# 电子电路与系统基础II

## 习题课第六讲　一阶动态电路的时域分析

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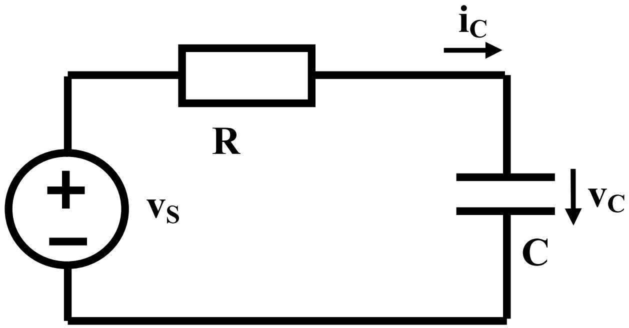
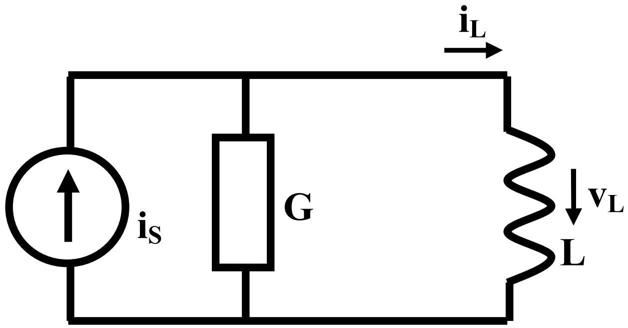
# 作业一、RC对偶GL




- 图示的一阶**RC**电路对偶一阶**GL**电路（习惯称之为**RL**电路），对一阶**RC**电路成立的结论对一阶**RL**电路同样成立，只需对偶量互换即可

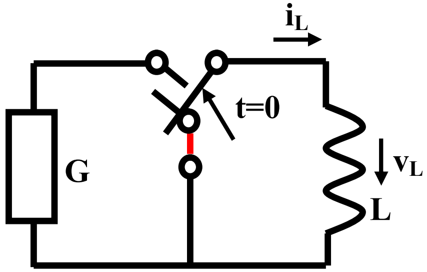
$$v_C(t)=V_0\cdot e^{-\frac{t}{\tau}}+\int_0^t v_S(\lambda)\cdot e^{\frac{\lambda-t}{\tau}}d\frac{\lambda}{\tau}$$

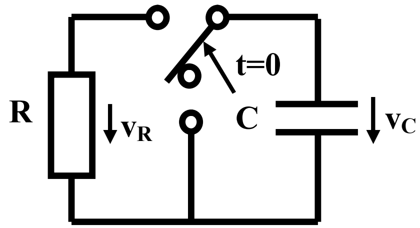
$$\tau = RC$$

$$i_L(t)=I_0\cdot e^{-\frac{t}{\tau}}+\int_0^t i_S(\lambda)\cdot e^{\frac{\lambda-t}{\tau}}d\frac{\lambda}{\tau}$$

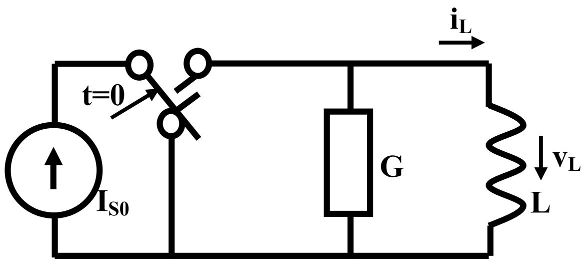
$$\tau = GL$$

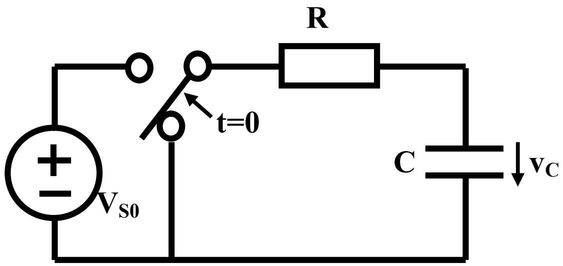
# 电感放磁、充磁曲线




- （1）练习9.2.2 分析图示一阶RL电路的零输入响应，假设开关在t=0时刻拨动，开关拨动前的电感初始电流为$I_0$
  - 给出电感电流放磁曲线，和放磁电压时域波形：表达式和曲线



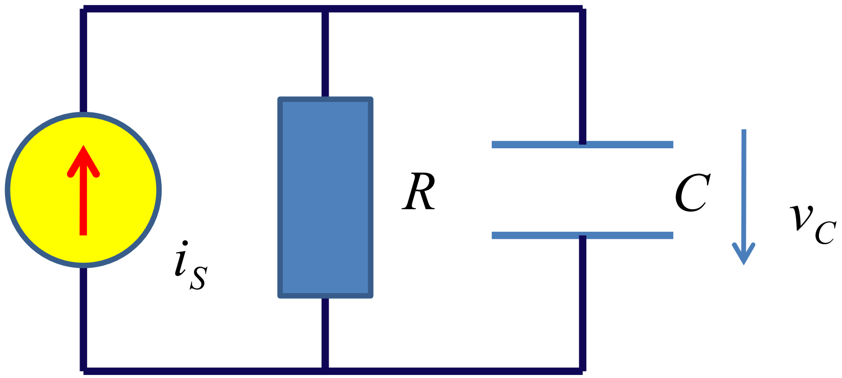



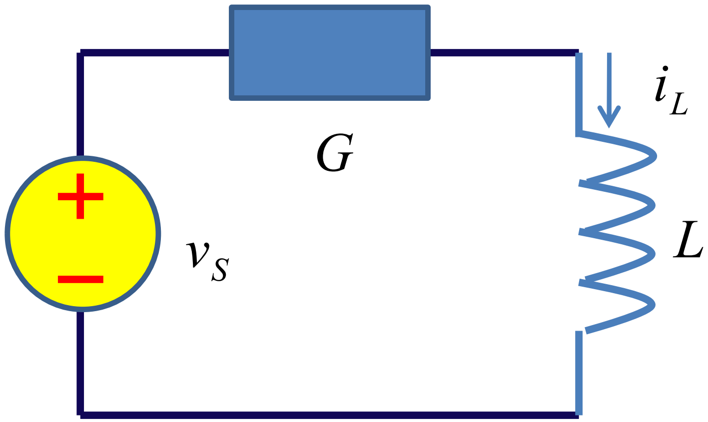

- （2）练习9.2.4 分析图示一阶RL电路的零状态响应，假设开关在t=0时刻换路，开关换路前放磁已经结束，电感初始电流为0
  - 给出电感电流充磁曲线，和充磁电压时域波形：表达式和曲线

# 电容和电感的对偶关系

| | 电容 | 电感 |
|---|---|---|
| 定义 | 电容是导体保持可移动电荷的能力，在单位电压作用下导体结构（结点）保持的电荷量就是电容量 | 电感是描述导线结构中流通电流产生与链接磁通的能力，在单位电流作用下导线结构（回路）链接的磁通量就是电感量 |
| 存在性 | 电路中总是有导体（结点）电荷集聚，（结点间）始终存在电容效应 | 电路中总是有导线（回路）链接磁通，（回路间）始终存在电感效应 |
| 线性时不变 | $C=\frac{Q}{V}$ $C_d=\frac{dQ}{dV}=C$ | $L=\frac{\Phi}{I}$ $L_d=\frac{d\Phi}{dI}=L$ |
| 元件约束 | $i(t)=\frac{dQ(t)}{dt}=C\frac{dv(t)}{dt}$ | $v(t)=\frac{d\Phi(t)}{dt}=L\frac{di(t)}{dt}$ |
| 非线性 | $Q(V_0+v(t))=C_0\cdot V_0+C_d(V_0)\cdot v(t)+\ldots$ | $\Phi(I_0+i(t))=L_0\cdot I_0+L_d(I_0)\cdot i(t)+\ldots$ |
| 元件约束 | $i(t)=\frac{dQ(t)}{dt}=C_d(V_0+v(t))\frac{dv(t)}{dt}$ | $v(t)=\frac{d\Phi(t)}{dt}=L_d(I_0+i(t))\frac{di(t)}{dt}$ |
| 线性时变 | $Q(t)=C(t)v(t)$ | $\Phi(t)=L(t)i(t)$ |
| 元件约束 | $i(t)=\frac{dQ(t)}{dt}=C(t)\frac{dv(t)}{dt}+v(t)\frac{dC(t)}{dt}$ | $v(t)=\frac{d\Phi(t)}{dt}=L(t)\frac{di(t)}{dt}+i(t)\frac{dL(t)}{dt}$ |

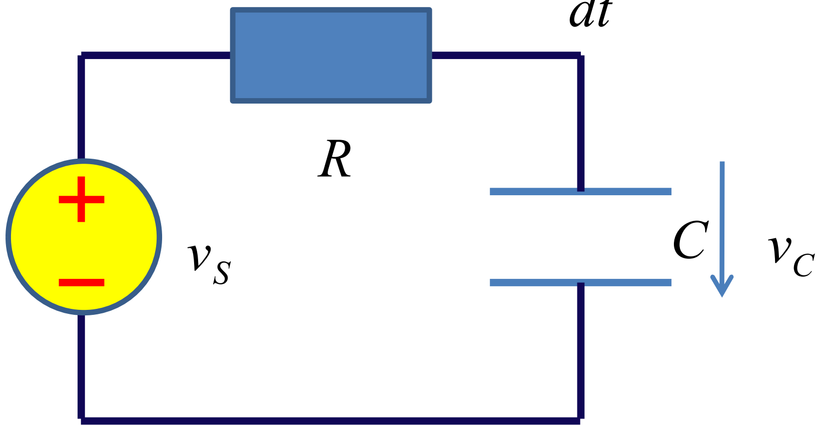
# 阻容电路 对偶 导感电路

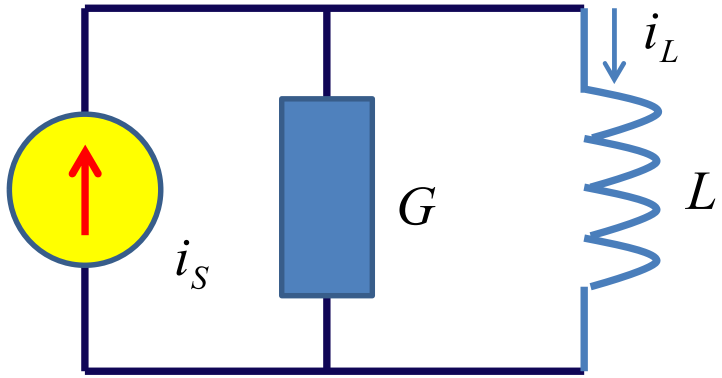


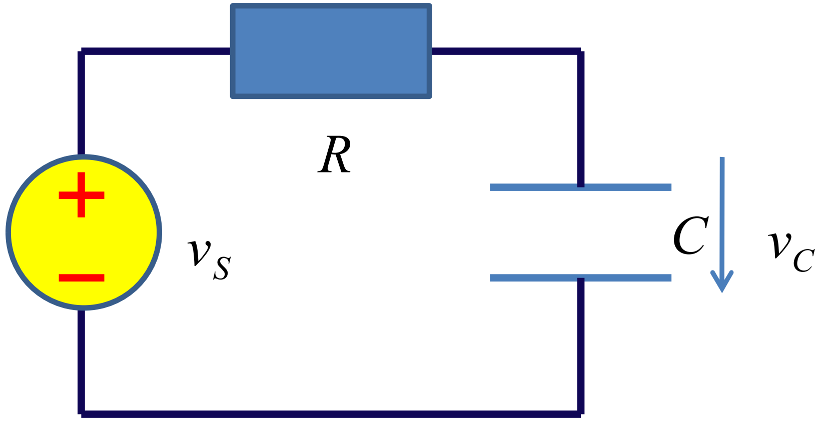



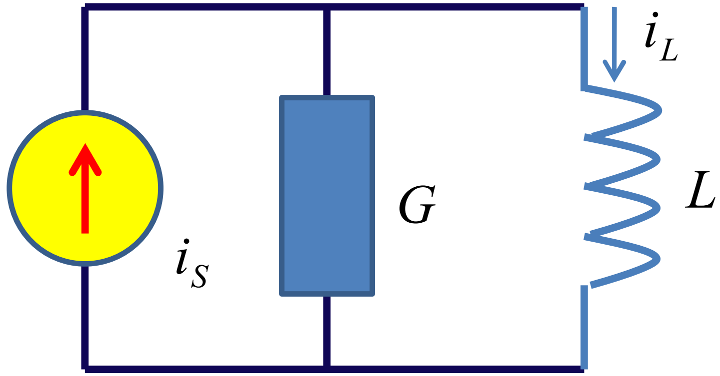

$$C\frac{dv_C}{dt}+\frac{v_C}{R}=i_S \qquad L\frac{di_L}{dt}+\frac{i_L}{G}=v_S$$

$$RC\frac{dv_C}{dt}+v_C=v_S \qquad GL\frac{di_L}{dt}+i_L=i_S$$






$$RC\frac{dv_C}{dt}+v_C=v_S \qquad \tau\frac{dx}{dt}+x=s \qquad GL\frac{di_L}{dt}+i_L=i_S$$

$$\tau = RC \qquad\qquad \tau = GL$$

# 微分方程求解

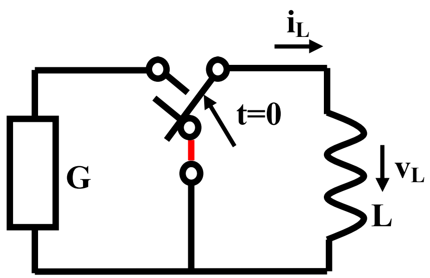
$$x(t)=\underline{\underline{X_0e^{-\frac{t}{\tau}}}}+\underline{\underline{\int_0^t s(\lambda)\cdot e^{\frac{\lambda-t}{\tau}}d\frac{\lambda}{\tau}}}$$

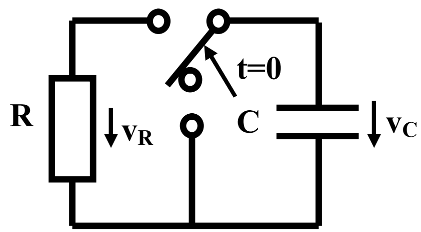
零输入响应　　零状态响应

$$v_C(t)=\underline{\underline{V_0e^{-\frac{t}{\tau}}}}+\underline{\underline{\int_0^t v_S(\lambda)\cdot e^{\frac{\lambda-t}{\tau}}d\frac{\lambda}{\tau}}}$$

$$i_L(t)=\underline{\underline{I_0e^{-\frac{t}{\tau}}}}+\underline{\underline{\int_0^t i_S(\lambda)\cdot e^{\frac{\lambda-t}{\tau}}d\frac{\lambda}{\tau}}}$$

# 零输入响应

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  - 给出电感电流放磁曲线，和放磁电压时域波形：表达式和曲线

$$i_L(t)=I_0e^{-\frac{t}{\tau}}$$

$$v_C(t)=V_0e^{-\frac{t}{\tau}}$$

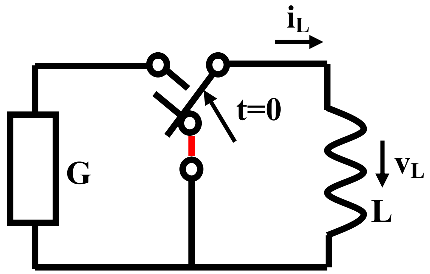
直接根据对偶关系写结果

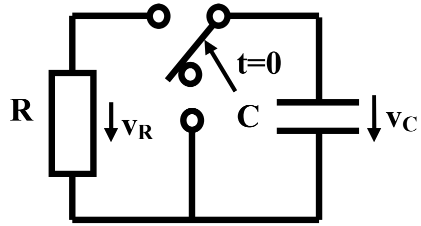
$$\tau=GL$$

$$\tau=RC$$

无需走求解微分方程的标准流程：直接由三要素给出最终结果

# 三要素法：通用于所有一阶LTI系统




$\tau = GL$ 时间常数 $\tau = RC$

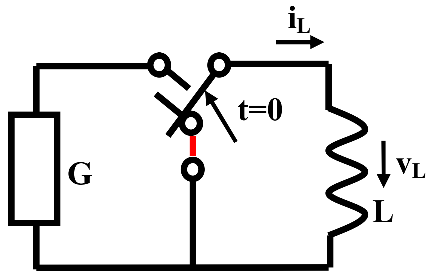
$i_L(0) = I_0$ 初值 $v_C(0) = V_0$

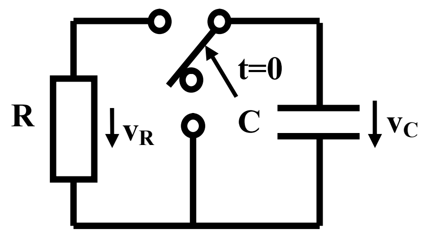
$i_{L\infty}(t) = 0$ 稳态解 $v_{C\infty}(t) = 0$

$$i_L(t) = i_{L\infty}(t) + \left(i_L(0) - i_{L\infty}(0)\right)e^{-\frac{t}{\tau}} = I_0 e^{-\frac{t}{\tau}}$$

$$v_C(t) = v_{C\infty}(t) + \left(v_C(0) - v_{C\infty}(0)\right)e^{-\frac{t}{\tau}} = V_0 e^{-\frac{t}{\tau}}$$

# 放磁电压、放电电流




电感磁通因放磁结点的存在以电压形式流失，有电压电感电流则改变

$$i_L(t)=\begin{cases} I_0 & t<0 \\ I_0 e^{-\frac{t}{\tau}} & t\geq 0 \end{cases}$$

电容电荷因放电回路的存在以电流形式流失，有电流电容电压则改变

$$v_C(t)=\begin{cases} V_0 & t<0 \\ V_0 e^{-\frac{t}{\tau}} & t\geq 0 \end{cases}$$

$$v_L(t)=L\frac{d}{dt}i_L(t)=\begin{cases} 0 & t<0 \\ -\frac{I_0}{G}e^{-\frac{t}{\tau}} & t\geq 0 \end{cases}$$

$$i_C(t)=C\frac{d}{dt}v_C(t)=\begin{cases} 0 & t<0 \\ -\frac{V_0}{R}e^{-\frac{t}{\tau}} & t\geq 0 \end{cases}$$

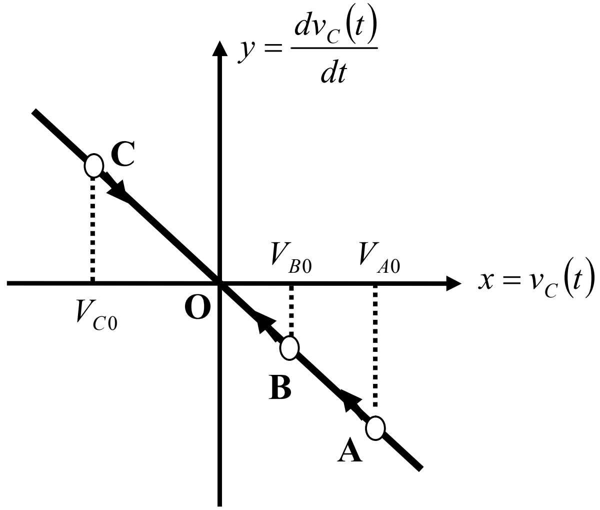
放磁电压　导致磁通流失，电感电流因而下降

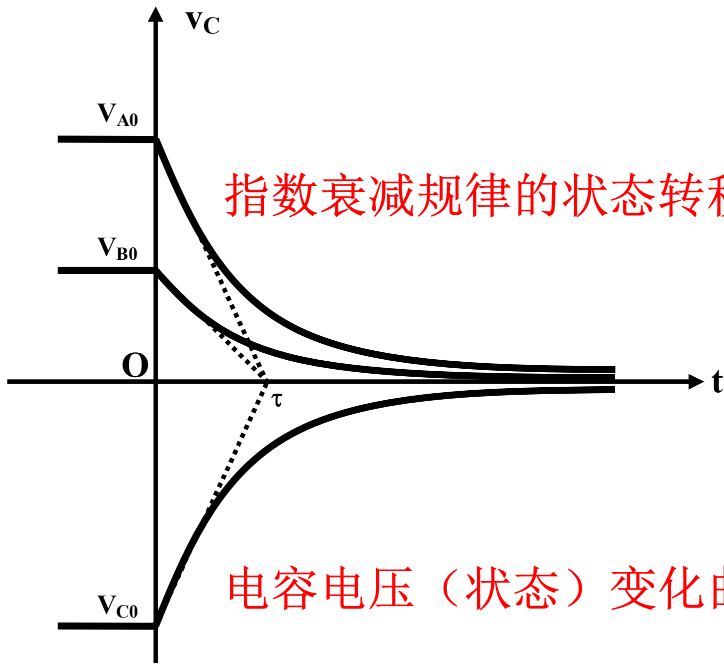
$$i_L(t)=I_0+\frac{1}{L}\int_0^t v_L(t)dt$$

放电电流　导致电荷流失，电容电压因而下降

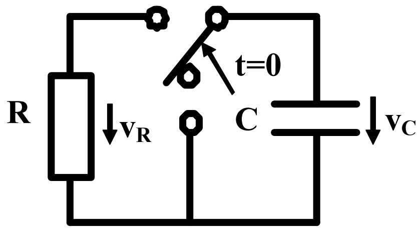
$$v_C(t)=V_0+\frac{1}{C}\int_0^t i_C(t)dt$$

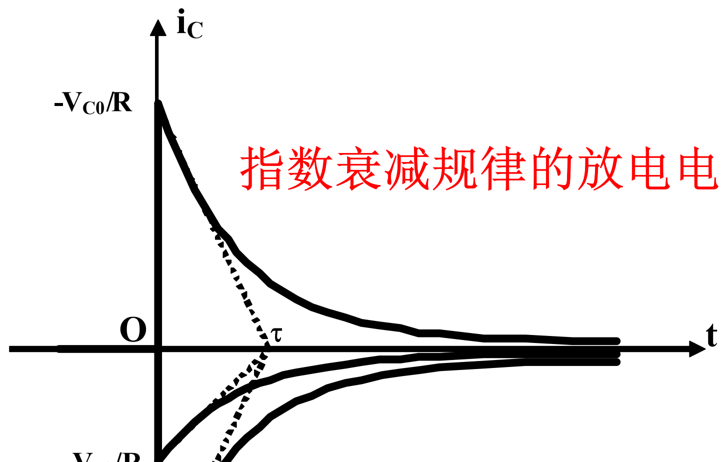
# 放电曲线



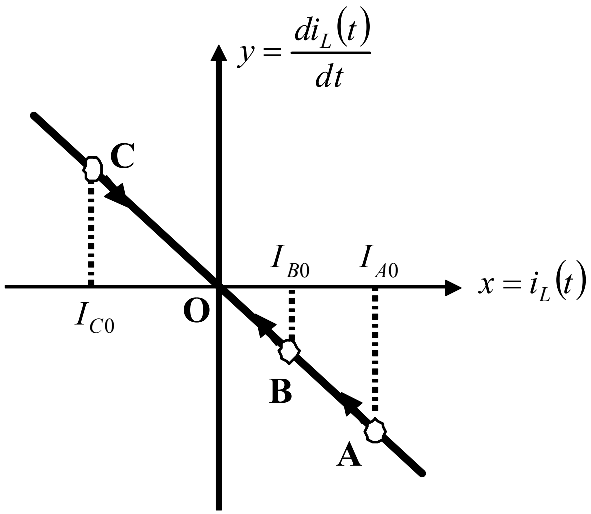

# 放电电流

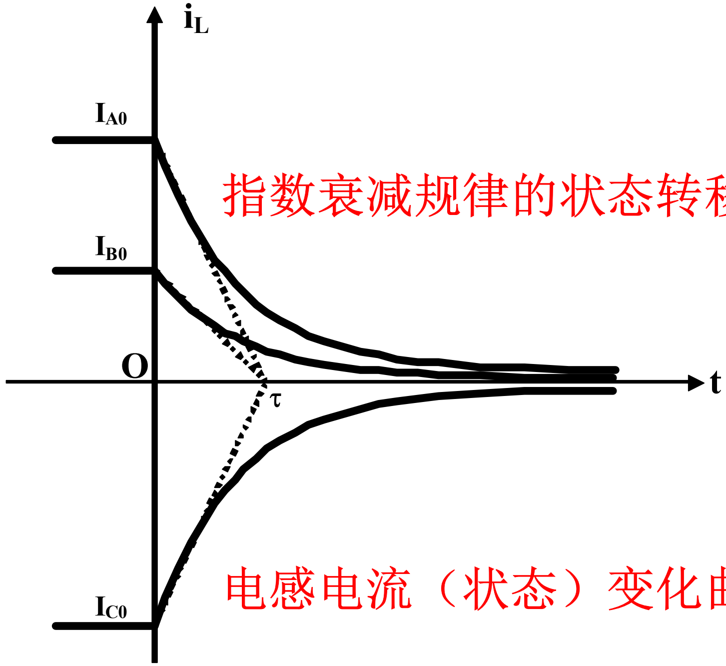



引起电容电压（状态）变化的原因是电容放电电流的存在，而电流却是因为电容电压加载电阻导致的

## 放磁曲线

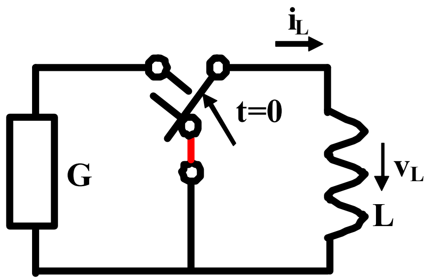


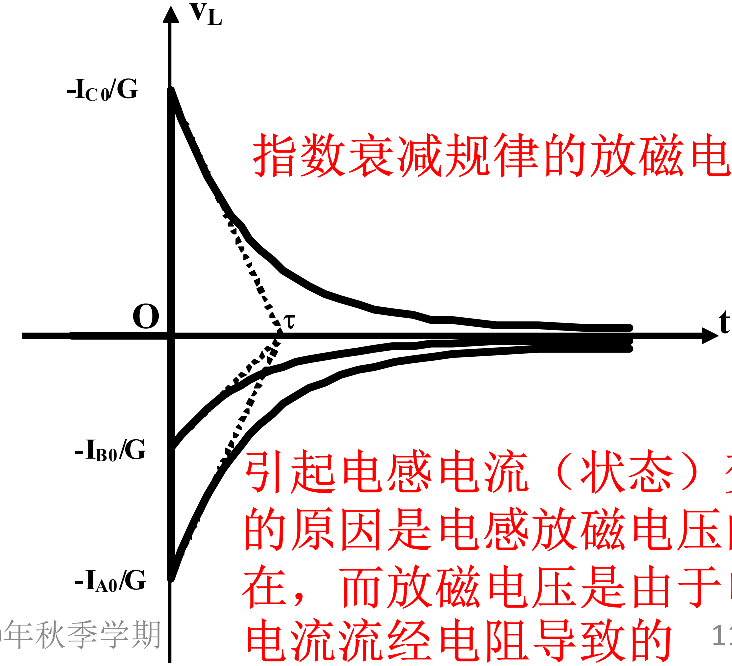




指数衰减规律的状态转移

电感电流（状态）变化曲线

## 放磁电压




指数衰减规律的放磁电压

引起电感电流（状态）变化的原因是电感放磁电压的存在，而放磁电压是由于电感电流流经电阻导致的

# 零状态响应：三要素法

- （2）练习9.2.4 分析图示一阶**RL**电路的零状态响应，假设开关在**t=0**时刻换路，开关换路前放磁已经结束，电感初始电流为**0**
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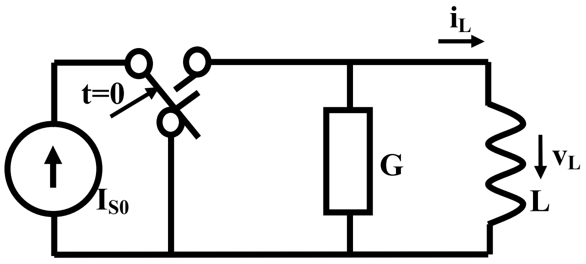
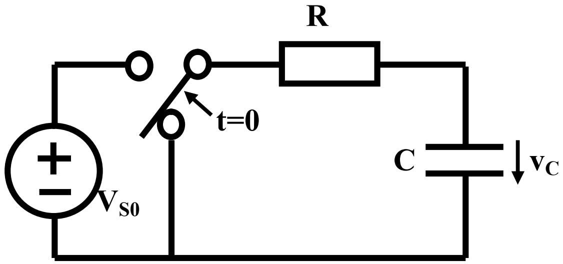


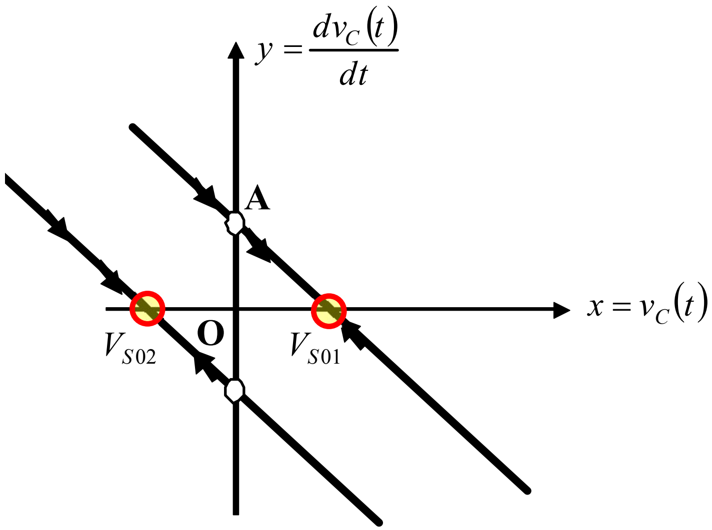



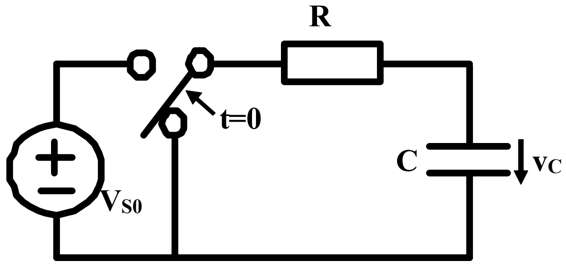

| | | |
|---|---|---|
| $\tau = GL$ | 时间常数 | $\tau = RC$ |
| $i_L(0) = 0$ | 初值 | $v_C(0) = 0$ |
| $i_{L\infty}(t) = I_{S0}$ | 稳态解 | $v_{C\infty}(t) = V_{S0}$ |

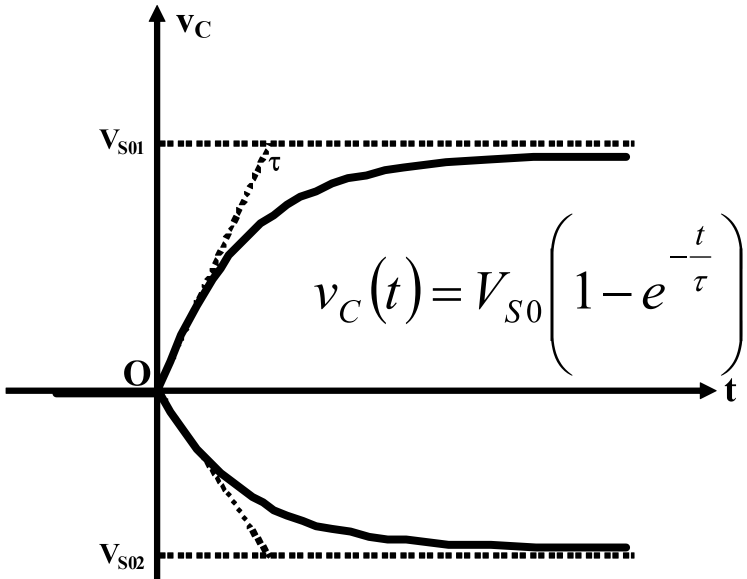
$$i_L(t) = i_{L\infty}(t) + \left(i_L(0) - i_{L\infty}(0)\right)e^{-\frac{t}{\tau}}$$
$$= I_{S0} - I_{S0}e^{-\frac{t}{\tau}} = I_{S0}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$v_C(t) = v_{C\infty}(t) + \left(v_C(0) - v_{C\infty}(0)\right)e^{-\frac{t}{\tau}}$$
$$= V_{S0} - V_{S0}e^{-\frac{t}{\tau}} = V_{S0}\left(1 - e^{-\frac{t}{\tau}}\right)$$

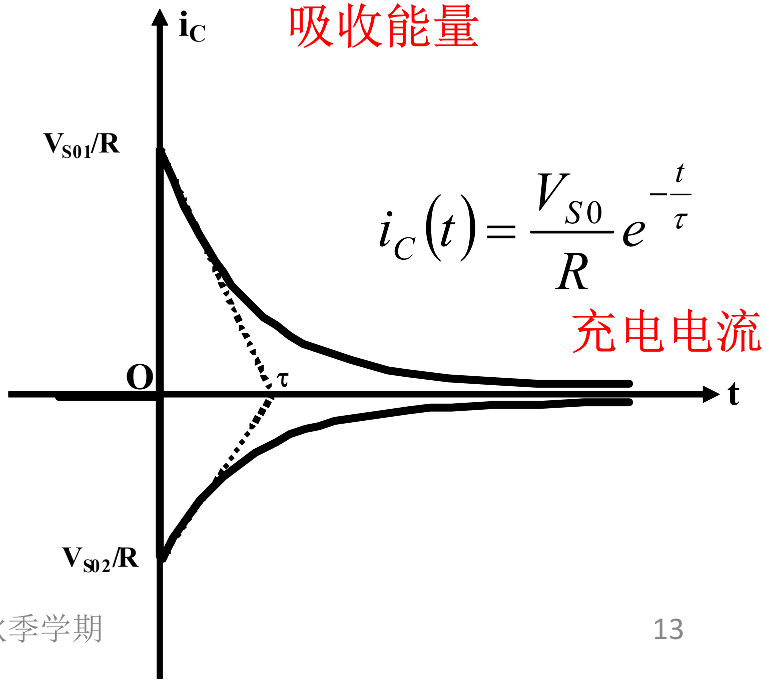
# 充电曲线、充电电流



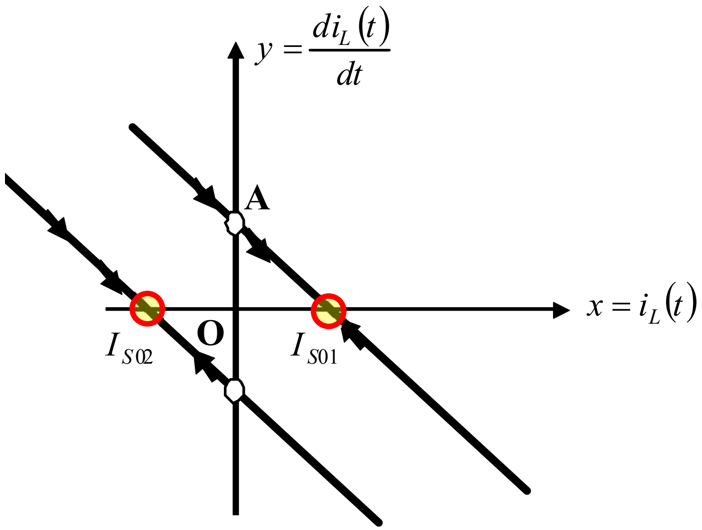



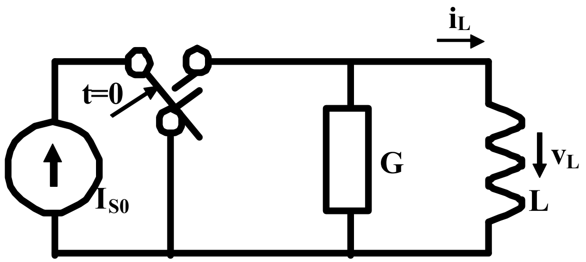

无论哪种情况，电源提供能量，电阻电容吸收能量

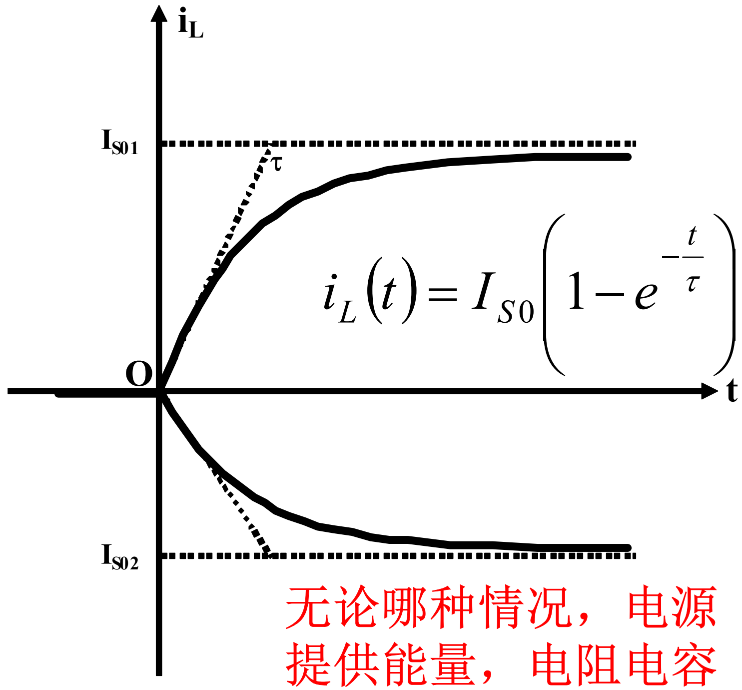



充电电流

# 充磁曲线、充磁电压

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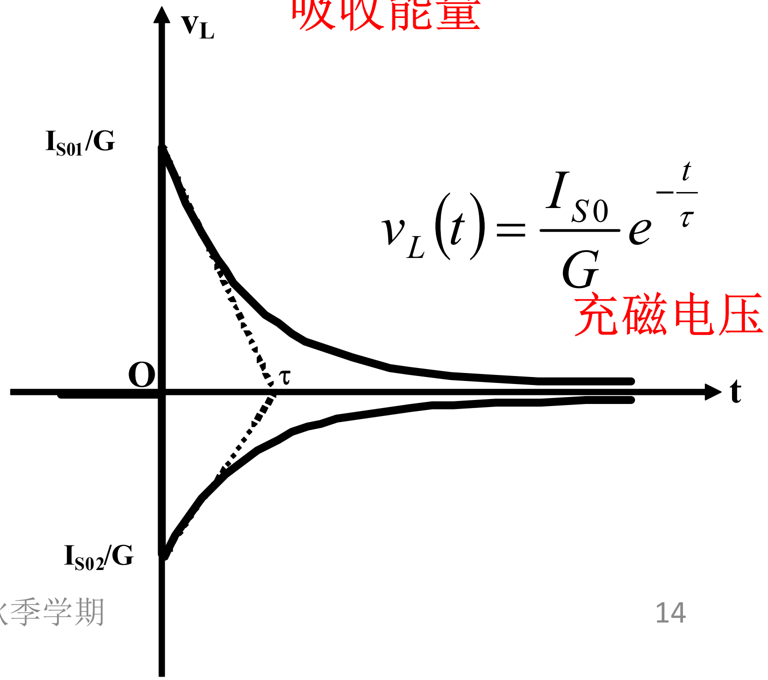


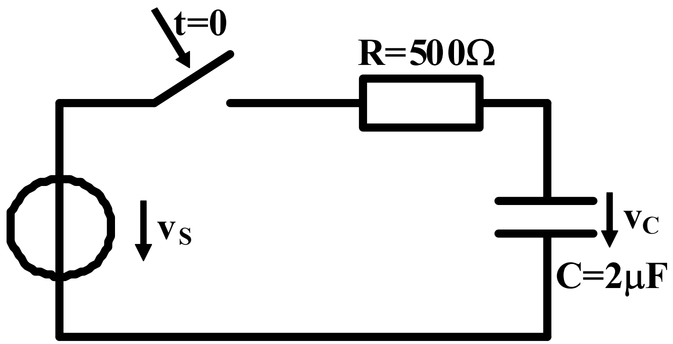
# 作业2 正弦激励

- 如图所示，t=0时刻开关闭合，正弦波电压激励源加载到一阶RC串联电路端口

$$v_S(t) = 2\cos\omega_0 t$$

  – 其中，$\omega_0 = 2\pi f_0 \quad f_0 = 500Hz$

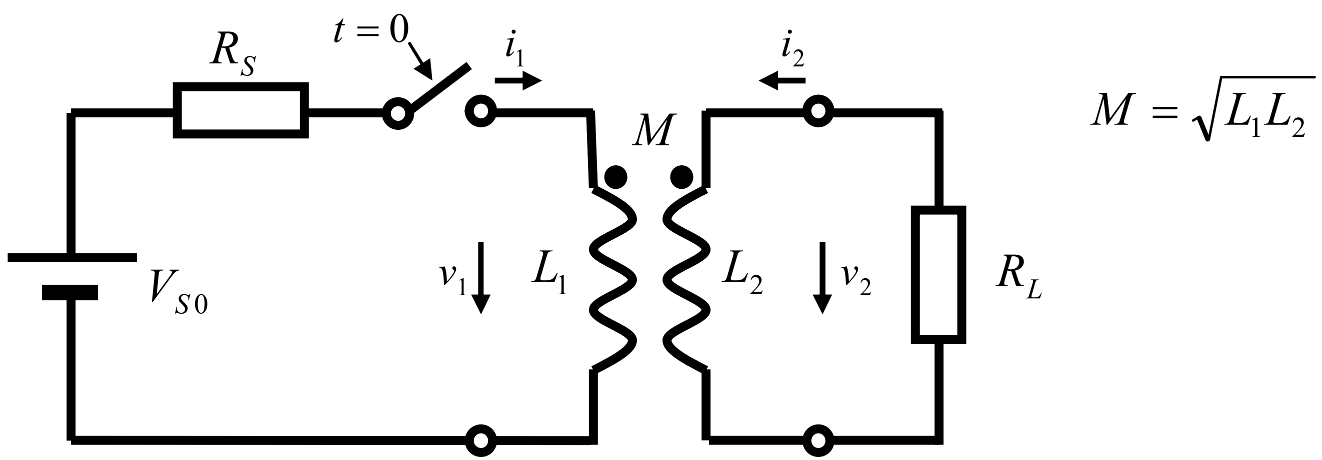
- 假设电容初始电压为0，$v_C(0)=0$，请给出电容电压时域表达式



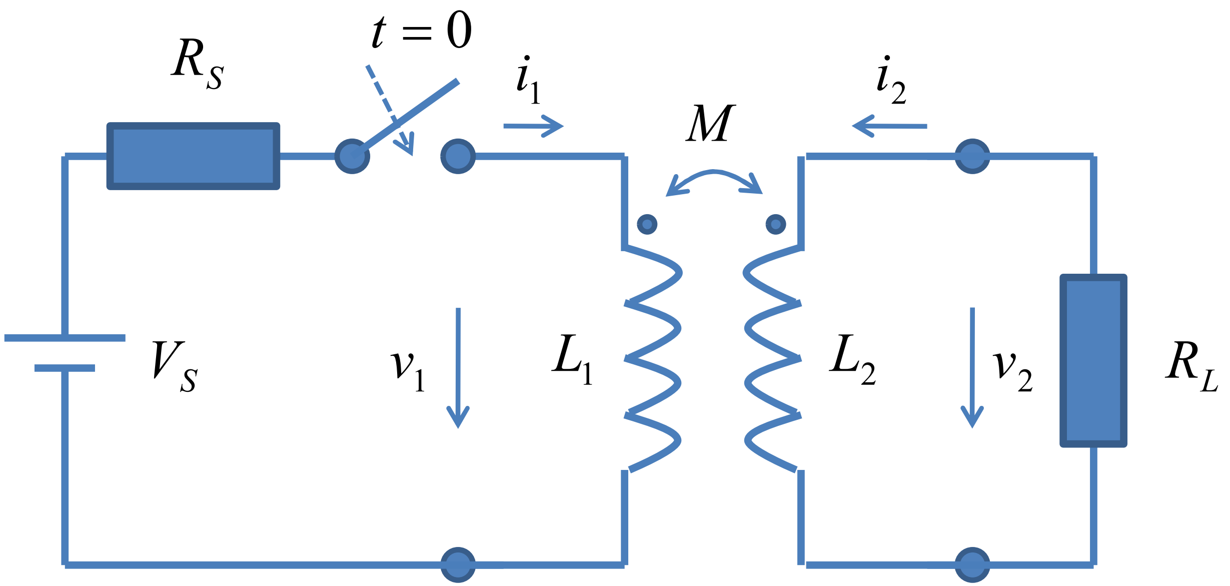

上上次习题课已讲
后向欧拉法仿真后，用三要素法给出理论结果

# 作业3 全耦合变压器是一阶元件

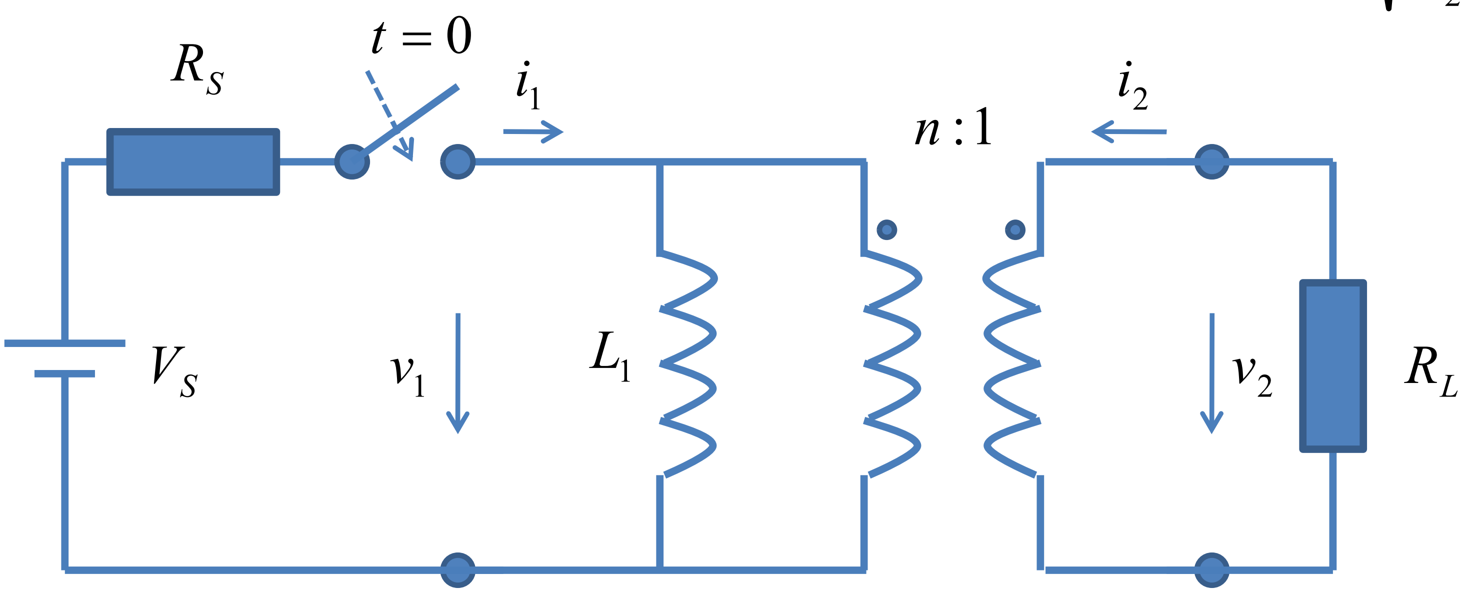
- 练习9.2.6 如图E9.2.9所示，这是一个全耦合变压器电路。开关在t=0时刻闭合，求变压器两个端口的电压时域表达式。



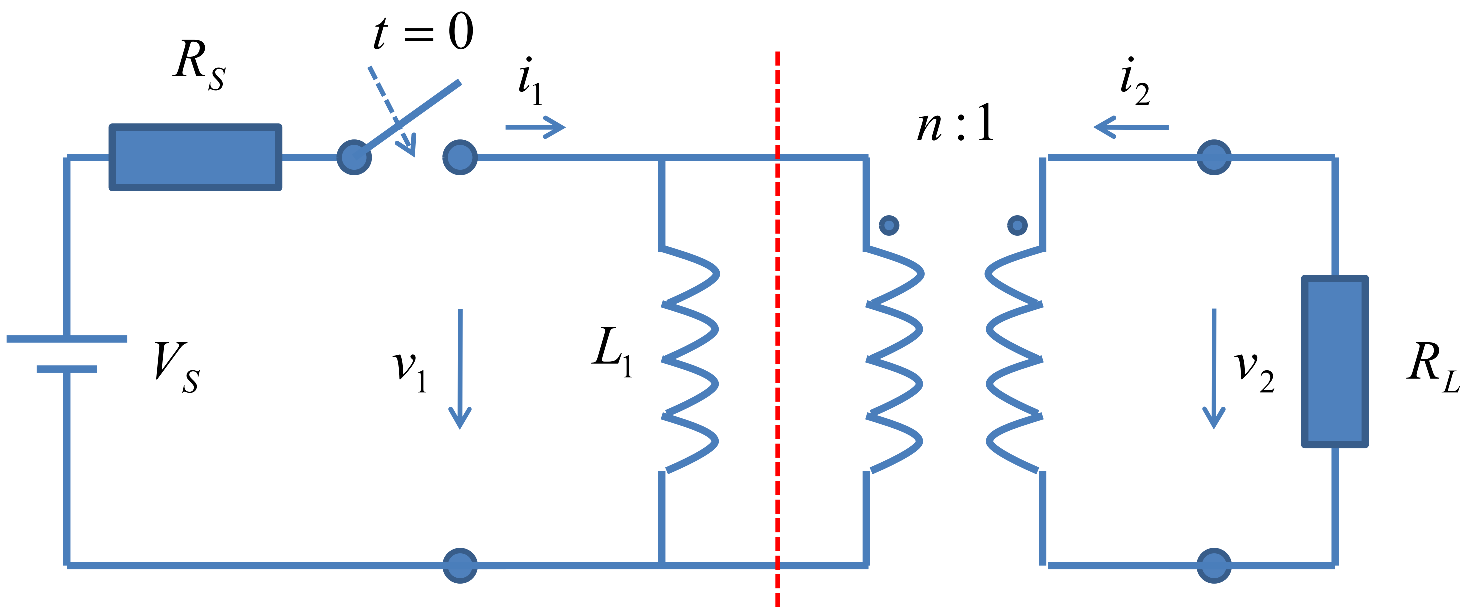
# 输入回路等效电路

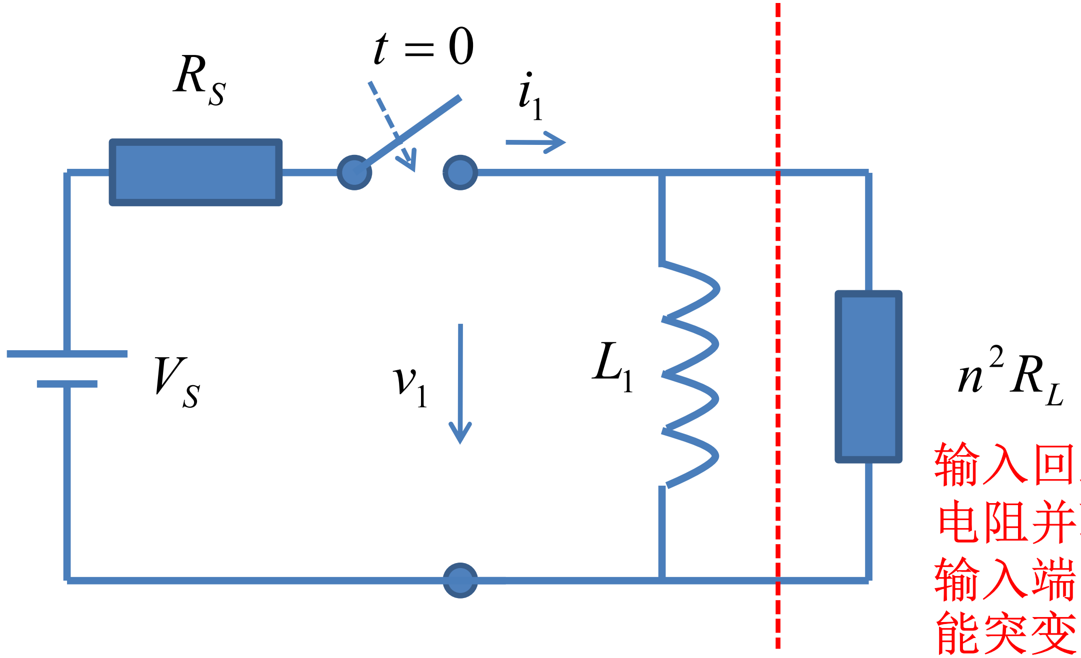



$$n=\sqrt{\frac{L_1}{L_2}}$$



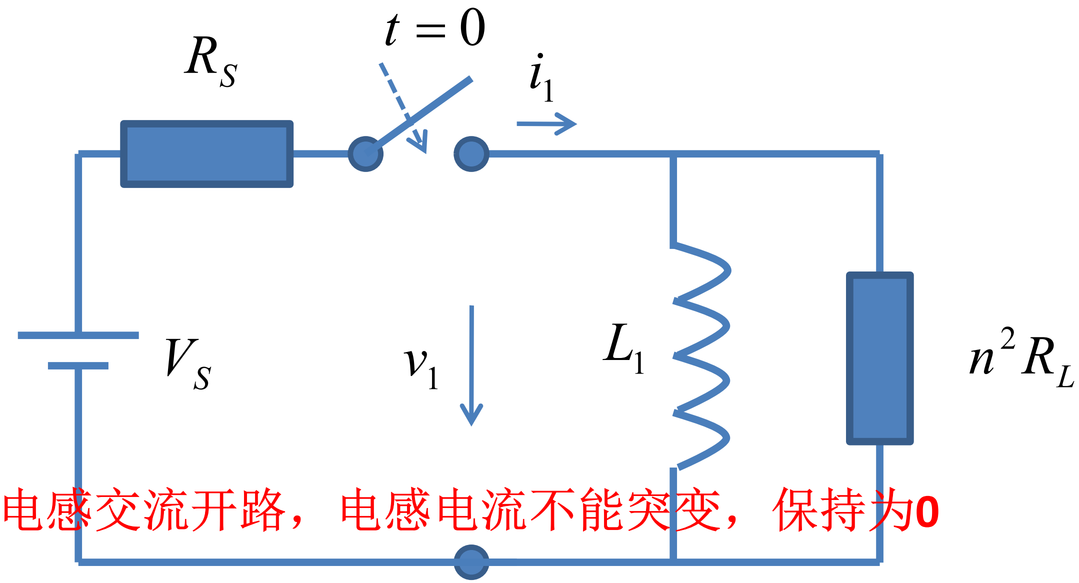

全耦合变压器为一阶电路




输入回路等效电路为电感和电阻并联：由于负载作用，输入端口电流可以突变：不能突变的是什么？

# 三要素法




$$i_1(0^+)=\frac{V_S}{R_S+n^2R_L}$$

开关闭合瞬间：电感交流开路，电感电流不能突变，保持为**0**

$$i_{1\infty}(t)=\frac{V_S}{R_S}$$

稳态：电感直流短路

$$\tau=GL_1=\left(\frac{1}{R_S}+\frac{1}{n^2R_L}\right)L_1$$

$$=\left(\frac{1}{R_S}+\frac{L_2}{L_1R_L}\right)L_1$$

$$=G_SL_1+G_LL_2=\tau_1+\tau_2$$

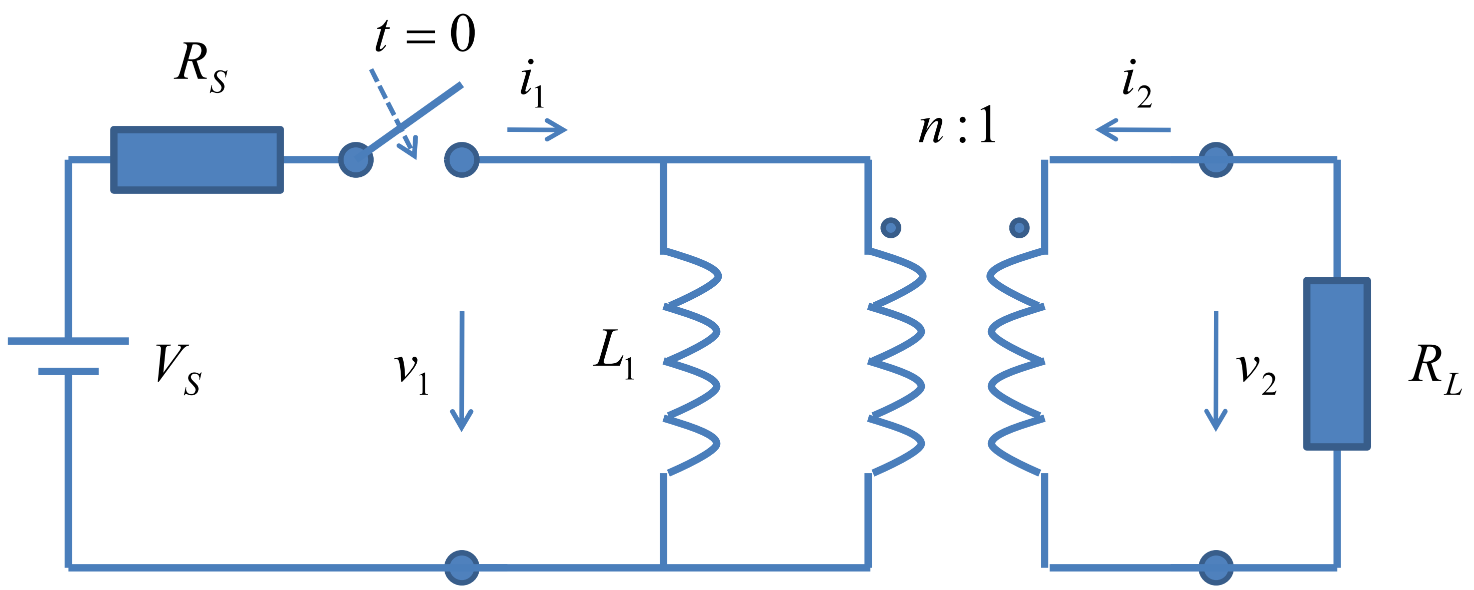
$$i_1(t)=i_{1\infty}(t)+\left(i_1(0^+)-i_{1\infty}(0^+)\right)e^{-\frac{t}{\tau}}$$

$$=\frac{V_S}{R_S}+\left(\frac{V_S}{R_S+n^2R_L}-\frac{V_S}{R_S}\right)e^{-\frac{t}{\tau}}\qquad U(t)$$

$$v_1(t)=v_{1\infty}(t)+\left(v_1(0^+)-v_{1\infty}(0^+)\right)e^{-\frac{t}{\tau}}$$

$$=\frac{n^2R_L}{R_S+n^2R_L}V_Se^{-\frac{t}{\tau}}\qquad U(t)$$

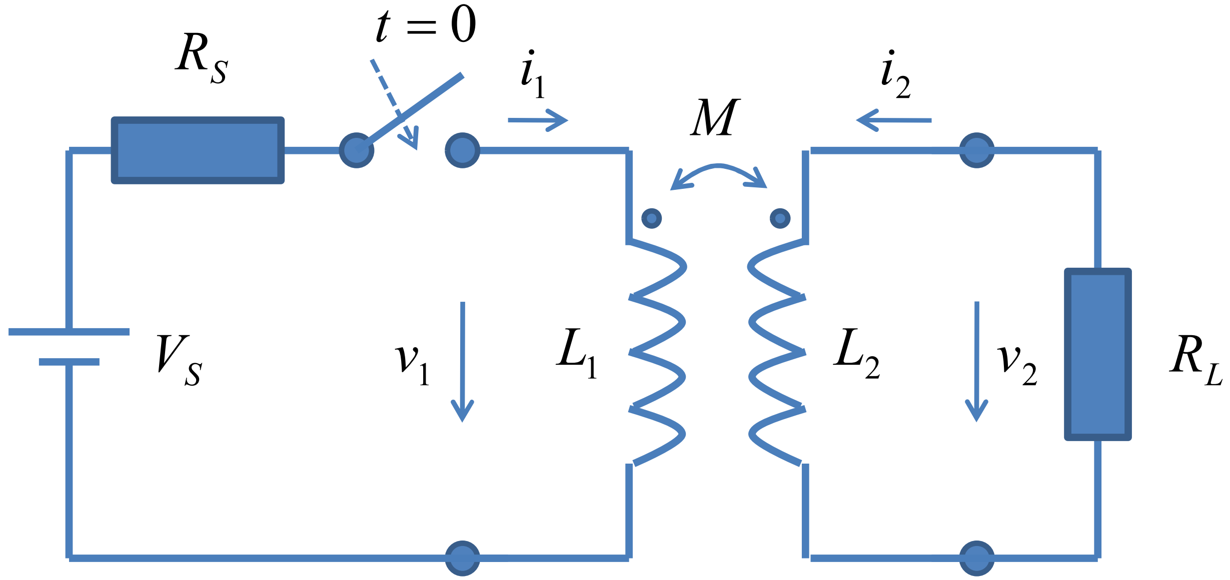
# 由输入回路压流直接获得输出回路压流



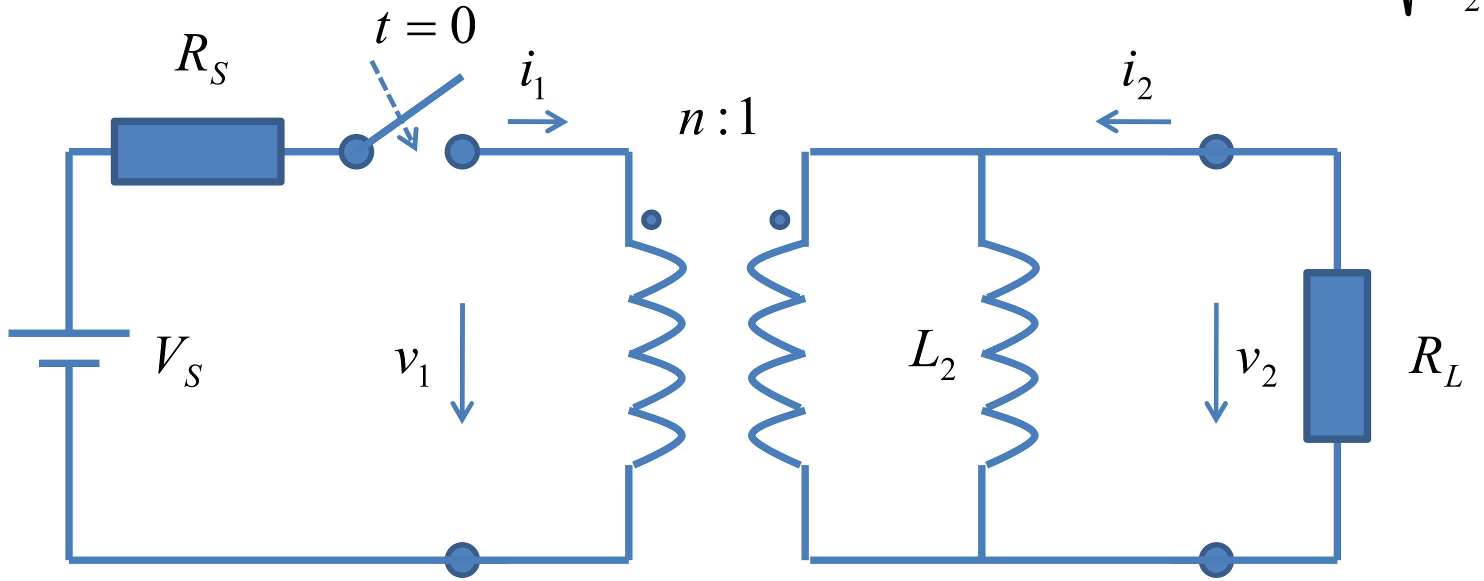

$$v_2(t)=\frac{1}{n}v_1(t)=\frac{nR_L}{R_S+n^2R_L}V_S e^{-\frac{t}{\tau}} \qquad U(t)$$

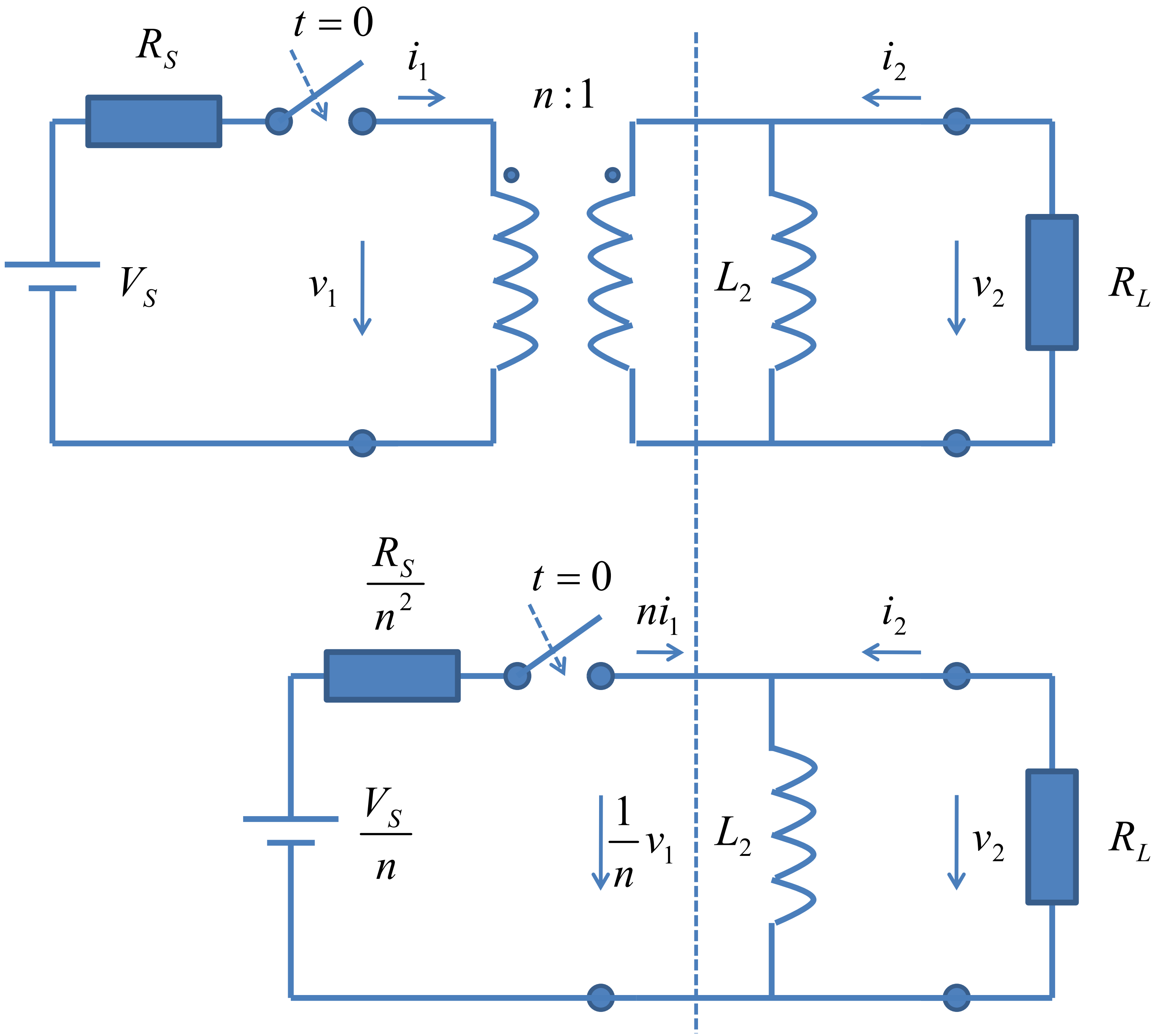
$$i_2(t)=-\frac{v_2(t)}{R_L}=-\frac{n}{R_S+n^2R_L}V_S e^{-\frac{t}{\tau}} \qquad U(t)$$

# 也可在输出回路等效分析



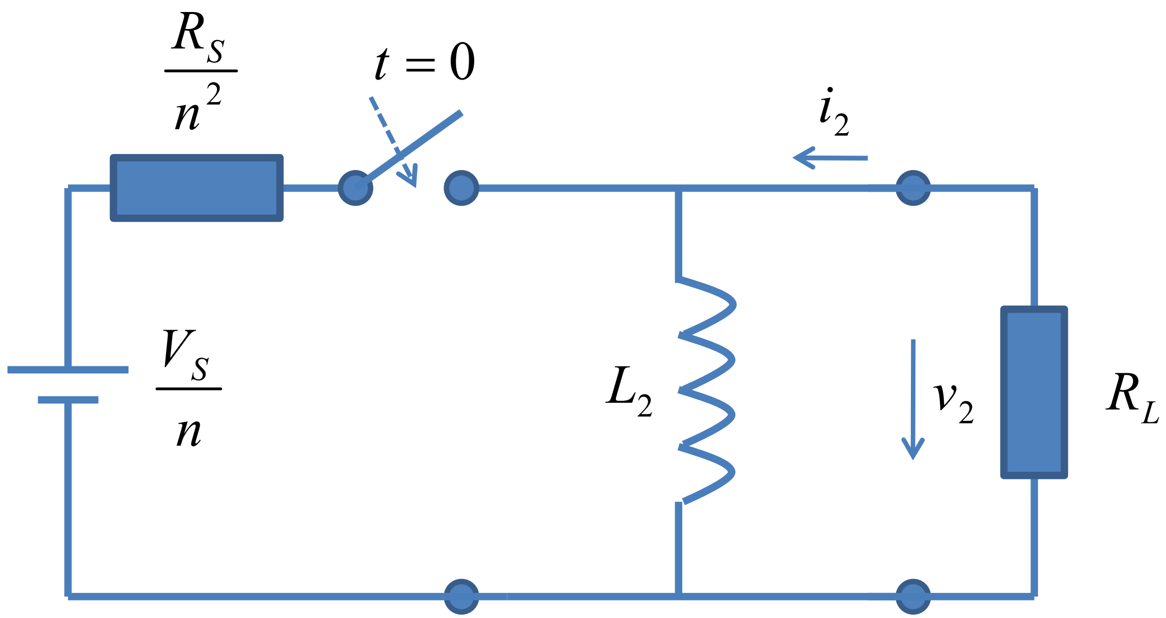

$$n=\sqrt{\frac{L_1}{L_2}}$$





$R_S$
$t=0$
$i_1$
$n:1$
$i_2$
$V_S$
$v_1$
$L_2$
$v_2$
$R_L$
$\frac{R_S}{n^2}$
$t=0$
$ni_1$
$i_2$
$\frac{V_S}{n}$
$\frac{1}{n}v_1$
$L_2$
$v_2$
$R_L$

# 三要素法




$$i_2(0^+) = -\frac{V_S/n}{R_S/n^2 + R_L}$$

$$i_{2\infty}(t) = 0$$

$$\tau = GL_2 = \left(\frac{n^2}{R_S} + \frac{1}{R_L}\right)L_2$$

$$= \left(\frac{L_1}{L_2 R_S} + \frac{1}{R_L}\right)L_2$$

$$= G_S L_1 + G_L L_2 = \tau_1 + \tau_2$$

无论如何等效，时间常数是一阶**LTI**系统的特征参量，不会发生改变

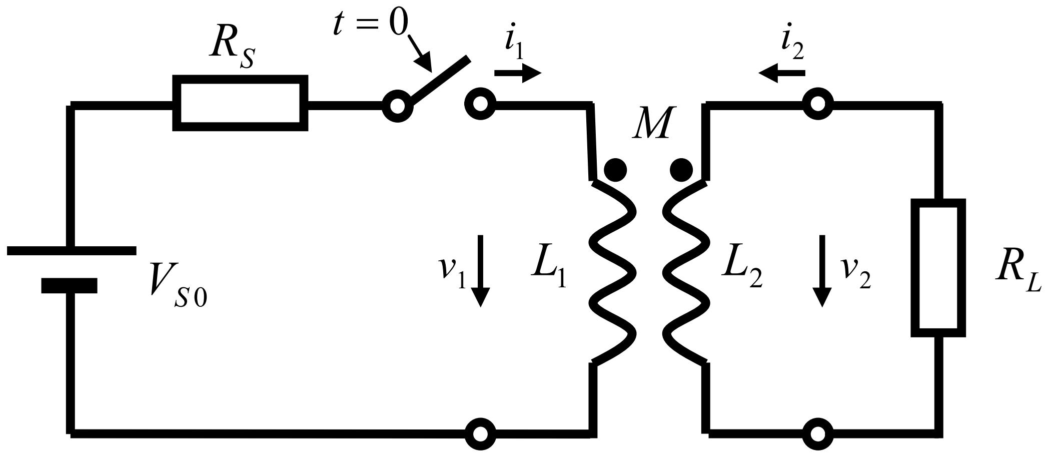
$$i_2(t) = i_{2\infty}(t) + \left(i_2(0^+) - i_{2\infty}(0^+)\right)e^{-\frac{t}{\tau}}$$

$$= -\frac{V_S/n}{R_S/n^2 + R_L}e^{-\frac{t}{\tau}} \qquad U(t)$$

无论如何等效，同一电量的最终表达式完全一致

$$v_2(t) = v_{2\infty}(t) + \left(v_2(0^+) - v_{2\infty}(0^+)\right)e^{-\frac{t}{\tau}}$$

$$= V_S/n\frac{R_L}{R_S/n^2 + R_L}e^{-\frac{t}{\tau}} \qquad U(t)$$




$$M=\sqrt{L_1L_2}$$

$$v_1(t)=\frac{n^2R_L}{R_S+n^2R_L}V_Se^{-\frac{t}{\tau}}\cdot U(t)$$

$$i_1(t)=\left(\frac{V_S}{R_S}+\left(\frac{V_S}{R_S+n^2R_L}-\frac{V_S}{R_S}\right)e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

$$v_2(t)=\frac{nR_L}{R_S+n^2R_L}V_Se^{-\frac{t}{\tau}}\cdot U(t)$$

$$i_2(t)=-\frac{n}{R_S+n^2R_L}V_Se^{-\frac{t}{\tau}}\cdot U(t)$$

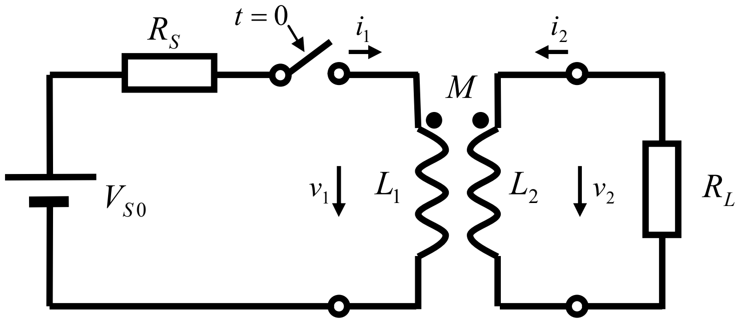
$$i_1(0^-)=0 \qquad i_1(0^+)=\frac{V_S}{R_S+n^2R_L}\neq 0$$

$$i_2(0^-)=0 \qquad i_2(0^+)=-\frac{nV_S}{R_S+n^2R_L}\neq 0$$

电感电流不是不能突变吗？

注意：这是二端口电感，具有阻抗变换作用，对于全耦合变压器，端口看入瞬间阻抗并非无穷大

# 非阶跃电流源充磁
# 电感储能不会突变




$M=\sqrt{L_1L_2}$

$$E_M(0^+)=0=E_M(0^-)$$

由于互感变压器吸收能量需要时间，故而存在瞬态过程

除非有冲激电压，无穷大功率，才能完成能量从电源到电感的瞬间转移。

$$p_M(0^+)=0=p_M(0^-)$$

**t=0**瞬间，电感无穷大，视为理想变压器，全耦合变压器在**t=0**瞬间是理想传输系统：端口**1**吸收的功率在端口**2**瞬间全部释放出去，变压器本身没有吸收功率

$$p_M(t)=v_1(t)i_1(t)+v_2(t)i_2(t)$$

$$=\frac{n^2R_L}{R_S+n^2R_L}V_Se^{-\frac{t}{\tau}}\cdot\left(\frac{V_S}{R_S}+\left(\frac{V_S}{R_S+n^2R_L}-\frac{V_S}{R_S}\right)e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

$$+\frac{nR_L}{R_S+n^2R_L}V_Se^{-\frac{t}{\tau}}\cdot\left(-\frac{n}{R_S+n^2R_L}V_Se^{-\frac{t}{\tau}}\right)\cdot U(t)$$

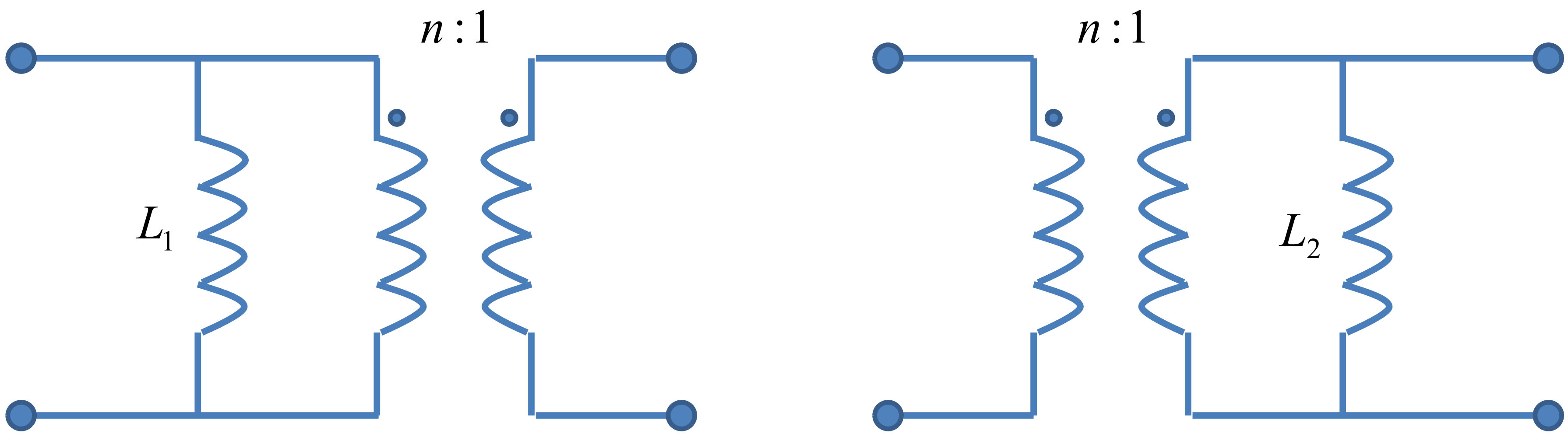
$$=\frac{V_S^2}{R_S}\frac{n^2R_L}{R_S+n^2R_L}\left(e^{-\frac{t}{\tau}}-e^{-2\frac{t}{\tau}}\right)\cdot U(t)$$

$$E_M(t)=\int_{-\infty}^{t}p_M(t)dt=\int_0^t\frac{V_S^2}{R_S}\frac{n^2R_L}{R_S+n^2R_L}\left(e^{-\frac{t}{\tau}}-e^{-2\frac{t}{\tau}}\right)dt=\frac{V_S^2}{R_S}\frac{n^2R_L}{R_S+n^2R_L}\left(-\tau\cdot\left(e^{-\frac{t}{\tau}}-1\right)+\frac{\tau}{2}\left(e^{-2\frac{t}{\tau}}-1\right)\right)$$

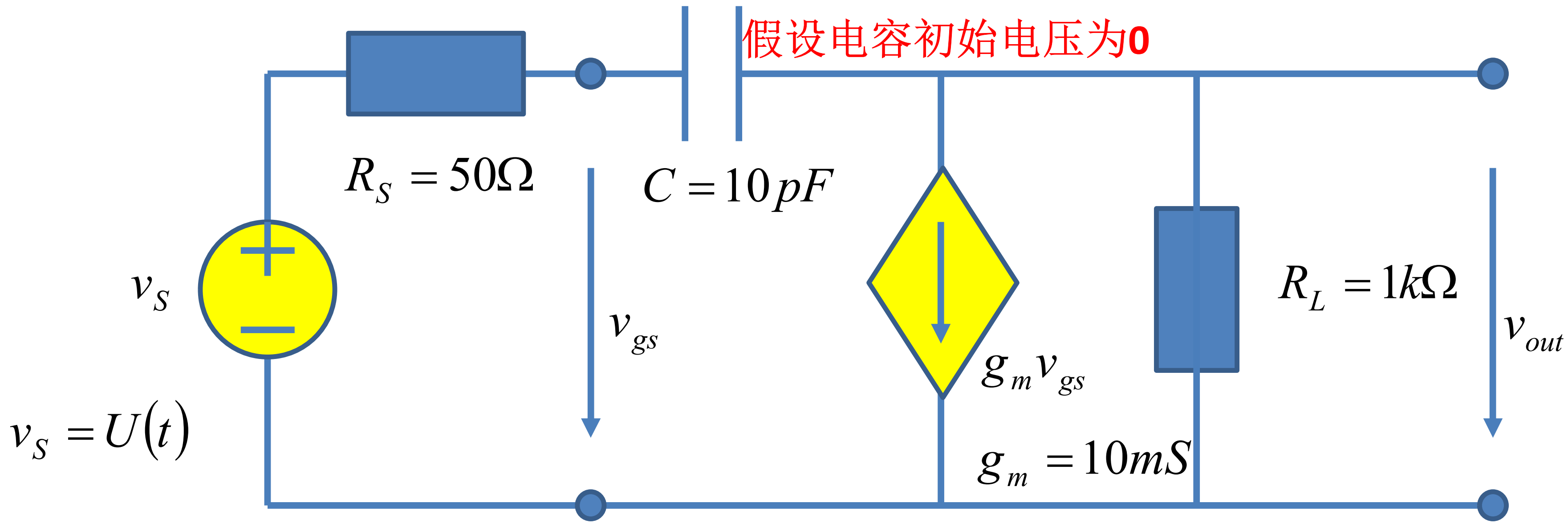
$$E_M(\infty)=\frac{V_S^2}{R_S}\frac{n^2R_L}{R_S+n^2R_L}\frac{\tau}{2}=\frac{V_S^2}{R_S}\frac{L_1R_L}{L_2R_S+L_1R_L}\frac{G_SL_1+G_LL_2}{2}=\frac{1}{2}L_1\frac{V_S^2}{R_S^2}=\frac{1}{2}L_1I_{1\infty}^2$$

# 全耦合变压器

- 储能在两个端口随意互换
  - 自行练习：开关闭合稳定后，开关又断开，请分析变压器两端电压变化曲线？



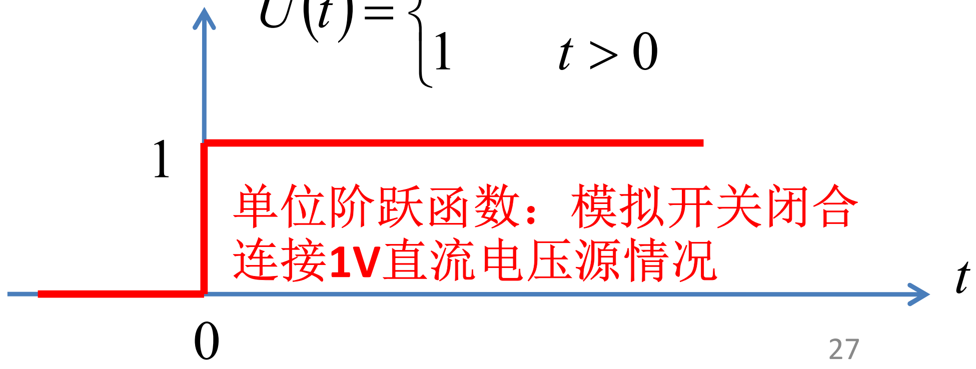
# 作业4 三要素法适用一阶RC电路



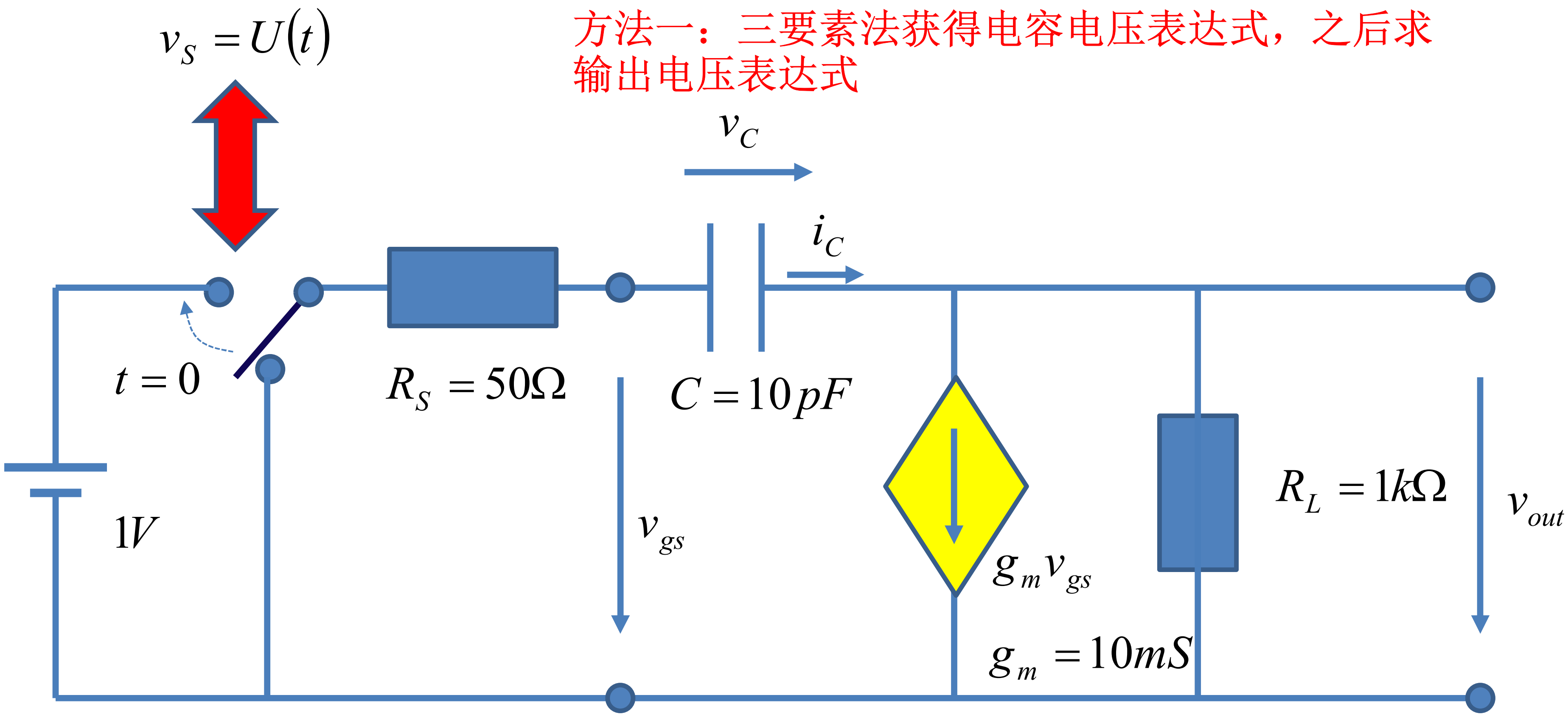

方法一、用三要素法获得电容时域波形**$v_C$(t)**，进而获得输出电压时域波形**$v_{out}$(t)**，表达式和曲线

方法二（选作）、用三要素法直接获得输出电压时域波形**$v_{out}$(t)**

$$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



方法一：三要素法获得电容电压表达式，之后求输出电压表达式




$$v_C(0^+) = v_C(0^-) = 0$$

电容上的电压正常情况下不会跳变：它是连续的

$$v_{C\infty}(t) = ?$$

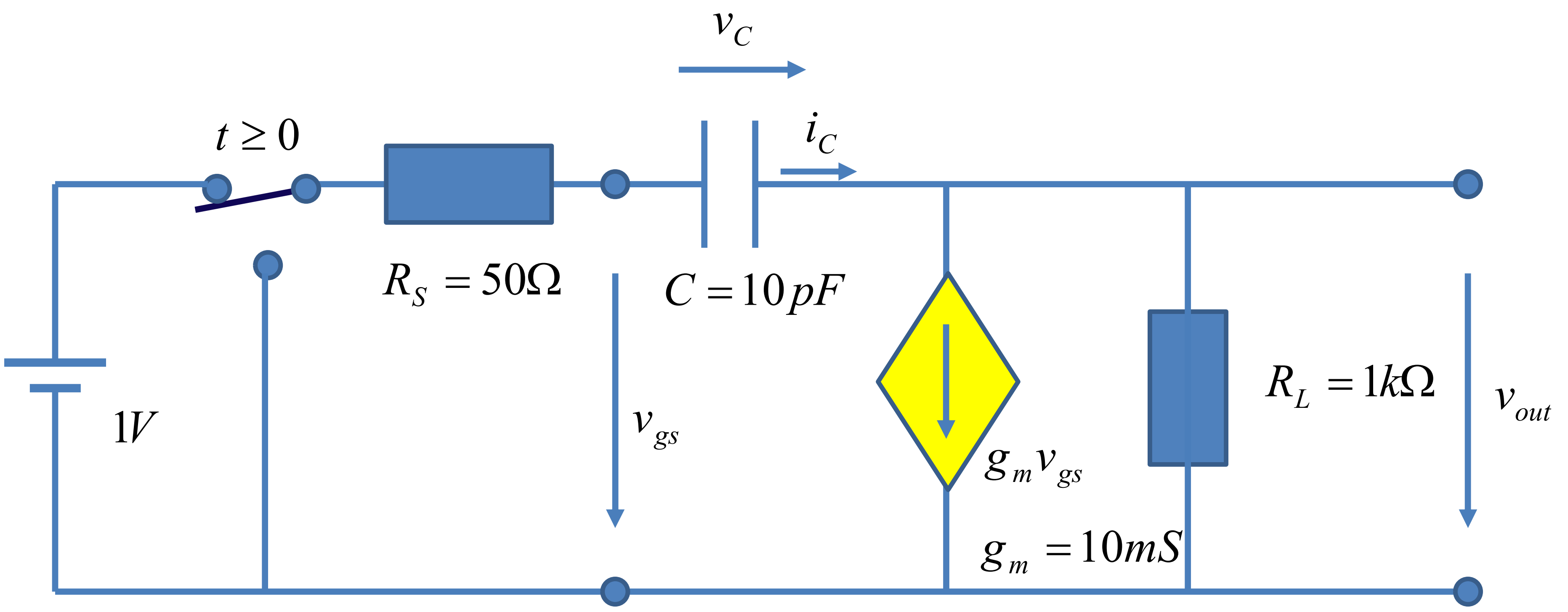
$$i_{C\infty}(t) = 0$$

**t=∞则直流电容开路**

$$v_{gs\infty}(t) = V_{S0} = 1(V)$$

$$g_m v_{gs\infty}(t) = 10mS \cdot 1V = 10mA$$

$$v_{out\infty}(t) = -(g_m v_{gs\infty}(t))R_L = -10mA \cdot 1k\Omega = -10V$$




$$v_{gs\infty}(t) = 1(V)$$

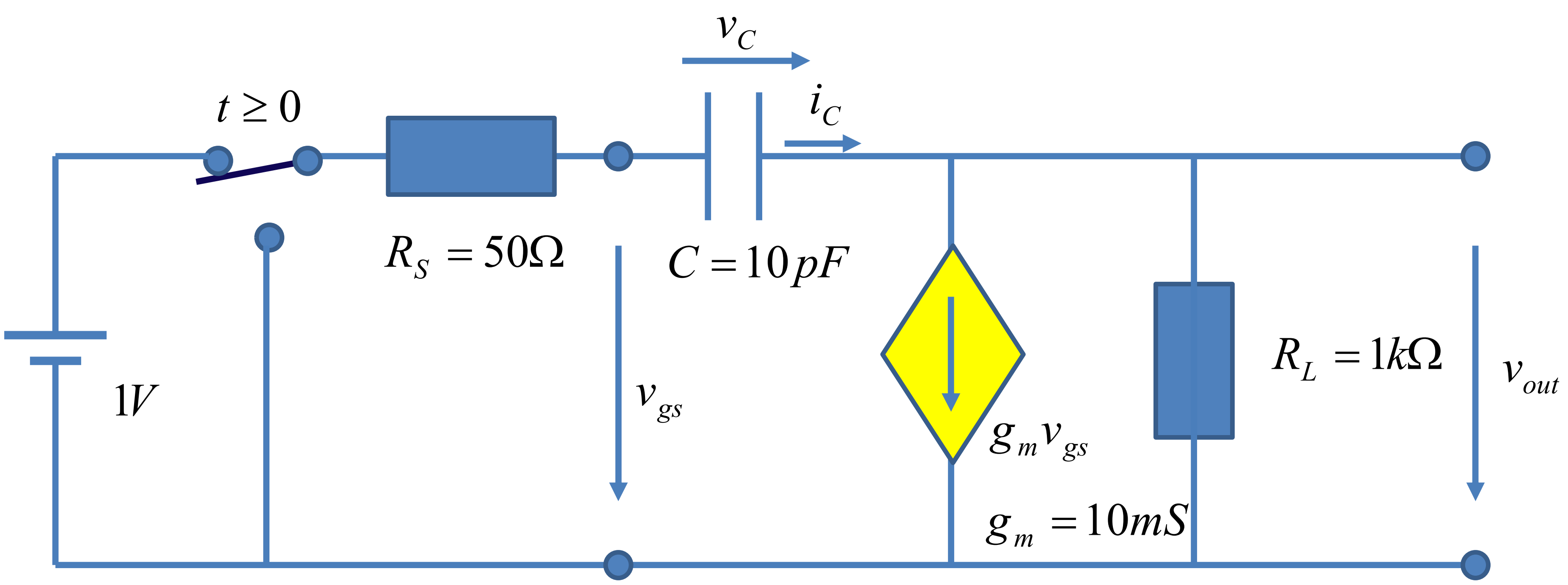
$$v_{out\infty}(t) = -10(V)$$

$$v_{C\infty}(t) = v_{gs\infty}(t) - v_{out\infty}(t) = 11(V)$$

$$\tau = ?R_{eq}C$$

$$R_{eq} = R_S + R_L + g_m R_S R_L$$
$$= 50 + 1000 + 10 \times 10^{-3} \cdot 50 \cdot 1000$$
$$= 1550(\Omega)$$

$$\tau = R_{eq}C = 1550 \times 10 \times 10^{-12} = 15.5(ns)$$



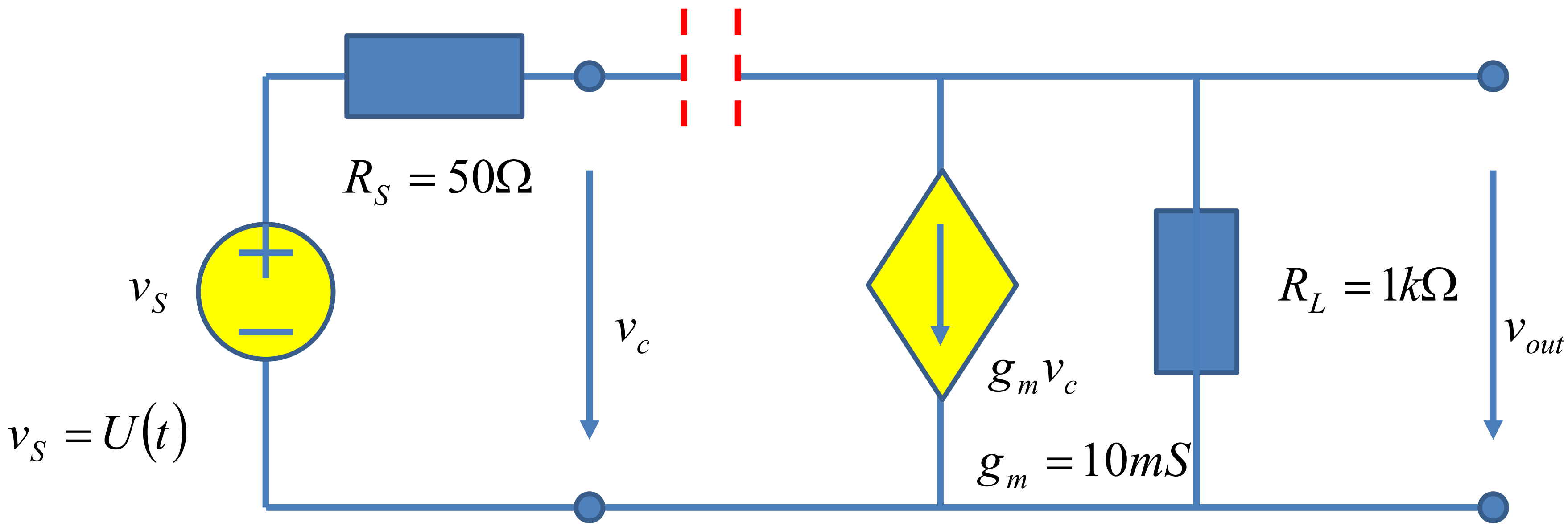

$v_C(0^+) = 0 \qquad v_{C\infty}(t) = 11(V) \qquad \tau = 15.5(ns)$

$$v_C(t) = v_{C\infty}(t) + \left(v_C(0^+) - v_{C\infty}(0^+)\right)e^{-\frac{t}{\tau}} = 11\left(1 - e^{-\frac{t}{15.5\times10^{-9}}}\right)$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = 7e^{-\frac{t}{15.5\times10^{-9}}}(mA)$$

$$v_{out}(t) = v_S(t) - i_C(t)R_S - v_C(t) = -10 + 10.65e^{-\frac{t}{15.5\times10^{-9}}}(V) \qquad (t \geq 0)$$

# 电容的影响




上学期电阻电路结果

输出是输入的即时响应：无记忆

$$v_{out}(t) = -g_m R_L v_S(t) = -10U(t)$$

本学期动态电路结果

=稳态+瞬态：有记忆

$$v_{out}(t) = \left(-10 + 10.65e^{-\frac{t}{15.5\times10^{-9}}}\right)U(t)$$

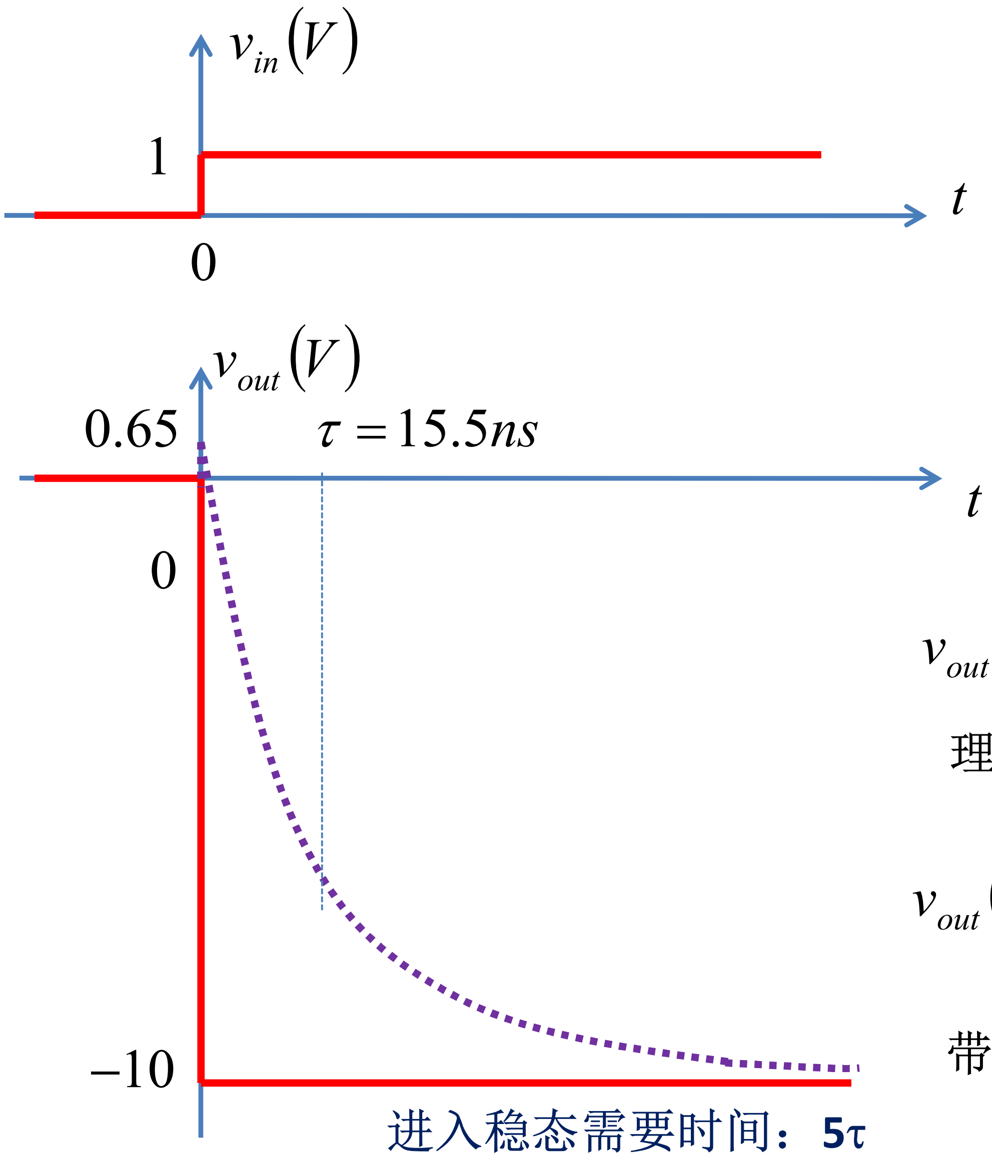
稳态响应　瞬态响应

在阶跃电压激励下：

稳态响应就是电阻电路结果

（电容开路，电感短路结果）

# 电容的影响：产生延时




$$v_{in}(t) = U(t)$$

输出想要达到理想的稳态，需要等待一段时间：这个等待时间一般称为延时

$$v_{out}(t) = -10U(t)$$

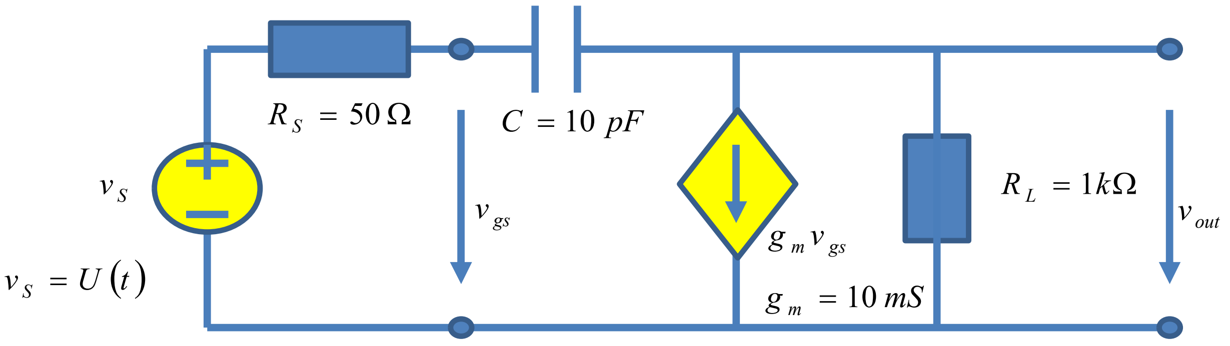
理想反相放大器输出响应

$$v_{out}(t) = \left(-10 + 10.65e^{-\frac{t}{15.5\times10^{-9}}}\right)U(t)$$

带跨接寄生电容的反相放大器输出响应

# 方法2：直接三要素

$$v_o(t)=v_{o\infty}(t)+\left(v_o(0^+)-v_{o\infty}(0^+)\right)e^{-\frac{t}{\tau}}\quad (t\ge 0)$$




$$\tau = CR_{eq} = C\left(R_S + R_L + g_m R_S R_L\right) = 15.5ns$$

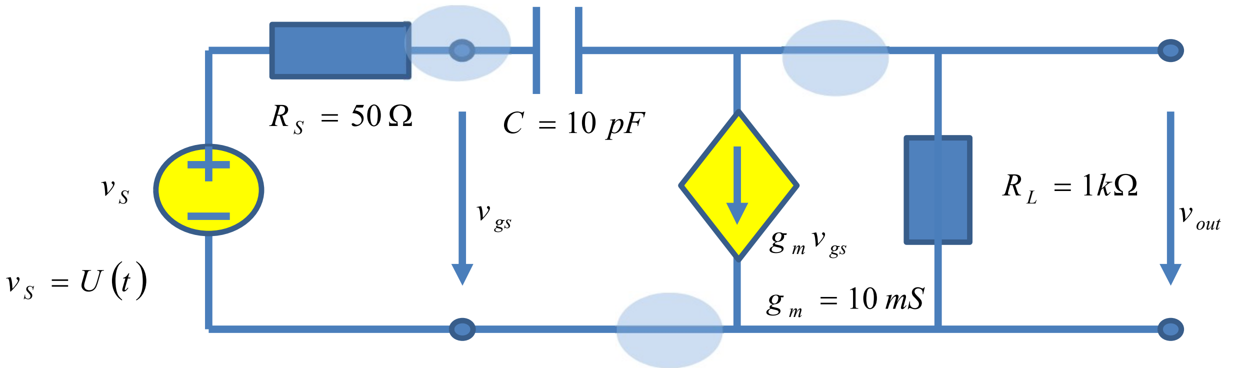
$$v_{o\infty}(t) = -g_m R_L U(t) = -10U(t) \qquad v_{o\infty}(0^+) = -g_m R_L = -10$$

$$v_o(0^+) = \frac{\frac{1}{g_m} \parallel R_L}{R_S + \frac{1}{g_m} \parallel R_L}\cdot 1 = \frac{\frac{1}{g_m+G_L}}{R_S + \frac{1}{g_m+G_L}} = \frac{R_L}{R_{eq}} = 0.645$$

$$v_o(t) = -10 + 10.645e^{-\frac{t}{\tau}} \qquad (t \ge 0)$$

# 方法3：微分方程求解

如果掌握了三要素法，就不要采用微分方程求解
这里列写这个过程，只是说明晶体管放大器跨接电容的作用！




结点电压法实质：以两个结点电压为未知量，列写两个结点的**KCL**方程

$$\frac{v_S - v_{gs}}{R_S} = i_C = C\frac{dv_C}{dt} = C\frac{d}{dt}\left(v_{gs} - v_{out}\right)$$

$$C\frac{d}{dt}\left(v_{gs} - v_{out}\right) = i_C = g_m v_{gs} + \frac{v_{out}}{R_L}$$

两个结点电压$\mathbf{v_{gs}}$和$\mathbf{v_{out}}$为未知量
两个结点的**KCL**方程

# 关于$v_{out}$的电路方程

$$\frac{v_S - v_{gs}}{R_S} = i_C = C\frac{dv_C}{dt} = C\frac{d}{dt}\left(v_{gs} - v_{out}\right)$$

$$C\frac{d}{dt}\left(v_{gs} - v_{out}\right) = i_C = g_m v_{gs} + \frac{v_{out}}{R_L}$$

$$\frac{v_S - v_{gs}}{R_S} = i_C = g_m v_{gs} + \frac{v_{out}}{R_L}$$

$$v_{gs} = \frac{\frac{v_S}{R_S} - \frac{v_{out}}{R_L}}{g_m + \frac{1}{R_S}}$$

$$C\frac{d}{dt}\left(\frac{\frac{v_S}{R_S} - \frac{v_{out}}{R_L}}{g_m + \frac{1}{R_S}} - v_{out}\right) = g_m \frac{\frac{v_S}{R_S} - \frac{v_{out}}{R_L}}{g_m + \frac{1}{R_S}} + \frac{v_{out}}{R_L}$$

$$\left(R_S + R_L + g_m R_S R_L\right)C\frac{dv_{out}}{dt} + v_{out} = R_L C\frac{dv_S}{dt} - g_m R_L v_S$$

$$\frac{dv_{out}}{dt} = -\frac{1}{R_{eq}C}v_{out} + \frac{R_L}{R_{eq}}\frac{dv_S}{dt} - \frac{g_m R_L}{R_{eq}C}v_S$$

$$\frac{dx(t)}{dt} = -\frac{1}{\tau}x(t) + s(t)$$

状态方程的一般格式

$$(R_S + R_L + g_m R_S R_L)C\frac{dv_{out}}{dt} + v_{out} = R_L C\frac{dv_S}{dt} - g_m R_L v_S \qquad v_S(t) = U(t)$$

$$\tau\frac{dv_{out}(t)}{dt} + v_{out}(t) = R_L C\cdot\delta(t) - g_m R_L\cdot U(t)$$

对比

$$RC\frac{dv_C(t)}{dt} + v_C(t) = v_S(t)$$

省略中间的积分过程

一阶RC电路

电阻电压+电容电压=电源电压

$$h_{RC,lowpass}(t) = \frac{1}{\tau}e^{-\frac{t}{\tau}}\cdot U(t)$$

$$g_{RC,lowpass}(t) = \left(1 - e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

$$v_{out}(t) = R_L C\cdot h_{RC}(t) - g_m R_L\cdot g_{RC}(t)$$

$$= R_L C\cdot\underline{\underline{\frac{1}{\tau}e^{-\frac{t}{\tau}}\cdot U(t)}} - g_m R_L\cdot\underline{\underline{\left(1 - e^{-\frac{t}{\tau}}\right)\cdot U(t)}}$$

$$v_{out}(t) = \left[R_L C\cdot\frac{1}{R_{eq}C}e^{-\frac{t}{\tau}} - g_m R_L\cdot\left(1 - e^{-\frac{t}{\tau}}\right)\right]\cdot U(t)$$

$$= \left[-g_m R_L + \left(\frac{R_L}{R_{eq}} - (-g_m R_L)\right)\cdot e^{-\frac{t}{\tau}}\right]\cdot U(t)$$

$$= \left[-10 + (0.65 - (-10))\cdot e^{-\frac{t}{\tau}}\right]\cdot U(t)$$

稳态响应　初值　指数衰减

虽然解形式一致，但问题变得复杂化了

不论是冲激、阶跃、正弦、方波，均可采用三要素法

跨接电容：低通+高通

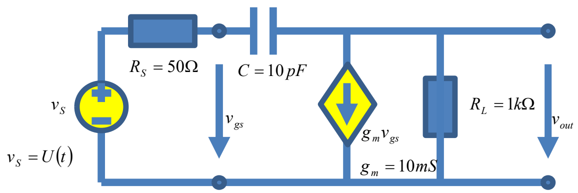
# 方法4 时频对应法

频域分析

$$\frac{\dot V_{out}}{\dot V_S}=\frac{R_L j\omega C-g_mR_L}{1+j\omega C\left(R_S+R_L+g_mR_SR_L\right)}=-g_mR_L\frac{1-j\omega\frac{C}{g_m}}{1+j\omega C\left(R_S+R_L+g_mR_SR_L\right)}$$

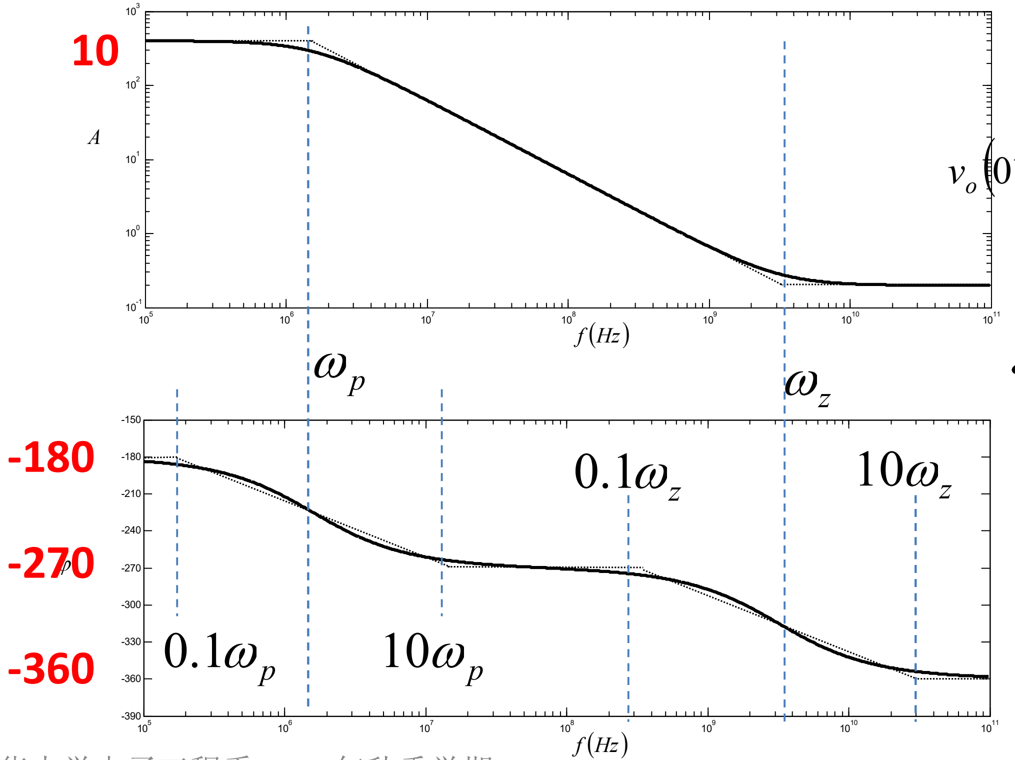
$$=-g_mR_L\frac{1-\frac{j\omega}{\omega_z}}{1+\frac{j\omega}{\omega_p}}$$

可用结点电压法等方法获得传递函数




$v_{o\infty}=-g_mR_L$

伯特图




$v_o\left(0^+\right)=\frac{R_L}{R_{eq}}$

0.645

$g_mR_L$

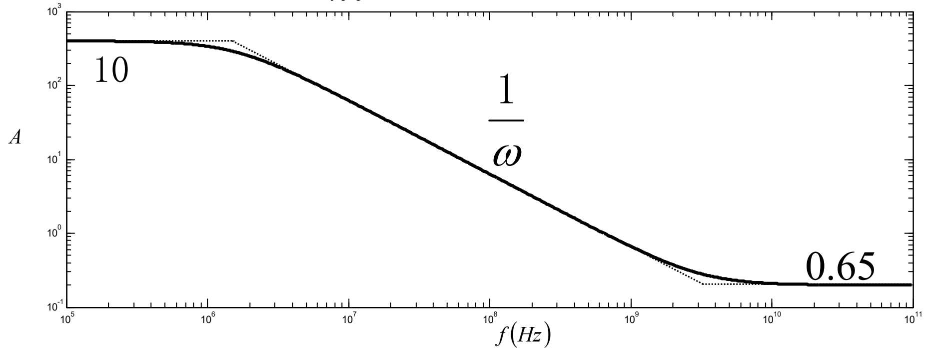
# 时频对应关系

$$H(j\omega)=\frac{\dot V_{out}}{\dot V_S}=-g_mR_L\frac{1-j\omega\frac{C}{g_m}}{1+j\omega C\left(R_S+R_L+g_mR_SR_L\right)}=-g_mR_L\frac{1-j\omega\frac{C}{g_m}}{1+j\omega CR_{eq}}$$

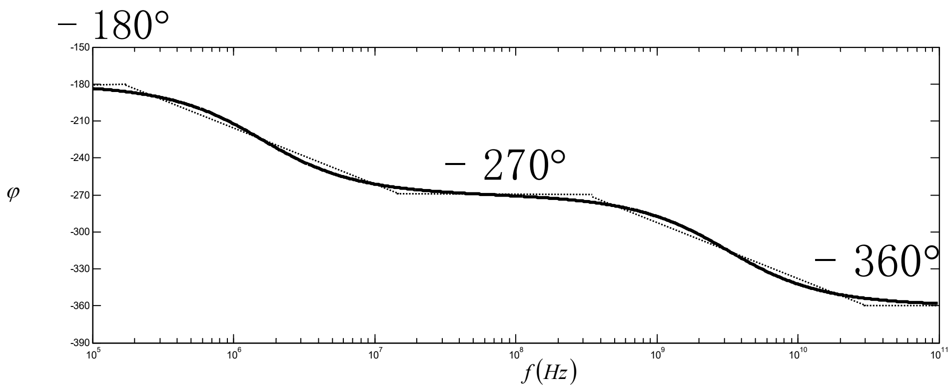
是否可以直接从频域传递函数获得阶跃或冲激响应呢？

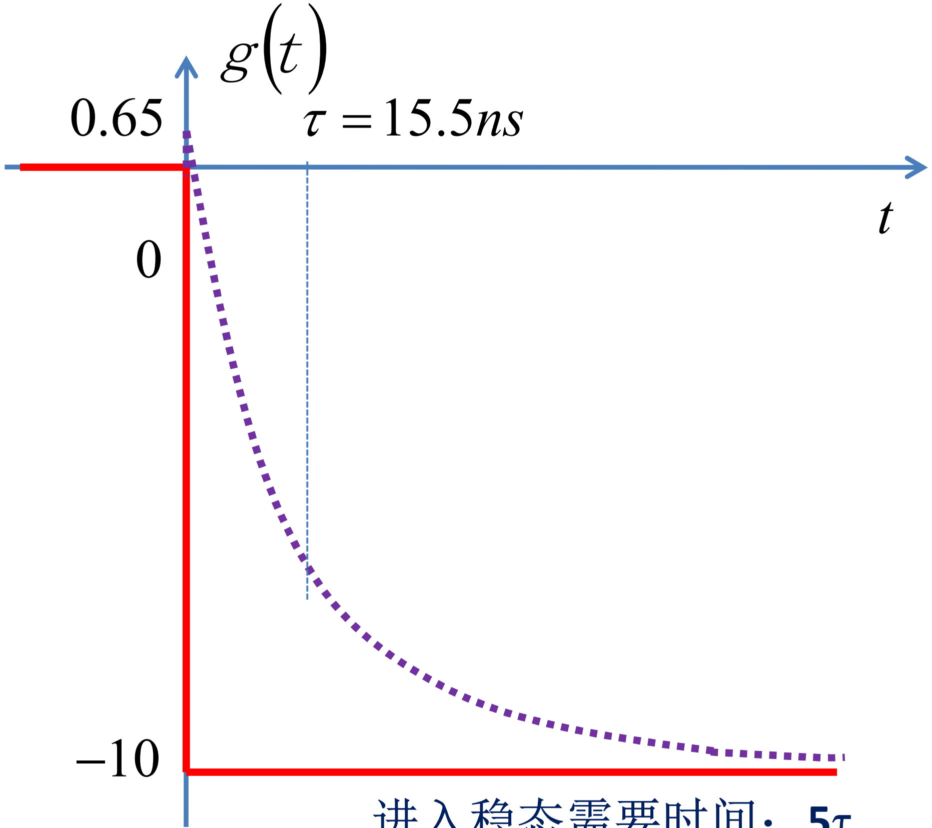
$$g(t)=\left(-10+10.65e^{-\frac{t}{15.5\times10^{-9}}}\right)U(t)=\left[-g_mR_L+\left(\frac{R_L}{R_{eq}}-\left(-g_mR_L\right)\right)\cdot e^{-\frac{t}{\tau}}\right]\cdot U(t)$$

$$f_0=\frac{1}{2\pi\tau}=10.27MHz$$




$$10\times f_0=0.65\times f_z$$




图属教材例9.2.10数值结果，和本题结果不匹配

进入稳态需要时间：5τ

# 时频对应关系表

典型一阶高低通

非典型一阶系统

| $h(t)$ | $H(s)$ | |
|---|---|---|
| $\delta(t)$ | $1$ | 直通电路 |
| $U(t)$ | $\frac{1}{s}$ | 纯电容 |
| $e^{-\omega_0 t}\cdot U(t)$ | $\frac{1}{s+\omega_0}$ | RC电路 |

其实是《信号与系统》中的拉普拉斯变换，不要求掌握，只需了解即可

# 一阶低通和一阶高通

变换表

$$H(j\omega)=A_0\frac{1}{1+j\omega\tau}\overset{\omega_0=\frac{1}{\tau}}{=}A_0\frac{\omega_0}{\omega_0+j\omega}\overset{s=j\omega}{=}A_0\frac{\omega_0}{s+\omega_0}$$

$$h(t)=A_0\omega_0e^{-\omega_0t}U(t)=A_0\frac{1}{\tau}e^{-\frac{t}{\tau}}\cdot U(t)$$

$$\frac{1}{s}H(s)=A_0\frac{1}{s}\frac{\omega_0}{s+\omega_0}=A_0\left(\frac{1}{s}-\frac{1}{s+\omega_0}\right)$$

$$g(t)=A_0\left(1-e^{-\omega_0t}\right)U(t)=A_0\left(1-e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

$$H(j\omega)=A_0\frac{j\omega\tau}{1+j\omega\tau}\overset{\omega_0=\frac{1}{\tau}}{=}A_0\frac{j\omega}{\omega_0+j\omega}\overset{s=j\omega}{=}A_0\frac{s}{s+\omega_0}=A_0-A_0\frac{\omega_0}{s+\omega_0}$$

$$h(t)=A_0\delta(t)-A_0\omega_0e^{-\omega_0t}\cdot U(t)=A_0\delta(t)-A_0\frac{1}{\tau}e^{-\frac{t}{\tau}}\cdot U(t)$$

$$\frac{1}{s}H(s)=A_0\frac{1}{s}\frac{s}{s+\omega_0}=A_0\frac{1}{s+\omega_0}$$

$$g(t)=A_0e^{-\omega_0t}\cdot U(t)=A_0e^{-\frac{t}{\tau}}\cdot U(t)$$

# 任意一阶系统

$$H(s)=\frac{\alpha s+\beta}{s+\omega_0}=\frac{\alpha(s+\omega_0)-\alpha\omega_0+\beta}{s+\omega_0}=\alpha+(\beta-\alpha\omega_0)\frac{1}{s+\omega_0}$$

$$h(t)=\alpha\cdot\delta(t)+(\beta-\alpha\omega_0)e^{-\omega_0 t}\cdot U(t)$$

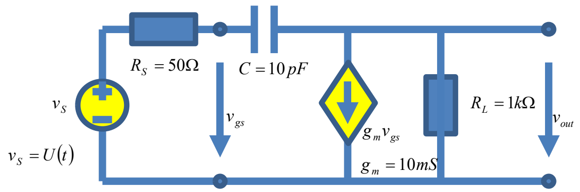
变换表

$$\frac{1}{s}H(s)=\frac{1}{s}\frac{\alpha s+\beta}{s+\omega_0}=\frac{\beta}{\omega_0}\frac{1}{s}+\left(\alpha-\frac{\beta}{\omega_0}\right)\frac{1}{s+\omega_0}$$

$$g(t)=\left(\frac{\beta}{\omega_0}+\left(\alpha-\frac{\beta}{\omega_0}\right)e^{-\omega_0 t}\right)\cdot U(t)$$

$$\frac{d}{dt}g(t)=\left(\frac{\beta}{\omega_0}+\left(\alpha-\frac{\beta}{\omega_0}\right)e^{-\omega_0 t}\right)\cdot\delta(t)+\left(-\omega_0\left(\alpha-\frac{\beta}{\omega_0}\right)e^{-\omega_0 t}\right)\cdot U(t)$$

$$=\alpha\cdot\delta(t)+(\beta-\alpha\omega_0)e^{-\omega_0 t}\cdot U(t)=h(t)$$




# 时频对应关系

$$H(j\omega)=\frac{\dot V_{out}}{\dot V_S}=-g_mR_L\frac{1-j\omega\frac{C}{g_m}}{1+j\omega C(R_S+R_L+g_mR_SR_L)}=-g_mR_L\frac{1-j\omega\frac{C}{g_m}}{1+j\omega CR_{eq}}$$

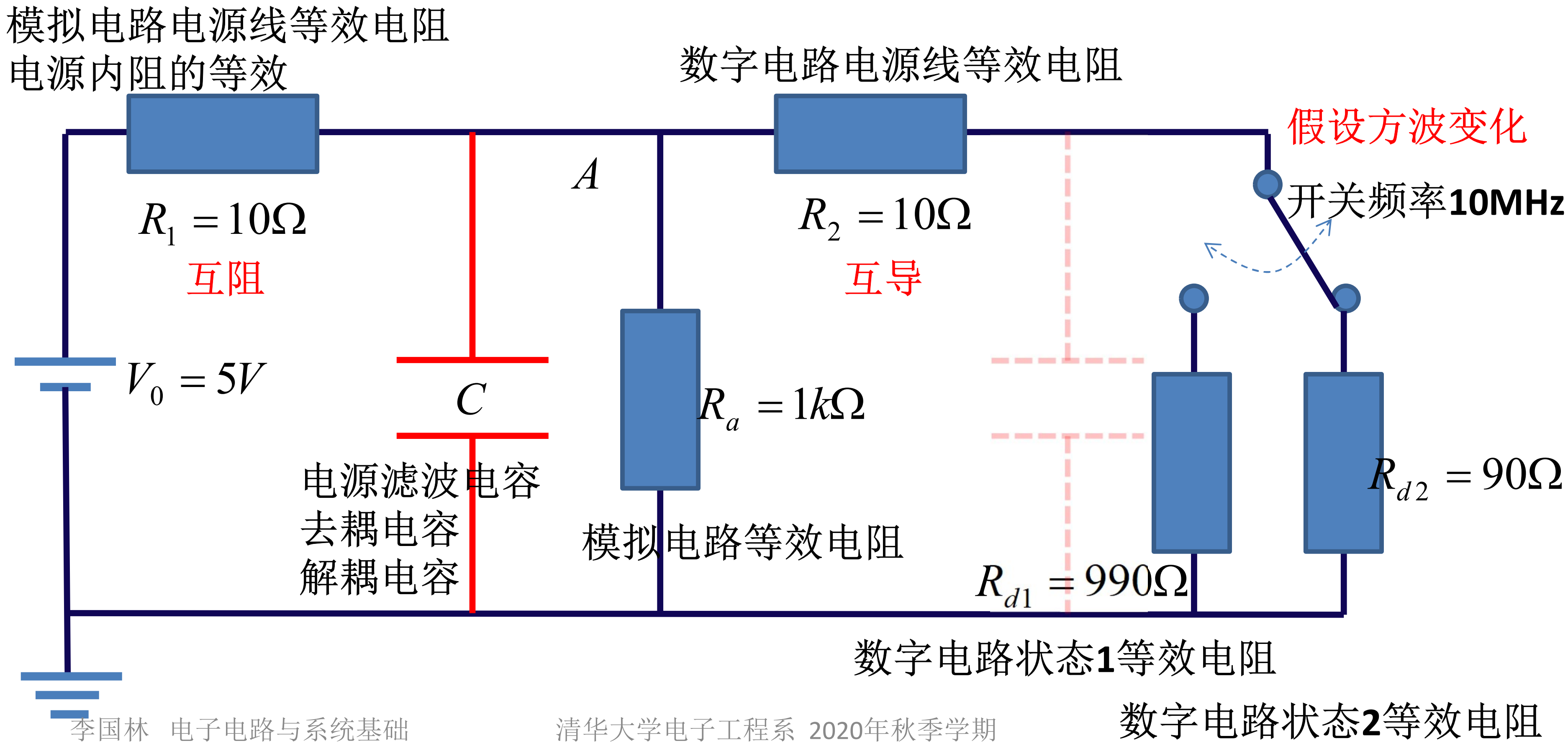
$$\frac{1}{s}H(s)=-g_mR_L\frac{1}{s}\frac{1-s\frac{C}{g_m}}{1+sCR_{eq}}=-g_mR_L\frac{1}{s}\frac{\omega_0-s\frac{1}{g_mR_{eq}}}{s+\omega_0}=-g_mR_L\left(\frac{1}{s}-\frac{1+\frac{1}{g_mR_{eq}}}{s+\omega_0}\right)$$

$$g(t)=-g_mR_L\left(U(t)-\left(1+\frac{1}{g_mR_{eq}}\right)e^{-\omega_0t}U(t)\right)$$
$$=\left(-g_mR_L+\left(g_mR_L+\frac{R_L}{R_{eq}}\right)e^{-\omega_0t}\right)U(t)$$

时频对应法就是拉普拉斯变换方法，本课程不要求掌握

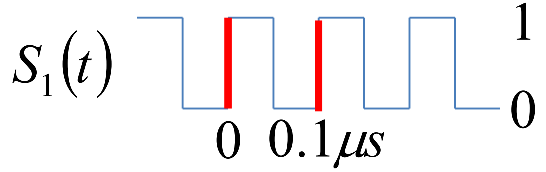
# 作业5：用电容做电源滤波

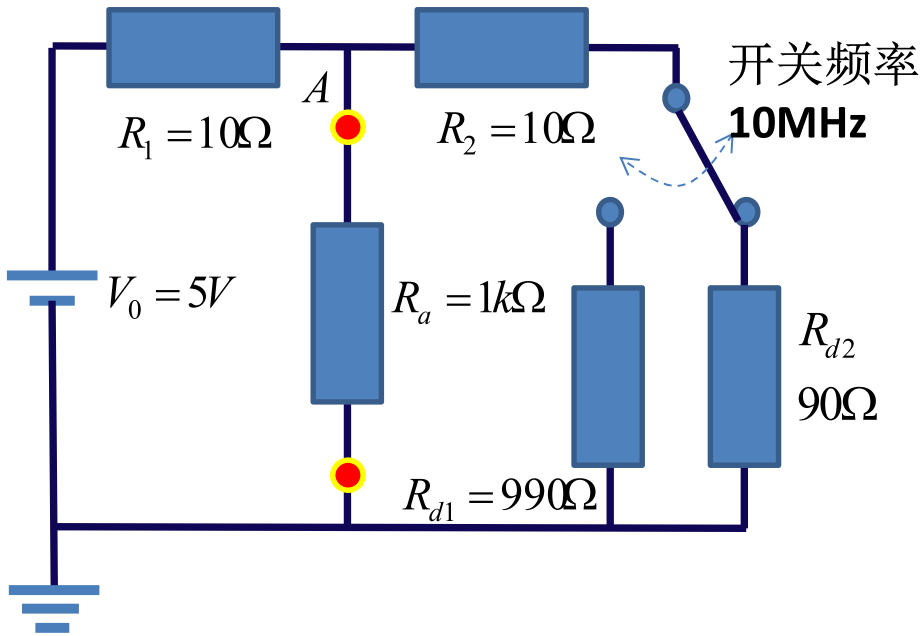
- （1）假设没有滤波电容，求模拟电路电源端A点的电压波形
- （2）多大的电容，可以使得A点电压波形起伏是没有电容时的1/10

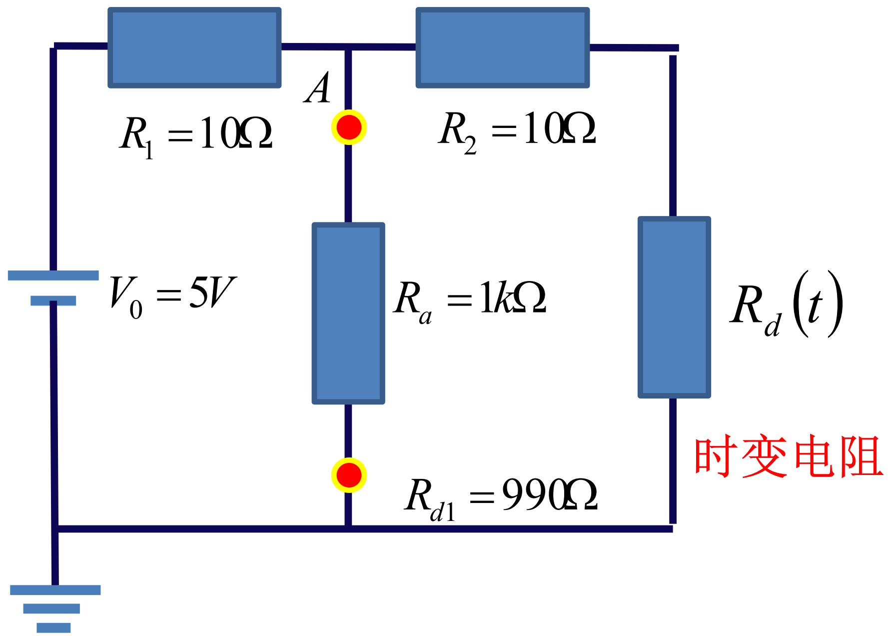


# 时变电阻

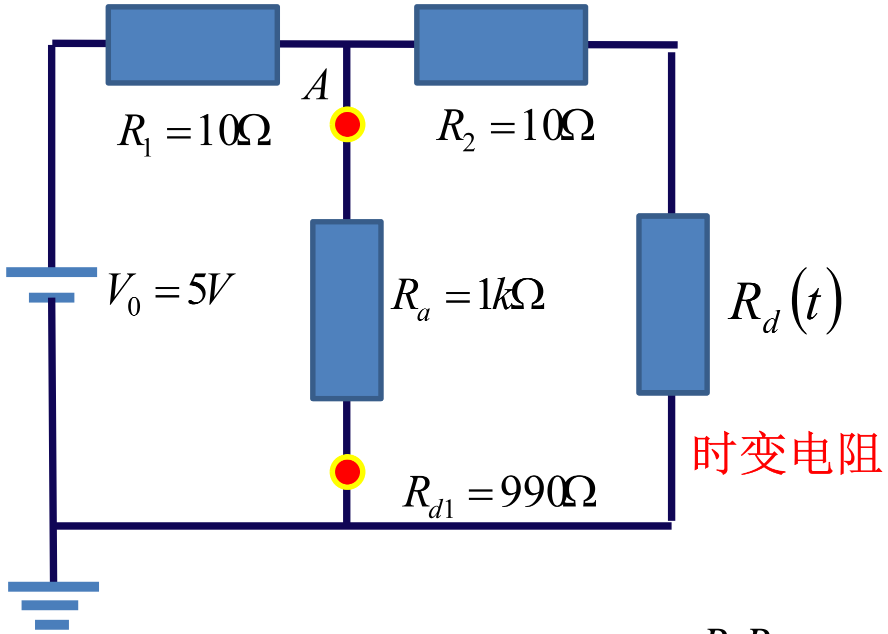
$$R_d(t) = R_{d1}S_1(t) + R_{d2}(1 - S_1(t))$$

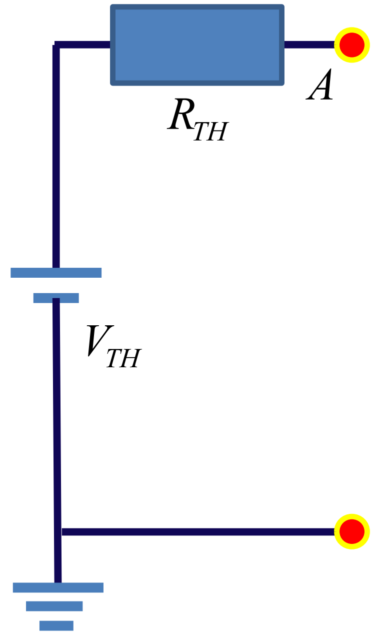


# 戴维南等效



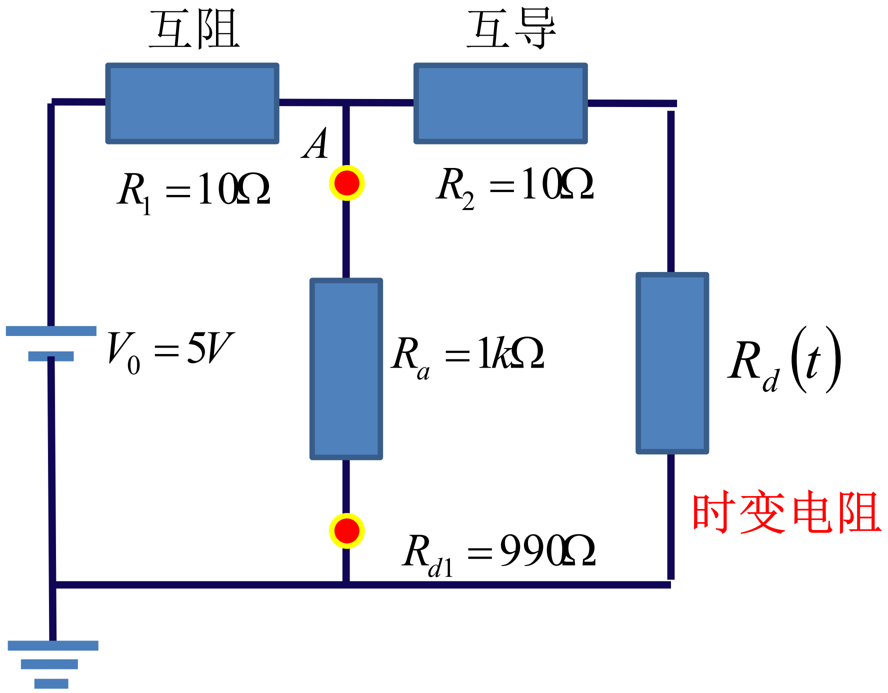

$$R_{TH} = R_1 \| R_a \| (R_2 + R_d) = \frac{R_a R_1}{R_1 + R_a + \frac{R_a R_1}{R_2 + R_d}}$$

$$V_{TH} = \frac{(R_2 + R_d) \| R_a}{R_1 + (R_2 + R_d) \| R_a} V_0 = \frac{R_a}{R_1 + R_a + \frac{R_1 R_a}{R_2 + R_d}} V_0$$

可以直接数值计算，但表达式可以更清晰地表明互阻和互导的影响

# 互阻和互导的影响

$$R_{TH}=R_1\|R_a\|(R_2+R_d)=\frac{R_aR_1}{R_1+R_a+\frac{R_aR_1}{R_2+R_d}}$$




$$V_{TH}=\frac{(R_2+R_d)\|R_a}{R_1+(R_2+R_d)\|R_a}V_0=\frac{R_a}{R_1+R_a+\frac{R_1R_a}{R_2+R_d}}V_0$$

$$R_{TH}\overset{R_1=0}{=}0$$

$$V_{TH}\overset{R_1=0}{=}V_0$$

如果没有互阻（互阻$R_1=0$），AG端口等效为恒压源，数字芯片的变化（时变电阻$R_d$）无法影响模拟芯片电压

互阻$R_1$和互导$G_2$导致数字芯片和模拟芯片相互耦合，数字芯片的电流变化导致模拟芯片电源电压波动：不开避免的干扰

（1）电源内阻不为0

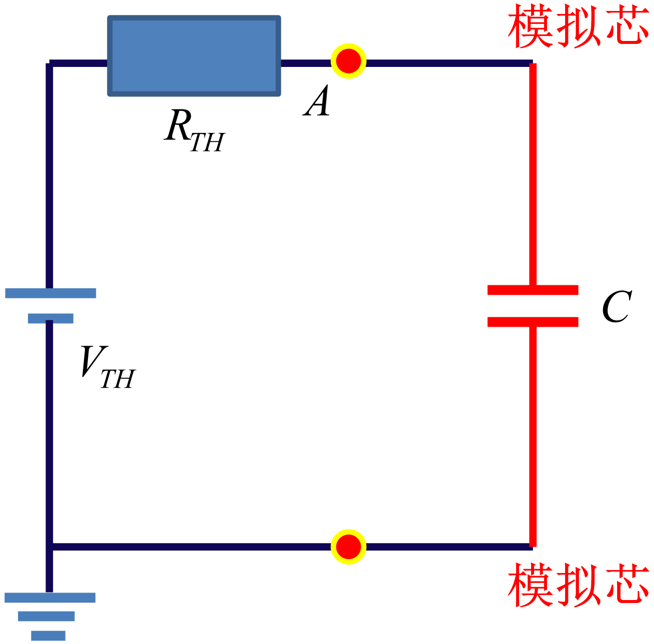
（2）无法为每个芯片单独供电

$$R_{TH}\overset{R_2=\infty}{=}\frac{R_aR_1}{R_1+R_a}$$

$$V_{TH}\overset{R_2=\infty}{=}\frac{R_a}{R_1+R_a}V_0$$

如果没有互导（$G_2=0$，分别单独提供供电电源），AG端口等效源电压和源内阻均不受数字芯片（时变电阻$R_d$）的影响，即数字芯片的电流变化无法影响模拟芯片的电源电压

# 滤波电容　降低波动　解除耦合



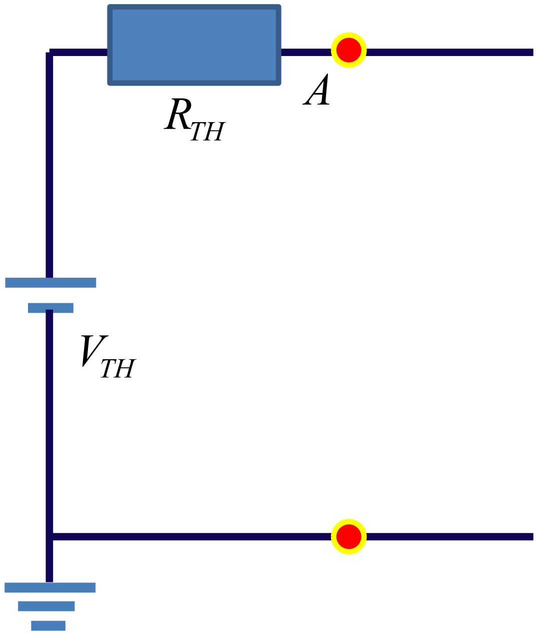

$$R_{TH}(t)=\frac{R_aR_1}{R_1+R_a+\dfrac{R_aR_1}{R_2+R_d(t)}}$$

$$V_{TH}(t)=\frac{R_a}{R_1+R_a+\dfrac{R_1R_a}{R_2+R_d(t)}}V_0$$

电容**C**具有电压保持功能，具有求平均功能，只要电容足够大，$\mathbf{V_{TH}}$和$\mathbf{R_{TH}}$的变化就会被电容抹平：电源滤波，芯片解耦

数字芯片的电流变化用时变电阻$\mathbf{R_d(t)}$抽象，$\mathbf{R_d(t)}$对模拟芯片的影响通过$\mathbf{R_1}$、$\mathbf{R_2}$实现

# 未加电容时，A点波动情况




$$R_{TH}(t)=\frac{R_aR_1}{R_1+R_a+\frac{R_aR_1}{R_2+R_d(t)}}$$

$$V_{TH}(t)=\frac{R_a}{R_1+R_a+\frac{R_1R_a}{R_2+R_d(t)}}V_0$$

$$R_d(t)=R_{d1}S_1(t)+R_{d2}\left(1-S_1(t)\right)$$

$$V_{TH,1}(t)=\frac{R_a}{R_1+R_a+\frac{R_1R_a}{R_2+R_{d1}}}V_0=\frac{1000}{10+1000+\frac{10\cdot 1000}{10+990}}\times 5=\frac{1000}{10+1000+\underline{\underline{10}}}\times 5=4.902V$$

$$V_{TH,2}(t)=\frac{R_a}{R_1+R_a+\frac{R_1R_a}{R_2+R_{d2}}}V_0=\frac{1000}{10+1000+\frac{10\cdot 1000}{10+90}}\times 5=\frac{1000}{10+1000+\underline{\underline{100}}}\times 5=4.505V$$

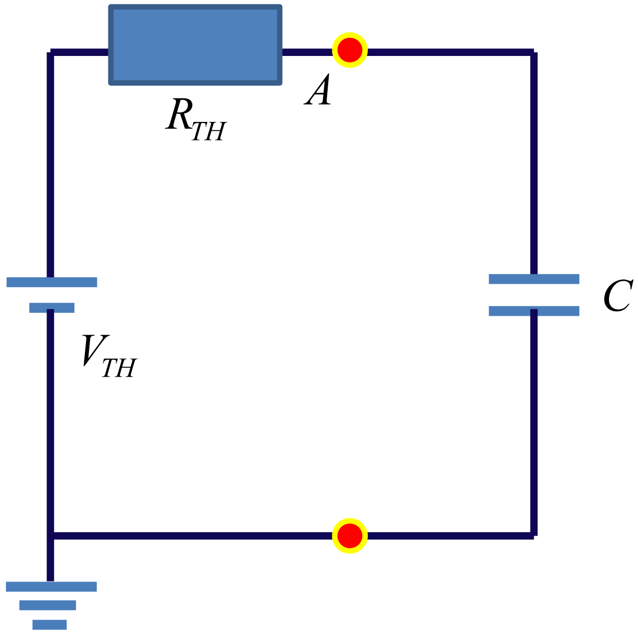
$R_{TH1}=9.804\Omega$

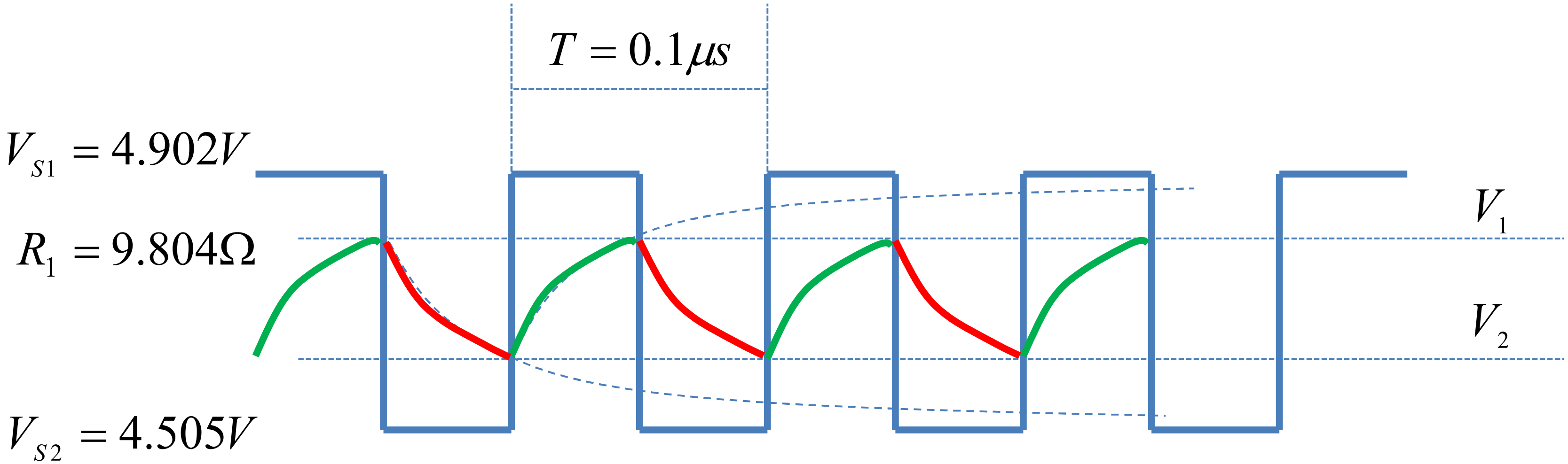
$R_{TH2}=9.010\Omega$

等效内阻同时变化

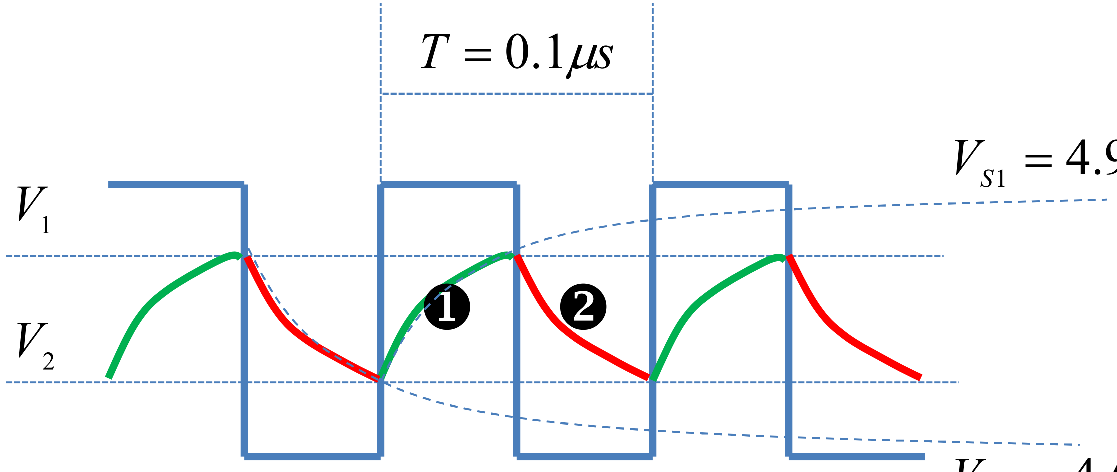
$\Delta V_{A0}=4.902-4.505=0.397V$ 电压波动**0.4V**

# 加电容，A点波动变缓



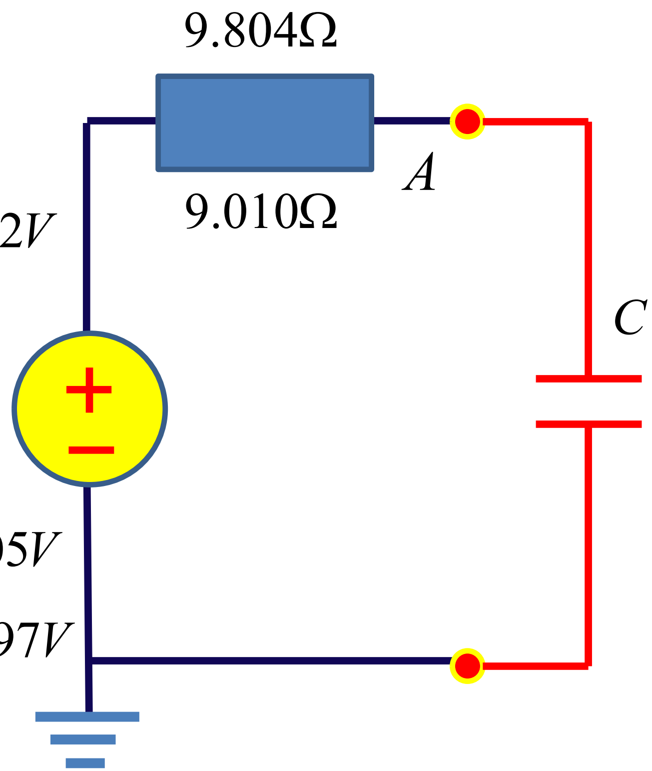




$R_2 = 9.010\Omega$ $\quad \Delta V_{A0} = 0.397V$ $\quad \Delta V_A = V_1 - V_2 \le 0.1\Delta V_{A0} = 0.0397V$




$$\Delta V_A = V_1 - V_2 \le 0.1\Delta V_{A0} = 0.0397V$$

$$\Delta V_{A0} = 0.397V$$




❶ $$v_r(t) = V_{S1} + (V_2 - V_{S1})e^{-\frac{\Delta t_1}{\tau_1}}$$

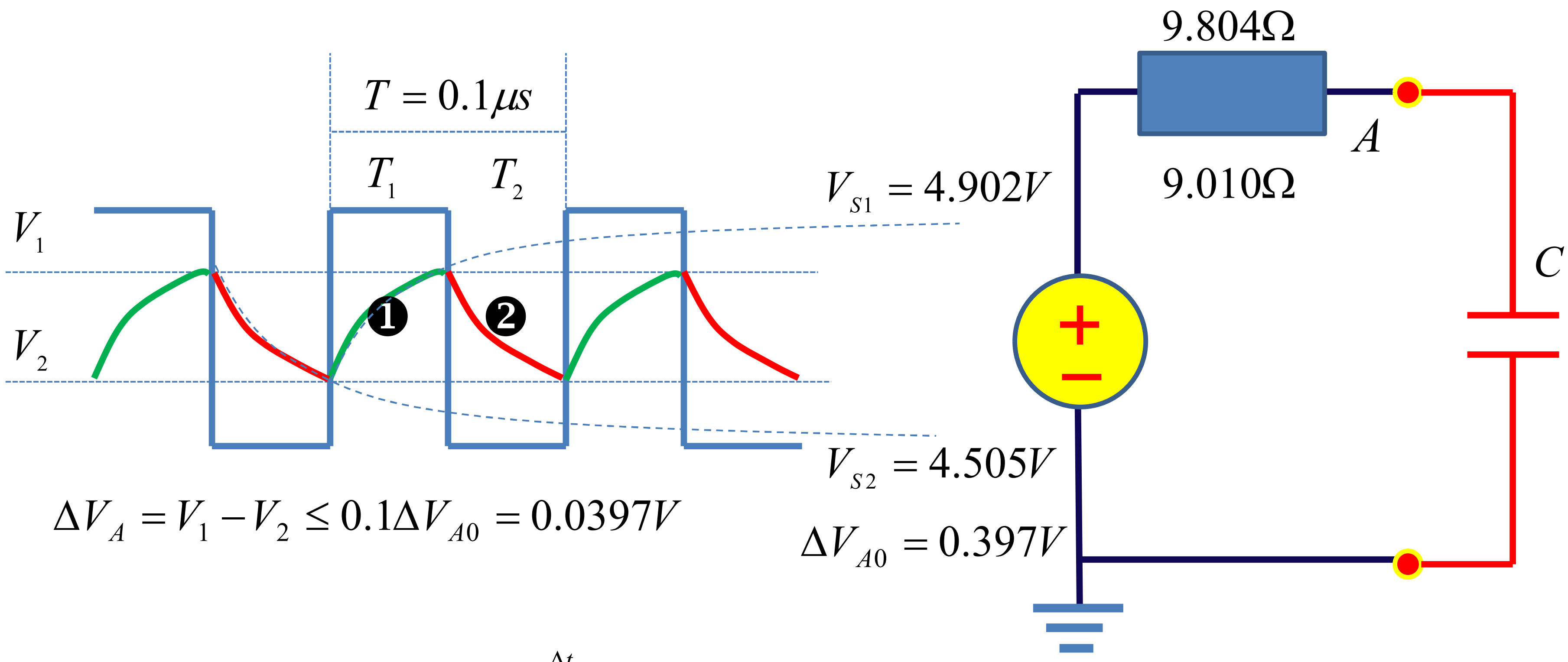
$$\tau_1 = R_1 C$$

$$R_1 = 9.804(\Omega)$$

❷ $$v_f(t) = V_{S2} + (V_1 - V_{S2})e^{-\frac{\Delta t_2}{\tau_2}}$$

$$\tau_2 = R_2 C$$

$$R_2 = 9.010(\Omega)$$




$$\Delta V_A = V_1 - V_2 \le 0.1\Delta V_{A0} = 0.0397V$$

❶ $$v_r(t) = V_{S1} + (V_2 - V_{S1})e^{-\frac{\Delta t_1}{\tau_1}}$$

$$V_1 = V_{S1} + (V_2 - V_{S1})e^{-\frac{0.5T}{\tau_1}}$$

❷ $$v_f(t) = V_{S2} + (V_1 - V_{S2})e^{-\frac{\Delta t_2}{\tau_2}}$$

$$V_2 = V_{S2} + (V_1 - V_{S2})e^{-\frac{0.5T}{\tau_2}}$$

$$\tau_1 = R_1 C$$

$$\tau_2 = R_2 C$$

$$\Delta V = V_1 - V_2 \le 0.1\Delta V_S$$

$$T = 0.1\mu s$$

$$C = ?$$

$$V_1 = V_{S1} + (V_2 - V_{S1})e^{-\frac{0.5T}{\tau_1}} = V_2 a_1 + V_{S1}(1 - a_1) \qquad a_1 = e^{-\frac{0.5T}{\tau_1}}$$

$$V_2 = V_{S2} + (V_1 - V_{S2})e^{-\frac{0.5T}{\tau_2}} = V_1 a_2 + V_{S2}(1 - a_2) \qquad a_2 = e^{-\frac{0.5T}{\tau_2}}$$

$$V_1 - V_2 a_1 = V_{S1}(1 - a_1) \qquad V_1 = \frac{V_{S1}(1-a_1) + V_{S2}(1-a_2)a_1}{1 - a_1 a_2}$$

$$V_2 - V_1 a_2 = V_{S2}(1 - a_2) \qquad V_2 = \frac{V_{S2}(1-a_2) + V_{S1}(1-a_1)a_2}{1 - a_1 a_2}$$

$$\Delta V_A = V_1 - V_2 = (V_{S1} - V_{S2})\frac{(1-a_1)(1-a_2)}{1 - a_1 a_2} \le 0.1(V_{S1} - V_{S2})$$

$$\frac{(1-a_1)(1-a_2)}{1 - a_1 a_2} \le 0.1$$

$$\frac{(1-a_1)(1-a_2)}{1-a_1a_2}\le 0.1 \qquad a_1=e^{-\frac{0.5T}{R_1C}} \qquad a_2=e^{-\frac{0.5T}{R_2C}}$$

非线性方程求解，可以用牛顿拉夫逊迭代法，过于复杂

假设C足够大，时间常数足够大，充放电时间足够长，可以做如下估算：

$$a_1=e^{-\frac{0.5T}{R_1C}}\approx 1-\frac{0.5T}{R_1C} \qquad a_2=e^{-\frac{0.5T}{R_2C}}\approx 1-\frac{0.5T}{R_2C}$$

$$0.1\ge\frac{(1-a_1)(1-a_2)}{1-a_1a_2}\approx\frac{\frac{0.5T}{\tau_1}\frac{0.5T}{\tau_2}}{\frac{0.5T}{\tau_1}+\frac{0.5T}{\tau_2}-\frac{0.5T}{\tau_1}\frac{0.5T}{\tau_2}}\approx\frac{\frac{0.5T}{\tau_1}\frac{0.5T}{\tau_2}}{\frac{0.5T}{\tau_1}+\frac{0.5T}{\tau_2}}=\frac{0.5T}{\tau_1+\tau_2}=\frac{0.5T}{C(R_1+R_2)}$$

$$C\ge\frac{0.5T}{0.1(R_1+R_2)}=\frac{5T}{R_1+R_2}=\frac{5\times 0.1\mu s}{9.8+9.0}=0.0266\mu F$$

$$C \geq \frac{0.5T}{0.1(R_1+R_2)} = \frac{5T}{R_1+R_2} = \frac{5\times 0.1\mu s}{9.8+9.0} = 0.0266\mu F$$ 取 $$C = 0.03\mu F$$

验证正确性

$$\tau_1 = R_1C = 9.8\times 0.03\mu F = 0.294\mu s$$

$$\tau_2 = R_2C = 9.0\times 0.03\mu F = 0.270\mu s$$

$$a_1 = e^{-\frac{0.5T}{\tau_1}} = e^{-\frac{0.05}{0.294}} = e^{-0.1701} = 0.847 \qquad a_2 = e^{-\frac{0.5T}{\tau_2}} = e^{-\frac{0.05}{0.270}} = e^{-0.1852} = 0.831$$

$$V_1 = \frac{V_{S1}(1-a_1)+V_{S2}(1-a_2)a_1}{1-a_1a_2} = \frac{4.902\times 0.153+4.505\times 0.169\times 0.847}{0.296} = 4.712V$$

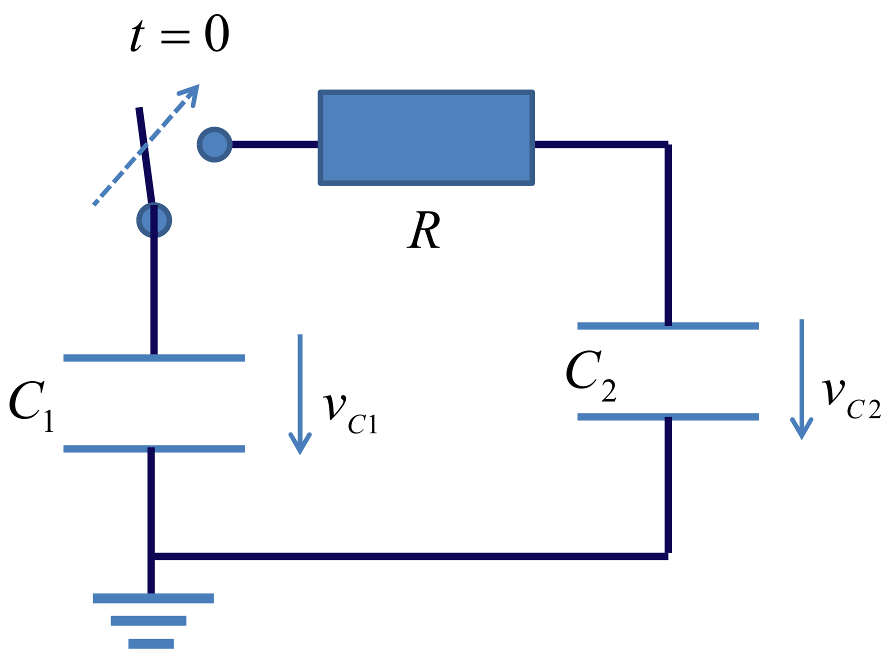
$$V_2 = \frac{V_{S2}(1-a_2)+V_{S1}(1-a_1)a_2}{1-a_1a_2} = \frac{4.505\times 0.169+4.902\times 0.153\times 0.831}{0.296} = 4.678V$$

$$\Delta V_A = V_1 - V_2 = 4.712-4.678 = 0.034V \qquad \Delta V_{A0} = 4.902-4.505 = 0.397V$$

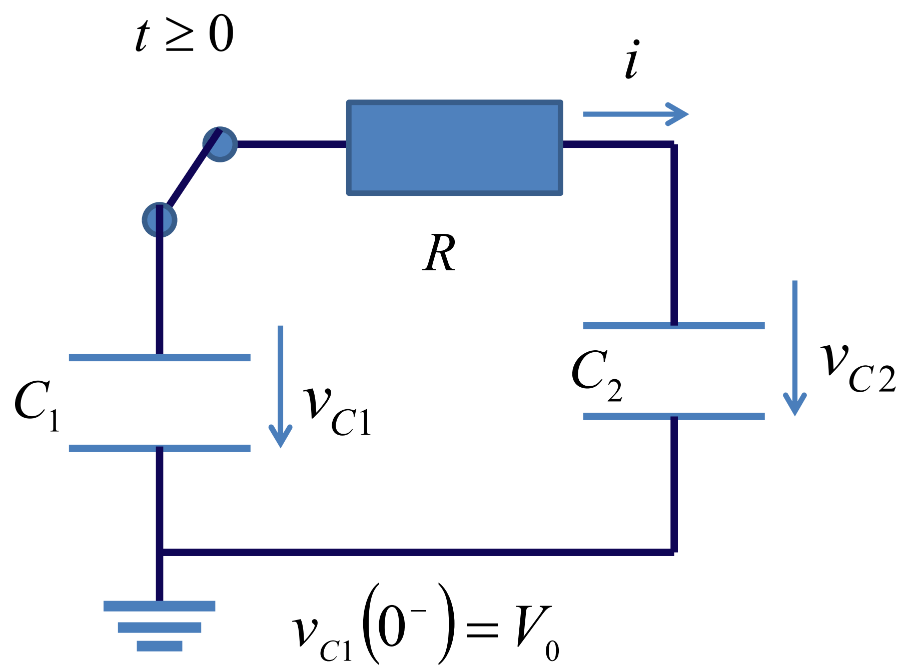
确实满足 $\Delta V_A \leq 0.1V_{A0}$

**C=0.3μF，ΔV<0.01ΔV$_S$：电容越大，滤波效果越好**

# 作业6 电容电荷的重新分配




- **t<0**时刻，$C_1$电容初始电压为$V_0$，$C_2$初始电压为**0**
- 在**t=0**时刻，开关闭合，求电容$C_1$、$C_2$两端电压变化规律，写出表达式，画出时域波形
  - 电荷重新分配过程中，电阻消耗多少能量？能量是否守恒？
- 考察**R**越来越小趋于**0**的变化过程中，回路电流是如何变化的？电容电荷的重新分配情况怎样？
  - 当**R=0**（短接线连接）时，电容电荷的重新分配是瞬间完成的，电容电压发生突变！出现无界电流！




$$v_{C1}(0^-) = V_0$$
$$v_{C2}(0^-) = 0$$

$$v_{C1} = iR + v_{C2}$$

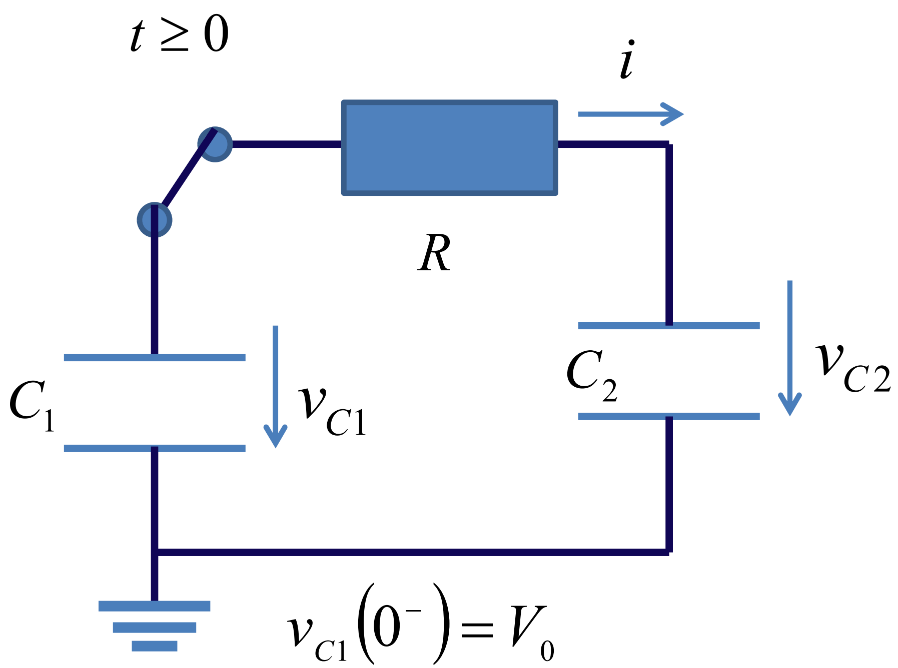
$$-i = C_1 \frac{dv_{C1}}{dt}$$

$$i = C_2 \frac{dv_{C2}}{dt}$$

$$\frac{dv_{C1}}{dt} = R\frac{di}{dt} + \frac{dv_{C2}}{dt}$$

$$-\frac{i}{C_1} = R\frac{di}{dt} + \frac{i}{C_2}$$

$$R\frac{di}{dt} + i\left(\frac{1}{C_2} + \frac{1}{C_1}\right) = 0 \qquad R\frac{di}{dt} + \frac{i}{C} = 0 \qquad RC\frac{di}{dt} + i = 0$$




$$v_{C1}(0^-) = V_0$$
$$v_{C2}(0^-) = 0$$

$$RC\frac{di}{dt} + i = 0$$ 零输入

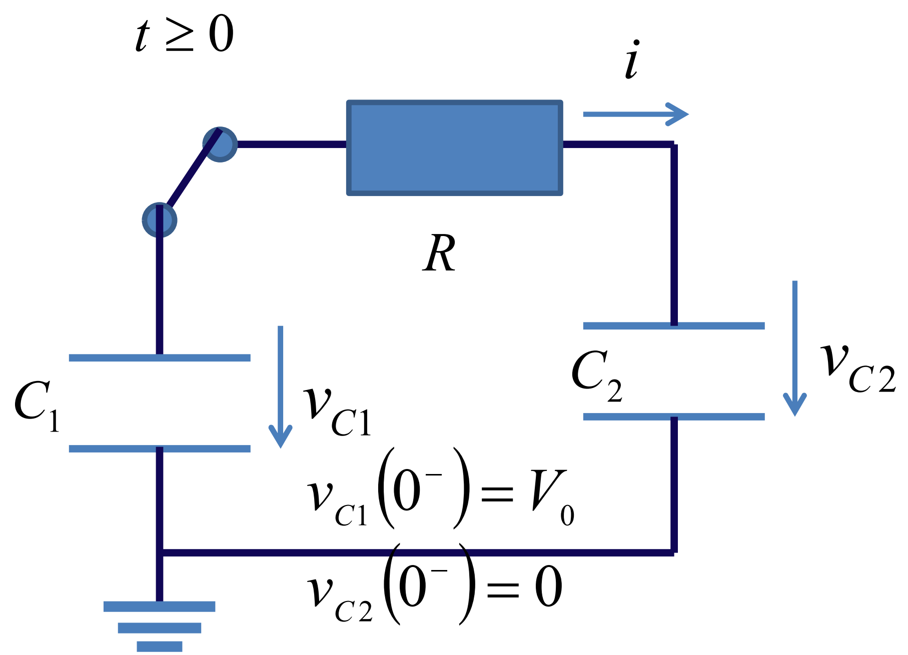
$$\frac{di}{dt} = -\frac{1}{RC}i$$

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$i = i(0^+)e^{-\frac{t}{\tau}}$$ 零输入响应

$$i(0^+) = \frac{v_{C1}(0^+) - v_{C2}(0^+)}{R} = \frac{v_{C1}(0^-) - v_{C2}(0^-)}{R} = \frac{V_0}{R}$$

$$i(t) = \frac{V_0}{R}e^{-\frac{t}{\tau}} \qquad (t \geq 0)$$




$$v_{C1}(0^-)=V_0$$

$$v_{C2}(0^-)=0$$

$$i(t)=\frac{V_0}{R}e^{-\frac{t}{\tau}} \qquad (t\geq 0)$$

$$\tau = RC$$

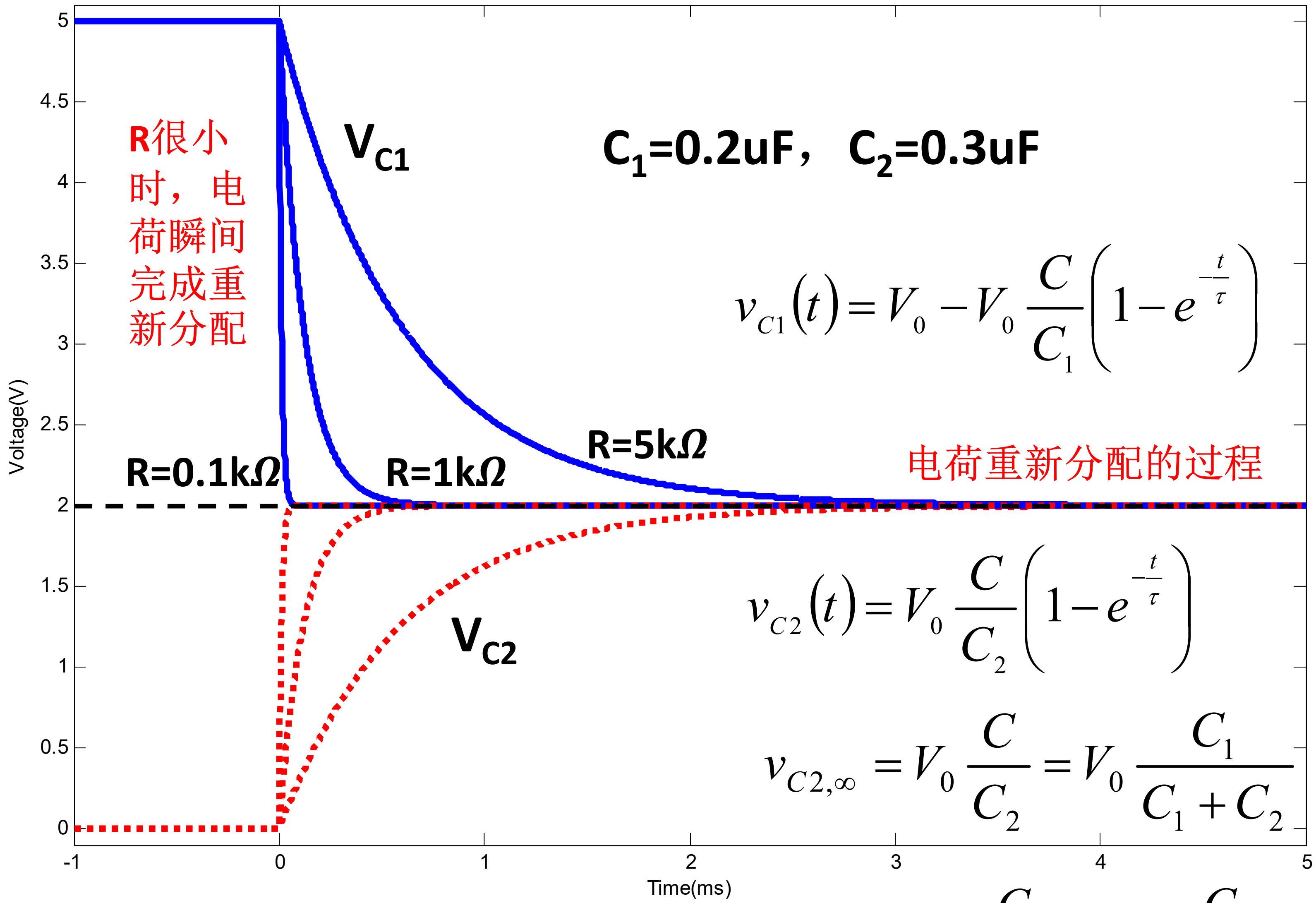
$$C=\frac{C_1C_2}{C_1+C_2}$$

$$v_{C1}(t)=v_{C1}(0)+\frac{1}{C_1}\int_0^t(-i(t))dt=V_0-\frac{1}{C_1}\frac{V_0}{R}(-\tau)e^{-\frac{t}{\tau}}\Big|_0^t$$

$$=V_0-\frac{1}{C_1}\frac{V_0}{R}(-RC)\left(e^{-\frac{t}{\tau}}-1\right)=V_0-V_0\frac{C}{C_1}\left(1-e^{-\frac{t}{\tau}}\right)$$

$$v_{C2}(t)=v_{C1}(0)+\frac{1}{C_2}\int_0^t i(t)dt=0+\frac{1}{C_2}\frac{V_0}{R}(-\tau)e^{-\frac{t}{\tau}}\Big|_0^t=V_0\frac{C}{C_2}\left(1-e^{-\frac{t}{\tau}}\right)$$

# 电容两端电压波形




$$v_{C1}(t) = V_0 - V_0 \frac{C}{C_1}\left(1 - e^{-\frac{t}{\tau}}\right)$$

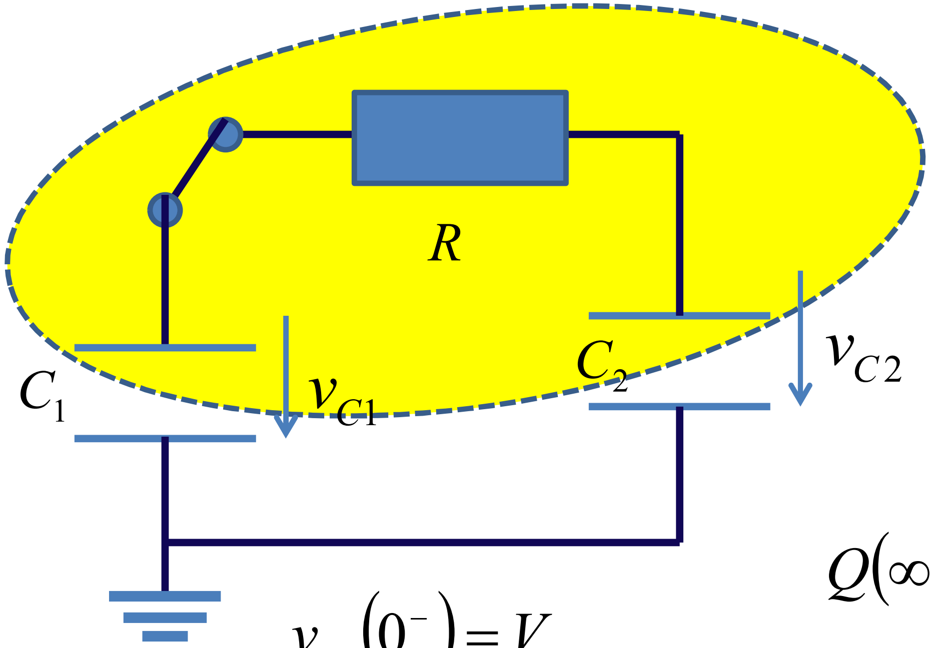
$$\frac{C_1 V_0}{C_1 + C_2}$$

$$v_{C2}(t) = V_0 \frac{C}{C_2}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$v_{C2,\infty} = V_0 \frac{C}{C_2} = V_0 \frac{C_1}{C_1 + C_2}$$

$$v_{C1,\infty} = V_0 - V_0 \frac{C}{C_1} = V_0 \frac{C_1}{C_1 + C_2}$$

# 解的解析：电荷重新分配




$$v_{C1}(0^-)=V_0$$
$$v_{C2}(0^-)=0$$

$$Q(0^-)=C_1v_{C1}(0^-)+C_2v_{C2}(0^-)=C_1V_0$$

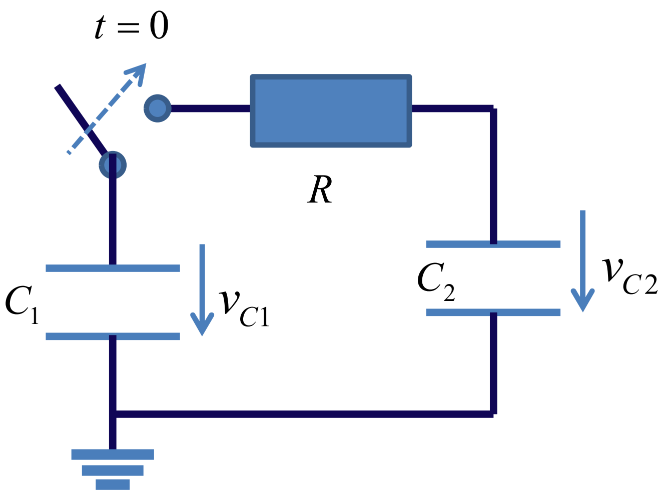
$$Q(0^+)=Q(0^-)=C_1V_0$$

可验证电荷守恒

$$Q(\infty)=C_1v_{C1}(\infty)+C_2v_{C2}(\infty)$$
$$=C_1\left(\frac{C_1}{C_1+C_2}V_0\right)+C_2\left(\frac{C_1}{C_1+C_2}V_0\right)=C_1V_0$$

两个电容上极板没有其他释放电荷的通路，于是两个极板上的总电荷始终守恒不变
电荷重新分配结束后，两个电容电压相等，于是不再有电荷转移

# R→0




$$i(t)=\frac{V_0}{R}e^{-\frac{t}{RC}}\cdot U(t)$$

$$=CV_0\left(\frac{1}{\tau}e^{-\frac{t}{\tau}}\cdot U(t)\right)$$

$$\xrightarrow{R\to 0} CV_0\cdot\delta(t)$$

**t=0**趋于无穷大，面积为$\mathbf{CV_0}$

$$i(t)=\frac{V_0}{R}e^{-\frac{t}{RC}}\cdot U(t) \qquad C=\frac{C_1C_2}{C_1+C_2}$$

$$Q(t)=\int_{-\infty}^{t}i(t)dt=\int_{-\infty}^{t}\frac{V_0}{R}e^{-\frac{t}{RC}}\cdot U(t)\cdot dt$$

$$=\int_{0}^{t}\frac{V_0}{R}e^{-\frac{t}{RC}}\cdot dt=CV_0\int_{0}^{t}\frac{1}{RC}e^{-\frac{t}{RC}}\cdot dt$$

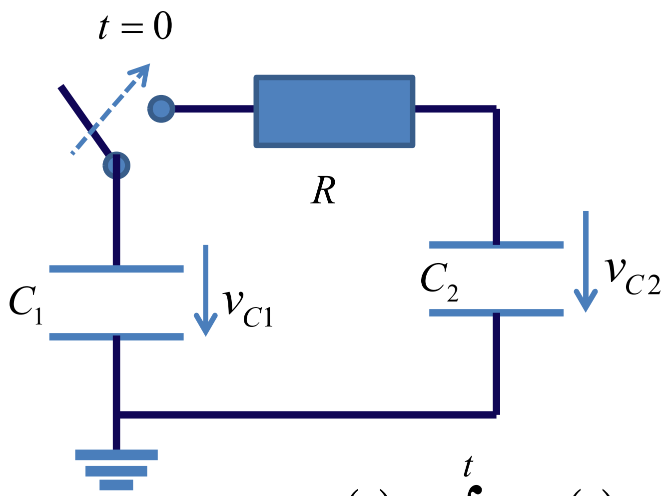
$$=-CV_0\cdot e^{-\frac{t}{RC}}\Big|_0^t=-CV_0\cdot\left(e^{-\frac{t}{RC}}-1\right)$$

$$=CV_0\left(1-e^{-\frac{t}{RC}}\right)$$

物理上如何理解这个结论？

$$Q(\infty)=CV_0$$

电荷转移是瞬间完成的




$$i(t)=\frac{V_0}{R}e^{-\frac{t}{\tau}}\xrightarrow{R\to 0}CV_0\cdot\delta(t)$$

$$p_R(t)=i^2(t)R=\frac{V_0^2}{R}e^{-2\frac{t}{\tau}}=\frac{1}{2}C\frac{V_0^2}{0.5\tau}e^{-\frac{t}{0.5\tau}}\xrightarrow{R\to 0}\frac{1}{2}CV_0^2\cdot\delta(t)$$

电阻为0，也有能耗！

$$E_R(t)=\int_0^t p_R(t)dt=\frac{V_0^2}{R}\left(\frac{\tau}{2}\right)\left(1-e^{-2\frac{t}{\tau}}\right)=\frac{1}{2}CV_0^2\left(1-e^{-2\frac{t}{\tau}}\right)\overset{\tau\to 0}{=}\frac{1}{2}CV_0^2$$

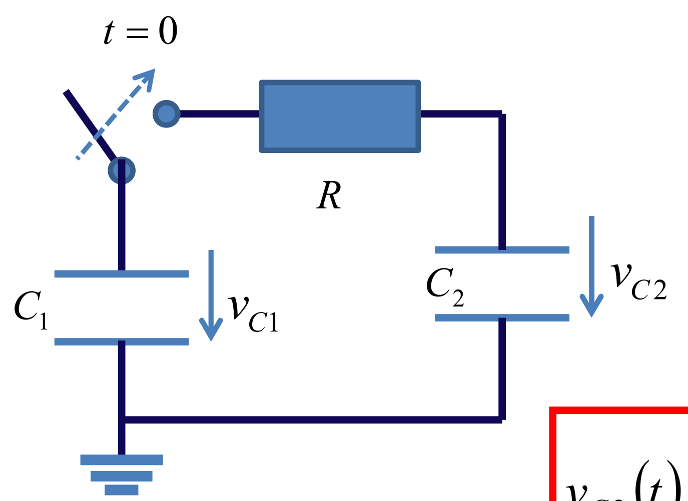
$$E_{C1}(t)=\frac{1}{2}C_1v_{C1}^2(t)=\frac{1}{2}C_1V_0^2\left(\frac{C_1}{C_1+C_2}+\frac{C_2}{C_1+C_2}e^{-\frac{t}{\tau}}\right)^2\overset{\tau\to 0}{=}\frac{1}{2}C_1V_{1\infty}^2$$

$$E_{C2}(t)=\frac{1}{2}C_2v_{C2}^2(t)=\frac{1}{2}C_2V_0^2\left(\frac{C_1}{C_1+C_2}-\frac{C_1}{C_1+C_2}e^{-\frac{t}{\tau}}\right)^2\overset{\tau\to 0}{=}\frac{1}{2}C_2V_{2\infty}^2$$

$$E_R(t)+E_{C1}(t)+E_{C2}(t)=\frac{1}{2}C_1V_0^2=E_{C1}(0^-)+E_{C2}(0^-)$$

考虑电阻影响后，能量始终守恒

**R→0：冲激电流抽象本身就代表了能耗**




# 可以直接用三要素法获得解的表达式

$v_{C2}(0)=0$ ❶

$v_{C2,\infty}(t)=\dfrac{C_1}{C_1+C_2}V_0$ ❷

$\tau=R\dfrac{C_1C_2}{C_1+C_2}$ ❸

前提：会应用电荷守恒

$$C_1V_0+0=C_1V_\infty+C_2V_\infty$$

$$V_\infty=\frac{C_1}{C_1+C_2}V_0$$

电容$C_2$充电

$$v_{C2}(t)=v_{C2,\infty}(t)+\left(v_{C2}(0)-v_{C2,\infty}(0)\right)e^{-\frac{t}{\tau}}$$
$$=\frac{C_1}{C_1+C_2}V_0\left(1-e^{-\frac{t}{\tau}}\right)$$

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$v_{C1}(0)=V_0$ ❶

$v_{C1,\infty}(t)=\dfrac{C_1}{C_1+C_2}V_0$ ❷

$\tau=R\dfrac{C_1C_2}{C_1+C_2}$ ❸

电容$C_1$放电

R=0，τ=0：电荷分配瞬间完成

$$v_{C1}(t)=v_{C1,\infty}(t)+\left(v_{C1}(0)-v_{C1,\infty}(0)\right)e^{-\frac{t}{\tau}}$$
$$=\frac{C_1}{C_1+C_2}V_0+\left(V_0-\frac{C_1}{C_1+C_2}V_0\right)e^{-\frac{t}{\tau}}$$
$$=\frac{C_1}{C_1+C_2}V_0+\frac{C_2}{C_1+C_2}V_0e^{-\frac{t}{\tau}}$$