电子电路与系统基础II

习题课第七讲 一阶动态电路的时频分析

李国林
清华大学电子工程系

冲激响应和阶跃响应

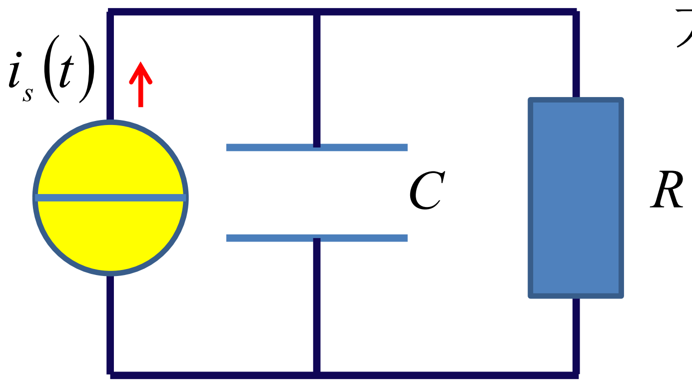
- **01**、证明对于一阶**RC**电路，冲激响应的积分为阶跃响应，阶跃响应的微分为冲激响应。

问题分析：

$$f\left(U(t)\right)=g(t) \qquad f\left(\delta(t)\right)=h(t)$$

阶跃激励　阶跃响应　　　冲激激励　冲激响应

$$U(t)=\int_{-\infty}^{t}\delta(t)\,dt \qquad \delta(t)=\frac{dU(t)}{dt}$$



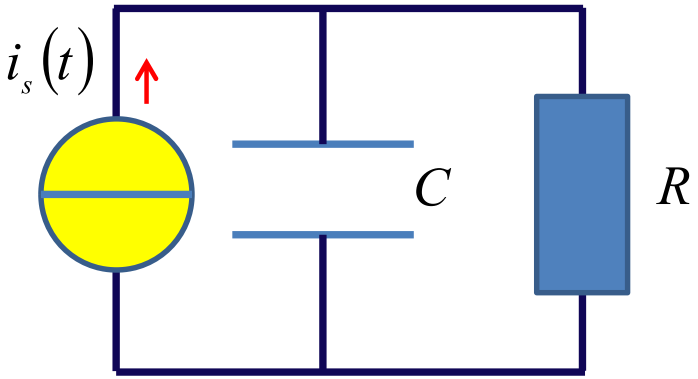
方法1：给出一阶**RC**电路具体的**g(t)**和**h(t)**，确认结论

$$\int_{-\infty}^{t}h(\lambda)d\lambda=g(t) \qquad \frac{d}{dt}g(t)=h(t)$$

方法2：利用**LTI**特性

$$f\left(ax+by\right)=af\left(x\right)+bf\left(y\right)$$

• 方法1：首先求出阶跃响应g(t)和冲激响应h(t)



设定某一电路
求对应该电路的
冲激响应和阶跃响应

阶跃电流 $i_s(t)=I_{S0}U(t)$ 作用

三要素法求解：初值$v_C(0)=0$，终值$v_{C\infty}(t)=I_{S0}R$，…

$$v_C(t)=v_{C\infty}(t)+\left(v_C(0)-v_{C\infty}(0)\right)e^{-\frac{t}{\tau}} \quad (t\geq 0)$$

$$=I_{S0}R\left(1-e^{-\frac{t}{\tau}}\right)U(t)$$

由均匀性可知 $g(t)=R\left(1-e^{-\frac{t}{\tau}}\right)\cdot U(t)$

冲激电流 $i_s(t)=CV_0\delta(t)$ 作用下

三要素法求解：
初值$v_C(0^+)=V_0$，终值$v_{C\infty}(t)=0$，…

$$v_C(t)=v_{C\infty}(t)+\left(v_C(0)-v_{C\infty}(0)\right)e^{-\frac{t}{\tau}} \quad (t\geq 0)$$

$$=V_0e^{-\frac{t}{\tau}}U(t)$$

由均匀性可知 $h(t)=\frac{1}{C}e^{-\frac{t}{\tau}}\cdot U(t)$

- 方法1：然后确认满足关系

$$\int_{-\infty}^{t} h(\lambda)d\lambda = g(t) \qquad \frac{d}{dt}g(t) = h(t)$$

$$g(t) = R\left(1 - e^{-\frac{t}{\tau}}\right)\cdot U(t) \qquad h(t) = \frac{1}{C}e^{-\frac{t}{\tau}}\cdot U(t)$$

（1）

$$\int_{-\infty}^{t} h(\lambda)d\lambda = \int_{-\infty}^{t}\frac{1}{C}e^{-\frac{\lambda}{\tau}}U(\lambda)d\lambda = \frac{1}{C}\int_{0}^{t}e^{-\frac{\lambda}{\tau}}d\lambda = -\frac{\tau}{C}e^{-\frac{\lambda}{\tau}}\Big|_{0}^{t}$$

$$= -\frac{RC}{C}\left(e^{-\frac{t}{\tau}} - 1\right)_{t\geq 0} = R\left(1 - e^{-\frac{t}{\tau}}\right)_{t\geq 0} = R\left(1 - e^{-\frac{t}{\tau}}\right)U(t) = g(t)$$

（2）

$$\frac{d}{dt}g(t) = \frac{d}{dt}\left[R\left(1 - e^{-\frac{t}{\tau}}\right)U(t)\right] = R\left[U(t)\frac{d}{dt}\left(1 - e^{-\frac{t}{\tau}}\right) + \left(1 - e^{-\frac{t}{\tau}}\right)\frac{d}{dt}U(t)\right]$$

$$= R\left[U(t)\frac{1}{\tau}e^{-\frac{t}{\tau}} + \delta(t)\left(1 - e^{-\frac{t}{\tau}}\right)\right] = \frac{R}{\tau}e^{-\frac{t}{\tau}}U(t) = \frac{1}{C}e^{-\frac{t}{\tau}}U(t) = h(t)$$

- 方法2：利用线性电路系统的线性特性

由于一阶RC电路是线性时不变系统，故而满足叠加性和均匀性

$$g(t)=f\left(U(t)\right)=f\left(\int_{-\infty}^{t}\delta(t)\,dt\right)\underset{\int dt\ linear\ operator}{\overset{f(\cdot)\ linear\ operator}{=}}\int_{-\infty}^{t}f\left(\delta(t)\right)dt=\int_{-\infty}^{t}h(t)\,dt$$

条件1：　$U(t)=\int_{-\infty}^{t}\delta(t)\,dt$

条件2：　$f(\cdot)$ 为线性时不变算子**(operator)**，
显然，此为线性时不变电路

算子：函数到函数的映射，如积分、微分
函数：数到数的映射，如各种初等函数

该结论对任意线性时不变电路、系统都成立！

- 方法2：利用线性电路(系统)的线性特性

$$\frac{d}{dt}g(t)=\lim_{\Delta t\to 0}\frac{g(t+\Delta t)-g(t)}{\Delta t}=\lim_{\Delta t\to 0}\frac{f(U(t+\Delta t))-f(U(t))}{\Delta t}$$

$$\overset{\text{叠加性}}{=}\lim_{\Delta t\to 0}\frac{f(U(t+\Delta t)-U(t))}{\Delta t}\overset{\text{均匀性}}{=}\lim_{\Delta t\to 0}f\left(\frac{U(t+\Delta t)-U(t)}{\Delta t}\right)$$

$$=f\left(\lim_{\Delta t\to 0}\frac{U(t+\Delta t)-U(t)}{\Delta t}\right)=f(\delta(t))=h(t)$$

f和微分为线性算子

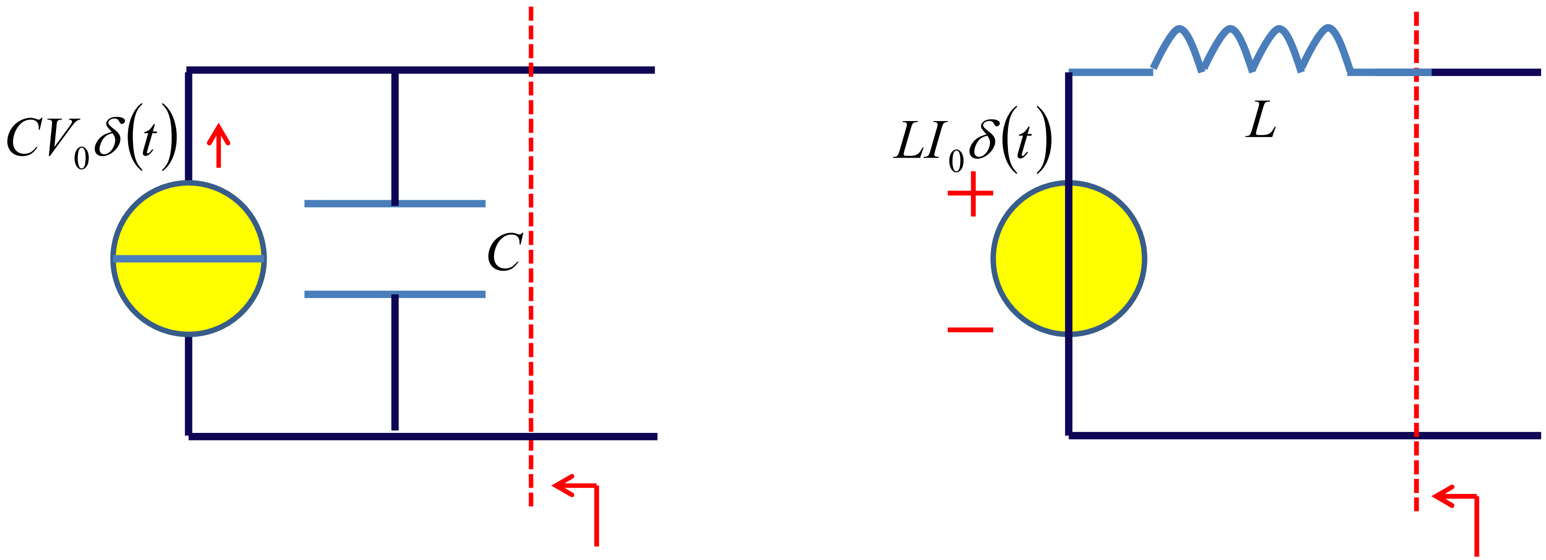
易犯问题

- **g(t)**和**h(t)**的具体表达求解中，激励源不同，导致物理量纲不对
  - 如求**g(t)**时，激励为阶跃电压源；
  - 求**h(t)**时，激励为冲激电流源；
  - 两者不符
  - 激励源，无论是阶跃，还是冲激，其位置性质不能改变，要么都是电压源，要么都是电流源；否则就乱了：两个说的不是一回事
    - 跨阻传递的冲激响应和阶跃响应直接对应
    - 电压传递的冲激响应和阶跃响应直接对应
    - 电流传递…
    - 跨导传递…

- **02、具有初始状态的电容和电感的源等效**
  - 请用诺顿源形式重新表述电容初始状态，用戴维南源形式重新表述电感初始状态

核心概念：等效源，戴维南等效、诺顿等效

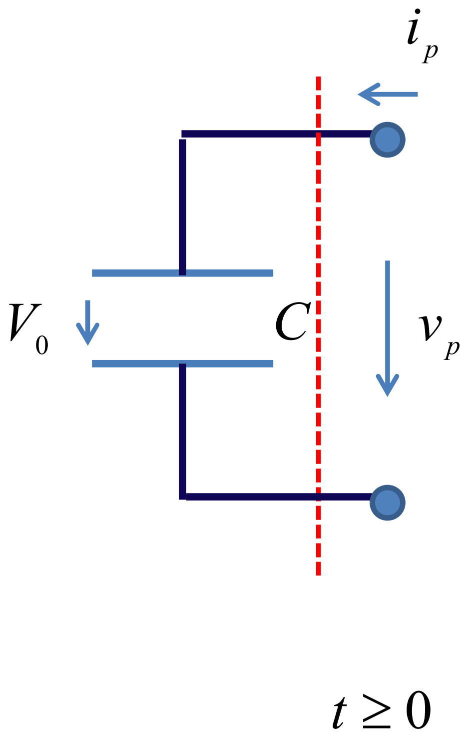
从端口的电压、电流关系进行等效
戴维南：电压源，诺顿：电流源

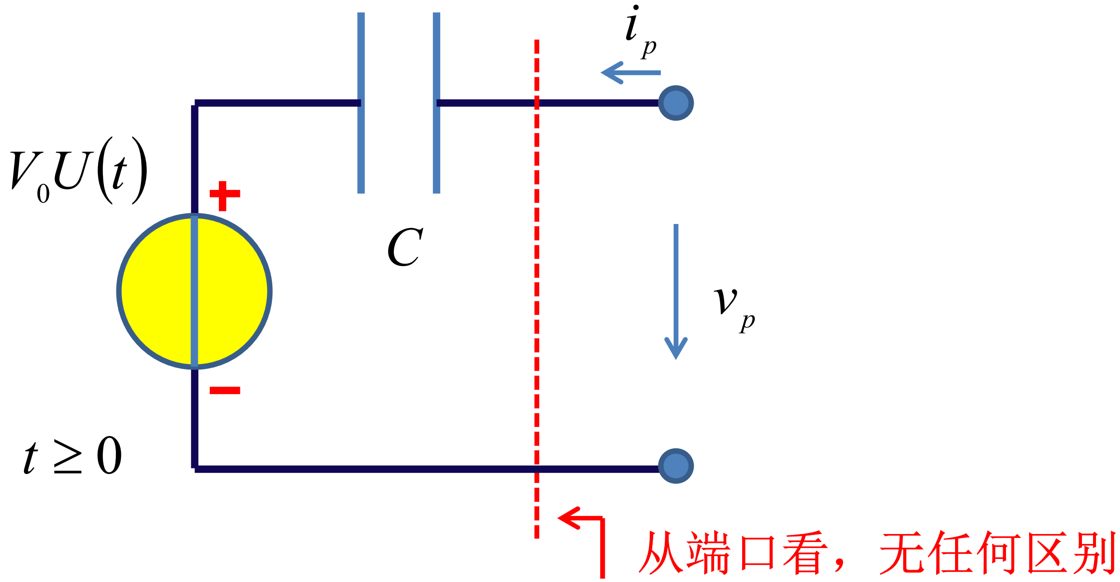


具有初始电压的电容
戴维南等效

直接由戴维南定理分析获得

源电压：开路电压V_0 $t \geq 0$
源内阻：内部独立源不起作用（内部无源）时的阻抗
无初始电压的电容

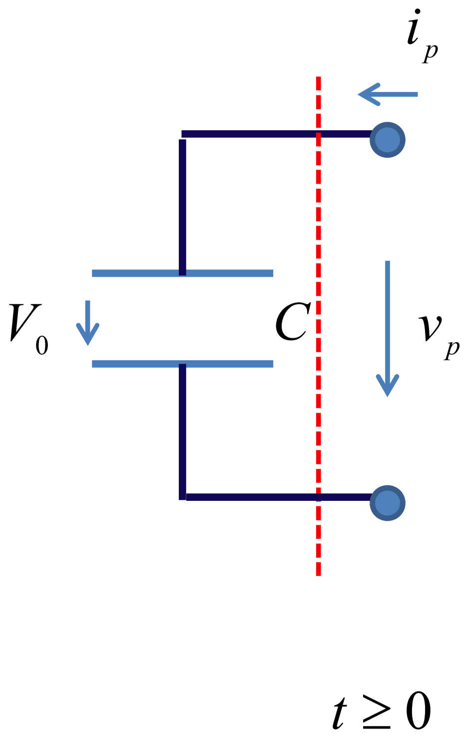


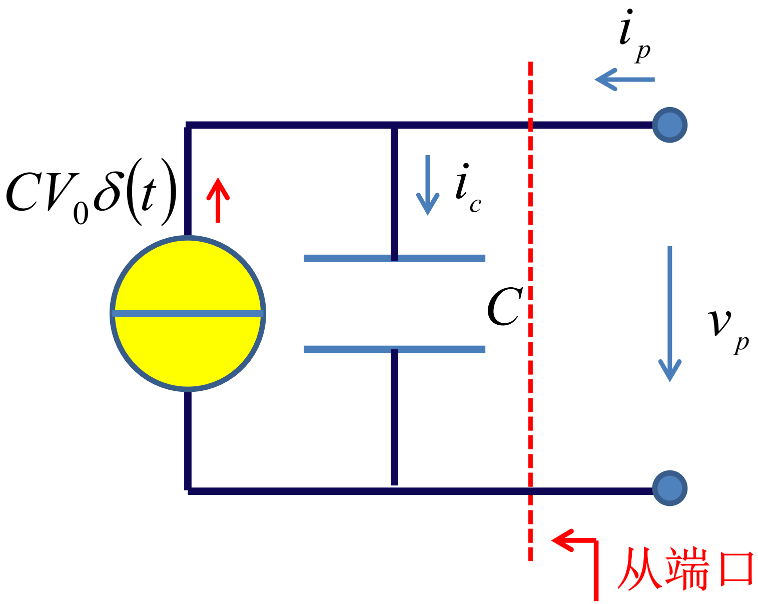


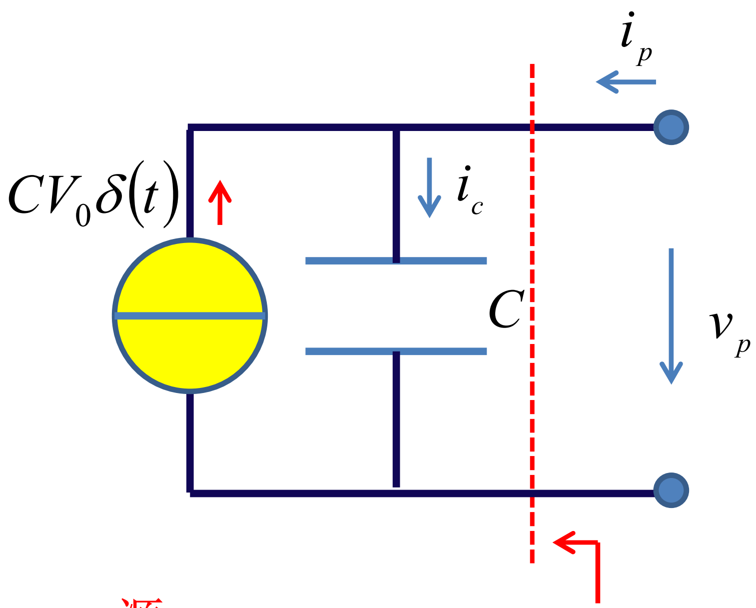
具有初始电压的电容 诺顿等效

直接由诺顿定理分析获得

源电流：短路电流-$CV_0\delta(t)$ $t \geq 0$
源内阻：内部独立源不起作用（内部无源）时的阻抗
无初始电压的电容



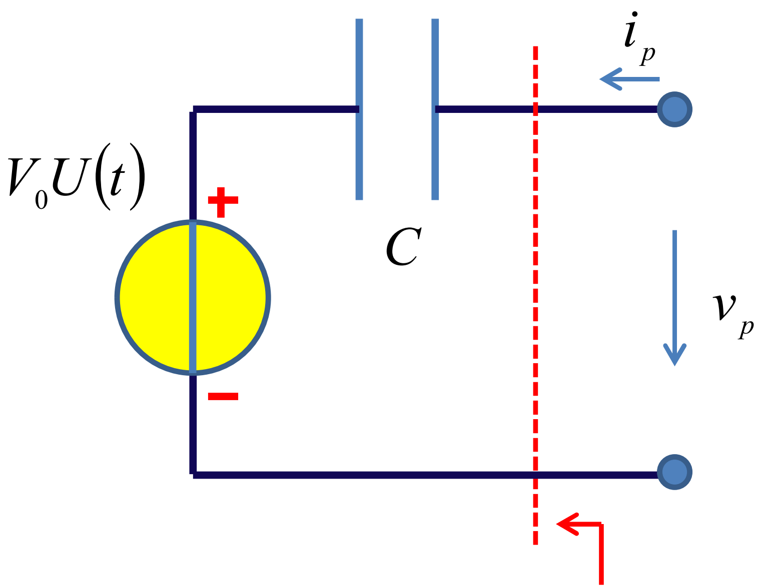




网络端口约束条件

$$i_p(t) = C\frac{dv_p(t)}{dt} - CV_0\delta(t)$$

$t \ge 0$ 假设电容无初始电压，初始电压由冲激电流源提供



网络端口约束条件

$$v_p(t) = V_0U(t) + \frac{1}{C}\int_0^t i_p(t)dt$$

$t \ge 0$ 假设电容无初始电压，初始电压由阶跃电压源提供

$$i_p(t)=C\frac{dv_p(t)}{dt}-CV_0\delta(t)$$

等价方程

$$v_p(t)=V_0U(t)+\frac{1}{C}\int_0^t i_p(t)dt$$

$$\int_0^t i_p(t)dt=C\int_0^t dv_p(t)-CV_0\int_0^t \delta(t)dt$$

$$\int_{0^-}^t i_p(t)dt=C\left(v_p(t)-v_p\left(0^-\right)\right)-CV_0U(t)$$

$$=Cv_p(t)-CV_0U(t)$$

$$\int_{0^+}^t i_p(t)dt=C\left(v_p(t)-v_p\left(0^+\right)\right)-0$$

$$=Cv_p(t)-CV_0 \qquad t\geq 0$$

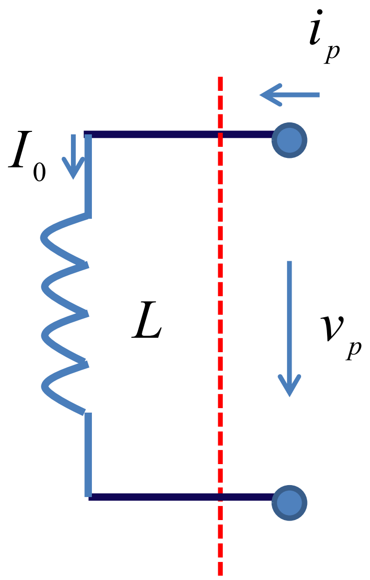
$$v_p(t)=V_0U(t)+\frac{1}{C}\int_0^t i_p(t)dt$$

$$\frac{d}{dt}v_p(t)=V_0\frac{d}{dt}U(t)+\frac{1}{C}\frac{d}{dt}\int_0^t i_p(t)dt$$

$$\frac{d}{dt}v_p(t)=V_0\delta(t)+\frac{1}{C}i_p(t)$$

$$i_p(t)=C\frac{dv_p(t)}{dt}-CV_0\delta(t)$$

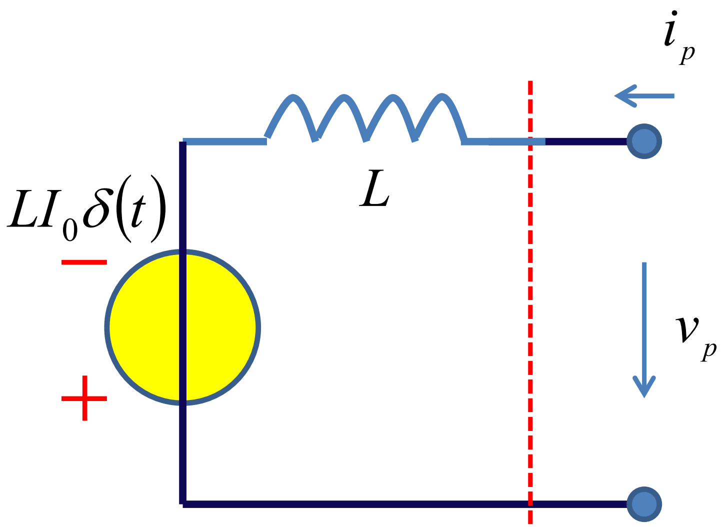
具有初始电流的电感等效电路



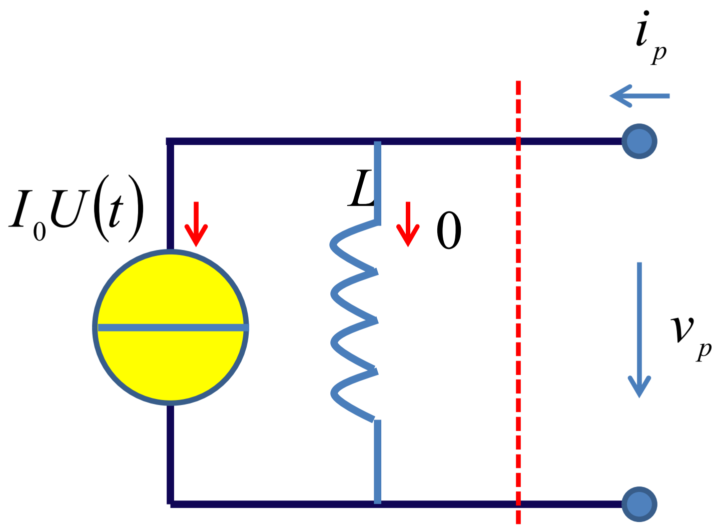
源：初始电流为I_0
阻：电感L

$$i_p(t) = I_0 + \frac{1}{L}\int_0^t v_p(t)dt$$

$t \geq 0$

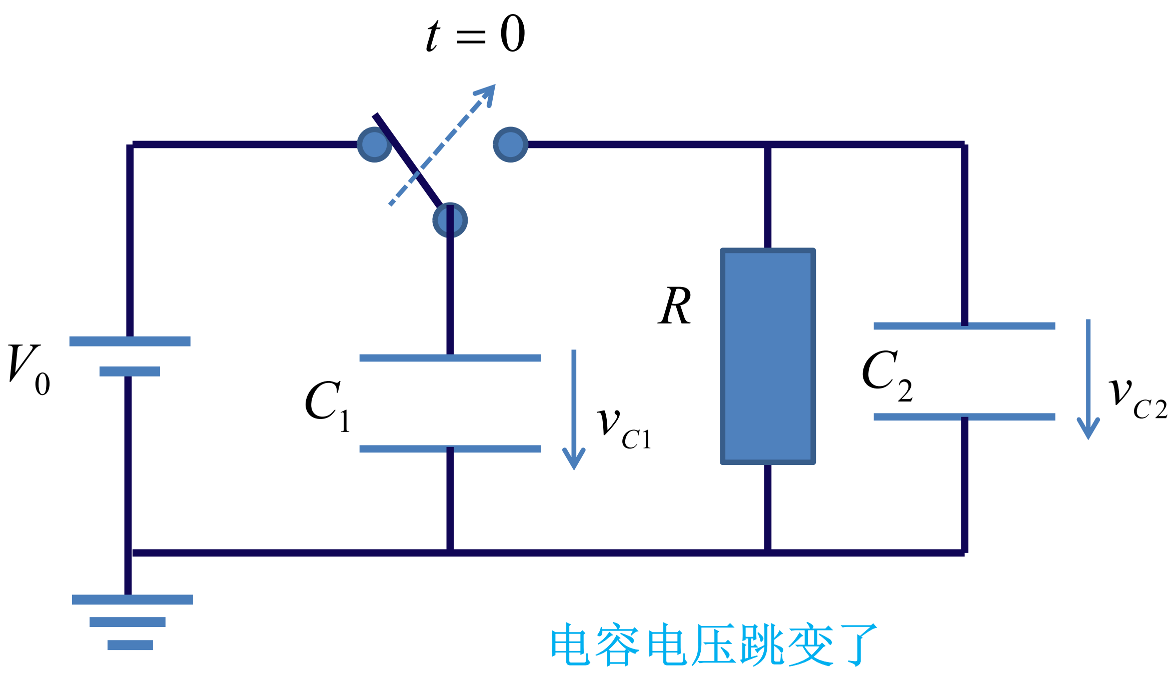


源：初始电流为I_0
抗：电感L

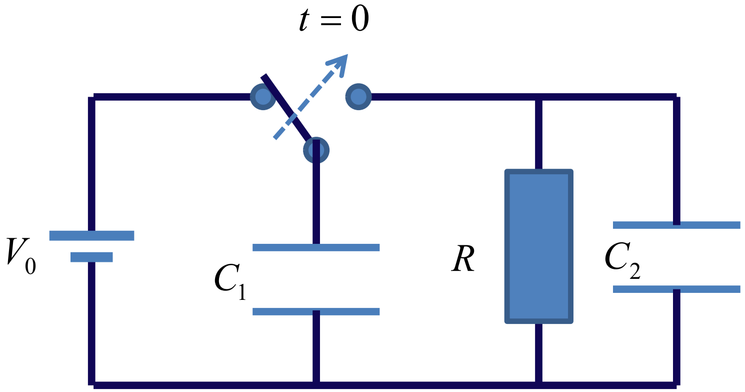


源：初始电流为I_0
抗：电感L

作业03 电容电压跳变了！



- 在t=0时刻，将开关拨向右侧电路，求电容C_1、C_2两端电压变化规律，写出表达式，画出时域波形

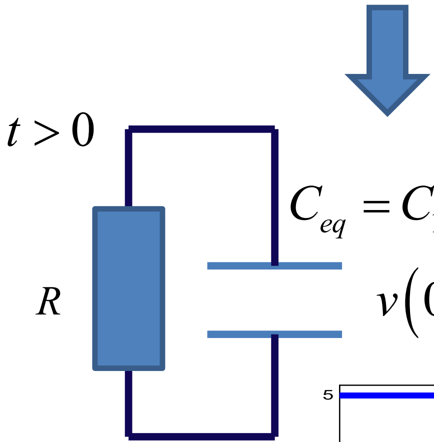


$$V_{C1}\left(0^-\right)=V_0 \qquad V_{C2}\left(0^-\right)=0$$

$$C_1V_{C1}(0^-)+C_2V_{C2}(0^-)=C_1V_{C1}(0^+)+C_2V_{C2}(0^+)$$

电荷守恒

$$V_{C1}\left(0^+\right)=V_{C2}\left(0^+\right)=\frac{C_1V_0}{C_1+C_2}$$



$$C_{eq}=C_1+C_2$$

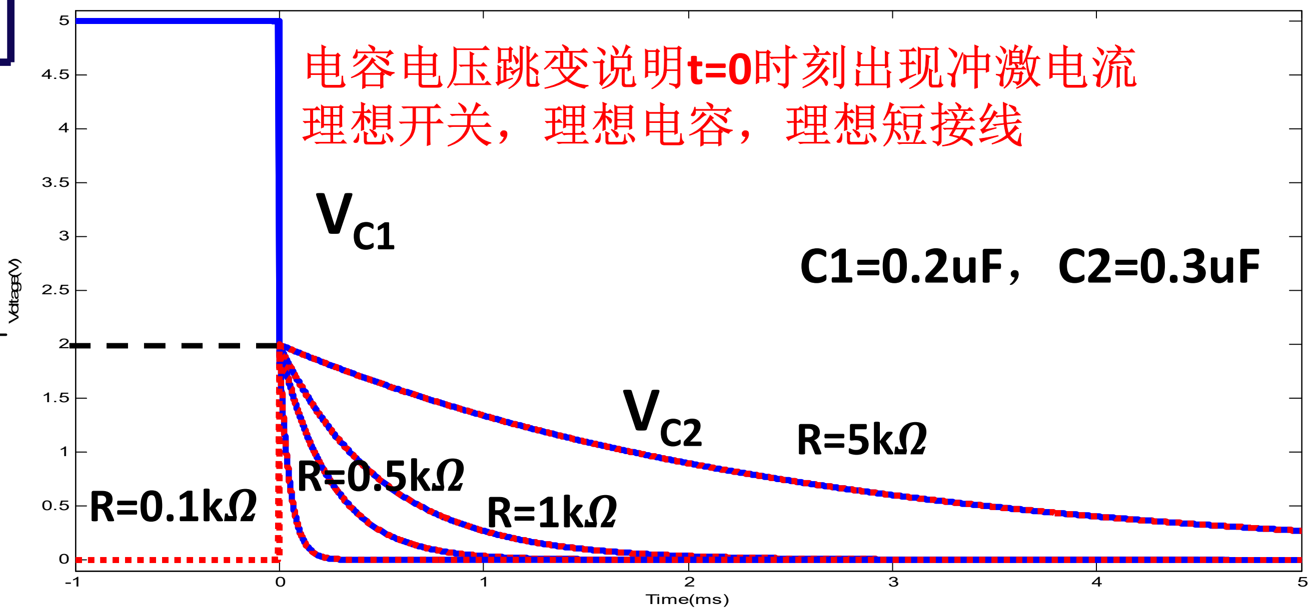
$$v\left(0^+\right)=\frac{C_1V_0}{C_1+C_2}$$

$$V_{C1}\left(t\right)=V_{C2}\left(t\right)=\frac{C_1V_0}{C_1+C_2}e^{-\frac{t}{\tau}},t>0$$

指数率放电

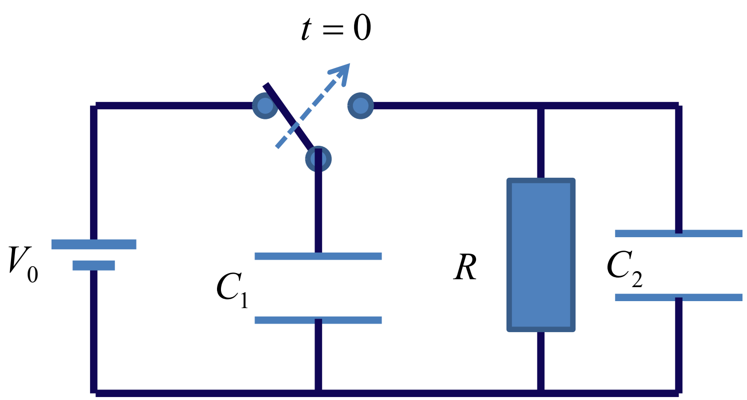
$$\tau=R\left(C_1+C_2\right)$$

$$v\left(0^+\right)=\frac{C_1V_0}{C_1+C_2}$$

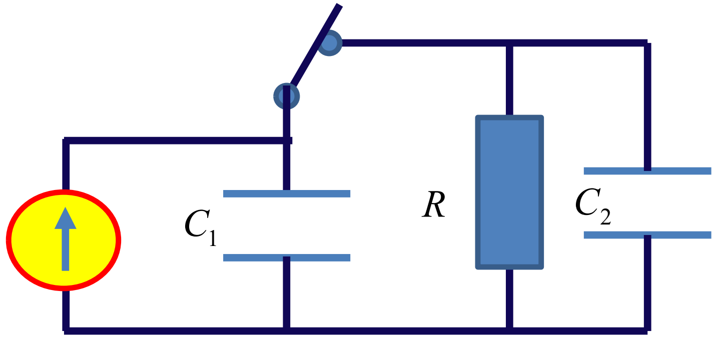


电容两端电压波形

等效源方法求初值



$$V_{C1}(0^-) = V_0 \qquad V_{C2}(0^-) = 0$$



$C_1V_0\delta(t)$

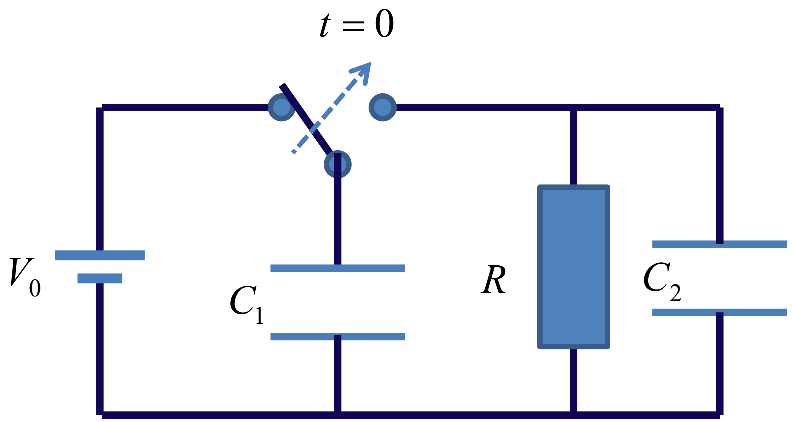
$$V_{C1}(0^+) = V_{C2}(0^+) = \frac{1}{C_1 + C_2}\int_{0^-}^{0^+} C_1V_0\delta(t)dt$$

$$= \frac{C_1V_0}{C_1 + C_2}\int_{0^-}^{0^+} \delta(t)dt = \frac{C_1V_0}{C_1 + C_2}$$

用等效源方法可以方便处理多电容/多电感串并联初值分析

（包括下节习题课讨论的电容型DAC等电容网络分析）

问题：能量丢失了？



$$E_{C1}\left(0^-\right)=\frac{1}{2}C_1V_0^2 \qquad E_{C2}\left(0^-\right)=0$$

$$E_{C1}\left(0^+\right)=\frac{1}{2}C_1\left(\frac{C_1V_0}{C_1+C_2}\right)^2$$

$$E_{C2}\left(0^+\right)=\frac{1}{2}C_2\left(\frac{C_1V_0}{C_1+C_2}\right)^2$$

$$E_C\left(0^+\right)=E_{C1}\left(0^+\right)+E_{C2}\left(0^+\right)$$

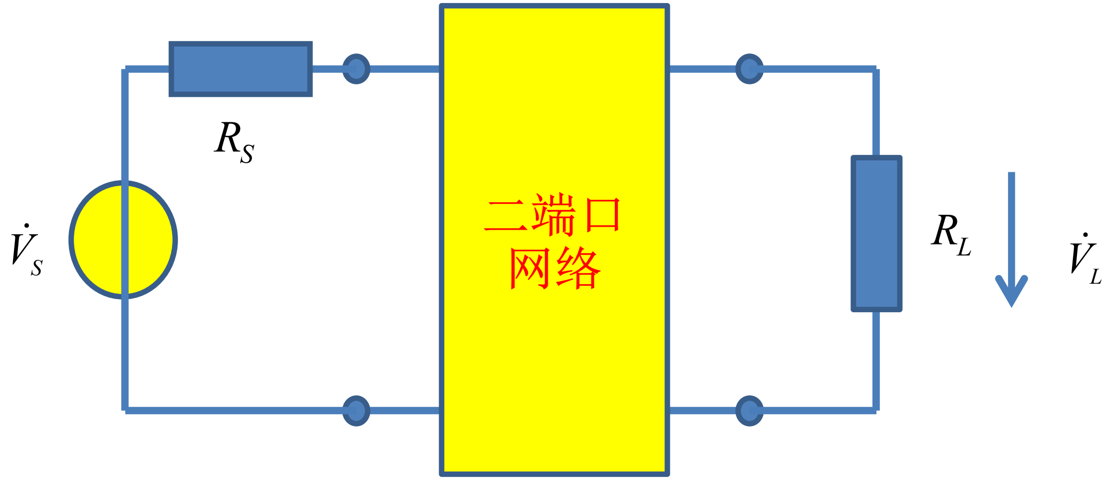
$$=\frac{1}{2}C_1V_0^2\frac{C_1}{C_1+C_2}=\frac{C_1}{C_1+C_2}E_C\left(0^-\right)<E_C\left(0^-\right)$$

能量丢失了？丢失了多少？
能量到了哪里？

作业04　一阶滤波器设计

- 设计一个**RC**低通滤波器，使得其**3dB**带宽为**10MHz**，已知信源内阻为**50Ω**，负载电阻为**50Ω**
  - 画出其幅频特性和相频特性（画伯特图）
  - 请再设计一个高通滤波器，**3dB**频点也在**10MHz**，画出伯特图。
  - 思考：如果用**RL**滤波器，滤波器形态怎样？参数如何设定？

滤波器是线性二端口动态网络



$$\left|H(j\omega)\right|^2 = \frac{\left|\dot{V}_L\right|^2 / R_L}{\left|\dot{V}_S\right|^2 / 4R_S} = \frac{P_L}{P_{S\max}} = G_p(\omega)$$

传递函数定义：

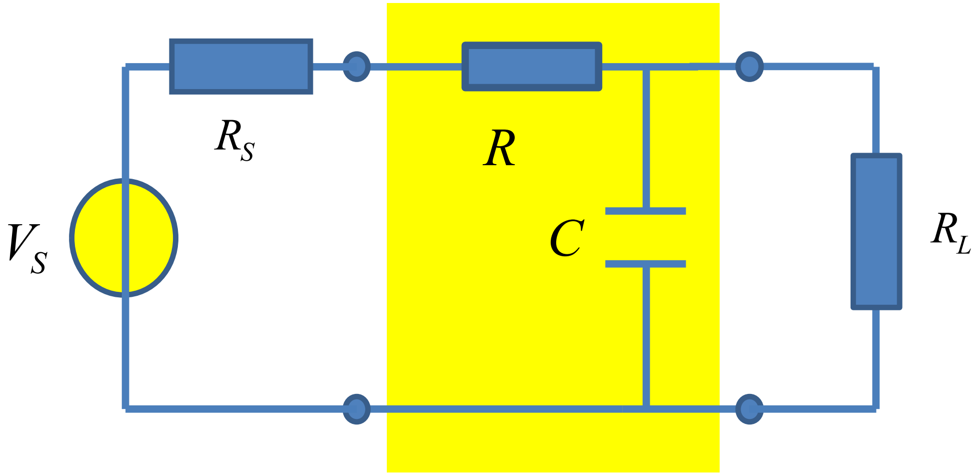
$$H(j\omega) = \frac{\dot{V}_L}{\dot{V}_S}$$

低频应用下的放大、滤波，或信源内阻为零，或负载电阻为无穷（输出开路）情况下，以电压传输为研究对象，做如是定义

$$H(j\omega) = 2\sqrt{\frac{R_S}{R_L}}\frac{\dot{V}_L}{\dot{V}_S}$$

射频应用下的放大、滤波，同时存在信源内阻和负载电阻，以功率传输为考察对象

一般RC低通设计思路



$$H(j\omega)=2\sqrt{\frac{R_S}{R_L}}\frac{\dot{V}_L}{\dot{V}_S}$$

$$\frac{\dot{V}_L}{\dot{V}_S}=\frac{\dfrac{R_L}{1+j\omega CR_L}}{R_S+R+\dfrac{R_L}{1+j\omega CR_L}}=\frac{R_L}{(R_S+R+R_L)+j\omega CR_L(R_S+R)}$$

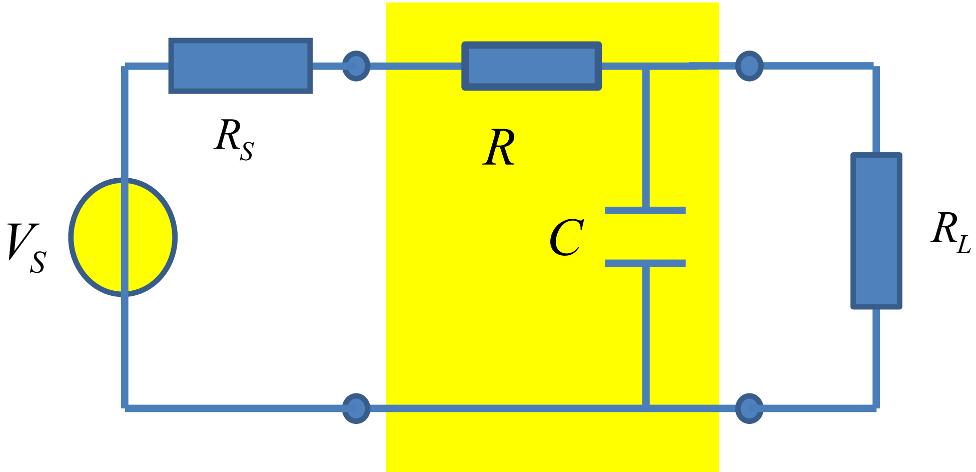
$$=\frac{R_L}{R_S+R+R_L}\frac{1}{1+j\omega C\dfrac{R_L(R_S+R)}{R_S+R+R_L}}=A_0\frac{1}{1+j\omega\tau}$$

$$A_0=\frac{R_L}{R_S+R+R_L}$$

$$\tau=C\frac{R_L(R_S+R)}{R_S+R+R_L}$$

$$H(j\omega)=\frac{2\sqrt{R_SR_L}}{R_S+R+R_L}\frac{1}{1+j\omega C\dfrac{R_L(R_S+R)}{R_S+R+R_L}}=H_0\frac{1}{1+j\omega\tau}\overset{\substack{s=j\omega\\ \omega_0=\frac{1}{\tau}}}{=}H_0\frac{\omega_0}{s+\omega_0}$$

一阶低通的标准形态



$$C=\frac{1}{2\pi BW_{3dB}\left(R_L \parallel R_S\right)}=\frac{1}{2\times 3.14\times 10M\times 25}=637\,pF$$

$$H(j\omega)=\frac{2\sqrt{R_S R_L}}{R_S+R+R_L}\frac{1}{1+j\omega C\dfrac{R_L\left(R_S+R\right)}{R_S+R+R_L}}=H_0\frac{1}{1+j\omega\tau}\overset{\substack{s=j\omega\\ \omega_0=\frac{1}{\tau}}}{=}H_0\frac{\omega_0}{s+\omega_0}$$

$$BW_{3dB}=10MHz=\frac{1}{2\pi\tau}=\frac{1}{2\pi C\left(R_L \parallel \left(R_S+R\right)\right)}$$

C、R两个自由度

$$H_0=\frac{2\sqrt{R_S R_L}}{R_S+R+R_L}=H(j0)\le 1$$

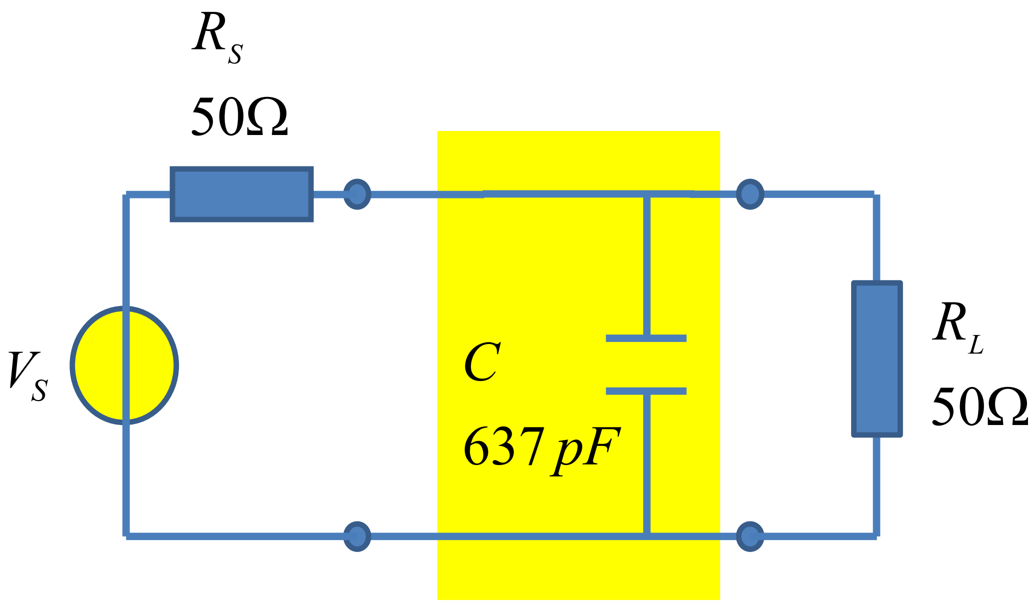
$$R=0: A_0=1$$

从最大功率传输角度，令**R=0**，无损滤波器

最终设计方案

$$H(j\omega)=2\sqrt{\frac{R_S}{R_L}}\frac{\dot{V}_L}{\dot{V}_S}=\frac{\omega_0}{s+\omega_0}=\frac{1}{1+\frac{j\omega}{\omega_0}}$$

1、有信源内阻和负载电阻的无源滤波器设计，无源滤波器二端口网络应该是无损网络，如果信源内阻和负载电阻不等，可能还需阻抗变换网络



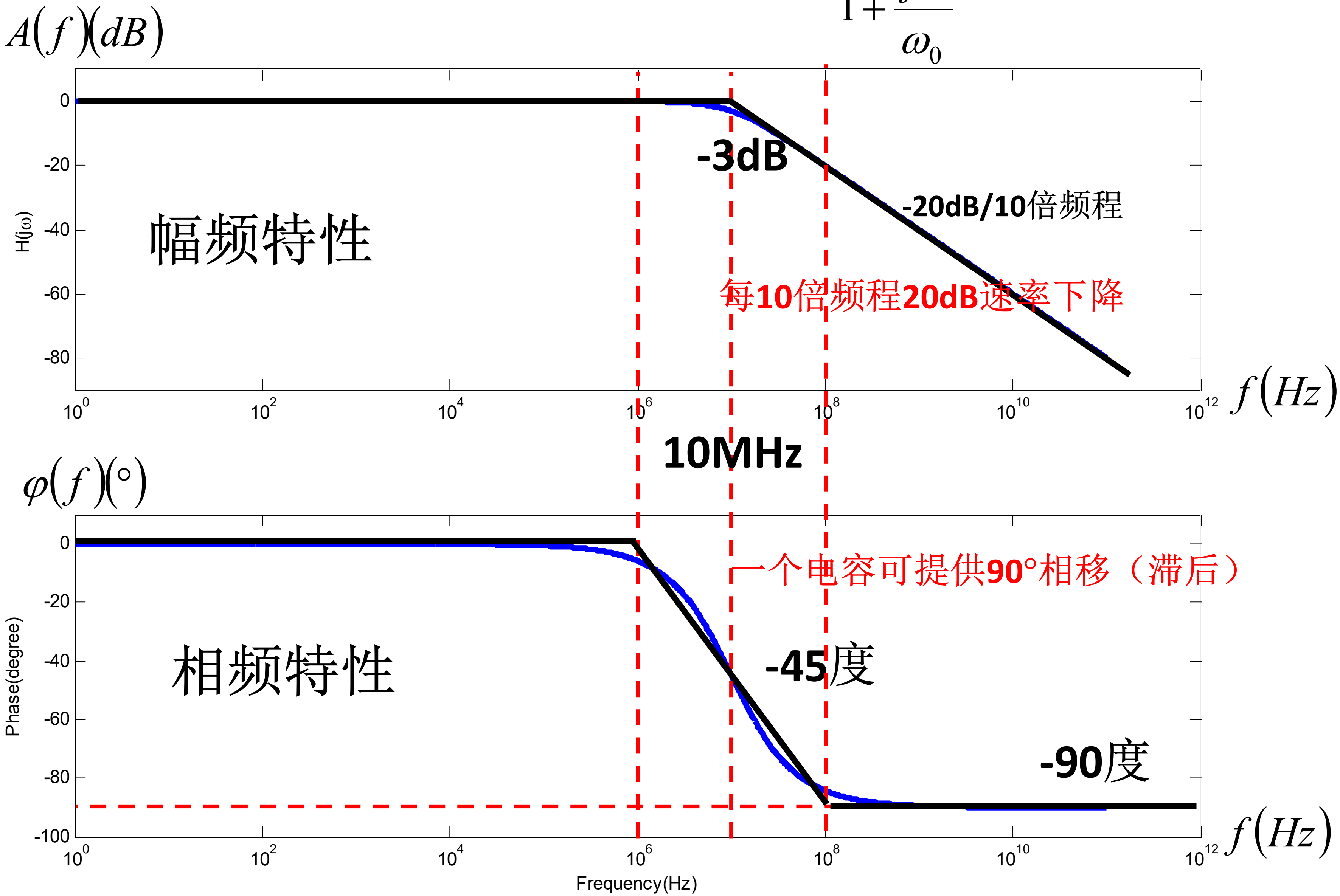
2、所设计的滤波器针对特定信源内阻和负载电阻，否则**3dB**频点会发生偏离

$$\tau=(R_L \parallel R_S)C=25\times 637p=15.9ns$$

$$BW_{3dB}=\frac{1}{2\pi\tau}=10MHz$$

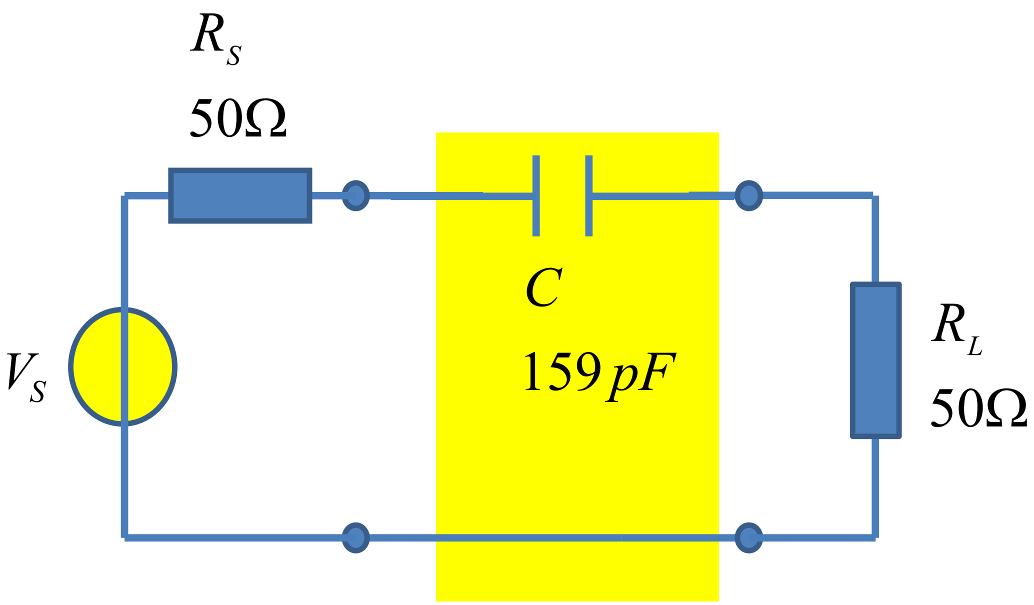
一阶低通滤波器

$$H(j\omega)=\frac{1}{1+\dfrac{j\omega}{\omega_0}}=A(\omega)e^{j\varphi(\omega)}$$



高通方案

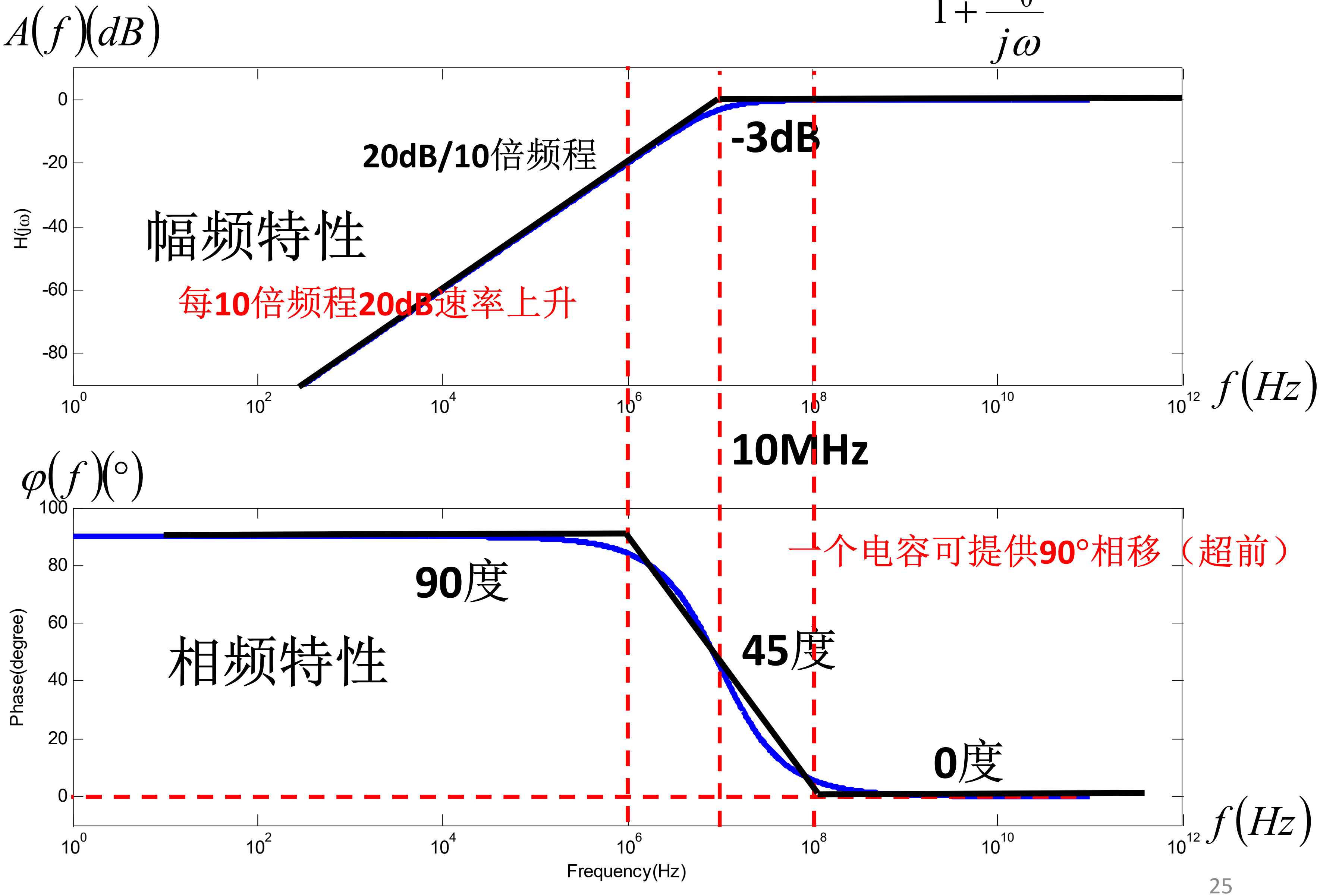
$$H(j\omega)=2\sqrt{\frac{R_S}{R_L}}\frac{\dot{V}_L}{\dot{V}_S}=\frac{s}{s+\omega_0}$$



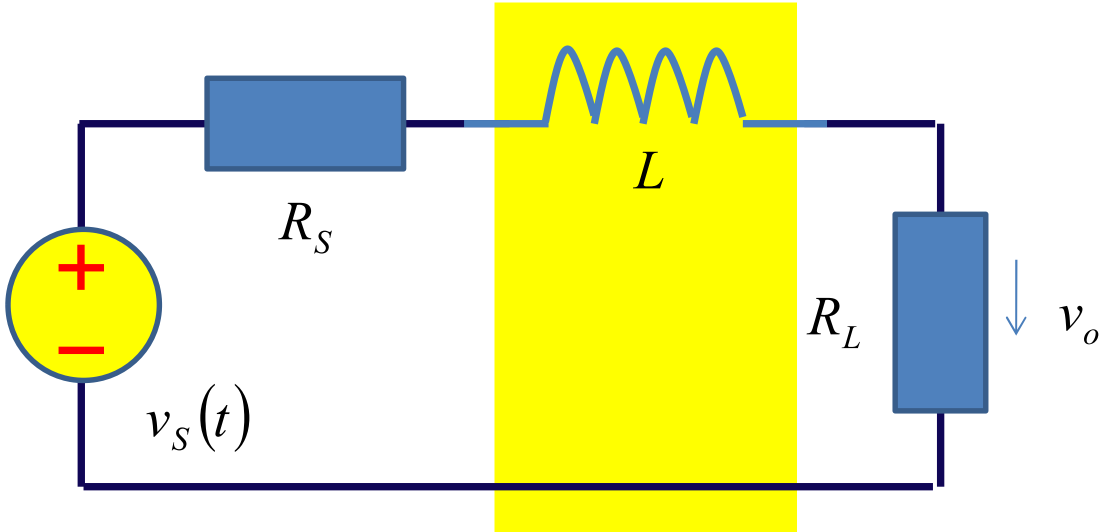
$$C=\frac{\tau}{R_L+R_S}=\frac{1}{2\pi\cdot f_{3dB}\cdot(R_L+R_S)}=\frac{1}{2\times3.14\times10M\times(50+50)}=159\,pF$$

一阶高通滤波器

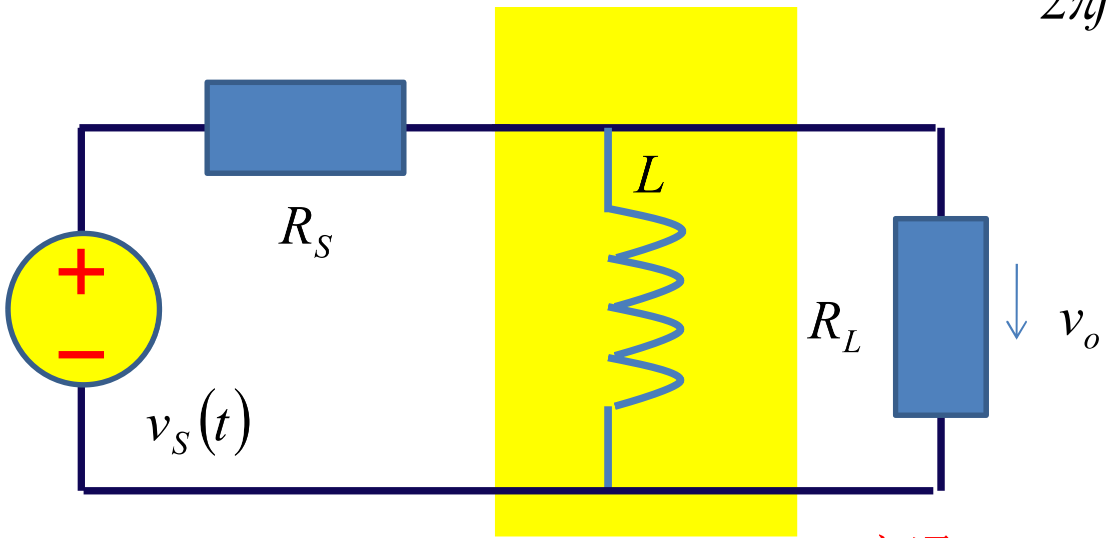
$$H(j\omega)=\frac{1}{1+\frac{\omega_0}{j\omega}}$$



一阶RL滤波器



低通



高通

$$L = \frac{\tau}{G} = \frac{R_S + R_L}{2\pi \cdot BW_{3dB}}$$

$$2\pi f_{3dB} = \omega_0 = \frac{1}{\tau} = \begin{cases} \dfrac{1}{RC} \\ \dfrac{1}{GL} \end{cases}$$

$$L = \frac{\tau}{G} = \frac{R_S \parallel R_L}{2\pi \cdot f_{3dB}}$$

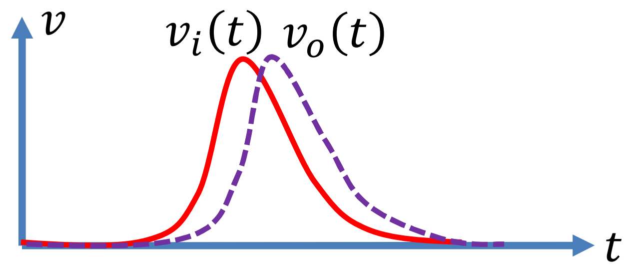
思考题

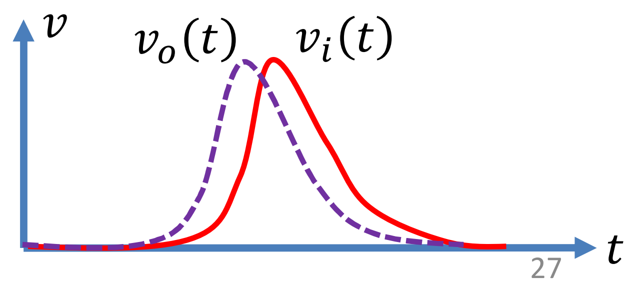
$$\varphi_{一阶低通}(\omega) = -arctan\omega\tau$$

$$\tau_g = -\frac{d\varphi(\omega)}{d\omega} \overset{\omega=0}{\triangleq} \tau$$

- 典型一阶/二阶低通/高通/全通网络的相频特性都是负斜率的，其群延时为正值，说明输出脉冲信号落后于输入脉冲信号

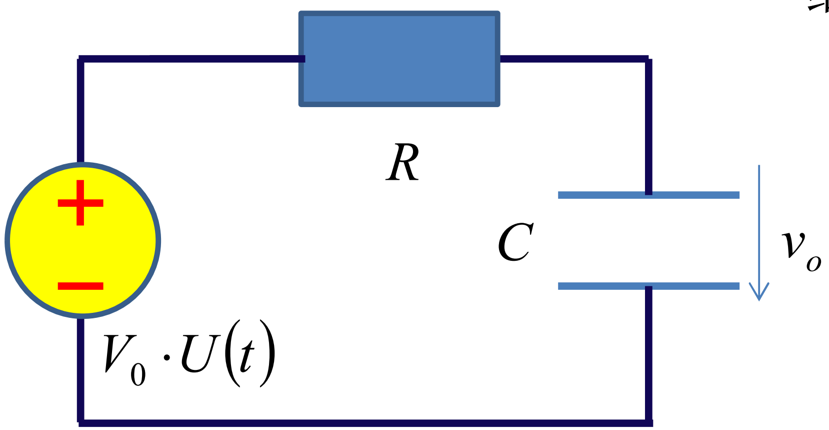
- 思考：是否可以设计出一个负群延时网络，使得输出脉冲超前于输入脉冲？这个电路网络是否违背因果律？

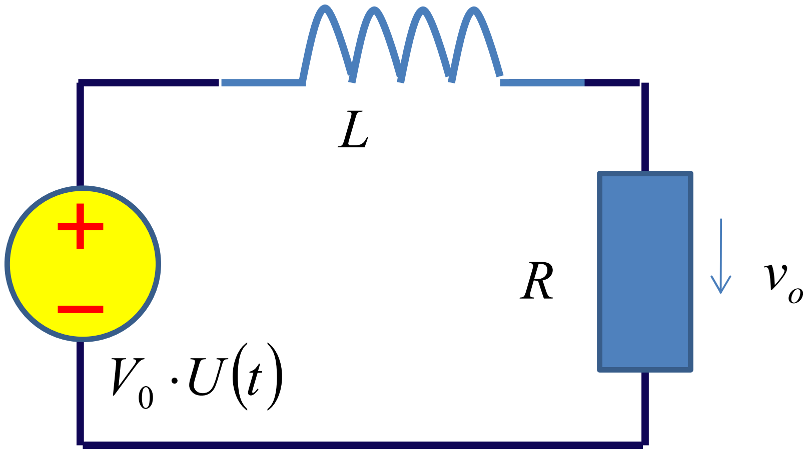


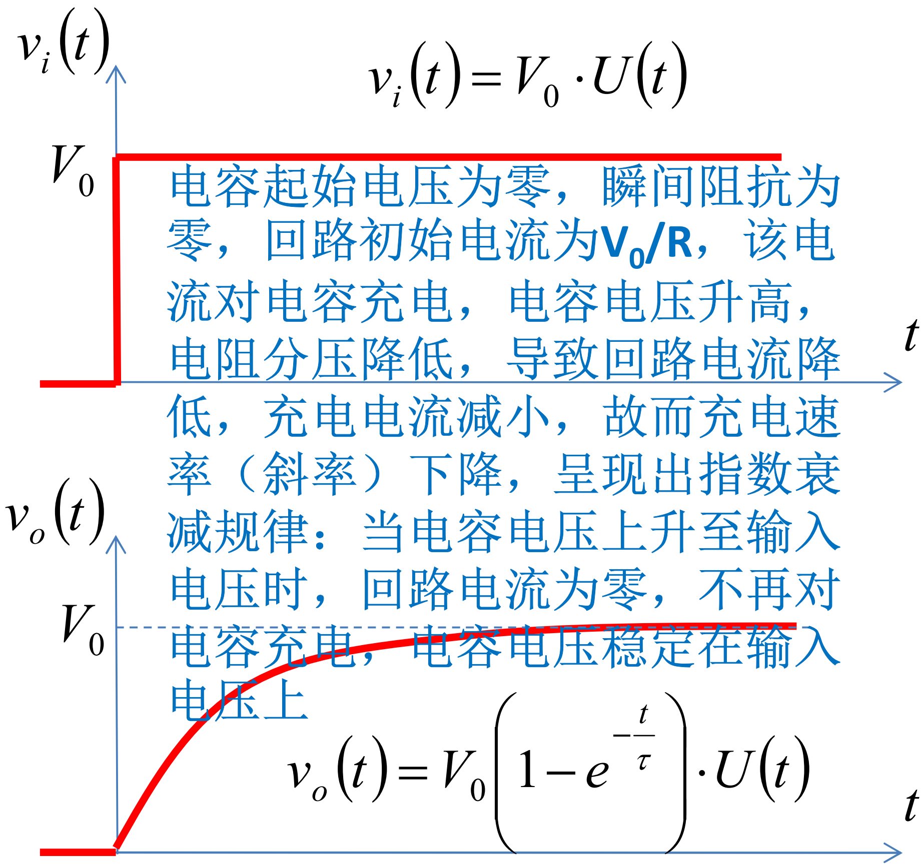


作业05

- 仿照对一阶**RC**低通阶跃响应曲线的理解和描述，给出关于一阶**RL**低通的阶跃响应曲线的理解与描述
- 同理，画出一阶**RL**高通电路，对照一阶**RC**高通网络，给出对一阶**RL**高通的阶跃响应曲线的理解与描述

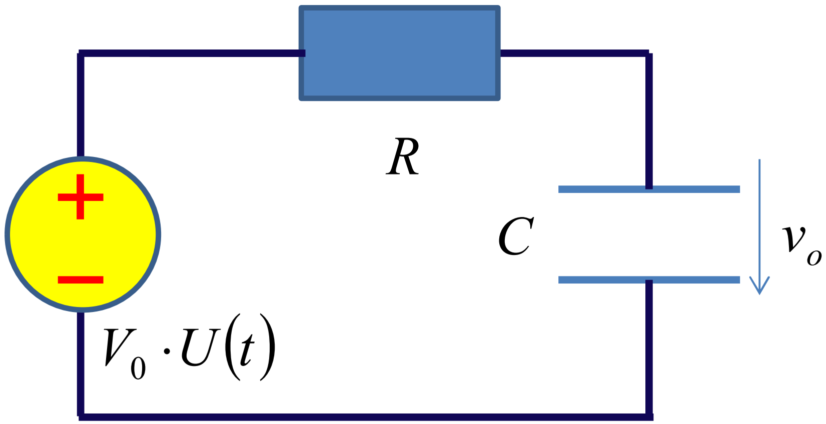






电容起始电压为零，瞬间阻抗为零，回路初始电流为V_0/R，该电流对电容充电，电容电压升高，电阻分压降低，导致回路电流降低，充电电流减小，故而充电速率（斜率）下降，呈现出指数衰减规律：当电容电压上升至输入电压时，回路电流为零，不再对电容充电，电容电压稳定在输入电压上

一阶RC低通



$$v_c(0)=0 \qquad v_{c\infty}(t)=V_0$$

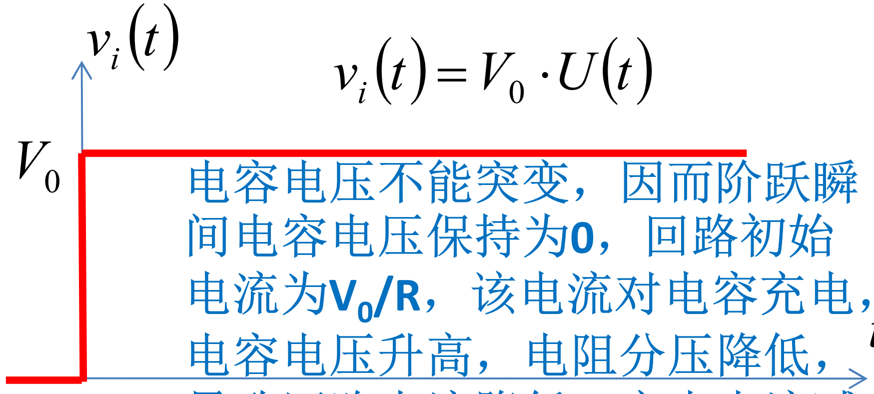
$$\tau = RC$$

$$v_c(t)=v_{c\infty}(t)+\left(v_c(0)-v_{c\infty}(0)\right)e^{-\frac{t}{\tau}}\ (t\geq 0)$$

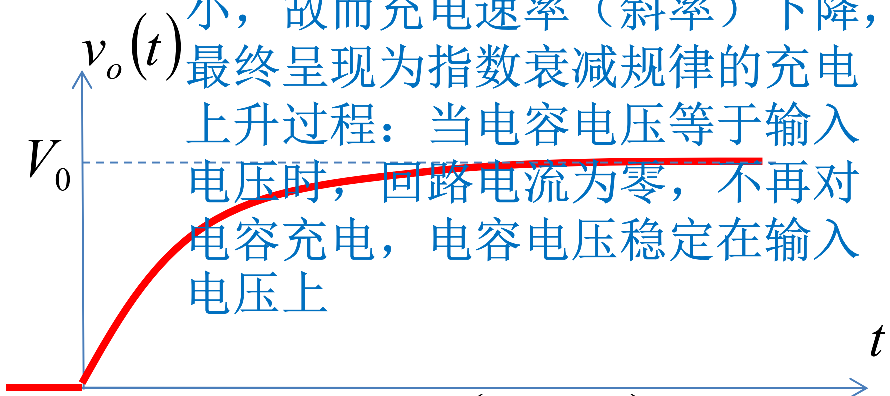
$$=V_0-V_0e^{-\frac{t}{\tau}}\ (t\geq 0)$$

$$=V_0\left(1-e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

$$v_i(t)=V_0\cdot U(t)$$

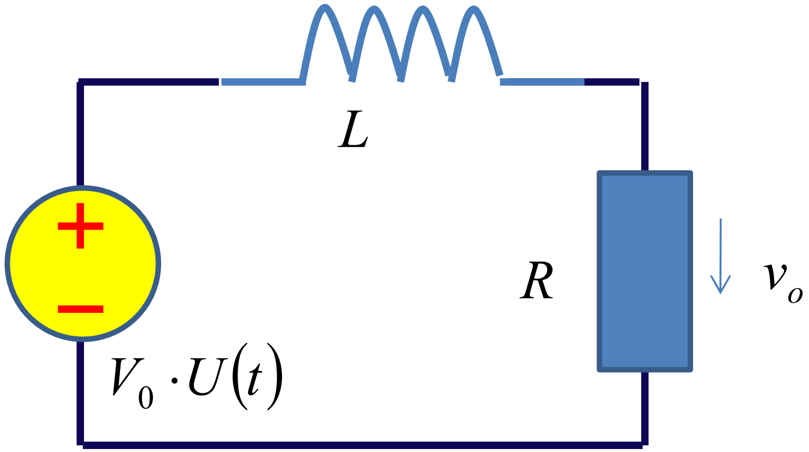


电容电压不能突变，因而阶跃瞬间电容电压保持为**0**，回路初始电流为**V$_0$/R**，该电流对电容充电，电容电压升高，电阻分压降低，导致回路电流降低，充电电流减小，故而充电速率（斜率）下降，最终呈现为指数衰减规律的充电上升过程：当电容电压等于输入电压时，回路电流为零，不再对电容充电，电容电压稳定在输入电压上



$$v_o(t)=V_0\left(1-e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

一阶RL低通



$$v_R\left(0^+\right)=0 \qquad v_{R\infty}(t)=V_0$$

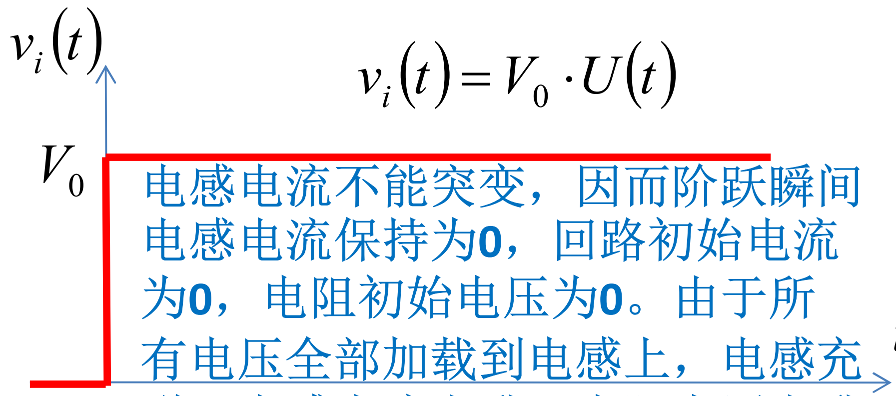
$$\tau = GL = \frac{L}{R}$$

$$v_R(t)=v_{R\infty}(t)+\left(v_R\left(0^+\right)-v_{R\infty}\left(0^+\right)\right)e^{-\frac{t}{\tau}}{}_{(t\geq 0)}$$

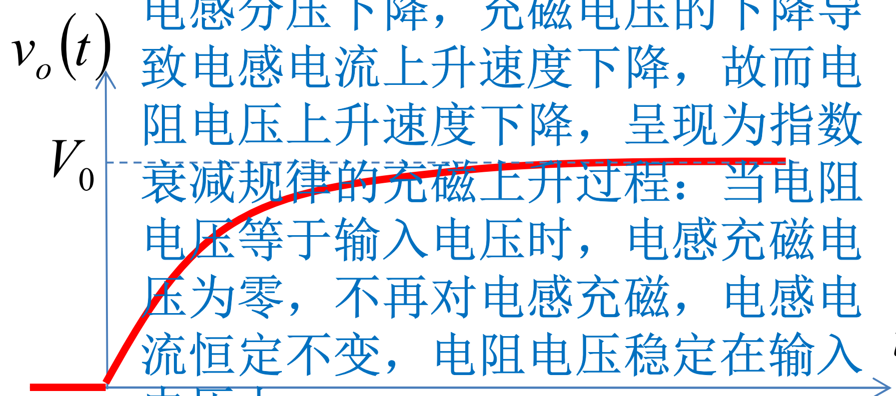
$$=V_0-V_0e^{-\frac{t}{\tau}}{}_{(t\geq 0)}$$

$$=V_0\left(1-e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

$$v_i(t)=V_0\cdot U(t)$$

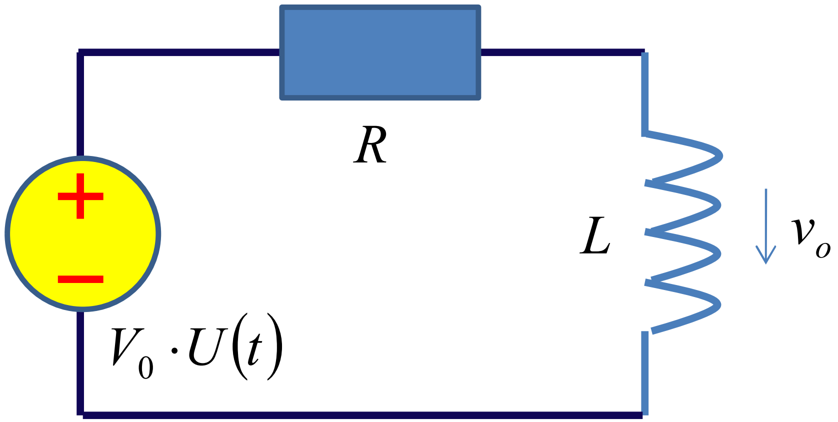


电感电流不能突变，因而阶跃瞬间电感电流保持为0，回路初始电流为0，电阻初始电压为0。由于所有电压全部加载到电感上，电感充磁，电感电流上升，电阻电压上升，电感分压下降，充磁电压的下降导致电感电流上升速度下降，故而电阻电压上升速度下降，呈现为指数衰减规律的充磁上升过程：当电阻电压等于输入电压时，电感充磁电压为零，不再对电感充磁，电感电流恒定不变，电阻电压稳定在输入电压上



$$v_o(t)=V_0\left(1-e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

一阶RL高通

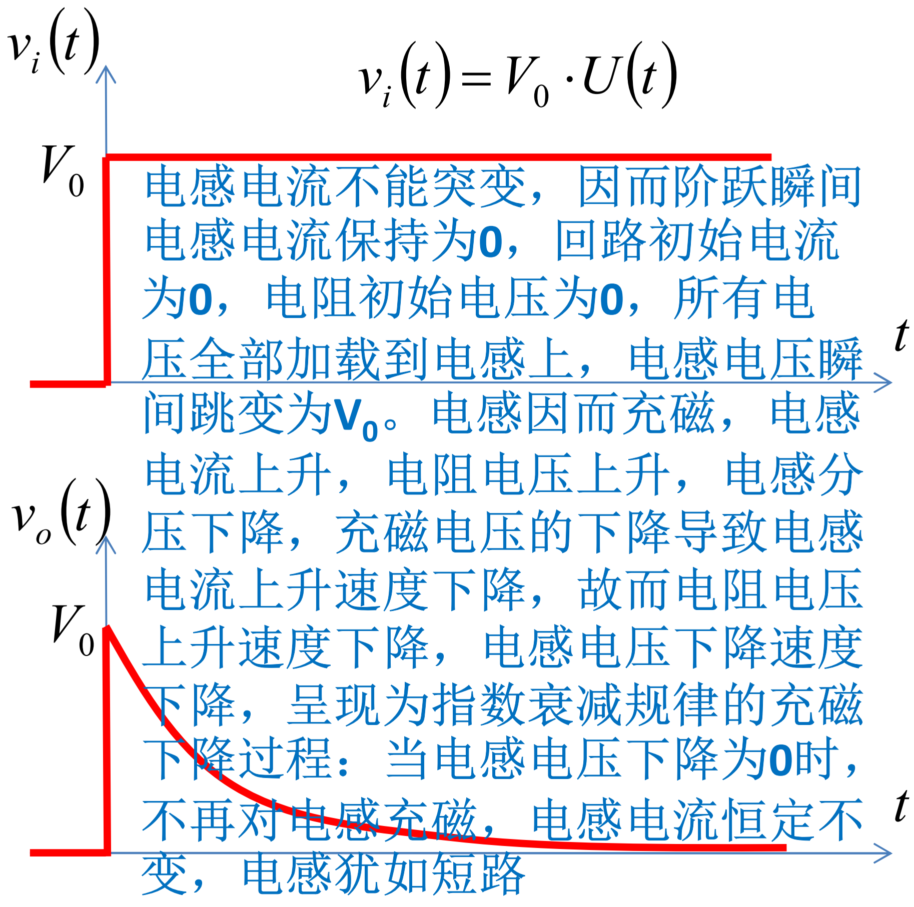


$$v_L(0^+) = V_0 \qquad v_{L\infty}(t) = 0$$

$$\tau = GL = \frac{L}{R}$$

$$v_L(t) = v_{L\infty}(t) + \left(v_L(0^+) - v_{L\infty}(0^+)\right)e^{-\frac{t}{\tau}}\ (t \geq 0)$$

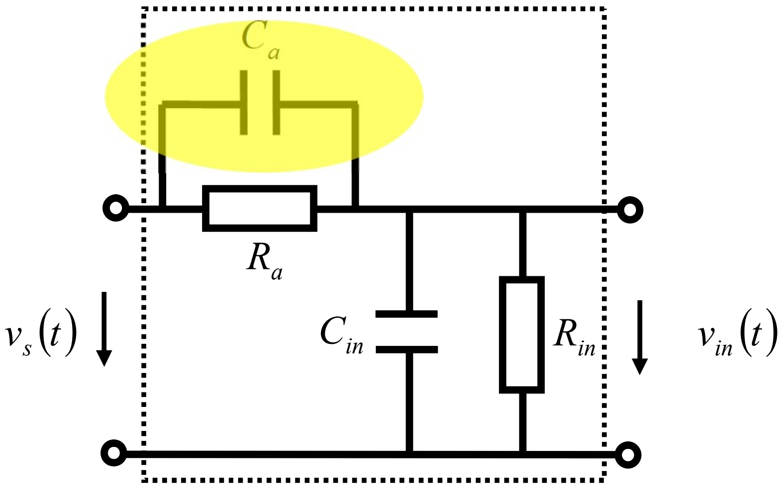
$$= V_0 e^{-\frac{t}{\tau}} \cdot U(t)$$



电感电流不能突变，因而阶跃瞬间电感电流保持为**0**，回路初始电流为**0**，电阻初始电压为**0**，所有电压全部加载到电感上，电感电压瞬间跳变为**V_0**。电感因而充磁，电感电流上升，电阻电压上升，电感分压下降，充磁电压的下降导致电感电流上升速度下降，故而电阻电压上升速度下降，电感电压下降速度下降，呈现为指数衰减规律的充磁下降过程：当电感电压下降为**0**时，不再对电感充磁，电感电流恒定不变，电感犹如短路

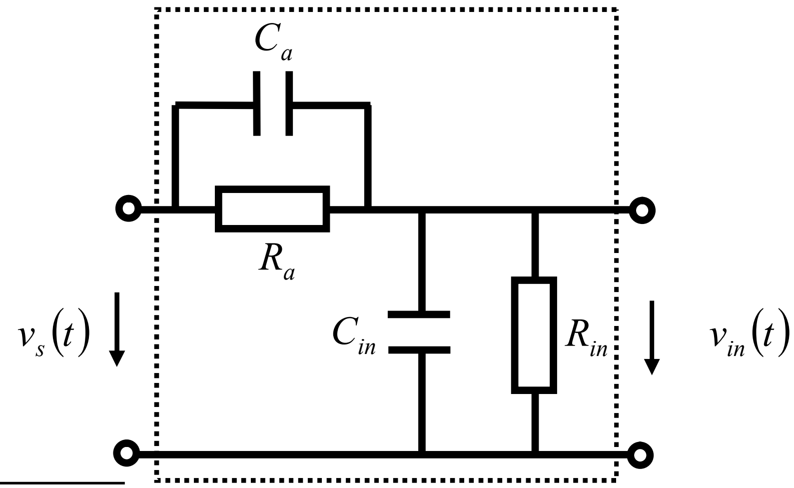
$$v_o(t) = V_0 e^{-\frac{t}{\tau}} \cdot U(t)$$

作业06 示波器探头补偿电容



- 1、从传递函数的幅频特性说明补偿电容最佳取值C_{aopt}=?
- 2、从时域阶跃响应波形说明补偿电容最佳取值C_{aopt}=?
  – 三要素法获得阶跃响应的一般表达式，之后分析说明
- 3、假设示波器输入电阻R_{in}为1MΩ，输入电容C_{in}为10pF，衰减电阻R_a为9MΩ，补偿电容C_a最佳值C_{aopt}为多少？画出C_a=0.5C_{aopt},C_{aopt},2C_{aopt}三种情况下的阶跃响应曲线

传递函数：频域看最佳



$$H(j\omega)=\frac{\dot{V}_{in}}{\dot{V}_s}=\frac{R_{in}||C_{in}}{R_a||C_a+R_{in}||C_{in}}$$

$$=\frac{\dfrac{R_{in}}{1+j\omega R_{in}C_{in}}}{\dfrac{R_a}{1+j\omega R_aC_a}+\dfrac{R_{in}}{1+j\omega R_{in}C_{in}}}=\frac{R_{in}(1+j\omega R_aC_a)}{R_a(1+j\omega R_{in}C_{in})+R_{in}(1+j\omega R_aC_a)}$$

$$=\frac{R_{in}(1+j\omega R_aC_a)}{R_a+R_{in}+j\omega R_aR_{in}(C_{in}+C_a)}=\frac{R_{in}}{R_a+R_{in}}\frac{(1+j\omega R_aC_a)}{1+j\omega\dfrac{R_aR_{in}}{R_a+R_{in}}(C_{in}+C_a)}$$

$$=\frac{R_{in}}{R_a+R_{in}}\frac{1+j\omega R_aC_a}{1+j\omega(R_a||R_{in})(C_{in}||C_a)}=\frac{R_{in}}{R_a+R_{in}}\frac{1+j\omega\tau_a}{1+j\omega\tau}$$

$$=\frac{R_{in}}{R_a+R_{in}}\frac{\sqrt{1+(\omega\tau_a)^2}}{\sqrt{1+(\omega\tau)^2}}e^{j(\arctan\omega\tau_a-\arctan\omega\tau)}\overset{\tau=\tau_a}{=}\frac{R_{in}}{R_a+R_{in}}$$

$$H(j\omega)=A_0e^{-j\omega\tau_0}$$

理想传输系统

形成理想传输系统特性

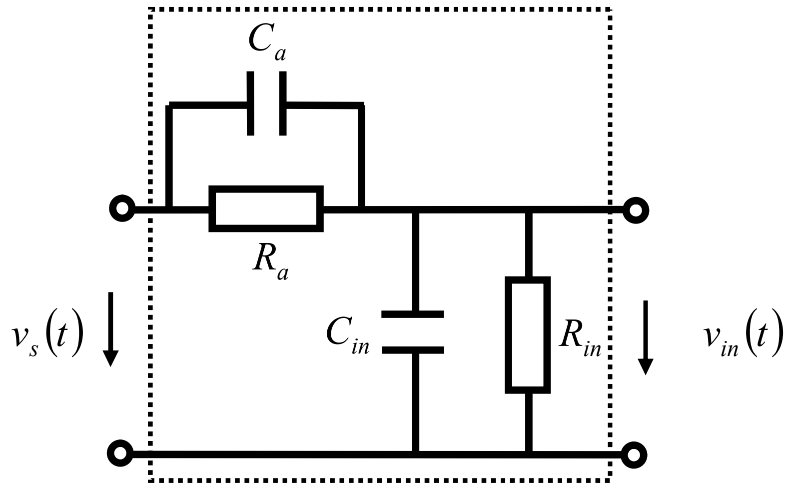
传函和频率无关，或者说带宽无穷大：理想衰减器，理想衰减传输系统

$$R_aC_a=(R_a\parallel R_{in})(C_{in}\parallel C_a)=\frac{R_aR_{in}}{R_a+R_{in}}(C_{in}+C_a)$$

$$C_aR_a=R_{in}C_{in}$$

$$C_{a,opt}=\frac{R_{in}}{R_a}C_{in}$$

阶跃响应：时域看最佳



$$v_s(t)=V_{S0}\cdot U(t) \qquad v_{in,\infty}(t)=\frac{R_{in}}{R_{in}+R_a}V_{S0}$$

$$\tau=RC=(R_a\parallel R_{in})\cdot(C_a\parallel C_{in})=\frac{R_aR_{in}}{R_a+R_{in}}(C_a+C_{in})$$

$$t=0^+ \qquad C_{in}v_{in}(0^+)=Q_0=\left(\frac{C_{in}C_a}{C_{in}+C_a}\right)v_s(0^+)=\frac{C_{in}C_a}{C_{in}+C_a}V_{S0}$$

$$v_{in}(0^+)=\frac{C_a}{C_{in}+C_a}V_{S0}$$

冲激电流瞬间为电容充电：电容极板电荷量

$$v_{in}(t)=v_{in,\infty}(t)+\left(v_{in}(0^+)-v_{in,\infty}(0^+)\right)e^{-\frac{t}{\tau}}$$

$$\overset{v_{in}(0^+)=v_{in,\infty}(0^+)}{=} \quad v_{in,\infty}(t)=\frac{R_{in}}{R_{in}+R_a}V_{S0}$$

$$\frac{C_a}{C_{in}+C_a}=\frac{R_{in}}{R_{in}+R_a}$$

$$C_aR_a=R_{in}C_{in}$$

$$C_{a,opt}=\frac{R_{in}}{R_a}C_{in}$$

输出是输入的简单分压
输出波形和输入波形对比，无任何失真
无失真传输！！！！示波器观测波形，
自然希望无失真传输到示波器输入端口

恰好补偿

- **3、假设示波器输入电阻R_{in}为1MΩ，输入电容C_{in}为10pF，衰减电阻R_a为9MΩ，补偿电容C_a最佳值C_{aopt}为多少？画出C_a=0.5C_{aopt},C_{aopt},2C_{aopt}三种情况下的阶跃响应曲线**

$$C_a = C_{a,opt} = 1.11pF$$

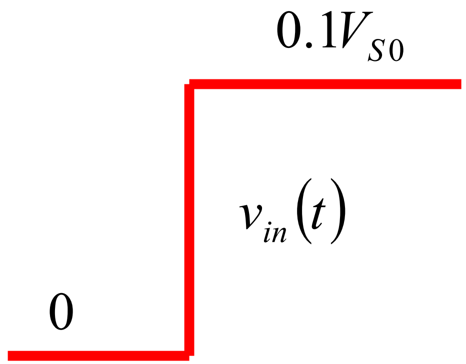
$$C_{a,opt} = \frac{R_{in}}{R_a} C_{in} = \frac{1M\Omega}{9M\Omega} \times 10pF = 1.11pF$$

$$\frac{\dot{V}_{in}}{\dot{V}_s} = \frac{R_{in}}{R_a + R_{in}} \frac{1 + j\omega R_a C_a}{1 + j\omega (R_a \| R_{in})(C_{in} \| C_a)} = \frac{1}{10}$$

$$v_{in}(t) = \frac{R_{in}}{R_{in} + R_a} \cdot V_{S0} U(t) = \frac{1}{10} V_{S0} U(t)$$

$$v_s(t) = V_{S0} U(t)$$

$$v_{in}(t) = 0.1 V_{S0} U(t)$$



恰好补偿，一步到位

欠补偿

$$C_a = 0.5C_{a,opt} = 0.555pF$$

$$v_{in}(t) = V_{S0}\left(\frac{R_{in}}{R_{in}+R_a} + \left(\frac{C_a}{C_{in}+C_a} - \frac{R_{in}}{R_{in}+R_a}\right)e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

$$= V_{S0}\left(0.1 + \left(\frac{0.555}{10+0.555} - 0.1\right)e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

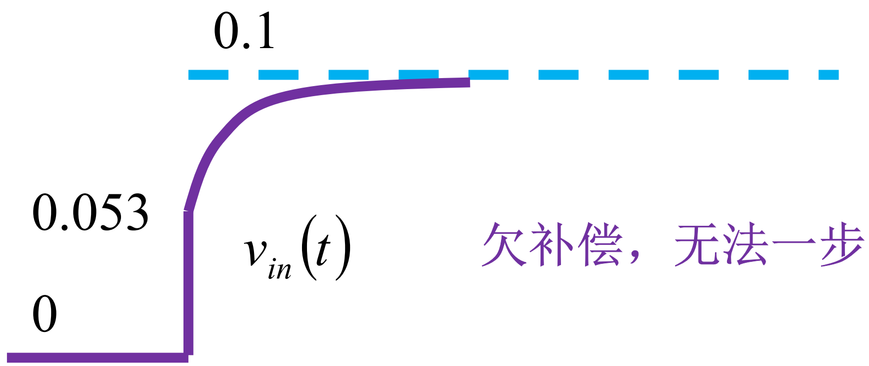
$$= V_{S0}\left(0.1 - 0.047e^{-\frac{t}{9.5\mu}}\right)\cdot U(t)$$

$$= 0.1V_{S0}\left(1 - 0.47e^{-\frac{t}{9.5\mu}}\right)\cdot U(t)$$

$$\tau = RC = (R_a \parallel R_{in})\cdot(C_a \parallel C_{in})$$

$$= \frac{R_a R_{in}}{R_a + R_{in}}(C_a + C_{in})$$

$$= 0.9M\Omega \times 10.555pF = 9.5\mu s$$



欠补偿，无法一步到位

过补偿

$$C_a = 2C_{a,opt} = 2.222pF$$

$$v_{in}(t) = V_{S0}\left(\frac{R_{in}}{R_{in}+R_a} + \left(\frac{C_a}{C_{in}+C_a} - \frac{R_{in}}{R_{in}+R_a}\right)e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

$$= V_{S0}\left(0.1 + \left(\frac{2.222}{10+2.222} - 0.1\right)e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

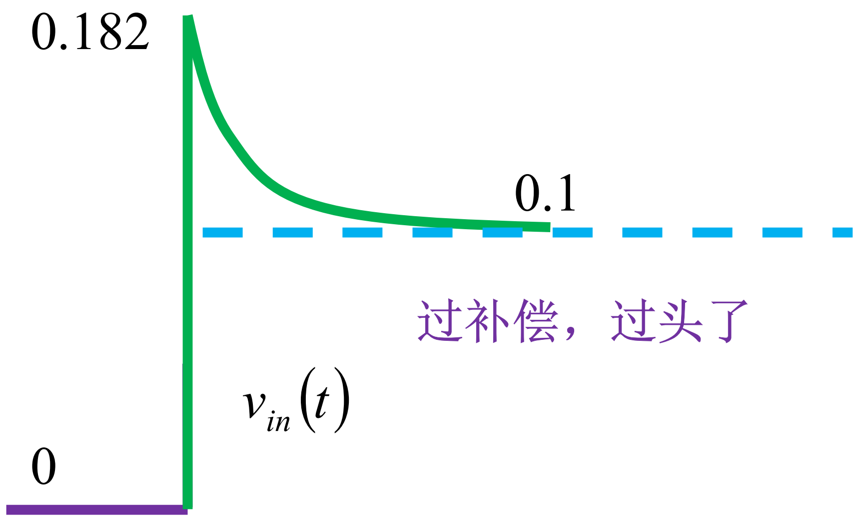
$$= V_{S0}\left(0.1 + 0.082e^{-\frac{t}{11\mu}}\right)\cdot U(t)$$

$$= 0.1V_{S0}\left(1 + 0.82e^{-\frac{t}{11\mu}}\right)\cdot U(t)$$

$$\tau = RC = (R_a \parallel R_{in})\cdot(C_a \parallel C_{in})$$

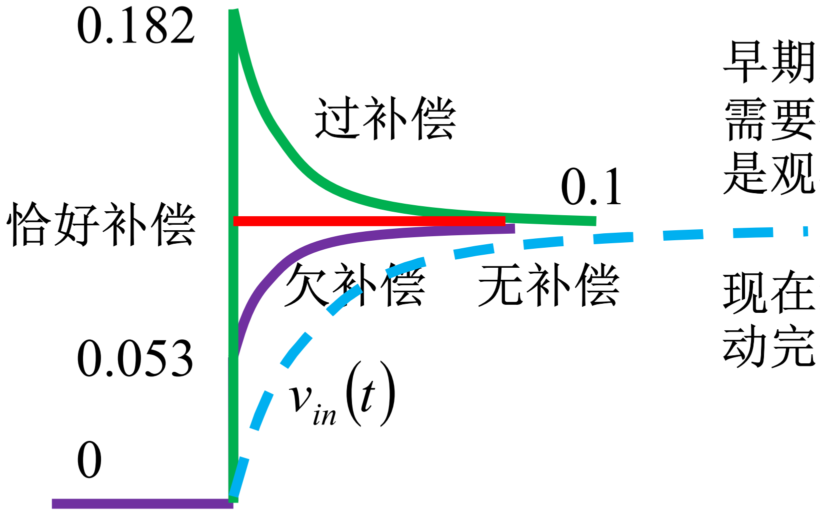
$$= \frac{R_a R_{in}}{R_a + R_{in}}(C_a + C_{in})$$

$$= 0.9M\Omega \times 12.222pF = 11\mu s$$



三种补偿效果

$$v_{in}(t)=\frac{1}{10}\left(1+0.82e^{-\frac{t}{11\mu}}\right)U(t)$$ 过补偿



早期示波器的探头补偿电容需要手工调准，调准方法就是观测方波激励下的响应

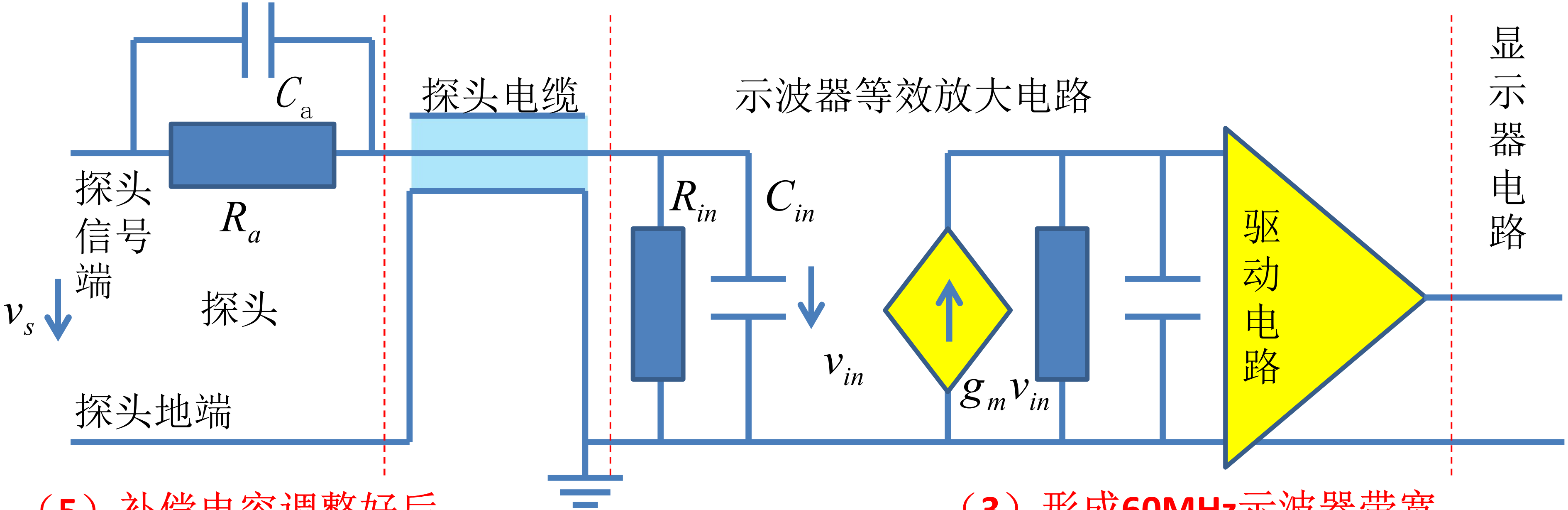
现在示波器的探头补偿可自动完成

$$v_{in}(t)=\frac{1}{10}\left(1-0.47e^{-\frac{t}{9.5\mu}}\right)U(t)$$ 欠补偿

示波器测试系统简化模型

（0）测试系统对被测电路产生不利影响

（1）衰减电阻隔离示波器探头电缆的影响



（5）补偿电容调整好后，低阻抗测试点测试精度高

（3）形成**60MHz**示波器带宽

$$\tau_{inner}=\frac{1}{2\pi BW}=\frac{1}{2\times 3.14\times 60MHz}=2.65ns$$

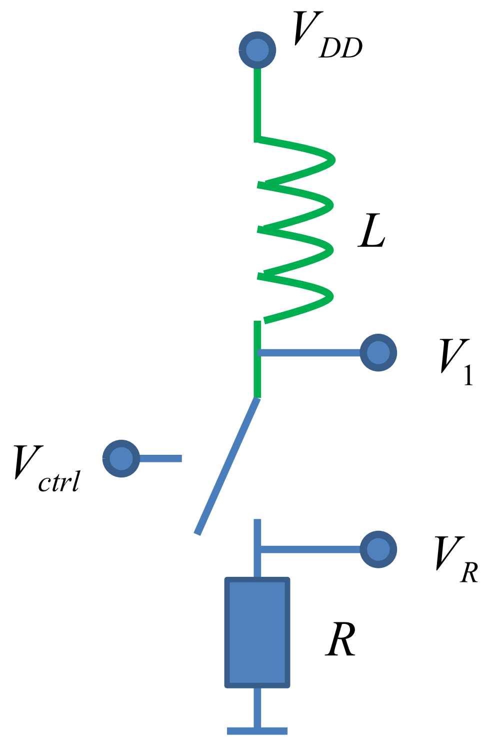
$$\tau_{in}=(R_{in}\,\|\,R_a)C_{in}=(1M\Omega\,\|\,9M\Omega)\times 10pF=9\mu s$$

（2）无补偿电容

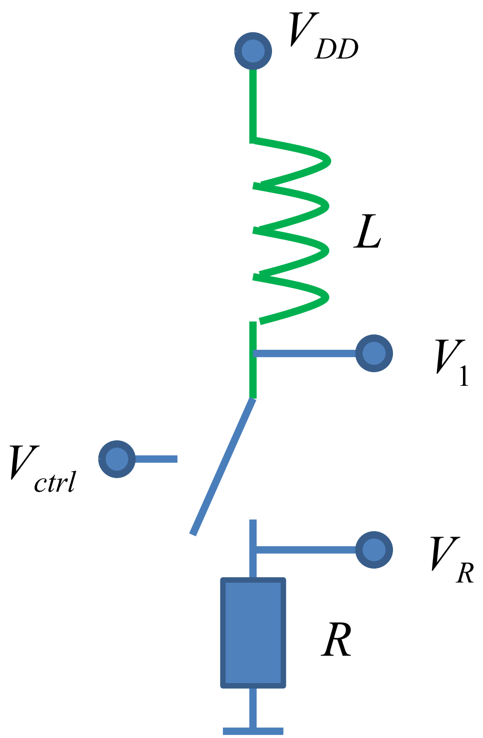
$(18\mathrm{k}Hz)$

（4）系统带宽严重受限于探头，故而需要补偿

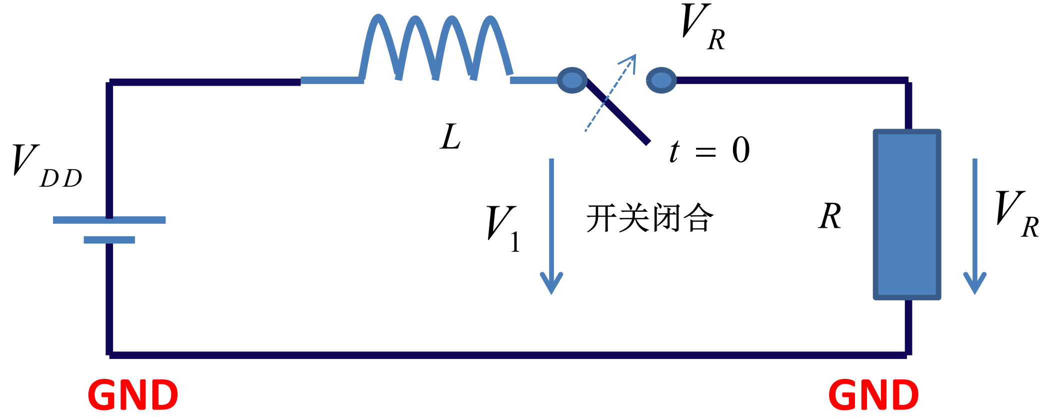
作业07 电感断流产生冲激电压



- 这是一个继电器示意电路，晶体管开关可以接通电路，为负载电阻供电
  - 假设开关是理想开关
- 请分析开关闭合瞬间，负载电阻上的电压变化情况
- 请分析开关断开瞬间，晶体管开关两端电压变化情况
  - 晶体管开关很容易被击穿，请给出你的解决办法

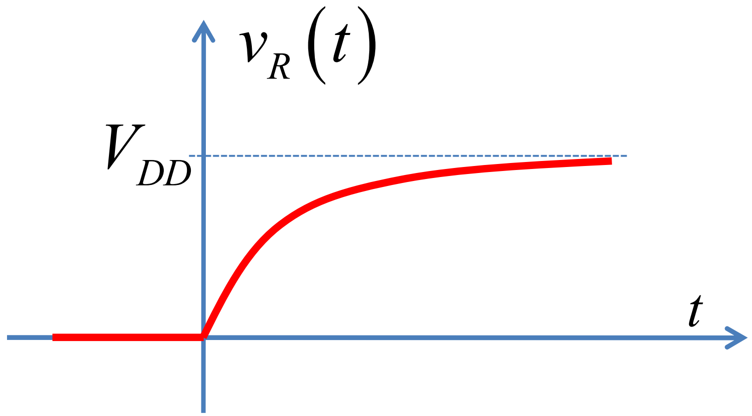


等效



开关闭合：三要素法

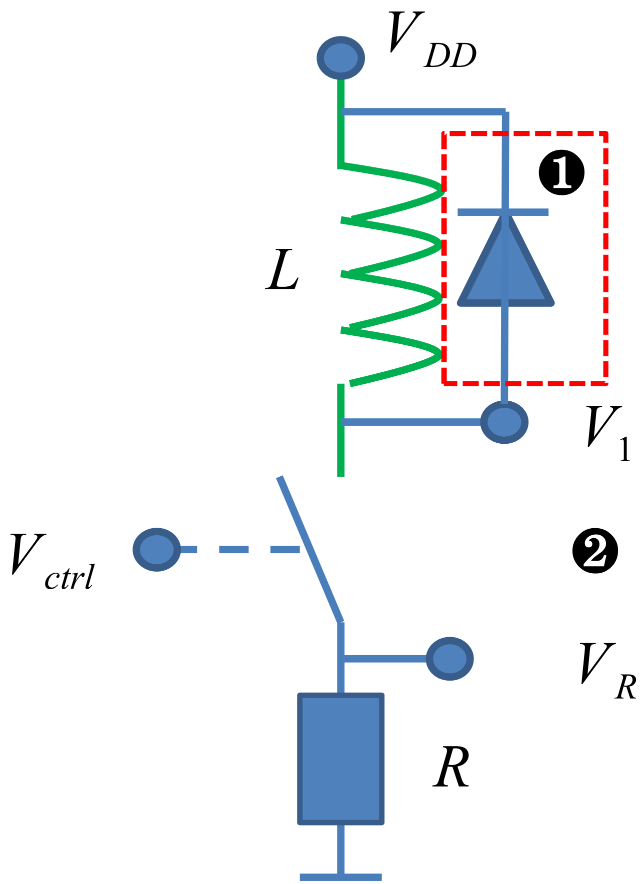
$$i_L(0^+) = i_L(0^-) = 0 \quad i_{L\infty}(t) = \frac{V_{DD}}{R} \quad \tau = GL = \frac{L}{R}$$

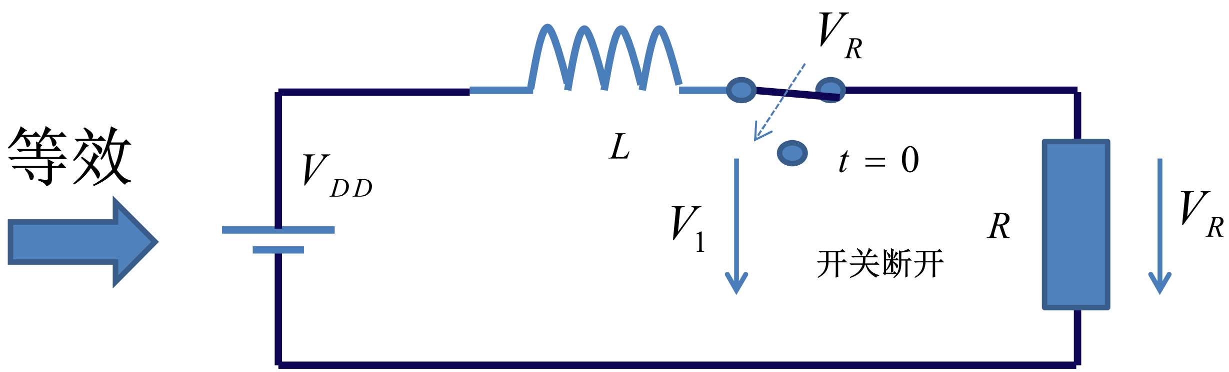


$$i_L(t) = \frac{V_{DD}}{R}\left(1 - e^{-\frac{t}{GL}}\right)U(t)$$

$$v_R(t) = i_L(t)R = V_{DD}\left(1 - e^{-\frac{t}{GL}}\right)U(t)$$

一阶低通系统的阶跃响应





$i_L(0^-)=\frac{V_{DD}}{R}$　　$i_L(0^+)=0$　　强行断开

$v_1(0^-)=V_{DD}$,　　$v_1(0^+)=V_{DD}-L\frac{di_L(0^+)}{dt}=V_{DD}+L\frac{V_{DD}}{R}\delta(t)$

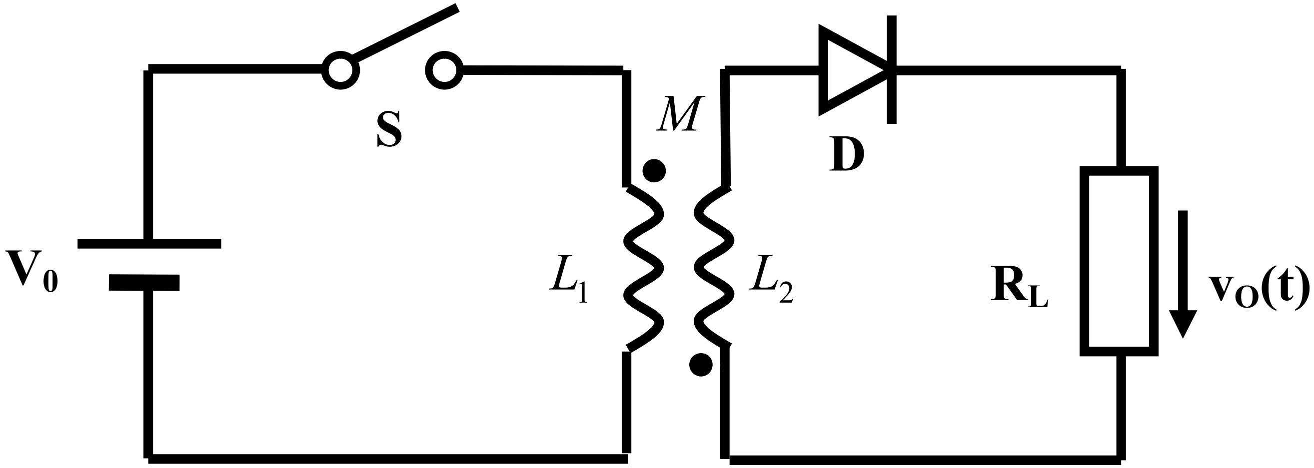
开关断开瞬间，晶体管漏极产生冲激电压

晶体管瞬间击穿损毁
（机械开关，空气击穿，产生电火花）

保护措施，提供电感放电通路：

电感电流不能突变，续流二极管为它提供电流通路

作业08

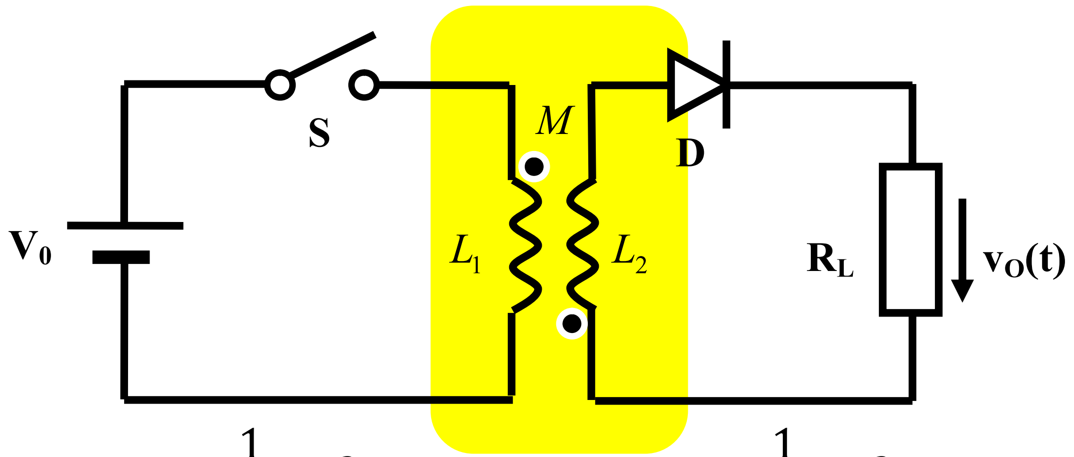


仅一端有电流的变压器退化为一阶电感

- 如图所示，开关起始是断开且电路已经进入稳态，假设开关S在t=0时刻闭合，在$t=t_0$时刻又断开，请分析负载电阻R_L上的电压波形$v_O(t)$，其中二极管为理想整流二极管，正偏导通则导通电压为0，反偏截止则开路。

$$v_1 = +L_1 \frac{d}{dt} i_1 - M \frac{d}{dt} i_2$$

$$v_2 = -M \frac{d}{dt} i_1 + L_2 \frac{d}{dt} i_2$$

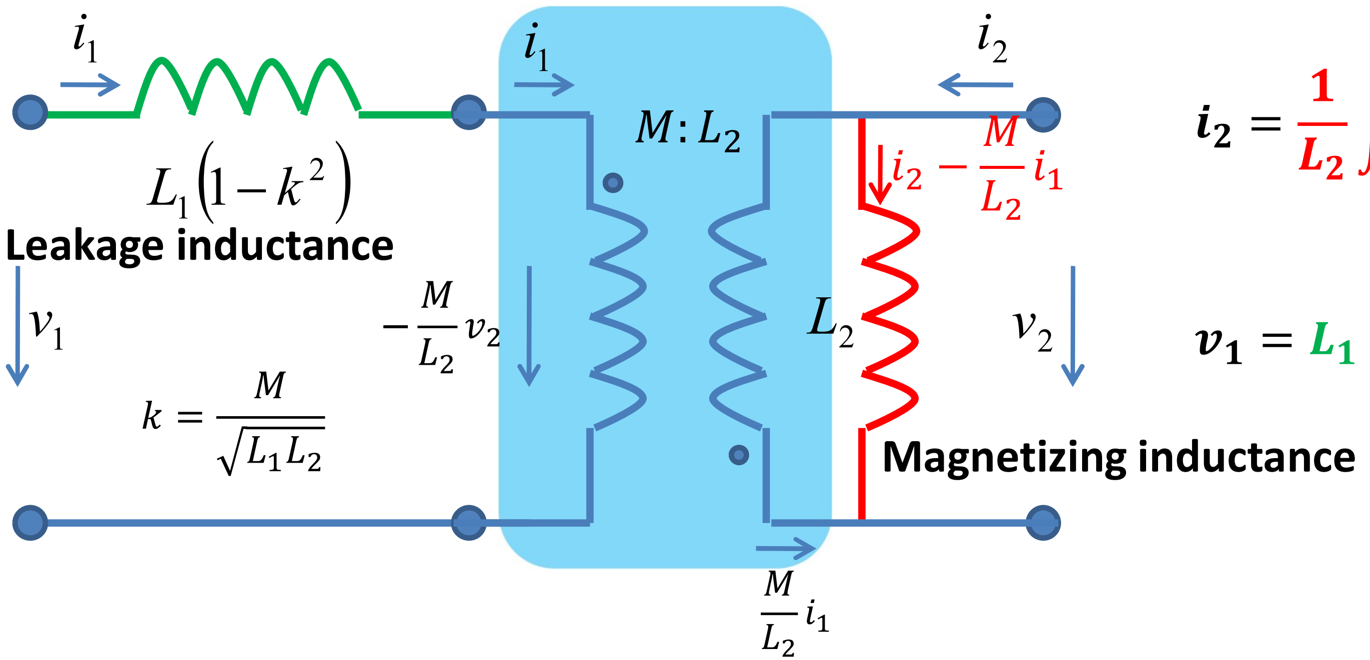


漏磁电感

励磁电感

$$E(t) = \frac{1}{2} L_1 i_1^2(t) - M i_1(t) i_2(t) + \frac{1}{2} L_2 i_2^2(t)$$

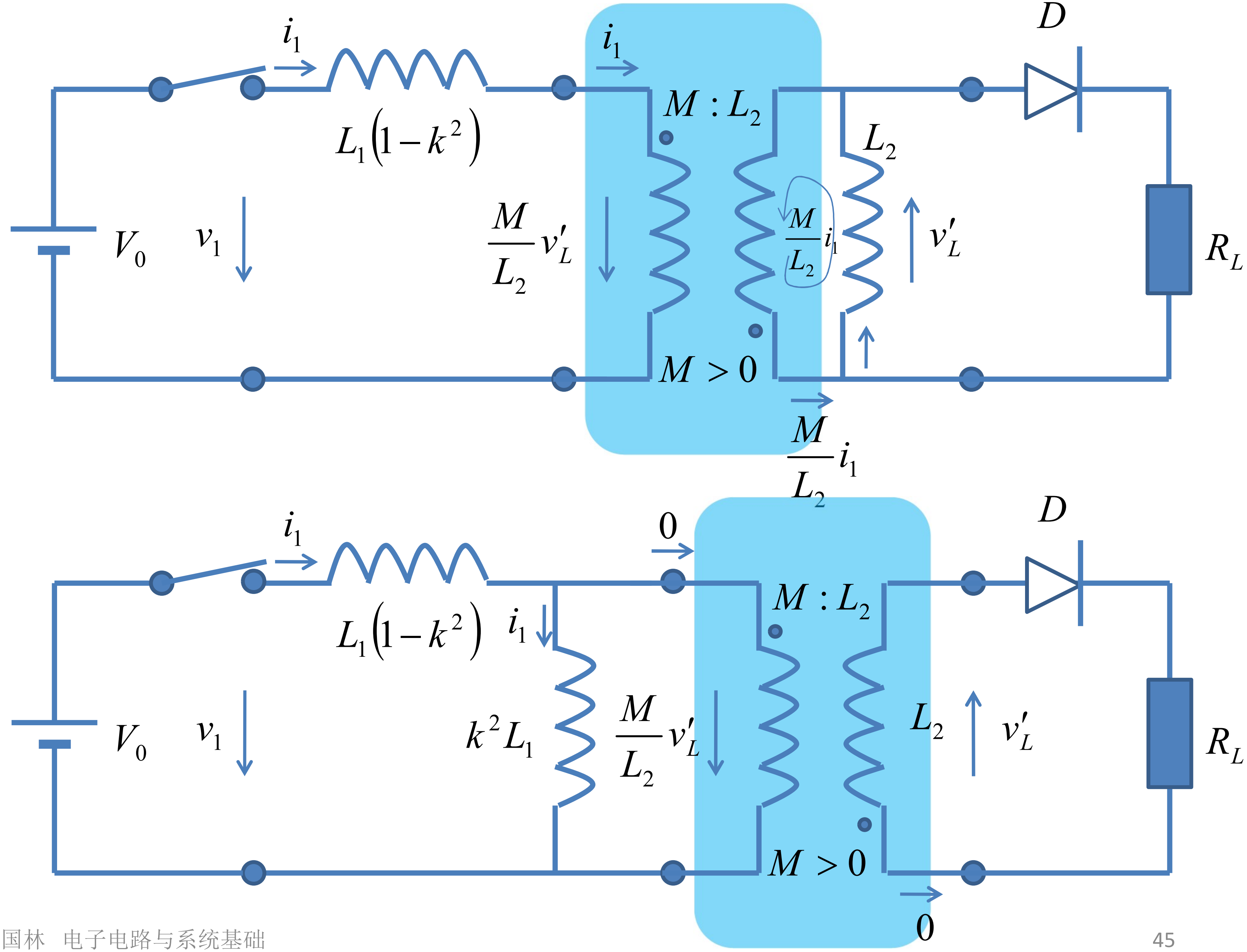
$$E(t) = \frac{1}{2}(1-k^2) L_1 \cdot i_1^2(t) + \frac{1}{2} L_2 \cdot \left(i_2(t) - \frac{M}{L_2} i_1(t) \right)^2$$



$$i_2 = \frac{1}{L_2} \int v_2 \, dt + \frac{M}{L_2} i_1$$

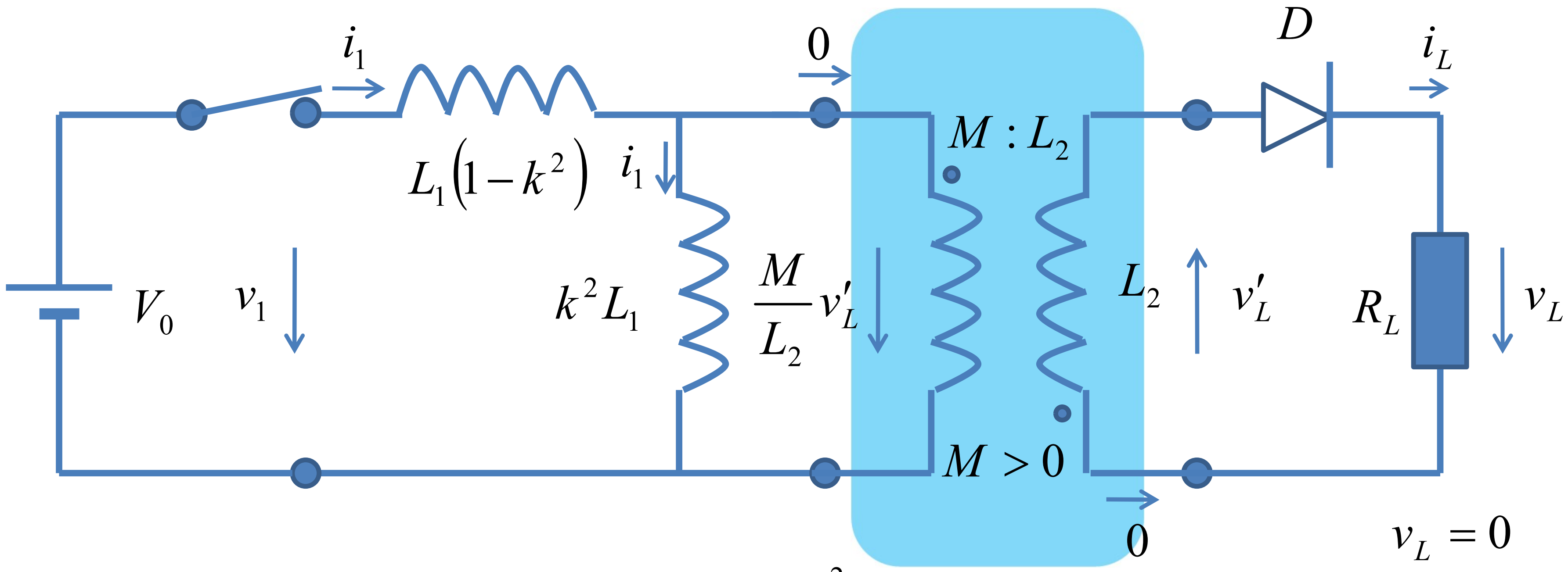
$$v_1 = L_1 \left(1 - \frac{M^2}{L_1 L_2} \right) \frac{d}{dt} i_1 - \frac{M}{L_2} v_2$$

开关闭合



i1
L1(1−k2)
M : L2
L2
D
V0
v1
M/L2 v'L
M/L2 i1
v'L
RL
M > 0
M/L2 i1
i1
0
M : L2
D
L1(1−k2)
i1
k2L1
M/L2 v'L
L2
v'L
V0
v1
RL
M > 0
0

开关闭合： 0<t<t₀



$v_L = 0$

$i_L = 0$

$$v_1(t) = V_0$$

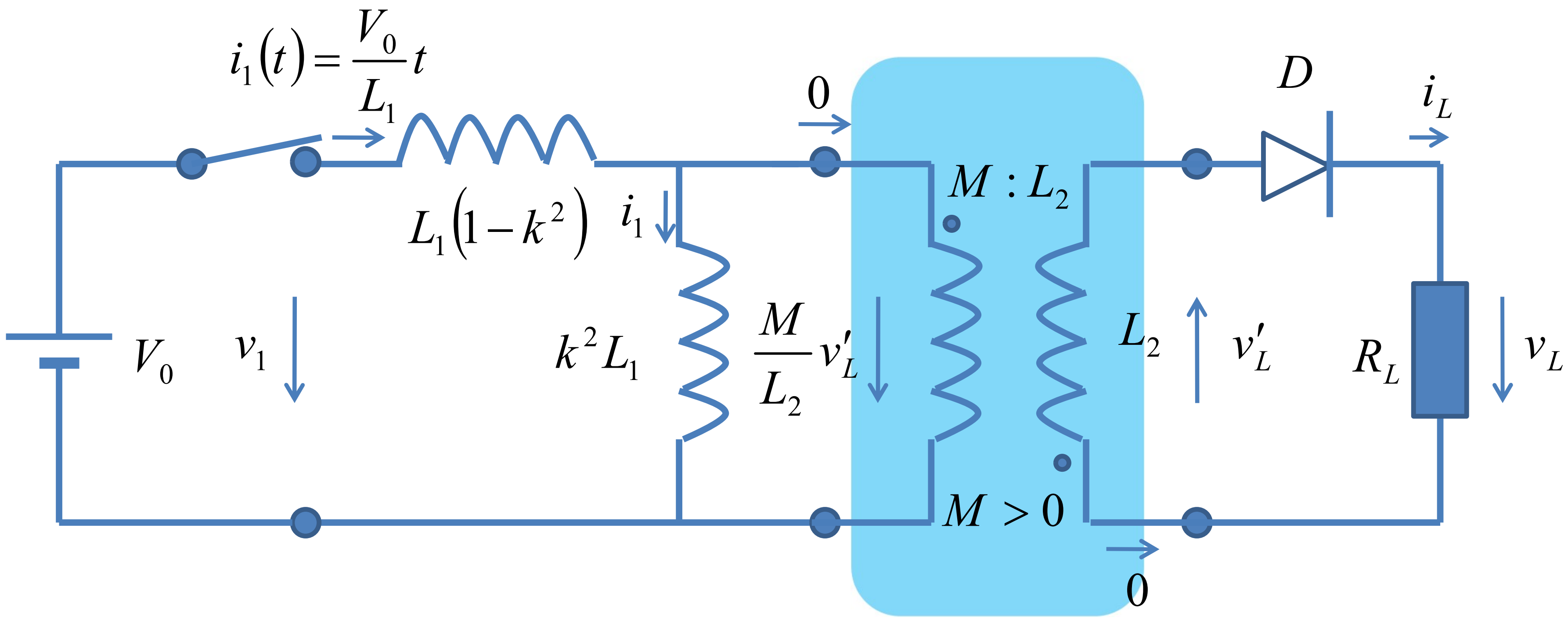
$$\frac{M}{L_2}v'_L = \frac{k^2L_1}{L_1}V_0$$

$$i_1(t) = I_{01} + \frac{1}{L_1}\int_0^t v_1(t)dt = \frac{V_0}{L_1}t$$

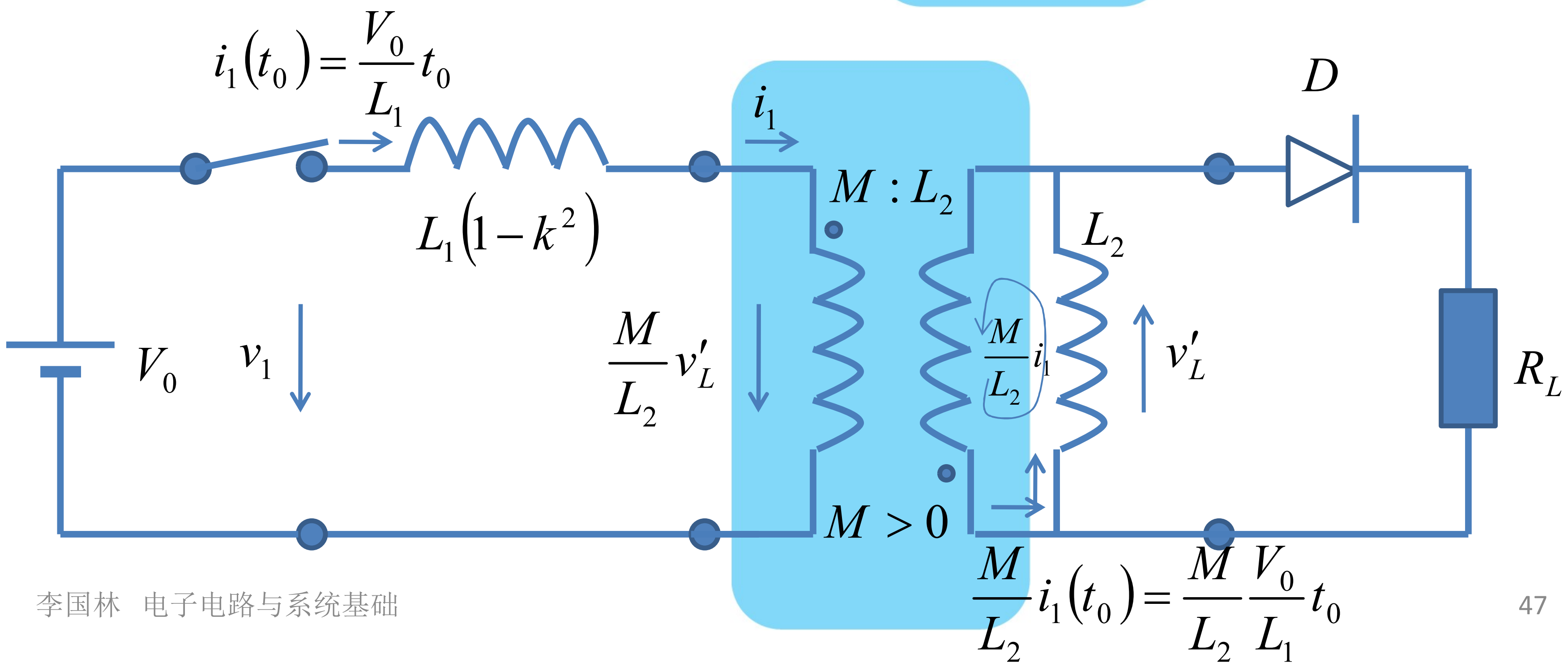
$$v'_L = \frac{L_2}{M}k^2V_0 = \frac{L_2}{M}\frac{M^2}{L_1L_2}V_0 = \frac{M}{L_1}V_0$$

$$v_2 = -v'_L = -\frac{M}{L_1}V_0$$

全部加载在反偏二极管上

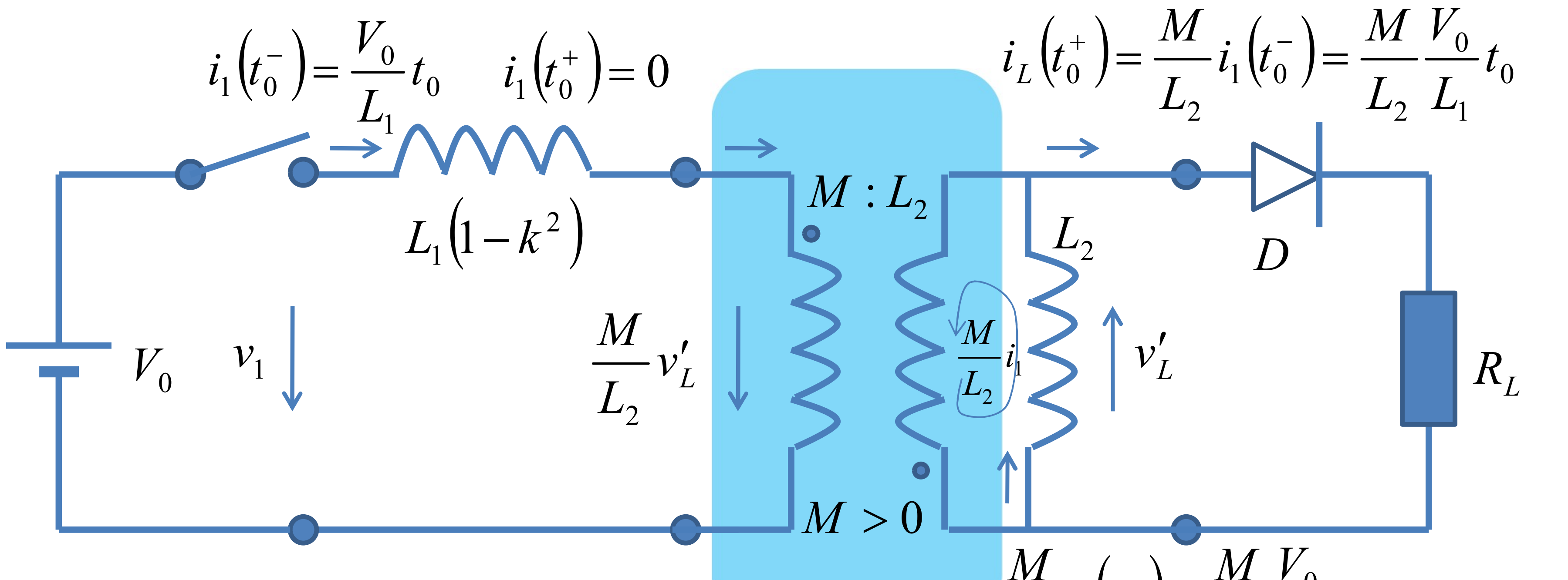


$i_1(t) = \frac{V_0}{L_1} t$
0
D
i_L
$L_1(1-k^2)$
i_1
$M : L_2$
V_0
v_1
$k^2 L_1$
$\frac{M}{L_2} v'_L$
L_2
v'_L
R_L
v_L
$M > 0$
0



$i_1(t_0) = \frac{V_0}{L_1} t_0$
D
i_1
$M : L_2$
$L_1(1-k^2)$
L_2
V_0
v_1
$\frac{M}{L_2} v'_L$
$\frac{M}{L_2} i_1$
v'_L
R_L
$M > 0$
$\frac{M}{L_2} i_1(t_0) = \frac{M}{L_2} \frac{V_0}{L_1} t_0$

开关断开瞬间



$$\frac{M}{L_2} i_1(t_0^-) = \frac{M}{L_2}\frac{V_0}{L_1} t_0$$

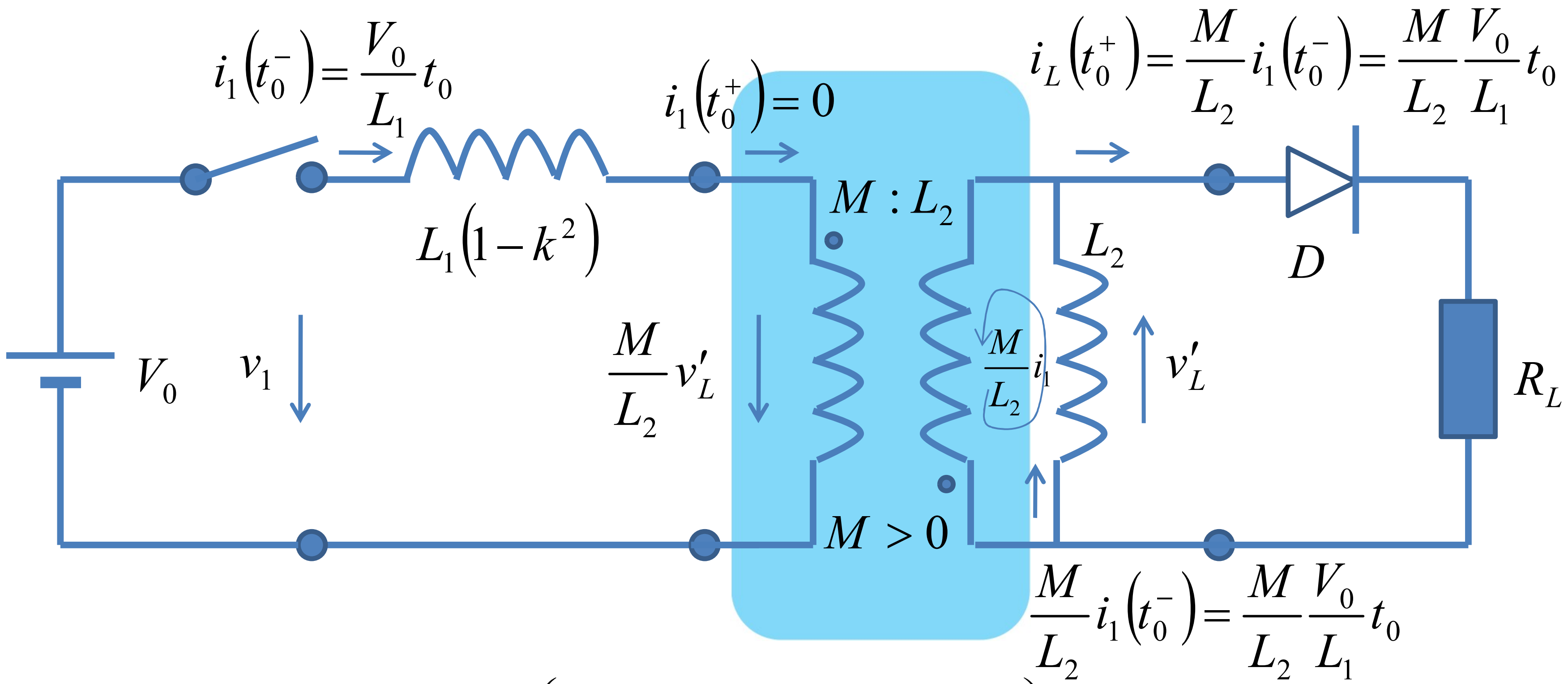
$$v_1(t_0^+) = (1-k^2)L_1 \left.\frac{di_1}{dt}\right|_{t=t_0} + \frac{M}{L_2} v_L'(t_0^+)$$

$$v_L'(t_0^+) = -\frac{M}{L_2}\frac{V_0}{L_1} R_L t_0$$

$$v_1(t_0^+) = (1-k^2)L_1 \frac{V_0}{L_1} t_0(-\delta(t-t_0)) + \frac{M}{L_2}\left(-\frac{M}{L_2}\frac{V_0}{L_1} R_L t_0\right)$$

$$= -(1-k^2)V_0 t_0 \delta(t-t_0) - k^2 \frac{R_L t_0}{L_2} V_0 = -(1-k^2)V_0 t_0 \delta(t-t_0) - k^2 \frac{t_0}{\tau_2} V_0$$

开关断开



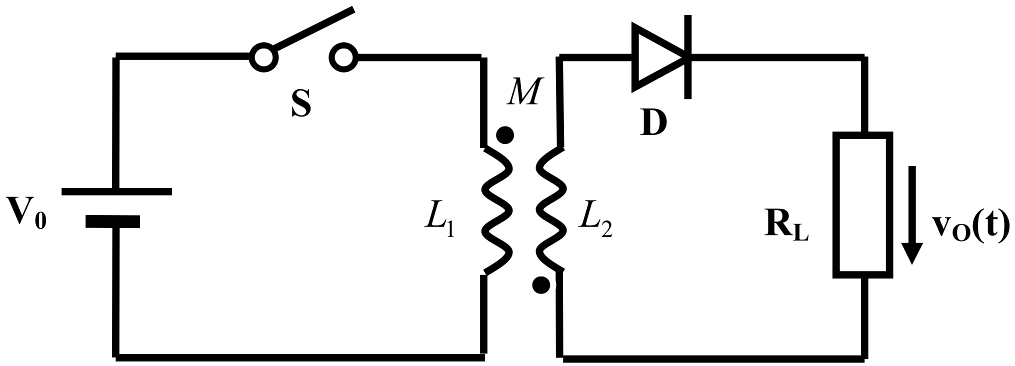
$$v_1(t)=-(1-k^2)V_0t_0\delta(t-t_0)+\frac{M}{L_2}\cdot\left(-\frac{M}{L_2}\frac{V_0}{L_1}t_0R_Le^{-\frac{t-t_0}{\tau_2}}U(t-t_0)\right)$$

$$=-(1-k^2)V_0t_0\delta(t-t_0)-k^2\frac{t_0}{\tau_2}V_0e^{-\frac{t-t_0}{\tau_2}}U(t-t_0)$$

$$v_L(t)=v_L(t_0^+)e^{-\frac{t-t_0}{\tau_2}}U(t-t_0)$$

$$=\frac{M}{L_2}\frac{V_0}{L_1}t_0R_Le^{-\frac{t-t_0}{\tau_2}}U(t-t_0)$$

思考：为了防止晶体管开关击穿损毁，如何加保护电路？

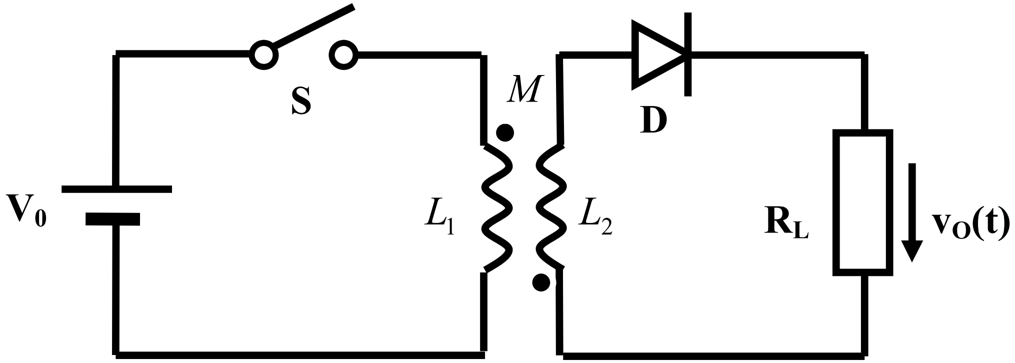


$$v_1(t) = \begin{cases} 0 & t < 0 \\ V_0 \quad \text{恒压充磁} & 0 < t < t_0 \\ -(1-k^2)V_0 t_0 \delta(t-t_0) - k^2 \dfrac{V_0 t_0}{\tau_2} e^{-\frac{t-t_0}{\tau_2}} U(t-t_0) & t \geq t_0 \end{cases}$$

漏磁电感储能瞬间释放，开关击穿　　励磁电感储能在回路**2**释放

$$i_1(t) = \begin{cases} 0 & t < 0 \\ \dfrac{V_0}{L_1} t \quad \text{电流线性增长} & 0 < t < t_0 \\ 0 & t > t_0 \end{cases}$$

回路**1**电流中断，励磁能量转移到回路**2**释放



$$i_L(t)=\frac{M}{L_2}\frac{V_0}{L_1}t_0 e^{-\frac{t-t_0}{\tau_2}}U(t-t_0)=\frac{M}{L_2}\cdot\frac{V_0 t_0}{L_1}\cdot e^{-\frac{t-t_0}{\tau_2}}U(t-t_0)$$

$$v_L(t)=\frac{M}{L_2}\cdot\frac{V_0 t_0}{L_1}\cdot R_L\cdot e^{-\frac{t-t_0}{\tau_2}}U(t-t_0)$$

初级回路电流中断后：部分能量在次级回路释放，被负载电阻吸收

DC-DC转换电路的变压器结构：储能，隔离，电压变换