电子电路与系统基础II

习题课第四讲　动态元件和动态电路分析

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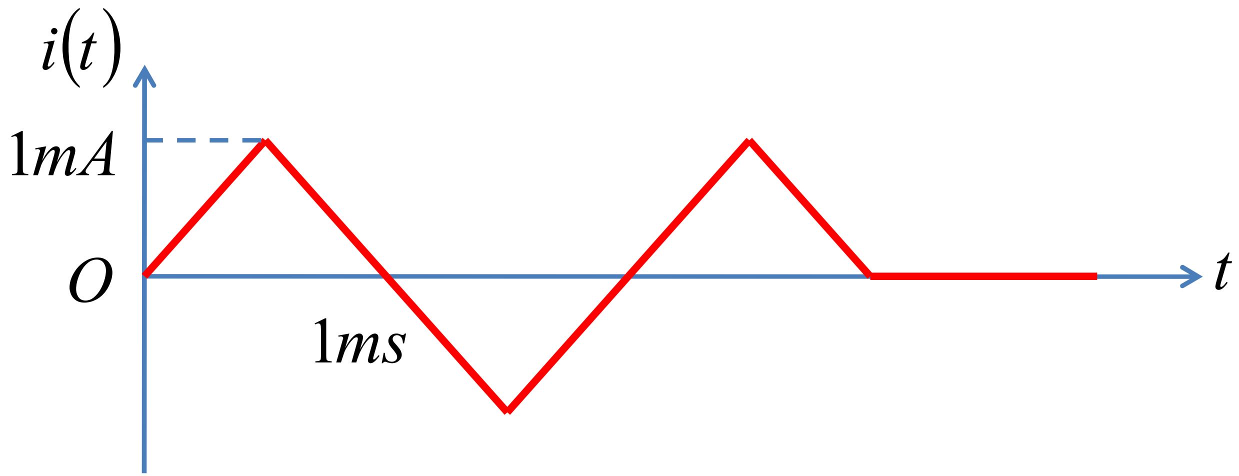
大纲

- 第二讲习题 电容和电感
  - 电容电感特性与状态方程列写及数值法求解
- 第三讲习题 动态电路分析方法（部分）
  - 相图和相量法

第2讲 电容和电感特性

作业1 电容电压、电荷与电能存储

- 某电容器电容容值为1μF，电容初始电压为5V，加在电容两端电流源的电流变化规律如图所示
  - （1）求电容上最终存储的电荷量为多大
  - （2）列写电流、电容电压、电荷量、电容存储电能随时间变化的表达式（教材例题缺）
  - （3）画出电流、电压、电荷、电能时域波形

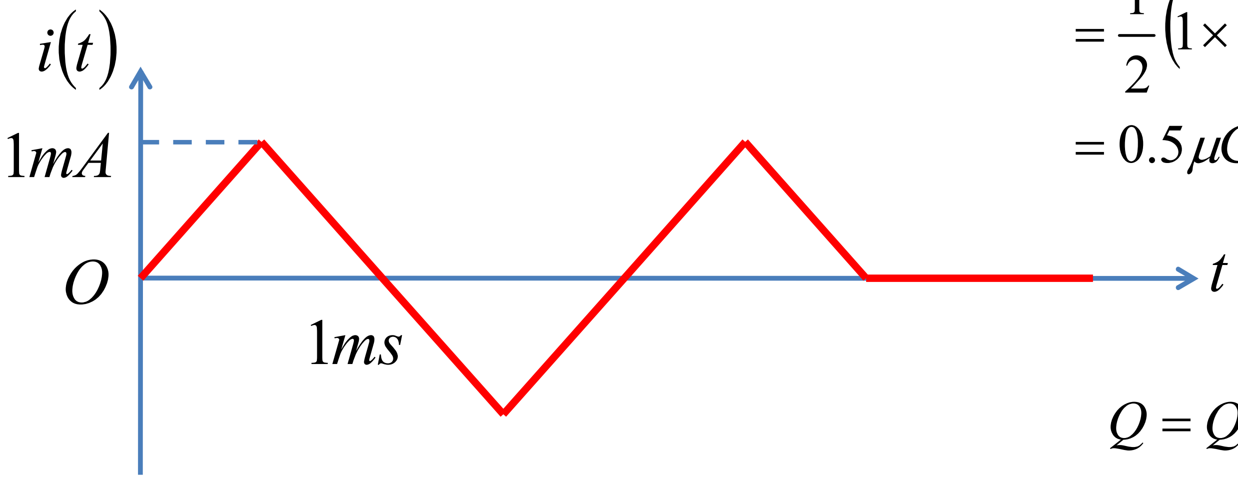


电容电量

$$v(1ms) = V_0 + \frac{1}{C}\int_0^{1ms} i(\tau)\cdot d\tau$$

$$= 5 + \frac{1}{1\times10^{-6}}\frac{1}{2}\left(1\times10^{-3}\times1\times10^{-3}\right)$$

$$= 5.5(V)$$



$$Q = C\cdot V = 1\times10^{-6}\times5.5 = 5.5\mu C$$

$$Q_0 = C\cdot V_0 = 5\mu C$$

$$\Delta Q = \int_0^{1ms} i(\tau)\cdot d\tau$$

$$= \frac{1}{2}\left(1\times10^{-3}\times1\times10^{-3}\right)$$

$$= 0.5\mu C$$

$$Q = Q_0 + \Delta Q = 5.5\mu C$$

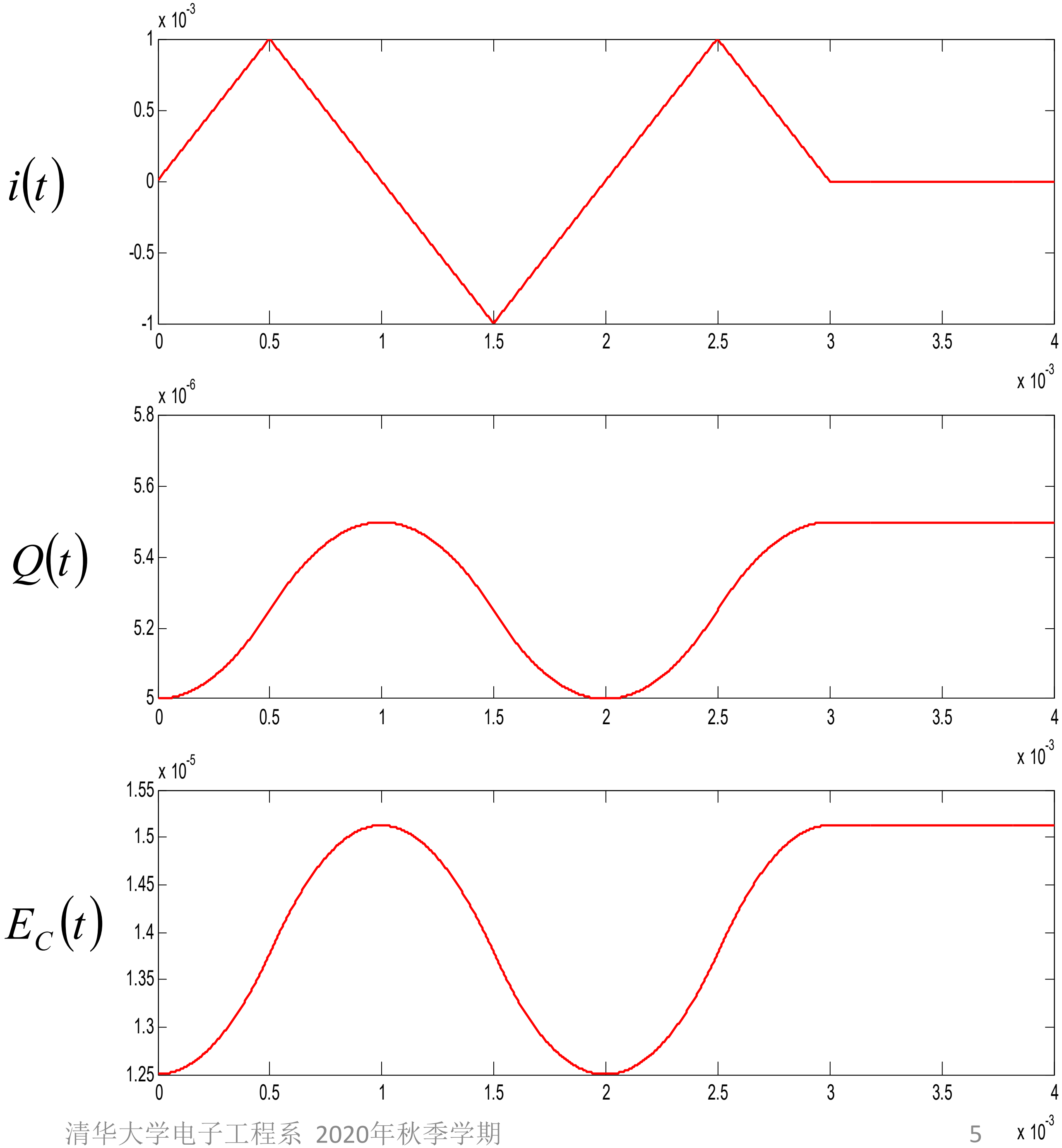
求数学表达式

$$Q(t)=\int_{-\infty}^{t} i(\tau)\cdot d\tau$$

$$=Q_0+\int_{0}^{t} i(\tau)\cdot d\tau$$

$$v(t)=\frac{Q(t)}{C}$$

$$E_C(t)=\frac{1}{2}Cv^2(t)$$



数学表达式

$$i(t)=\begin{cases} 2t\ mA & 0\le t<0.5\ ms \\ -2(t-1)\ mA & 0.5\le t<1.5\ ms \\ 2(t-2)\ mA & 1.5\le t<2.5\ ms \\ -2(t-3)\ mA & 2.5\le t<3\ ms \\ 0 & t\ge 3\ ms \end{cases}$$

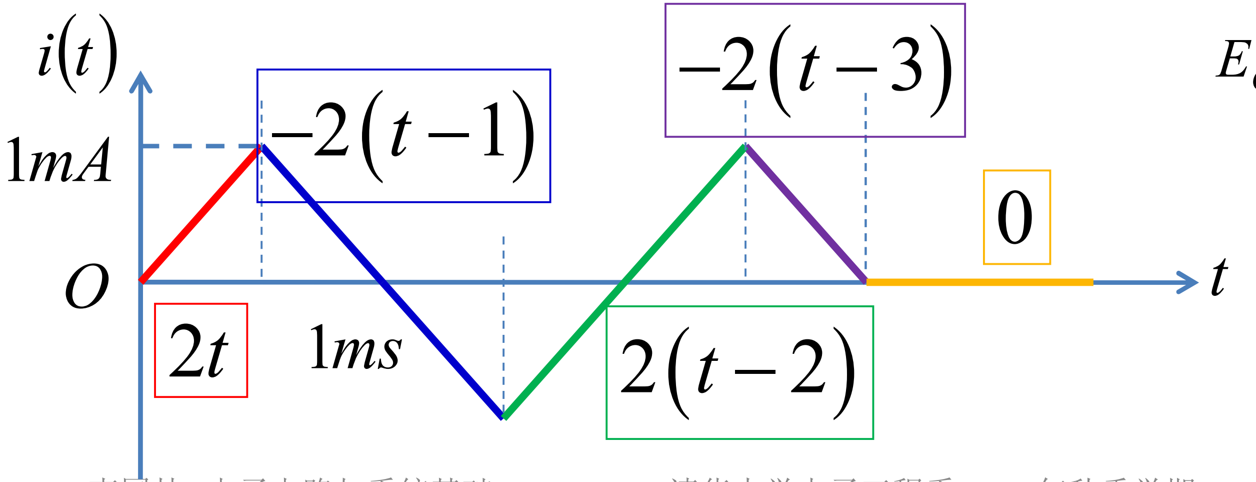
特别注意：时间单位ms，电流单位mA

$$Q(t)=Q_0+\int_0^t i(\tau)\cdot d\tau$$

$$v(t)=\frac{Q(t)}{C}$$

$$=V_0+\frac{1}{C}\int_0^t i(\tau)\cdot d\tau$$

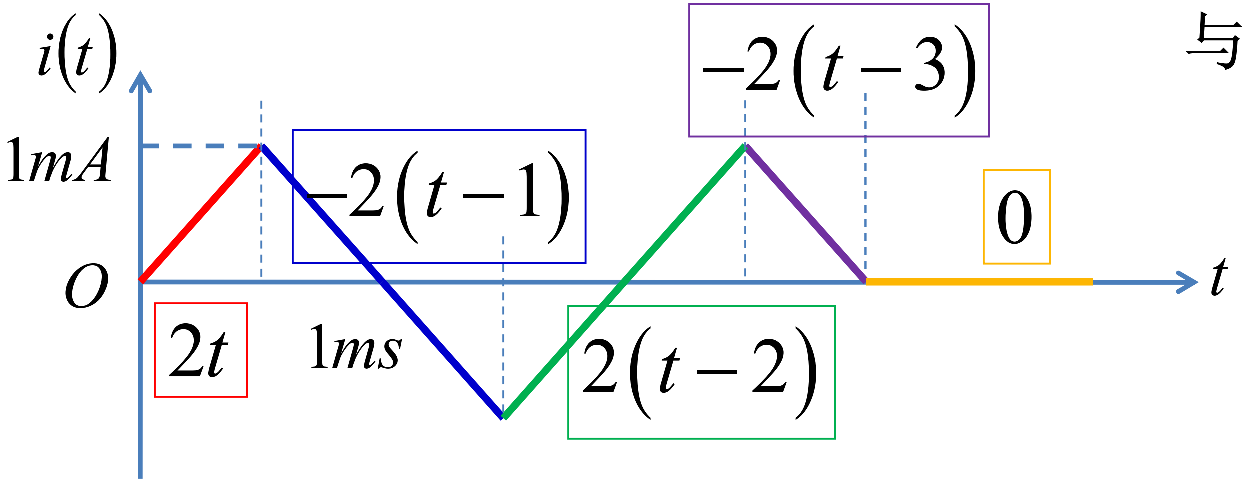
$$E_C(t)=\frac{1}{2}Cv^2(t)$$



$$Q(t)=Q(0)+\int_0^t i(t)dt=CV(t)$$

$$=\begin{cases} 5+t^2\ \mu C & 0\le t<0.5\ ms \\ 5.5-(t-1)^2\ \mu C & 0.5\le t<1.5\ ms \\ 5+(t-2)^2\ \mu C & 1.5\le t<2.5\ ms \\ 5.5-(t-3)^2\ \mu C & 2.5\le t<3\ ms \\ 5.5\ \mu C & t\ge 3\ ms \end{cases}$$

$$i(t)=\begin{cases} 2t\ mA & 0\le t<0.5\ ms \\ -2(t-1)\ mA & 0.5\le t<1.5\ ms \\ 2(t-2)\ mA & 1.5\le t<2.5\ ms \\ -2(t-3)\ mA & 2.5\le t<3\ ms \\ 0 & t\ge 3\ ms \end{cases}$$



- 分段线性电流的积分为分段二次函数
- 线性时不变电容，电压与电荷相同变化规律

$$V(t)=\frac{Q(t)}{C}$$

$$=\ldots(V)$$

$$Q(t)=Q(0)+\int_0^t i(t)\,dt=CV(t)$$

$$=\begin{cases} 5+t^2\ \mu C & 0\le t<0.5\ ms \\ 5.5-(t-1)^2\ \mu C & 0.5\le t<1.5\ ms \\ 5+(t-2)^2\ \mu C & 1.5\le t<2.5\ ms \\ 5.5-(t-3)^2\ \mu C & 2.5\le t<3\ ms \\ 5.5\ \mu C & t\ge 3\ ms \end{cases}$$

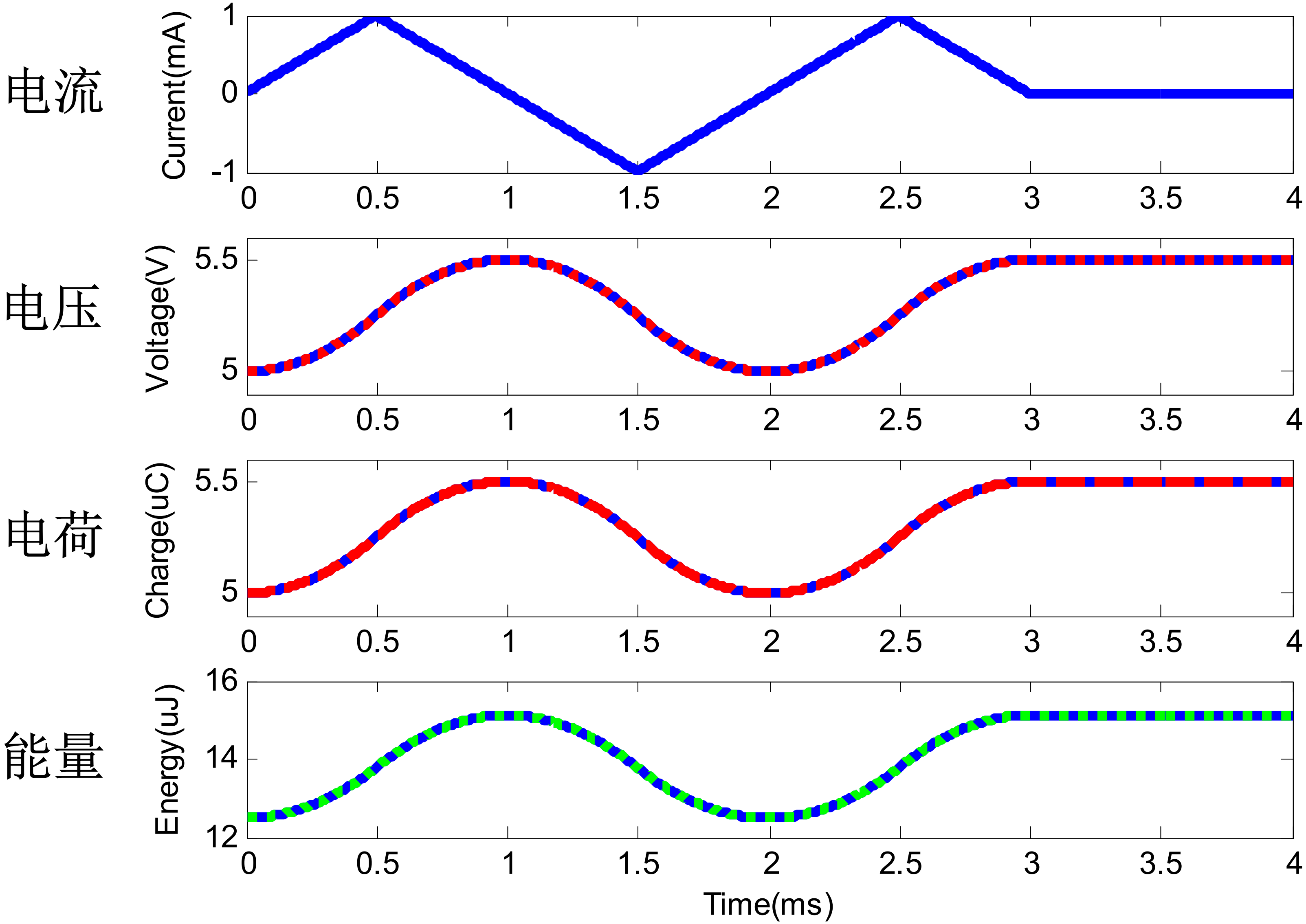
$$V(t)=V(0)+\frac{1}{C}\int_0^t i(t)\,dt$$

$$=\begin{cases} 5+t^2\ V & 0\le t<0.5\ ms \\ 5.5-(t-1)^2\ V & 0.5\le t<1.5\ ms \\ 5+(t-2)^2\ V & 1.5\le t<2.5\ ms \\ 5.5-(t-3)^2\ V & 2.5\le t<3\ ms \\ 5.5\,V & t\ge 3\ ms \end{cases}$$

$$E_C(t)=\frac{1}{2}Cv^2(t)$$

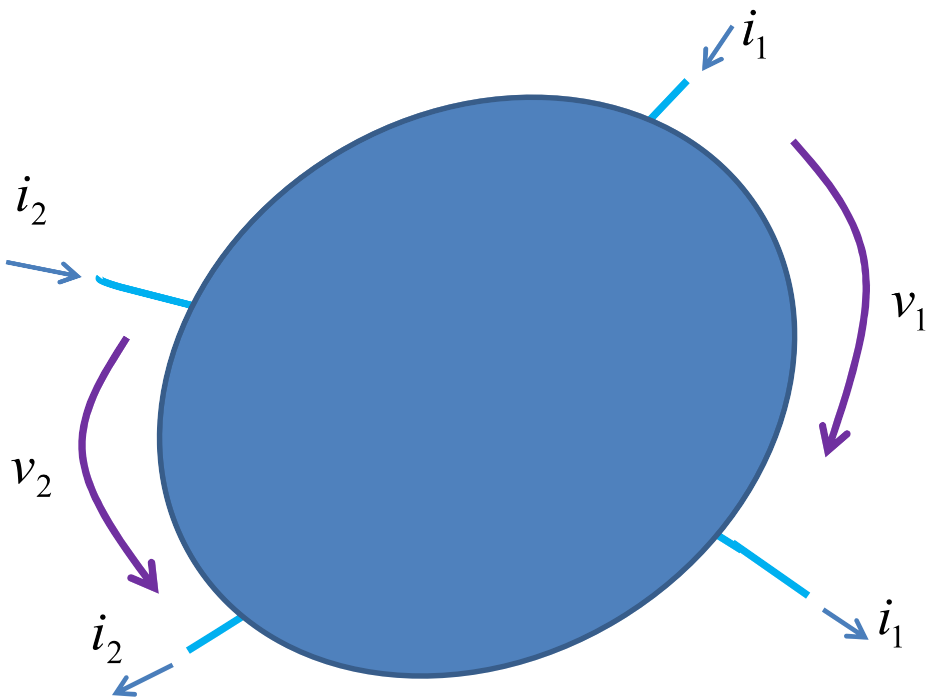
$$=\begin{cases} 0.5\left(5+t^2\right)^2\ \mu J & 0\le t\le 0.5ms \\ 0.5\left(5.5-(t-1)^2\right)^2\ \mu J & 0.5ms\le t\le 1.5ms \\ 0.5\left(5+(t-2)^2\right)^2\ \mu J & 1.5ms\le t\le 2.5ms \\ 0.5\left(5.5-(t-3)^2\right)^2\ \mu J & 2.5ms\le t\le 3ms \\ 5.5\ \mu J & t\ge 3ms \end{cases}$$

- 电容电能存储正比于电容电压的平方


电流
电压
电荷
能量
Current(mA)
1
0
-1
Voltage(V)
5.5
5
Charge(uC)
5.5
5
Energy(uJ)
16
14
12
0
0.5
1
1.5
2
2.5
3
3.5
4
Time(ms)

作业2：同名端判定

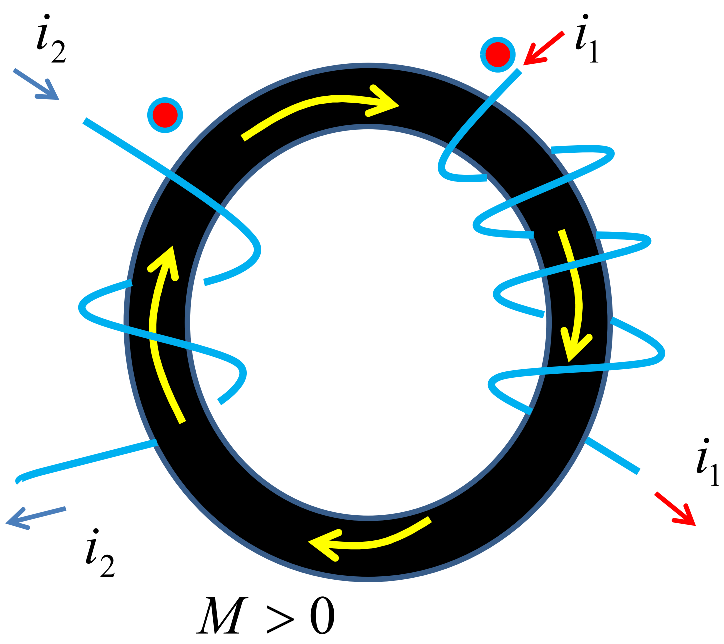
- 如果知道线圈绕向，可以判定同名端
- 如果被封装为黑匣子，内部绕向未知，请设计一个测试方案，判定同名端

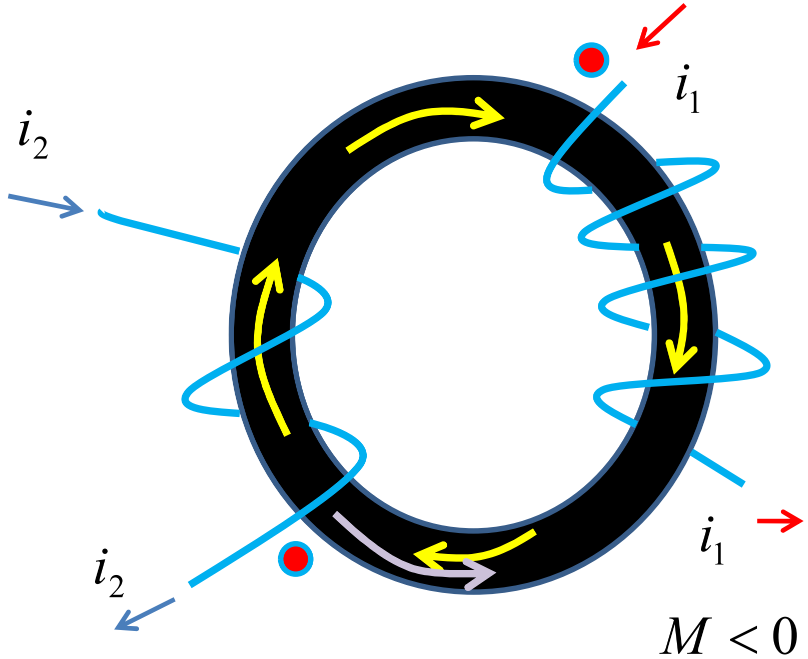


知道绕向：流入电流使得磁通加强的两个端点是同名端

不知绕向：？？？

考察感生电动势





同名端方向和端口电压、电流关联参考方向一致，互感大于**0**，否则小于**0**

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

令端口2开路

$$i_2 = 0 \qquad \frac{v_2}{v_1} = \frac{M}{L_1}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} L_1 \\ M \end{bmatrix} \frac{d}{dt} i_1$$

不知绕向：只需判定两个端口电压变化是否具有同相性即可：同相则同名

根据前述规律设计测试电路如下：

输入回路接电压源并串联开关和限流电阻，电源极性为正极接**a1**，输出回路接电压表，假定上接正极

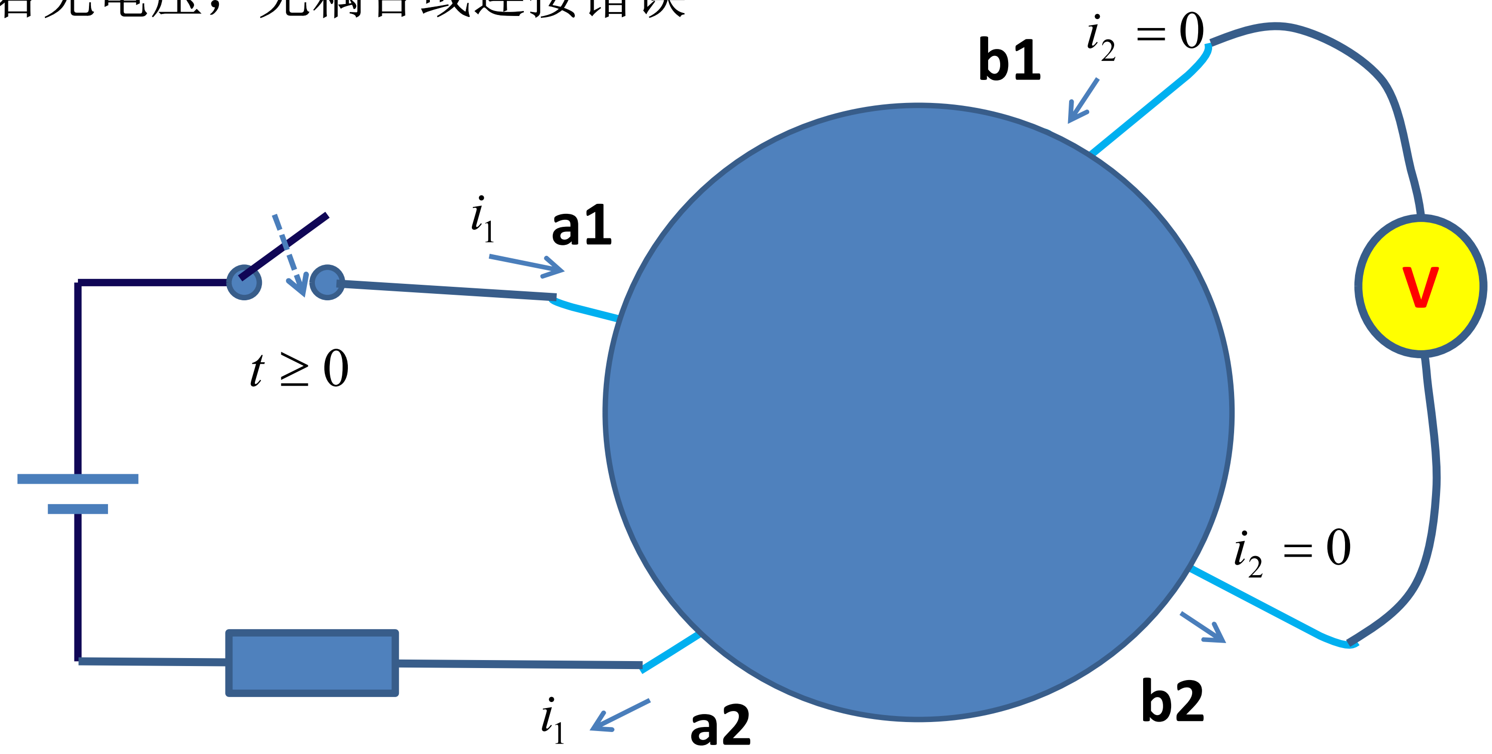
测试时，开关闭合，观察电压表视数

1）若为正电压，则**a1**和**b1**为同名端

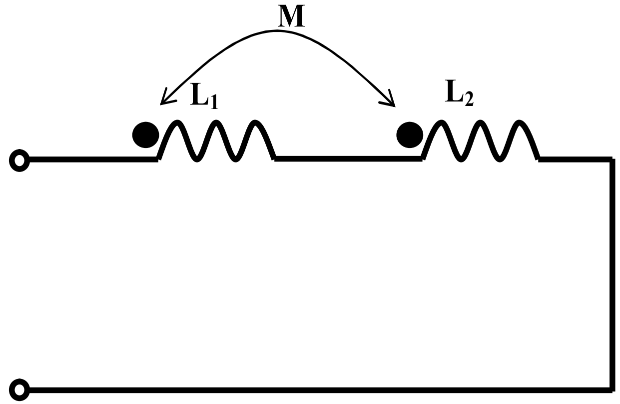
2）若为负电压，则**a1**和**b2**为同名端

3）若无电压，无耦合或连接错误

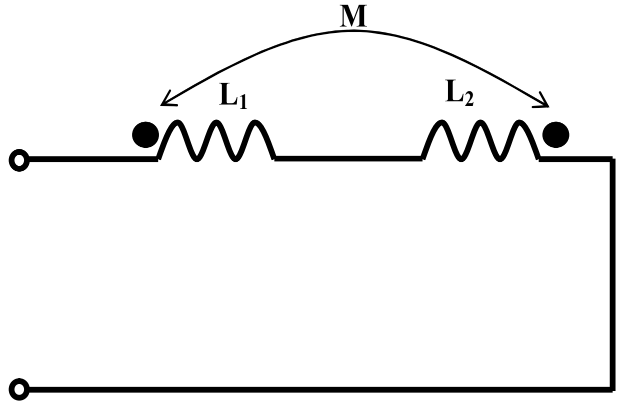
$$\frac{v_2}{v_1}=\frac{M}{L_1}>?<0$$



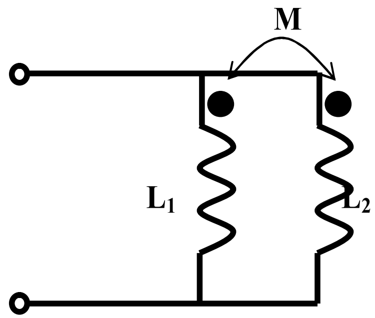
作业3：等效电感计算



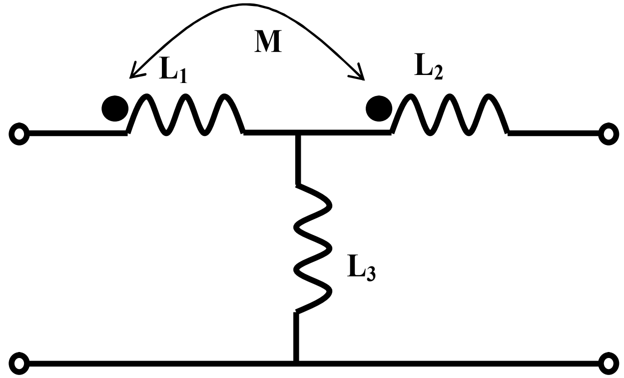
（a）电感串联：带互感



（b）电感串联：带互感，绕线反向



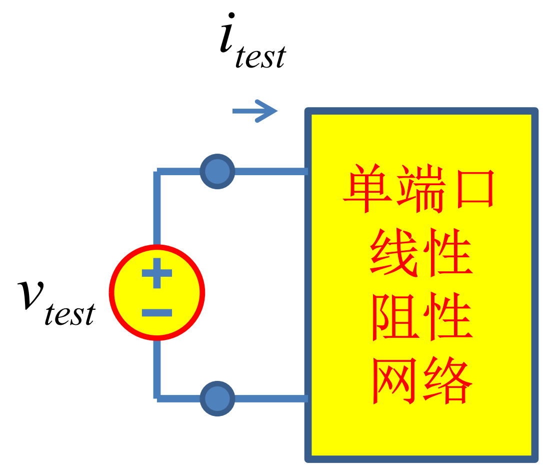
（c）电感并联：带互感



（d）二端口电感等效

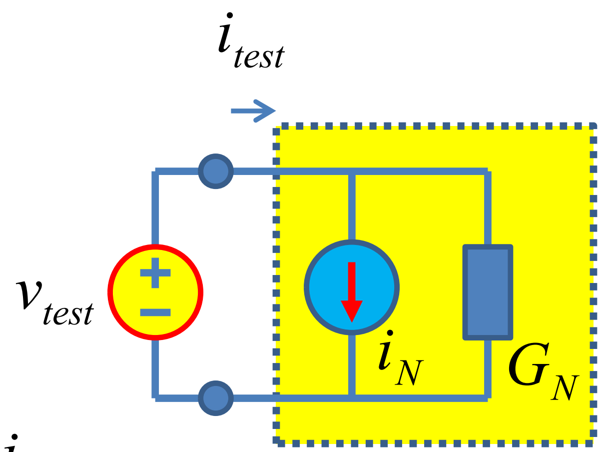
回顾：单端口线性电阻网络等效电路模型

加压求流、加流求压



加压求流

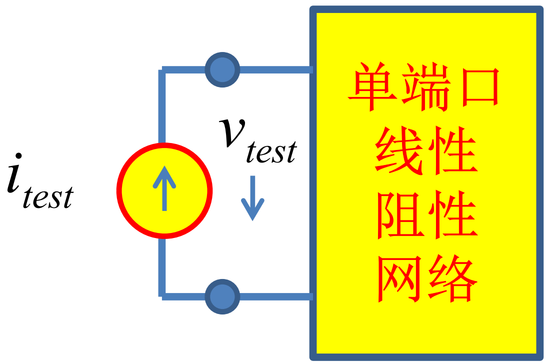
$$i_{test}=\alpha\cdot v_{test}+\beta$$



$$i_{test}=G_N\cdot v_{test}+i_N$$

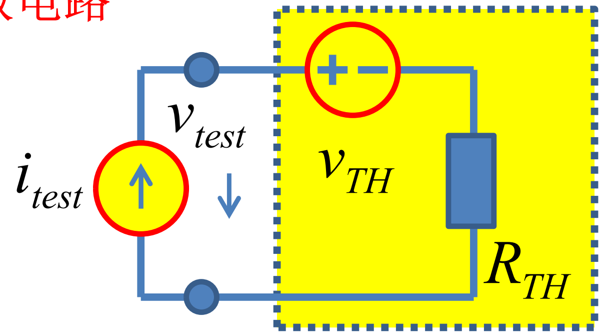
$$i_N=\beta \quad G_N=\alpha$$

加压测试获得压控形式的等效电路
加流测试获得流控形式的等效电路



加流求压

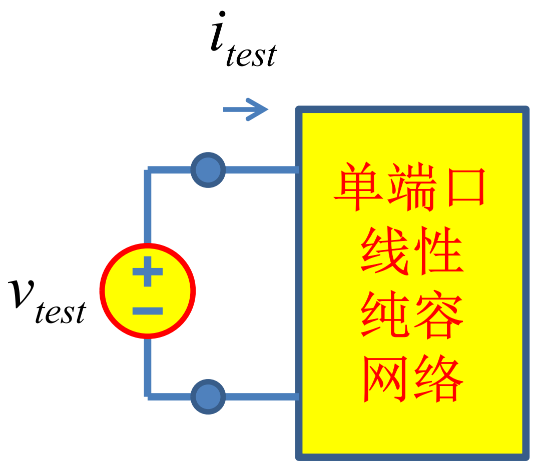
$$v_{test}=\alpha\cdot i_{test}+\beta$$



$$v_{test}=R_{TH}\cdot i_{test}+v_{TH}$$

$$v_{TH}=\beta \quad R_{TH}=\alpha$$

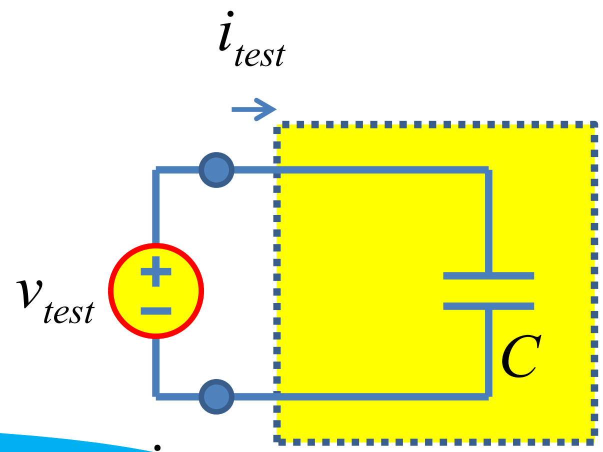
单端口纯容或纯感网络



加压求流

$$i_{test}=\alpha\cdot\frac{dv_{test}}{dt}$$

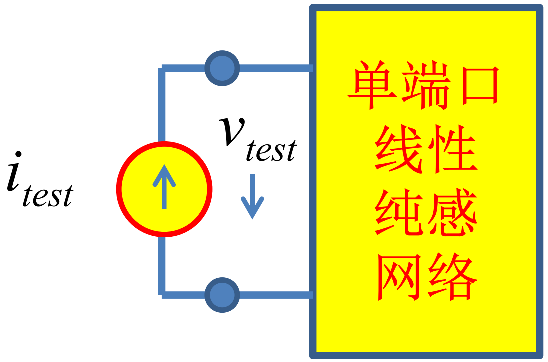
$$i_{test}=C\cdot\frac{dv_{test}}{dt}$$



$$\dot{I}_{test}=\alpha\cdot j\omega\dot{V}_{test} \qquad \dot{I}_{test}=j\omega C\dot{V}_{test}$$

$$C=\alpha$$

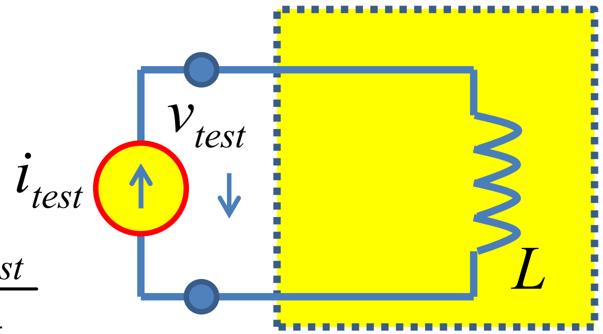
原理上可以这样分析



加流求压

$$v_{test}=\alpha\cdot\frac{di_{test}}{dt}$$

$$v_{test}=L\cdot\frac{di_{test}}{dt}$$

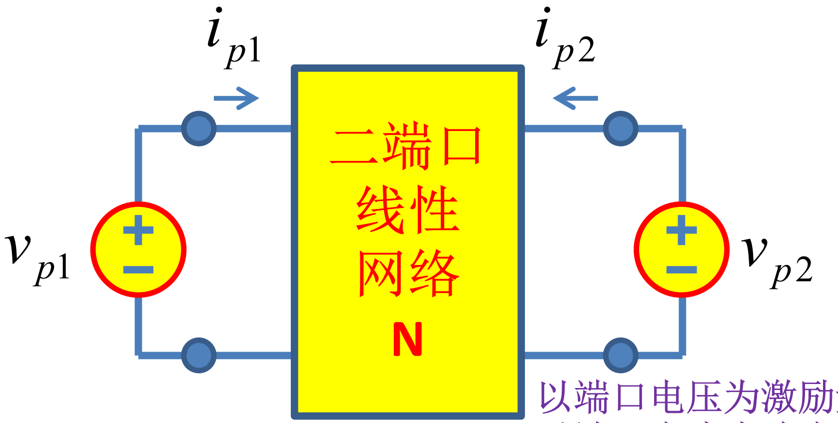


$$\dot{V}_{test}=\alpha\cdot j\omega\dot{I}_{test} \qquad \dot{V}_{test}=j\omega L\dot{I}_{test}$$

$$L=\alpha$$

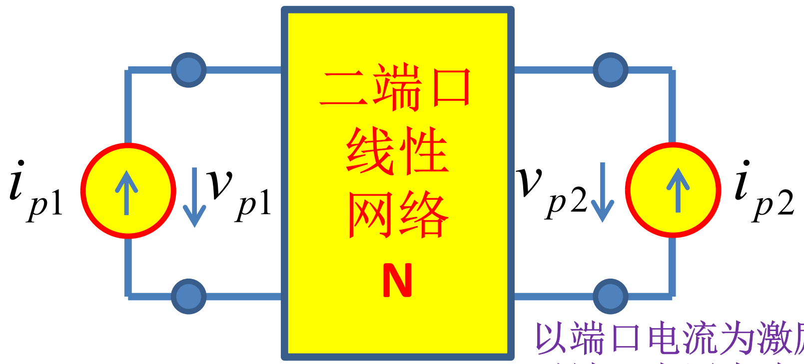
端口加压、加流方法

对LTI二端口网络的测量：4种基本测量手段



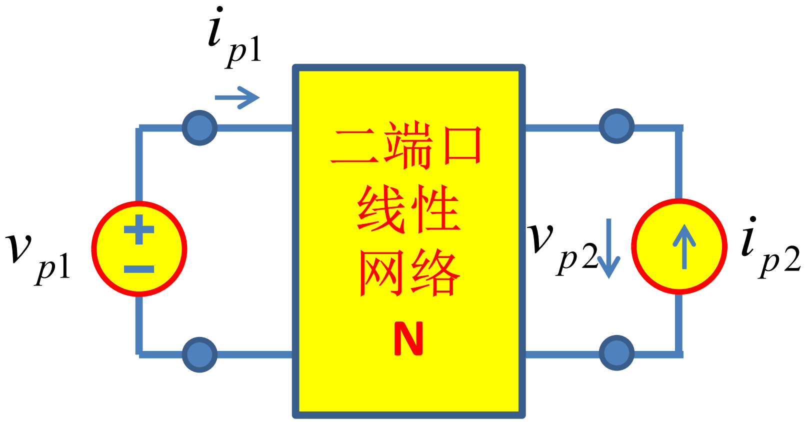
以端口电压为激励量，
以端口电流为响应量

两个端口同时加独立变化的测试电压
y参量：在相量域，复数Y参量矩阵

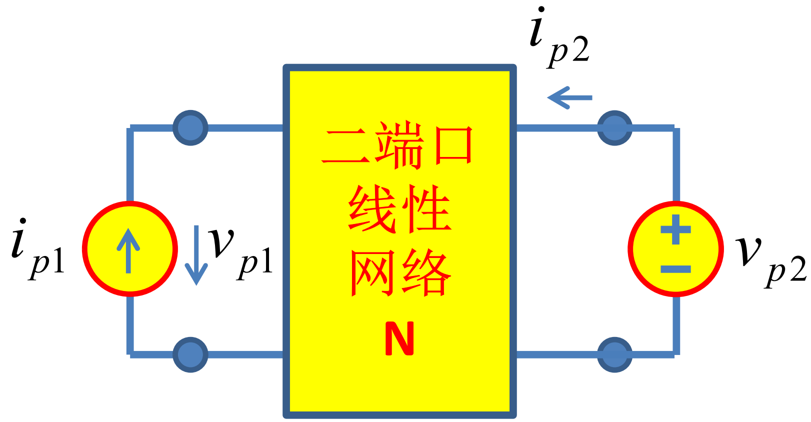


以端口电流为激励量，
以端口电压为响应量

两个端口同时加独立变化的测试电流
z参量：在相量域，复数Z参量矩阵

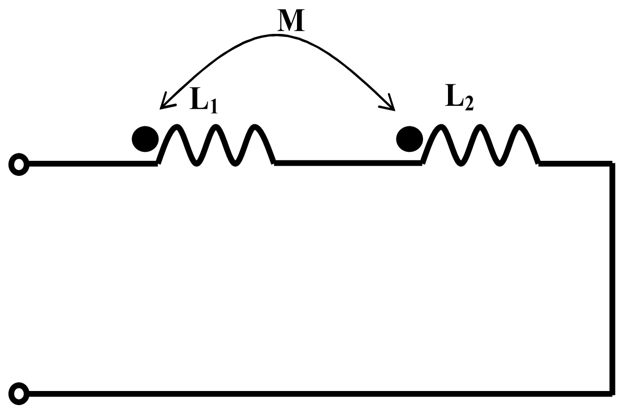


端口1加测试电压、端口2加测试电流
g参量：在相量域，复数g参量矩阵

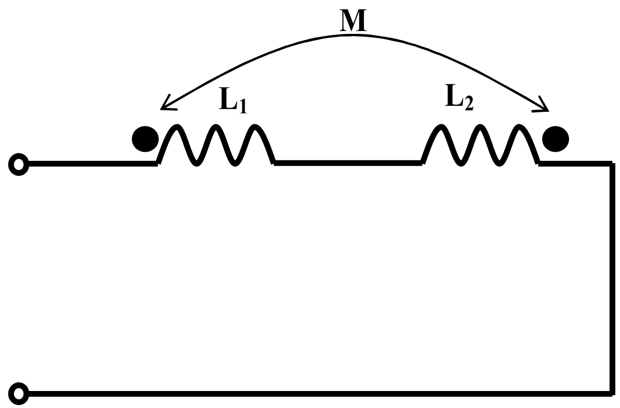


端口1加测试电流、端口2加测试电压
h参量：在相量域，复数h参量矩阵

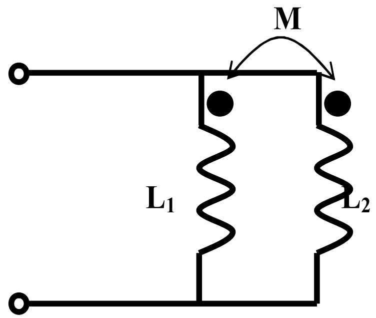
纯感网络，加流求压



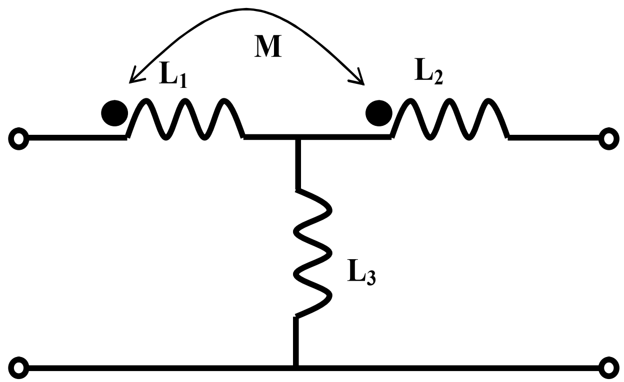
（a）电感串联：带互感



（b）电感串联：带互感，绕线反向

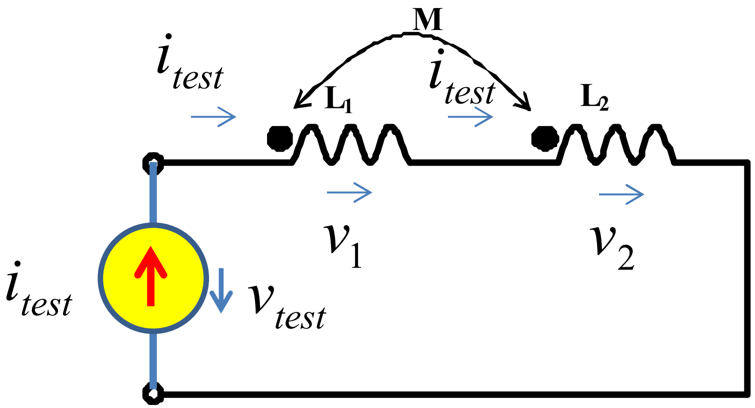


（c）电感并联：带互感



（d）二端口电感等效

带互感的电感串联（1）



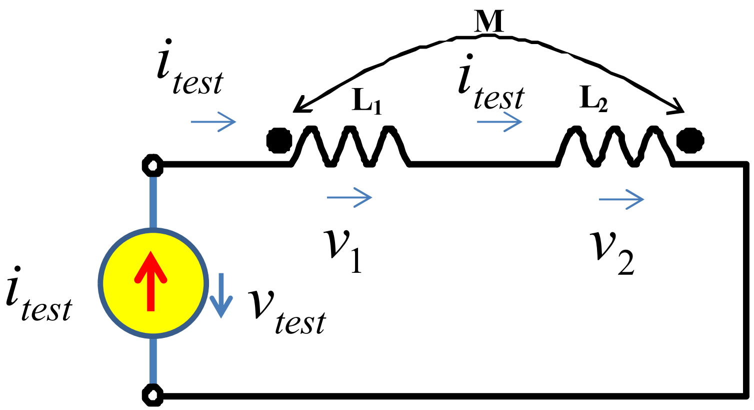
$$L = L_1 + L_2 + 2M$$

$$\begin{aligned} v_{test} &= v_1 + v_2 \\ &= \left(L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}\right) + \left(L_2 \frac{di_2}{dt} + M\frac{di_1}{dt}\right) \\ &= \left(L_1 \frac{di_{test}}{dt} + M\frac{di_{test}}{dt}\right) + \left(L_2 \frac{di_{test}}{dt} + M\frac{di_{test}}{dt}\right) \\ &= \left(L_1 + L_2 + 2M\right)\frac{di_{test}}{dt} \\ &= L\frac{di_{test}}{dt} \end{aligned}$$

$$\begin{aligned} L^{\text{磁芯电感}} &= N_1^2\Xi + N_2^2\Xi + 2kN_1N_2\Xi \\ &\overset{k=1}{=} \left(N_1^2 + N_2^2 + 2N_1N_2\right)\Xi \\ &= \left(N_1 + N_2\right)^2\Xi = N^2\Xi \end{aligned}$$

在磁环上绕**N=N1+N2**圈的电感，可视为全耦合的L_1和L_2的电感串联，总电感**L= L_1+L_2+2M**

带互感的电感串联（2）



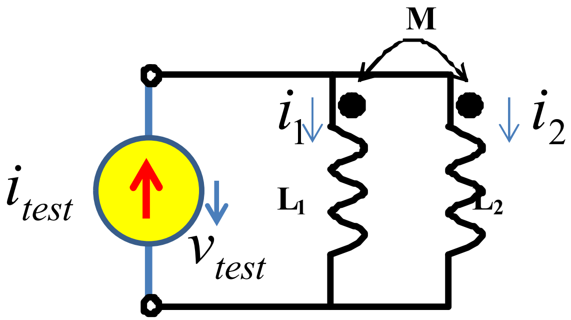
$$L = L_1 + L_2 - 2M$$

$$\begin{aligned}
v_{test} &= v_1 + v_2 \\
&= \left(L_1\frac{di_1}{dt} - M\frac{di_2}{dt}\right) + \left(L_2\frac{di_2}{dt} - M\frac{di_1}{dt}\right) \\
&= \left(L_1\frac{di_{test}}{dt} - M\frac{di_{test}}{dt}\right) + \left(L_2\frac{di_{test}}{dt} - M\frac{di_{test}}{dt}\right) \\
&= \left(L_1 + L_2 - 2M\right)\frac{di_{test}}{dt} \\
&= L\frac{di_{test}}{dt}
\end{aligned}$$

$$\begin{aligned}
L &\overset{\text{磁芯电感}}{=} N_1^{\ 2}\Xi + N_2^{\ 2}\Xi - 2kN_1N_2\Xi \\
&\overset{k=1}{=} \left(N_1^{\ 2} + N_2^{\ 2} - 2N_1N_2\right)\Xi \\
&= \left(N_1 - N_2\right)^2\Xi
\end{aligned}$$

在磁环上绕**N1**圈，再反向绕**N1**圈，则形成无电感导线回路。

带互感的电感并联（1）



$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = v_{test}$$

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = v_{test}$$

$$\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_{test}$$

$$i_{test} = i_1 + i_2$$

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_{test}$$

$$= \frac{1}{L_1 L_2 - M^2} \begin{bmatrix} L_2 & -M \\ -M & L_1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_{test}$$

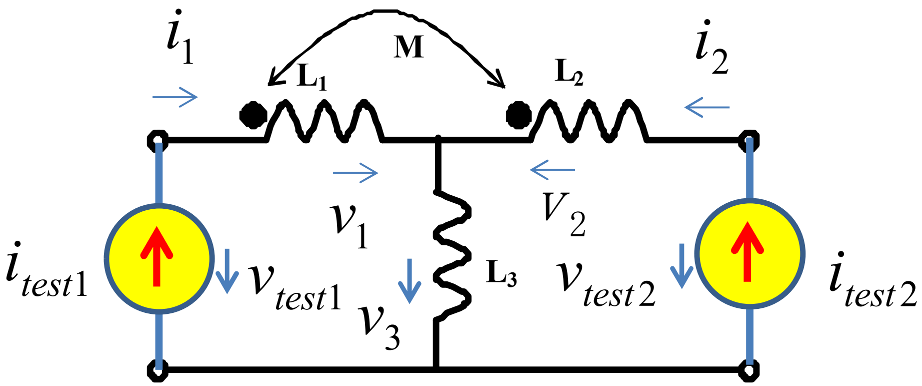
$$= \frac{1}{L_1 L_2 - M^2} \begin{bmatrix} L_2 - M \\ L_1 - M \end{bmatrix} v_{test}$$

$$\frac{di_{test}}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{L_2 - M + L_1 - M}{L_1 L_2 - M^2} v_{test}$$

$$v_{test} = \frac{L_1 L_2 - M^2}{L_2 + L_1 - 2M} \frac{di_{test}}{dt}$$

$$L = \frac{L_1 L_2 - M^2}{L_2 + L_1 - 2M}$$

二端口电感等效



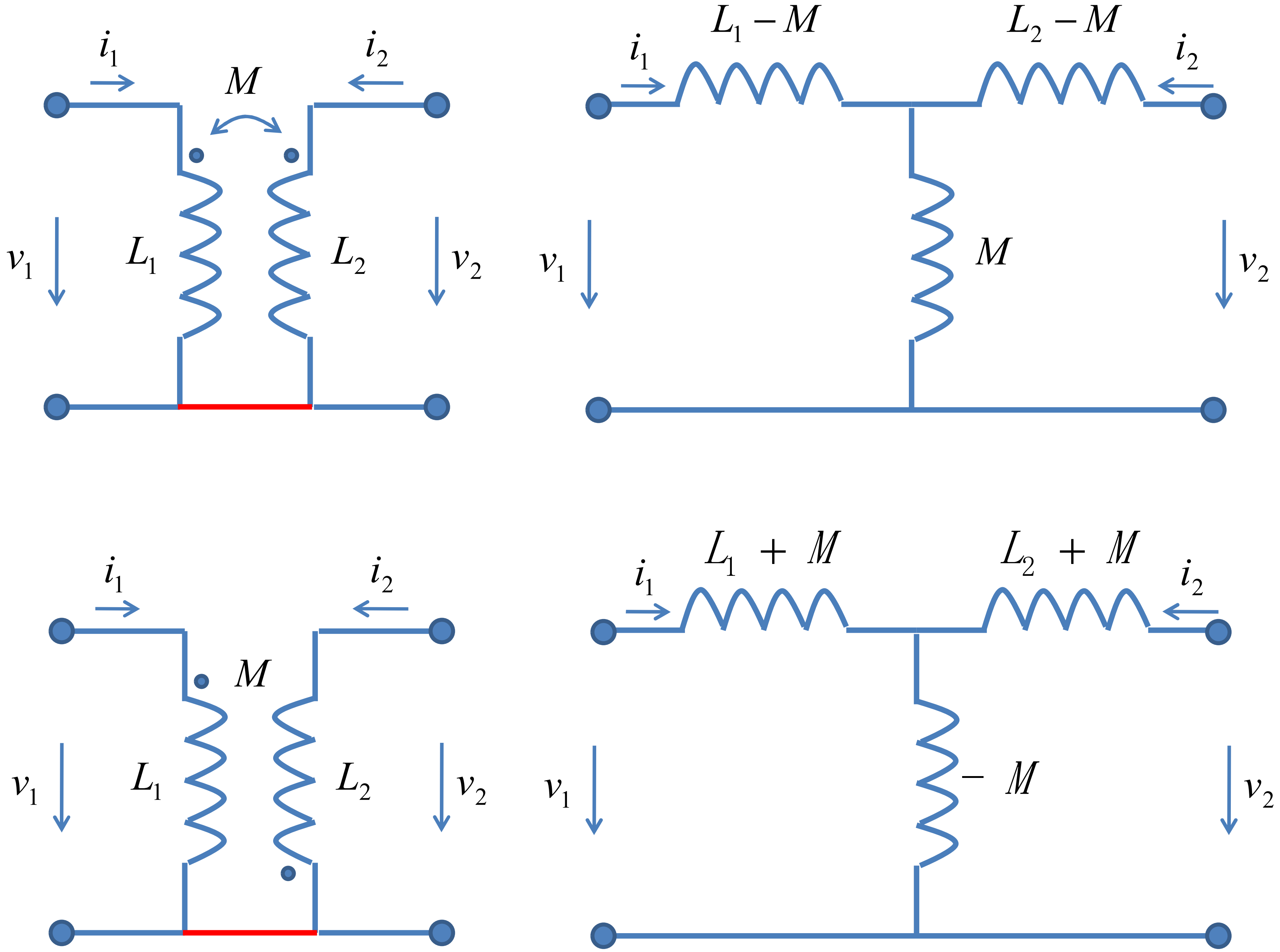
$$\begin{bmatrix} v_{test1} \\ v_{test2} \end{bmatrix} = \begin{bmatrix} L_1+L_3 & L_3-M \\ L_3-M & L_2+L_3 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{test1} \\ i_{test2} \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} L_1+L_3 & L_3-M \\ L_3-M & L_2+L_3 \end{bmatrix}$$

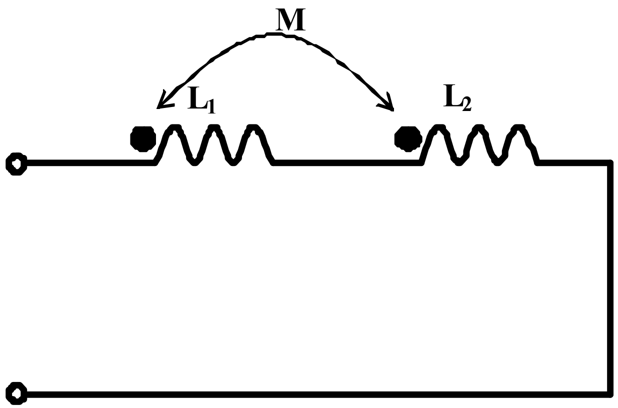
$$v_{test1} = v_1 + v_3 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} + L_3 \frac{d(i_1+i_2)}{dt} = (L_1+L_3)\frac{di_1}{dt} + (L_3-M)\frac{di_2}{dt}$$

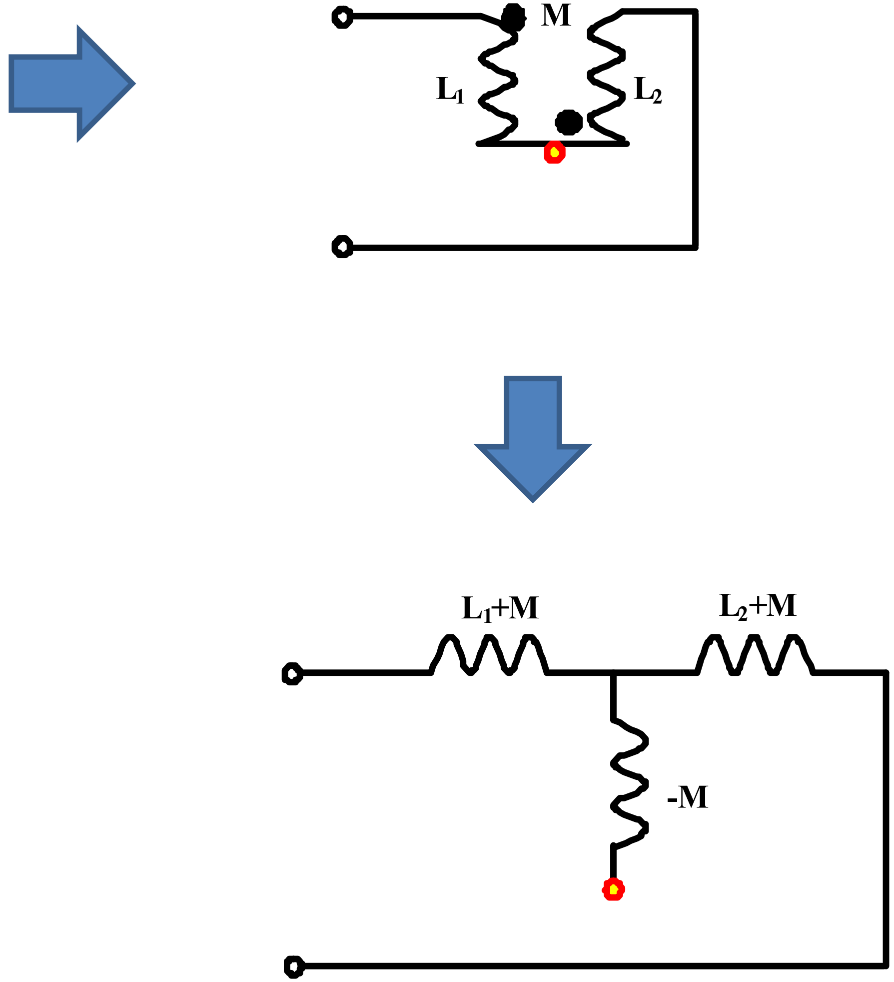
$$v_{test2} = v_2 + v_3 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} + L_3 \frac{d(i_1+i_2)}{dt} = (L_2+L_3)\frac{di_2}{dt} + (L_3-M)\frac{di_1}{dt}$$

T型网络等效分析

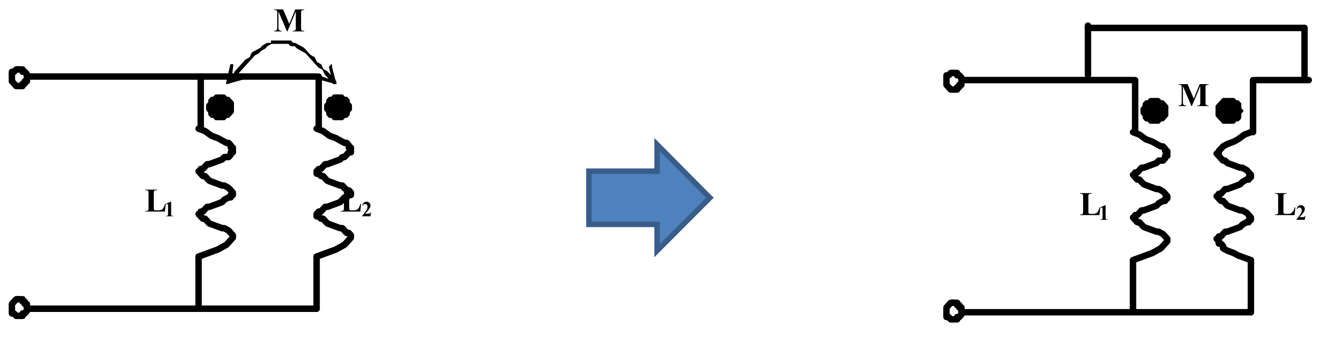


i1
i2
M
v1
L1
L2
v2
L1 − M
L2 − M
i1
i2
v1
M
v2
i1
i2
M
v1
L1
L2
v2
L1 + M
L2 + M
i1
i2
v1
− M
v2





$$L = (L_1 + M) + (L_2 + M)$$
$$= L_1 + L_2 + 2M$$



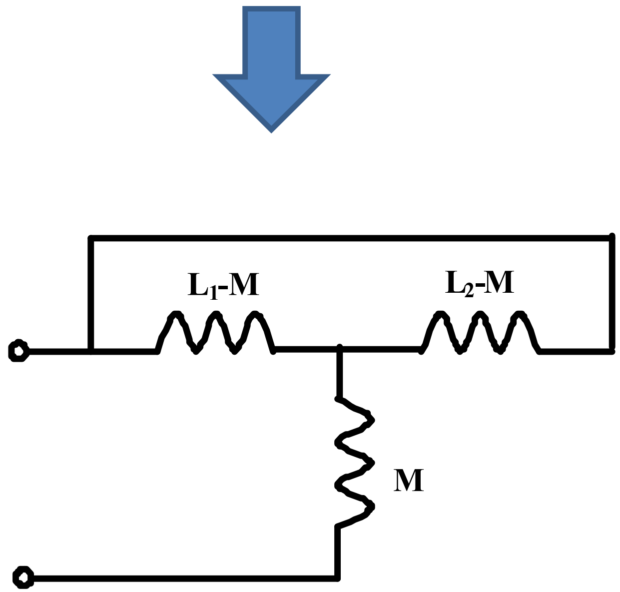
$$L=\left(\left(L_1-M\right)\|\left(L_2-M\right)\right)\text{串M}$$

$$=\frac{1}{\dfrac{1}{L_1-M}+\dfrac{1}{L_2-M}}+M$$

$$=\frac{\left(L_1-M\right)\left(L_2-M\right)}{L_1-M+L_2-M}+M$$

$$=\frac{L_1L_2-ML_2-ML_1+M^2}{L_1+L_2-2M}+\frac{\left(L_1+L_2-2M\right)M}{L_1+L_2-2M}$$

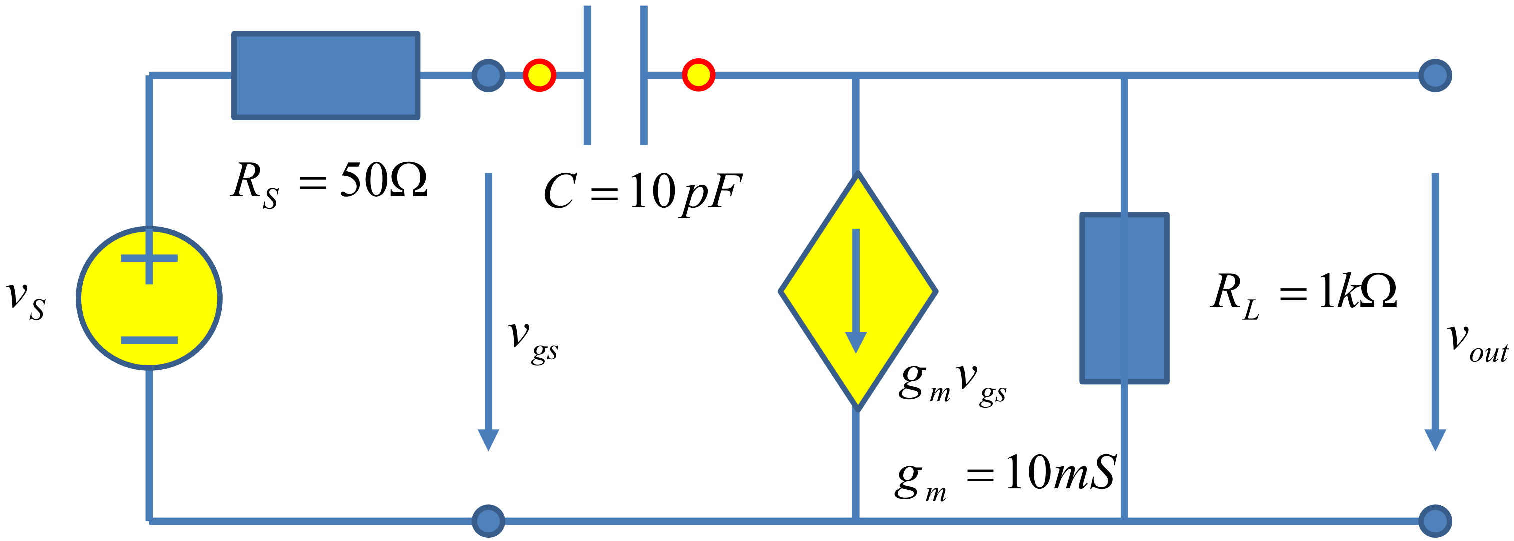
$$=\frac{L_1L_2-M^2}{L_1+L_2-2M}$$



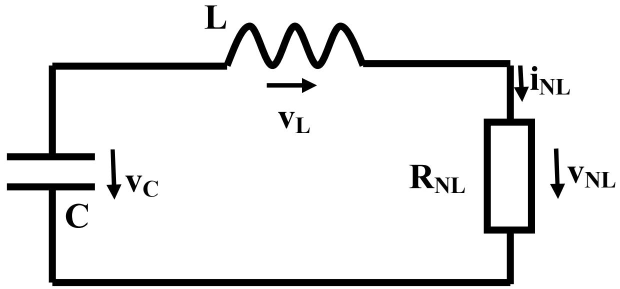
作业4　列写电路方程

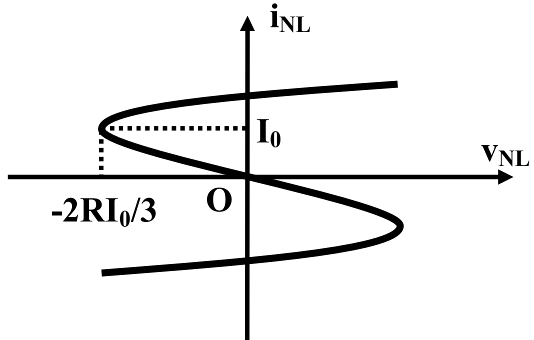
（下节课和三要素法作业一并考察）

- 带跨接电容的跨导放大器
  - 将电容之外的电阻电路等效为戴维南源，列写关于电容电压v_c的电路方程（ v_c为未知量x ）
  - 列写关于v_{out}的电路方程（v_{out}为未知量x）



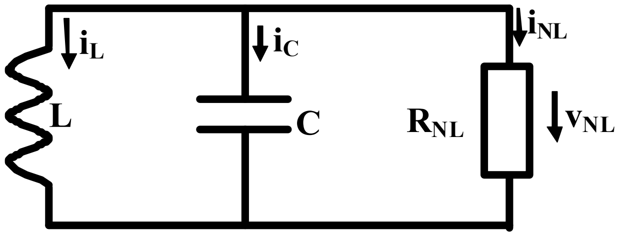
作业5（选作）：后向欧拉法仿真

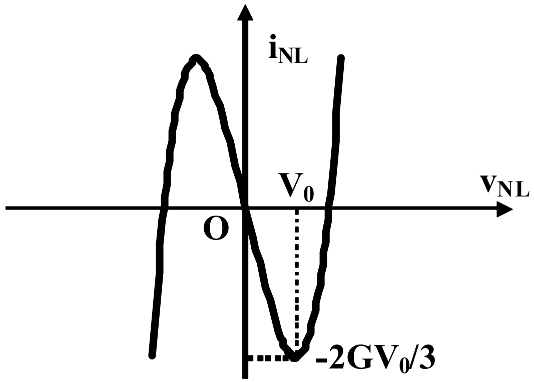




$$v_{NL} = -Ri_{NL} + \frac{R}{3I_0^2} i_{NL}^3$$

课堂仿真例：**L=100μH，C=20pF，R=100Ω，I_0=1mA，$v_C(0)$=10mV，$i_L(0)$=0**

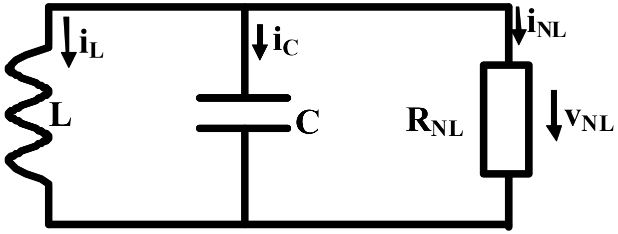


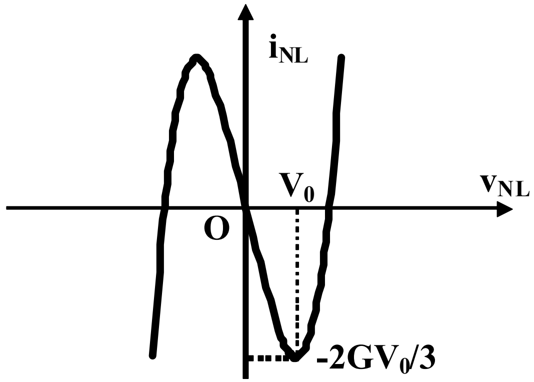


$$i_{NL} = -Gv_{NL} + \frac{G}{3V_0^2} v_{NL}^3$$

作业仿真例：**C=100pF，L=20μH，G=100μS，V_0=1V，$v_C(0)$=0，$i_L(0)$=10mA**

列写状态方程





$$i_{NL} = -Gv_{NL} + \frac{G}{3V_0^2} v_{NL}^3$$

$$0 = i_L + i_C + i_{NL} = i_L + C\frac{dv_C}{dt} - Gv_C + \frac{G}{3V_0^2} v_C^3$$

$$v_C = v_L = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{1}{L} v_C$$

$$\frac{dv_C}{dt} = -\frac{1}{C} i_L + \frac{G}{C} v_C - \frac{G}{3CV_0^2} v_C^3$$

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v_C \end{bmatrix} = \begin{bmatrix} \frac{1}{L} v_C \\ -\frac{1}{C} i_L + \frac{G}{C} v_C - \frac{G}{3CV_0^2} v_C^3 \end{bmatrix}$$

非线性时不变二阶动态系统

$$\frac{d}{dt}\mathbf{x} = f(\mathbf{x})$$

后向欧拉法

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v_C \end{bmatrix} = \begin{bmatrix} \frac{1}{L}v_C \\ -\frac{1}{C}i_L + \frac{G}{C}v_C - \frac{G}{3CV_0^2}v_C^3 \end{bmatrix}$$

$$\frac{d}{dt}\mathbf{x} = f(\mathbf{x})$$

$$\mathbf{x}(t_{k+1}) = \mathbf{x}(t_k) + \Delta t \cdot f(\mathbf{x}(t_{k+1}))$$

$$\begin{bmatrix} i_L(t_{k+1}) \\ v_C(t_{k+1}) \end{bmatrix} = \begin{bmatrix} i_L(t_k) \\ v_C(t_k) \end{bmatrix} + \Delta t \cdot \begin{bmatrix} \frac{1}{L}v_C(t_{k+1}) \\ -\frac{1}{C}i_L(t_{k+1}) + \frac{G}{C}v_C(t_{k+1}) - \frac{G}{3CV_0^2}v_C^3(t_{k+1}) \end{bmatrix}$$

$$i_L(t_{k+1}) = i_L(t_k) + \frac{\Delta t}{L}v_C(t_{k+1}) = i_L(t_k) + \frac{1}{R_L}v_C(t_{k+1})$$

$$v_C(t_{k+1}) = v_C(t_k) - \frac{\Delta t}{C}i_L(t_{k+1}) + \frac{\Delta t G}{C}v_C(t_{k+1}) - \frac{\Delta t G}{3CV_0^2}v_C^3(t_{k+1})$$

$$= v_C(t_k) - R_C\left(i_L(t_k) + \frac{1}{R_L}v_C(t_{k+1})\right) + \frac{R_C}{R}v_C(t_{k+1}) - \frac{R_C}{3RV_0^2}v_C^3(t_{k+1})$$

$$f(v_C(t_{k+1})) = \frac{R_C}{3RV_0^2}v_C^3(t_{k+1}) + \left(1 + \frac{R_C}{R_L} - \frac{R_C}{R}\right)v_C(t_{k+1}) - v_C(t_k) + R_C i_L(t_k)$$

此非线性方程可以用等效电路（电容戴维南源，电感诺顿源）同样获得

牛顿-拉夫逊迭代法求非线性代数方程

$$f\left(v_C\left(t_{k+1}\right)\right)=\frac{R_C}{3RV_0^2}v_C^3\left(t_{k+1}\right)+\left(1+\frac{R_C}{R_L}-\frac{R_C}{R}\right)v_C\left(t_{k+1}\right)-v_C\left(t_k\right)+R_C i_L\left(t_k\right)$$

$$f'\left(v_C\left(t_{k+1}\right)\right)=\frac{R_C}{R}\frac{v_C^2\left(t_{k+1}\right)}{V_0^2}+\left(1+\frac{R_C}{R_L}-\frac{R_C}{R}\right)$$

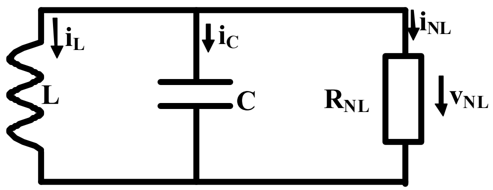
$$v_C^{(0)}\left(t_{k+1}\right)=v_C\left(t_k\right)$$

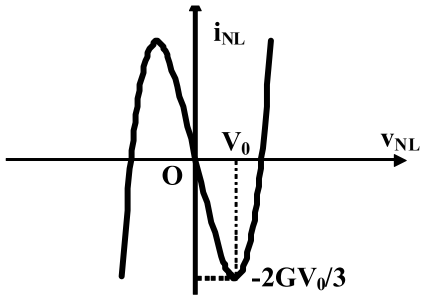
用前一时刻的状态做为这一时刻迭代初始值

$$v_C^{(j+1)}\left(t_{k+1}\right)=v_C^{(j)}\left(t_{k+1}\right)-\frac{f\left(v_C^{(j)}\left(t_{k+1}\right)\right)}{f'\left(v_C^{(j)}\left(t_{k+1}\right)\right)}$$

牛顿拉夫逊迭代格式

matlab初始设置






```
clear all                %清空内存
                         %电路参量设置
G=100E-6;                %N型负阻参量
V0=1;
R=1/G;

L=20E-6;                 %并联电感
C=100E-12;               %并联电容

vC(1)=0;                 %电容初始电压
iL(1)=10E-3;             %电感初始电流
tt(1)=0;                 %时间起点

Dt=1E-10;                %时间步长
RC=Dt/C;                 %后向欧拉法时间离散化电容等效电压源内阻
RL=L/Dt;                 %后向欧拉法时间离散化电感等效电流源内阻
```

作业仿真例：**C=100pF，L=20μH，G=100μS，V_0=1V，$v_C(0)$=0，$i_L(0)$=10mA**

如果看不懂可请求帮助：**help**

- >> help clear
- CLEAR Clear variables and functions from memory.
- CLEAR removes all variables from the workspace.
- CLEAR VARIABLES does the same thing.
- CLEAR GLOBAL removes all global variables.
- CLEAR FUNCTIONS removes all compiled M- and MEX-functions.
-
- CLEAR ALL removes all variables, globals, functions and MEX links.
- CLEAR ALL at the command prompt also removes the Java packages import
- list.
-
- CLEAR IMPORT removes the Java packages import list at the command
- prompt. It cannot be used in a function.
-
- CLEAR CLASSES is the same as CLEAR ALL except that class definitions
- are also cleared…

后向欧拉法：前一个时间点的状态是后一个时间状态的激励源

```
k=1;
for t=Dt:Dt:2E-5                    %后向欧拉法时间步进计算
   k=k+1;
   tt(k)=t;
                                    %非线性代数方程的牛顿拉夫逊迭代法求解
   vC(k)=vC(k-1);                   %迭代初始值设置为上个时间点的数值解
   flag=0;
   while flag==0
      f=(1+RC/RL-RC/R)*vC(k)+RC/R/3/V0^2*vC(k)^3-vC(k-1)+RC*iL(k-1);  %非线性方程
      fp=1+RC/RL-RC/R+RC/R*(vC(k)/V0)^2;                               %微分斜率
      vC(k)=vC(k)-f/fp;                                                %牛顿拉夫逊迭代

      if abs(f)<1E-9                %迭代结束标记
         flag=1;
      end
   end

   iL(k)=iL(k-1)+vC(k)/RL;          %求电感电流
end
```

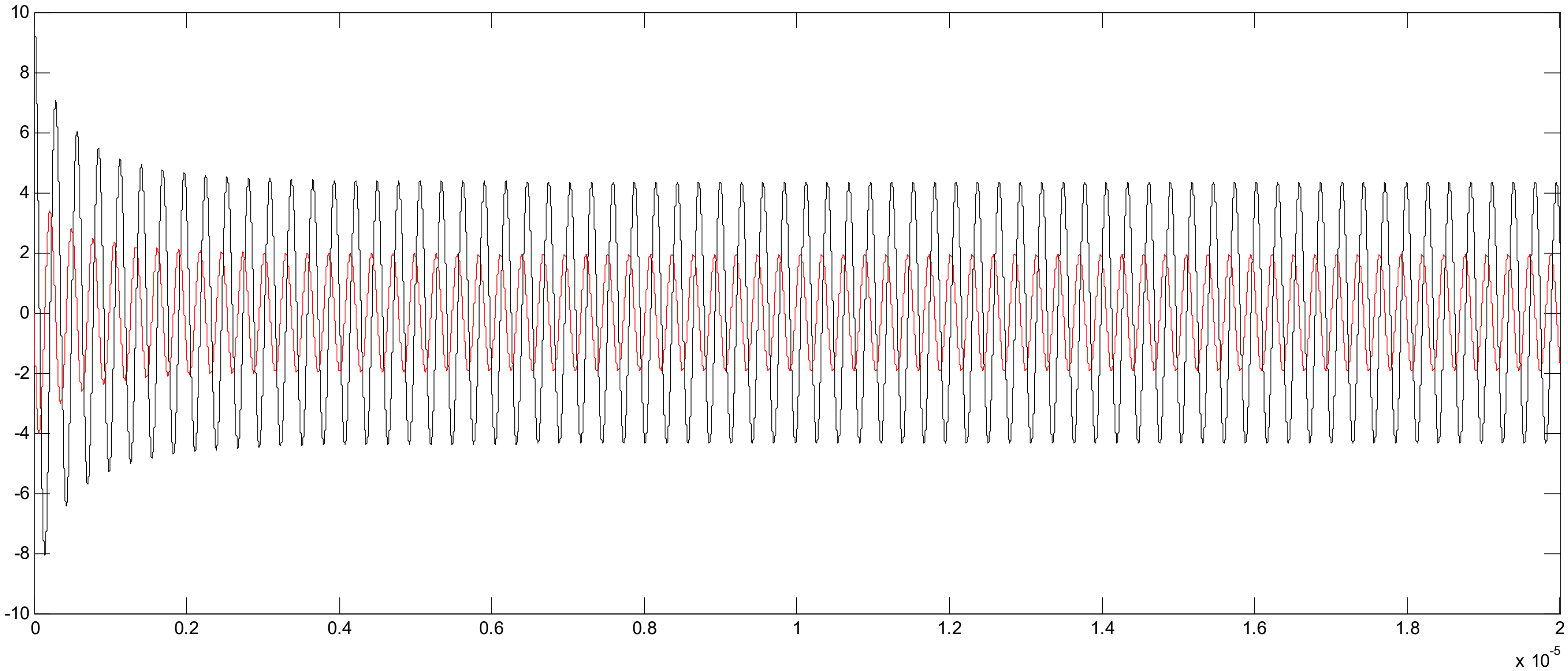
$$f\left(v_C\left(t_{k+1}\right)\right)=\left(1+\frac{R_C}{R_L}-\frac{R_C}{R}\right)v_C\left(t_{k+1}\right)+\frac{R_C}{3RV_0^2}v_C^3\left(t_{k+1}\right)-v_C\left(t_k\right)+R_C i_L\left(t_k\right)$$

$$i_L\left(t_{k+1}\right)=i_L\left(t_k\right)+\frac{1}{R_L}v_C\left(t_{k+1}\right)$$

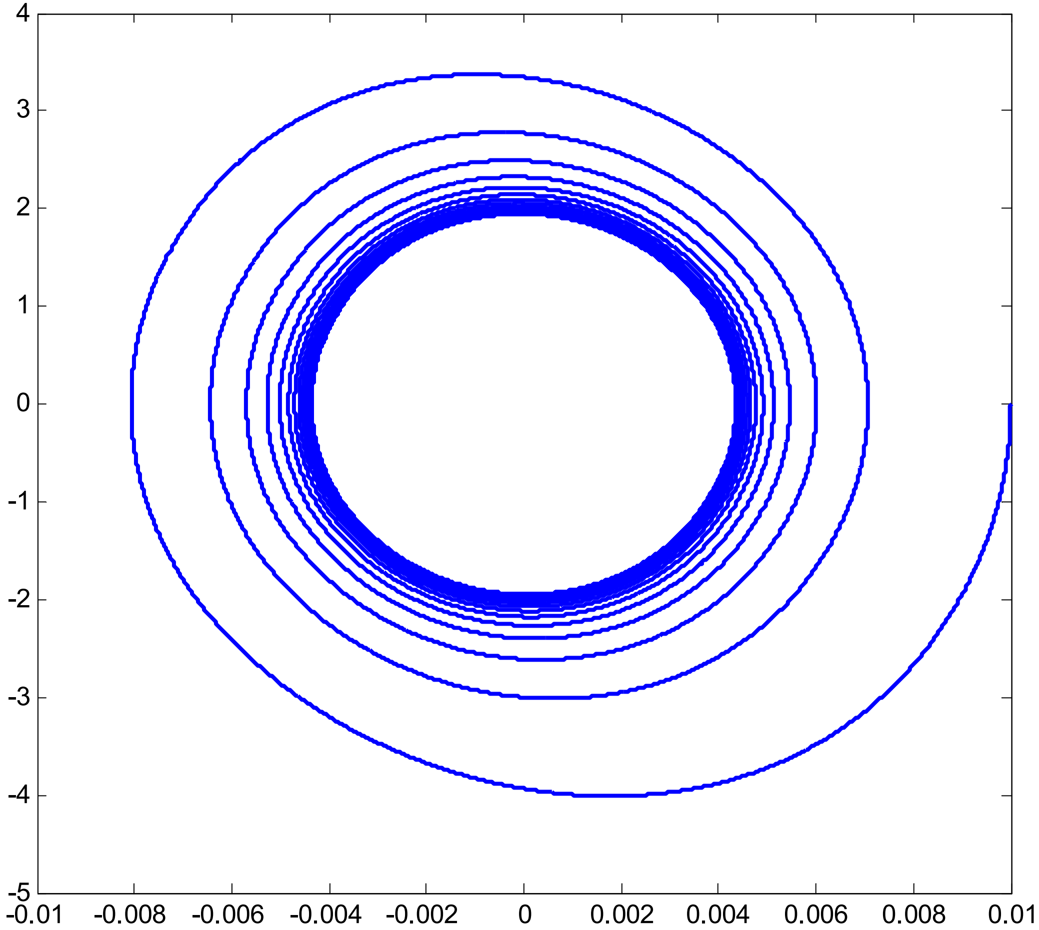
输出：作图

```
figure(1)
hold on                 %保持在一张图上画数条曲线
plot(tt,vC,'r')         %电容电压时域波形（单位V）
plot(tt,iL*1E3,'k')     %电感电流时域波形（单位mA）

figure(2)               %相图
plot(iL,vC)
```

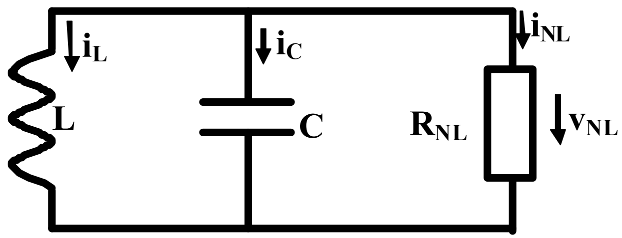


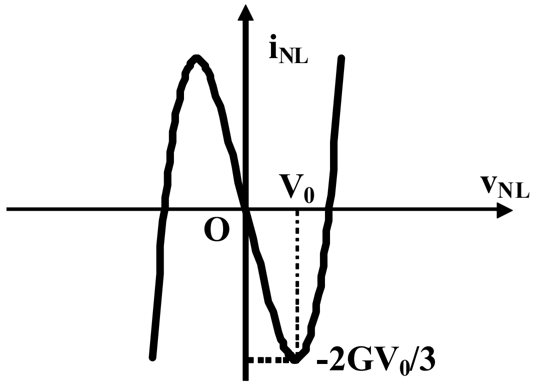
相轨迹



相轨迹收敛到一个圆形极限环上，说明这是一个正弦波振荡器；

由于初始状态较大，使得非线性电阻起始阶段工作在正阻区，故而起始呈现振幅衰减振荡波形，随着振荡幅度的降低，负导效应增强。当负导效应和正导效应相互抵偿时，则电路中只剩下纯的**LC**谐振腔，能量转换呈现正弦形态



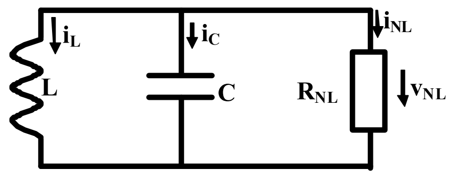


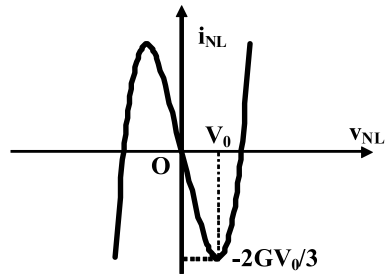
张弛振荡

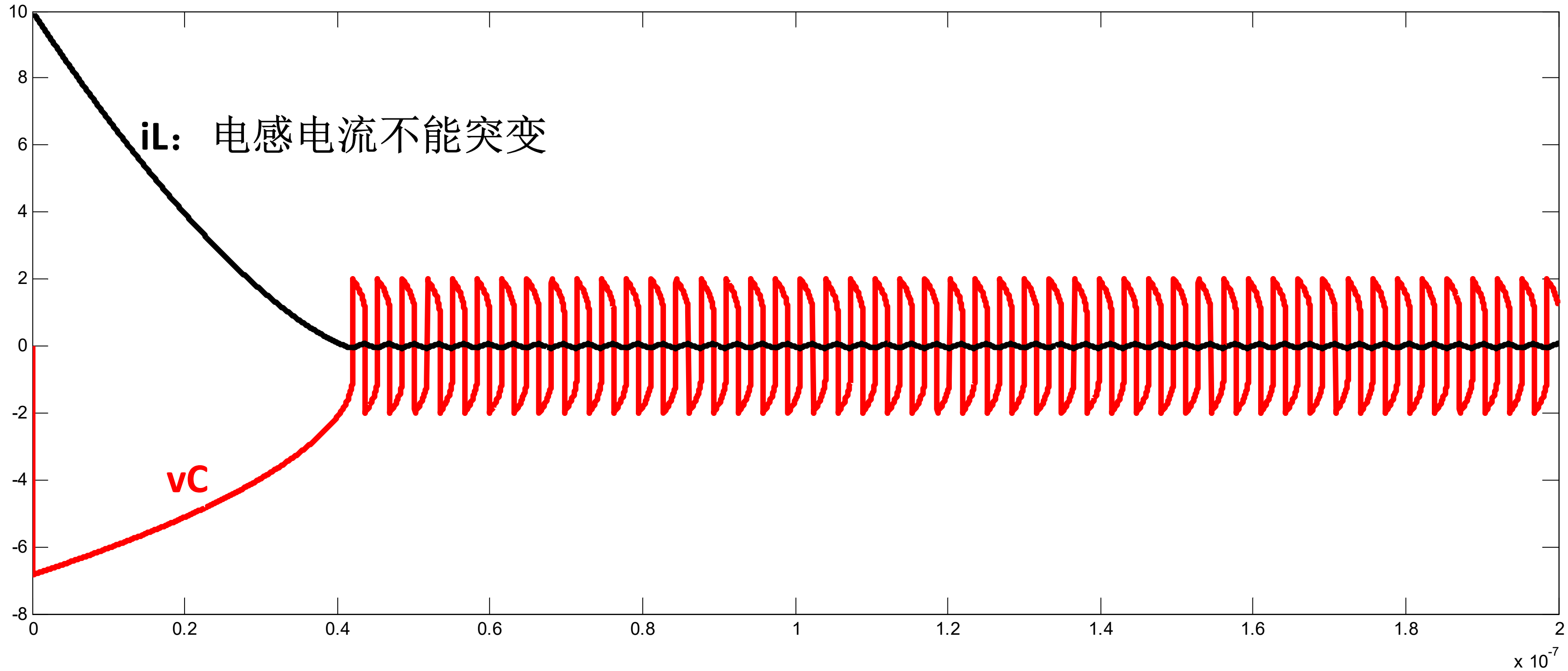
$$C = 100pF \to 0.1fF$$

极小电容视为开路

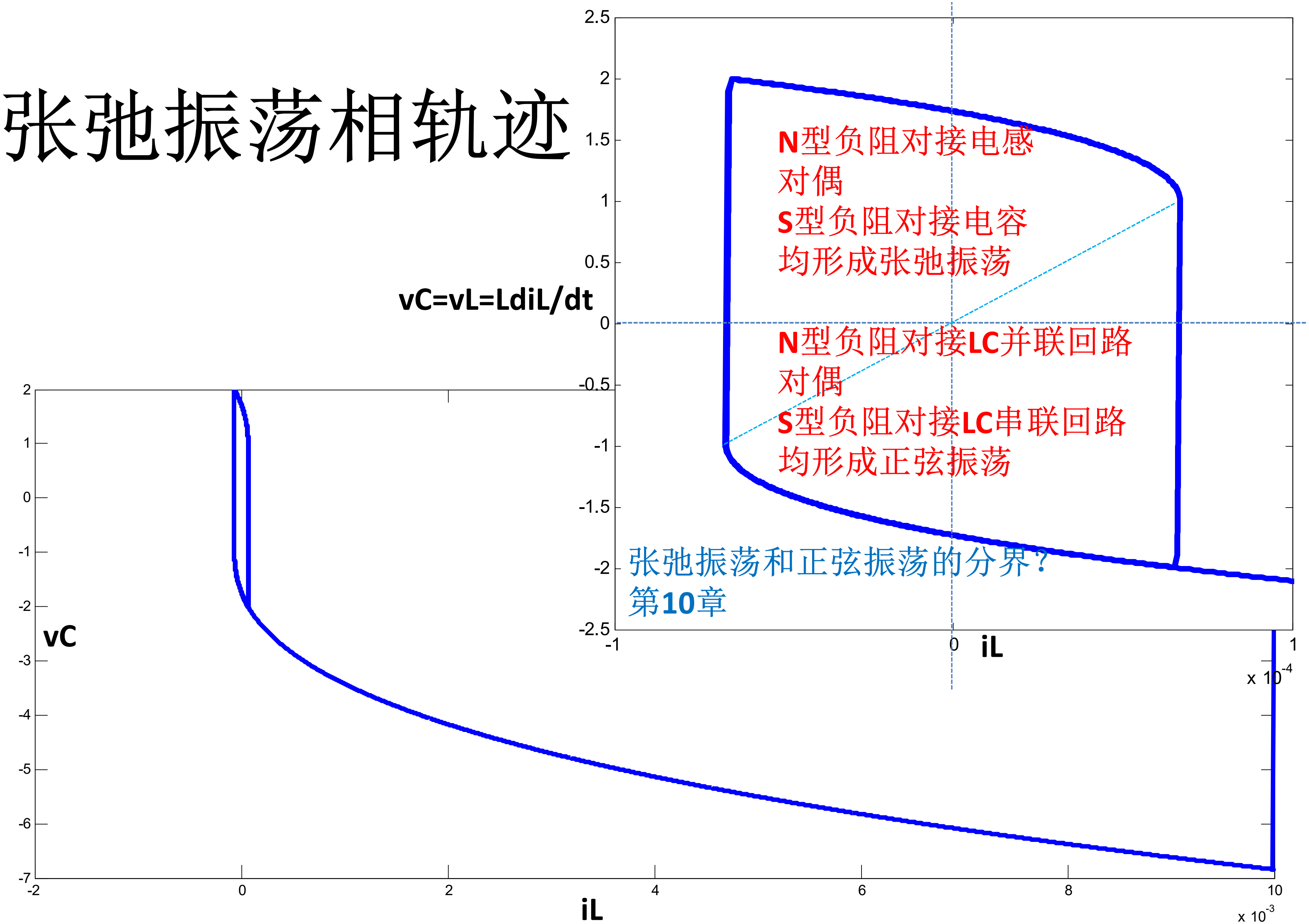
电感+N型负阻：张弛振荡







张弛振荡相轨迹

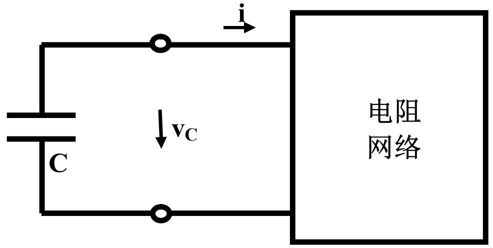


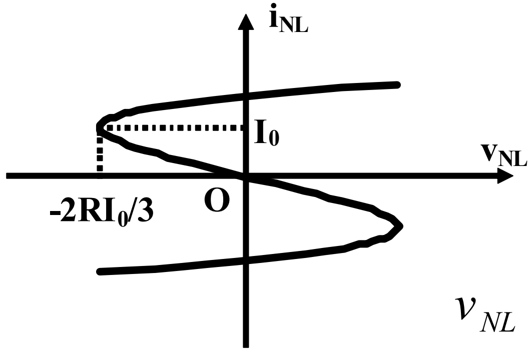
vC=vL=LdiL/dt
N型负阻对接电感
对偶
S型负阻对接电容
均形成张弛振荡
N型负阻对接LC并联回路
对偶
S型负阻对接LC串联回路
均形成正弦振荡
张弛振荡和正弦振荡的分界?
第10章
iL
x 10-4
vC
iL
x 10-3

第3讲 动态电路基本分析方法

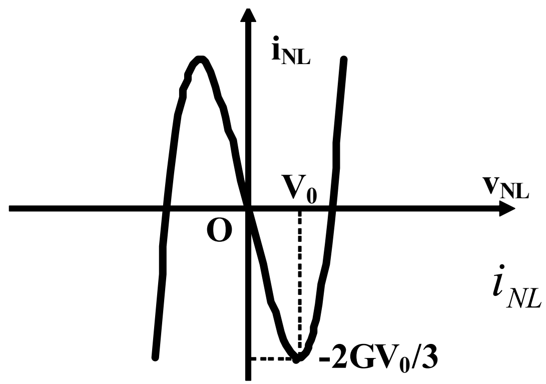
作业1 一阶动态系统的相轨迹

- （练习8.2.9） 图示单电容一阶动态系统中的电阻网络，分别为如下五种情况，请画出相图，并说明平衡点在什么位置？是否稳定？是否会出现振荡？
  - 线性电阻R
  - 线性负阻-R
  - 戴维南源，源电压为V_{S0}，源内阻为R_S
  - S型负阻
  - N型负阻



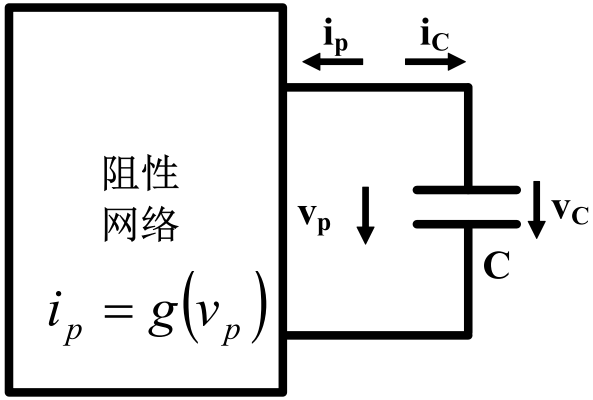


$$v_{NL} = -Ri_{NL} + \frac{R}{3I_0^2} i_{NL}^3$$



$$i_{NL} = -Gv_{NL} + \frac{G}{3V_0^2} v_{NL}^3$$

一阶RC电路：相图



$$C\frac{dv_C}{dt} = i_C = -i_p = -g(v_p) = -g(v_C)$$

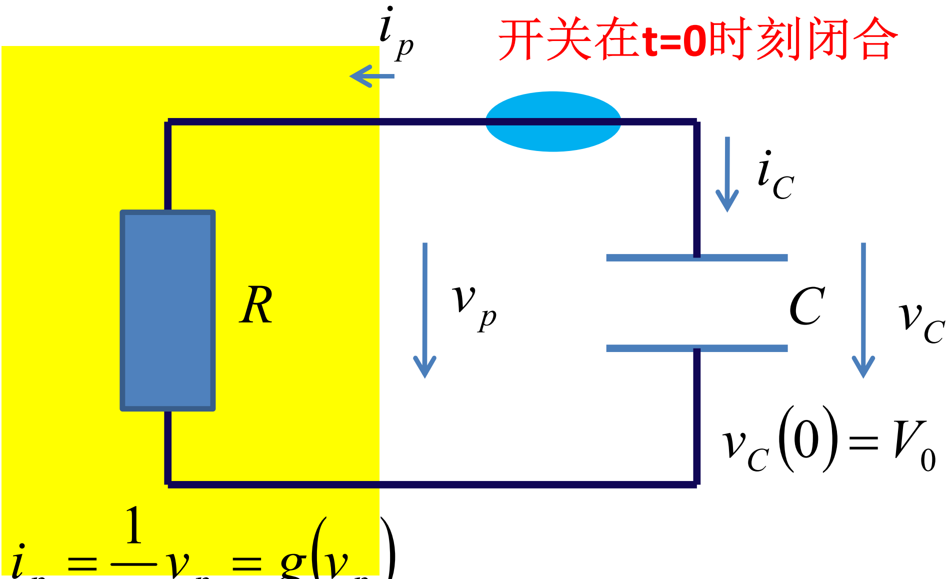
$$\frac{dv_C(t)}{dt} = -\frac{1}{C}g(v_C(t))$$ 状态方程

$$x = v_C(t)$$

$$y = \frac{dv_C(t)}{dt}$$

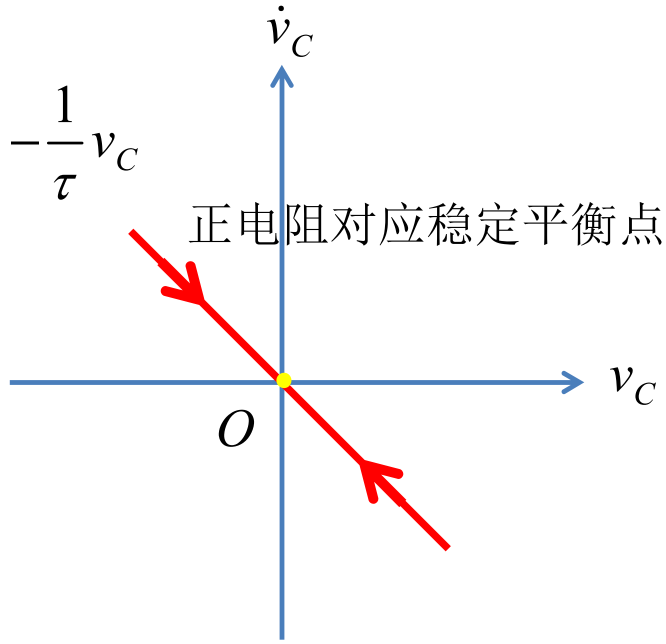
$$y = -\frac{1}{C}g(x)$$ 相轨迹

（1）电阻网络为线性电阻



$$i_p=\frac{1}{R}v_p=g(v_p)$$

$$\dot{v}_C=-\frac{1}{\tau}v_C$$



所谓稳定平衡点

$$\frac{dv_C(t)}{dt}=-\frac{1}{C}g(v_C(t))=-\frac{1}{C}\frac{v_C(t)}{R}=-\frac{1}{RC}v_C(t)=-\frac{1}{\tau}v_C(t)$$

$$t\to\infty$$

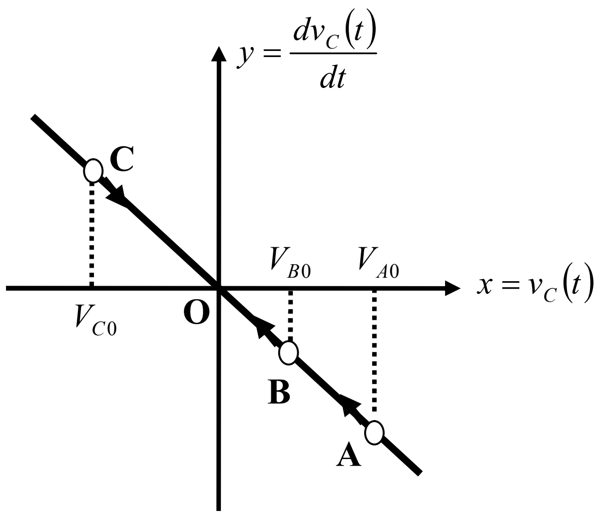
$$v_C(t)\to 0$$

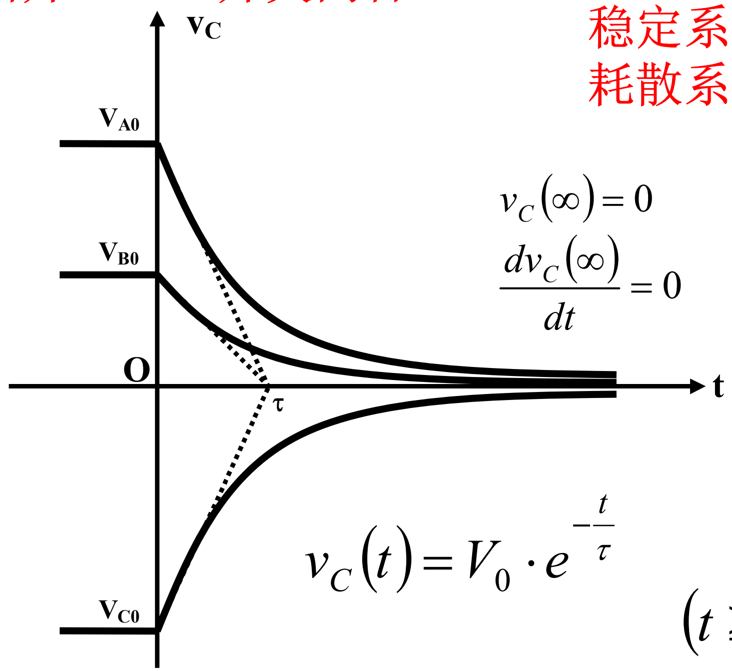
$$\frac{dv_C(t)}{dt}\to 0$$

$$y=-\frac{1}{\tau}x$$

时间常数 $\tau=RC$

不同初值，放电曲线形态一致





$$v_C(\infty)=0$$

$$\frac{dv_C(\infty)}{dt}=0$$

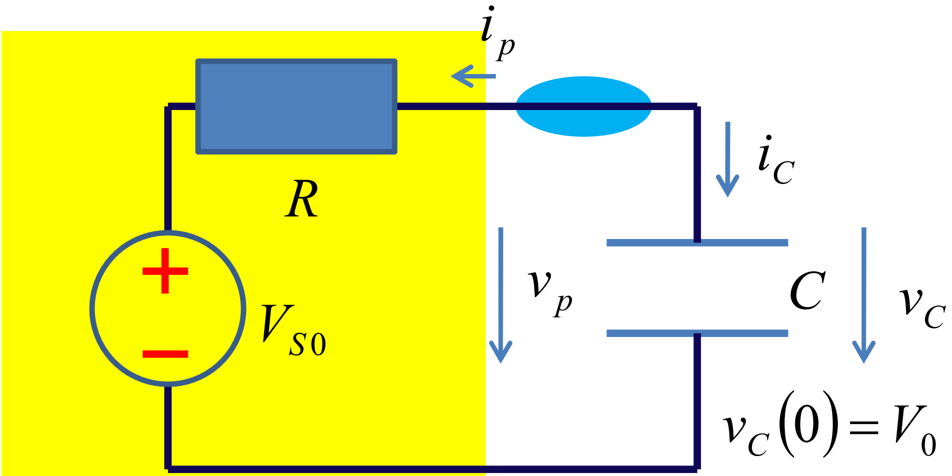
$$v_C(t)=V_0\cdot e^{-\frac{t}{\tau}}\quad (t\geq 0)$$

相轨迹的斜率**-1/τ**，代表了状态转移速度

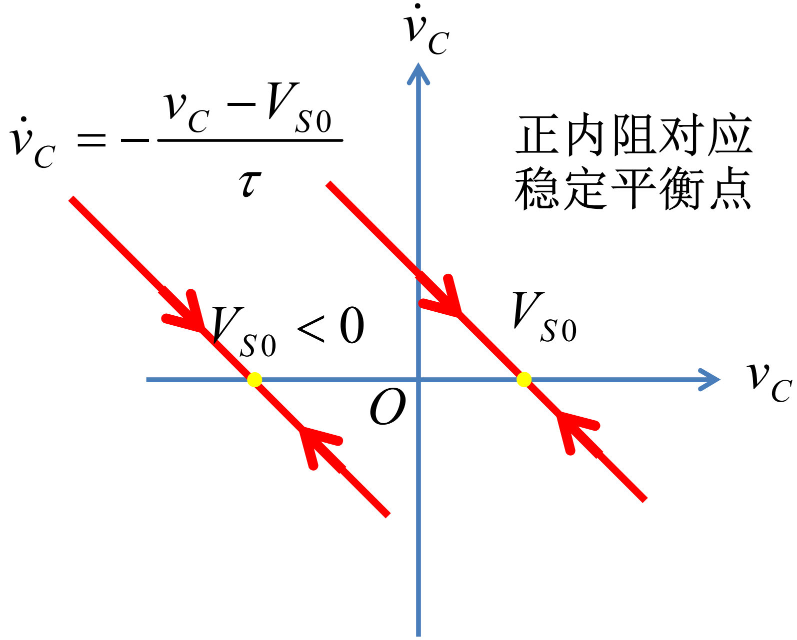
时间常数越小，相轨迹越陡，状态转移速度越快，从一个状态转移到下一个状态用的时间就越短

R=0,τ=0，瞬间完成放电（冲激电流）

（2） 电阻网络为直流戴维南源



$$i_p = \frac{v_p - V_{S0}}{R} = g(v_p)$$



稳定平衡点：直流工作点

$$\frac{dv_C(t)}{dt} = -\frac{1}{C}g(v_C(t)) = -\frac{1}{C}\frac{v_C(t)-V_{S0}}{R} = -\frac{1}{\tau}v_C(t)+\frac{1}{\tau}V_{S0}$$

$$t \to \infty$$

$$v_C(t) \to V_{S0}$$

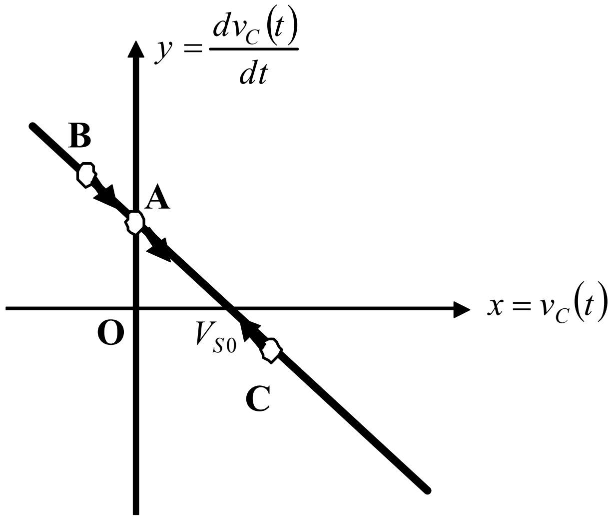
$$\frac{dv_C(t)}{dt} \to 0$$

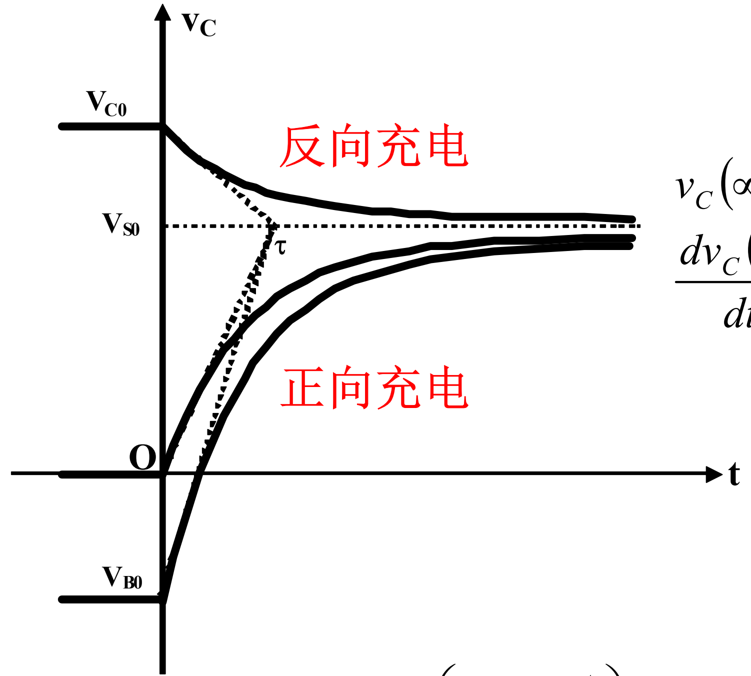
$$y = -\frac{x-V_{S0}}{\tau}$$

时间常数 $\tau = RC$

不同初值，形态一致

开关断开　　开关闭合





$$v_C(\infty)=V_{S0}$$

$$\frac{dv_C(\infty)}{dt}=0$$

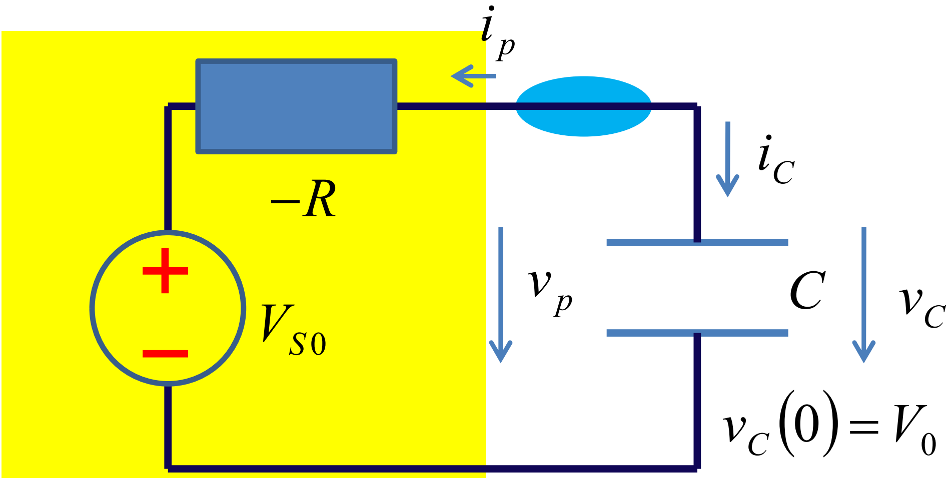
$$\frac{dv_C(t)}{dt}=-\frac{1}{\tau}v_C(t)+\frac{1}{\tau}V_{S0}$$

$$y=-\frac{x-V_{S0}}{\tau}$$

$$v_C(t)=V_0+(V_{S0}-V_0)\left(1-e^{-\frac{t}{\tau}}\right)\cdot U(t)$$

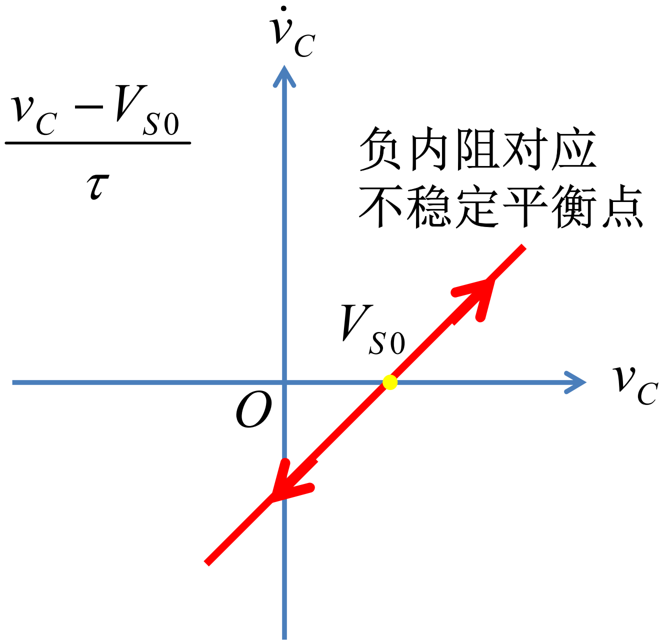
$$=\begin{cases} V_0 & t<0 \\ V_{S0}+(V_0-V_{S0})e^{-\frac{t}{\tau}} & t\geq 0 \end{cases}$$

（3） 戴维南源内阻为负阻



$$i_p=\frac{v_p-V_{S0}}{-R}=g(v_p)$$

$$\dot{v}_C=\frac{v_C-V_{S0}}{\tau}$$



不稳定平衡点：直流工作点

$$\frac{dv_C(t)}{dt}=-\frac{1}{C}g(v_C(t))=-\frac{1}{C}\frac{v_C(t)-V_{S0}}{-R}=\frac{1}{\tau}v_C(t)-\frac{1}{\tau}V_{S0}$$

$t\to-\infty$

假设时间可以倒流

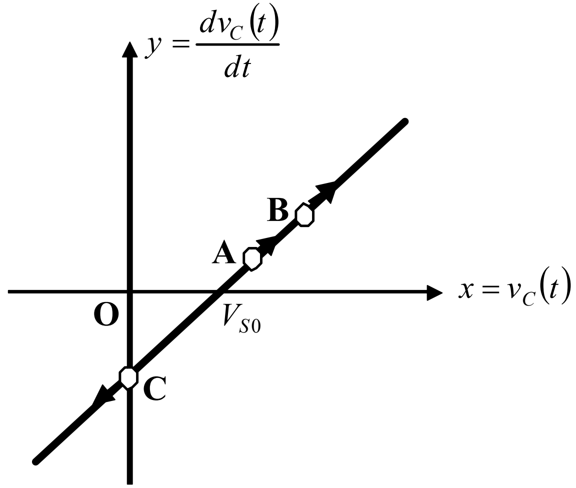
$v_C(t)\to V_{S0}$

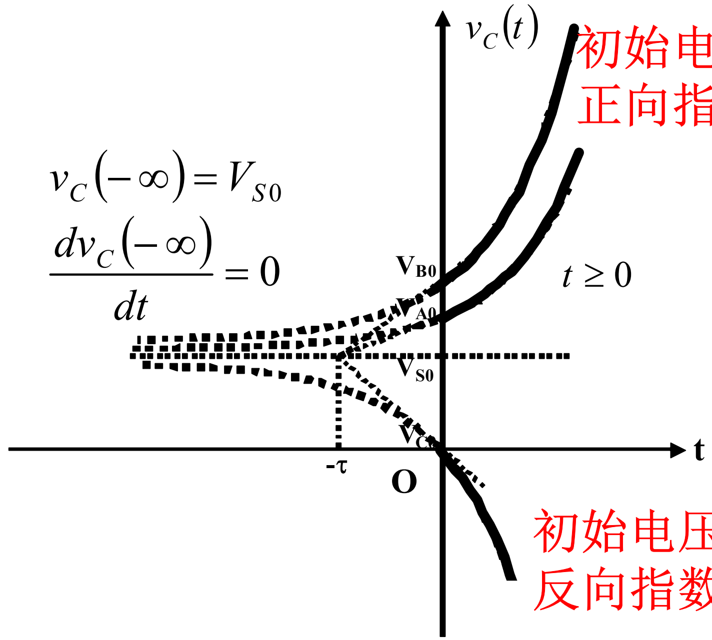
$$\frac{dv_C(t)}{dt}\to 0$$

$$y=\frac{x-V_{S0}}{\tau}$$

时间常数 $\tau=RC$

不同初值，形态一致





$$\frac{dv_C(t)}{dt}=\frac{v_C(t)-V_{S0}}{\tau}$$

$$=\frac{V_0-V_{S0}}{\tau}e^{\frac{t}{\tau}}$$

$$v_C(t)=v_{C,\infty}(t)+\left(v_C(0^+)-v_{C,\infty}(0^+)\right)e^{\frac{t}{\tau}}$$

$$=V_{S0}+\left(V_0-V_{S0}\right)e^{\frac{t}{\tau}}\qquad (t\geq 0)$$

1、代入方程，成立，确实为解

2、代入t=0，确实为初值$v_C(0)=V_0$

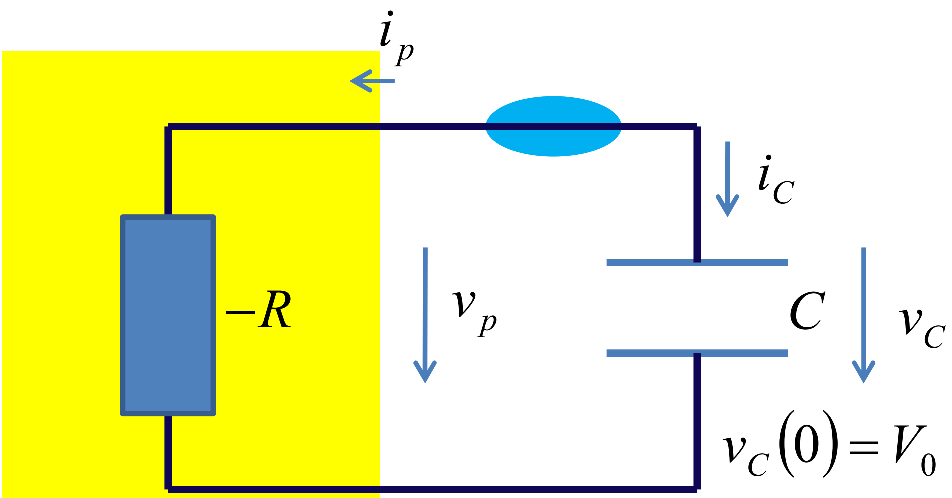
三要素形式没有任何本质区别

1、指数增长规律

2、稳态值为t→-∞时的不稳定平衡状态

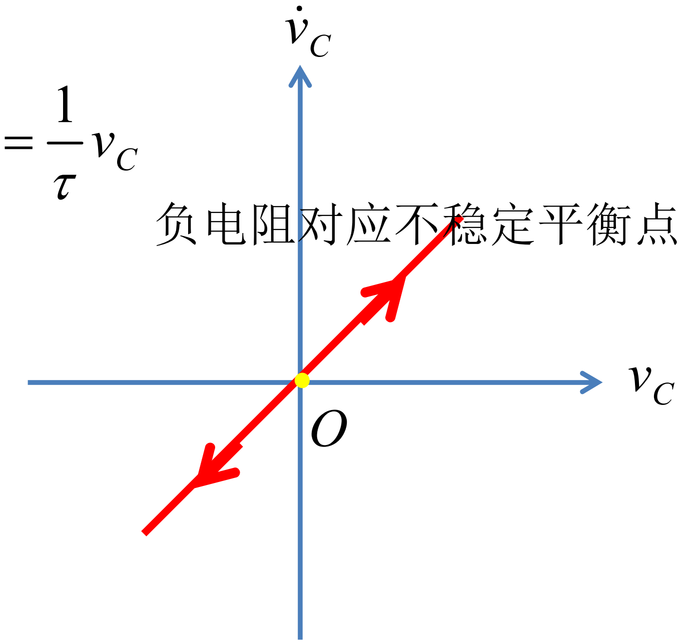
（3.2）电阻网络为线性负阻

戴维南源电压为零



$$i_p=\frac{1}{-R}v_p=g(v_p)$$

$$\dot{v}_C=\frac{1}{\tau}v_C$$



不稳定平衡点

$$\frac{dv_C(t)}{dt}=-\frac{1}{C}g(v_C(t))=-\frac{1}{C}\frac{v_C(t)}{-R}=\frac{1}{RC}v_C(t)=\frac{1}{\tau}v_C(t)$$

$$t\to-\infty$$

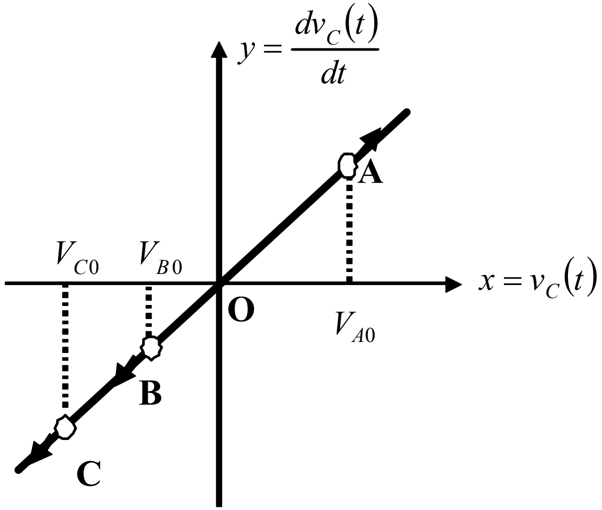
$$v_C(t)\to 0$$

$$\frac{dv_C(t)}{dt}\to 0$$

$$y=\frac{1}{\tau}x$$

时间常数 $\tau=RC$

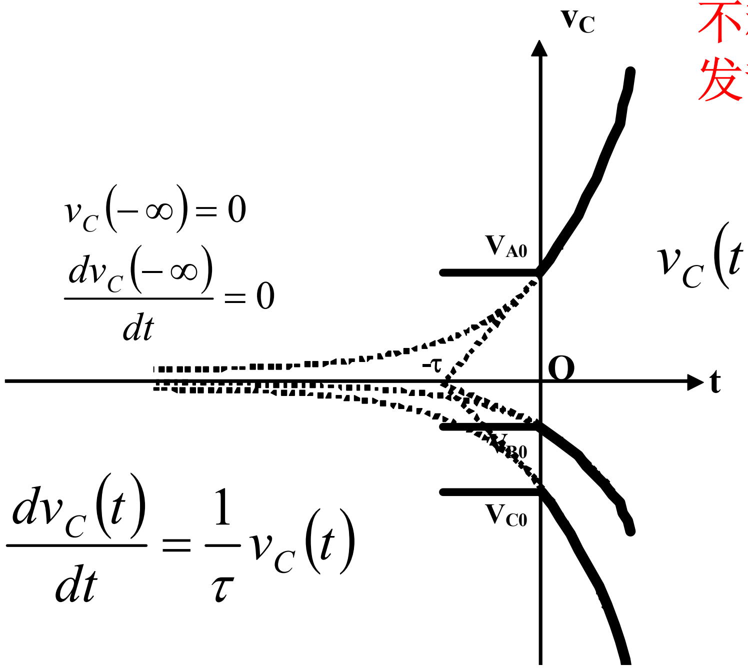
负阻为电容充电：越充越快



$$\frac{dv_C(t)}{dt}=\frac{v_C(t)-V_{S0}}{\tau}\overset{V_{S0}=0}{=}\frac{v_C(t)}{\tau}$$

$$v_C(-\infty)=0$$

$$\frac{dv_C(-\infty)}{dt}=0$$



不稳定系统
发散系统

$$v_C(t)=V_0\cdot e^{\frac{t}{\tau}}$$

$$(t\ge 0)$$

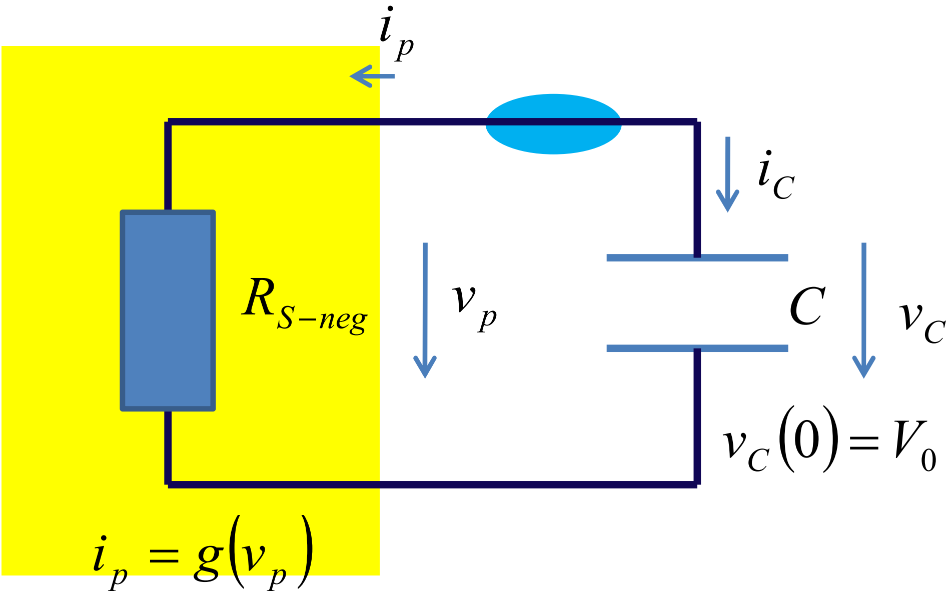
$$\frac{dv_C(t)}{dt}=\frac{1}{\tau}v_C(t)$$

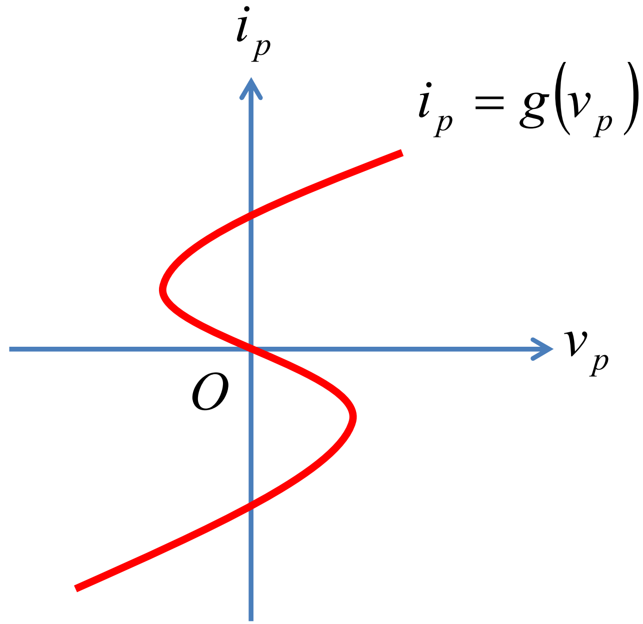
指数增长规律

直流偏置清零
以V_{S0}为参考0电位即可

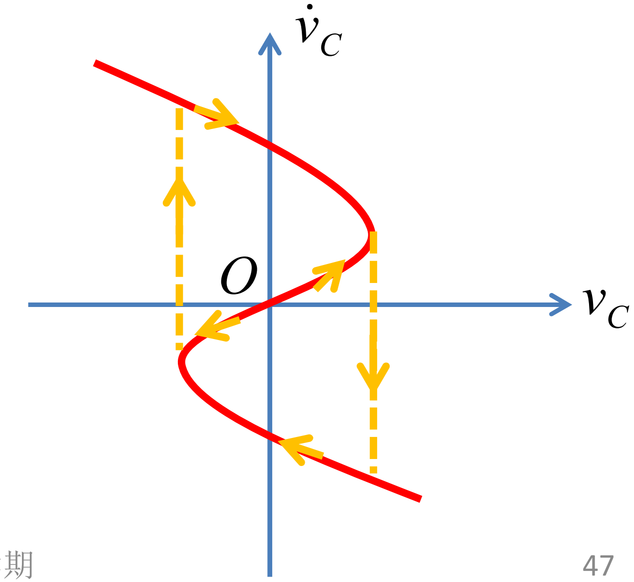
$$\left(v_C(t)-V_{S0}\right)=\left(V_0-V_{S0}\right)\cdot e^{\frac{t}{\tau}}$$

（4） S型负阻

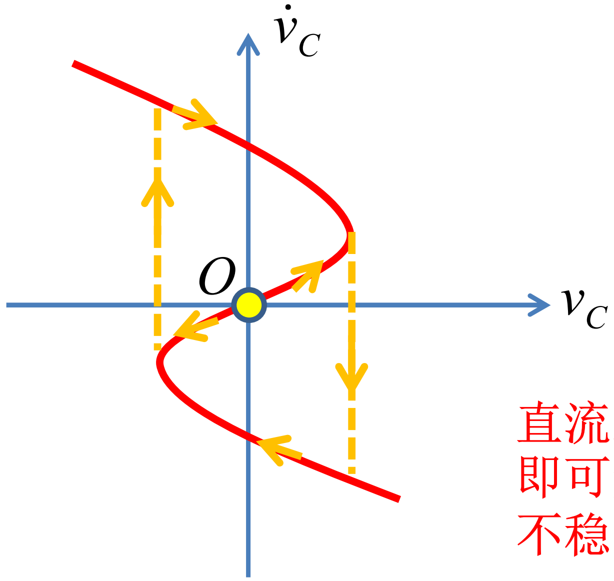


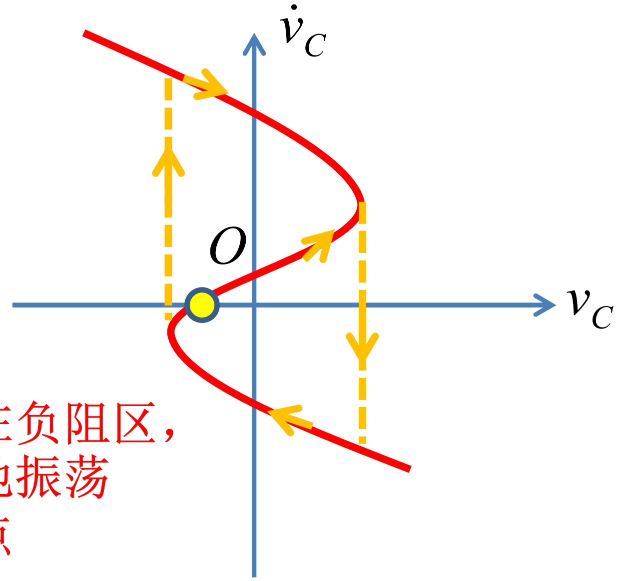


$$\frac{dv_C(t)}{dt}=-\frac{1}{C}g(v_C(t))$$

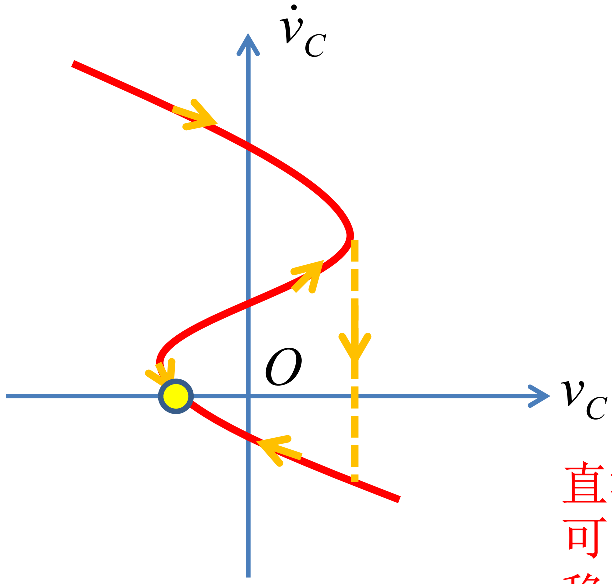


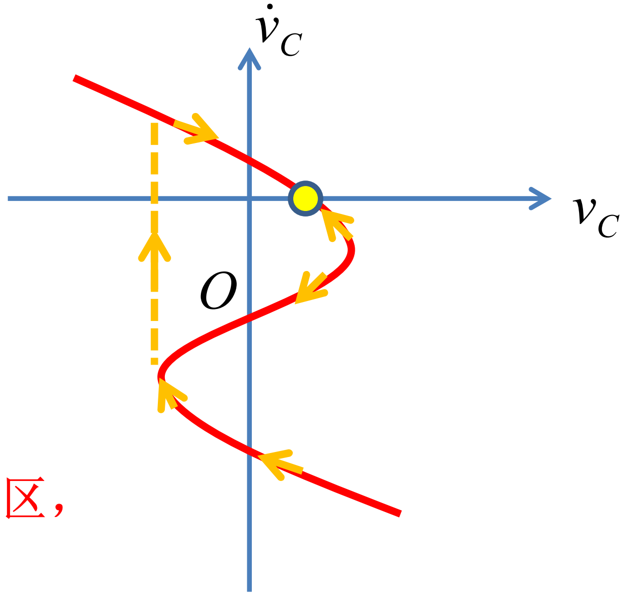
S型负阻的不同直流工作点





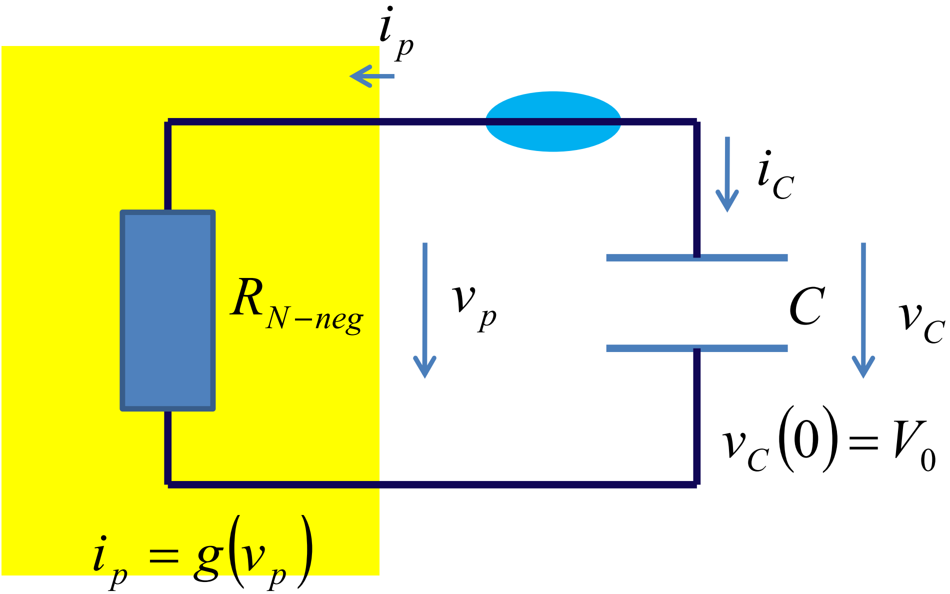
直流工作点在负阻区，
即可形成张弛振荡
不稳定平衡点

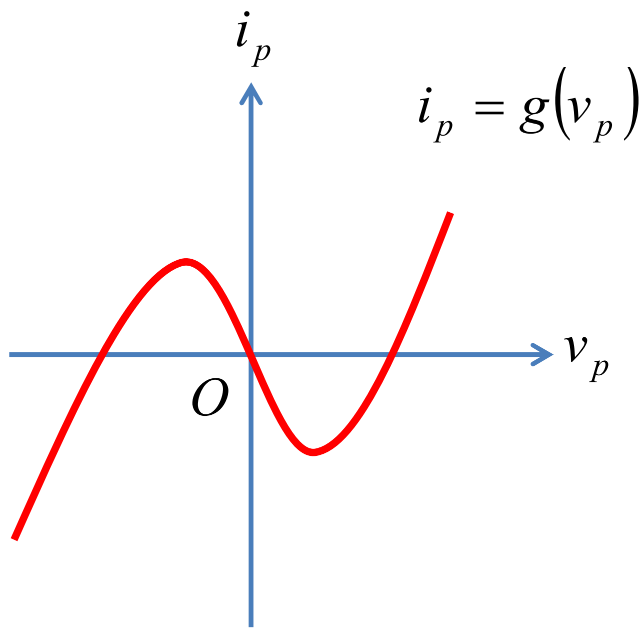




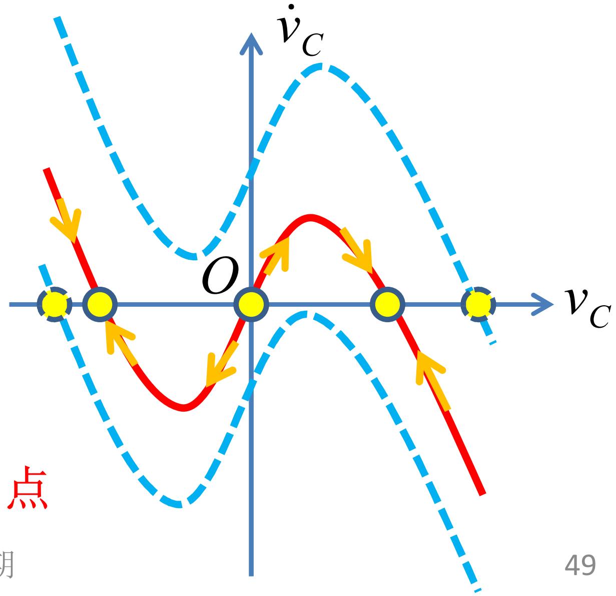
直流工作点在正阻区，
可能形成单脉冲
稳定平衡点

（5） N型负阻



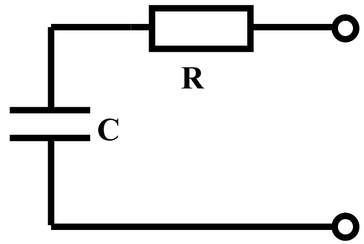


$$\frac{dv_C(t)}{dt} = -\frac{1}{C} g(v_C(t))$$



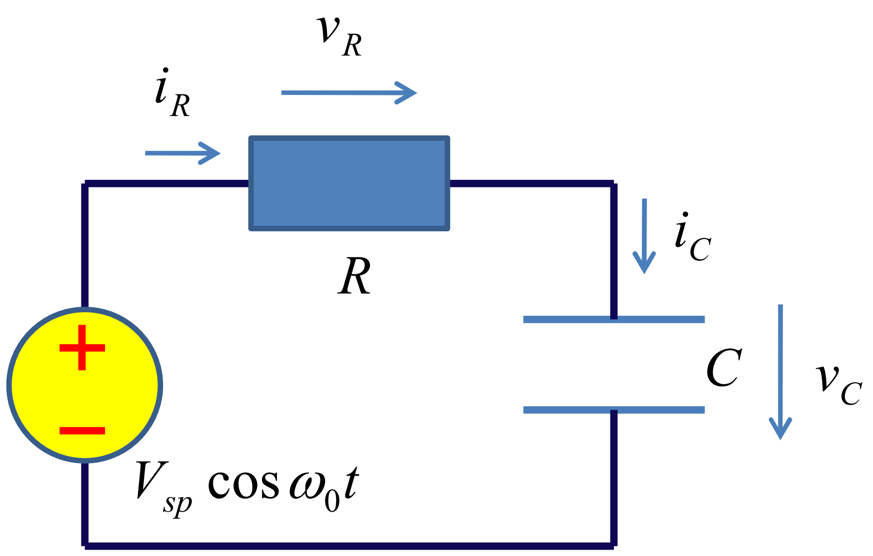
直流工作点在负阻区
可形成两个记忆状态
必然同时存在两个正阻工作点

作业2 串联RC



- 对于图示RC串联电路
  - （练习8.3.8）在单端口加载正弦波激励电压源，测得电阻上正弦波电压幅度为3V，电容上正弦波电压幅度为4V，问激励电压源正弦波电压幅度为多少？保持正弦激励电压源幅度不变，但频率增加为原来频率的2倍，此时测得电阻上电压幅度为多少？电容上的电压幅度为多少？
  - （练习8.3.9）在单端口加载正弦波电压 $v_S(t)=V_{Sp}\cos\omega t$ ，电容上分压为多少？电阻上分压为多少？是否满足两个分压之和等于总电压（KVL方程）？在频域分析中如何理解两个分压之和等于总电压（KVL方程）？

RC串联



$$v_S = V_{sp}\cos(\omega_0 t)$$

$$v_R = 3\cos(\omega_0 t + \varphi_R)$$

$$V_{sp} = ?$$

$$v_C = 4\cos(\omega_0 t + \varphi_C)$$

在单端口加载正弦波激励电压源，测得电阻上正弦波电压幅度为**3V**，电容上正弦波电压幅度为**4V**，问激励电压源正弦波电压幅度为多少？保持正弦激励电压源幅度不变，但频率增加为原来频率的**2**倍，此时测得电阻上电压幅度为多少？电容上的电压幅度为多少？

$$\dot{I} = I_p\angle\varphi_I$$

$$i_C = i_R = I_p\cos(\omega_0 t + \varphi_I)$$

$$\dot{V}_R = R\dot{I}$$

$$V_{Rp} = 3 = I_p R$$

$$\varphi_R = \varphi_I$$

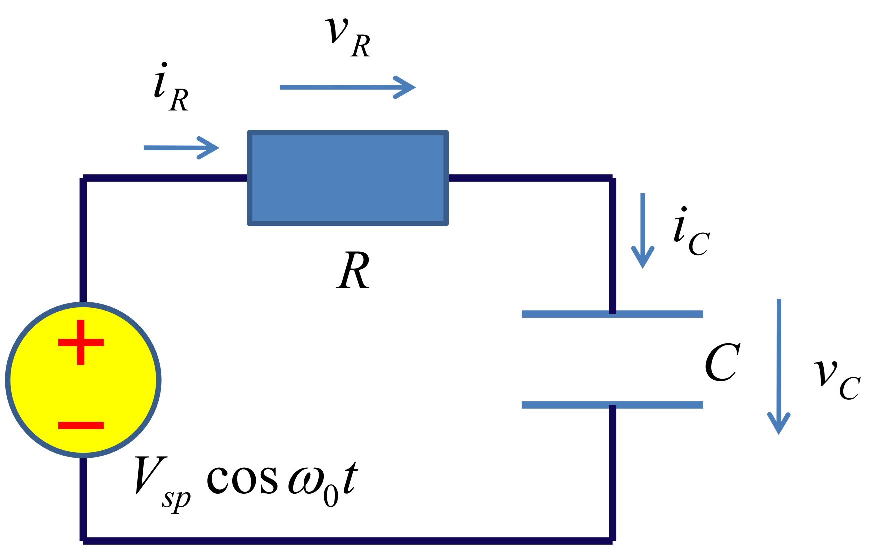
电阻压流同频同相

$$\dot{V}_C = \frac{\dot{I}}{j\omega C} = \frac{I_p}{\omega C}\angle(\varphi_I - 90°)$$

$$V_{Cp} = 4 = \frac{I_p}{\omega_0 C}$$

$$\varphi_R = \varphi_I - 90°$$

电容压流同频滞后**90°**



$$\dot{I} = I_p \angle \varphi_I \qquad i_C = i_R = I_p \cos(\omega_0 t + \varphi_I)$$

$$\dot{V}_R = R\dot{I} \qquad V_{Rp} = 3 = I_p R \qquad \varphi_R = \varphi_I$$

$$\dot{V}_C = \frac{\dot{I}}{j\omega C} = \frac{I_p}{\omega C}\angle(\varphi_I - 90°) \qquad V_{Cp} = 4 = \frac{I_p}{\omega_0 C} \qquad \varphi_R = \varphi_I - 90°$$

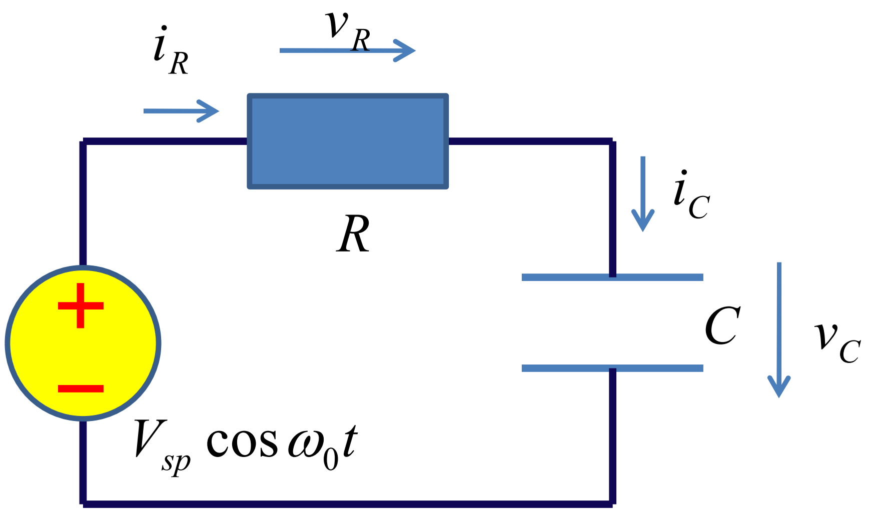
$$\dot{V}_S = \dot{V}_R + \dot{V}_C = \dot{I}\left(R + \frac{1}{j\omega_0 C}\right) = \dot{I}\sqrt{R^2 + \left(\frac{1}{\omega_0 C}\right)^2}\angle - \arctan\frac{1}{\omega_0 RC}$$

$$= I_p\sqrt{R^2 + \left(\frac{1}{\omega_0 C}\right)^2}\angle\left(\varphi_I - \arctan\frac{1}{\omega_0 RC}\right) = \sqrt{(I_p R)^2 + \left(\frac{I_p}{\omega_0 C}\right)^2}\angle\left(\varphi_I - \arctan\frac{1}{\omega_0 RC}\right)$$

$$= \sqrt{3^2 + 4^2}\angle\left(\varphi_I - \arctan\frac{1}{\omega_0 RC}\right) = 5\angle\left(\varphi_I - \arctan\frac{1}{\omega_0 RC}\right)$$

$$V_{sp} = 5 = \sqrt{V_{Rp}^2 + V_{Cp}^2} = \sqrt{3^2 + 4^2}$$

相位差**90°**，组成直角三角形



在单端口加载正弦波激励电压源，测得电阻上正弦波电压幅度为**3V**，电容上正弦波电压幅度为**4V**，问激励电压源正弦波电压幅度为多少？保持正弦激励电压源幅度不变，但频率增加为原来频率的**2**倍，此时测得电阻上电压幅度为多少？电容上的电压幅度为多少？

$$\dot{V}_S = \dot{V}_R + \dot{V}_C = \dot{I}\left(R + \frac{1}{j\omega_0 C}\right)$$

$$= \sqrt{3^2 + 4^2}\angle\ldots = 5\angle\ldots$$

$$\dot{V}_S = \dot{V}_R + \dot{V}_C = \dot{I}_2\left(R + \frac{1}{j2\omega_0 C}\right)$$

$$= 5\angle\ldots = \sqrt{V_{Rp2}^2 + V_{Cp2}^2}\angle\ldots$$

$$\frac{V_{Rp1}}{V_{Cp1}} = \frac{3}{4} = \frac{I_{p1}R}{I_{p1}/\omega_0 C} = \omega_0 RC$$

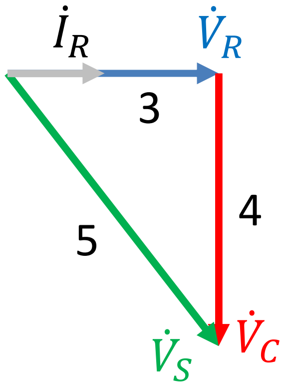
$$\frac{V_{Rp2}}{V_{Cp2}} = \frac{I_{p2}R}{I_{p2}/2\omega_0 C} = 2\omega_0 RC = 1.5$$

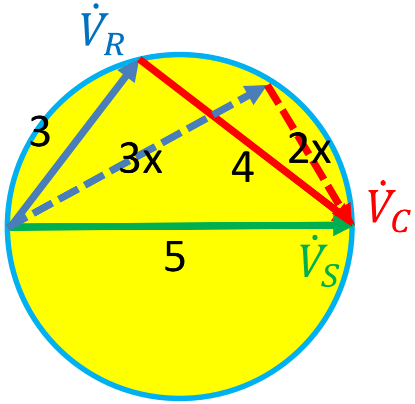
$$5 = \sqrt{V_{Rp2}^2 + V_{Cp2}^2} = V_{Cp2}\sqrt{1.5^2 + 1^2} = 1.8V_{Cp2}$$

$$V_{Cp2} = 5/1.8 = 2.77(V)$$ ← 4V

$$V_{Rp2} = 1.5 \times V_{Cp2} = 4.16(V)$$ ← 3V

相量图解法





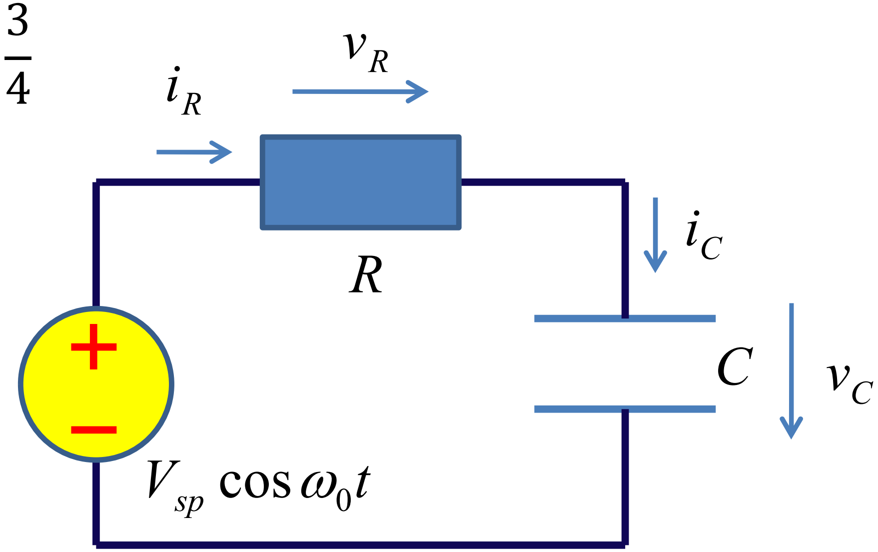
$$\frac{|\dot{V}_R|}{|\dot{V}_C|}=\frac{R}{\frac{1}{2\omega_0 C}}=\frac{3}{2}=\frac{3x}{2x}$$

$$5=|\dot{V}_S|=\sqrt{|\dot{V}_R|^2+|\dot{V}_C|^2}$$

$$=\sqrt{(3x)^2+(2x)^2}=\sqrt{13}x$$

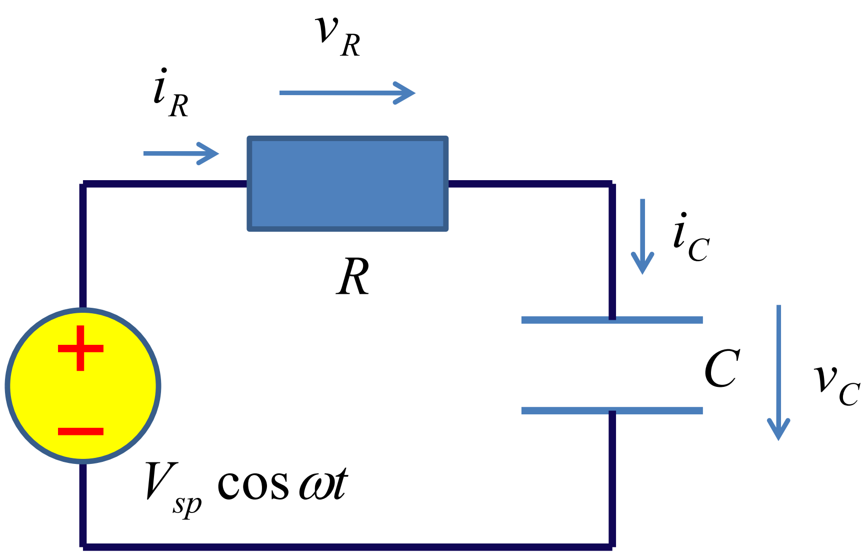
$$x=\frac{5}{\sqrt{13}}$$

$$\frac{|\dot{V}_R|}{|\dot{V}_C|}=\frac{R}{\frac{1}{\omega_0 C}}=\frac{3}{4}$$



$$|\dot{V}_R|=3x=\frac{15}{\sqrt{13}}=4.16V$$

$$|\dot{V}_C|=2x=\frac{10}{\sqrt{13}}=2.77V$$



$$\dot{I}=\frac{\dot{V}_S}{R+\frac{1}{j\omega C}}$$

$$\dot{V}_R=\dot{I}R=\frac{R}{R+\frac{1}{j\omega C}}\dot{V}_S=\frac{j\omega RC}{1+j\omega RC}\dot{V}_S$$

$$=\frac{\omega RC}{\sqrt{1+(\omega RC)^2}}\angle\left(\frac{\pi}{2}-\arctan\omega RC\right)\dot{V}_S$$

在单端口加载正弦波电压 $v_S(t)=V_{Sp}\cos\omega t$ ，电容上分压为多少？电阻上分压为多少？是否满足两个分压之和等于总电压（**KVL**方程）？在频域分析中如何理解两个分压之和等于总电压（**KVL**方程）？

$$v_R(t)=V_{sp}\frac{\omega RC}{\sqrt{1+(\omega RC)^2}}\cos\left(\omega t+\frac{\pi}{2}-\arctan\omega RC\right)$$

$$v_C(t)=V_{sp}\frac{1}{\sqrt{1+(\omega RC)^2}}\cos(\omega t-\arctan\omega RC)$$

KVL方程在时域和频域均满足

频域表述更简单一些

$$\dot{V}_R = \dot{I}R = \frac{R}{R+\frac{1}{j\omega C}}\dot{V}_S = \frac{j\omega RC}{1+j\omega RC}\dot{V}_S$$

$$v_R(t) = V_{sp}\frac{\omega RC}{\sqrt{1+(\omega RC)^2}}\cos\left(\omega t+\frac{\pi}{2}-\arctan\omega RC\right)$$

$$\dot{V}_C = \dot{I}\frac{1}{j\omega C} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}\dot{V}_S = \frac{1}{1+j\omega RC}\dot{V}_S$$

$$v_C(t) = V_{sp}\frac{1}{\sqrt{1+(\omega RC)^2}}\cos(\omega t-\arctan\omega RC)$$

$$\dot{V}_R + \dot{V}_C = \dot{V}_S$$

记住：相量域电量是复数，不能只考察幅度，还必须考虑相位影响

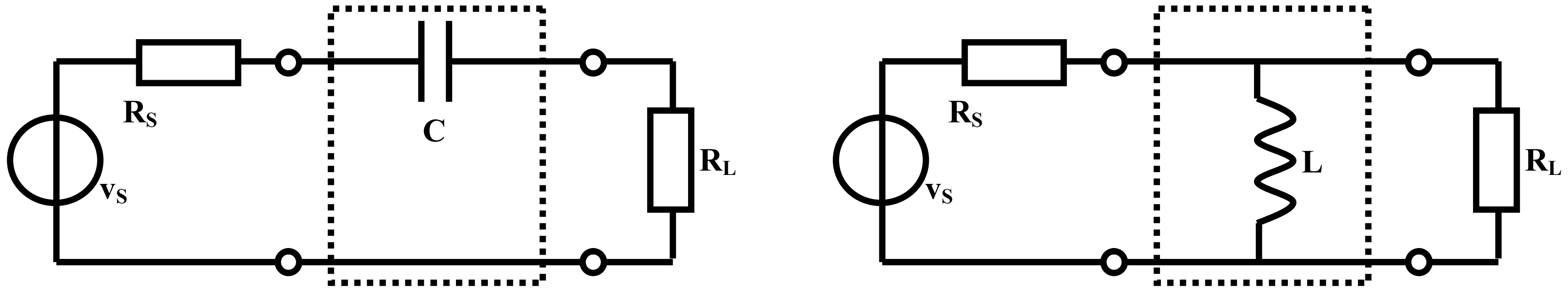
$V_{sp} \neq V_{Rp}+V_{Cp}$： 矢量叠加，平行四边形法则运算

$$v_R(t)+v_C(t) = -V_{sp}\frac{\omega RC}{\sqrt{1+(\omega RC)^2}}\sin(\omega t-\arctan\omega RC)+V_{sp}\frac{1}{\sqrt{1+(\omega RC)^2}}\cos(\omega t-\arctan\omega RC)$$

$$= -V_{sp}\sin\varphi\sin(\omega t-\arctan\omega RC)+V_{sp}\cos\varphi\cos(\omega t-\arctan\omega RC)$$

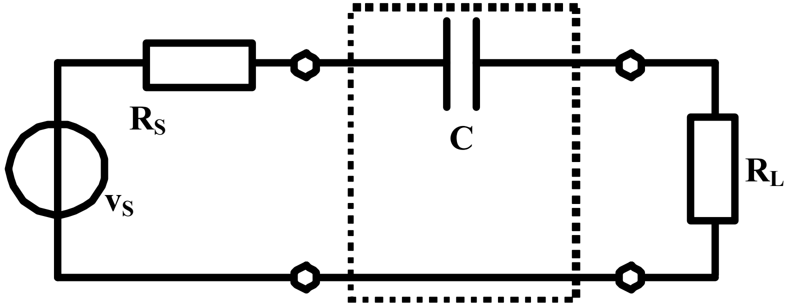
$$= V_{sp}\cos(\omega t-\arctan\omega RC+\varphi) = V_{sp}\cos\omega t = v_S(t)$$

作业3 耦合电容和高频扼流圈

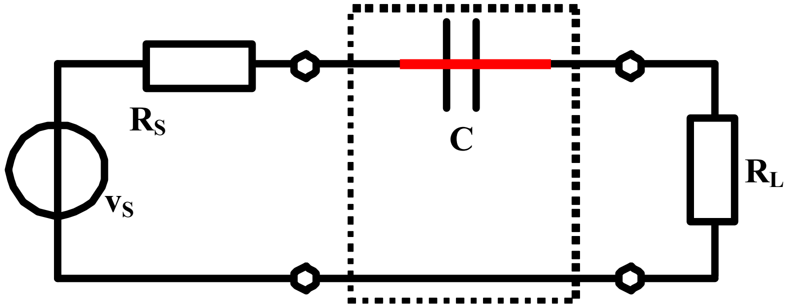


- （练习8.3.22）
  - 如图a所示，这是一个用耦合电容耦合激励源和负载的简单电路模型。请分析确认：什么频率下可认为耦合电容是交流短路的？什么频率下可认为耦合电容是直流开路的？
  - 如图b所示，这是一个高频扼流圈例子，一端接电源的高频扼流圈在此处被处理为接地。请分析确认：在什么频率下可认为高频扼流圈是直流短路的？什么频率下可认为高频扼流圈是交流开路的？

耦合电容



耦合电容高频短路，电路模型为



$$v_L = \frac{R_L}{R_S + R_L} v_S = \eta v_S$$

$$\omega_0 = \frac{1}{\tau} = \frac{1}{RC} = \frac{1}{(R_S + R_L)C}$$

$$\dot{V}_L = \frac{R_L}{R_S + \frac{1}{j\omega C} + R_L} \dot{V}_S$$

$$= \frac{R_L}{R_S + R_L} \frac{1}{1 + \frac{1}{j\omega C(R_S + R_L)}} \dot{V}_S$$

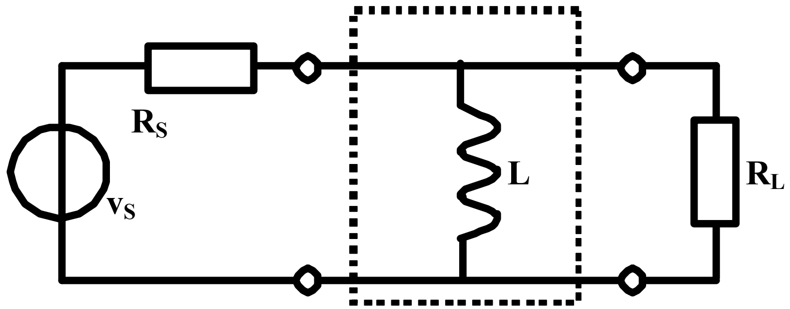
$$= \eta \frac{1}{1 + \frac{1}{j\omega\tau}} \dot{V}_S = \eta \frac{1}{1 + \frac{\omega_0}{j\omega}} \dot{V}_S$$

$$\overset{\omega >> \omega_0}{\approx} \eta \dot{V}_S$$

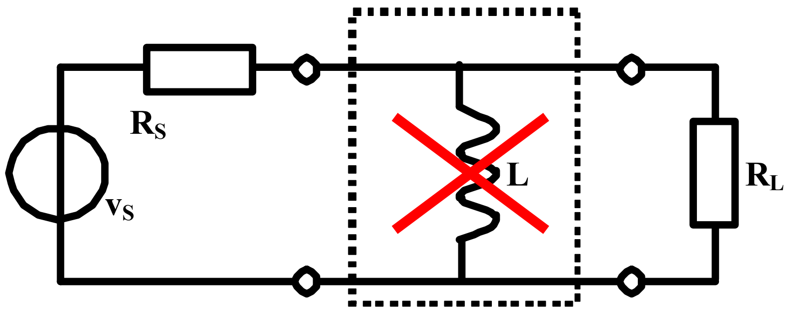
负载电压是输入电压的高频分量

$\omega >> \omega_0$时，耦合电容视为高频短路

高频扼流圈



高频扼流圈高频开路，电路模型为



$$v_L = \frac{R_L}{R_S + R_L} v_S = \eta v_S$$

$$\dot{V}_L = \frac{R_L \parallel j\omega L}{R_S + R_L \parallel j\omega L}\dot{V}_S = \frac{\dfrac{j\omega L R_L}{R_L + j\omega L}}{R_S + \dfrac{j\omega R_L L}{R_L + j\omega L}}\dot{V}_S$$

$$= \frac{j\omega L R_L}{R_S(R_L + j\omega L) + j\omega L R_L}\dot{V}_S = \frac{j\omega L R_L}{R_S R_L + j\omega L(R_S + R_L)}\dot{V}_S$$

$$= \frac{R_L}{R_S + R_L}\frac{j\omega L}{\dfrac{R_S R_L}{R_S + R_L} + j\omega L}\dot{V}_S = \frac{R_L}{R_S + R_L}\frac{j\omega L}{R_S \parallel R_L + j\omega L}\dot{V}_S$$

$$= \frac{R_L}{R_S + R_L}\frac{j\omega G_{SL} L}{1 + j\omega G_{SL} L}\dot{V}_S = \eta \frac{1}{1 + \dfrac{\omega_0}{j\omega}}\dot{V}_S \overset{\omega >> \omega_0}{\approx} \eta \dot{V}_S$$

$$\omega_0 = \frac{1}{\tau} = \frac{1}{G_{SL} L} = \frac{R_S \parallel R_L}{L}$$

负载电压是输入电压的高频分量

$\omega >> \omega_0$时，高频扼流圈视为高频开路

高频? $\omega >> \omega_0$

- $\omega > 10\omega_0$

$$\dot{V}_L = \eta \frac{1}{1+\frac{\omega_0}{j\omega}} \dot{V}_S$$

$$\dot{V}_L = \eta \frac{1}{1+\frac{\omega_0}{j10\omega_0}} \dot{V}_S = \eta \frac{1}{1-j0.1} \dot{V}_S$$

$$= \eta \frac{e^{j5.7^\circ}}{1.005} \dot{V}_S = \eta \cdot 0.995 \cdot e^{j5.7^\circ} \cdot \dot{V}_S$$

0.5%误差 **3.2%误差**

$$v_S = V_{sp} \cos(\omega_0 t)$$

$$v_L = \eta V_{sp} \cos(\omega_0 t)$$

耦合电容短路，
高频扼流圈开路

$$v_L = 0.995\eta V_{sp} \cos(\omega_0 t + 5.7^\circ)$$

足够接近耦合电容短路，
高频扼流圈开路

低频?ω<<ω$_0$

- ω<0.1ω$_0$

$$\dot{V}_L = \eta \frac{1}{1+\frac{\omega_0}{j\omega}} \dot{V}_S$$

$$\dot{V}_L = \eta \frac{1}{1+\frac{\omega_0}{j0.1\omega_0}} \dot{V}_S = \eta \frac{1}{1-j10} \dot{V}_S$$

$$= \eta \frac{e^{j84.3°}}{10.05} \dot{V}_S = \eta \cdot 0.0995 \cdot e^{j84.3°} \cdot \dot{V}_S$$

1 %的功率泄漏是否可忽略不计？

$$v_S = V_{sp}\cos(\omega_0 t)$$

$$v_L = 0$$

耦合电容开路，
高频扼流圈短路

$$v_L = 0.1\eta V_{sp}\cos(\omega_0 t + 90° - 5.7°)$$

足够接近耦合电容直流开路，高频扼流圈直流短路

如何直接写出一阶系统系统传函表达式

- 先判断低通高通

如果不是简单的低通和高通，则老老实实地在相量域求解方程获得传递函数

$$H_{LP}(j\omega) = H_0 \frac{1}{1 + j\omega\tau}$$

$$H_{HP}(j\omega) = H_0 \frac{j\omega\tau}{1 + j\omega\tau}$$

- 求**H0**：中心频点的传递系数

$$H_0 = H_{LP}(j0)_{\text{电容开路，电感短路}}$$

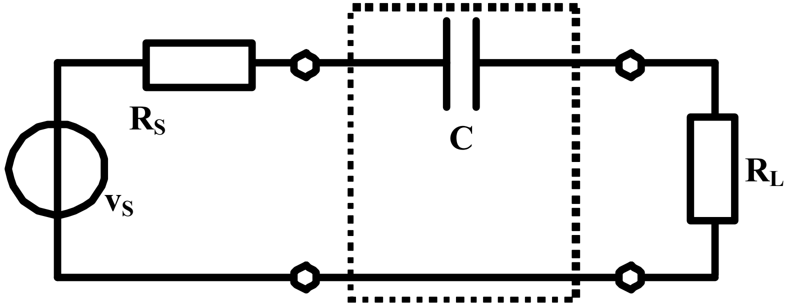
$$H_0 = H_{HP}(j\infty)_{\text{电容短路，电感开路}}$$

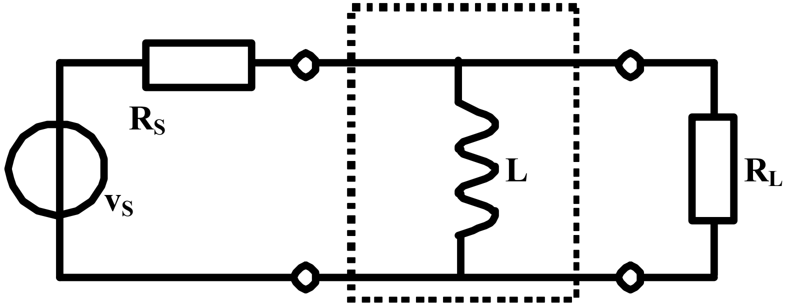
- 求τ：一阶系统的时间常数

$$\tau = RC$$

$$\tau = GL = \frac{L}{R}$$

R是C或L两端看入的等效电阻





$$H_{HP}(j\omega) = H_0 \frac{j\omega\tau}{1 + j\omega\tau}$$

$$H_{HP}(j\omega) = H_0 \frac{j\omega\tau}{1 + j\omega\tau}$$

$$H_0 = \frac{R_L}{R_S + R_L}$$

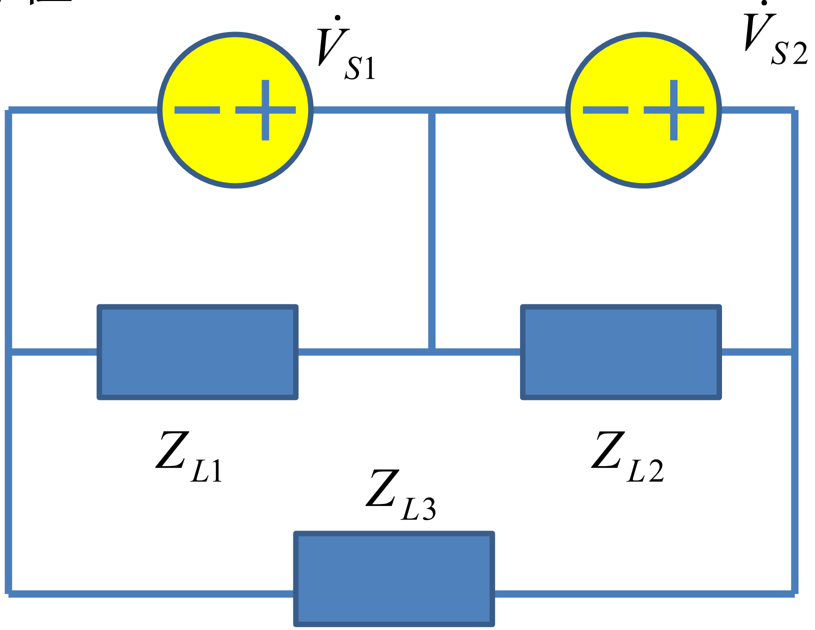
$$H_0 = \frac{R_L}{R_S + R_L}$$

$$\tau = (R_S + R_L)C$$

$$\tau = (G_S + G_L)L$$

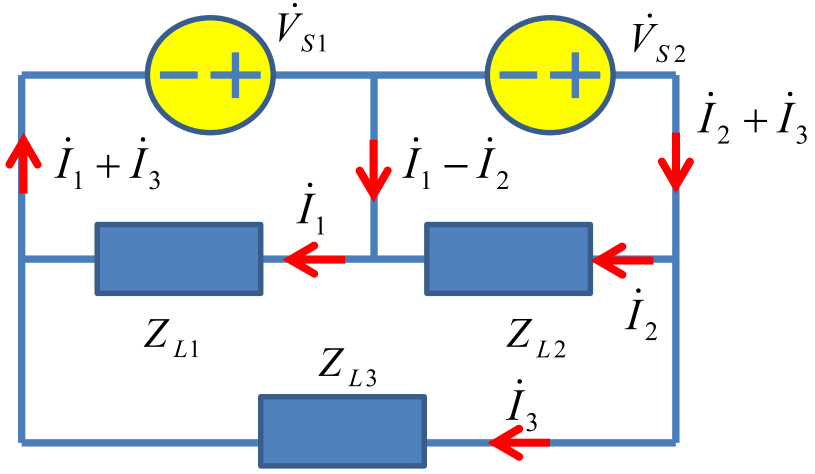
作业4：复功率

- （练习8.3.5）如图所示电路中有两个电压源和三个负载，负载1吸收的功率为1.8kW和600var，负载2吸收功率为1.5kVA，功率因数0.8超前（电流超前电压），负载3为（12Ω）||（j48Ω）
  - 如果$V_{s1}=V_{s2}=120\angle 0°V_{rms}$，求两个电源发送的平均功率和无功功率
  - 确认复功率守恒



- 负载1吸收功率为1.8kW和600var，负载2吸收功率为1.5kVA，功率因数0.8 超前（电流超前电压），负载3为（12Ω）||（j48Ω）， $V_{s1}=V_{s2}=120\angle 0° V_{rms}$

回路电流法



$$\dot{I}_{1,rms}^* = \frac{S_1}{\dot{V}_{1,rms}} = \frac{1800+j600}{120} = 15+j5(A_{rms})$$

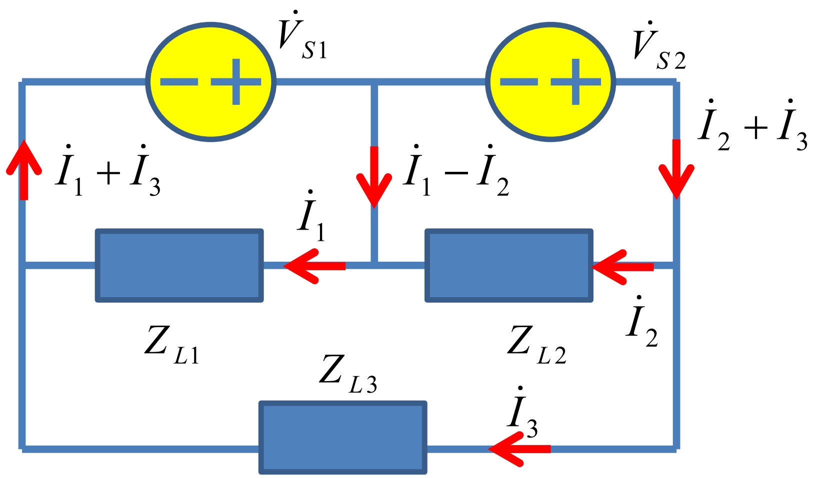
$$\dot{I}_{2,rms}^* = \frac{S_2}{\dot{V}_{2,rms}} = \frac{|S_2|\angle(\varphi_{V2}-\varphi_{I2})}{|V_{2,rms}|\angle\varphi_2} = \frac{1500\angle \arctan\frac{-0.6}{0.8}}{120\angle 0}$$

$$= \frac{1500\times 0.8 - j1500\times 0.6}{120} = \frac{1200-j900}{120} = 10-j7.5(A_{rms})$$

$$\dot{I}_{3,rms} = \frac{\dot{V}_{1,rms}+\dot{V}_{2,rms}}{Z_{L3}} = \frac{240}{12\parallel j48} = \frac{240}{\frac{12\times j48}{12+j48}} = \frac{240(12+j48)}{12\times j48} = \frac{5(1+j4)}{j} = 20-j5(A_{rms})$$

$$S_3 = \dot{V}_{3,rms}\dot{I}_{3,rms}^* = (240)\times(20+j5) = 4800+j1200$$

$$S_L = S_1+S_2+S_3 = (1800+j600)+(1200-j900)+(4800+j1200)$$
$$= 7800W + j900\,\text{var}$$



$\dot{I}_{1,rms} = 15 - j5(A_{rms})$ $\dot{I}^*_{1,rms} = 15 + j5(A_{rms})$

$\dot{I}_{2,rms} = 10 + j7.5(A_{rms})$ $\dot{I}^*_{2,rms} = 10 - j7.5(A_{rms})$

$\dot{I}_{3,rms} = 20 - j5(A_{rms})$

$$S_{S1} = V_1\left(\dot{I}_{1,rms} + \dot{I}_{3,rms}\right)^* = 120\times(15 - j5 + 20 - j5)^* = 120\times(35 + j10) = 4200 + j1200$$

$$S_{S2} = V_2\left(\dot{I}_{2,rms} + \dot{I}_{3,rms}\right)^* = 120\times(10 + j7.5 + 20 - j5)^* = 120\times(30 - j2.5) = 3600 - j300$$

$P_{S1} = 4200W$ $P_{S2} = 3600W$ $P_S = P_{S1} + P_{S2} = 7800W$

$Q_{S1} = 1200\,\text{var}$ $Q_{S1} = -300\,\text{var}$ $Q_S = Q_{S1} + Q_{S2} = 900\,\text{var}$

复功 $S_{S1} + S_{S2} = S_S = S_L = S_1 + S_2 + S_3$

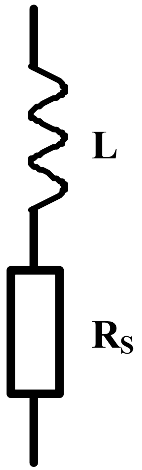
实功 $P_{S1} + P_{S2} = P_S = P_L = P_1 + P_2 + P_3$

虚功 $Q_{S1} + Q_{S2} = Q_S = Q_L = Q_1 + Q_2 + Q_3$

能量守恒

作业5、6：电桥

- 5、（习题8.4，8.5）用电桥测电感



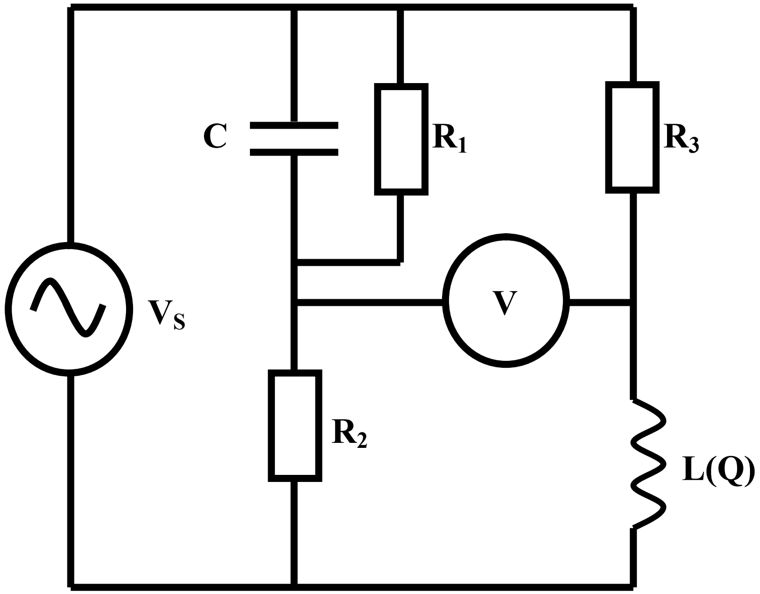
$$Q=\frac{\omega L}{R_S}$$

$$\text{品质因数}=\frac{|\text{虚功}|}{\text{实功}}=\begin{cases}\frac{|\text{串联电抗}|}{\text{串联电阻}}\\ \frac{|\text{并联电纳}|}{\text{并联电导}}\end{cases}$$

$$Z_L=R_S+j\omega L$$

真实电感存在寄生效应，这里假设频率较低，
只考虑寄生电阻效应

Maxwell Bridge



$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{R_2 R_3}{\frac{R_1}{1 + j\omega_0 R_1 C}} = \frac{R_2 R_3}{R_1}\left(1 + j\omega_0 R_1 C\right)$$

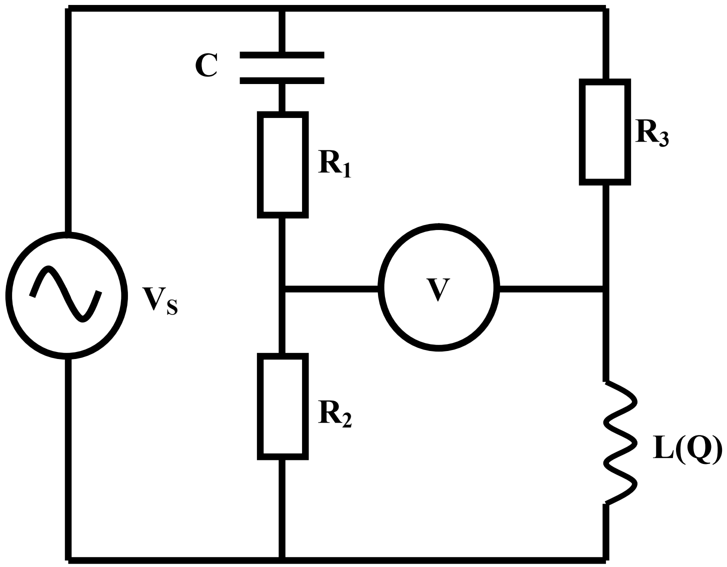
$$= \frac{R_2 R_3}{R_1} + j\omega_0 R_2 R_3 C = R_S + j\omega_0 L$$

$$L = R_2 R_3 C$$

$$Q = \frac{\omega_0 L}{R_S} = \frac{\omega_0 R_2 R_3 C}{\frac{R_2 R_3}{R_1}} = \omega_0 R_1 C = Q_1 = \frac{\omega_0 C}{G_1} = \frac{|\text{并联电纳}|}{\text{并联电导}}$$

Hay's Bridge

$$Z_1 Z_4 = Z_2 Z_3$$



$$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{R_2 R_3}{R_1 + \frac{1}{j\omega_0 C}} = \frac{R_2 R_3}{R_1} \frac{j\omega_0 R_1 C}{1 + j\omega_0 R_1 C}$$

$$= \frac{R_2 R_3}{R_1} \frac{\left((\omega_0 R_1 C)^2 + j\omega_0 R_1 C\right)}{1 + (\omega_0 R_1 C)^2} = R_S + j\omega_0 L$$

$$Q = \frac{\omega_0 L}{R_S} = \frac{1}{\omega_0 R_1 C} = Q_1 = \frac{|\text{串联电抗}|}{\text{串联 电阻}}$$

$$R_S = \frac{R_2 R_3}{R_1} \frac{(\omega_0 R_1 C)^2}{1 + (\omega_0 R_1 C)^2}$$

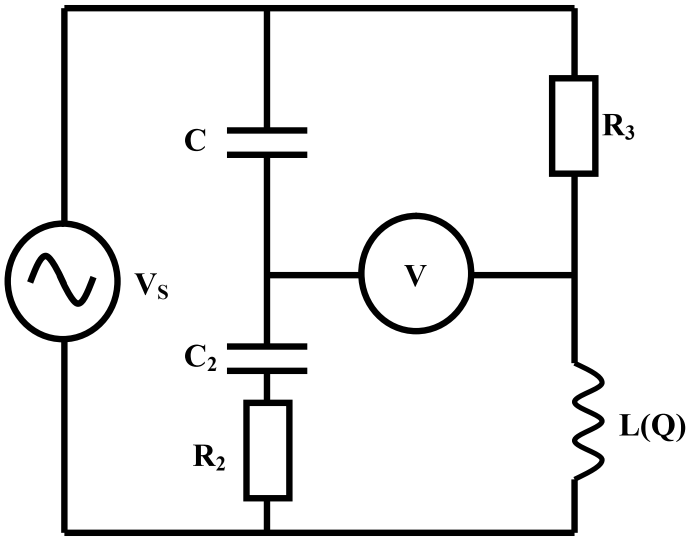
$$Q >> 1$$

$$L = \frac{R_2 R_3}{R_1} \frac{R_1 C}{1 + (\omega_0 R_1 C)^2} = \frac{R_2 R_3 C}{1 + (\omega_0 R_1 C)^2}$$

$$L = \frac{R_2 R_3 C}{1 + (\omega_0 R_1 C)^2} \approx R_2 R_3 C$$

$$Z_1 Z_4 = Z_2 Z_3$$

Owen's Bridge



$$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{\left(R_2 + \frac{1}{j\omega_0 C_2}\right) R_3}{\frac{1}{j\omega_0 C}} = \left(j\omega_0 R_2 C + \frac{C}{C_2}\right) R_3$$

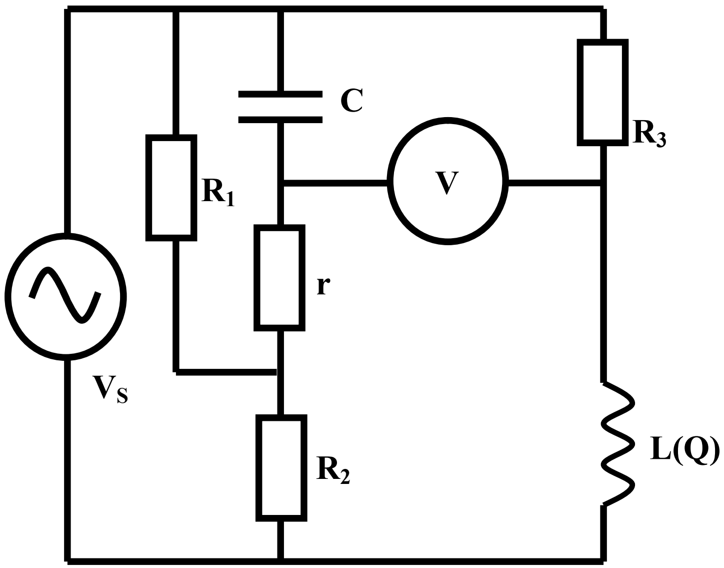
$$= \left(\frac{C}{C_2} R_3 + j\omega_0 R_3 R_2 C\right) = R_S + j\omega_0 L$$

$$R_S = \frac{C}{C_2} R_3$$

$$L = R_3 R_2 C$$

$$Q = \frac{\omega_0 L}{R_S} = \omega_0 R_2 C_2 = \frac{1}{Q_2} = \frac{1}{\frac{|串联电抗|}{串联电阻}}$$

Anderson’s Bridge



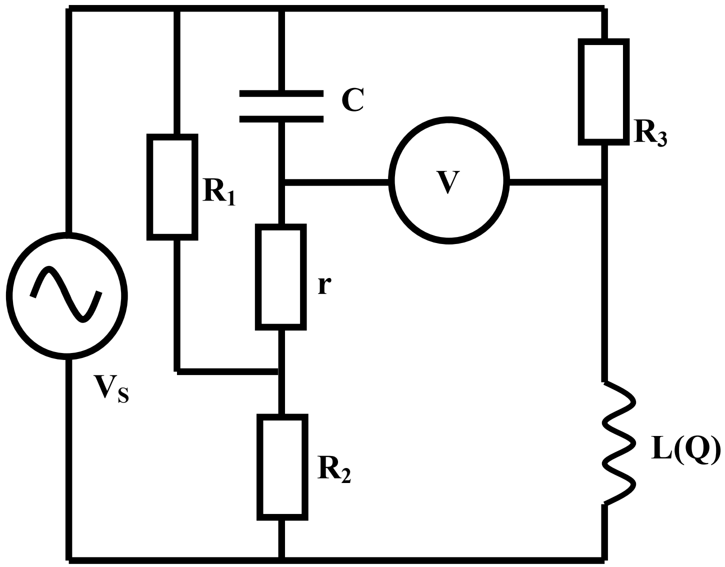
$$\dot{V}_{d3} = \frac{R_3}{Z_4 + R_3}\left(-\dot{V}_S\right)$$

$$\dot{V}_{dC} = -\dot{V}_S \frac{R_1 \parallel \left(r + \frac{1}{j\omega_0 C}\right)}{R_2 + R_1 \parallel \left(r + \frac{1}{j\omega_0 C}\right)} \frac{\frac{1}{j\omega_0 C}}{r + \frac{1}{j\omega_0 C}}$$

$$\dot{V}_{d3} = \dot{V}_{dC}$$

$$\frac{R_1 \parallel \left(r + \frac{1}{j\omega_0 C}\right)}{R_2 + R_1 \parallel \left(r + \frac{1}{j\omega_0 C}\right)} \frac{\frac{1}{j\omega_0 C}}{r + \frac{1}{j\omega_0 C}} = \frac{R_3}{Z_4 + R_3}$$

$$\frac{\dfrac{R_1\cdot\left(r+\dfrac{1}{j\omega_0 C}\right)}{R_1+\left(r+\dfrac{1}{j\omega_0 C}\right)}}{R_2+\dfrac{R_1\cdot\left(r+\dfrac{1}{j\omega_0 C}\right)}{R_1+\left(r+\dfrac{1}{j\omega_0 C}\right)}}\,\frac{\dfrac{1}{j\omega_0 C}}{r+\dfrac{1}{j\omega_0 C}}=\frac{R_1}{R_2R_1+R_2\left(r+\right.}$$



$$=\frac{R_1\cdot\dfrac{1}{j\omega_0 C}}{\left(R_2R_1+R_2r+R_1r\right)+\left(R_2+R_1\right)\dfrac{1}{j\omega_0 C}}=\frac{R_3}{Z_4+R_3}$$

$$=\frac{R_1}{\left(R_2R_1+R_2r+R_1r\right)j\omega_0 C+\left(R_2+R_1\right)}=\frac{R_3}{j\omega_0 L+R_S+R_3}$$

$$=\frac{1}{j\omega_0\left(R_2+\dfrac{R_2}{R_1}r+r\right)C+\dfrac{R_2}{R_1}+1}=\frac{1}{j\omega_0\dfrac{L}{R_3}+\dfrac{R_S}{R_3}+1}$$

$$R_S=\frac{R_2}{R_1}R_3$$

$$L=R_3\left(R_2+\frac{R_2}{R_1}r+r\right)C$$

$$=R_3R_2C\left(1+\frac{1}{R_1}r+\frac{1}{R_2}r\right)$$

$$=R_3R_2C\left(1+\frac{r}{R_1\parallel R_2}\right)$$