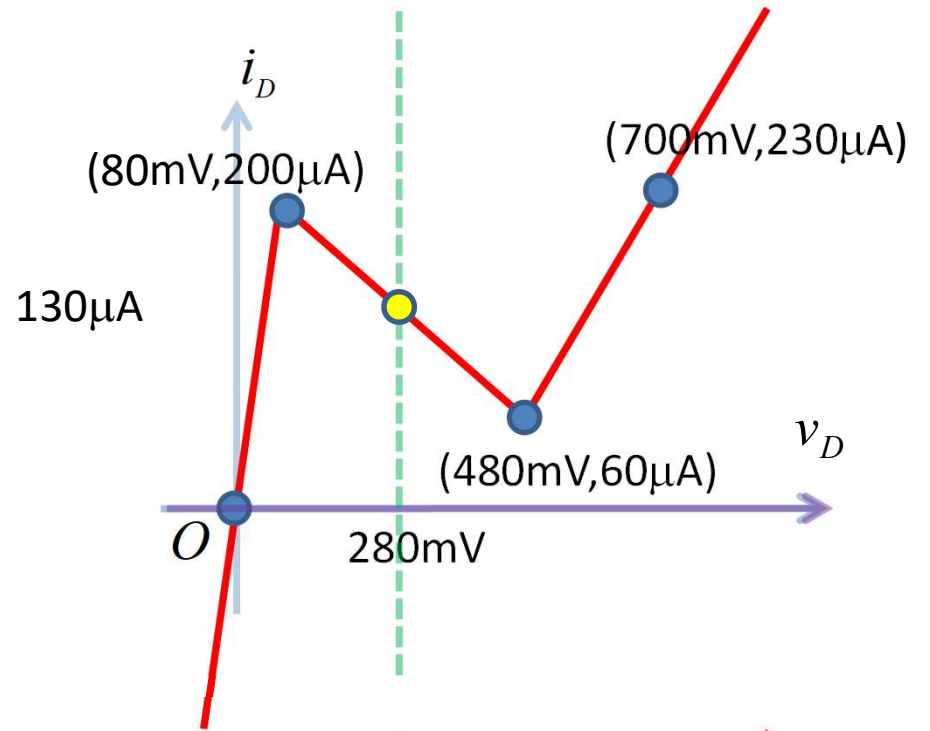
坐标变换



$$\frac{di_L}{dt} = \frac{1}{L} v_L = \frac{1}{L} v_p = \frac{1}{L} f(i_p) = \frac{1}{L} f(-i_L)$$

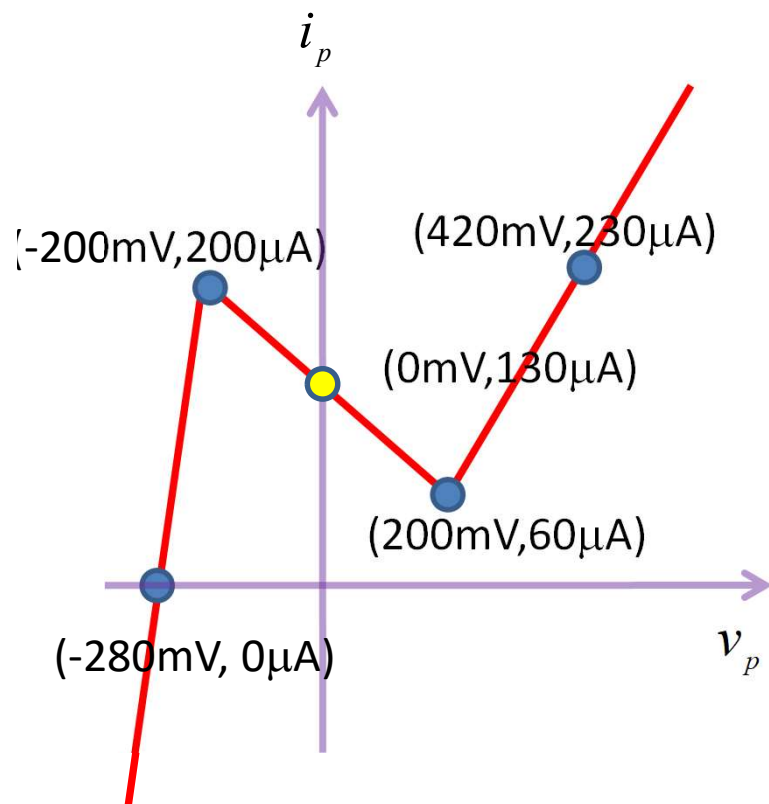
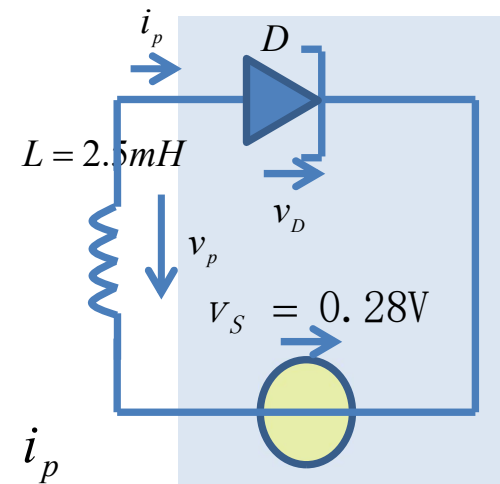
$$v_p = v_D - v_S = v_D - 0.28V$$

$$i_p = i_D$$



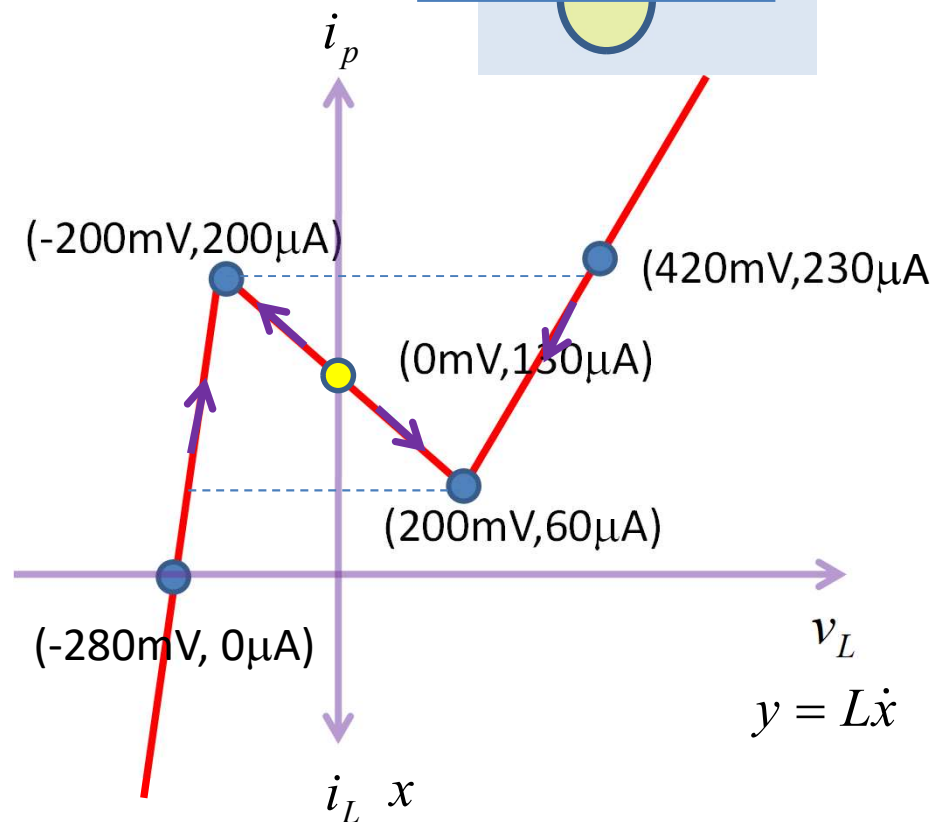
相图分析

$$\frac{di_L}{dt} = \frac{1}{L} f(-i_L)$$

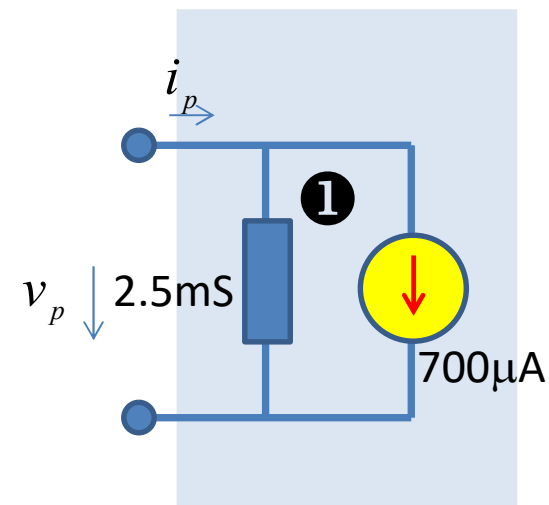
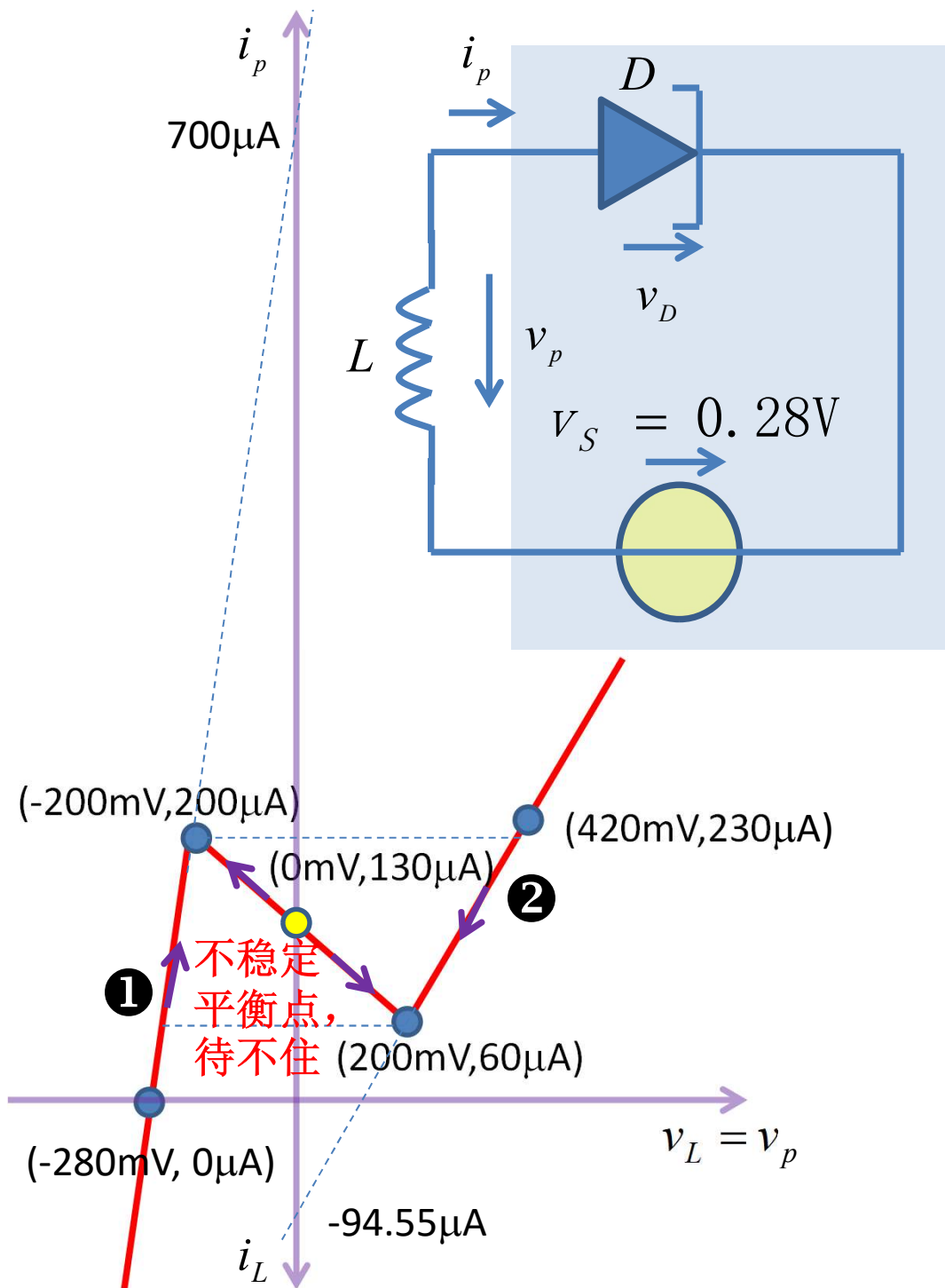


$$i_L = -i_p$$

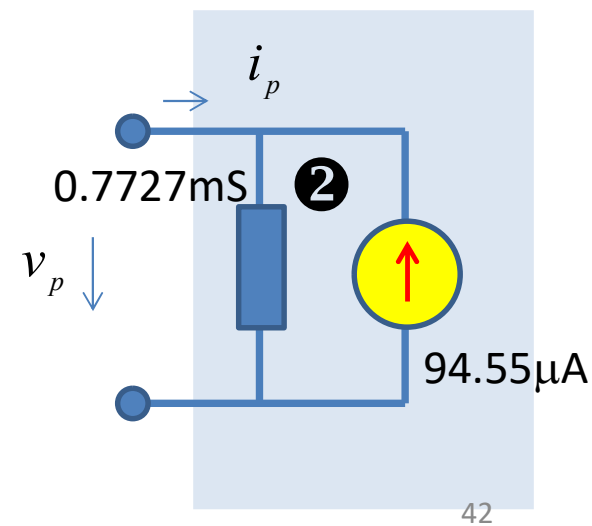
$$v_L = v_p$$



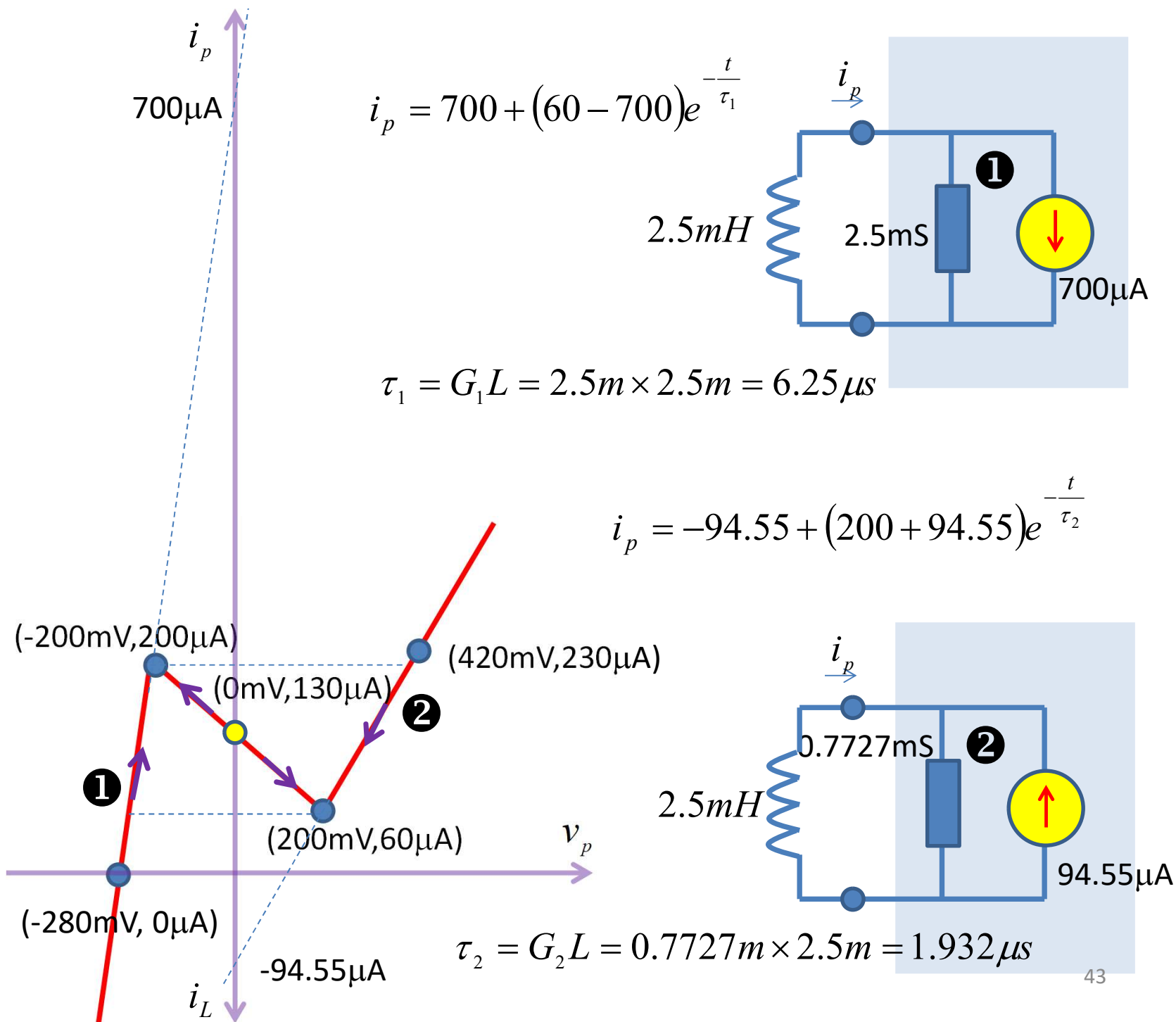
正阻区诺顿等效



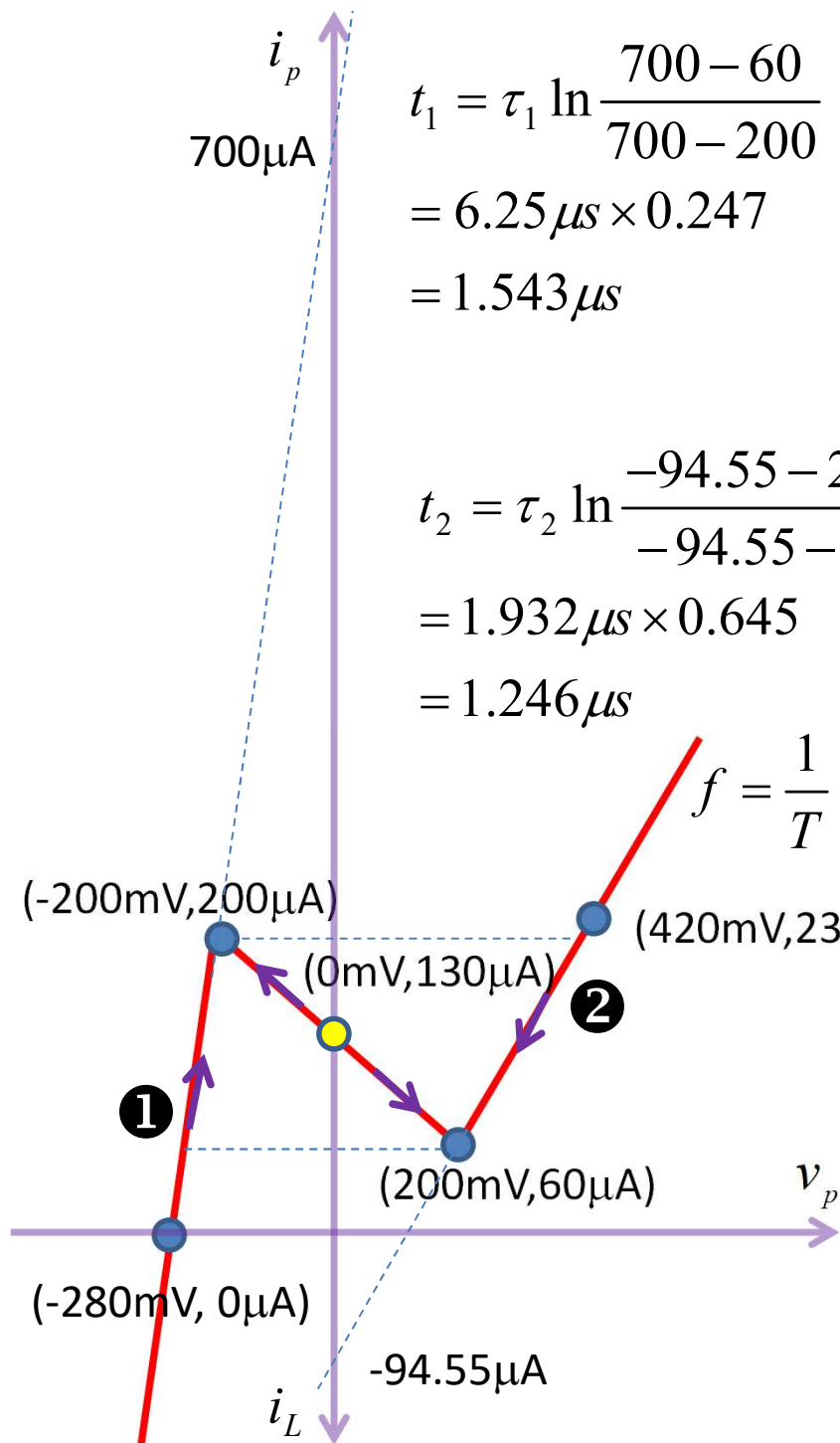
针对 v_p, i_p 进行建模



诺顿源为电感充磁



待 在 两个 暂 稳 态 的 时 间 与 振 荡 频 率



$$t_1 = \tau_1 \ln \frac{700 - 60}{700 - 200}$$

$$= 6.25 \mu\text{s} \times 0.247$$

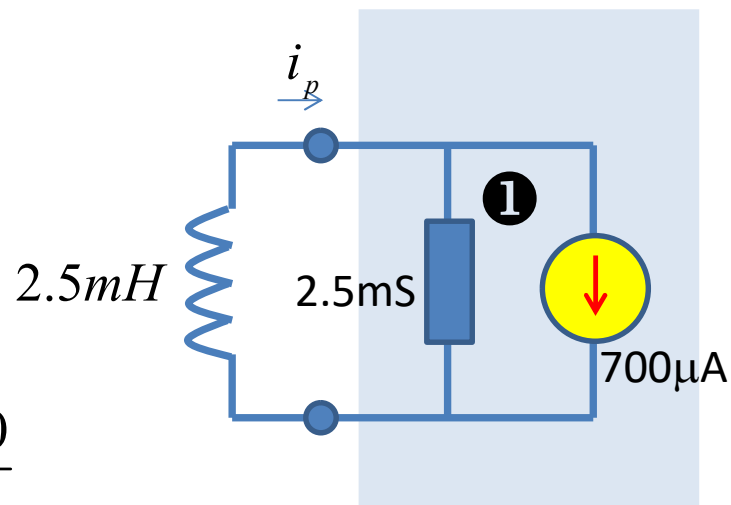
$$= 1.543 \mu\text{s}$$

$$t_2 = \tau_2 \ln \frac{-94.55 - 200}{-94.55 - 60}$$

$$= 1.932 \mu\text{s} \times 0.645$$

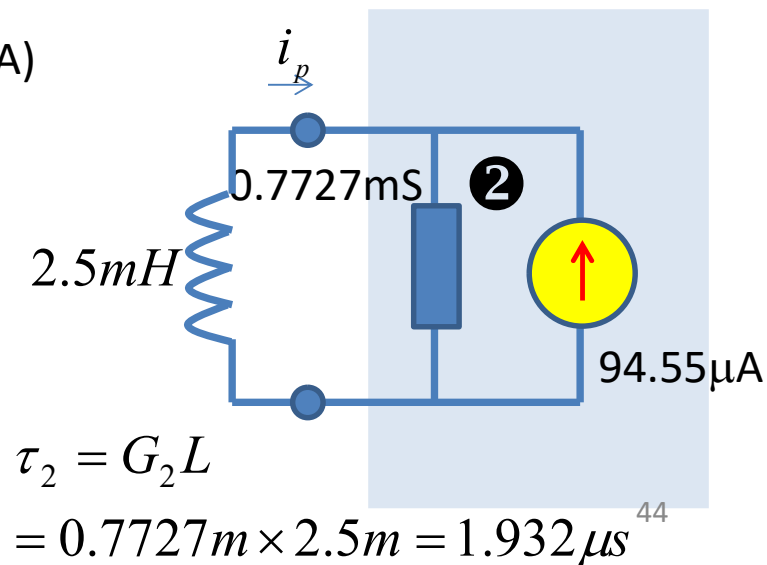
$$= 1.246 \mu\text{s}$$

$$f = \frac{1}{T} = \frac{1}{1.543 \mu + 1.246 \mu} = 359 \text{kHz}$$



$$\tau_1 = G_1 L$$

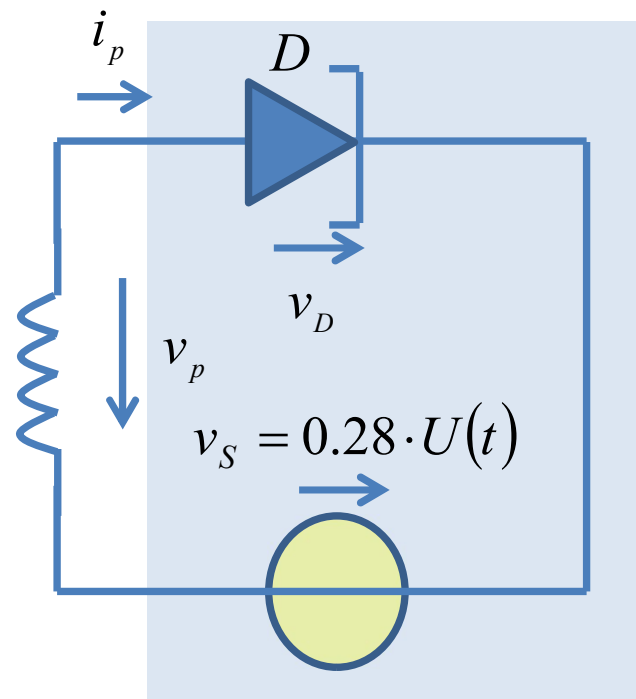
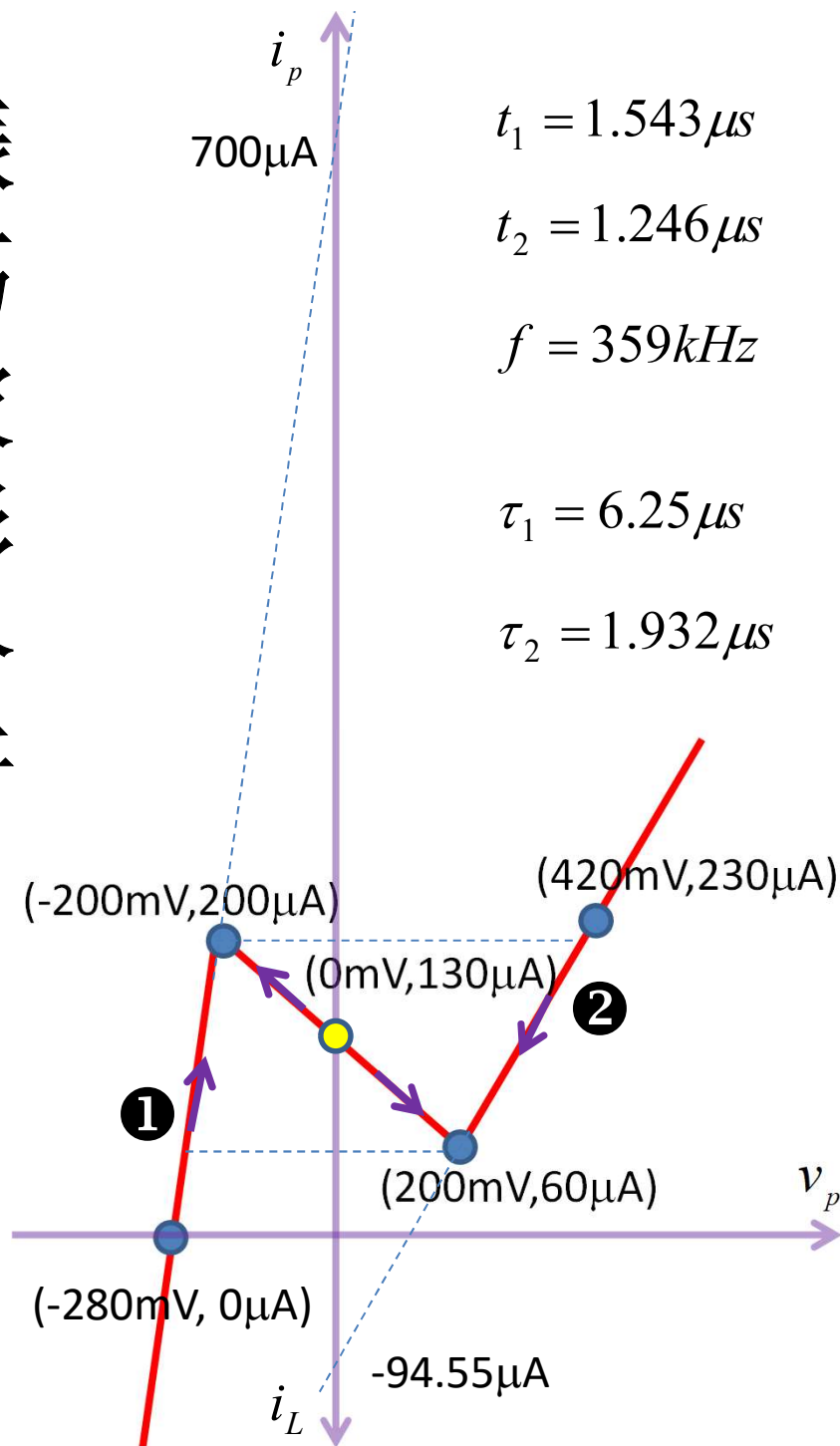
$$= 2.5\text{m} \times 2.5\text{m} = 6.25 \mu\text{s}$$



$$\tau_2 = G_2 L$$

$$= 0.7727\text{m} \times 2.5\text{m} = 1.932 \mu\text{s}$$

振荡波形分析



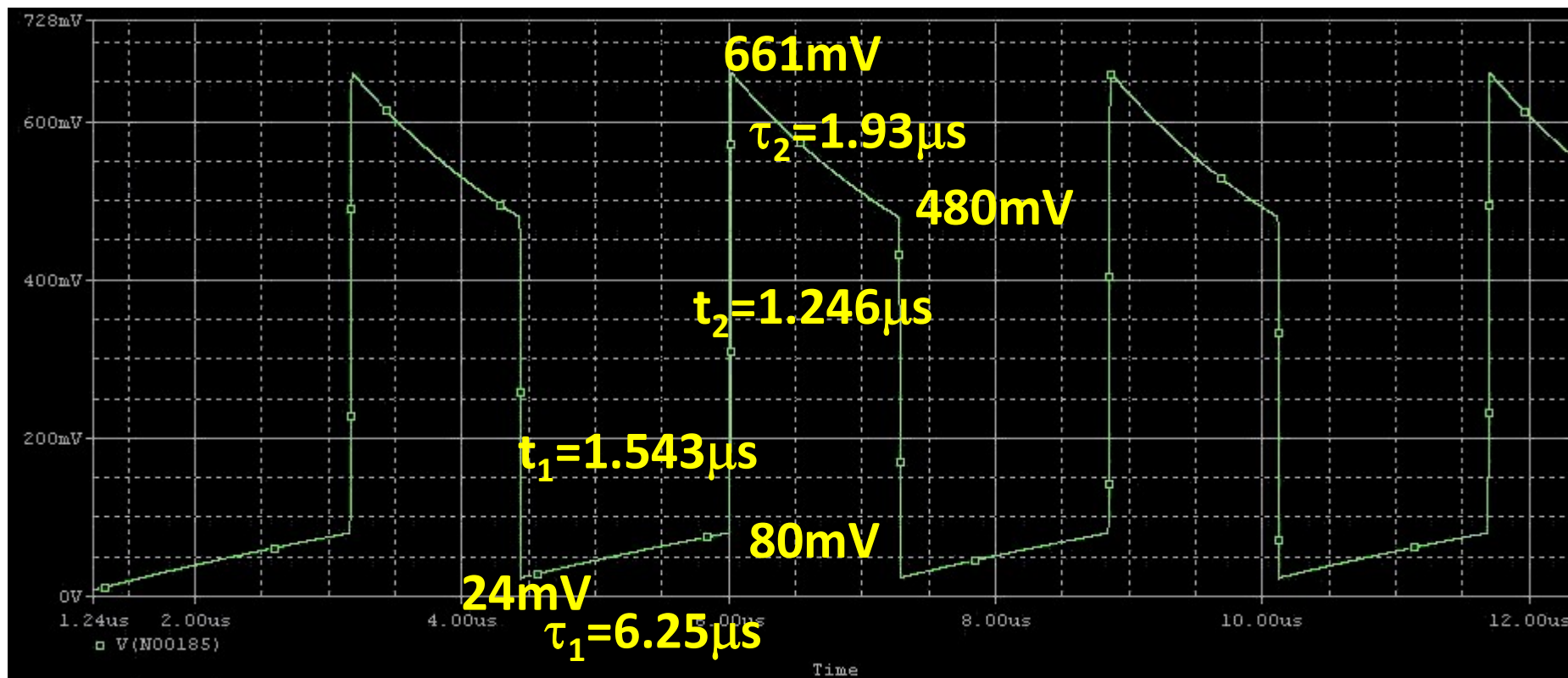
$i_p (\mu A)$
①: 60 $\mu A \rightarrow 200\mu A$
②: 200 $\mu A \rightarrow 60\mu A$

$v_p (mV)$
①: -256mV \rightarrow -200mV
②: 381mV \rightarrow 200mV

$v_D (mV)$
①: 24mV \rightarrow 80mV
②: 661mV \rightarrow 480mV

考试画波形图时，需要标注清楚转折点电压电流/时间点，时间常数等关键参量

振荡波形仿真结果



$$v_D (mV) \sim t (\mu s)$$

$$\tau_1 = 6.25 \mu s$$

$$\tau_2 = 1.932 \mu s$$

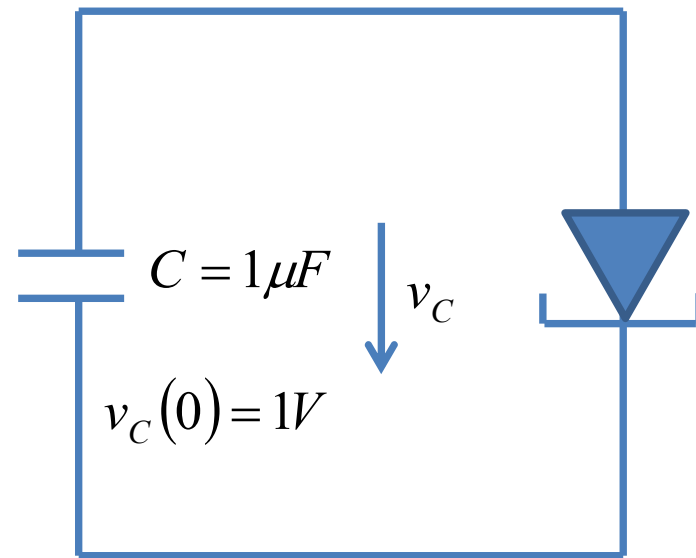
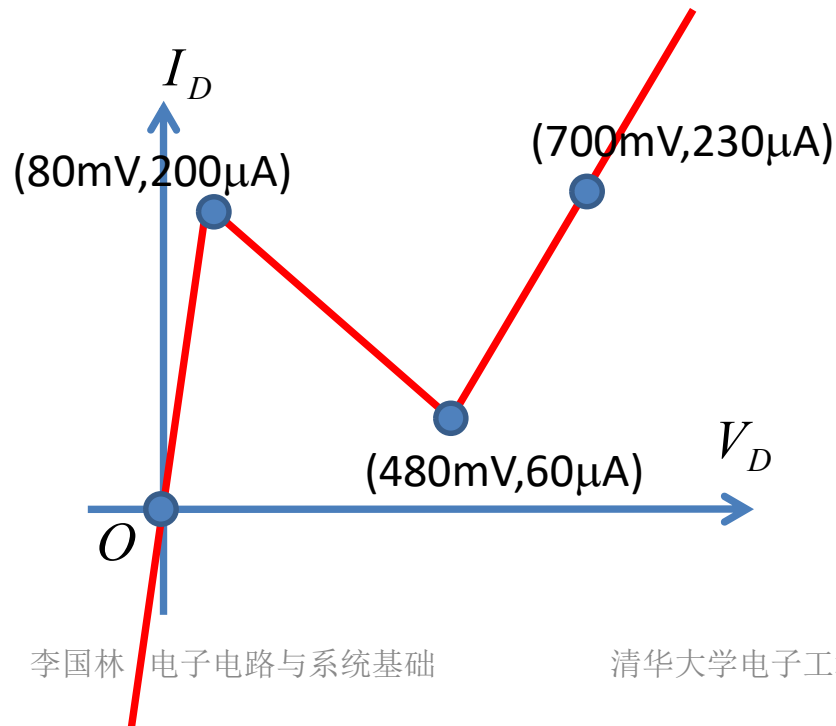
$$v_D (mV)$$

$$\textcircled{1}: 24mV \rightarrow 80mV$$

$$\textcircled{2}: 661mV \rightarrow 480mV$$

作业8 C和N型负阻：电容放电

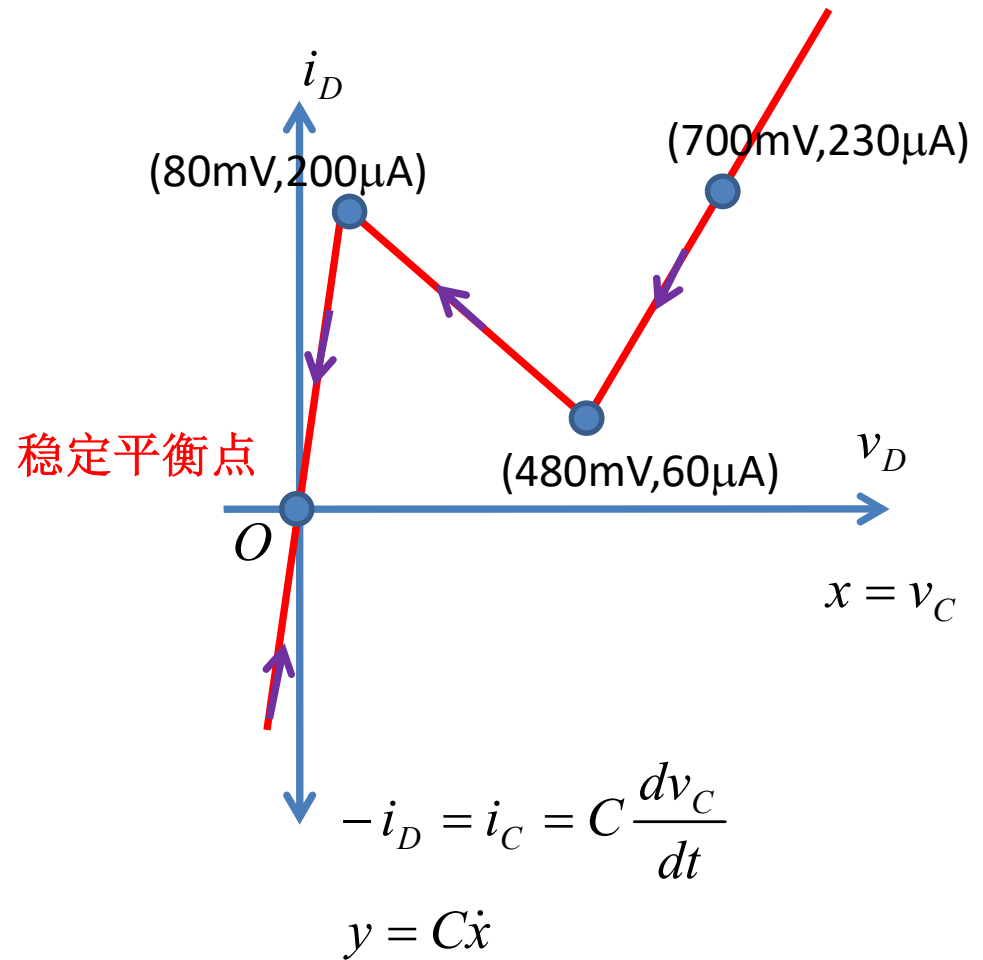
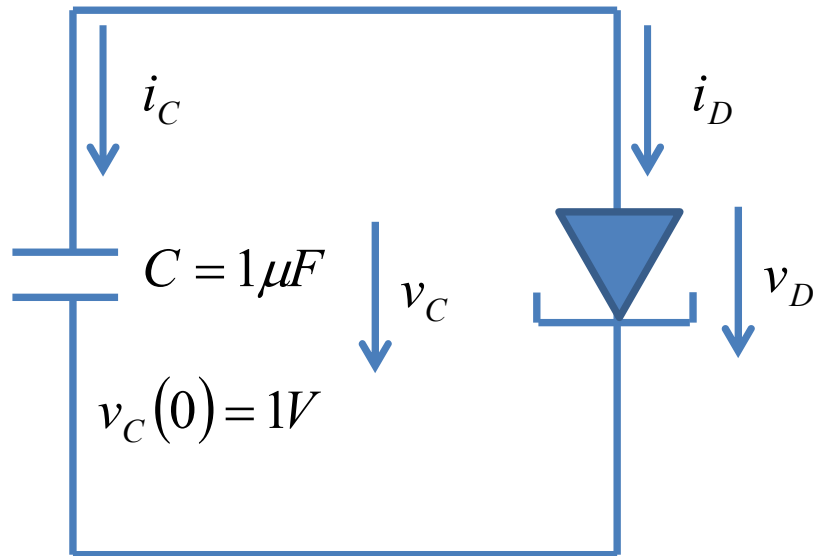
- 相图分析后，给出电容电压时域表达式和时域波形图



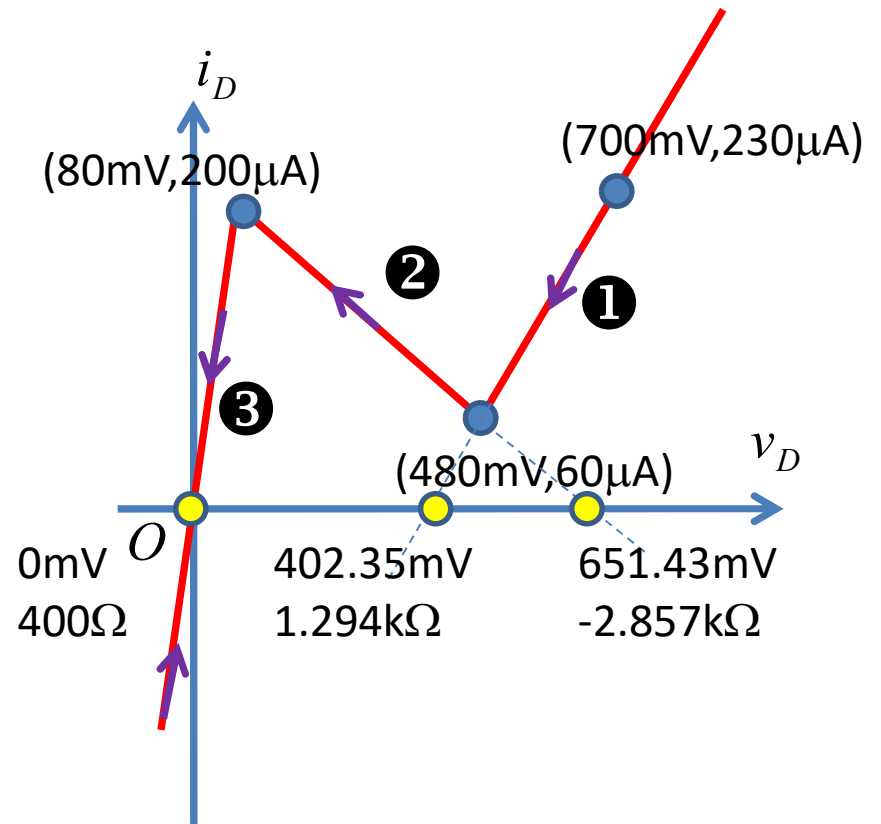
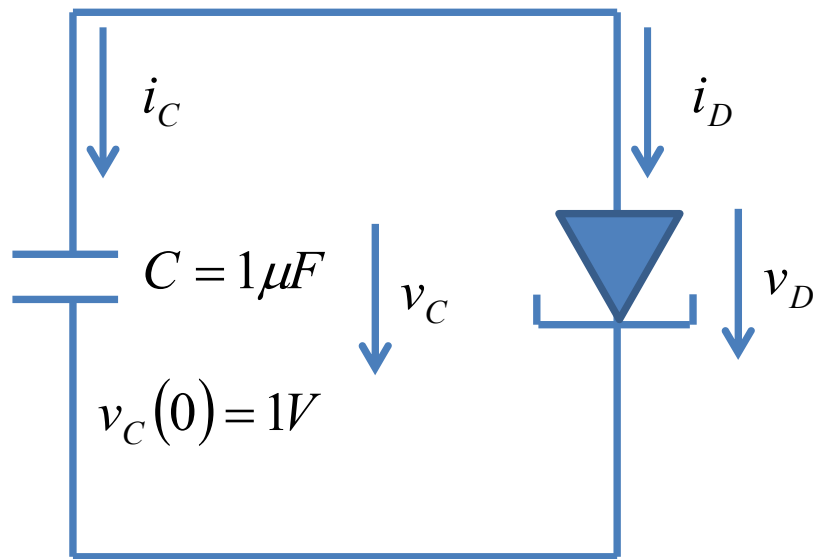
- 1、如果分析不清楚，请先PSPICE仿真，根据仿真结果分析（选作）
- 2、用PSPICE仿真确认你的分析结果（选作）

相图分析

$$C \frac{dv_C}{dt} = i_C = -i_D = -f(v_D)$$



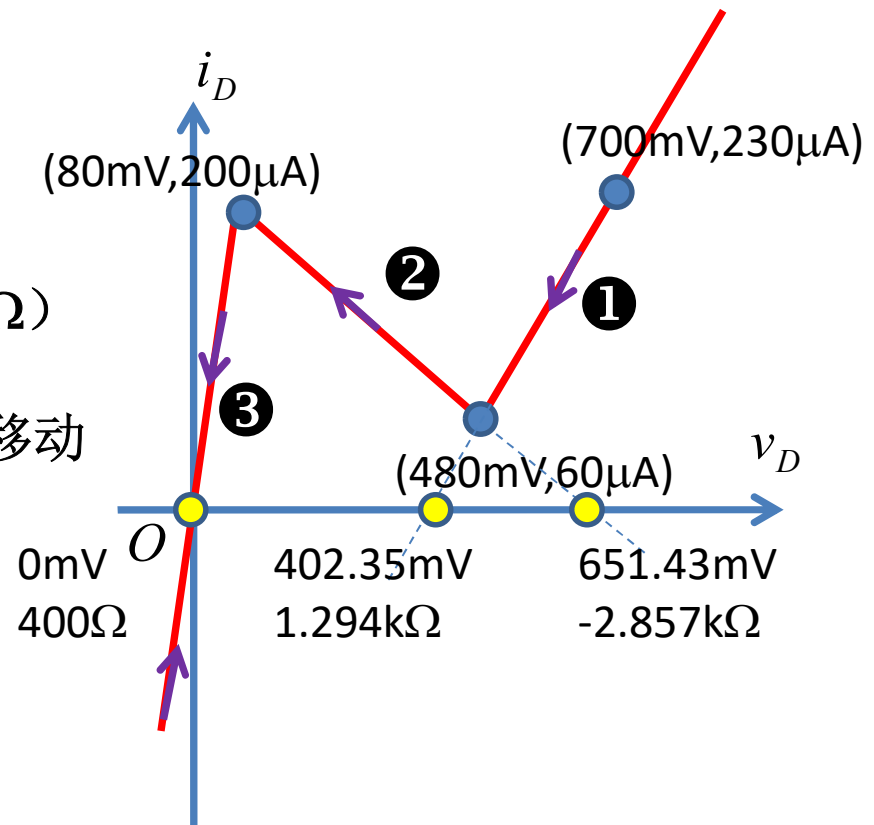
等效电路分析



- ①区：**戴维南电压源（**402.35mV**，**1.294kΩ**）对电容充电：从**1V**到**480mV**
 正内阻：朝稳定平衡点**402.35mV**移动
- ②区：**戴维南电压源（**651.43mV**，**-2.857kΩ**）对电容充电：从**480mV**到**80mV**
 负内阻：背离不稳定平衡点**651.43mV**移动
- ③区：**戴维南电压源（**0mV**，**400Ω**）对电容充电：从**80mV**到**0mV**
 正内阻：朝稳定平衡点**0mV**移动

1区 指数衰减规律

①区：戴维南电压源（402.35mV，1.294kΩ）
对电容充电：从1V到480mV
正内阻：朝稳定平衡点402.35mV移动



$$v_{C1}(t) = v_{C\infty 1}(t) + (v_{C1}(0) - v_{C\infty 1}(0))e^{-\frac{t}{\tau_1}}$$

$$= 402.35 + (1000 - 402.35)e^{-\frac{t}{1.294ms}}$$

$$= 402.35 + 597.65e^{-\frac{t}{1.294ms}} \quad t \geq 0$$

$$2.641ms \geq t \geq 0$$

$$\Delta t_1 = \tau_1 \ln \frac{V_{S01} - V_{00}}{V_{S01} - V_{01}} = 1.294ms \times \ln \frac{402.35 - 1000}{402.35 - 480} = 2.641ms$$

②区 指数增长规律

②区：戴维南电压源（651.43mV， -2.857kΩ）
对电容充电：从480mV到80mV
负内阻：背离不稳定平衡点651.43mV
移动

不稳定平衡：负无穷时间点对应点

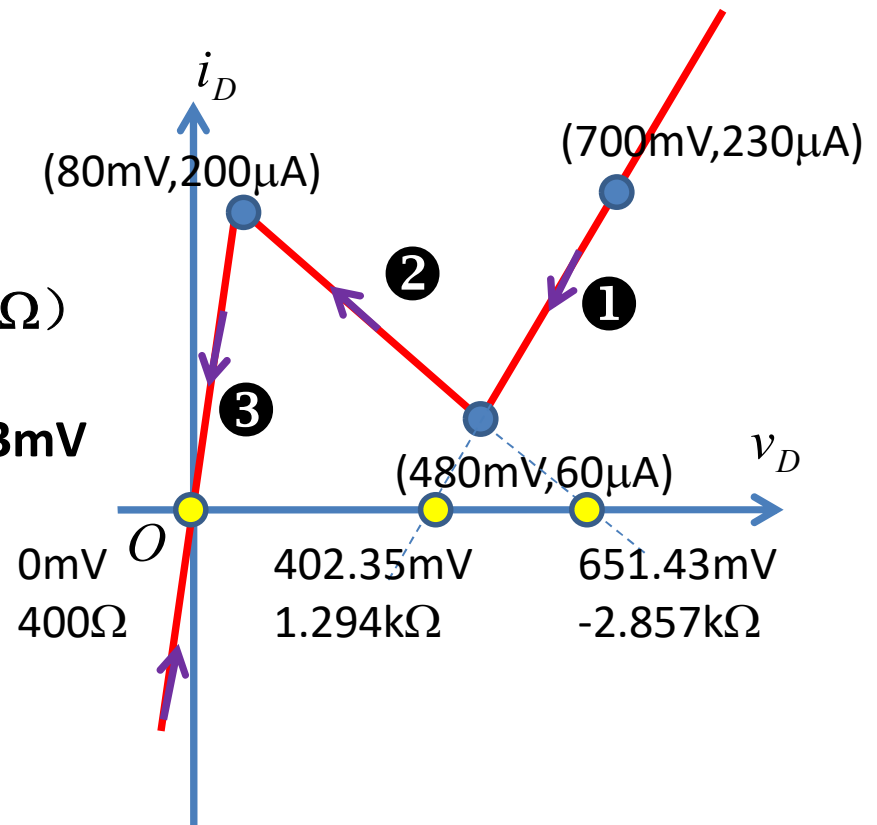
$$v_{C2}(t) = v_{C\infty 2}(t) + (v_{C2}(t_1) - v_{C\infty 2}(t_1))e^{-\frac{t-t_1}{\tau_2}}$$

$$= 651.43 + (480 - 651.43)e^{-\frac{t-t_1}{2.857ms}}$$

$$= 651.43 - 171.43e^{-\frac{t-2.641ms}{2.857ms}}$$

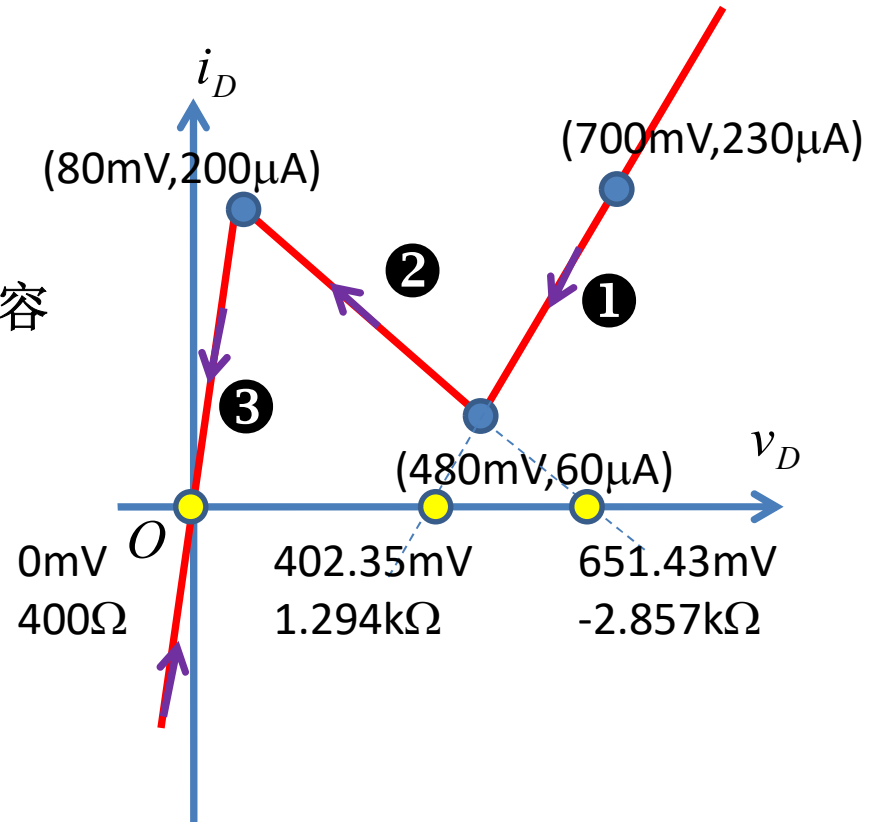
$$6.081ms \geq t \geq 2.641ms$$

$$\Delta t_2 = \tau_2 \ln \frac{V_{S02} - V_{01}}{V_{S02} - V_{02}} = -2.857ms \times \ln \frac{651.43 - 480}{651.43 - 80} = 3.440ms$$



③ 区 指数衰减规律

③ 区：戴维南电压源（0mV，400Ω）对电容充电：从80mV到0mV
正内阻：朝稳定平衡点0mV移动



$$v_{C3}(t) = v_{C\infty3}(t) + (v_{C3}(0) - v_{C\infty3}(0))e^{-\frac{t-t_2}{\tau_3}}$$

$$= 0 + (80 - 0)e^{-\frac{t-6.081ms}{0.4ms}}$$

$$= 80e^{-\frac{t-6.081ms}{0.4ms}} \quad t \geq 6.081ms$$

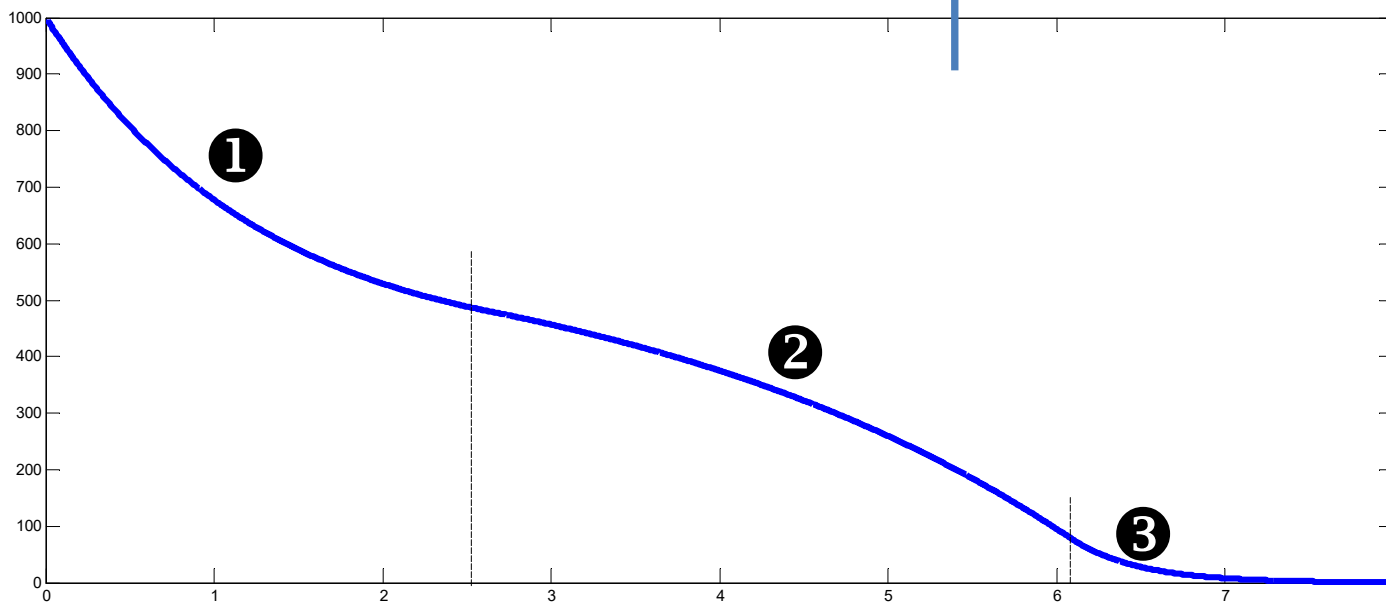
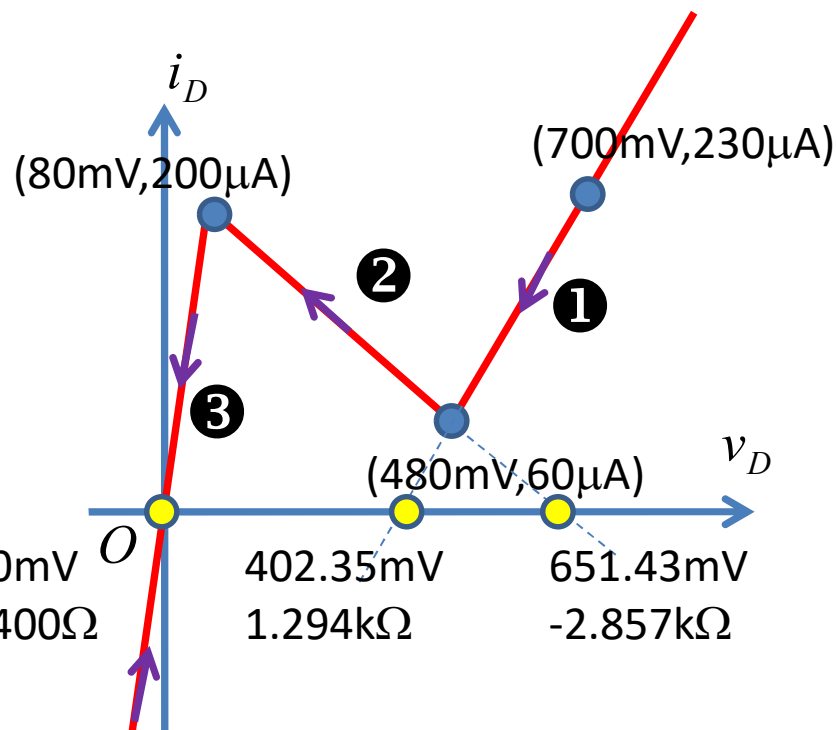
$5\tau_3 = 5 \times 0.4ms = 2ms$ **2ms后可视为进入稳态 (t>8ms)**

波形图

$$v_{C1}(t) = 402.35 + 597.65e^{-\frac{t}{1.294ms}} \quad 2.641ms \geq t \geq 0$$

$$v_{C2}(t) = 651.43 - 171.43e^{-\frac{t-2.641ms}{2.857ms}} \quad 6.081ms \geq t \geq 2.641ms$$

$$v_{C3}(t) = 80e^{-\frac{t-6.081ms}{0.4ms}} \quad t \geq 6.081ms$$



结论

- **S型负阻+电容**

- 直流工作点在负阻区，张弛振荡：无稳

- 只有一个不稳定平衡点

- 直流工作点在正阻区：单稳

- 只有一个稳定平衡点

- **N型负阻+电容**

- 有一个稳定平衡点：非线性电阻的充放电

- 有一个不稳平衡点，两个稳定平衡点：双稳

