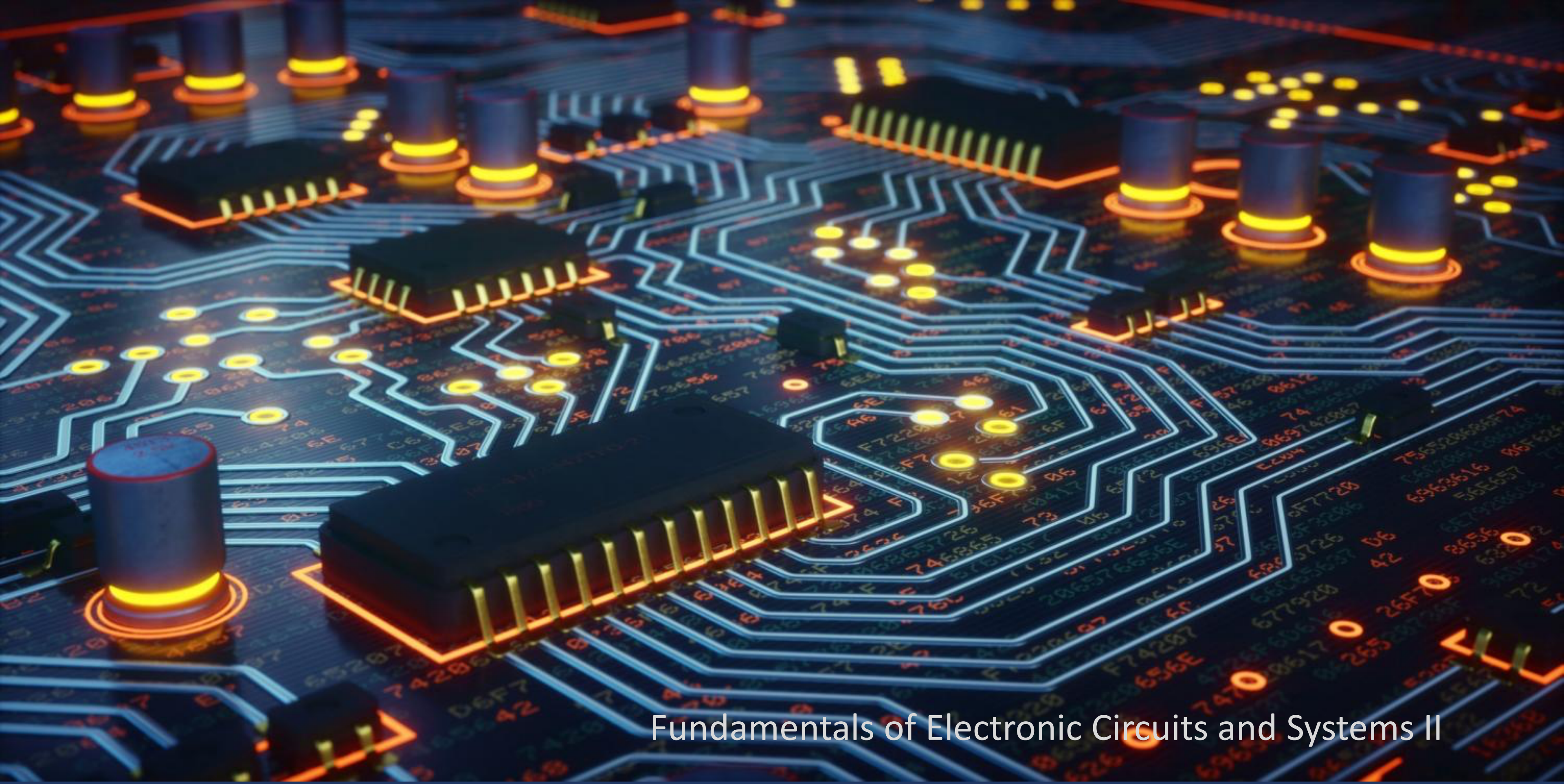


Fundamentals of Electronic Circuits and Systems II

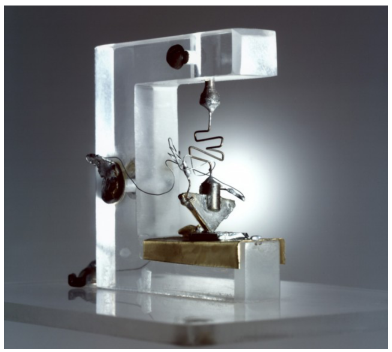
Bipolar Junction Transistors

Milin Zhang

Dept of EE, Tsinghua University

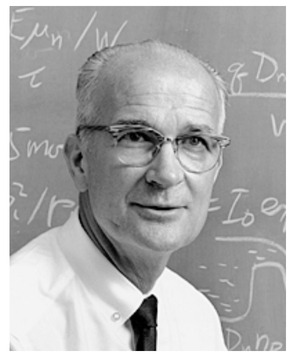
BJT – a milestone in IC history

Bipolar Junction Transistor = BJT



First Transistor @ 1947

Nobel Prize @1956



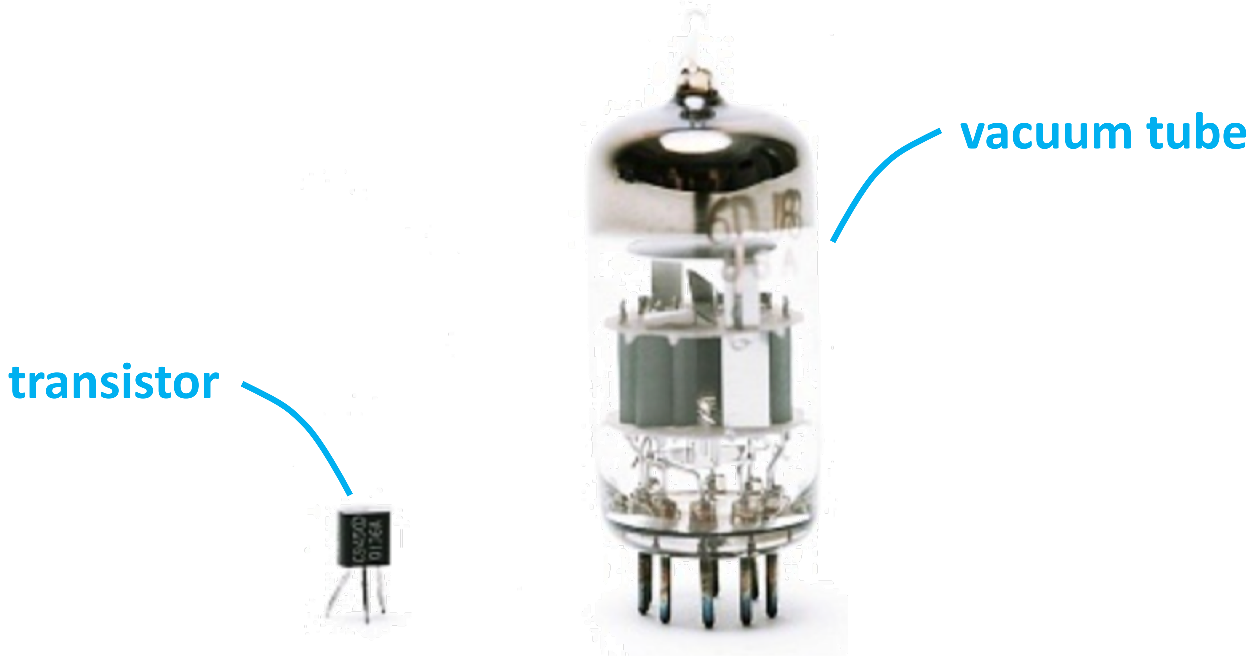
William Shockley

John Bardeen

Walter Brattain

BJT – a milestone in IC history

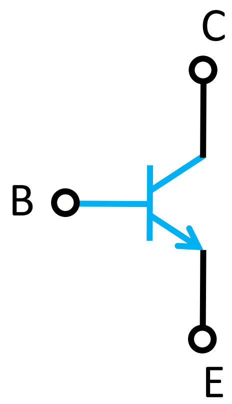
Bipolar Junction Transistor = BJT



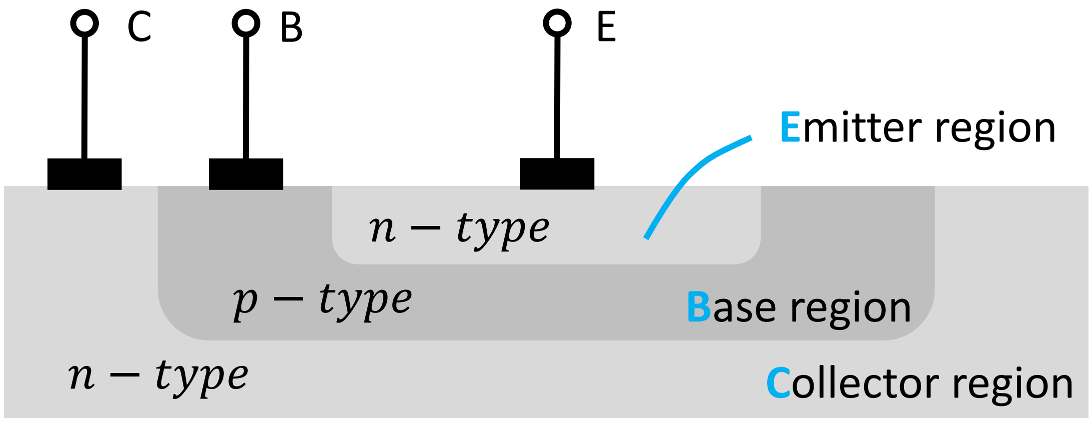
Outline

- Introduction to BJT
  - WHAT does it look like
  - HOW does it work
- The characteristic curves
- Circuit analysis techniques with BJT

Structure of BJT



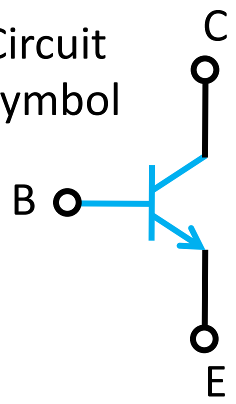
Circuit symbol



Cross section of an npn BJT

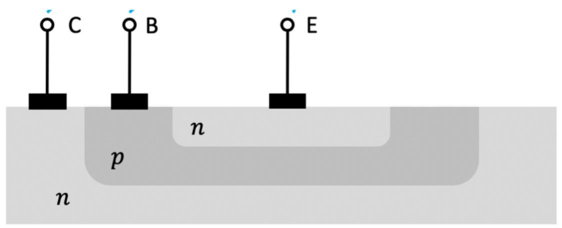
Structure of BJT

Circuit symbol

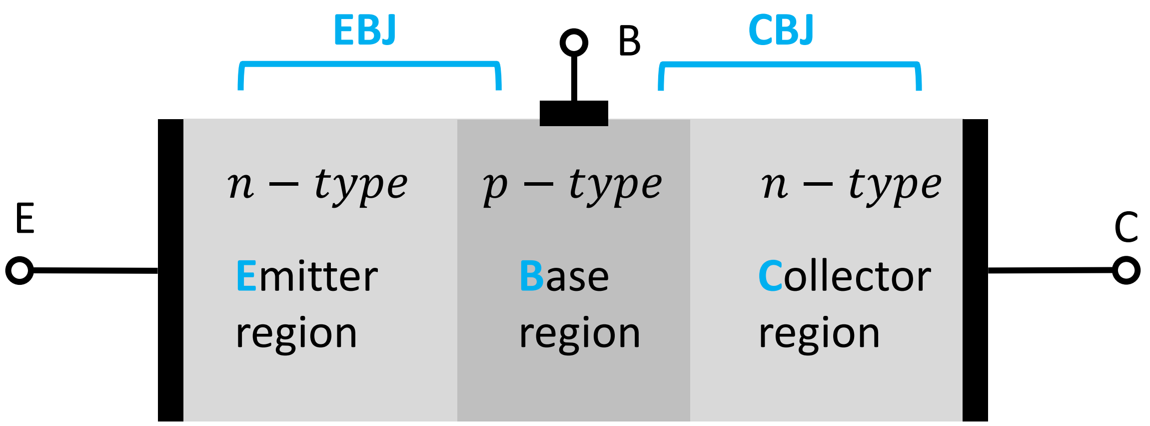


There are 2 pn junctions

- Emitter-base junction (EBJ)
- Collector-base junction (CBJ)

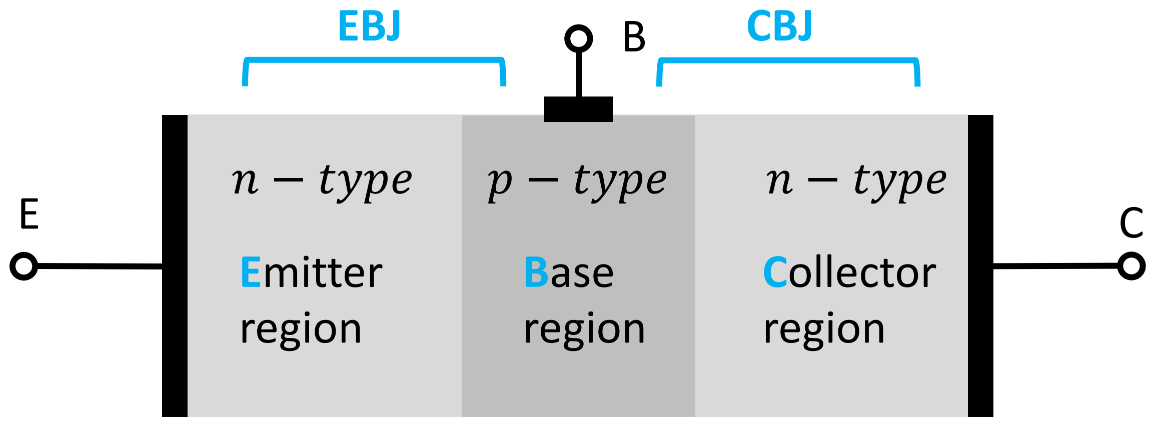


Cross section of an npn BJT



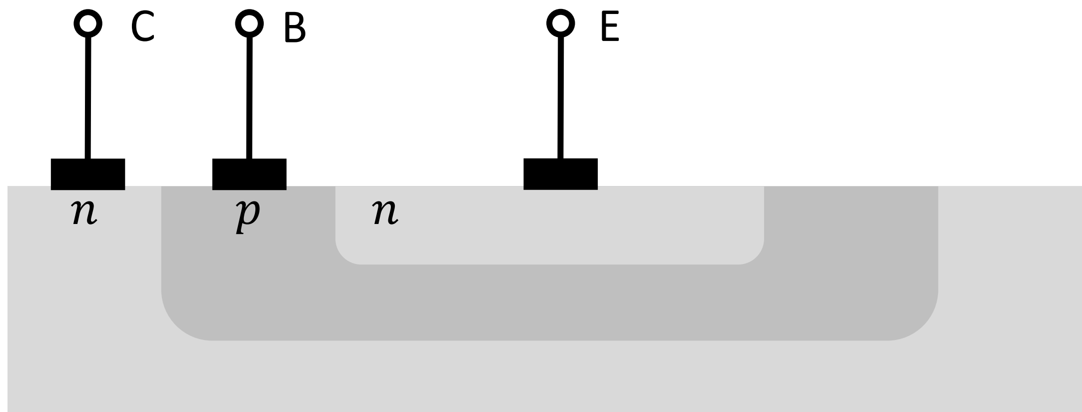
Simplified structure of an npn BJT

BJT modes of operations

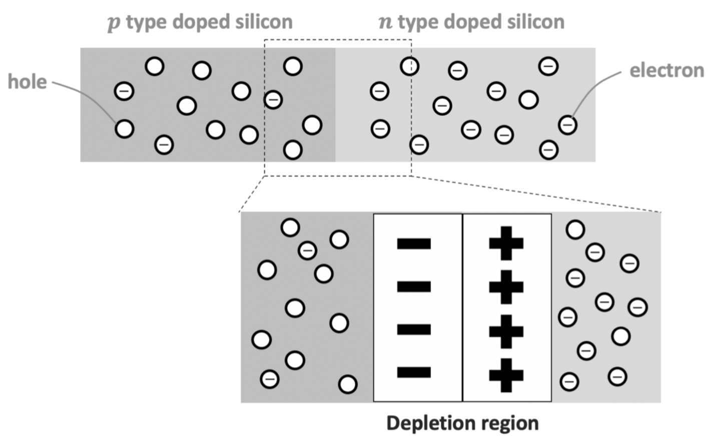


Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward
Reverse Active	Reverse	Forward

How does a BJT work?



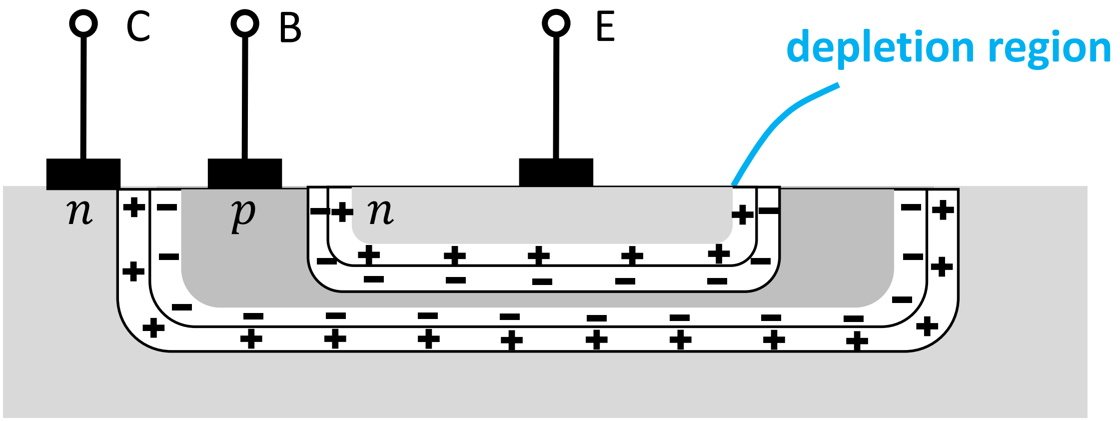
Recall: depletion in pn junction



Carrier-depletion region

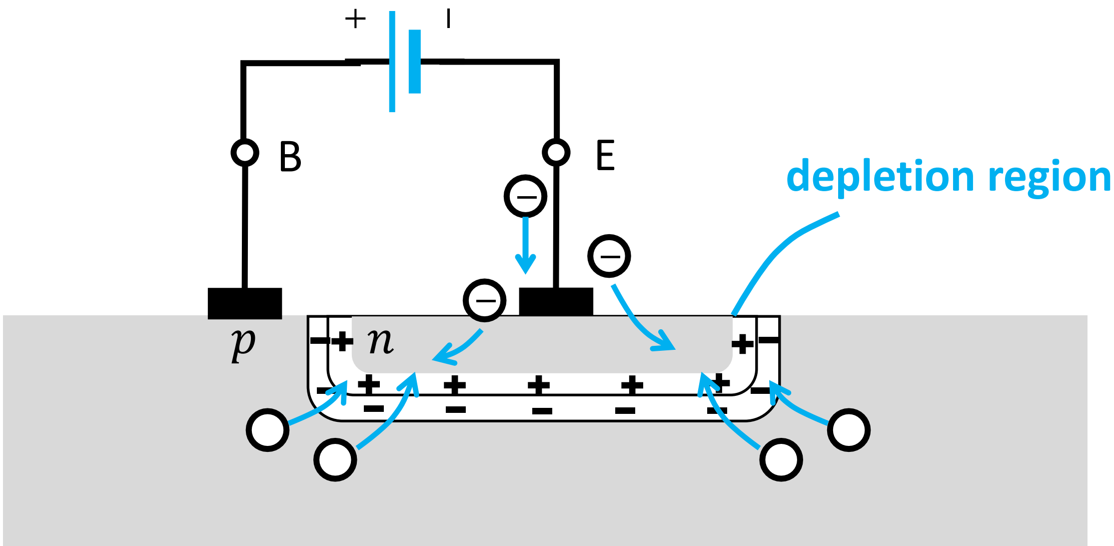
- **Exist in both sides**
- **Uncovered charges**
- **n is more positive than p**
- **Electronic field generated**

How does a BJT work?



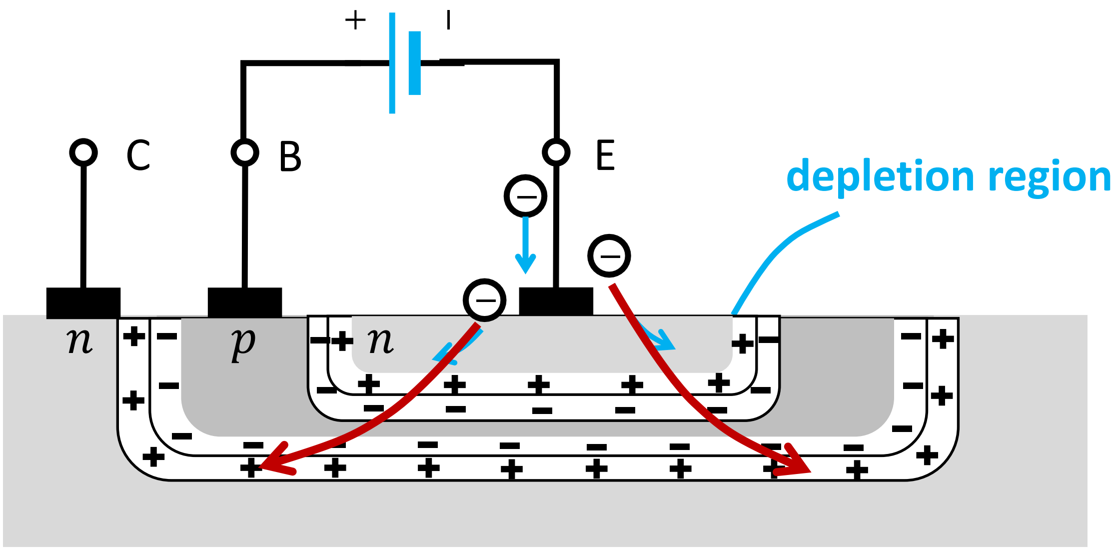
- There are two pn junctions
- Depletion regions are generated in both sides of both EBJ and CBJ

How does a BJT work?



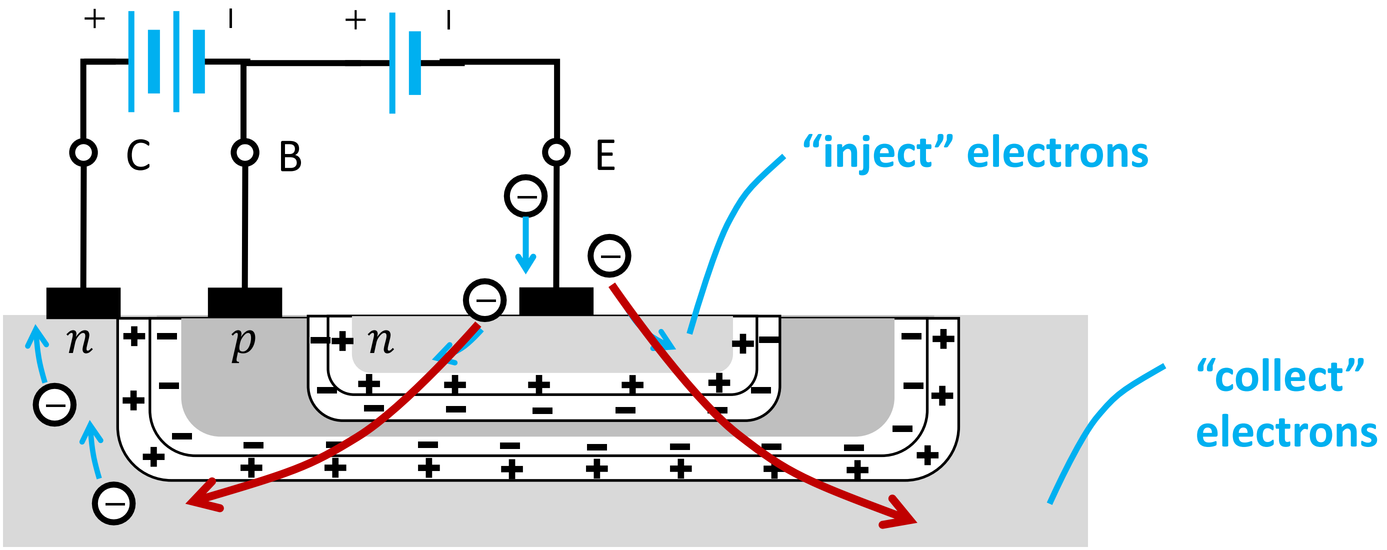
- A forward-bias applied to EBJ
- width of depletion region of EBJ ↓
- The base region is very thin. But there are too much electrons …

How does a BJT work?



- Electrons enter the collector region
- Emitter current is dominated by the electrons

How does a BJT work?



- A reversed-bias applied to CBJ
- Current generated in collector region
- The BJT is biased in **ACTIVE** mode

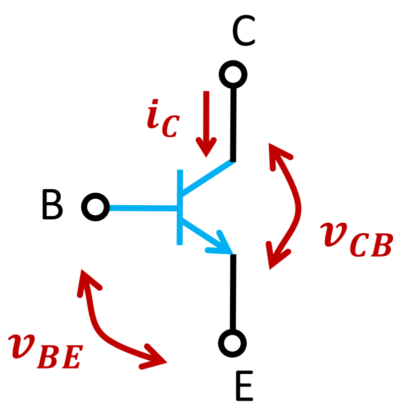
Currents of BJT in active mode

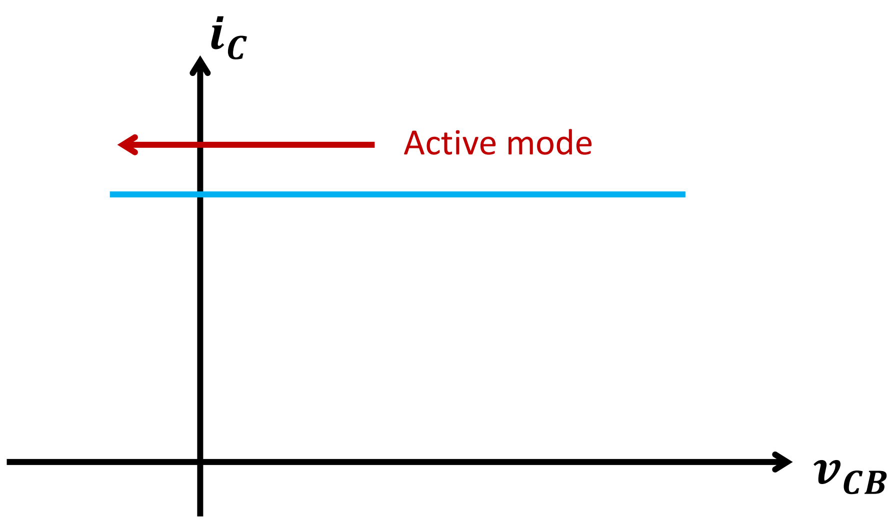
- **collector current** $i_C = I_S e^{\frac{v_{BE}}{V_T}}$

Saturation current $I_S = \frac{A_E q D_n n_i^2}{N_A W}$

- A_E: Cross-sectional area
- D_n: Electron diffusivity in base
- n_i: Carrier density
- W: Base width
- N_A: Doping concentration in the base

- **i_C is NOT dependent on v_{CB}**





Currents of BJT in active mode

- **collector current** $\boldsymbol{i_C = I_S e^{\frac{v_{BE}}{V_T}}}$ $\left(I_S = \frac{A_E q D_n n_i^2}{N_A W}\right)$

- **base current** $\boldsymbol{i_B = \frac{i_C}{\beta}}$

$$i_B = \frac{A_E q D_p n_i^2}{N_D L_P} e^{\frac{v_{BE}}{V_T}} + \frac{A_E q W n_i^2}{\tau_b N_A} e^{\frac{v_{BE}}{V_T}}$$

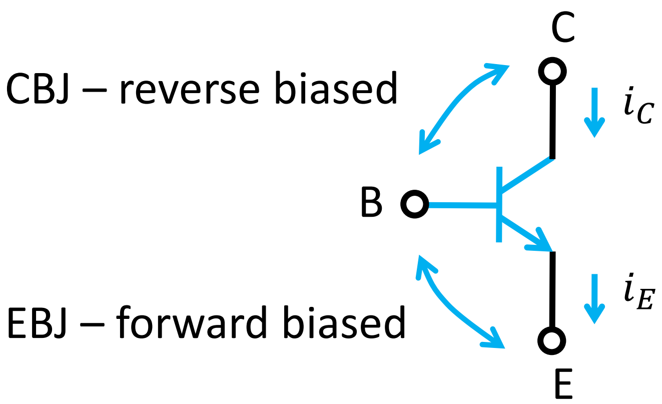
Only related to the physical structure of the BJT

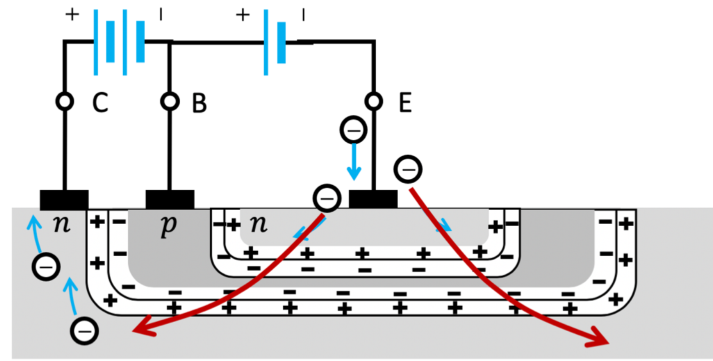
$$= \left(\frac{D_p N_A W}{D_n N_D L_P} + \frac{\mathrm{W}^2}{2 D_n \tau_b}\right) I_S e^{\frac{v_{BE}}{V_T}}$$

||

$\boldsymbol{\beta}$ – common-emitter current gain
typical value of β is 50 – 200

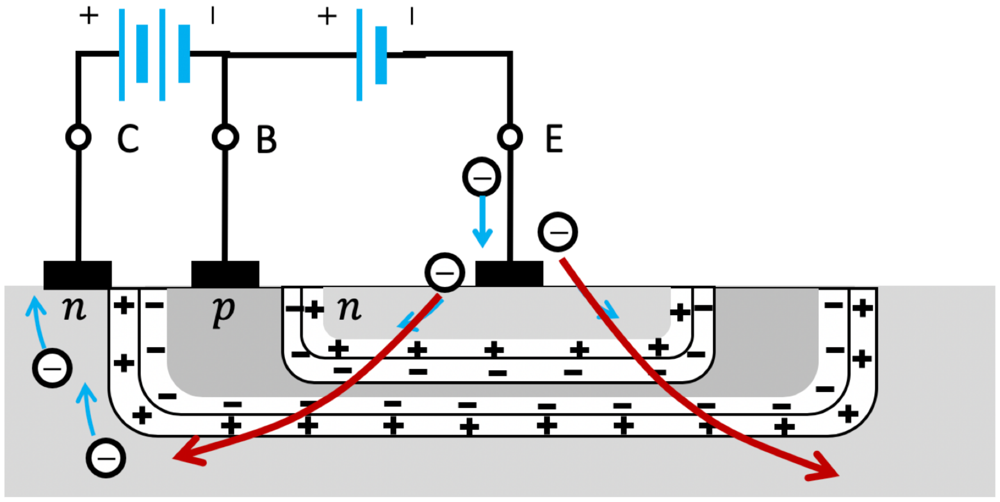
Summary: BJT in active mode

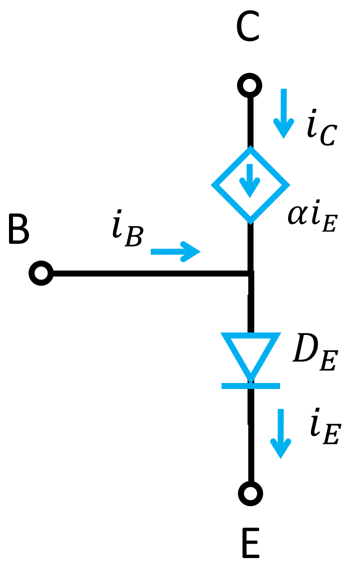




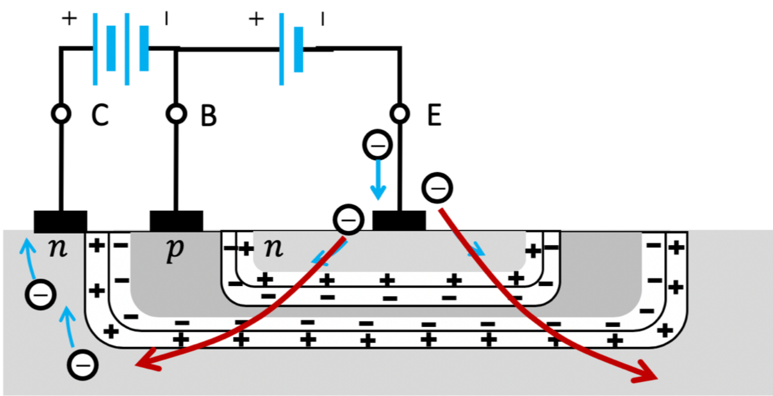
- **collector current** $i_C = I_S e^{\frac{v_{BE}}{V_T}}$
- **base current** $i_B = \frac{i_C}{\beta}$
- **emitter current** $i_E = i_B + i_C = \frac{\beta + 1}{\beta} i_C = \frac{1}{\alpha} i_C$

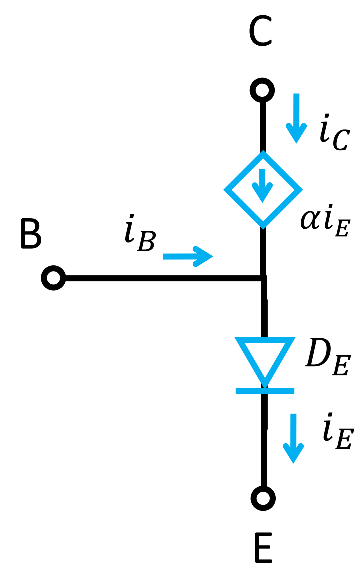
A model for active mode





A model for active mode





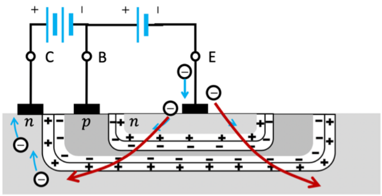
- Since

$$i_C = I_S e^{\frac{v_{BE}}{V_T}} \qquad i_E = I_{SE} e^{\frac{v_{BE}}{V_T}} \qquad i_E = \frac{i_C}{\alpha}$$

- Thus $\rightarrow$ $I_{SE} = \frac{I_S}{\alpha}$

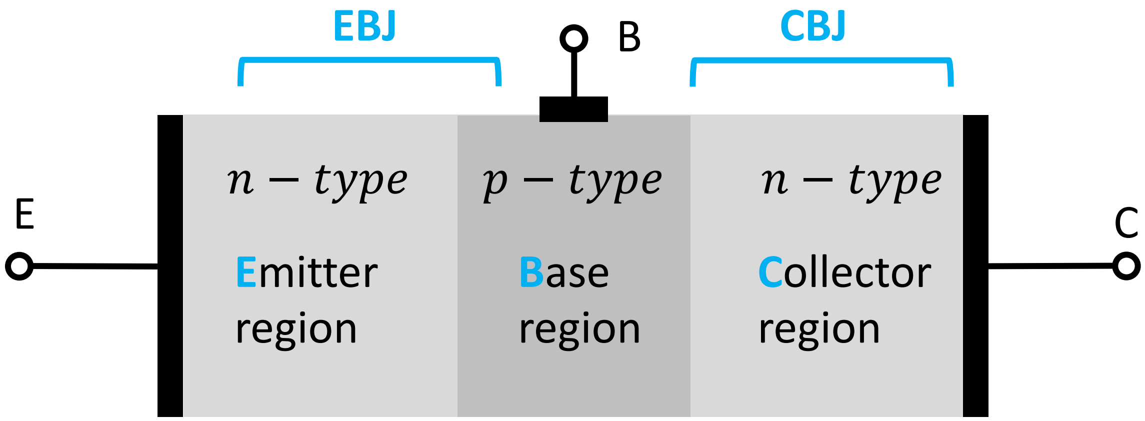
Outline

- Introduction to BJT
  - Device structure
  - How does it work?
    - Active mode



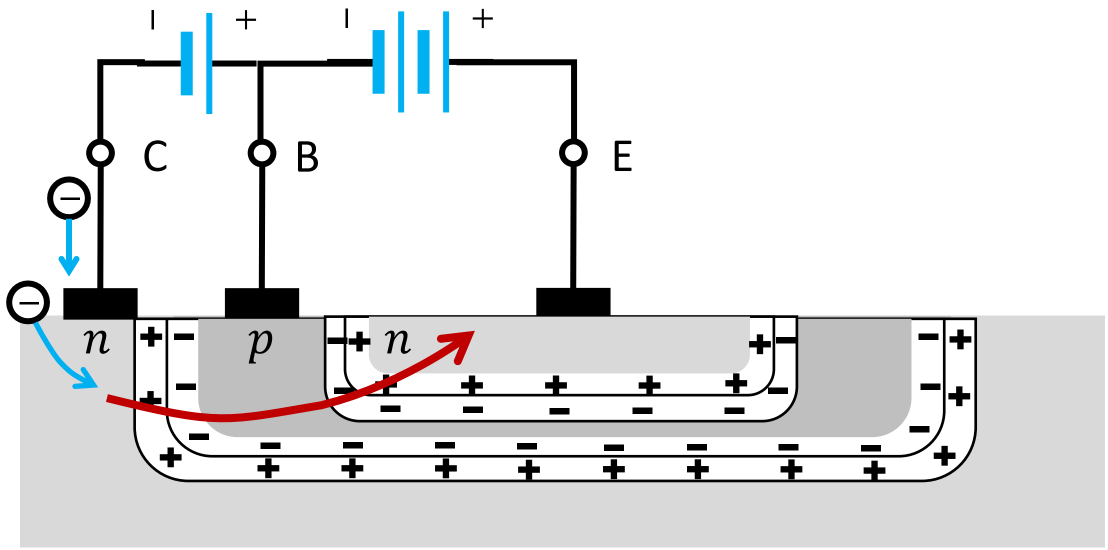
Active mode

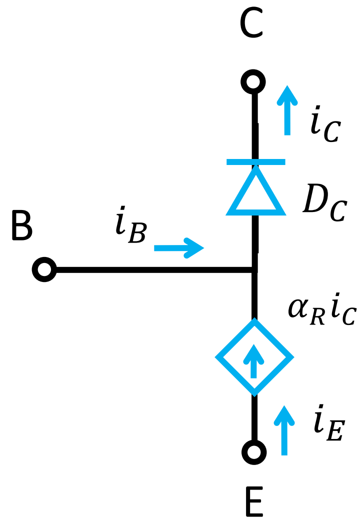
Recall: BJT modes of operations



Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward
Reverse Active	Reverse	Forward

BJT in reverse mode



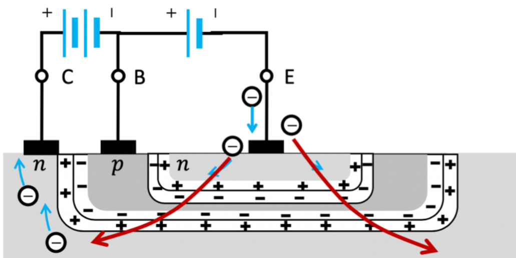


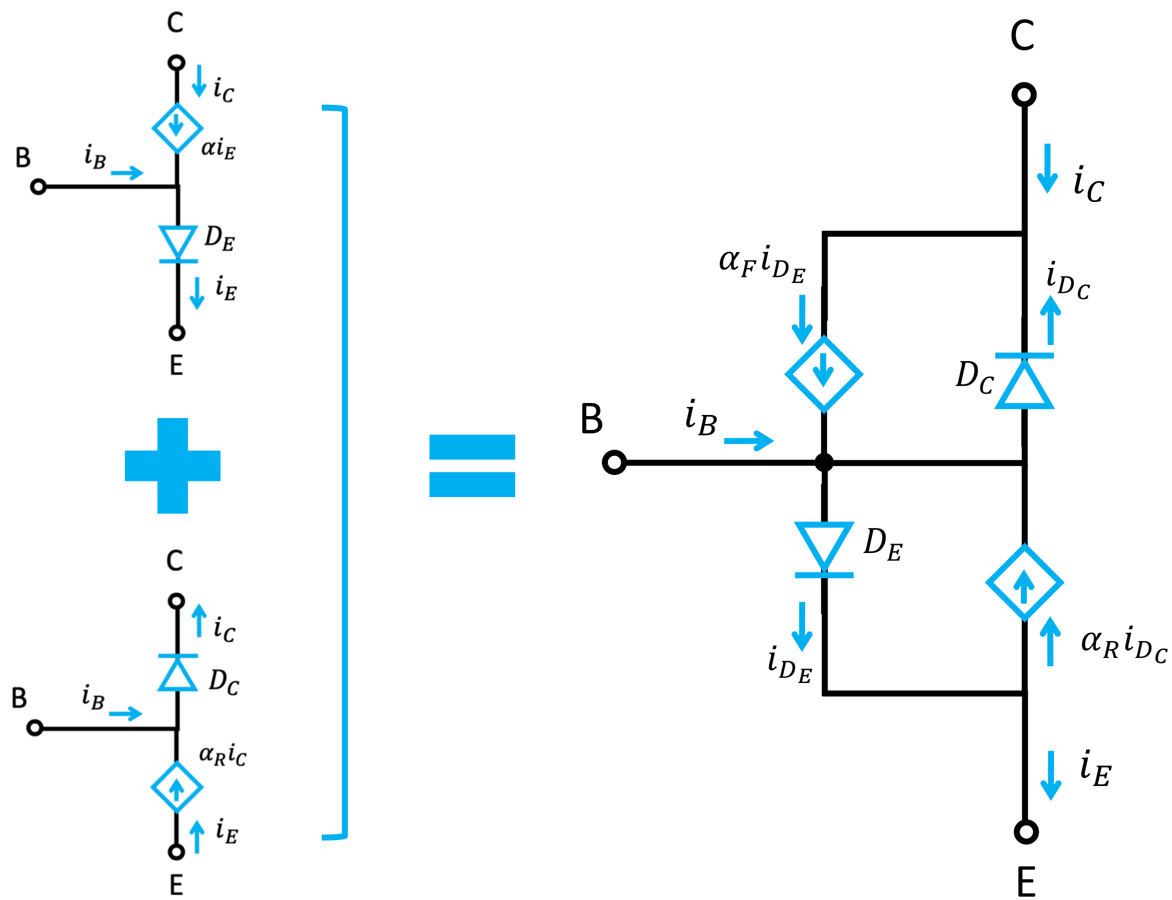
- α_R is much lower than α in active mode
- Typical value of α_R is 0.01 – 0.05

$$i_B = \frac{i_E}{\beta_R} \qquad i_E = \alpha_R i_C$$

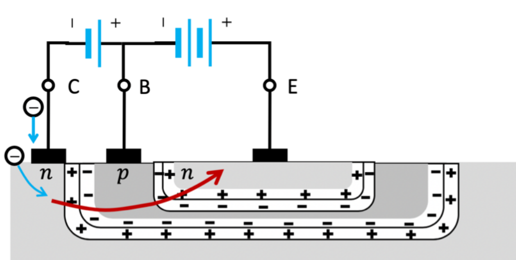
EM model for BJT

- Active mode





- Reverse mode



EM model for BJT

- According to KCL

$$i_C = -i_{D_C} + \alpha_F i_{D_E}$$

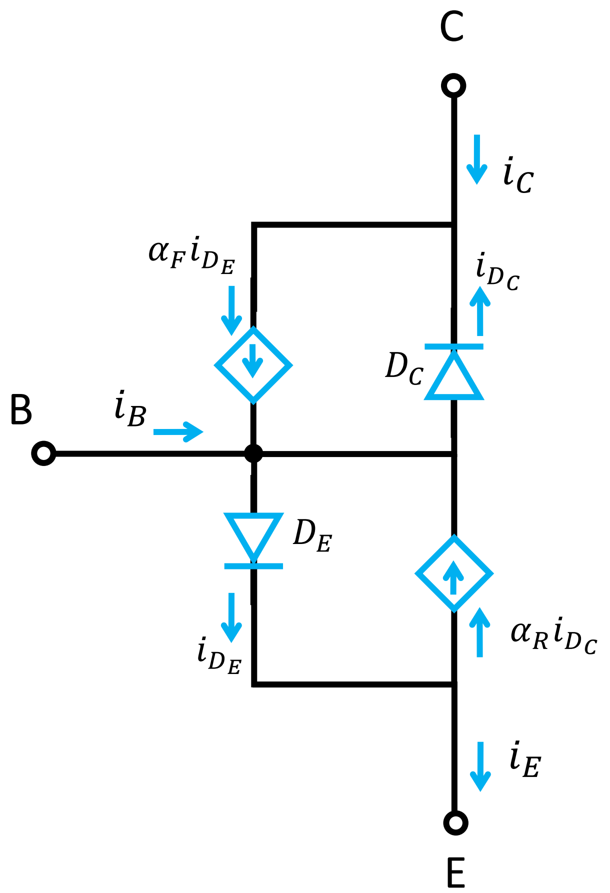
- According to $i - v$ characteristics of D_E and D_C

$$i_{D_C} = I_{SC}\left(e^{\frac{v_{BC}}{V_T}} - 1\right) = \frac{I_S}{\alpha_R}\left(e^{\frac{v_{BC}}{V_T}} - 1\right)$$

$$i_{D_E} = I_{SE}\left(e^{\frac{v_{BE}}{V_T}} - 1\right) = \frac{I_S}{\alpha_F}\left(e^{\frac{v_{BE}}{V_T}} - 1\right)$$

- Take i_{D_C} and i_{D_E} to i_C

$$i_C = -\frac{I_S}{\alpha_R}\left(e^{\frac{v_{BC}}{V_T}} - 1\right) + I_S\left(e^{\frac{v_{BE}}{V_T}} - 1\right)$$



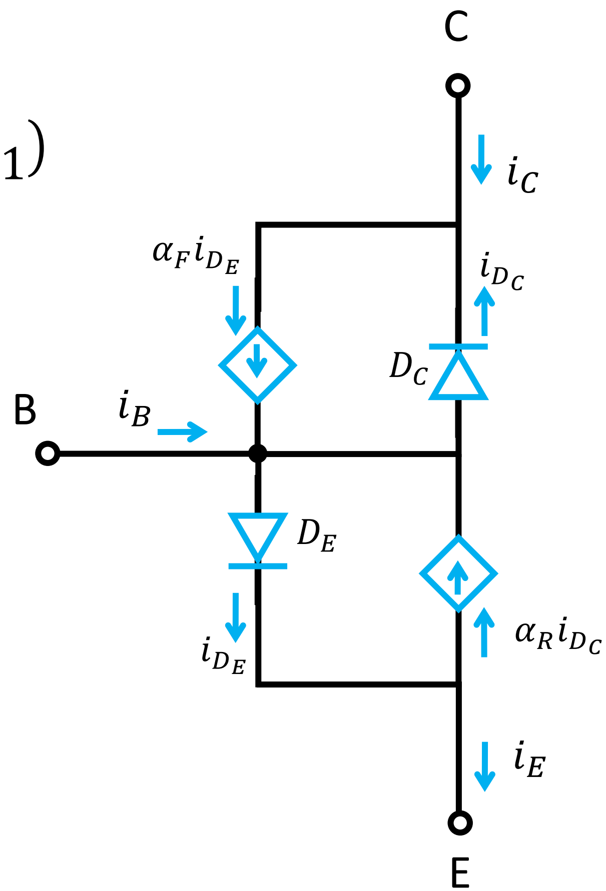
EM model for BJT

$$\begin{cases} i_E = i_{DE} - \alpha_R i_{DC} = \dfrac{I_S}{\alpha_F}\left(e^{\frac{v_{BE}}{V_T}} - 1\right) - I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right) \\ i_C = -i_{DC} + \alpha_F i_{DE} = -\dfrac{I_S}{\alpha_R}\left(e^{\frac{v_{BC}}{V_T}} - 1\right) + I_S\left(e^{\frac{v_{BE}}{V_T}} - 1\right) \end{cases}$$

$$i_C = -\frac{I_S}{\alpha_R}\left(e^{\frac{v_{BC}}{V_T}} - 1\right) + I_S\left(e^{\frac{v_{BE}}{V_T}} - 1\right)$$

$$+\alpha_F I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right) - \alpha_F I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right)$$

$$= \left(\alpha_F - \frac{1}{\alpha_R}\right) I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right) + \alpha_F i_E$$

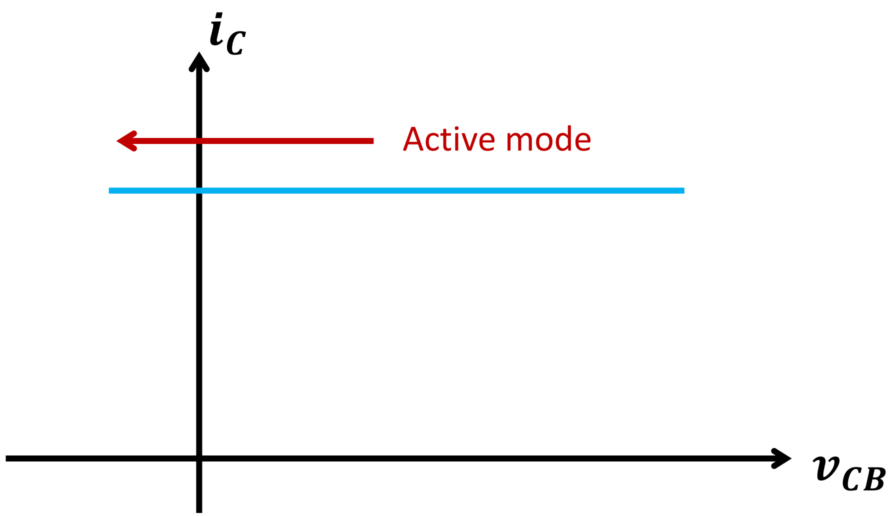


Recall: Currents of BJT in Active Mode

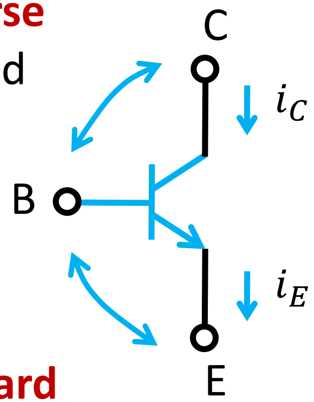
- **collector current** $i_C = I_S e^{\frac{v_{BE}}{V_T}}$

Saturation current $I_S = \frac{A_E q D_n n_i^2}{N_A W}$

- i_C **is NOT dependent on** v_{CB}



CBJ – **reverse** biased

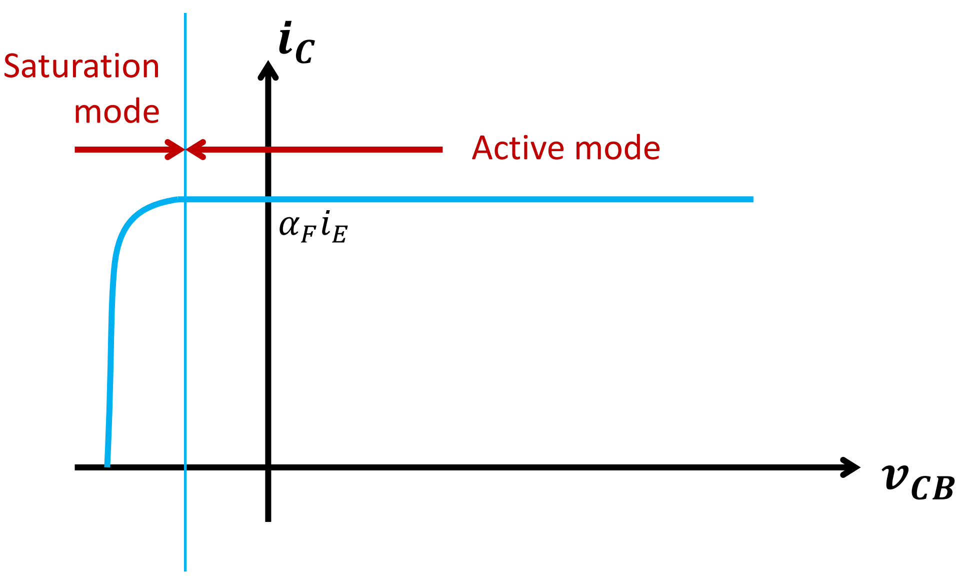


EBJ – **forward** biased

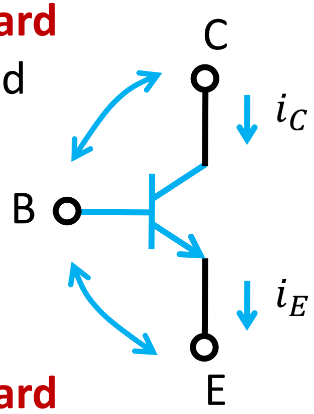
BJT in Saturation Mode

$$i_C = \left(\alpha_F - \frac{1}{\alpha_R}\right) I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right) + \alpha_F i_E$$

When v_{BC} is increasing $\rightarrow$ i_C decreases



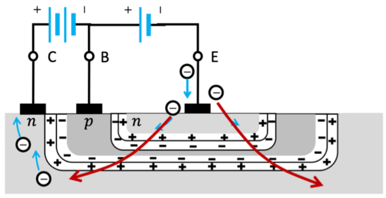
CBJ – **forward** biased



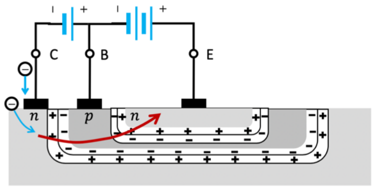
EBJ – **forward** biased

Outline

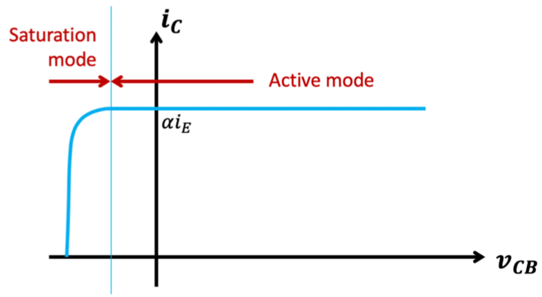
- Introduction to BJT
  - Device structure
  - How does it work?
    - Active mode
    - Reverse mode
    - Saturation mode
  - ***npn* v.s. *pnp***



Active mode

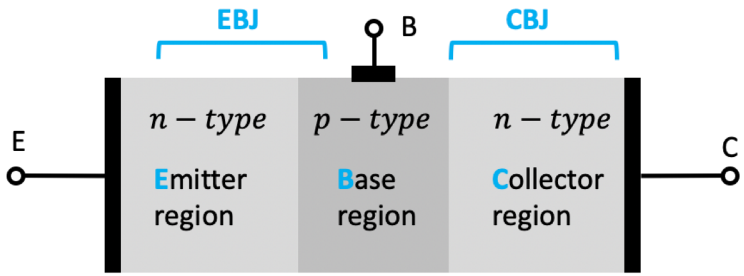


reverse mode

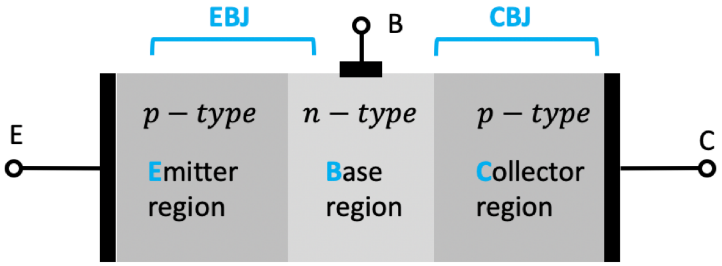


$\boldsymbol{npn}$ v.s. $\boldsymbol{pnp}$

$\boldsymbol{npn}$ type BJT



$\boldsymbol{pnp}$ type BJT



Mode	EBJ	CBJ
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Outline

- Introduction to BJT
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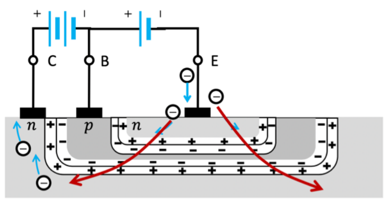
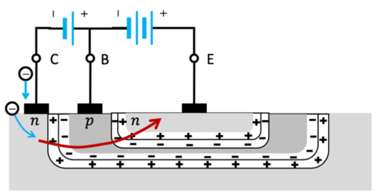
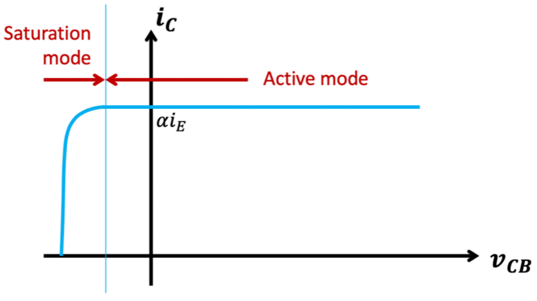
  - How does it work?
    - Active mode
    - Reverse mode
    - Saturation mode
  - npn v.s. pnp
- **Characteristics curves**

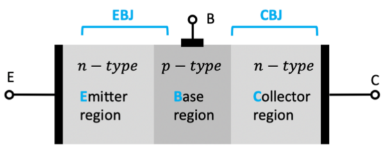


Active mode

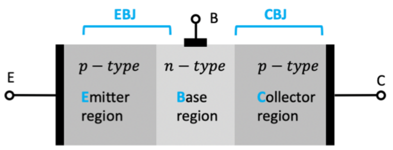


reverse mode



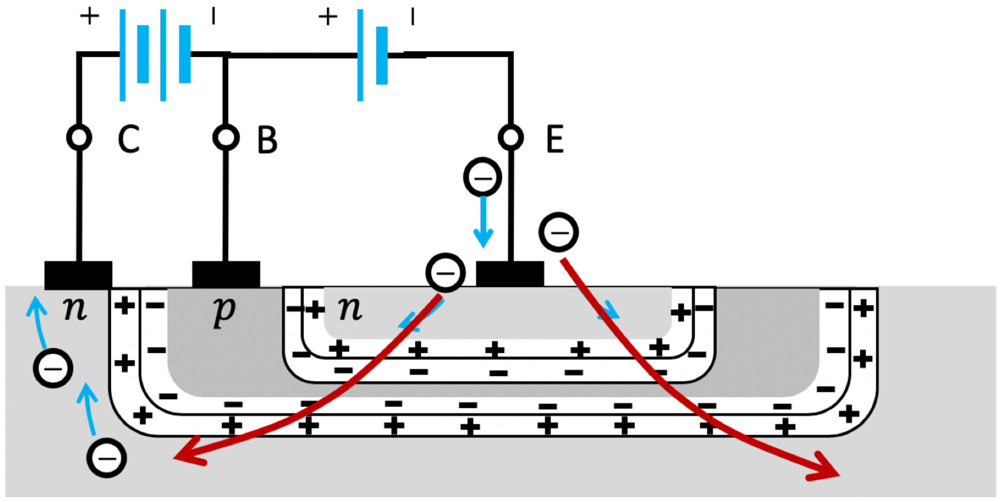


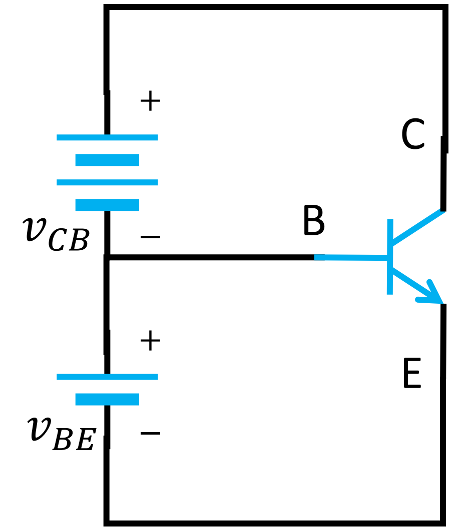
npn type BJT



pnp type BJT

From dev. structure to circuit model



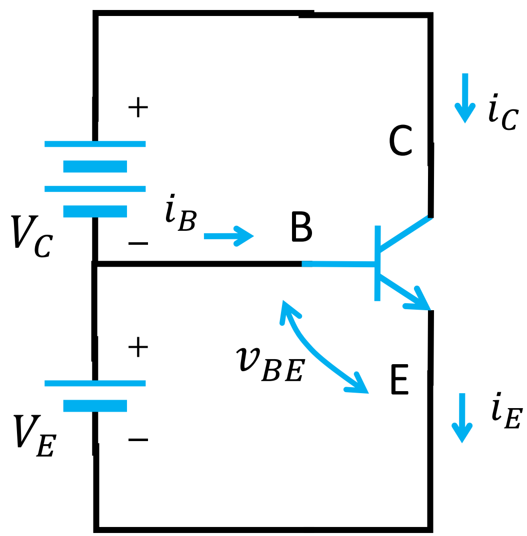


$$i_C = I_S e^{\frac{v_{BE}}{V_T}} \qquad \text{where } V_T = \frac{kT}{q}$$

$$i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}}$$

$$i_E = \frac{i_C}{\alpha} = \frac{I_S}{\alpha} e^{\frac{v_{BE}}{V_T}} \qquad \text{where } \alpha = \frac{\beta}{1+\beta}$$

$i - v$ characteristics



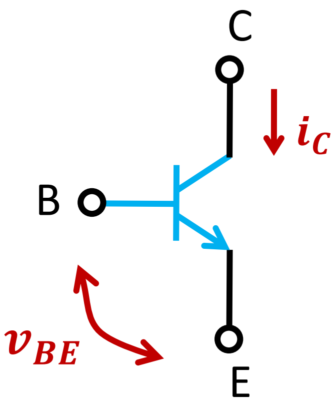
- As we already knew, in active mode

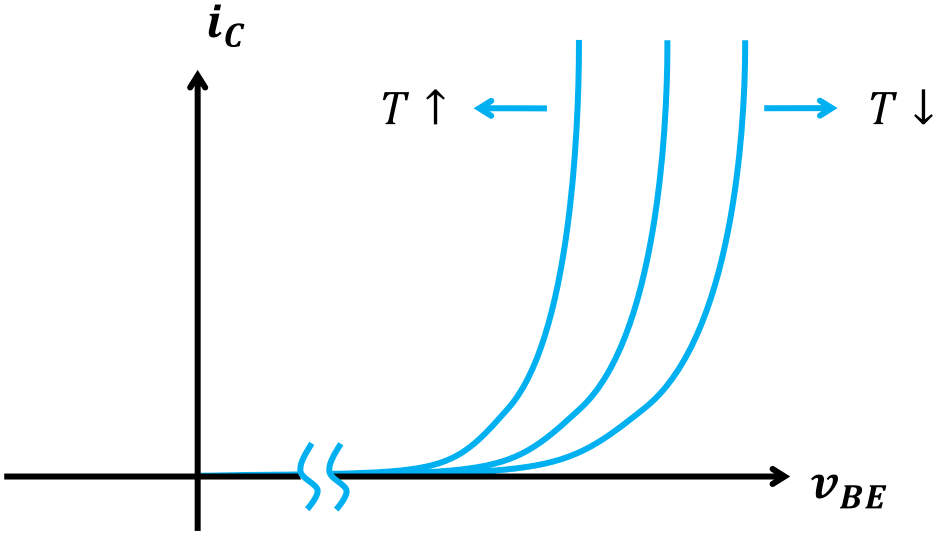
$$i_C = \beta i_B \qquad i_E = (1+\beta)\, i_B$$

- Mathematically $\quad i_E = i_B + i_C$
- Image the transistor is a super big node
- According to KCL $\quad i_E = i_B + i_C$

KCL/KVL works for transistor circuit anaylsis

$i_C - v_{BE}$ characteristics

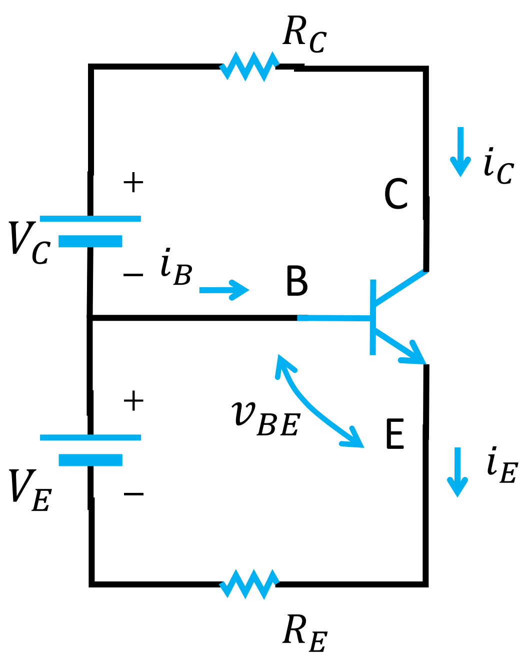




$$i_C = I_S e^{\frac{v_{BE}}{V_T}}$$

Example 1

QUESTION: The transistor has $\beta = 100$ and exhibits a v_{BE} of $0.7V$ at $i_C = 1mA$. Find the resistance of R_C and R_E when the transistor is biased in active region at $i_C = 2mA$ and $v_{CB} = 5V$ with $V_C = V_E = 15V$



- According to KVL

$$V_C = i_C R_C + v_{CB} \quad \rightarrow \quad R_C = 5k\Omega$$

- According to $\boldsymbol{i_C - v_{BE}}$ characteristics $\quad i_C = I_S e^{\frac{v_{BE}}{V_T}}$

Since $i_C = 1mA \quad @\, v_{BE} = 0.7V$

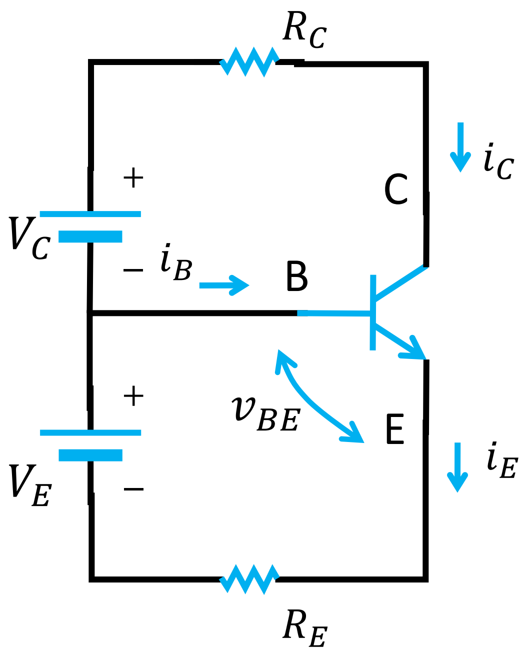
$$v_{BE}\Big|_{i_C=2mA} = 0.7 + V_T In\left(\frac{2mA}{1mA}\right) = 0.717V$$

- According to KVL

$$V_E = i_E R_E + v_{BE} \quad \rightarrow \quad i_E R_E = 15.717V$$

Example 1

QUESTION: The transistor has $\beta = 100$ and exhibits a v_{BE} of $0.7V$ at $i_C = 1mA$. Find the resistance of R_C and R_E when the transistor is biased in active region at $i_C = 2mA$ and $v_{CB} = 5V$ with $V_C = V_E = 15V$



- Since $\beta = 100$

$$\alpha = \frac{\beta}{\beta + 1} = 0.99$$

- According to $\boldsymbol{i_C - i_E}$ characteristics

$$i_E = \frac{i_C}{\alpha} = 2.02mA$$

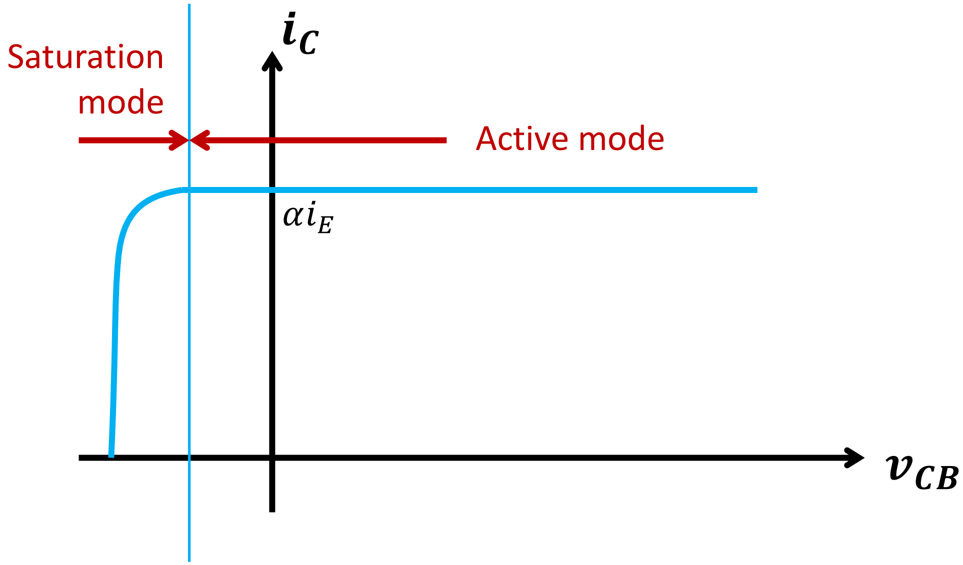
- According to $i - v$ characteristics of resistor

$$R_E = 7.07k\Omega$$

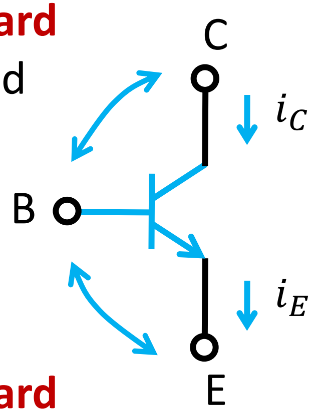
Recall: BJT in Saturation Mode

$$i_C = \left(\alpha_F - \frac{1}{\alpha_R}\right) I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right) + \alpha_F i_E$$

When v_{BC} is increasing $\rightarrow$ i_C decreases

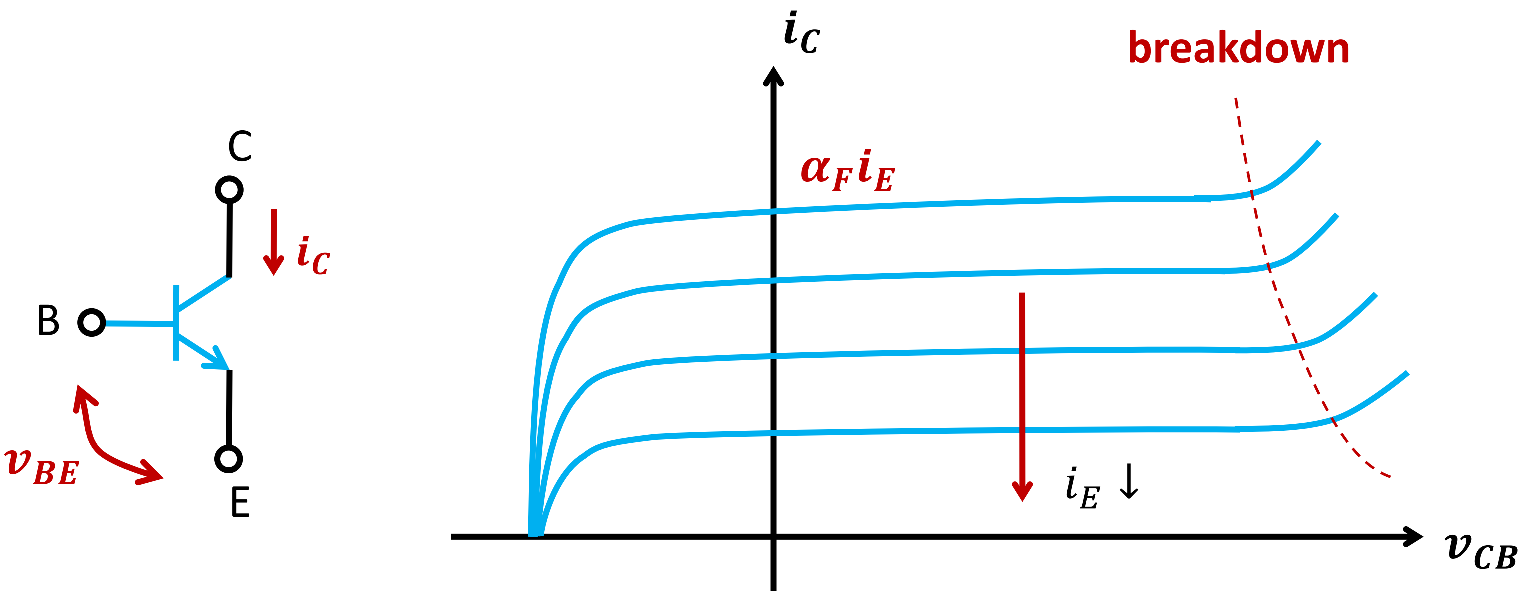


CBJ – **forward** biased



EBJ – **forward** biased

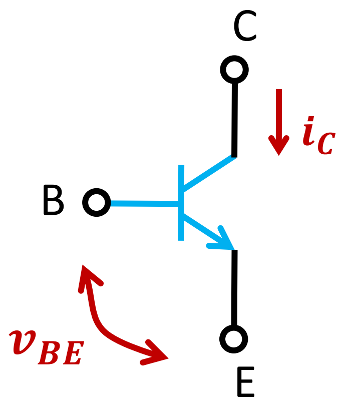
$i_C - v_{CB}$ Characteristics

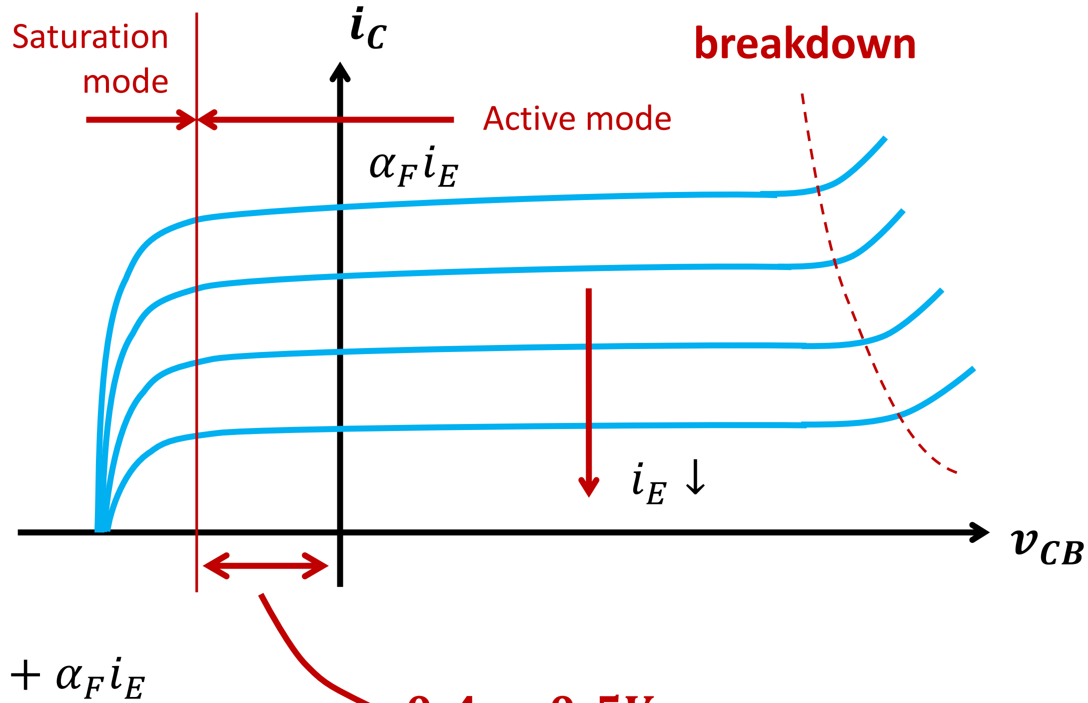


$$i_C = \left(\alpha_F - \frac{1}{\alpha_R}\right) I_S\left(e^{\frac{v_{BC}}{V_T}} - 1\right) + \alpha_F i_E$$

$i_C = \alpha_F i_E$ when $v_{CB} = 0$

$i_C - v_{CB}$ Characteristics





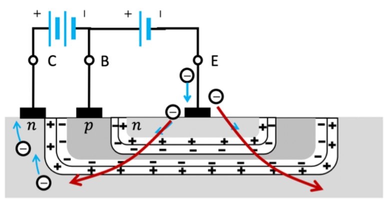
$$i_C = \left(\alpha_F - \frac{1}{\alpha_R}\right) I_S \left(e^{\frac{v_{BC}}{V_T}} - 1\right) + \alpha_F i_E$$

$0.4 - 0.5V$

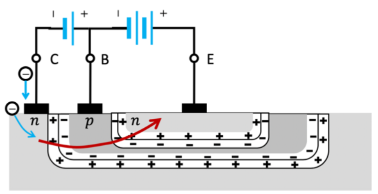
Apparent threshold

Outline

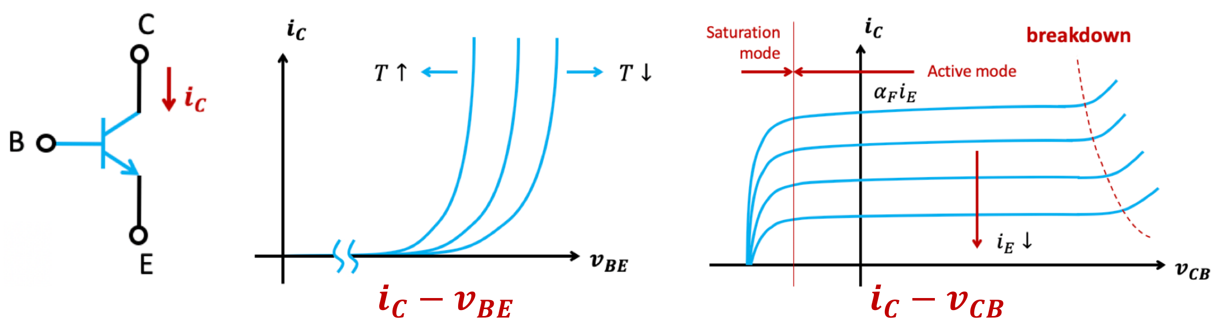
- Introduction to BJT
  - Device structure
  - How does it work?
    - Active mode
    - Reverse mode
    - Saturation mode
  - npn v.s. pnp
- The characteristic curves
  - $i - v$ characteristics



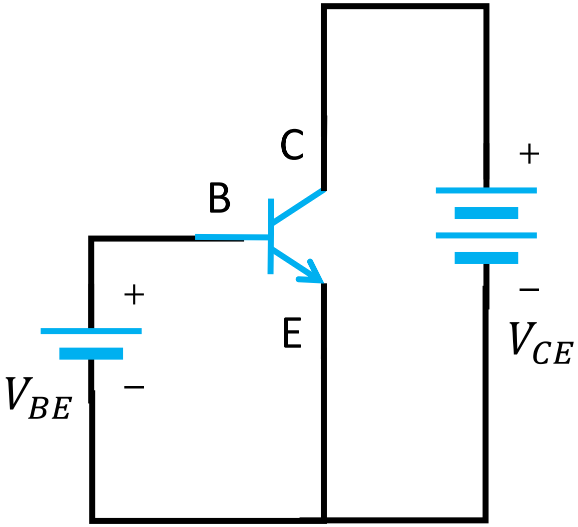
Active mode

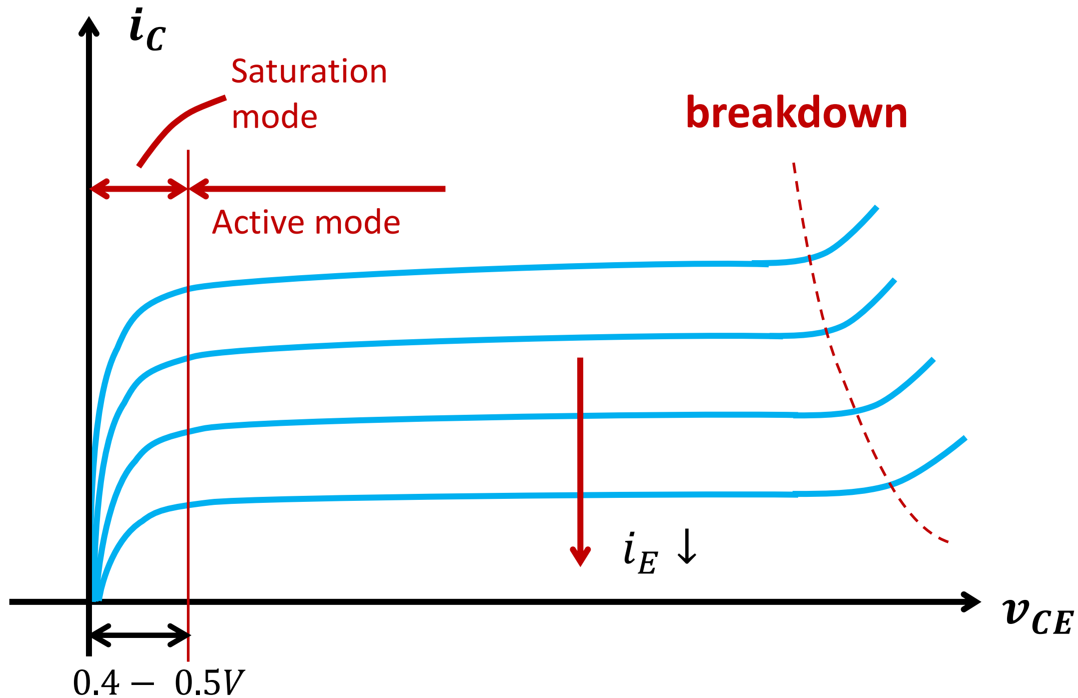


reverse mode

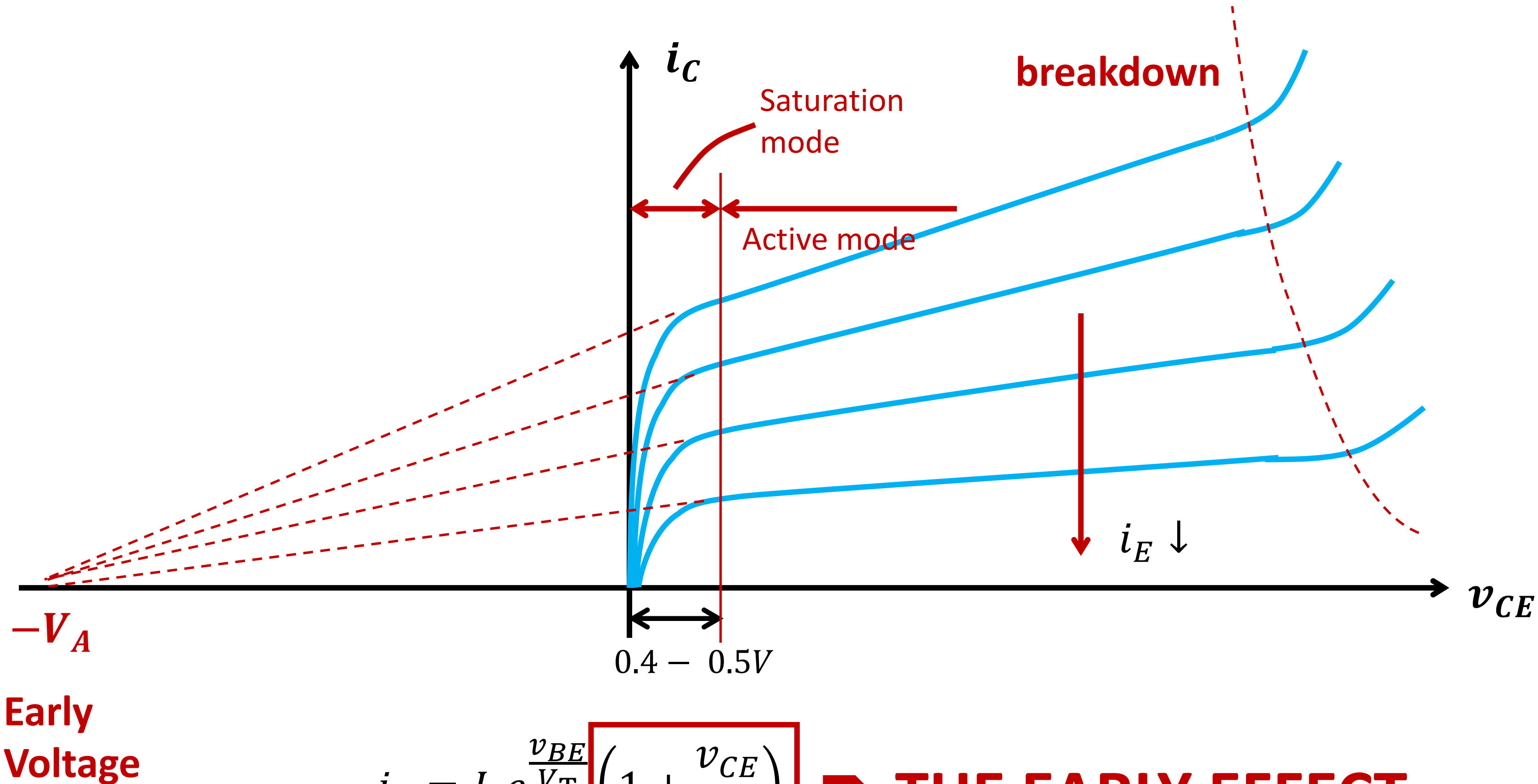


$i_C - v_{CE}$ Characteristics





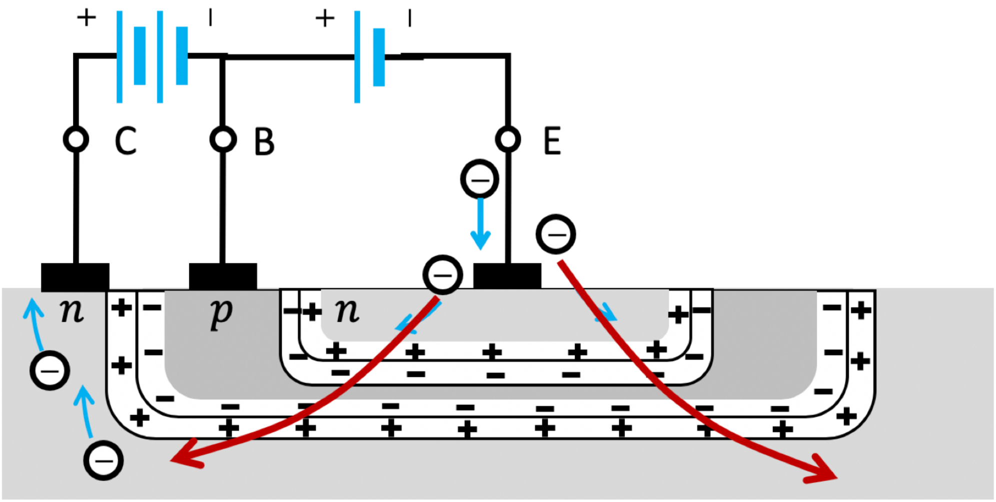
$i_C - v_{CE}$ Characteristics

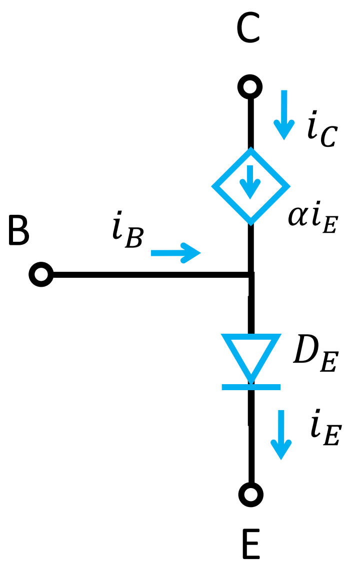


$$i_C = I_S e^{\frac{v_{BE}}{V_T}} \left(1 + \frac{v_{CE}}{V_A}\right)$$

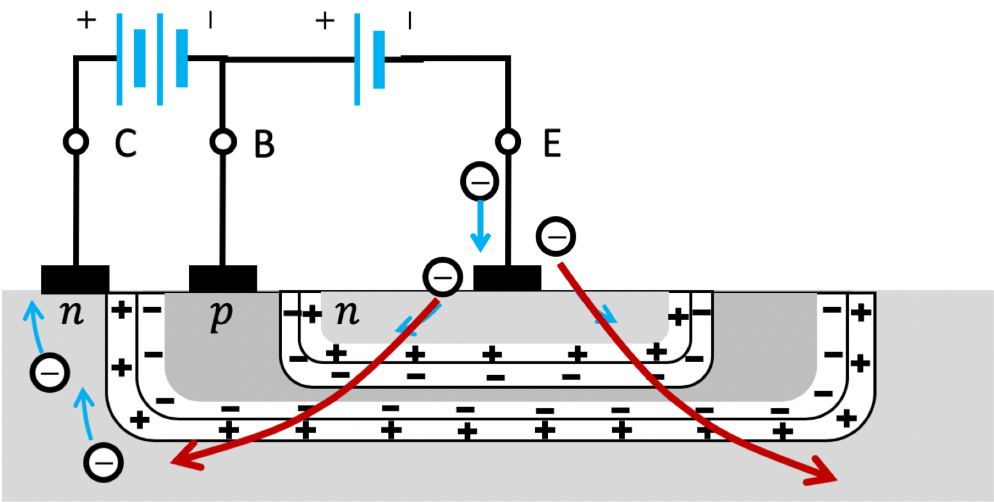
THE EARLY EFFECT

Recall: A Model for Active Mode





WHY the Early Effect



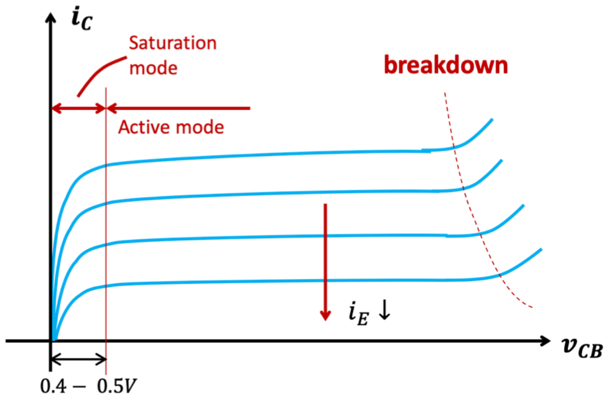
$$i_C = I_S e^{\frac{v_{BE}}{V_T}}$$

where $I_S = \frac{A_E q D_n n_i^2}{N_A W}$

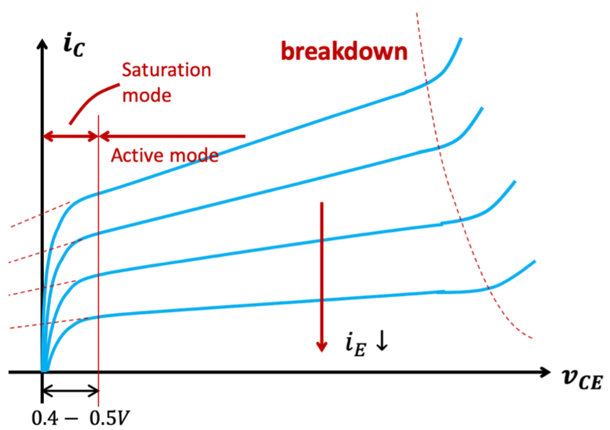
$v_{CE} \uparrow$ ➔ $v_{CB} \uparrow$ ➔ $W \downarrow$ ➔ $I_S \uparrow$ ➔ $i_C \uparrow$

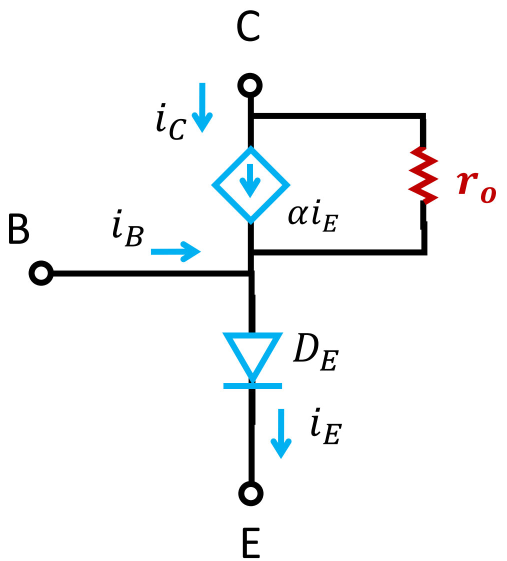
$i_C - v_{CE}$ Characteristics

Ideal case

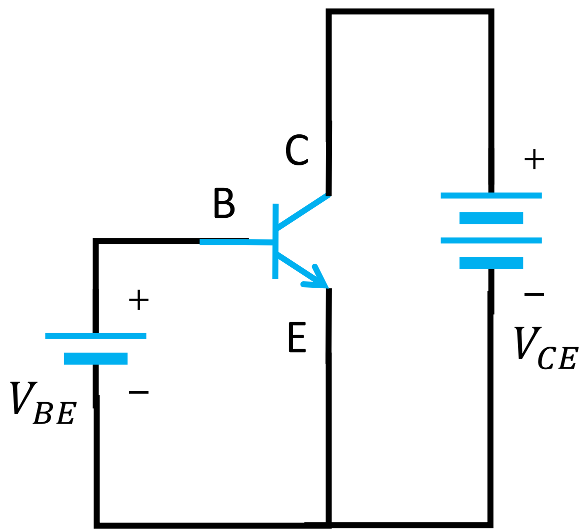


Consider Early Effect





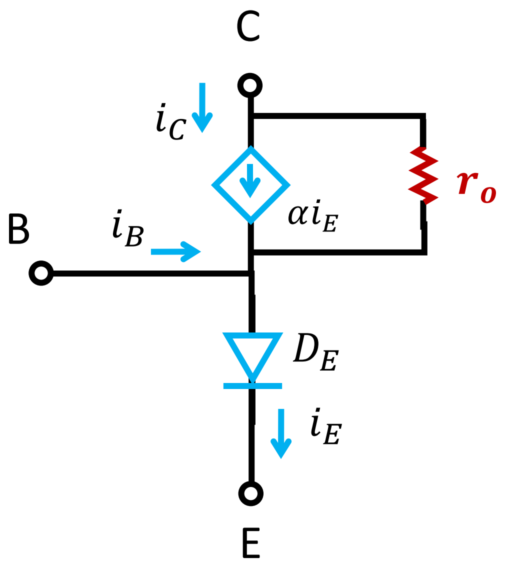
$i_C - v_{CE}$ Characteristics



$$i_C = I_S e^{\frac{v_{BE}}{V_T}}\left(1 + \frac{v_{CE}}{V_A}\right)$$

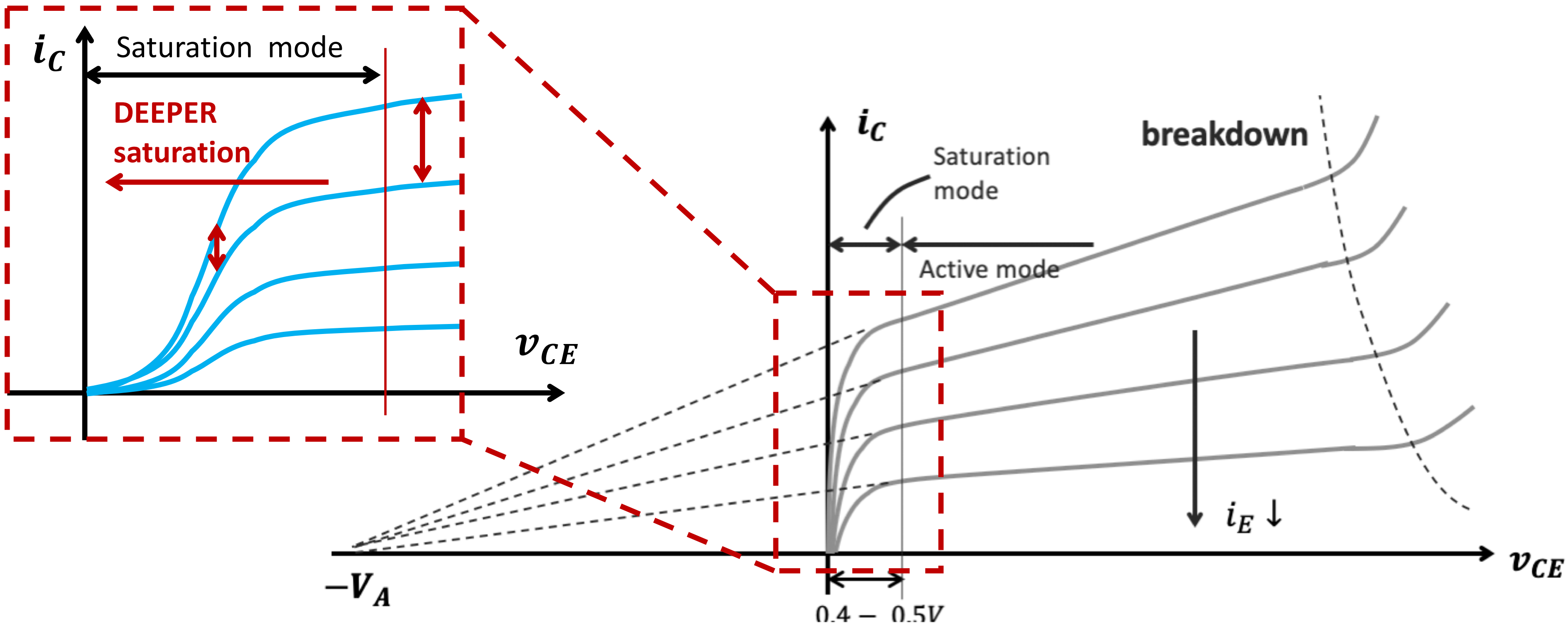
$$r_o \equiv \left[\left.\frac{\partial i_C}{\partial v_{CE}}\right|_{v_{BE}}\right]^{-1}$$

$$= \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$$



The nonzero slop of the $i_C - v_{CE}$ characteristics straight lines indicates that the output resistance looking into the collector is NOT infinite.

$i_C - v_{CE}$ Characteristics

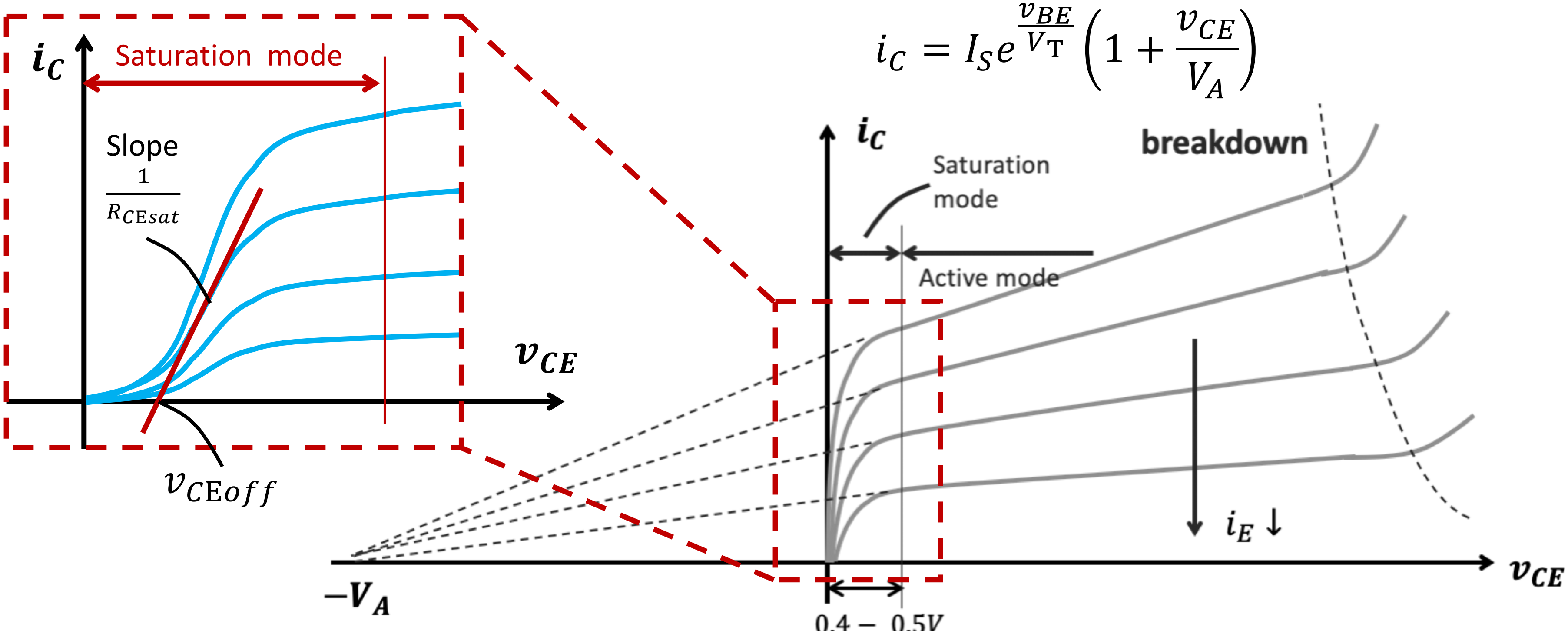


- We already have β_{DC} for large signal, defined as $\beta_{DC} = \frac{I_C}{I_B}$
- Define incremental β_{AC} as $\beta_{AC} = \left.\frac{\Delta i_C}{\Delta i_B}\right|_{v_{CE}}$

β_{AC} is smaller in saturation mode than in active mode

$i_C - v_{CE}$ Characteristics

$$i_C = I_S e^{\frac{v_{BE}}{V_T}}\left(1 + \frac{v_{CE}}{V_A}\right)$$



- The saturation voltage $v_{CEsat} = v_{CEoff} + I_{Csat}R_{CEsat}$

  Typical value $0.1 - 0.3V$

Where R_{CEsat} is defined as $R_{CEsat} \equiv \left.\frac{\partial v_{CE}}{\partial i_C}\right|_{\substack{i_B = I_B \\ i_C = I_{Csat}}}$

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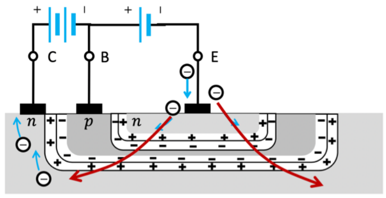
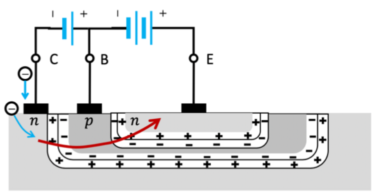
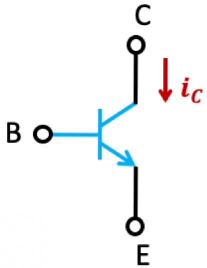


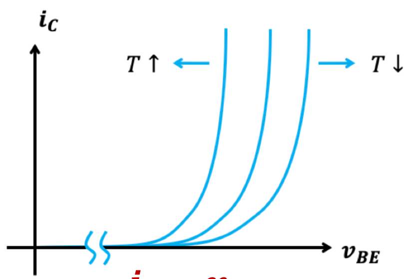
Active mode



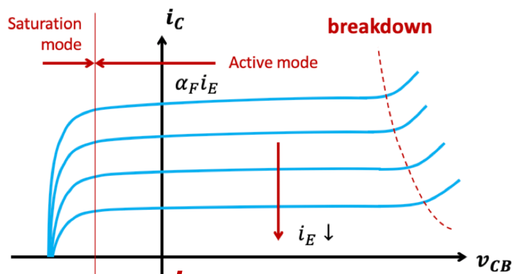
reverse mode

Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward
Reverse Active	Reverse	Forward

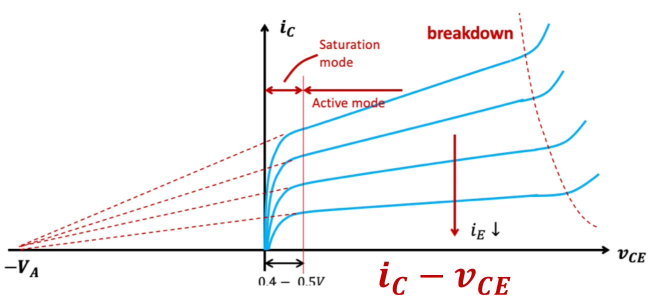




$i_C - v_{BE}$



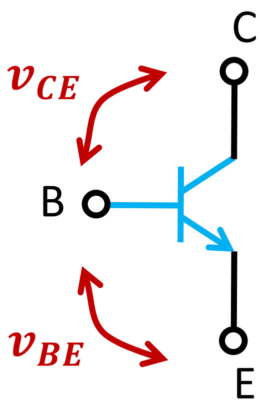
$i_C - v_{CB}$

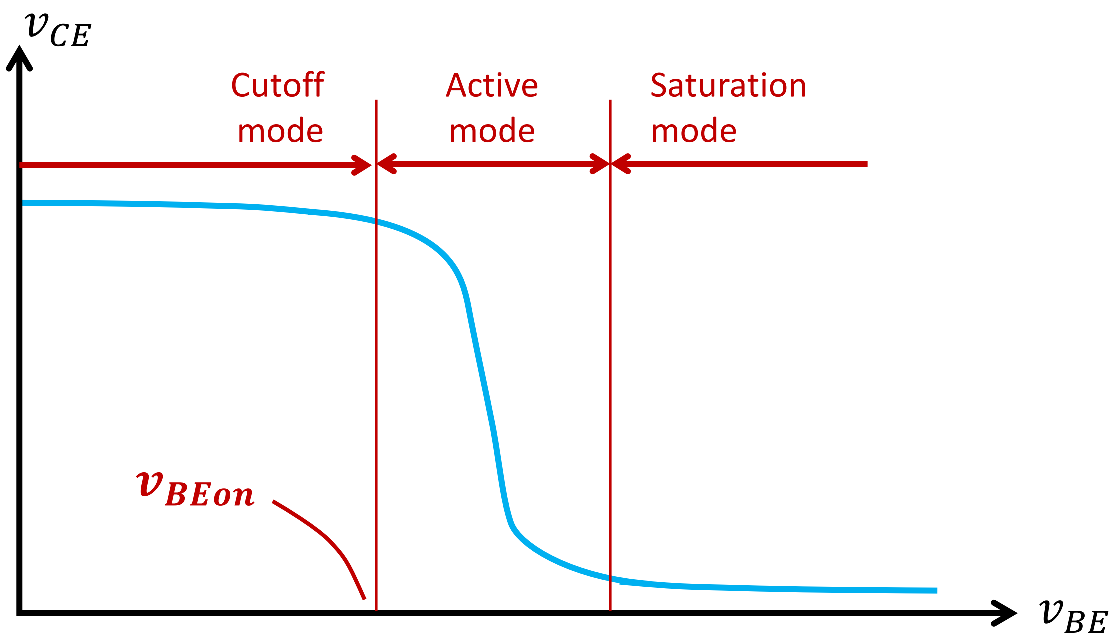


$i_C - v_{CE}$

  - **The transfer characteristic**

The Transfer Characteristic





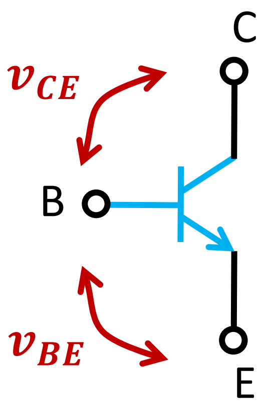
Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward
Reverse Active	Reverse	Forward

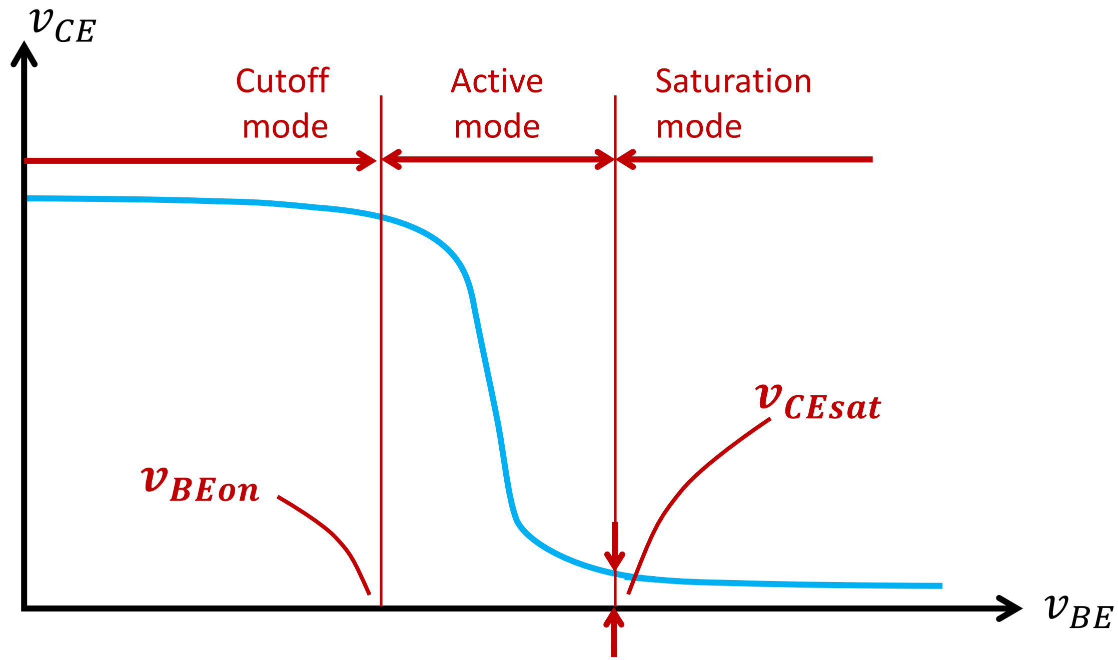
@ Cutoff mode

EBJ is off ➡ $v_{BE} < v_{BEon}$

Typical value of v_{BEon} is $0.5V$

The Transfer Characteristic





Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward
Reverse Active	Reverse	Forward

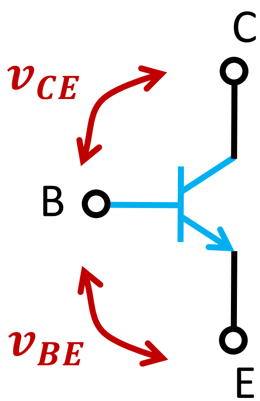
@ Saturation mode

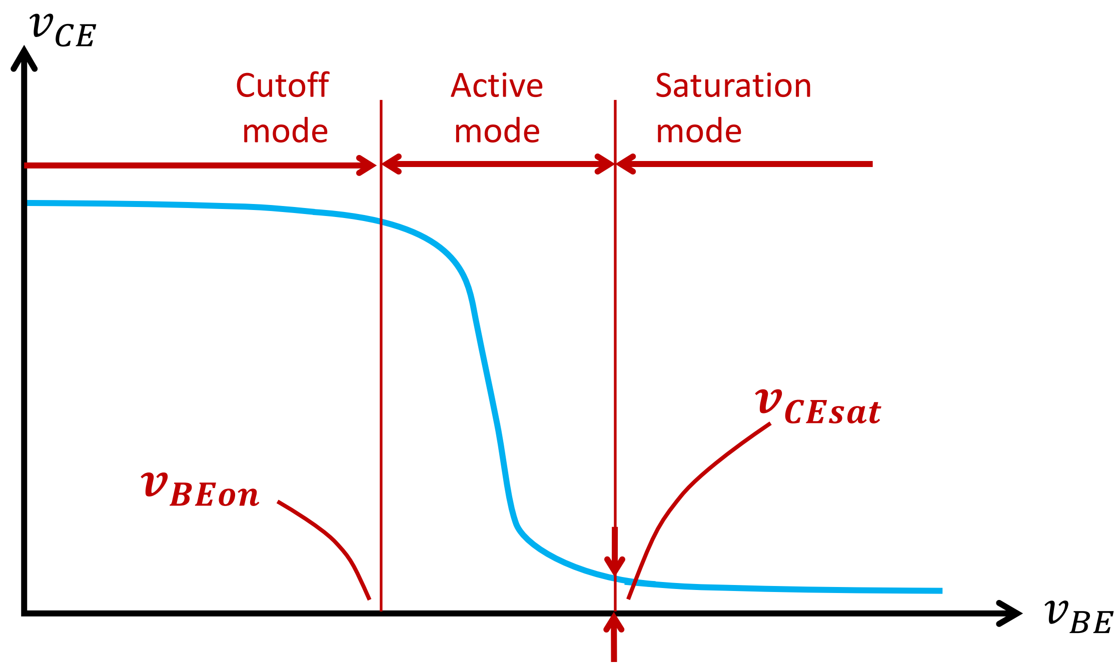
EBJ is on ➡ $v_{BE} > v_{BEon}$

CBJ is on ➡ $v_{BC} = v_{BE} - v_{CE} > v_{BCon}$

Typical value of v_{BCon} is $0.4V$

The Transfer Characteristic





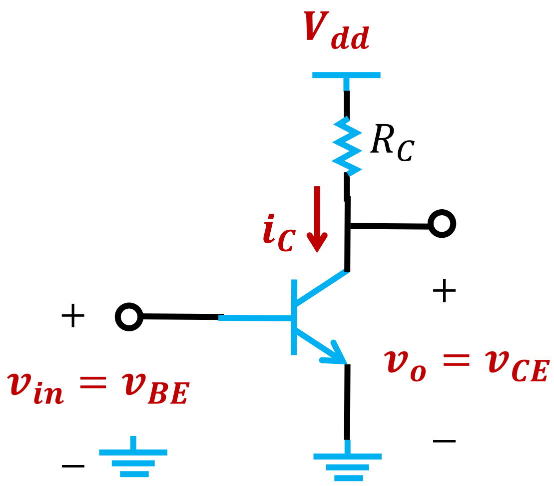
Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward
Reverse Active	Reverse	Forward

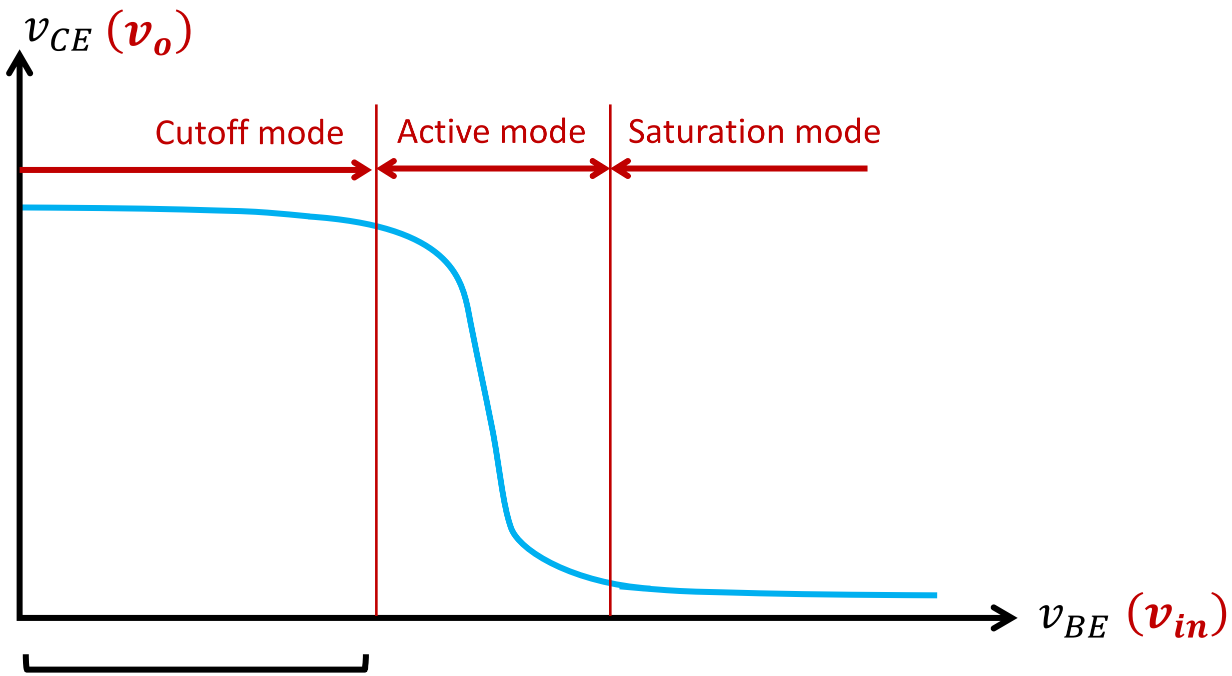
@ Active mode

EBJ is on ➡ $v_{BE} > v_{BEon}$

CBJ is off ➡ $v_{BC} = v_{BE} - v_{CE} < v_{BCon}$

The Transfer Characteristic



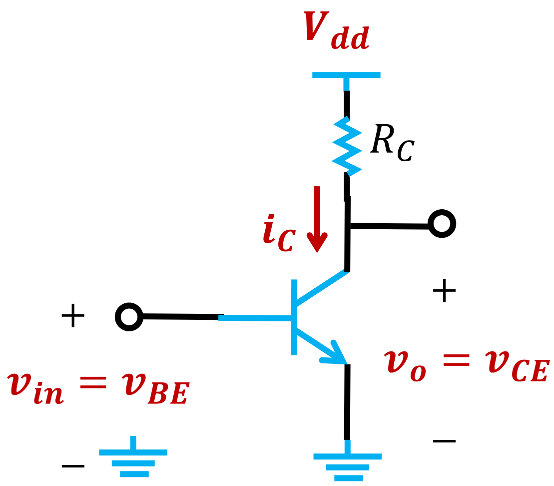


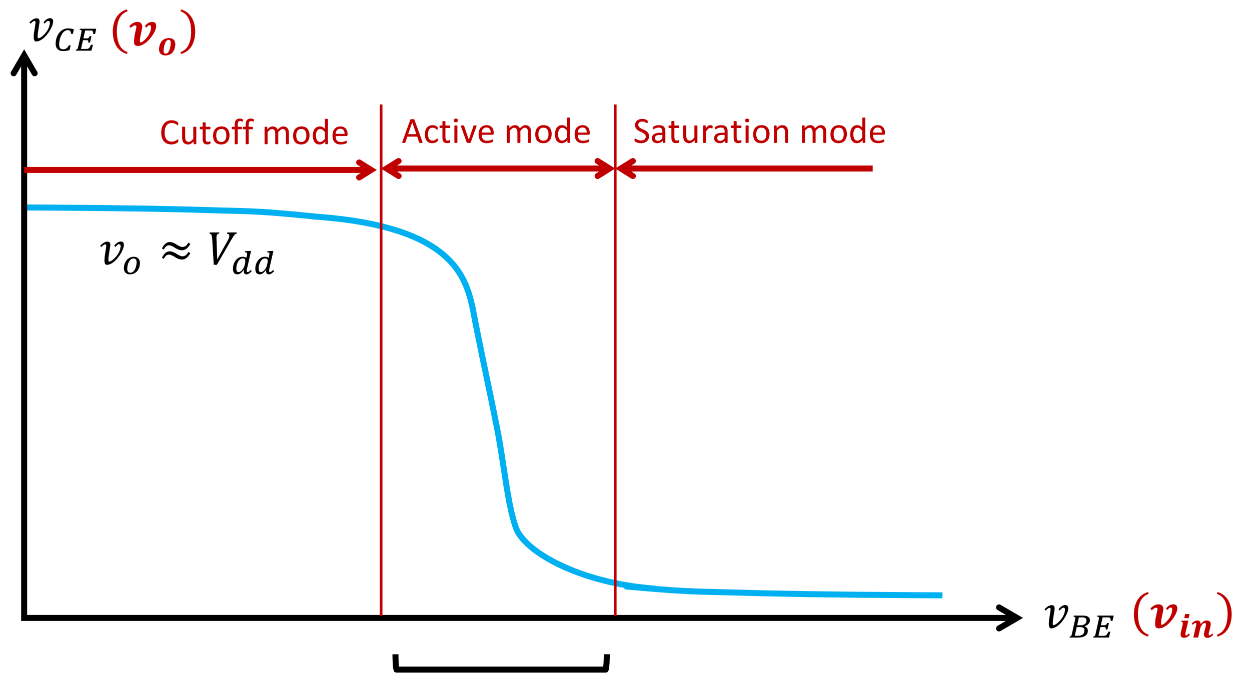
$v_{in} < v_{BEon}$

$i_C = 0$

$v_o = V_{dd} - i_C R_C = V_{dd}$

The Transfer Characteristic





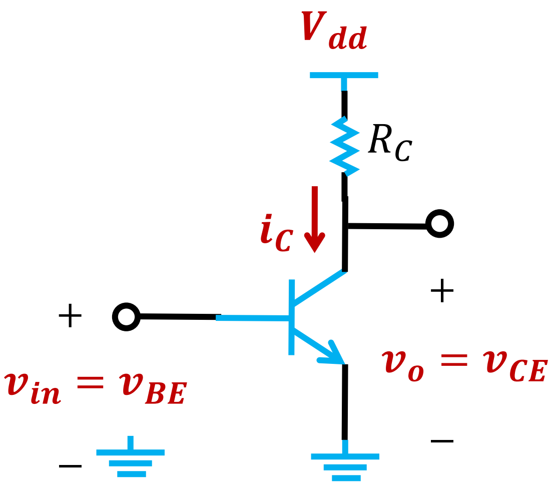
$$v_{in} > v_{BEon}$$

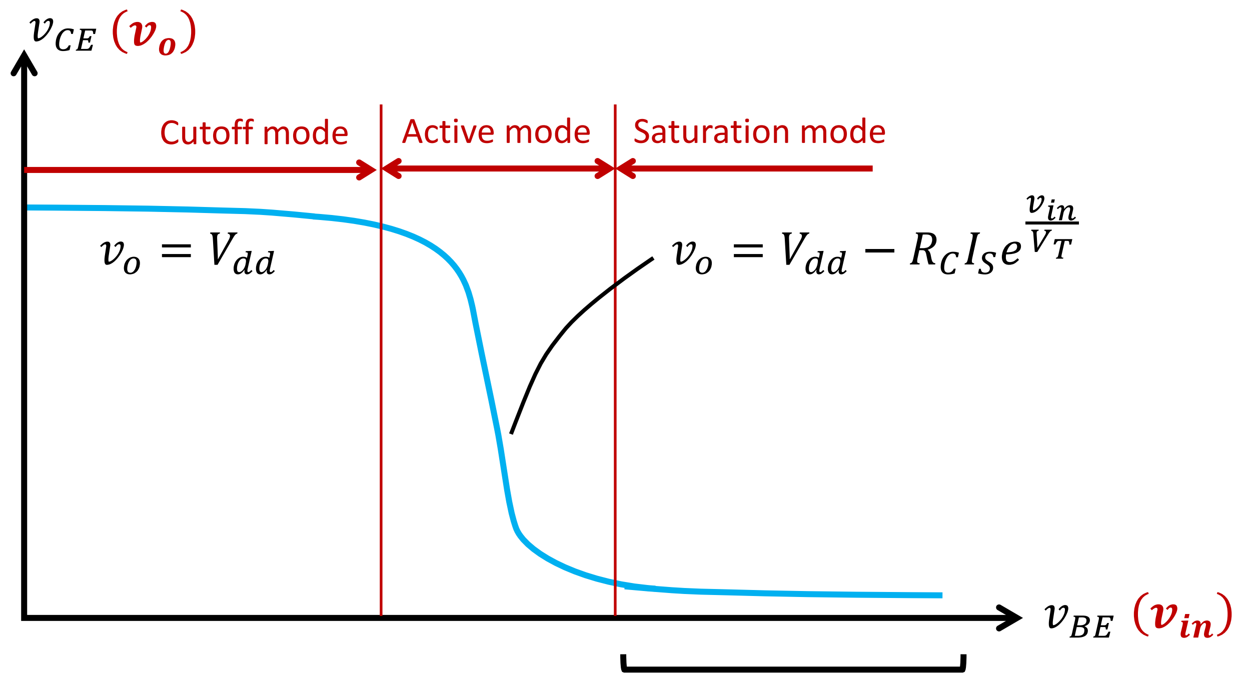
$$i_C = I_S e^{\frac{v_{BE}}{V_T}}$$

$$v_o = V_{dd} - i_C R_C$$

$$= V_{dd} - R_C I_S e^{\frac{v_{in}}{V_T}}$$

The Transfer Characteristic





$$v_o < v_{in} - v_{BCon}$$

$$v_{CE} = V_{CEsat}$$

$$I_{Csat} = \frac{V_{dd} - V_{CEsat}}{R_C}$$

Outline

- Introduction to BJT
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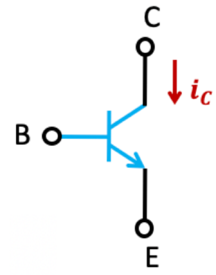
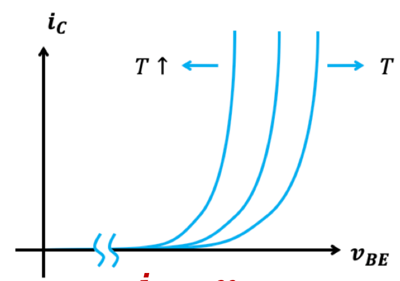
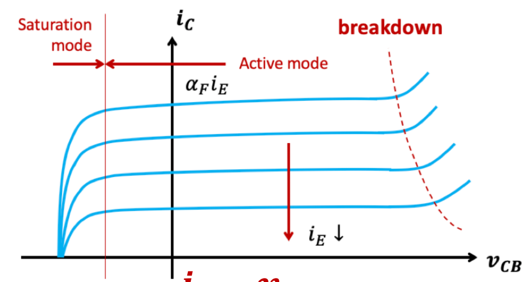
- The characteristic curves
  - $i - v$ characteristics
  - The transfer characteristic
- **Circuit analysis techniques with BJT**
  - **DC analysis techniques**



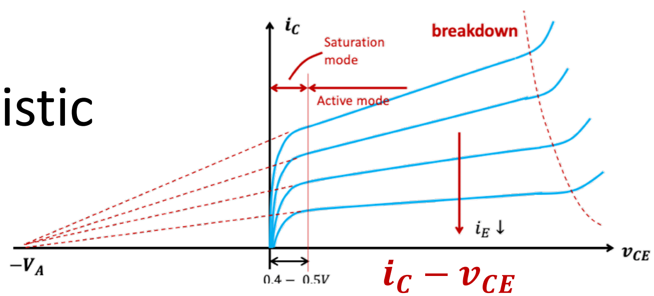
Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward
Reverse Active	Reverse	Forward



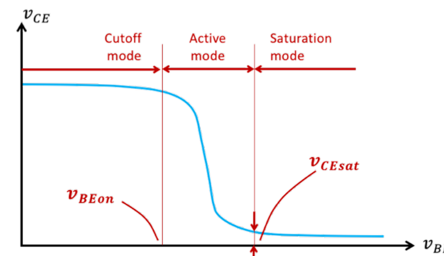
$i_C - v_{BE}$



$i_C - v_{CB}$

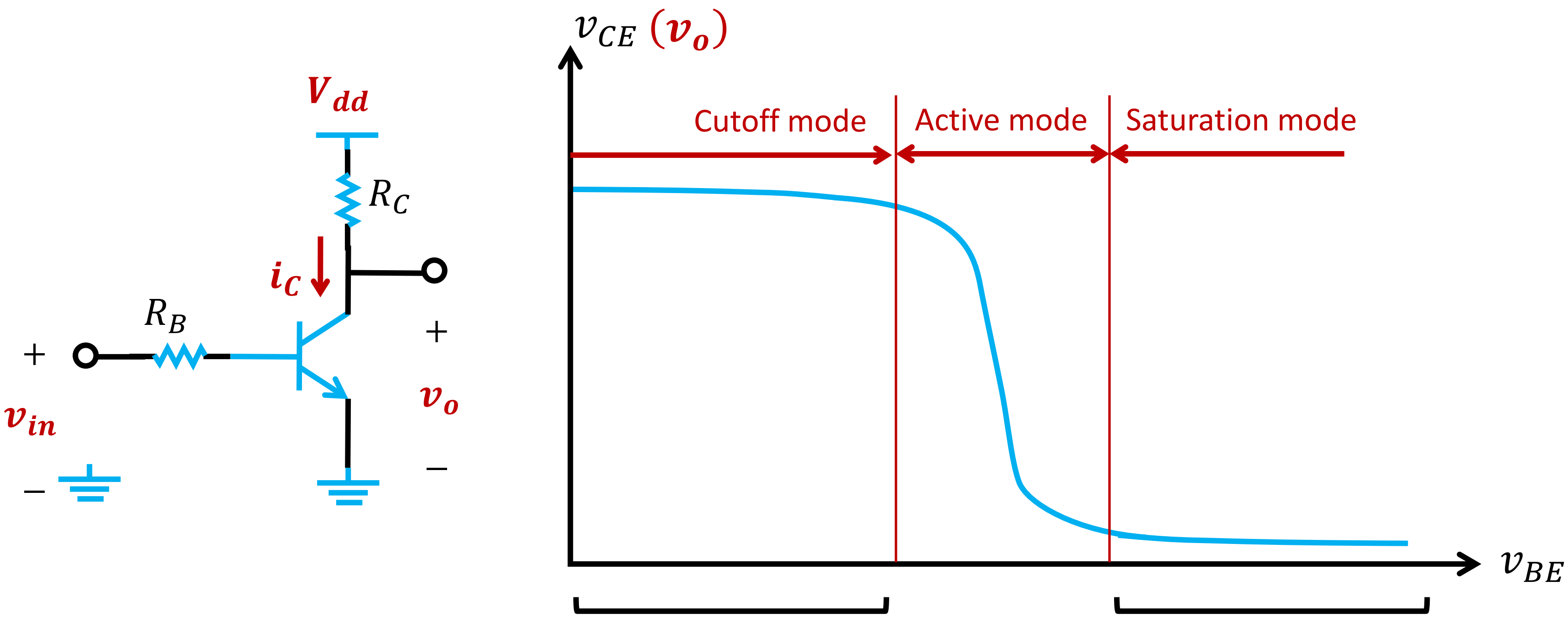


$i_C - v_{CE}$



$v_{BE} - v_{CE}$

Recall: Cutoff/Saturation Mode



$v_{in} < v_{BEon}$

$i_C = 0$

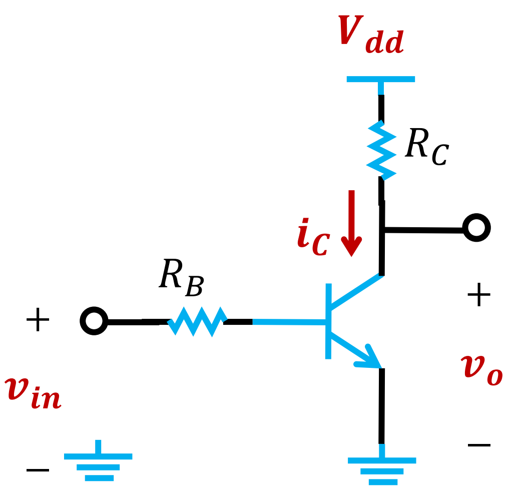
$v_o = V_{dd} - i_C R_C = V_{dd}$

$v_o < v_{in} - v_{BCon}$

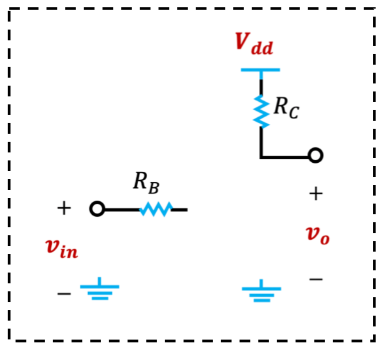
$v_{CE} = V_{CEsat}$

$$I_{Csat} = \frac{V_{dd} - V_{CEsat}}{R_C}$$

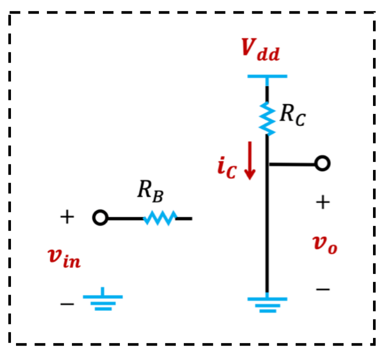
Transistor in Cutoff/Saturation Mode

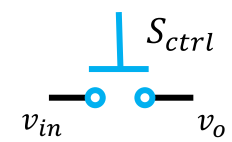


- Cutoff mode



- Saturation mode





$$v_o = v_{in}$$

When $S_{ctrl} = V_H$

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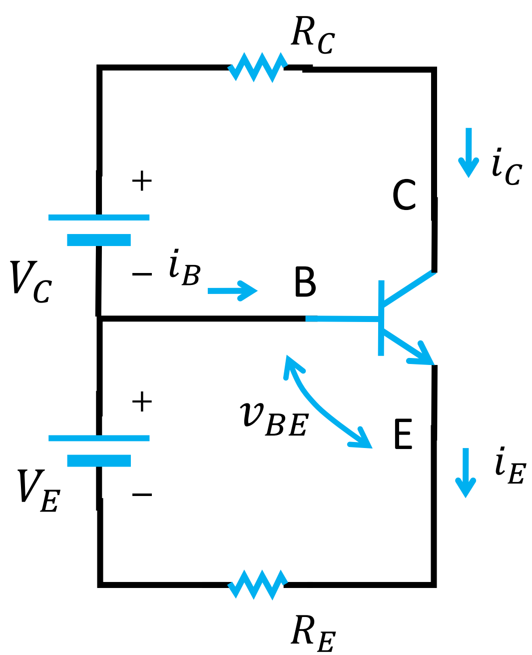
  - The transfer characteristic
- Circuit analysis techniques with BJT
  - DC analysis techniques
    - Transistor in cutoff/saturation mode
    - **Transistor in active mode**

Recall: Example 1

QUESTION: The transistor has $\beta = 100$ and exhibits a v_{BE} of $0.7V$ at $i_C = 1mA$. Find the resistance of R_C and R_E when the transistor is biased in active region at $i_C = 2mA$ and $v_{CB} = 5V$ with $V_C = V_E = 15V$



$$V_C = i_C R_C + v_{CB} \quad \rightarrow \quad R_C = 5k\Omega$$

Since $i_C = 1mA$ @ $v_{BE} = 0.7V$

$$v_{BE}\Big|_{i_C=2mA} = 0.7 + V_T In\left(\frac{2mA}{1mA}\right) = 0.717V$$

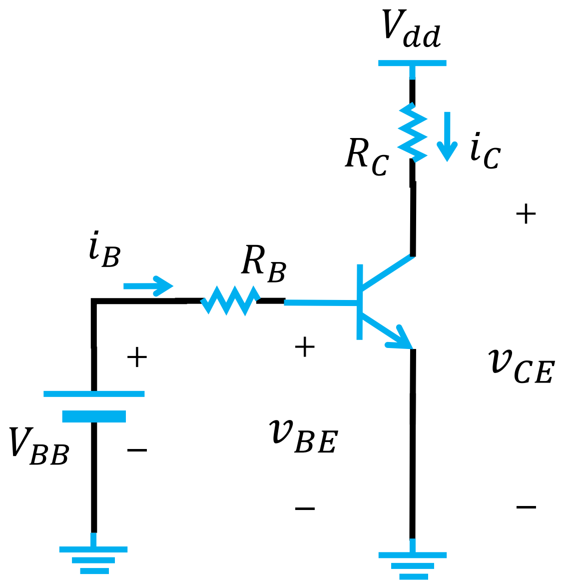
$$V_E = i_E R_E + v_{BE} \quad \rightarrow \quad i_E R_E = 15.717V$$

$$i_E = \frac{i_C}{\alpha} = 2.02mA$$ where $\alpha = \frac{\beta}{\beta+1} = 0.99$

$$R_E = 7.07k\Omega$$

Example 2

QUESTION: Find out the voltage of V_{BE} and V_{CE} with $R_B = 100k\Omega$, $R_C = 2k\Omega$, V_{dd}=10V, $V_{BB} = 5V$, and $\beta = 100$.



- According to KVL

$$V_{BB} = i_B R_B + v_{BE}$$

- According to $\boldsymbol{i_C - v_{BE}}$ characteristics $\quad i_C = I_S e^{\frac{v_{BE}}{V_T}}$

$$V_E = \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}} R_B + v_{BE}$$

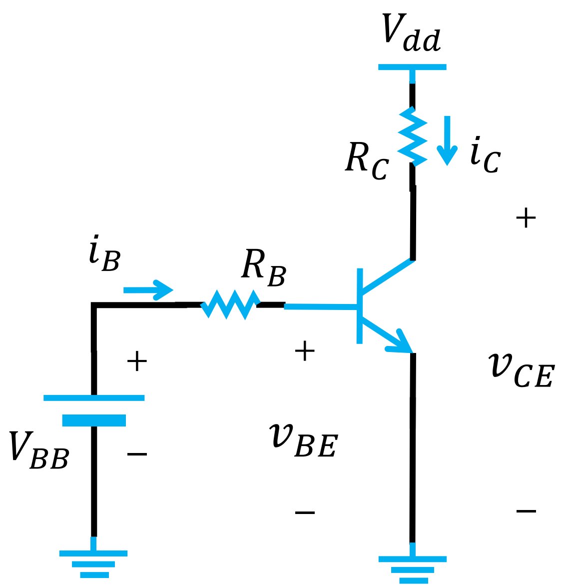
1 unknown in 1 equation

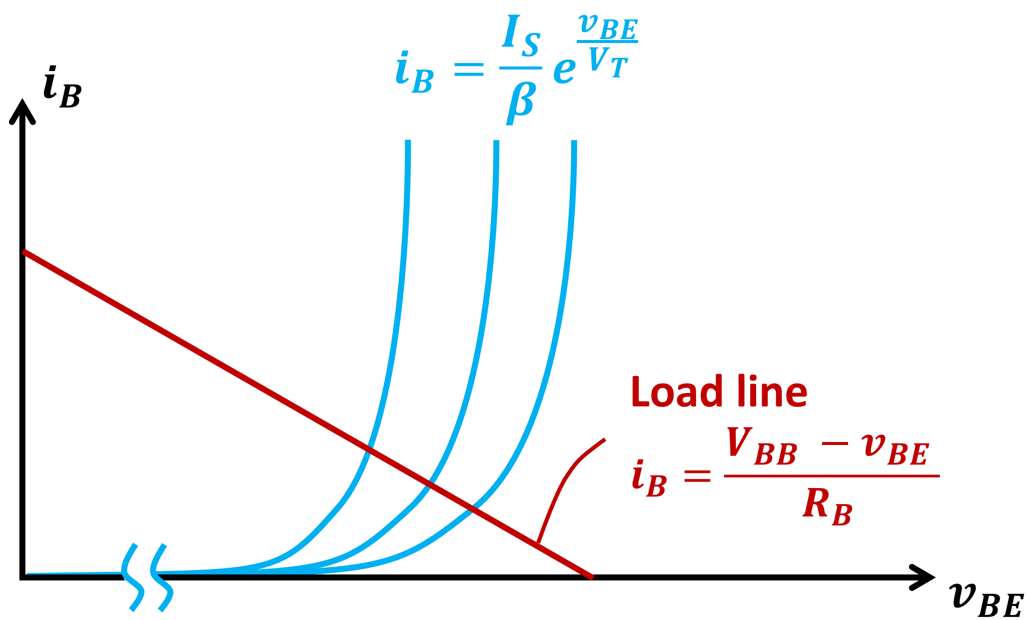
BUT WE CANNOT FIND AN ANALYTICAL SOLUTION

NUMERICAL SOLUTION AVAILABLE IN 20230253

Example 2

QUESTION: Find out the voltage of V_{BE} and V_{CE} with $R_B = 100k\Omega$, $R_C = 2k\Omega$, V_{dd}=10V, $V_{BB} = 5V$, and $\beta = 100$.

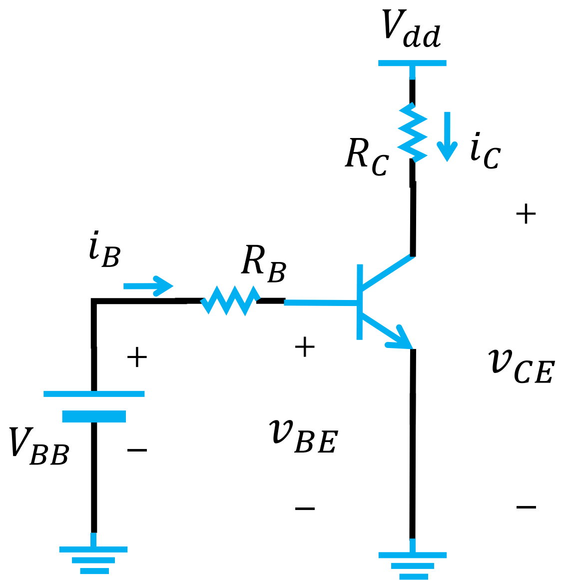


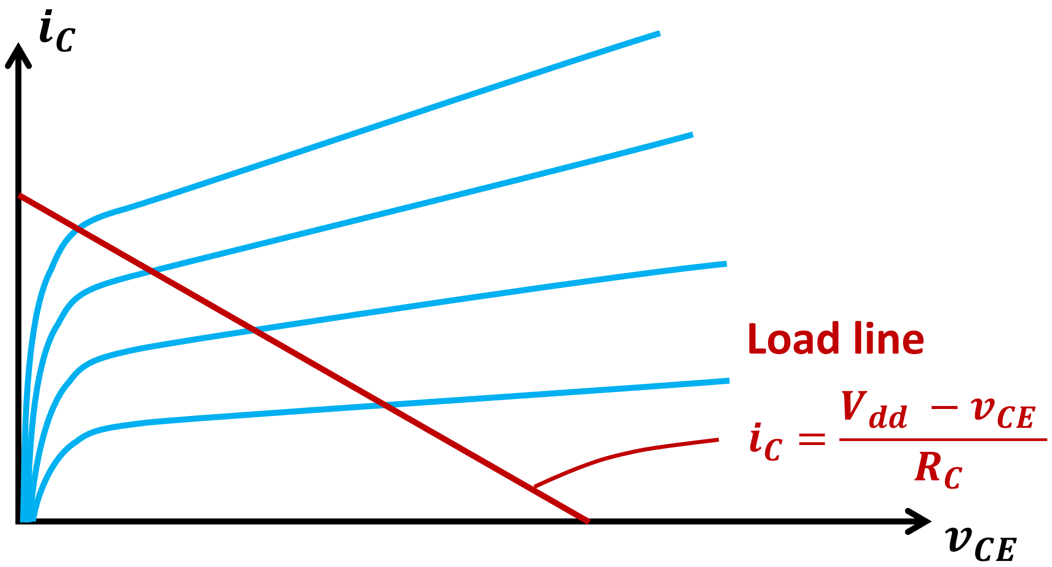


- According to KVL $\quad V_{BB} = i_B R_B + v_{BE}$

Example 2

QUESTION: Find out the voltage of V_{BE} and V_{CE} with $R_B = 100k\Omega$, $R_C = 2k\Omega$, V_{dd}=10V, $V_{BB} = 5V$, and $\beta = 100$.





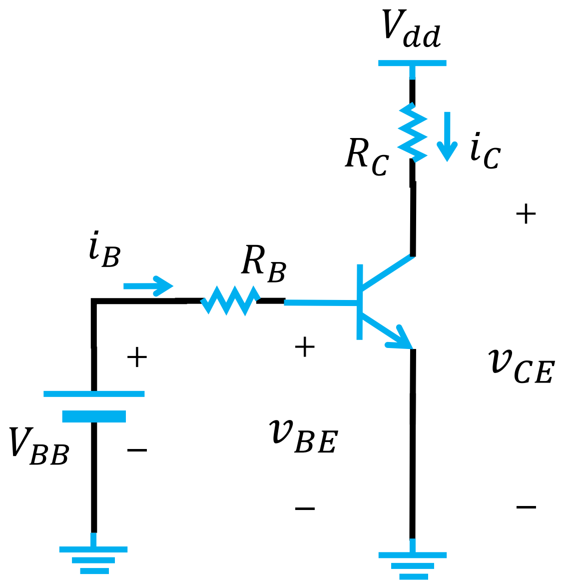
The graphical analysis tech is NOT quantitative.

- Find the load line according to KVL

$$V_{dd} = i_C R_C + v_{CE}$$

Example 2

QUESTION: Find out the voltage of V_{BE} and V_{CE} with $R_B = 100k\Omega$, $R_C = 2k\Omega$, V_{dd}=10V, $V_{BB} = 5V$, and $\beta = 100$. **Assume $|V_{BE}| = 0.7V$ in active mode.**



- Let's **ASSUME** the transistor is biased in **active mode**
- According to characteristics of the transistor

$$V_{BE} \approx 0.7V$$

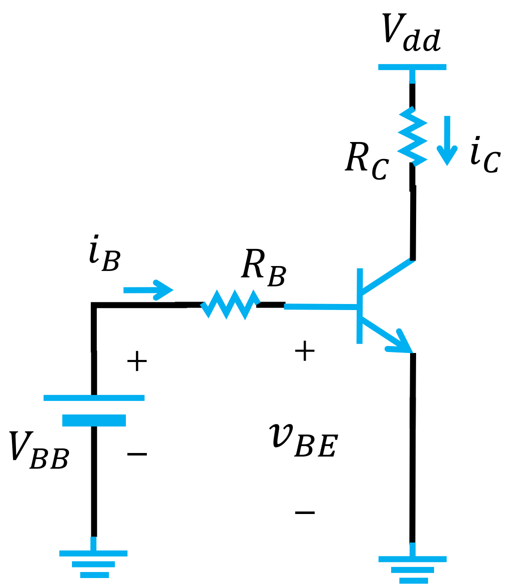
- According to KVL & transistor characteristics

$$\begin{cases} V_{BB} = i_B R_B + V_{BE} \\ V_{dd} = i_C R_C + V_{CE} \\ i_C = \beta i_B = 4.3mA \end{cases} \Rightarrow \begin{cases} i_B = 0.043mA \\ i_C = 4.3mA \\ V_{CE} = 1.4V \end{cases}$$

- **Check ASSUMPTION**

$$V_{BC} = -0.7V \quad < V_{BCon} \Rightarrow$$ **Active mode**

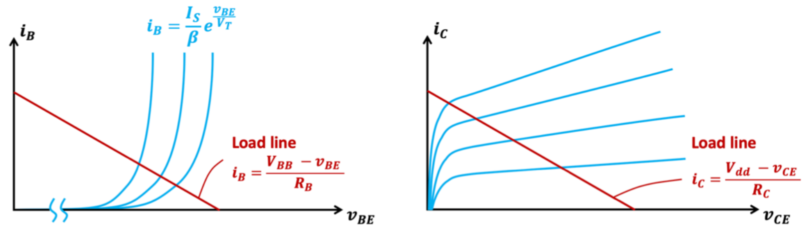
Summary: DC analysis for BJT



- **Method 1 – quantitative analysis**

$$V_E = \frac{I_S}{\alpha} e^{\frac{v_{BE}}{V_T}} R_E + v_{BE}$$ ☹ No analytical solution

- **Method 2 – graphical analysis**

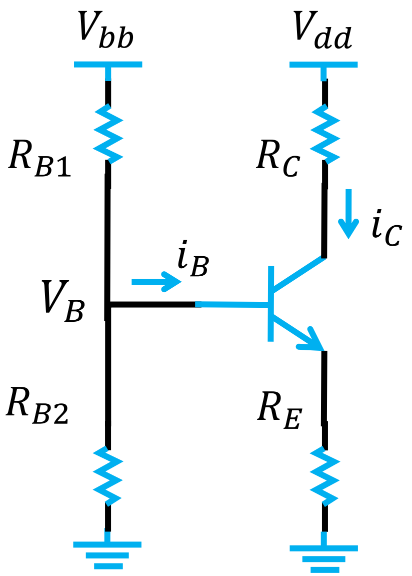


☺ Easy to operate
☹ not a quantitative result

- **Method 3 – $|V_{BE}| \approx V_{th} = 0.7V$ in active mode**
  - Step 1: assume the transistor is in active region
  - Step 2: solve the circuit
  - Step 3: check assumption

Example 3

QUESTION: Find out the voltage of V_{BE} and V_{CE} with $R_{B1} = 100k\Omega$, $R_{B2} = 50k\Omega$, $R_C = 5k\Omega$, $R_E = 3k\Omega$, $V_{dd} = V_{BB} = 15V$, and $\beta = 100$. Assume $|V_{BE}| = 0.7V$ in active mode.



- Assume the transistor is biased in active region

$$i_C = \beta i_B \qquad V_{BE} \approx V_{th} = 0.7V$$

- According to KVL & KCL

$$\begin{cases} \dfrac{V_{bb} - V_B}{R_{B1}} = \dfrac{V_B}{R_{B2}} + i_B \\ V_B = V_{BE} + (\beta + 1) i_B R_E \end{cases}$$

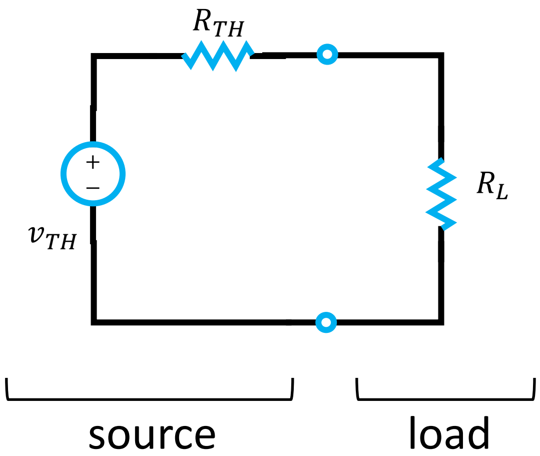
2 unknown in 2 equations

ANY better way?

Recall: Circuit equivalent

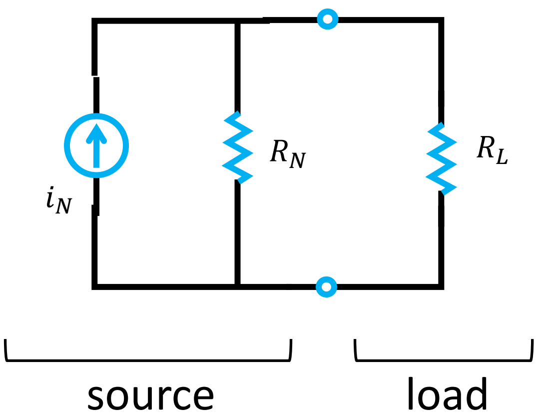
Thévenin's theorem

LINEAR two-terminal circuit can be replaced by an equivalent circuit composed of a voltage source and a series resistor



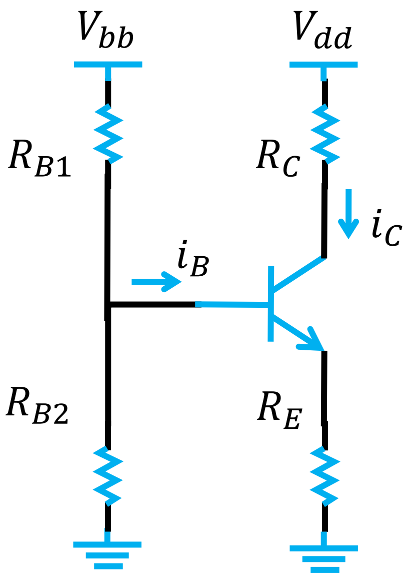
Norton's theorem

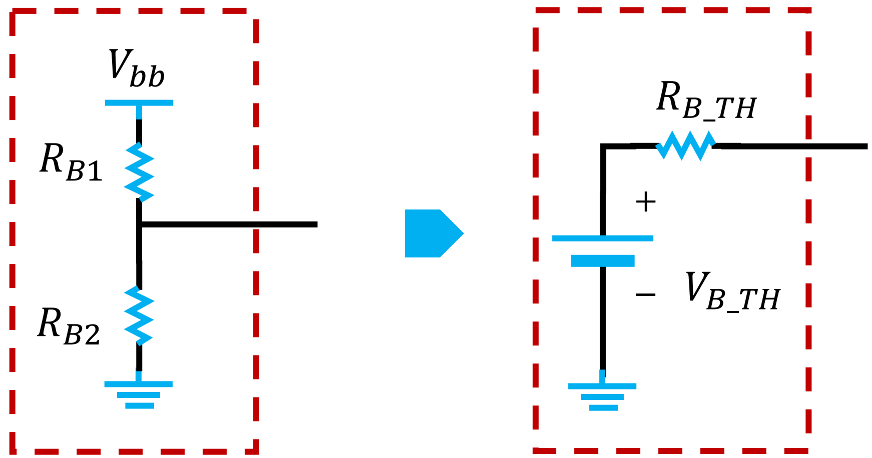
LINEAR two-terminal circuit can be replaced by an equivalent circuit composed of a current source and a parallel resistor



Example 3

QUESTION: Find out the voltage of V_{BE} and V_{CE} with $R_{B1} = 100k\Omega$, $R_{B2} = 50k\Omega$, $R_C = 5k\Omega$, $R_E = 3k\Omega$, $V_{dd} = V_{BB} = 15V$, and $\beta = 100$. Assume $|V_{BE}| = 0.7V$ in active mode.





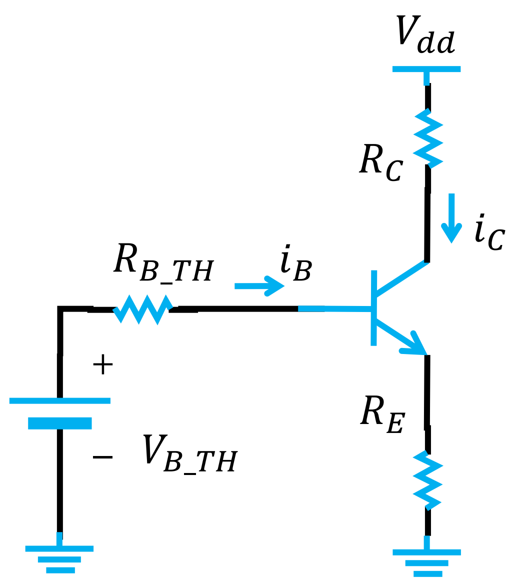
- According to Thévenin's theorem

$$R_{B\_TH} = R_{B1}||R_{B2} = 33.3k\Omega$$

$$V_{B\_TH} = \frac{R_{B2}}{R_{B1} + R_{B2}} V_{dd} = 5V$$

Example 3

QUESTION: Find out the voltage of V_{BE} and V_{CE} with $R_{B1} = 100k\Omega$, $R_{B2} = 50k\Omega$, $R_C = 5k\Omega$, $R_E = 3k\Omega$, $V_{dd} = V_{BB} = 15V$, and $\beta = 100$. Assume $|V_{BE}| = 0.7V$ in active mode.



$$\begin{cases} R_{B\_TH} = 33.3k\Omega \\ V_{B\_TH} = 5V \end{cases}$$

- Assume the transistor is biased in active region

$$i_C = \beta i_B \qquad V_{BE} \approx V_{th} = 0.7V$$

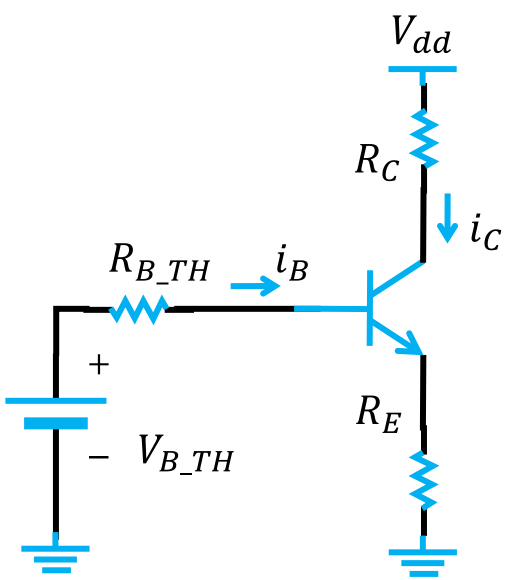
- According to KVL & KCL

$$\begin{cases} V_{B\_TH} = i_B R_{B\_TH} + V_{BE} + i_E R_E \\ V_C = V_{dd} - i_C R_C \\ i_E = i_B + i_C = (\beta + 1)\, i_B \end{cases}$$

$$\Rightarrow \begin{cases} i_B = 0.0128mA \\ V_C = 8.6V \\ V_E = i_E R_E = 3.88V \end{cases}$$

Example 3

QUESTION: Find out the voltage of V_{BE} and V_{CE} with $R_{B1} = 100k\Omega$, $R_{B2} = 50k\Omega$, $R_C = 5k\Omega$, $R_E = 3k\Omega$, $V_{dd} = V_{BB} = 15V$, and $\beta = 100$. Assume $|V_{BE}| = 0.7V$ in active mode.



$$\begin{cases} R_{B\_TH} = 33.3k\Omega \\ V_{B\_TH} = 5V \end{cases}$$

- The voltage of V_{BE} and V_{CE} can be calculated as

$$V_{BE} = V_{B\_TH} - i_B R_{B_{TH}} - V_E = 0.694V$$

$$V_{CE} = V_{dd} - i_C R_C - V_E = -3.664V$$

- **Check ASSUMPTION**

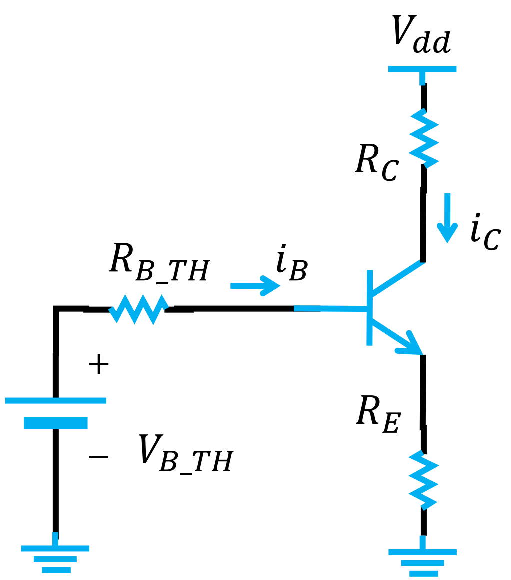
$$V_{BE} = 0.694V \quad > V_{BEon}$$

$$V_{BC} = -3.664V \quad < V_{BCon}$$

Active mode

Example 3

QUESTION: Find out the voltage of V_{BE} and V_{CE} with $R_{B1} = 100k\Omega$, $R_{B2} = 50k\Omega$, $R_C = 5k\Omega$, $R_E = 3k\Omega$, $V_{dd} = V_{BB} = 15V$, and $\beta = 100$. Assume $|V_{BE}| = 0.7V$ in active mode.



$$\begin{cases} R_{B\_TH} = 33.3k\Omega \\ V_{B\_TH} = 5V \end{cases}$$

The ASSUMPTION

$i_C = \beta i_B$

$V_{BE} \approx V_{th} = 0.7V$

DC analysis results

$V_{BE} = 0.694V$

$V_{BC} = -3.664V$

WHY the voltage of V_{BE} is different?

- A more precise method

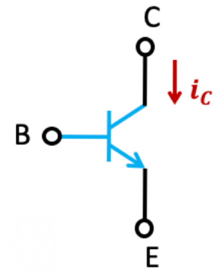
$$V_{B\_TH} = \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}} R_{B\_TH} + V_{BE} + \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}} R_E$$

1 unknown in 1 equation

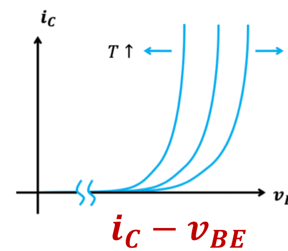
BUT WE CANNOT FIND AN ANALYTICAL SOLUTION

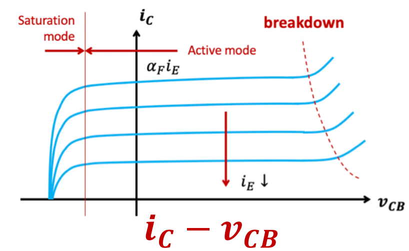
Outline

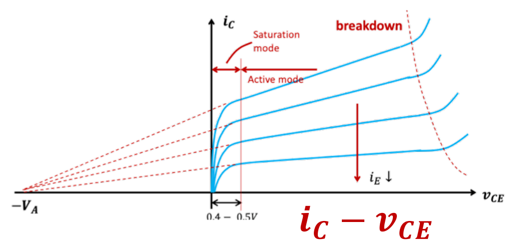
- Introduction to BJT
  - Device structure
  - How does it work?
    - Cutoff / Active / Reverse / Saturation mode
  - npn v.s. pnp
- The characteristic curves
  - $i - v$ characteristics
  - The transfer characteristic
- Circuit analysis techniques with BJT
  - DC analysis techniques
    - **Method 1 – quantitative analysis**
    - **Method 2 – graphical analysis**
    - **Method 3 – $|V_{BE}| \approx V_{th} = 0.7V$ in active mode**

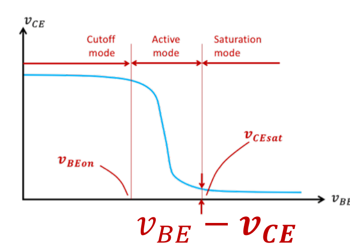


Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward
Reverse Active	Reverse	Forward









Outline

- Introduction to BJT
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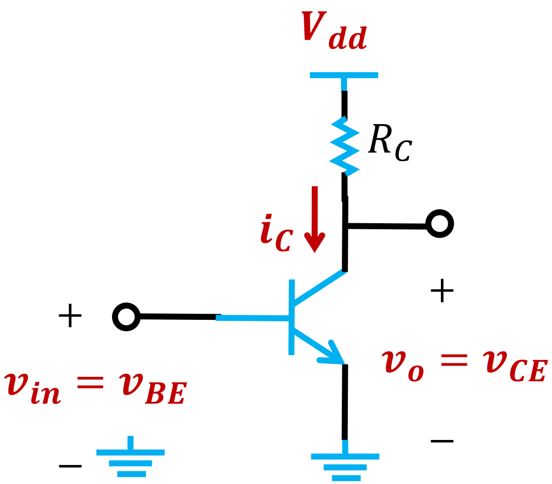
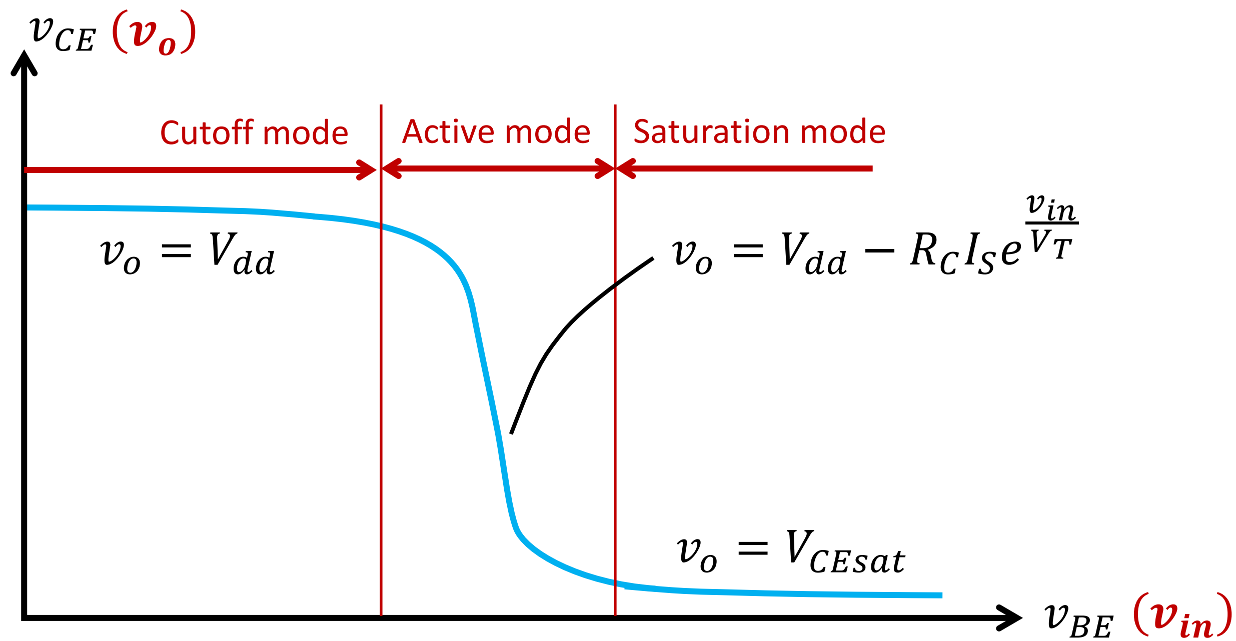
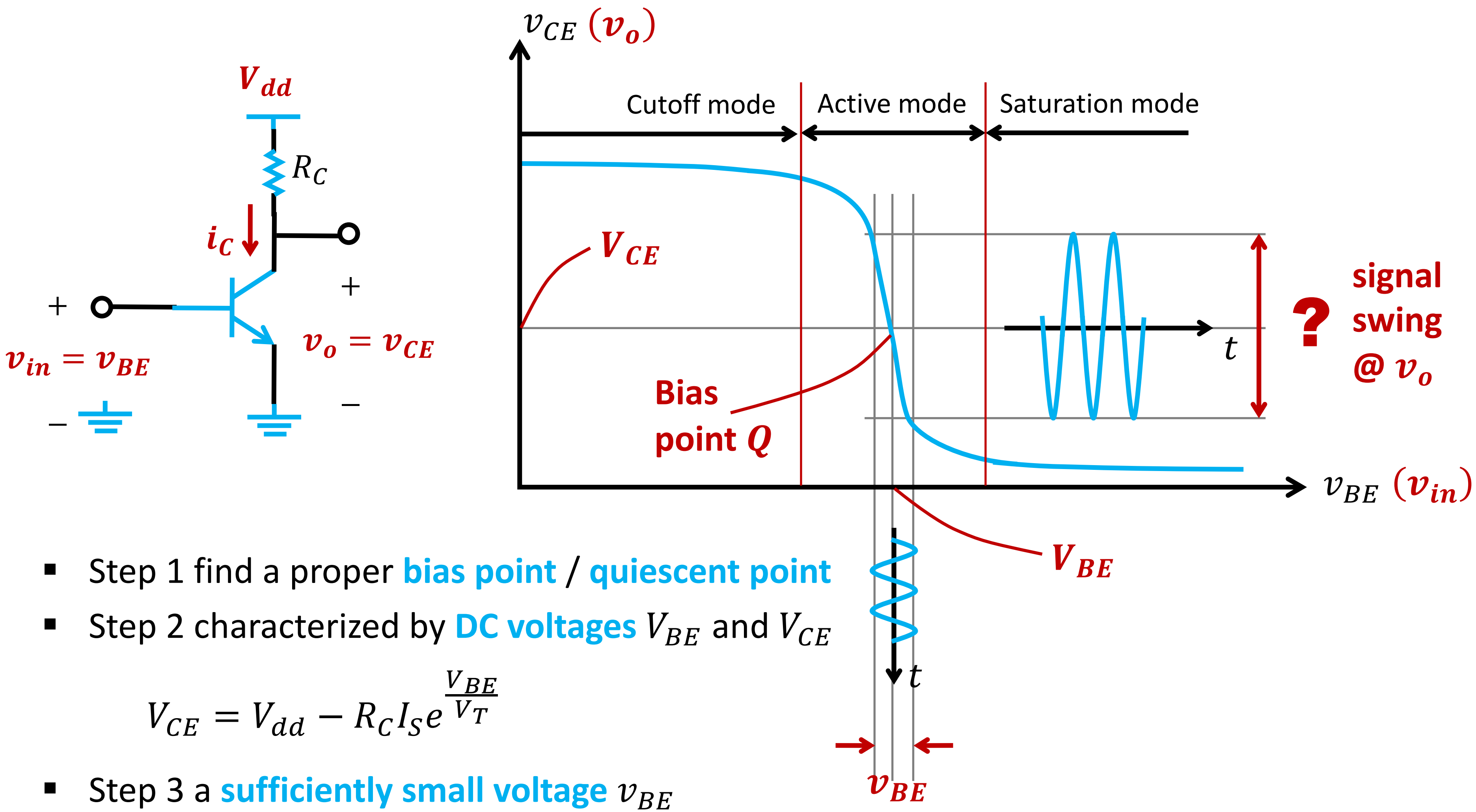
- Circuit analysis techniques with BJT
  - DC analysis techniques
  - **AC analysis techniques**

Recall: the Transfer Characteristic





Signal Amplification

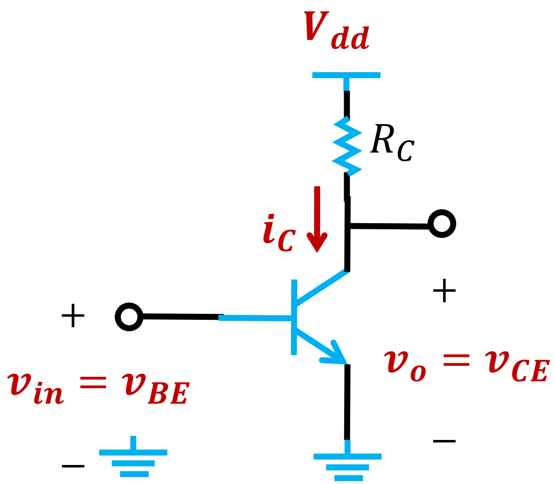


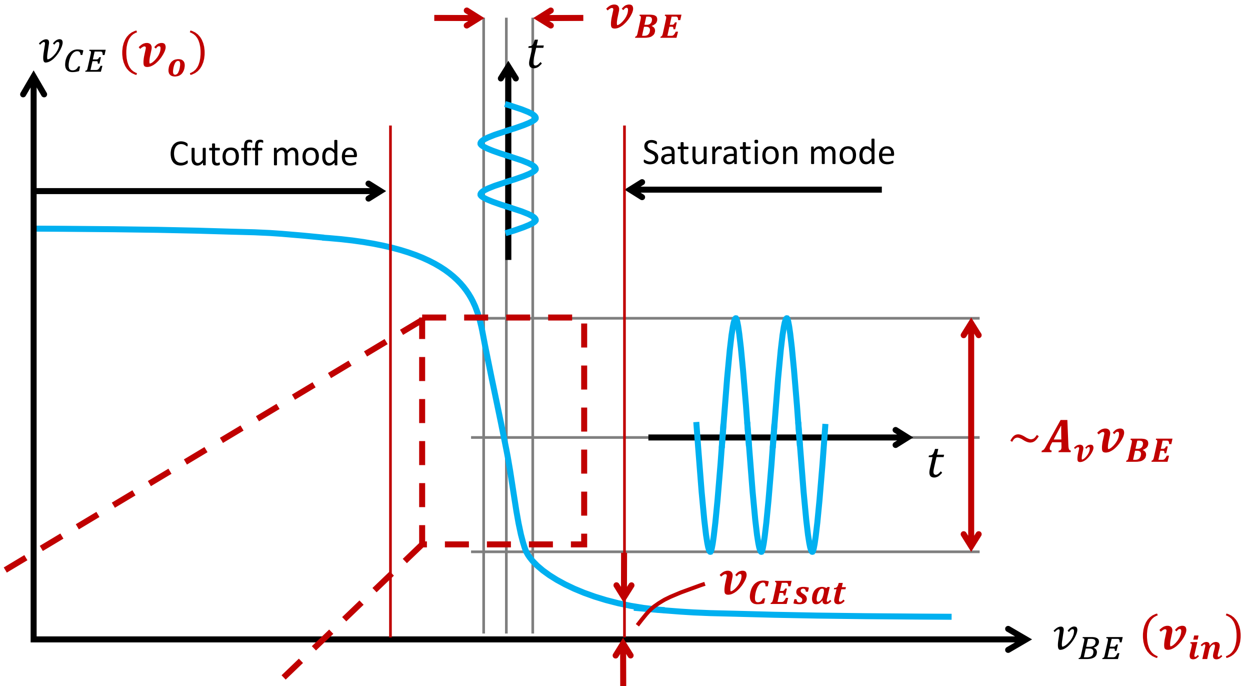
- Step 1 find a proper bias point / quiescent point
- Step 2 characterized by DC voltages V_{BE} and V_{CE}

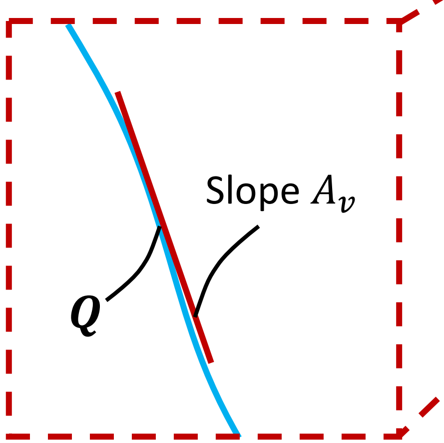
$$V_{CE} = V_{dd} - R_C I_S e^{\frac{V_{BE}}{V_T}}$$

- Step 3 a sufficiently small voltage v_{BE} superimposed on V_{BE}

Signal Amplification







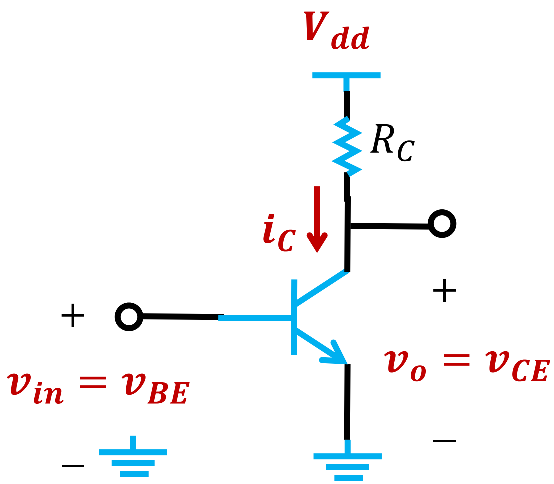
$$v_o = V_{dd} - R_C I_S e^{\frac{v_{in}}{V_T}}$$

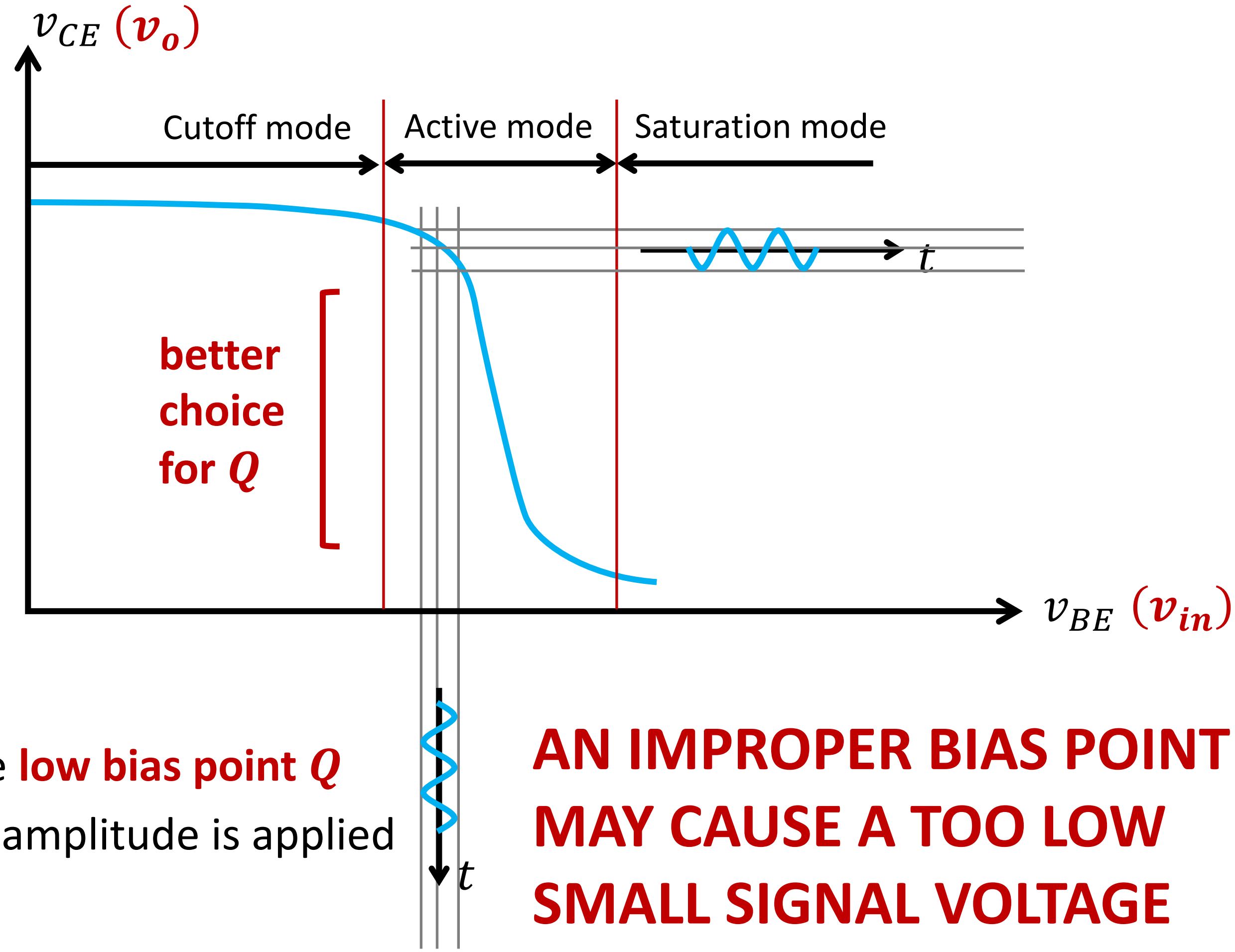
$$A_v = \left.\frac{dv_o}{dv_{in}}\right|_{v_{in}=V_{BE}} = -\frac{1}{V_T} R_C \underbrace{\boxed{I_S e^{\frac{V_{BE}}{V_T}}}}_{I_C} = -\frac{I_C R_C}{V_T}$$

$$= -\frac{V_{dd} - V_{CE}}{V_T} > -\frac{V_{dd} - V_{CEsat}}{V_T}$$

A_v is denoted as the small-signal voltage gain

Locating the Bias Point



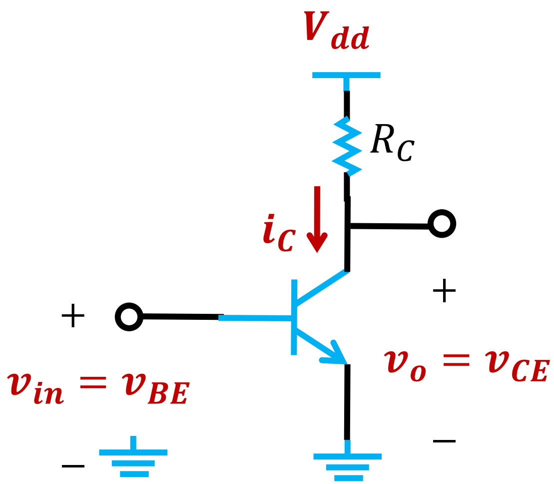


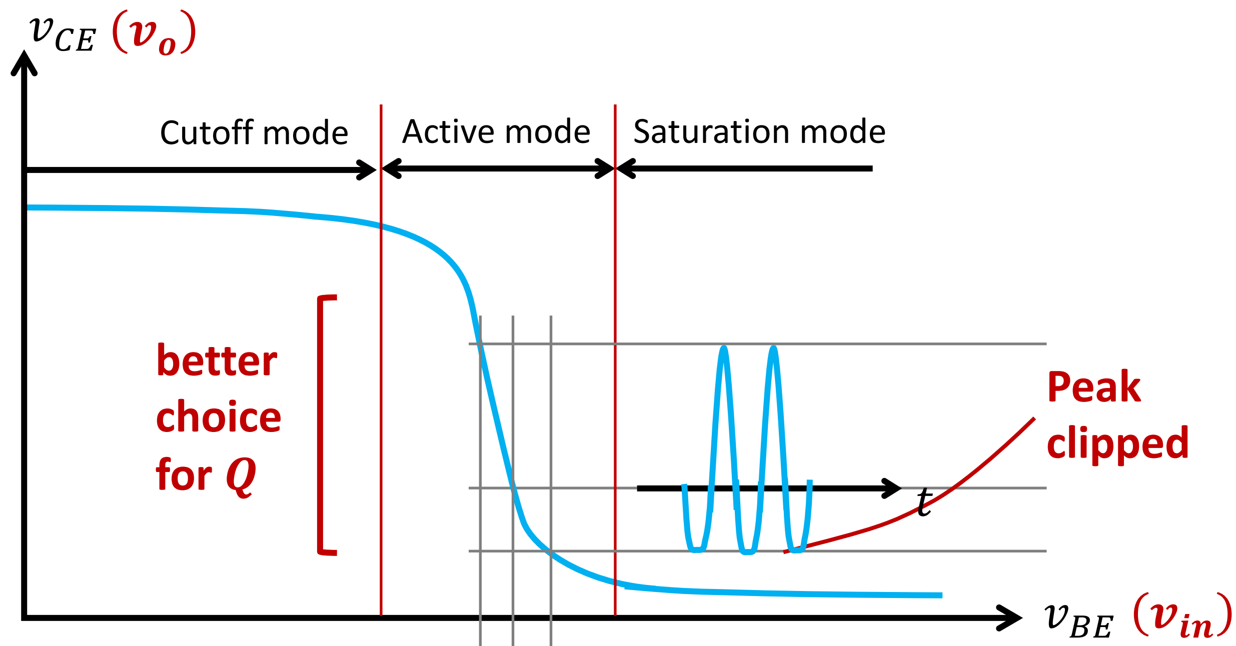
CASE 1

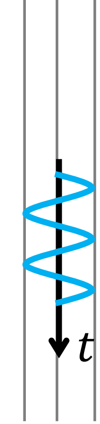
- BJT is biased @ relative **low bias point Q**
- A **sufficiently small** AC amplitude is applied

AN IMPROPER BIAS POINT MAY CAUSE A TOO LOW SMALL SIGNAL VOLTAGE GAIN

Locating the Bias Point





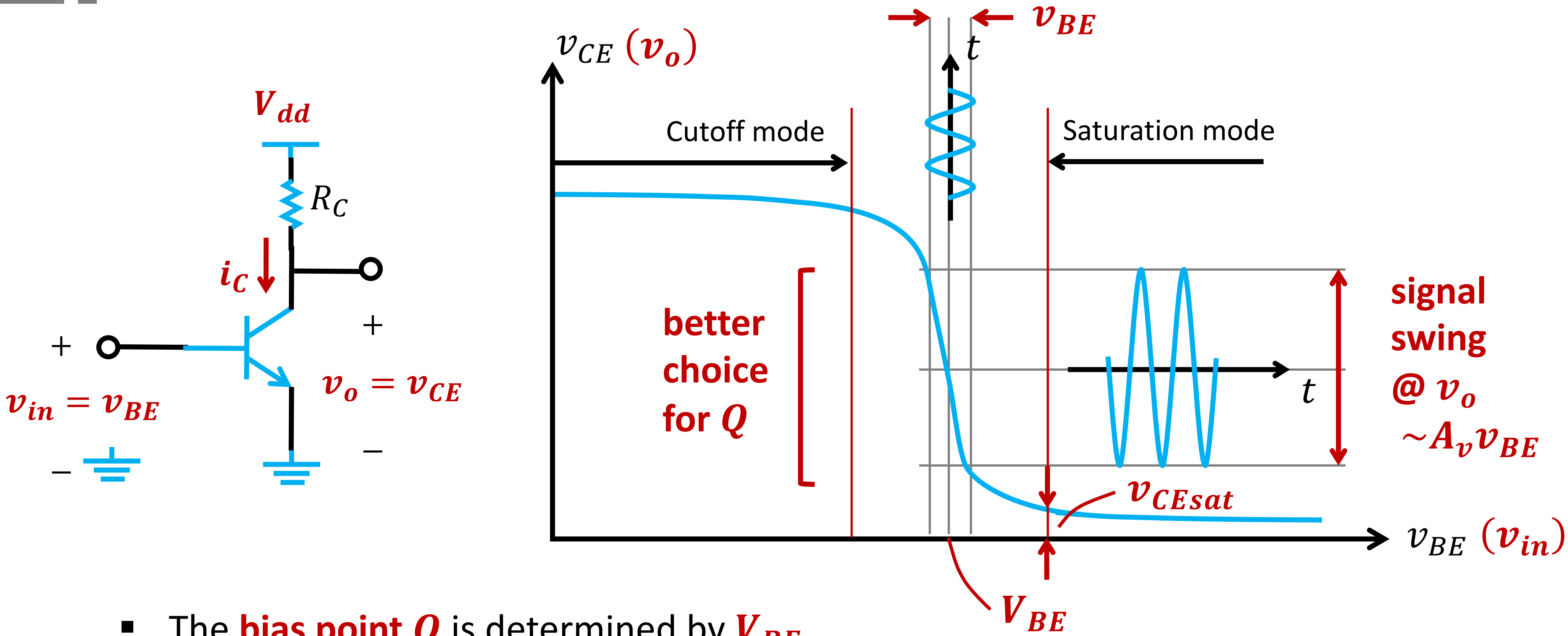


CASE 2

- BJT is biased @ a fine **quiescent point**
- A **larger** AC amplitude is applied
- The peak of the output is **clipped**

A NOT SMALL ENOUGH AC INPUT MAY CAUSE A DISASTER TO LINEARITY PERFORMANCE

Locating the Bias Point



- The **bias point Q** is determined by V_{BE}
- The **GAIN A_v** is determined by the location of Q
- The allowable **SIGNAL SWING at the output** must be considered

LOCATING THE BIAS POINT IS IMPORTANT IN SMALL SIGNAL AMPLIFICATION

? How to locate the bias point

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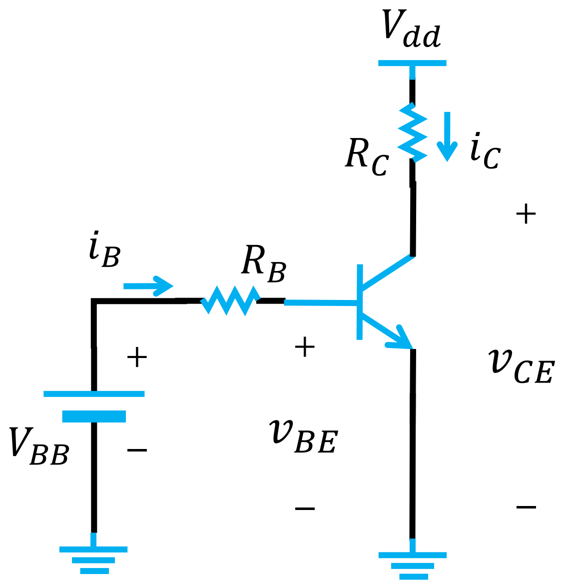
  - The transfer characteristic
- Circuit analysis techniques with BJT
  - DC analysis techniques
  - **AC analysis techniques**
    - Locating the bias point is important
    - **HOW to locate the bias point**

Recall: Example 2

QUESTION: Find out the voltage of V_{BE} and V_{CE} with $R_B = 100k\Omega$, $R_C = 2k\Omega$, V_{dd}=10V, $V_{BB} = 5V$, and $\beta = 100$. Assume $|V_{BE}| = 0.7V$ in active mode.



- Let's **ASSUME** the transistor is biased in **active mode**
- According to characteristics of the transistor

$$V_{BE} \approx 0.7V$$

- According to KVL & transistor characteristics

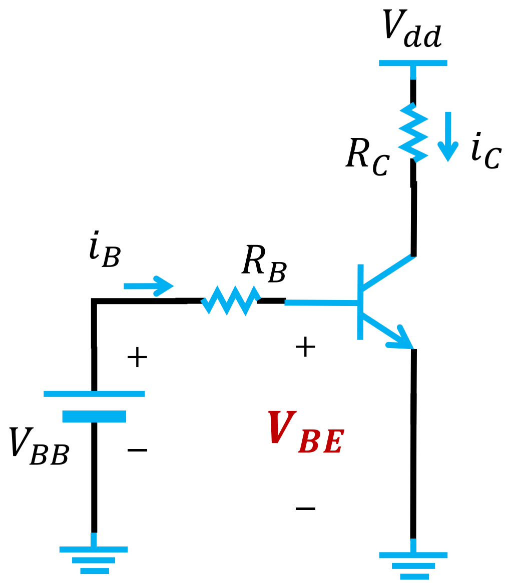
$$\begin{cases} V_{BB} = i_B R_B + V_{BE} \\ V_{dd} = i_C R_C + V_{CE} \\ i_C = \beta i_B = 4.3mA \end{cases} \Rightarrow \begin{cases} i_B = 0.043mA \\ i_C = 4.3mA \\ V_{CE} = 1.4V \end{cases}$$

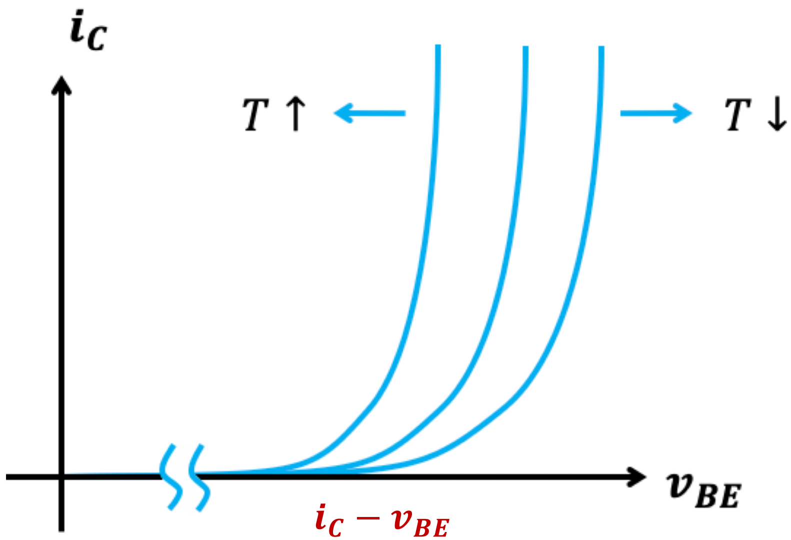
- **Check ASSUMPTION**

$$V_{BC} = -0.7V < V_{BCon} \Rightarrow$$ **Active mode**

HOW to Locate the Bias Point

- **Method 1 – Locate the bias point by fixing V_{BE}** ☹

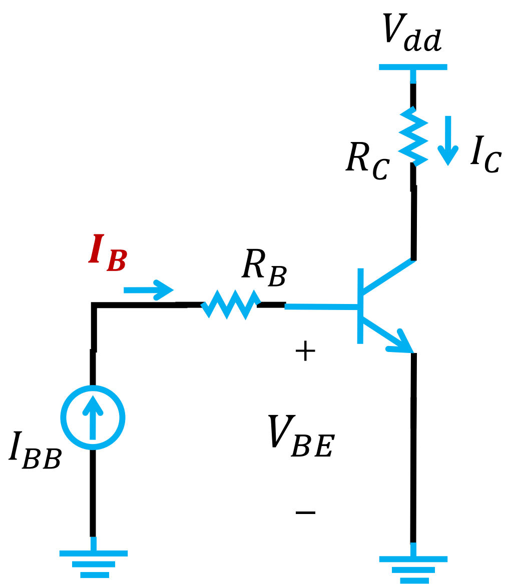




I_C is **too sensitive** to the change of V_{BE}

HOW to Locate the Bias Point

- **Method 2 – Locate the bias point by fixing I_B** ☹



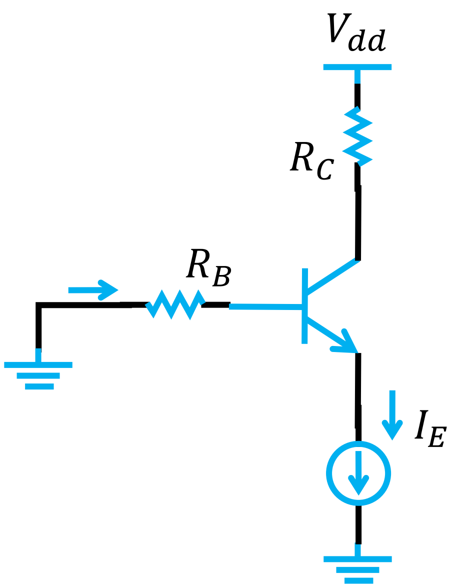
- According transistor characteristics, in active mode

$$I_C = \beta I_B$$

The **large variations of β** results in **large variation in I_C**

HOW to Locate the Bias Point

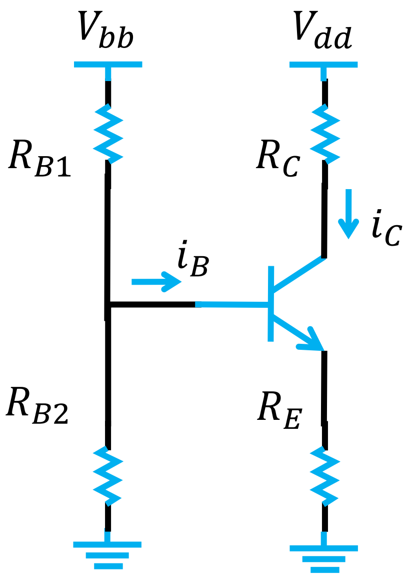
- **Method 3 – Using Constant-Current Source** ☺



- I_E is independent of β and R_B
- How to realize a current source?
  - Current mirror explained in next chapter

Recall: Example 3

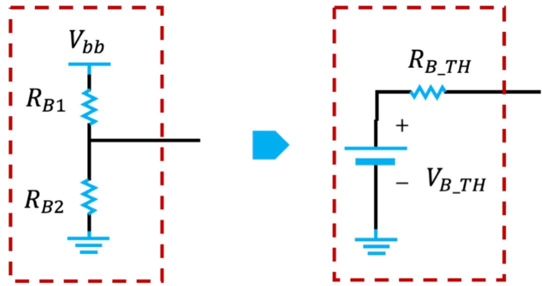
QUESTION: Find out the voltage of V_{BE} and V_{CE} with $R_{B1} = 100k\Omega$, $R_{B2} = 50k\Omega$, $R_C = 5k\Omega$, $R_E = 3k\Omega$, $V_{dd} = V_{BB} = 15V$, and $\beta = 100$. Assume $|V_{BE}| = 0.7V$ in active mode.



- According to Thévenin's theorem

$$R_{B\_TH} = R_{B1}||R_{B2} = 33.3k\Omega$$

$$V_{B\_TH} = \frac{R_{B2}}{R_{B1} + R_{B2}} V_{dd} = 5V$$



- Assume the transistor is biased in active region
- The voltage of V_{BE} and V_{CE} can be calculated as

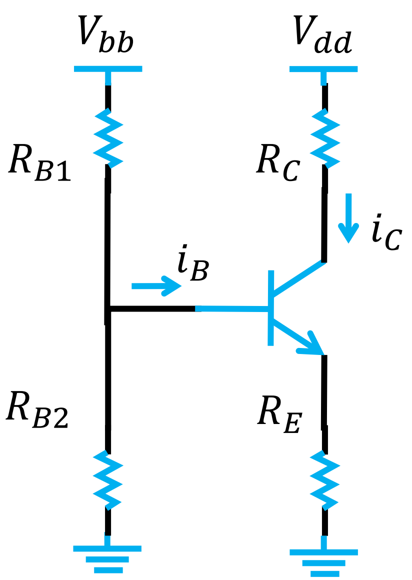
$$V_{BE} = V_{B\_TH} - i_B R_{B_{TH}} - V_E = 0.694V$$

$$V_{CE} = V_{dd} - i_C R_C - V_E = -3.664V$$

- Check assumption

HOW to Locate the Bias Point

- **Method 4 – Classical Discrete-Circuit Arrangement** ☺



- According to KVL & Thévenin's theorem

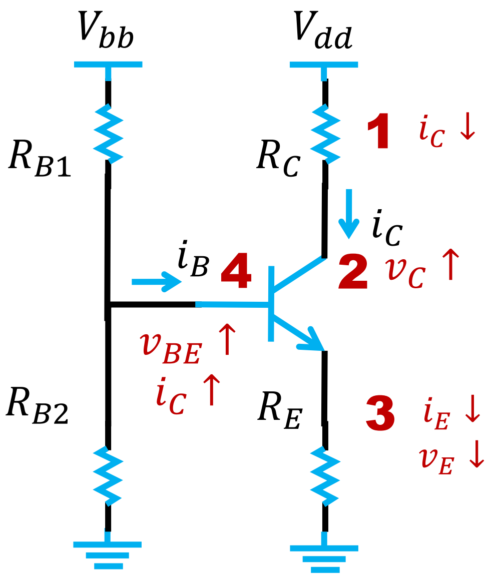
$$I_E = \frac{V_{B\_TH} - V_{BE}}{R_E + R_B/(\beta + 1)} \quad \text{where } V_{B\_TH} = \frac{R_{B2}}{R_{B1}+R_{B2}} V_{dd}$$

- Since $V_{B\_TH} \gg V_{BE}$, $R_E \gg \frac{R_B}{\beta+1}$

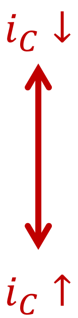
A better tolerance to the variation of V_{BE}

HOW to Locate the Bias Point

- **Method 4 – Classical Discrete-Circuit Arrangement** ☺



- What is R_E used for?
  - If there is a decrease @ i_C
  - The voltage drop on R_C decreases
  - i_E and v_E decrease correspondingly
  - v_{BE} increases, causing an increasing of i_C

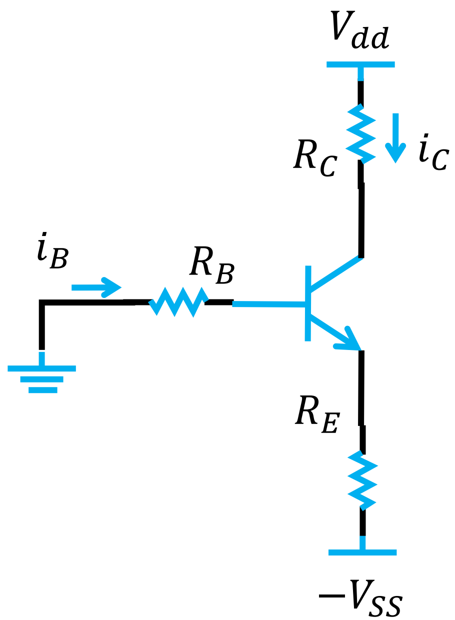


NEGATIVE FEEDBACK is observed

R_E stabilizes the bias current

HOW to Locate the Bias Point

- **Method 5 – Two-Power-Supply Version** ☺



- According to KVL & transistor characteristics

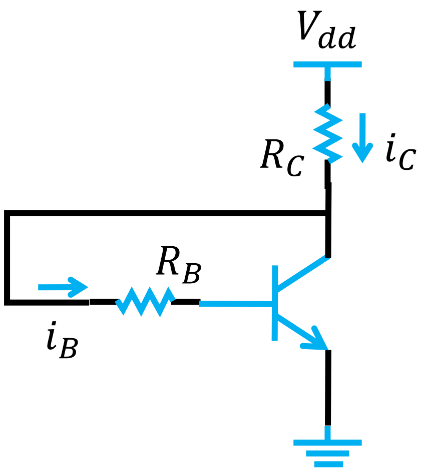
$$V_{SS} = I_B R_B + V_{BE} + I_E R_E$$

$$I_E = (\beta + 1) I_B$$ **if the transistor is in active mode**

$$i_E = \frac{V_{SS} - V_{BE}}{R_E + R_B/(\beta + 1)}$$

HOW to Locate the Bias Point

- **Method 6 – Collector-to-Base Feedback Resistor** ☺



- According to KVL & transistor characteristics

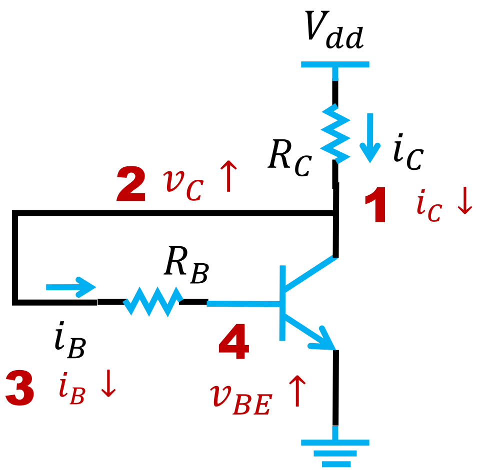
$$V_{dd} = I_C R_C + I_B R_B + V_{BE}$$

$I_E = (\beta + 1) I_B$ **if the transistor is in active mode**

$$i_E = \frac{V_{dd} - V_{BE}}{R_C + R_B/(\beta + 1)}$$

HOW to Locate the Bias Point

- **Method 6 – Collector-to-Base Feedback Resistor** ☺



- What is R_B used for?
  - If there is a decrease @ i_C — $i_C \downarrow$
  - The voltage drop on R_C decreases, v_C increases
  - i_B decrease correspondingly
  - v_{BE} increases, causing an increasing of i_C — $i_C \uparrow$

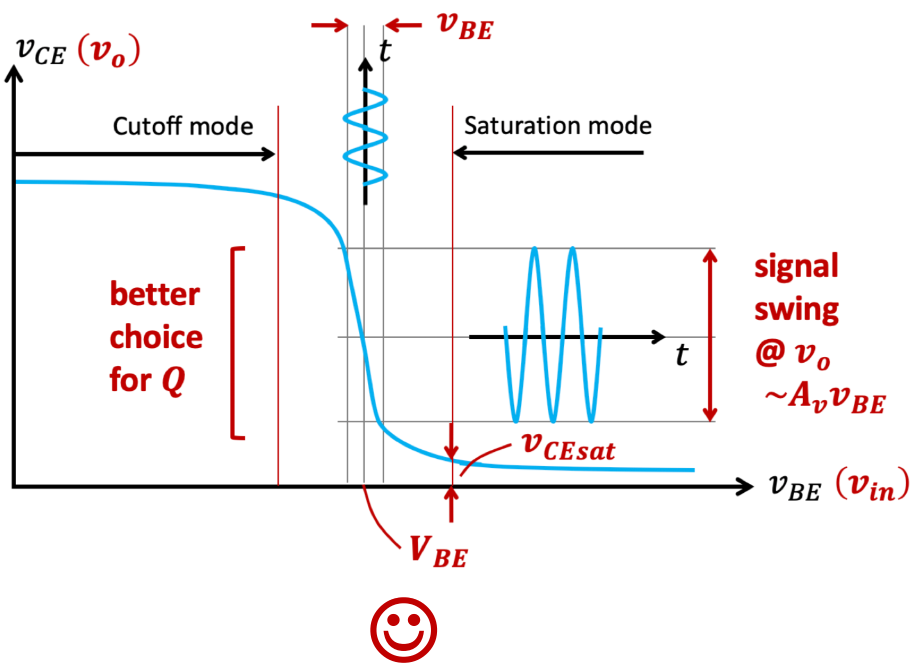
NEGATIVE FEEDBACK is observed

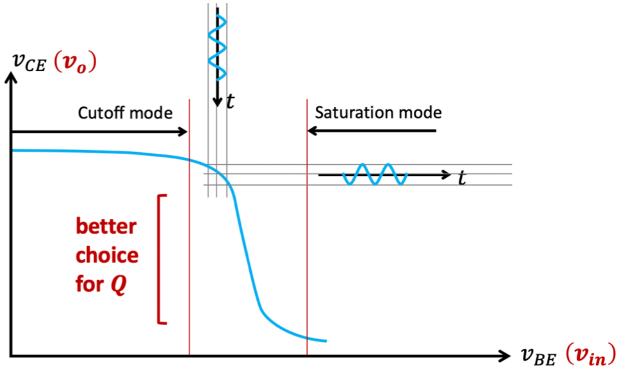
R_B stabilizes the bias point

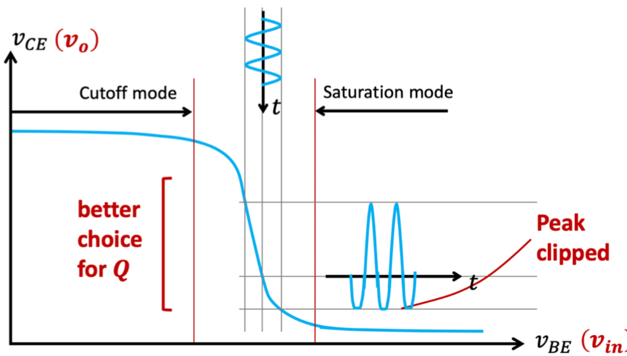
Summary: Locate the Bias Point

WHY? **Key to small signal amplification**

- Q is determined by $\boldsymbol{V_{BE}}$
- Q determines the **GAIN $\boldsymbol{A_v}$**
- Q has effects on the **OUTPUT SIGNAL SWING**



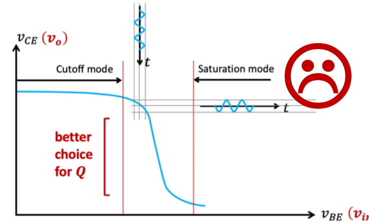


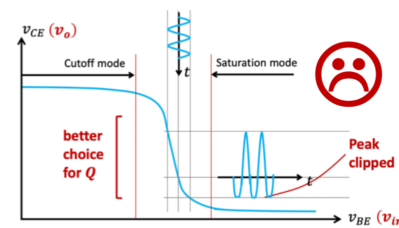


Summary: Locate the Bias Point

WHY?

Key to small signal amplification





- Q is determined by $\boldsymbol{V_{BE}}$
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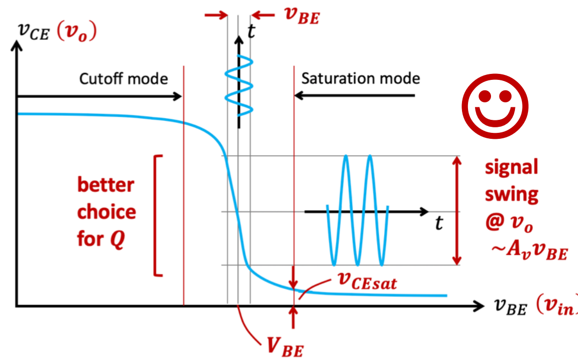
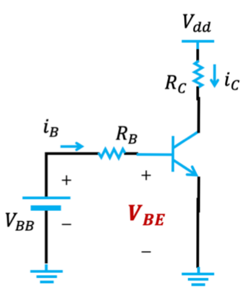
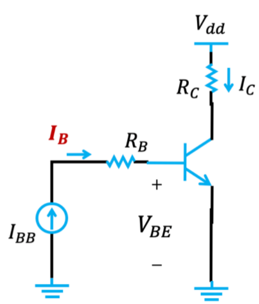


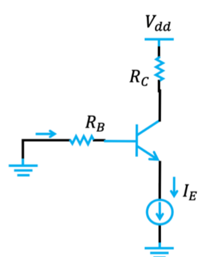
HOW?



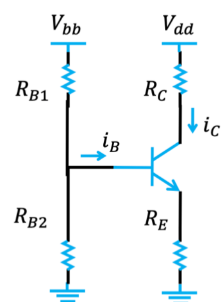
fixing V_{BE}



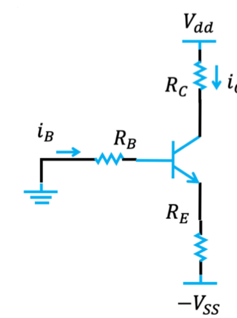
fixing I_B



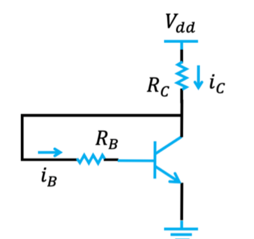
fixing I_E



Classical Discrete-Circuit Arrangement



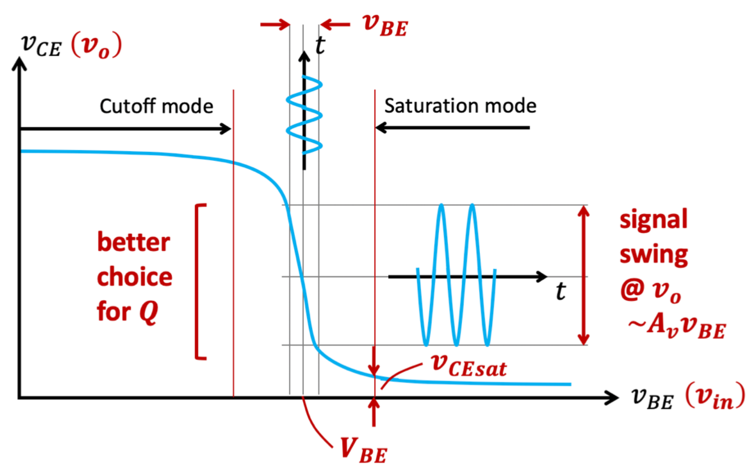
Two-Power-Supply Version



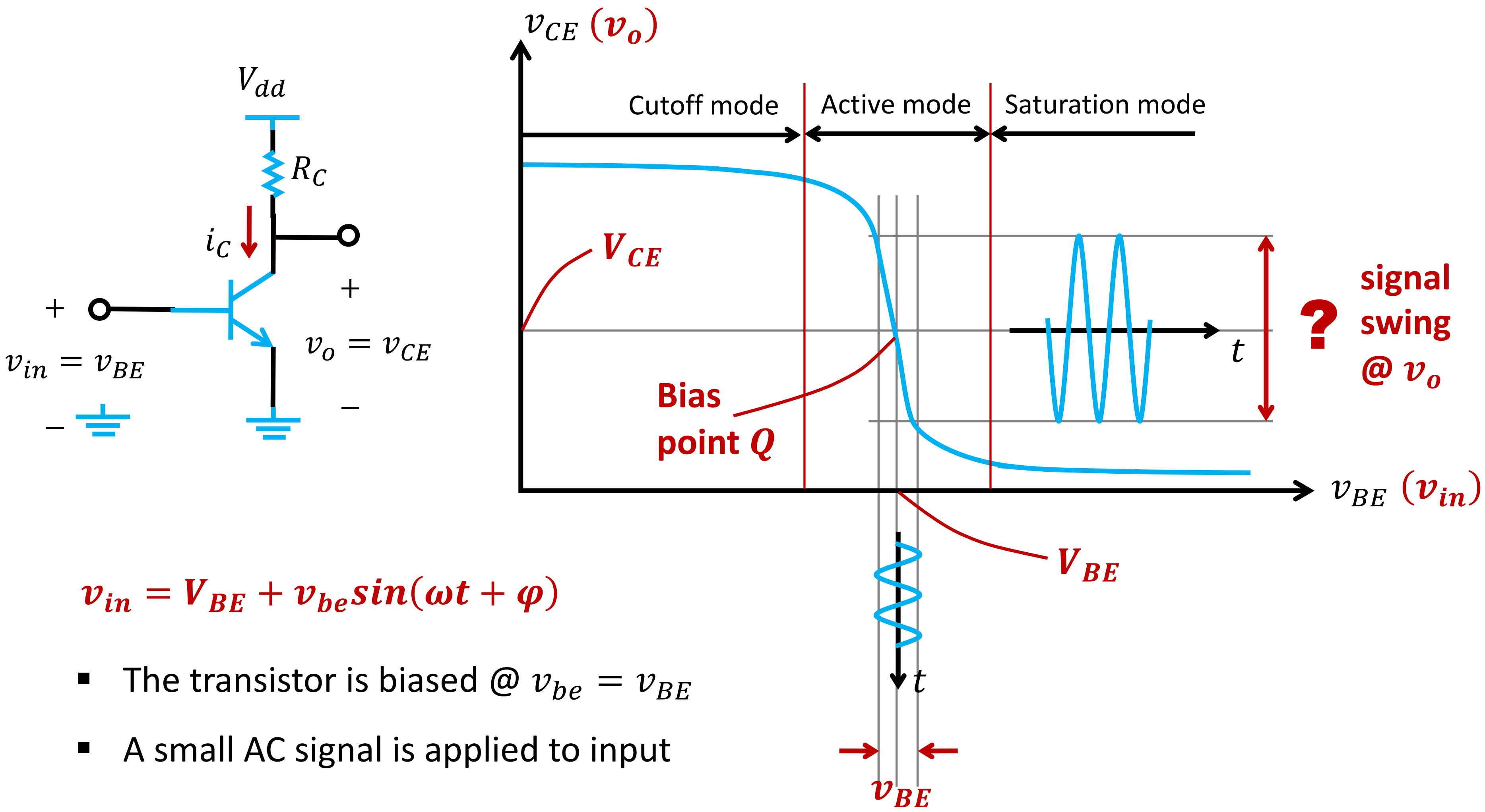
Collector-to-Base Feedback Resistor

Outline

- Introduction to BJT
  - Device structure
  - How does it work?
    - Cutoff / Active / Reverse / Saturation mode
  - npn v.s. pnp
- The characteristic curves
  - $i - v$ characteristics
  - The transfer characteristic
- Circuit analysis techniques with BJT
  - DC analysis techniques
  - AC analysis techniques
    - Locate the bias point
    - **Small-signal operation & model**



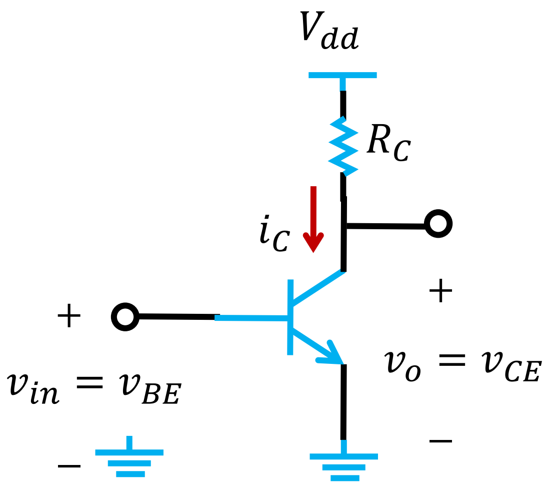
Small-Signal Operation



$$v_{in} = V_{BE} + v_{be}sin(\omega t + \varphi)$$

- The transistor is biased @ $v_{be} = v_{BE}$
- A small AC signal is applied to input

Small-Signal Operation



$$v_{in} = V_{BE} + v_{be} sin(\omega t + \varphi) = V_{BE} + v_{in,AC}$$

- According to the transistor characteristic

$$i_C = I_S e^{\frac{v_{BE}}{V_T}} = I_S e^{\frac{V_{BE} + v_{in,AC}}{V_T}} = \underbrace{I_S e^{\frac{V_{BE}}{V_T}}}_{I_C} e^{\frac{v_{in,AC}}{V_T}}$$

$$= I_C e^{\frac{v_{in,AC}}{V_T}}$$

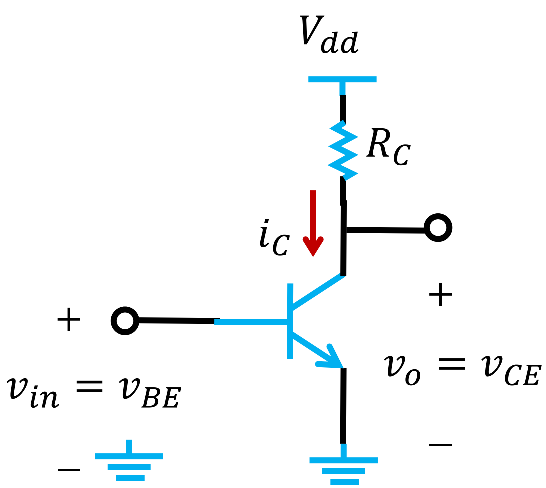
If $v_{in,AC} \ll V_T$

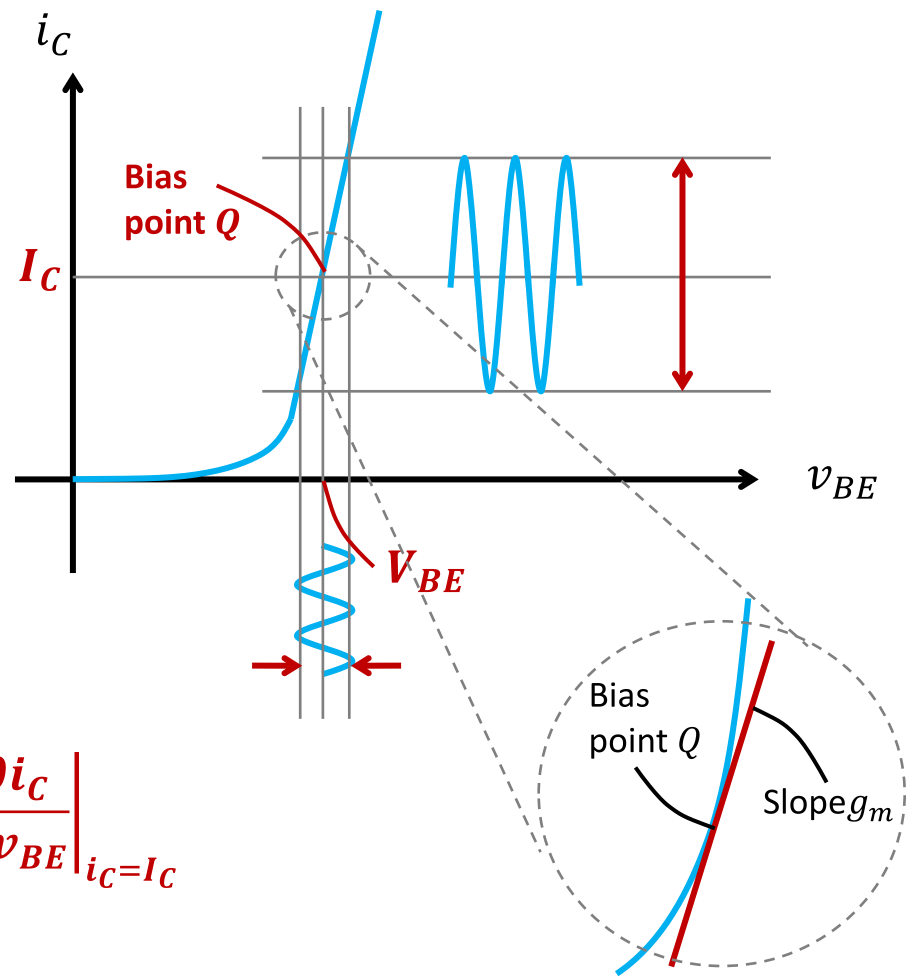
$$\approx I_C \left(1 + \frac{v_{in,AC}}{V_T}\right)$$

$$i_C = \underbrace{I_C}_{i_C@DC} + \underbrace{\frac{I_C}{V_T} v_{in,AC}}_{i_C@AC} = I_C + g_m v_{in,AC}$$

Define TRANSCONDUCTANCE $g_m = \frac{I_C}{V_T}$

Transconductance g_m



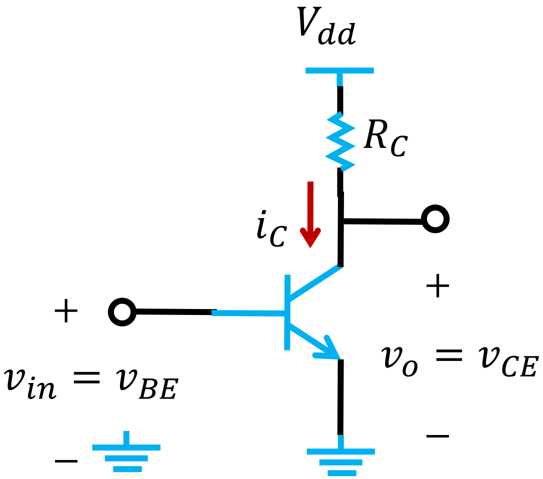


$$v_{in} = V_{BE} + v_{in,AC}$$

$$i_C = I_C + g_m v_{in,AC}$$

TRANSCONDUCTANCE $g_m = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{i_C = I_C}$

Voltage Gain A_v



- According to KVL

$$v_o = V_{dd} - i_C R_C = V_{dd} - \left(i_C\big|_{DC} + i_C\big|_{AC} \right) R_C$$

$$= \underbrace{V_{dd} - I_C R_C}_{V_{CE}} \underbrace{- i_C\big|_{AC} R_C}_{g_m v_{AC}}$$

$$v_{in} = V_{BE} + v_{AC}$$

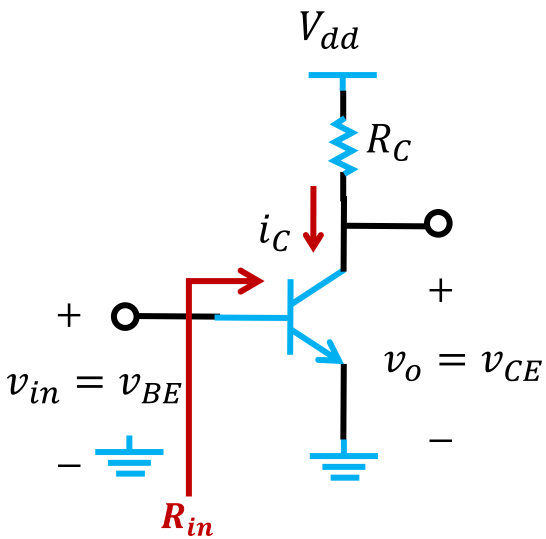
$$i_C = I_C + g_m v_{AC}$$

$$g_m = \frac{\partial i_C}{\partial v_{BE}}\bigg|_{i_C = I_C} = \frac{I_C}{V_T}$$

- Define voltage gain

$$A_v = \frac{v_o|_{AC}}{v_{in}|_{AC}} = \frac{-g_m v_{AC} R_C}{v_{AC}} = -g_m R_C$$

Input Resistance @ Base



$$v_{in} = V_{BE} + v_{AC}$$

$$i_C = I_C + g_m v_{AC}$$

- According to the definition of R_{in}

$$R_{in} = \frac{\Delta v_{in}}{\Delta i_{in}} = \frac{\Delta v_{BE}}{\Delta i_B}$$

- According to the transistor characteristic in active mode

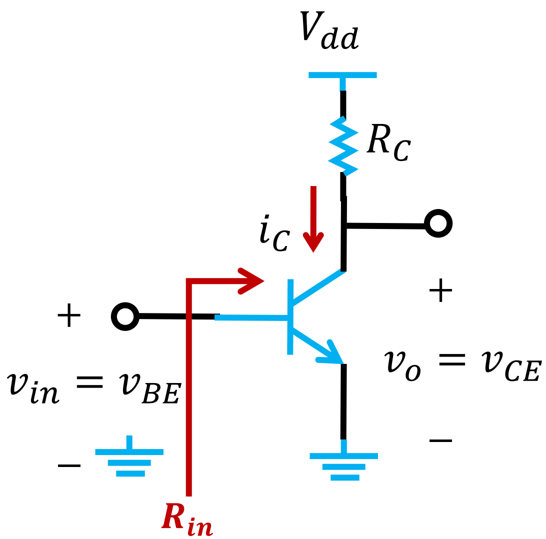
$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta}\frac{I_C}{V_T} v_{AC}$$

$$\Rightarrow \Delta i_B = \frac{1}{\beta}\frac{I_C}{V_T} v_{AC} = \frac{1}{\beta} g_m v_{AC}$$

- Thus,

$$R_{in} = \frac{\Delta v_{BE}}{\Delta i_B} = \frac{v_{AC}}{\Delta i_B} = \frac{V_T}{I_B} \quad \text{or} \quad = \frac{\beta}{g_m}$$

Summary: Small-Signal Operation



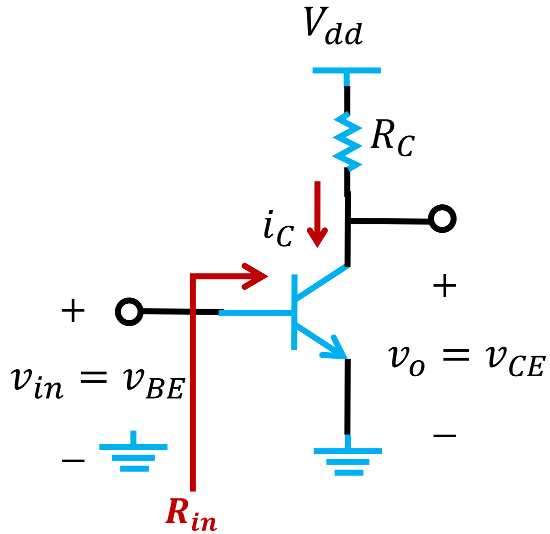
$$v_{in} = V_{BE} + v_{be} sin(\omega t + \varphi) = V_{BE} + v_{AC}$$

Voltage gain $A_v = \frac{v_o|_{AC}}{v_{in}|_{AC}} = -g_m R_C$

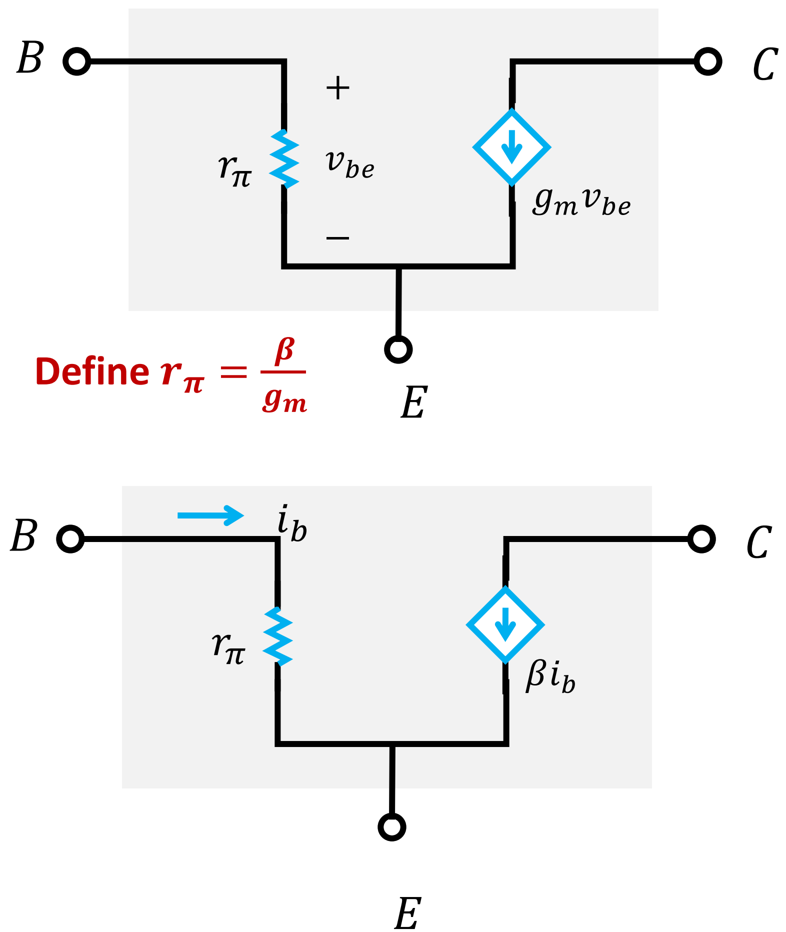
Transconductance $g_m = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{i_C = I_C} = \frac{I_C}{V_T}$

Input resistance @ Base $R_{in} = \frac{V_T}{I_B} = \frac{\beta}{g_m}$

Small-Signal Model



SIMPLIFIED HYBRID-π MODEL



Define $r_\pi = \frac{\beta}{g_m}$

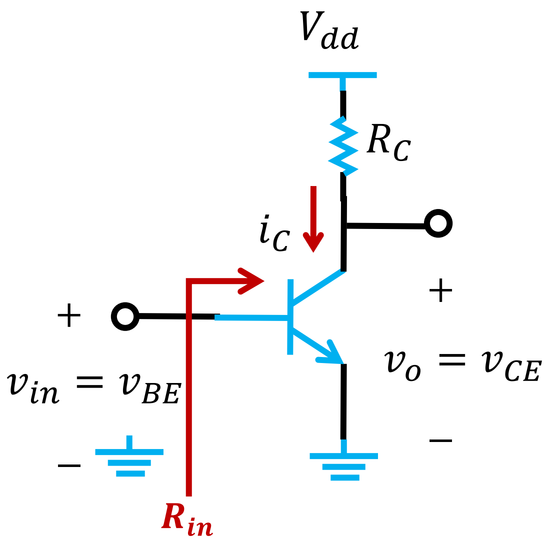
$$v_{in} = V_{BE} + v_{be} sin(\omega t + \varphi) = V_{BE} + v_{AC}$$

Voltage gain $A_v = \frac{v_o|_{AC}}{v_{in}|_{AC}} = -g_m R_C$

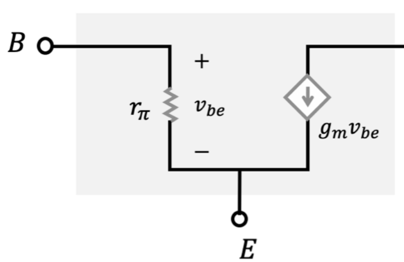
Transconductance $g_m = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{i_C = I_C} = \frac{I_C}{V_T}$

Input resistance @ Base $R_{in} = \frac{V_T}{I_B} = \frac{\beta}{g_m}$

Small-Signal Model



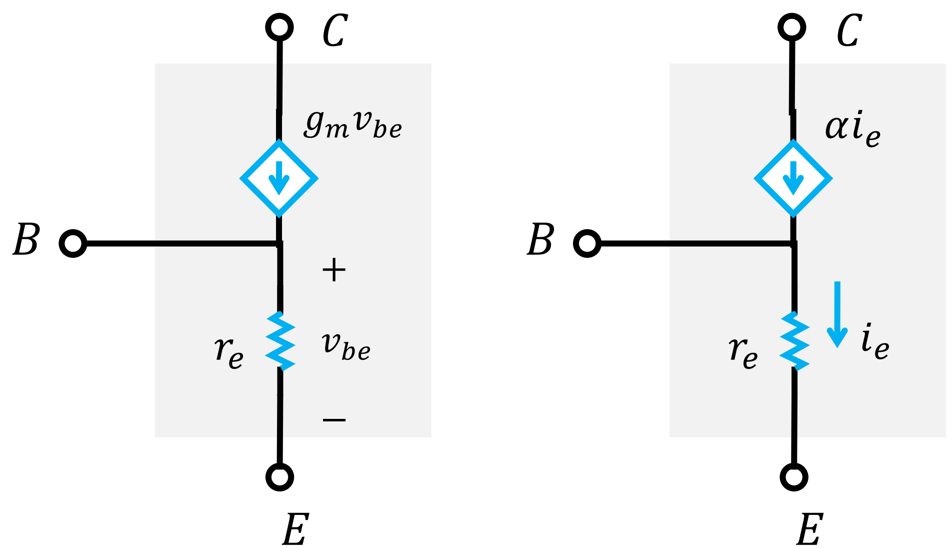
SIMPLIFIED HYBRID-π MODEL



$g_m = \frac{I_C}{V_T}$

$r_\pi = \frac{\beta}{g_m}$

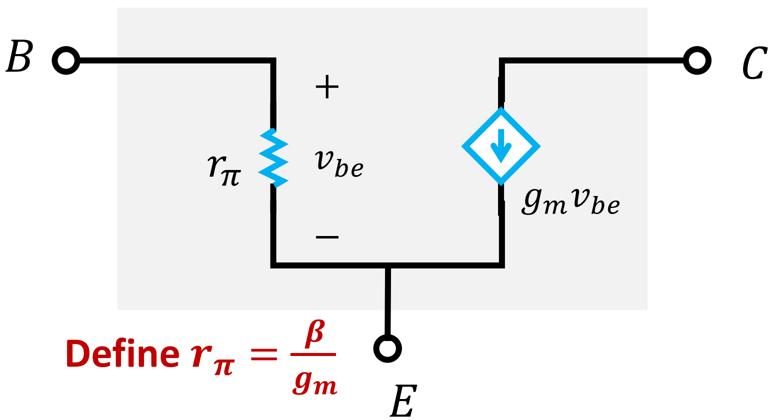
SIMPLIFIED T MODEL



Define $r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$

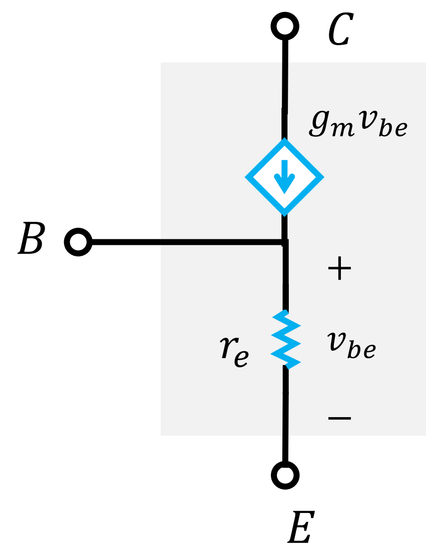
Small-Signal Model

SIMPLIFIED HYBRID-π MODEL



Define $\boldsymbol{r_\pi = \frac{\beta}{g_m}}$

SIMPLIFIED T MODEL



Define

$$\boldsymbol{r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}}$$

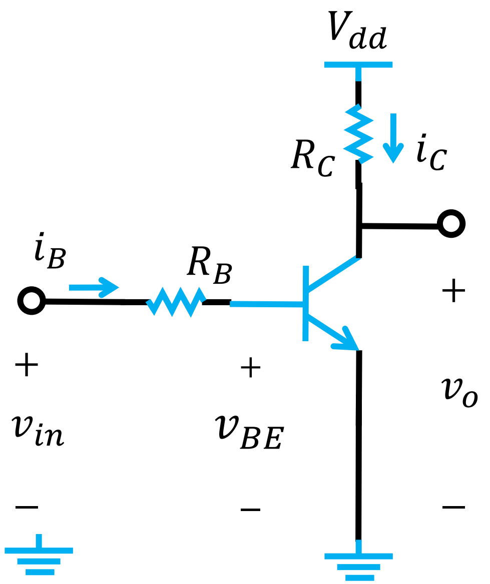
- According to hybrid-π model $\quad i_b = \dfrac{v_{be}}{r_\pi}$
- According to T model

$$i_b = \frac{v_{be}}{r_e} - g_m v_{be} = \frac{v_{be}}{r_e}(1 - g_m r_e) = \frac{v_{be}}{r_e}(1-\alpha) = \frac{v_{be}}{(1+\beta)r_e} = \frac{v_{be}}{r_\pi}$$

HYBRID-π MODEL and T MODEL are equivalent

Example 4

QUESTION: Find out the voltage of v_o with $v_{in} = 3V + 0.1sin(\omega t + \varphi)$, $R_B = 100k\Omega$, $R_C = 3k\Omega$, $V_{dd} = 10V$, and $\beta = 100$. The threshold voltage $V_{th} = 0.7V$.



- **Step 1: perform DC analysis**
- Let's **ASSUME** the transistor is biased in **active mode**
- According to KVL & transistor characteristics

$$\begin{cases} v_{in}\big|_{DC} = i_B R_B + V_{th} \\ V_C = V_{dd} - i_C R_C \\ i_C = \beta i_B \end{cases} \qquad \begin{cases} i_B = 0.023mA \\ V_C = 3.1V \end{cases}$$

- **Check ASSUMPTION**

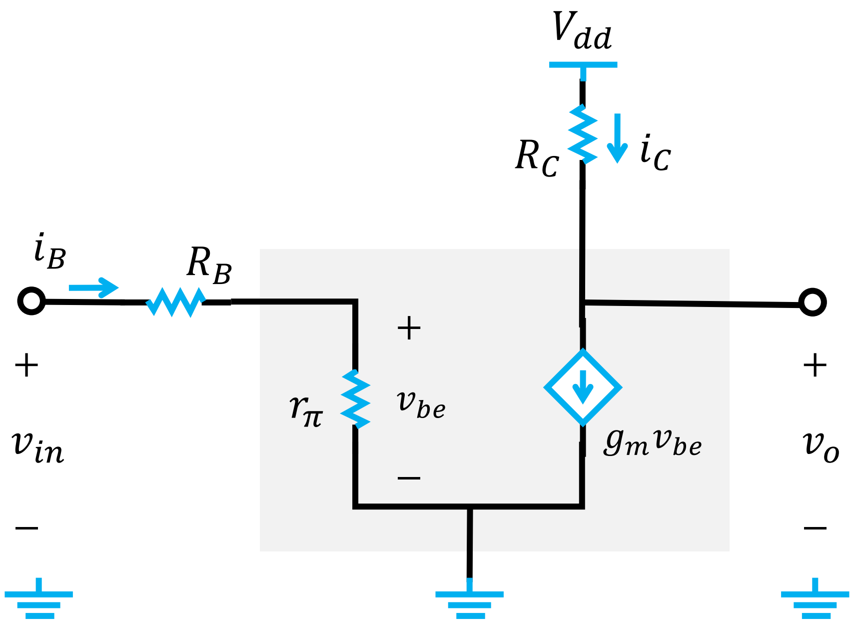
$$V_{BE} = 0.7V \quad > V_{BEon}$$

$$V_{BC} = -2.4V \quad < V_{BCon}$$

→ **Active mode**

Example 4

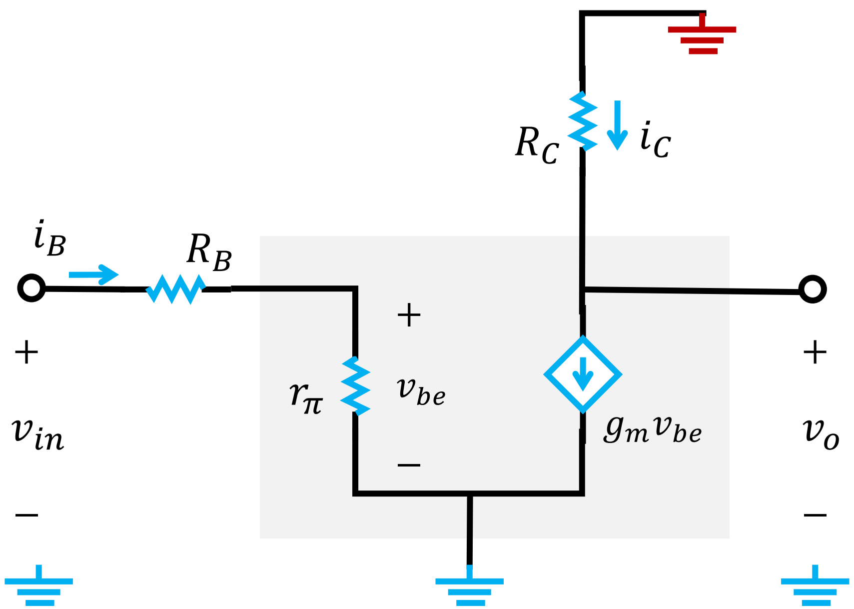
QUESTION: Find out the voltage of v_o with $v_{in} = 3V + 0.1sin(\omega t + \varphi)$, $R_B = 100k\Omega$, $R_C = 3k\Omega$, $V_{dd} = 10V$, and $\beta = 100$. The threshold voltage $V_{th} = 0.7V$.



- **Step 2: perform AC analysis**
  - **Step 2.1: replace the transistor with the small-signal model**

Example 4

QUESTION: Find out the voltage of v_o with $v_{in} = 3V + 0.1sin(\omega t + \varphi)$, $R_B = 100k\Omega$, $R_C = 3k\Omega$, $V_{dd} = 10V$, and $\beta = 100$. The threshold voltage $V_{th} = 0.7V$.



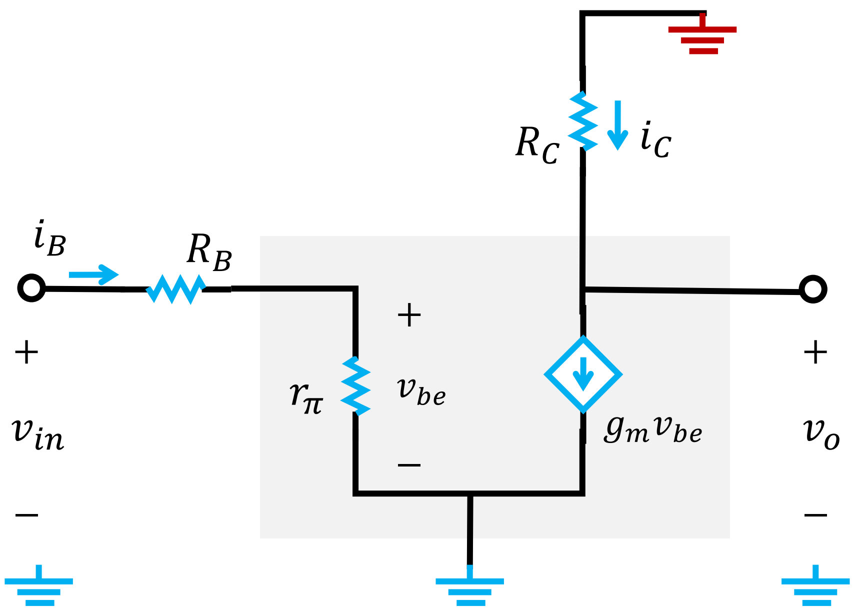
- **Step 2: perform AC analysis**
  - **Step 2.1: replace the transistor with the small-signal model**
  - **Step 2.2: turn off DC sources**
    - SHORT all voltage sources
    - OPEN all current sources
  - **Step 2.3: Calculate small-signal model parameters**

$$g_m = \frac{I_C}{V_T} = \frac{2.3mA}{25mV} = 92mA/V$$

$$r_\pi = \frac{\beta}{g_m} = 1.086k\Omega$$

Example 4

QUESTION: Find out the voltage of v_o with $v_{in} = 3V + 0.1sin(\omega t + \varphi)$, $R_B = 100k\Omega$, $R_C = 3k\Omega$, $V_{dd} = 10V$, and $\beta = 100$. The threshold voltage $V_{th} = 0.7V$.



- **Step 2: perform AC analysis**
  - **Step 2.4: Analyze resulting circuit**

$$v_o = -g_m v_{be} R_C$$

$$= -g_m R_C \frac{r_\pi}{R_B + r_\pi} v_{in}$$

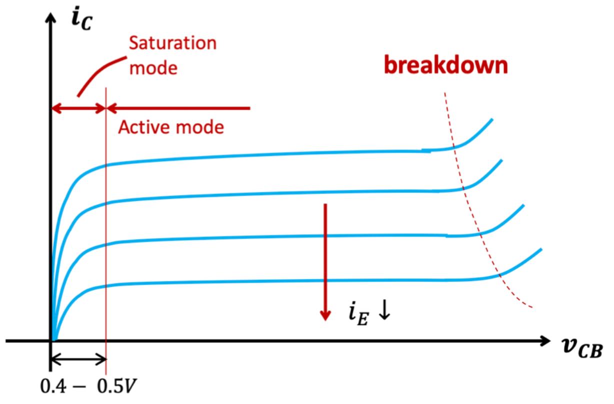
The voltage gain

$$A_v = \frac{v_o}{v_{in}} = -g_m R_C \frac{r_\pi}{R_B + r_\pi}$$

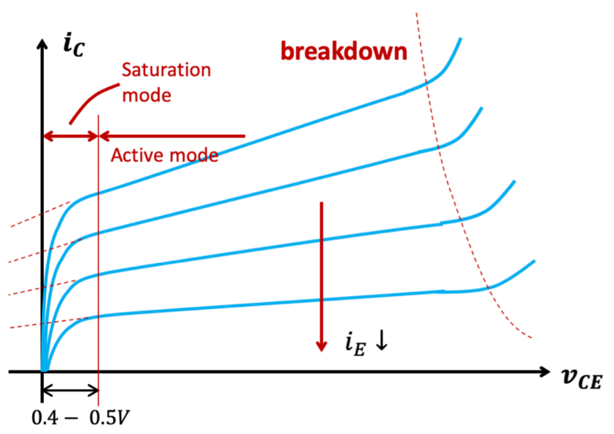
$$= -3.04$$

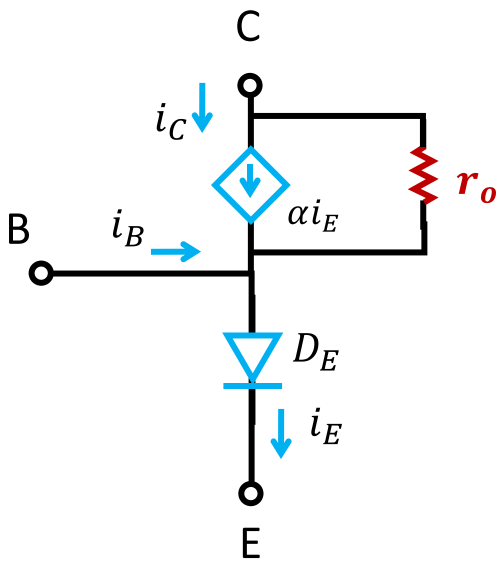
Recall: the Early Effect

Ideal case



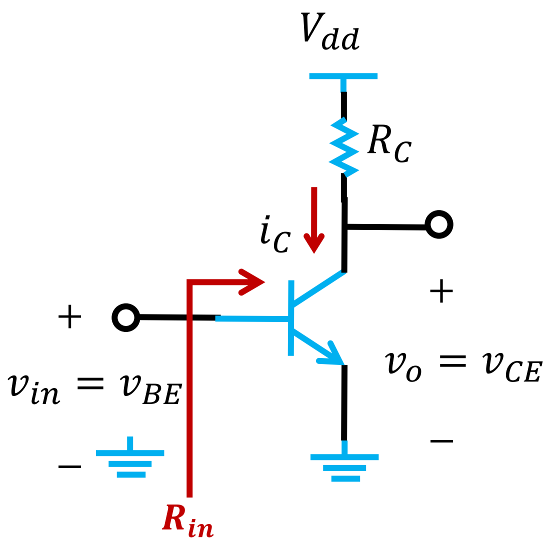
Consider Early Effect



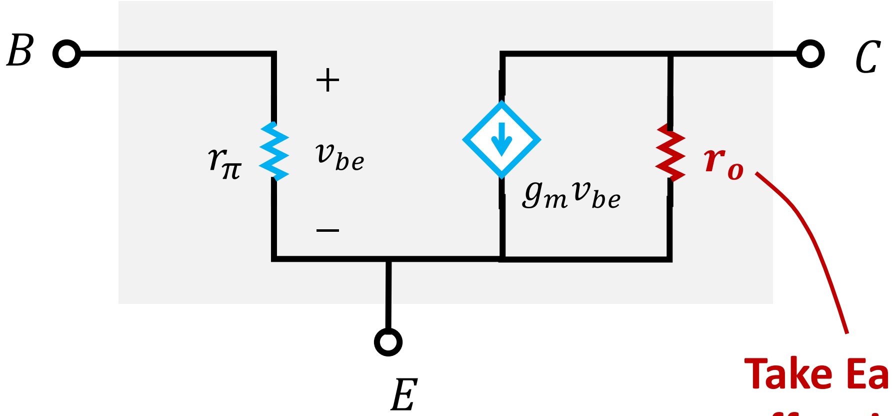


$$r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$$

Small-Signal Model



SIMPLIFIED HYBRID-π MODEL

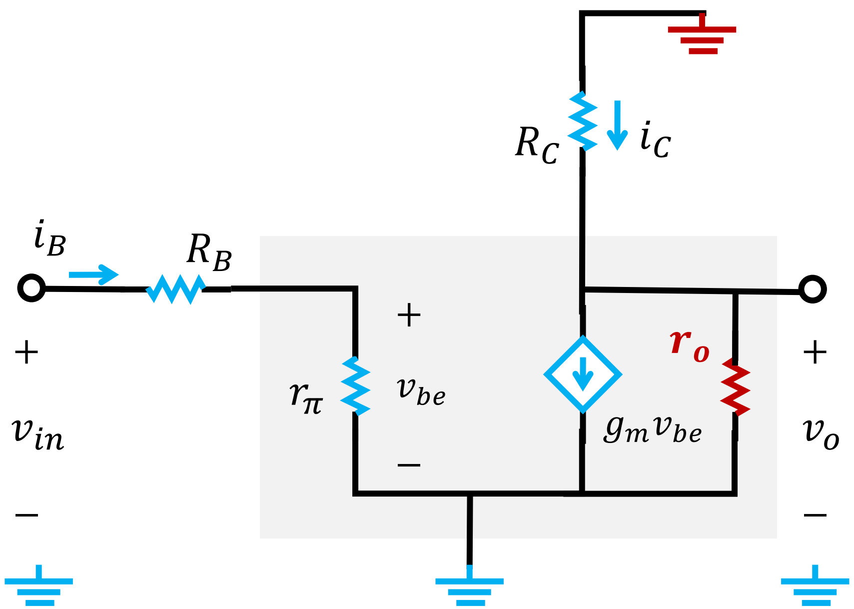


Take Early effect into account

$$r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$$

Example 4

QUESTION: Find out the voltage of v_o with $v_{in} = 3V + 0.1sin(\omega t + \varphi)$, $R_B = 100k\Omega$, $R_C = 3k\Omega$, $V_{dd} = 10V$, and $\beta = 100$, $\boldsymbol{V_A = 100V}$. The threshold voltage $V_{th} = 0.7V$.



- **Step 2: perform AC analysis**
  - **Step 2.4: Analyze resulting circuit**

$$v_o = -g_m v_{be}(R_C||r_o)$$

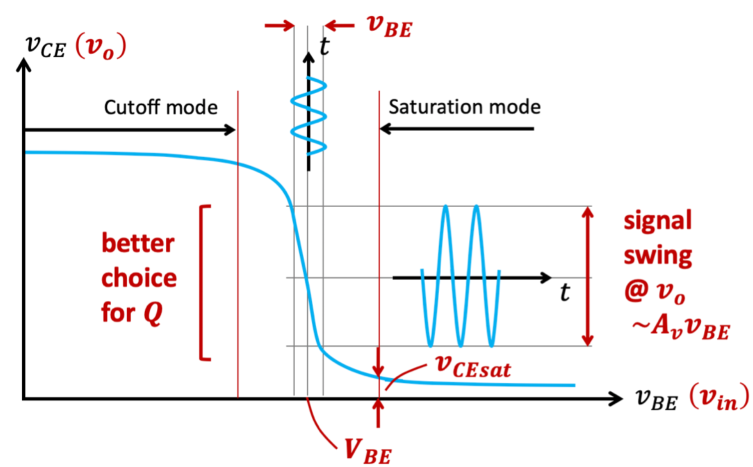
$$= -g_m(R_C||r_o)\frac{r_\pi}{R_B + r_\pi}v_{in}$$

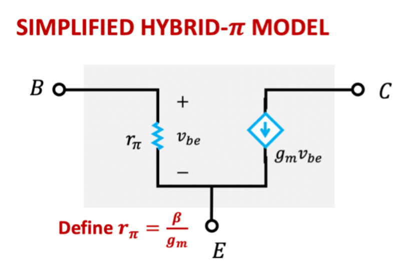
The voltage gain

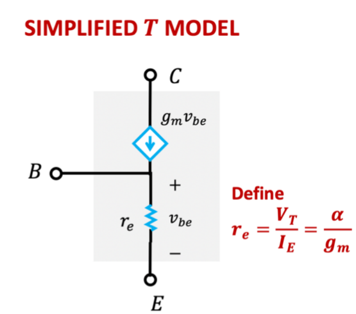
$$A_v = \frac{v_o}{v_{in}} = -g_m(R_C||r_o)\frac{r_\pi}{R_B + r_\pi}$$

Outline

- Introduction to BJT
  - Device structure
  - How does it work?
    - Cutoff / Active / Reverse / Saturation mode
  - npn v.s. pnp
- The characteristic curves
  - $i - v$ characteristics
  - The transfer characteristic
- Circuit analysis techniques with BJT
  - DC analysis techniques
  - AC analysis techniques
    - Locate the bias point
    - Small-signal operation & model
    - **Characterizing Amplifiers**

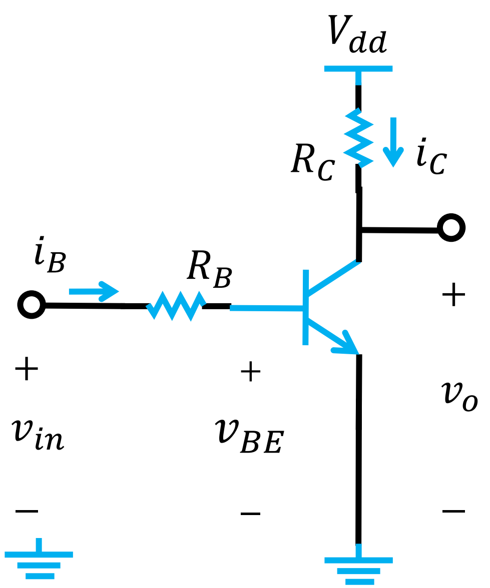






Recall: Example 4

QUESTION: Find out the voltage of v_o with $v_{in} = 3V + 0.1sin(\omega t + \varphi)$, $R_B = 100k\Omega$, $R_C = 3k\Omega$, $V_{dd} = 10V$, and $\beta = 100$. The threshold voltage $V_{th} = 0.7V$.



- Step 1: perform DC analysis

$V_{BE} = 0.7V \quad > V_{BEon}$

$V_{BC} = -2.4V \quad < V_{BCon}$

Active mode

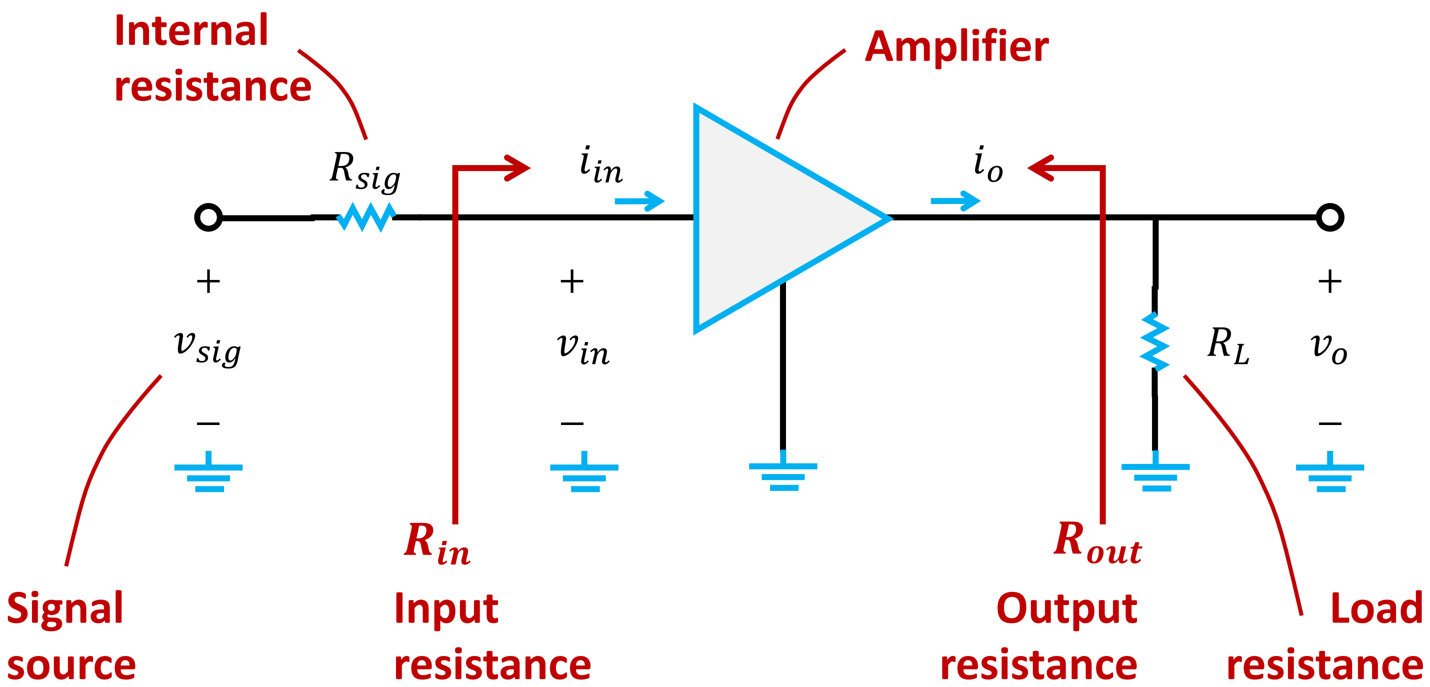
- Step 2: perform AC analysis

$$v_o = -g_m v_{be} R_C = -g_m R_C \frac{r_\pi}{R_B + r_\pi} v_{in}$$

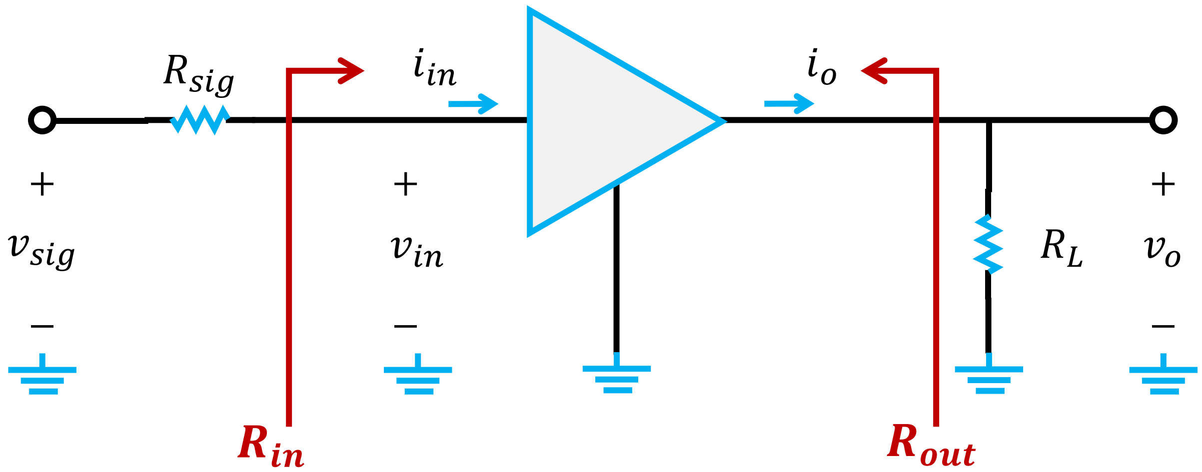
$$A_v = \frac{v_o}{v_{in}} = -g_m R_C \frac{r_\pi}{R_B + r_\pi} = -3.04$$

The AC amplitude of v_{in} is amplified

How to Describe an Amplifier



How to Describe an Amplifier

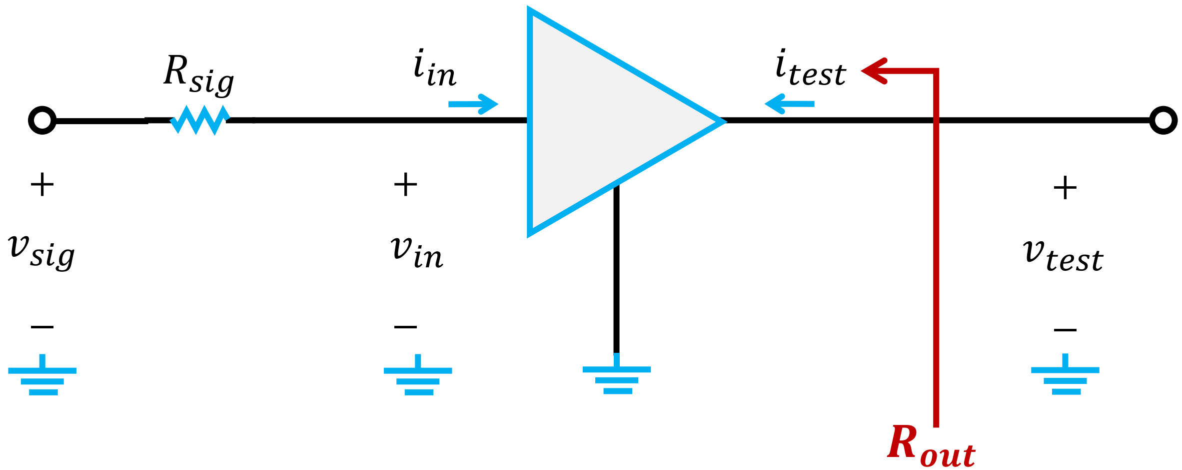


Define OPEN-CIRCUIT VOLTAGE GAIN $A_{vo} \equiv \left.\frac{v_o}{v_{in}}\right|_{R_L=\infty}$

Define INPUT RESISTANCE $R_{in} \equiv \frac{v_{in}}{i_{in}}$

How to calculate OUTPUT RESISTANCE R_{out}

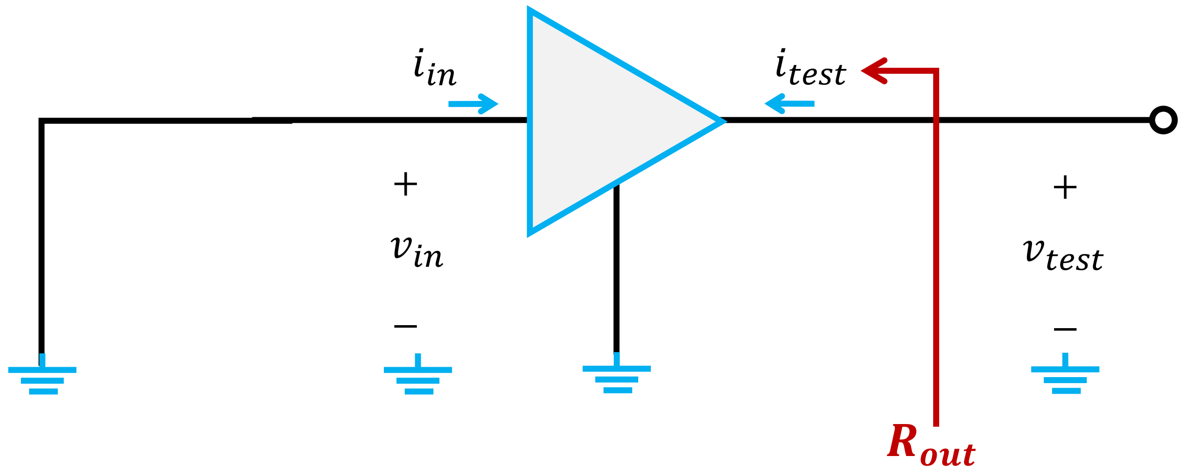
How to Describe an Amplifier



How to calculate OUTPUT RESISTANCE R_{out}

- Step 1: remove load and apply a testing voltage

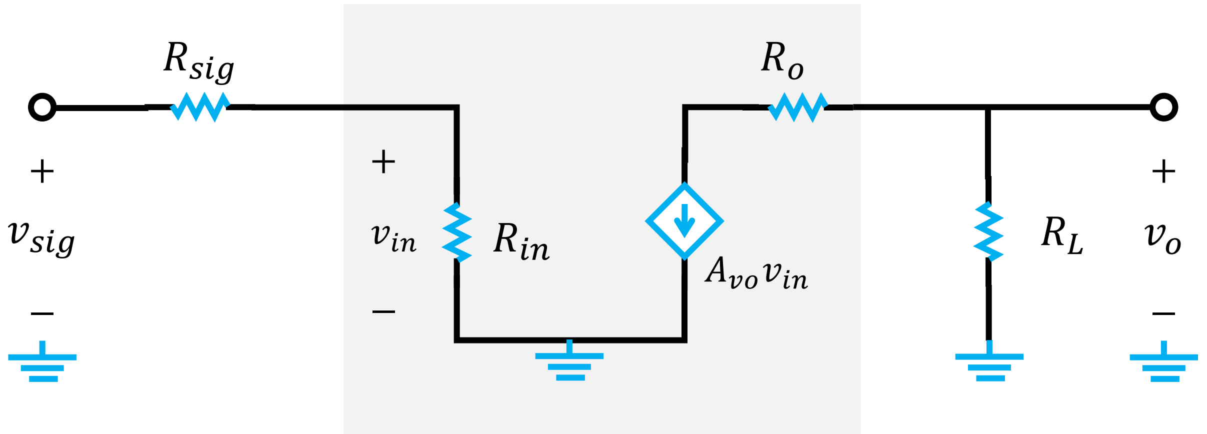
How to Describe an Amplifier



How to calculate OUTPUT RESISTANCE R_{out}

- Step 1: remove load and apply a testing voltage
- Step 2: turn off the input source
  - SHORT all voltage sources
  - OPEN all current sources
- Step 3: calculate the output resistance $R_{out} = \frac{v_{test}}{i_{test}}$

How to Describe an Amplifier



- According to KVL $$v_o = \frac{R_L}{R_L + R_o} A_{vo} v_{in}$$
- Define the voltage gain of the amplifier $$A_v \equiv \frac{v_o}{v_{in}} = \frac{R_L}{R_L + R_o} A_{vo}$$
- Define the overall voltage gain $$G_v \equiv \frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} \frac{R_L}{R_L + R_o} A_{vo}$$

Outline

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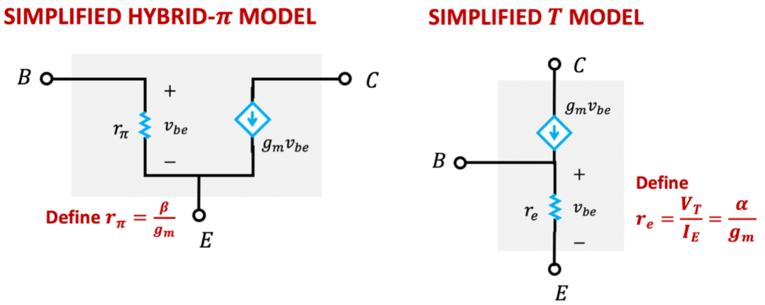
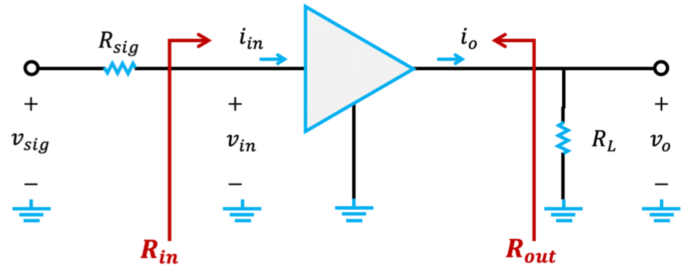
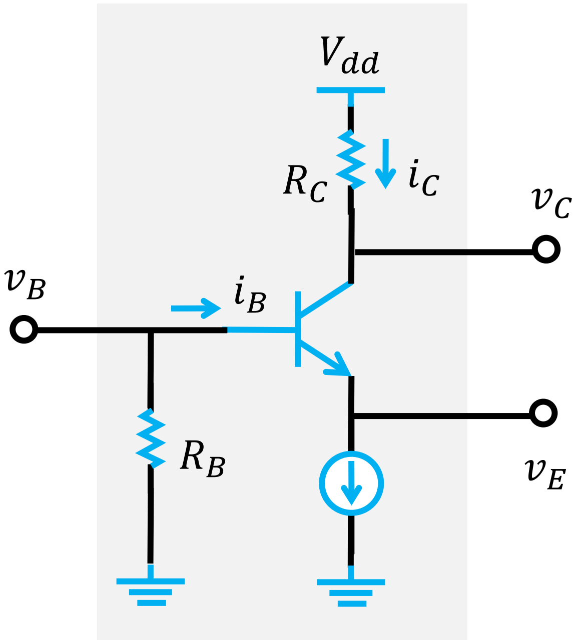
    - **Basic BJT Amplifier Configurations**



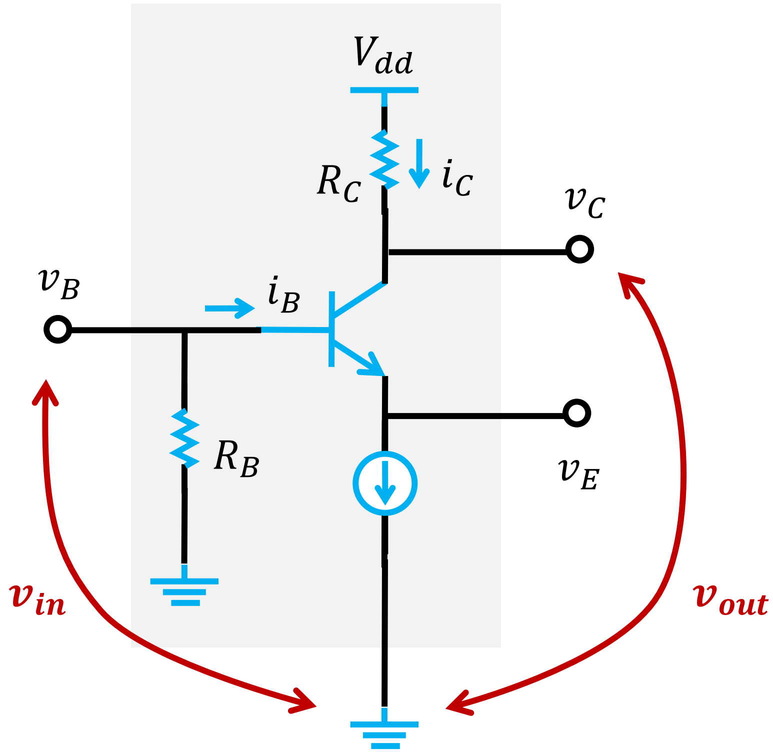


Basic BJT Amplifier Configurations



- Assume the transistor is biased in active mode
- There are three ports: **Base**, **Collector** and **Emitter**
- An amplifier requires two voltages: $\boldsymbol{v_{in}}$ and $\boldsymbol{v_o}$

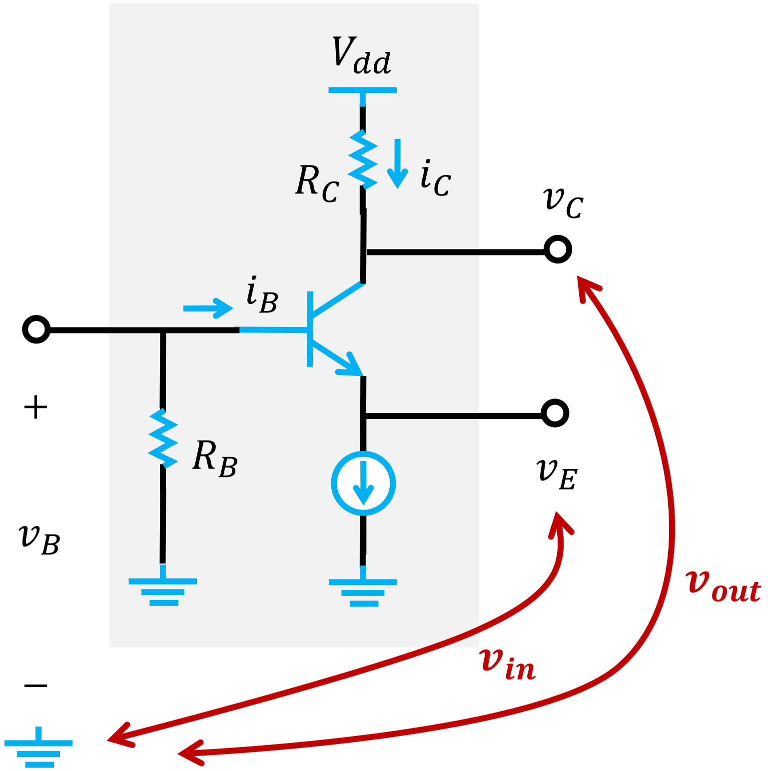
Basic BJT Amplifier Configurations



Common-Emitter (CE) Amplifier

- The Emitter is “shared” by v_{in} and v_o

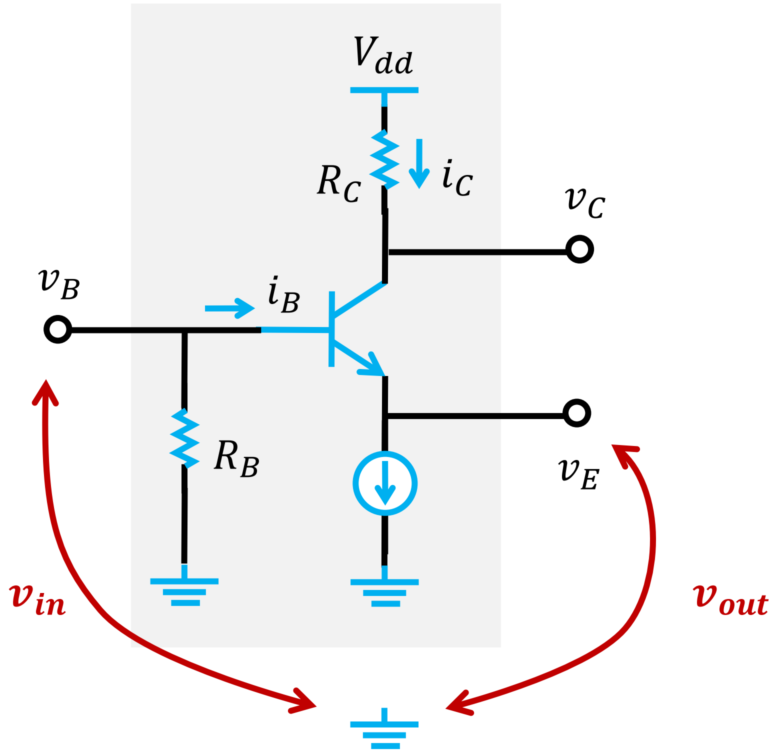
Basic BJT Amplifier Configurations



Common-Base (CB) Amplifier

- The Base is “shared” by v_{in} and v_o

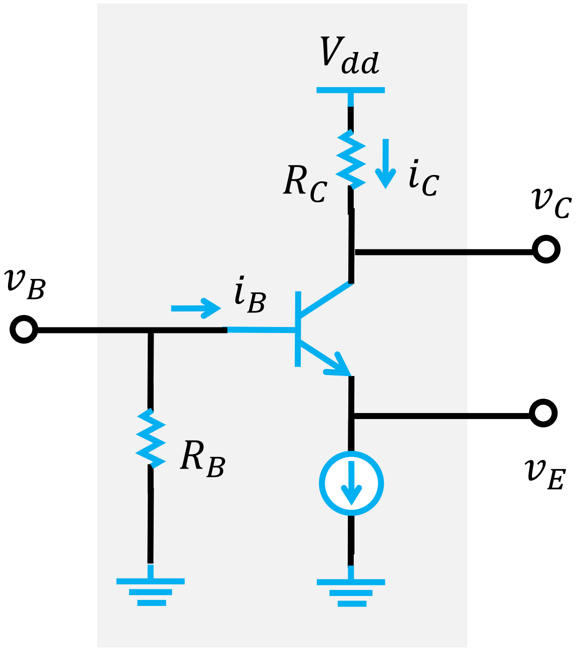
Basic BJT Amplifier Configurations



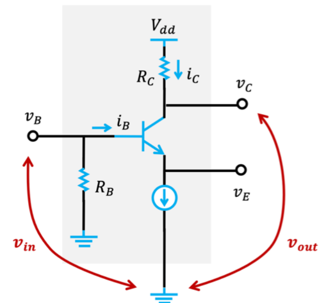
Common-Collector (CC) Amplifier

- The Collector is “shared” by v_{in} and v_o

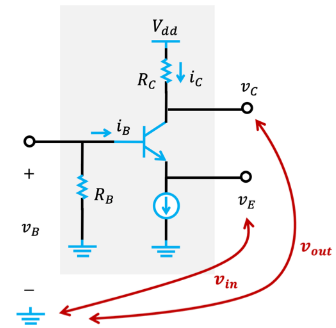
Basic BJT Amplifier Configurations



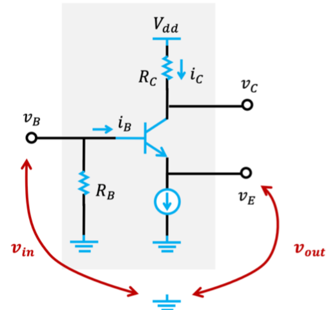
- Assume the transistor is biased in active mode
- There are three ports: **Base**, **Collector** and **Emitter**
- An amplifier requires two voltages: v_{in} and v_o



Common-Emitter (CE) Amplifier



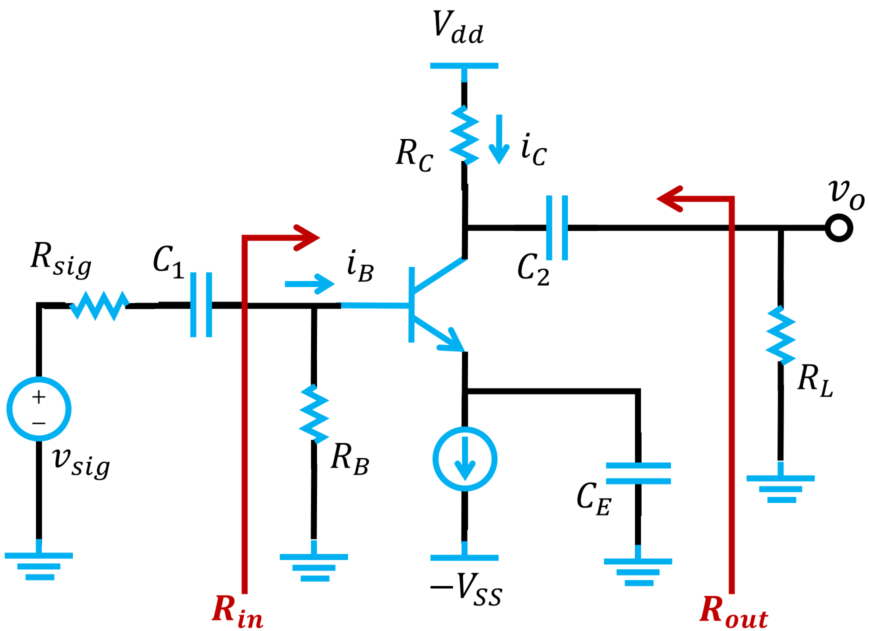
Common-Base (CB) Amplifier



Common-Collector (CC) Amplifier

Example 5: CE Amplifier

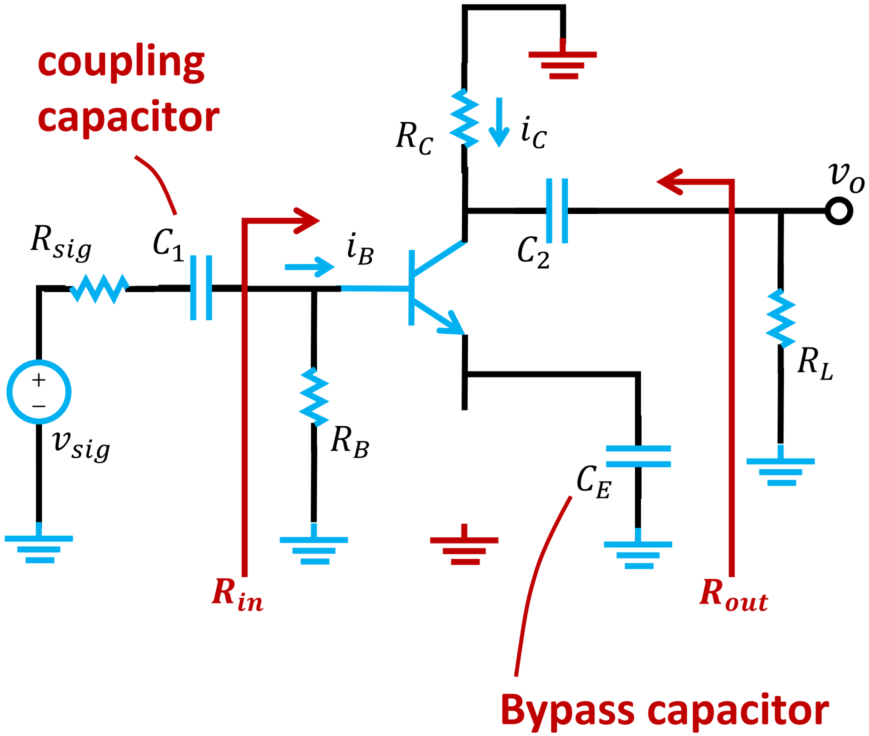
QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β.



- **Step 1: perform DC analysis**
  - Step 1.1: **ASSUME** the transistor is biased in **active mode**
  - Step 1.2: calculate all the DC currents and voltages according to KVL/KCL
  - Step 1.3: **Check ASSUMPTION**

Example 5: CE Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β.



- **Step 2: perform AC analysis**
  - Step 2.1: turn off DC sources
    - SHORT all voltage sources
    - OPEN all current sources

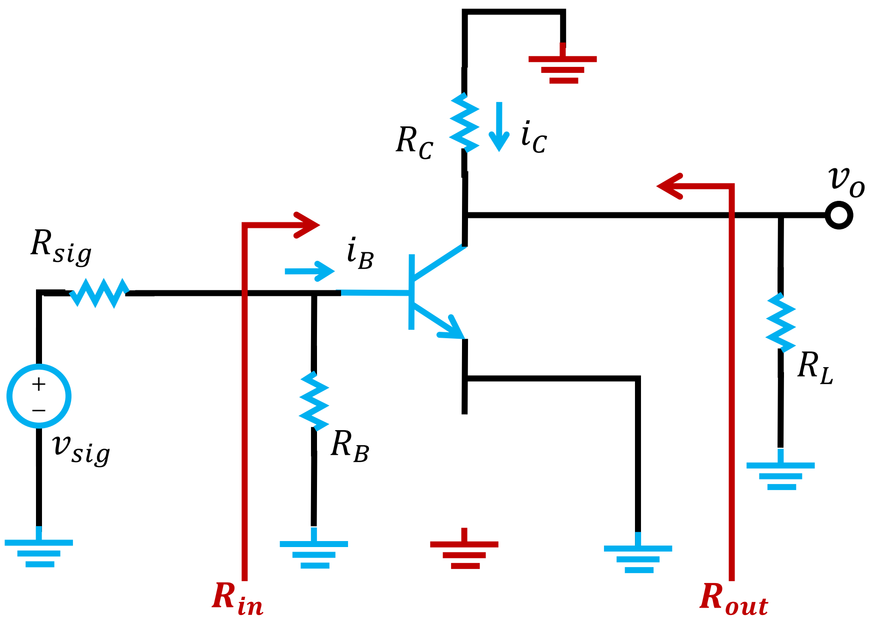
$$Z_{C_E} = \frac{1}{sC_E}$$

The bypass capacitor C_E provides a very LOW impedance to ground at all signal frequencies of interest

The coupling capacitor C_1 and C_2 act as perfect short circuit at all signal frequencies of interest

Example 5: CE Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2, C_E, V_{dd}, V_{SS}, and β.

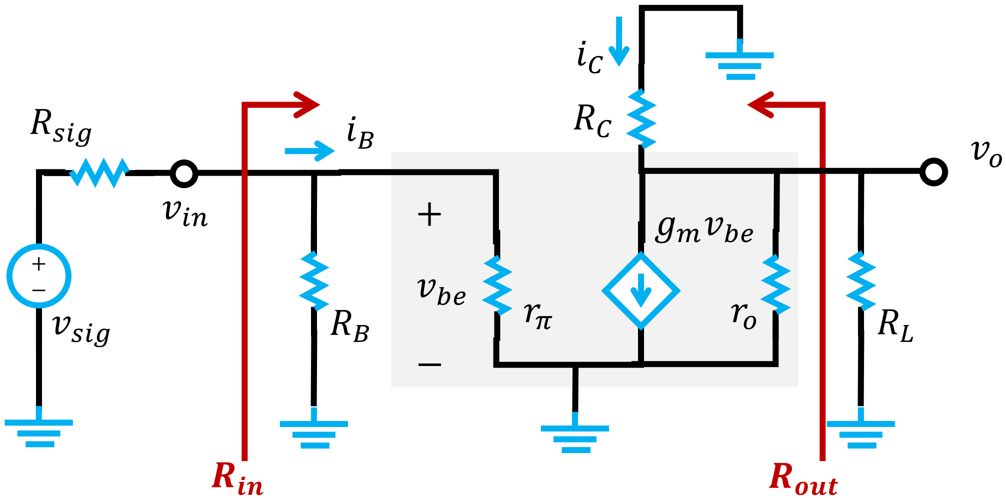


- **Step 2: perform AC analysis**
  - Step 2.1: turn off DC sources
    - SHORT all voltage sources
    - OPEN all current sources

  The capacitors can APPROXIMATELY be as short circuit @ AC

Example 5: CE Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β.



- **Step 2: perform AC analysis**
  - Step 2.1: turn off DC sources
    - SHORT all voltage sources
    - OPEN all current sources
  - Step 2.2: Calculate small-signal model parameters, β and r_π
  - Step 2.3: replace the transistor with the small-signal model
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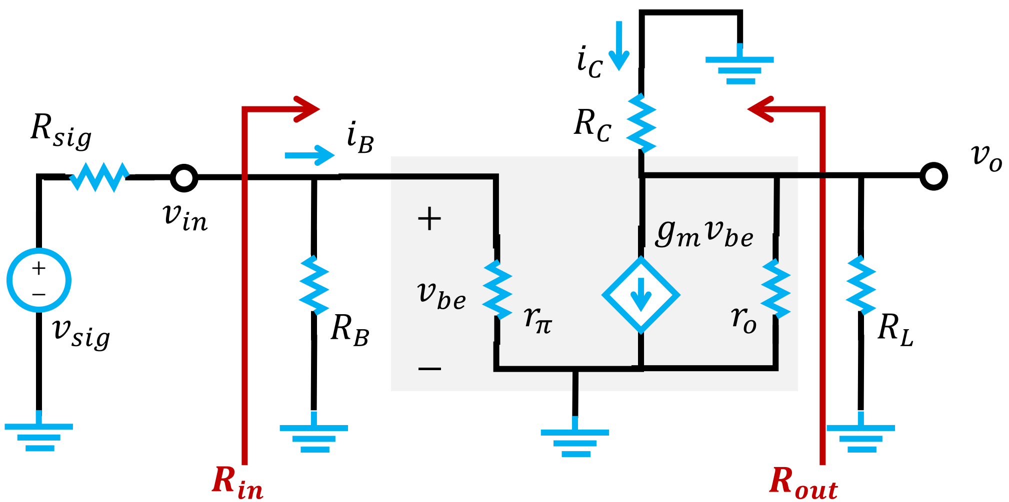
Example 5: CE Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β.



- According to KVL

$$v_{in} = v_{be} = \frac{R_B||r_\pi}{R_{sig} + R_B||r_\pi} v_{sig}$$

$$v_o = -g_m v_{be}(R_C||R_L||r_o)$$

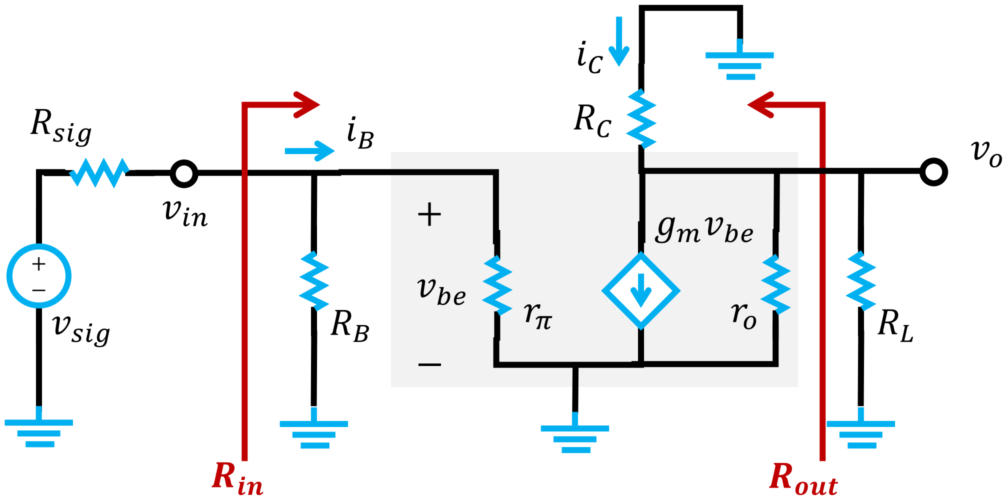
- The overall voltage gain

$$G_v = \frac{v_o}{v_{sig}}$$

$$= -\frac{R_B||r_\pi}{R_{sig}+R_B||r_\pi} g_m(R_C||R_L||r_o)$$

Example 5: CE Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β.



- Calculate the open-circuit voltage gain @ $R_L = \infty$

$$A_{vo} = \frac{v_o}{v_{in}} = -g_m(R_C||r_o)$$

Since $R_C \ll r_o$ ➡ $A_{vo} \approx -g_m R_C$

- The input resistance

$$R_{in} = R_B||r_\pi \approx r_\pi$$

Since $R_B \gg r_\pi$

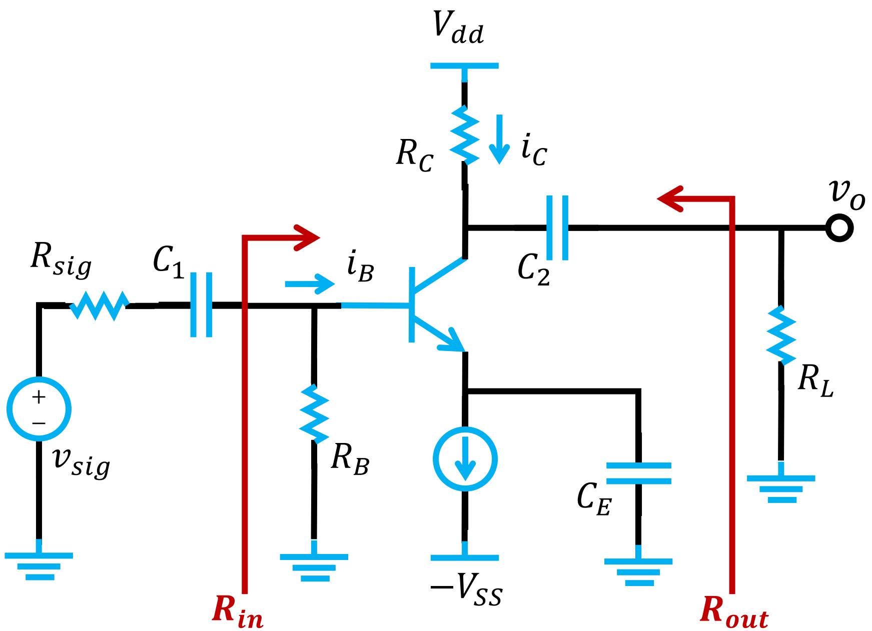
- The output resistance

$$R_{out} = R_C||r_o \approx R_C$$

Since $r_o \gg R_C$

Example 5: CE Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2, C_E, V_{dd}, V_{SS}, and β.



- The **VOLTAGE GAIN** is relative **high**

$$G_v = \frac{v_o}{v_{sig}} = -\frac{R_B||r_\pi}{R_{sig} + R_B||r_\pi} g_m(R_C||R_L||r_o)$$

$$A_{vo} = \frac{v_o}{v_{in}} = -g_m(R_C||r_o)$$

- The **INPUT RESISTANCE** is relative **low**

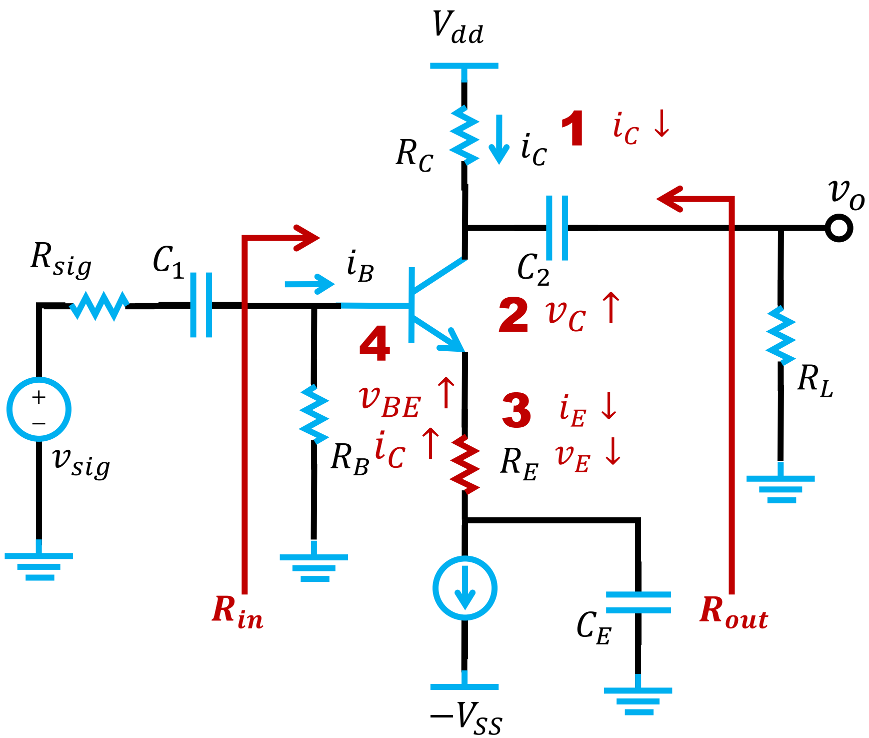
$$R_{in} = R_B||r_\pi \approx r_\pi$$

- The **OUTPUT RESISTANCE** is relative **high**

$$R_{out} = R_C||r_o \approx R_C$$

Example 6: CE Amp. w/ emitter res.

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, R_E, C_1, C_2,C_E, V_{dd}, V_{SS}, and β. Ignore the Early effect.

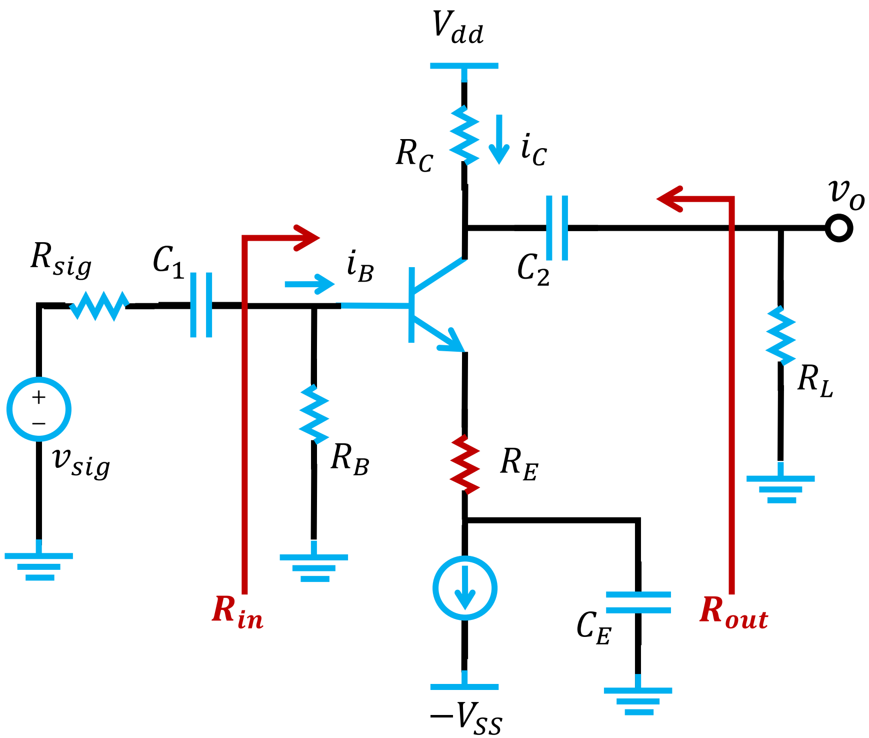


- What is R_E used for?
  - If there is a decrease @ i_C — $i_C \downarrow$
  - v_C increases due to R_C
  - i_E and v_E decrease correspondingly
  - v_{BE} increases, and i_C increases — $i_C \uparrow$

NEGATIVE FEEDBACK is observed

Example 6: CE Amp. w/ emitter res.

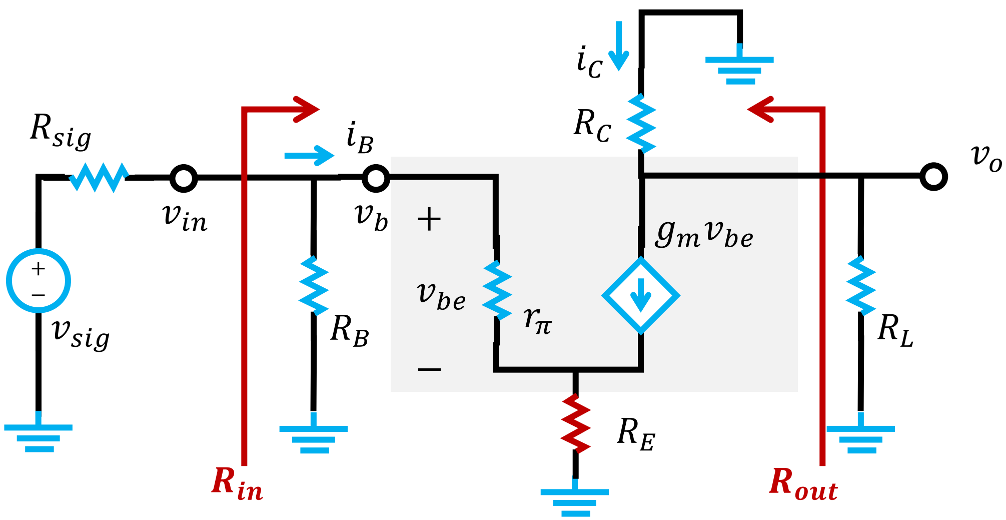
QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, R_E, C_1, C_2,C_E, V_{dd}, V_{SS}, and β. Ignore the Early effect.



- **Step 1: perform DC analysis (SKIP)**
- **Step 2: perform AC analysis**
  - Step 2.1: turn off DC sources **(SKIP)**
  - Step 2.2: Calculate small-signal model parameters, β and r_π **(SKIP)**
  - Step 2.3: replace the transistor with the small-signal model **(SKIP)**
  - **Step 2.4: Analyze the resulting circuit**

Example 6: CE Amp. w/ emitter res.

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, R_E, C_1, C_2,C_E, V_{dd}, V_{SS}, and β. Ignore the Early effect.



- According to KVL

$$v_{in} = (i_B + g_m r_\pi i_B)R_E + r_\pi i_B$$

$$v_{out} = -\beta i_B (R_C || R_L)$$

- The input resistance

$$R_{in} = R_B || \frac{v_{test}}{i_{test}}$$

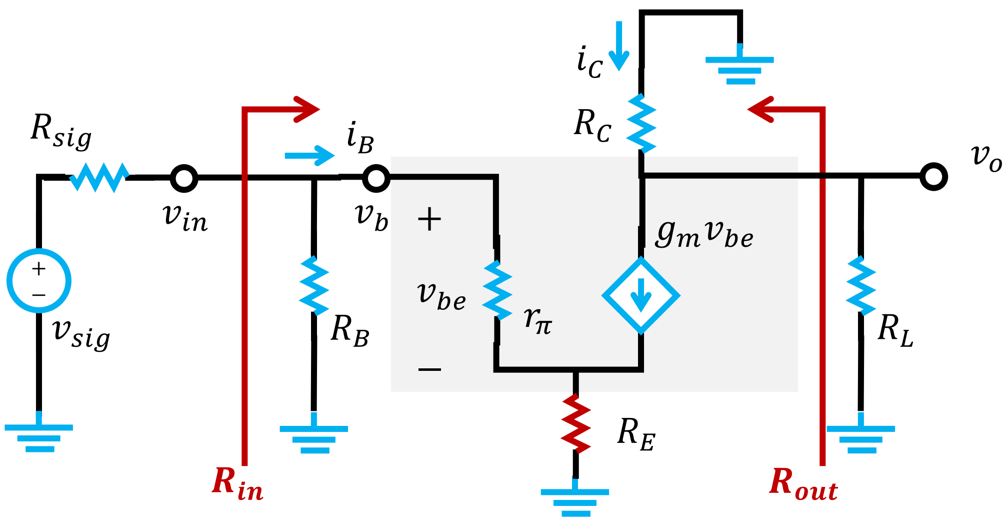
$$= R_B || \big((1 + g_m r_\pi)R_E + r_\pi\big)$$

Since $R_B \gg r_\pi$

$$\approx (1 + g_m r_\pi)R_E + r_\pi$$

Example 6: CE Amp. w/ emitter res.

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, R_E, C_1, C_2, C_E, V_{dd}, V_{SS}, and β. Ignore the Early effect.



- The overall voltage gain

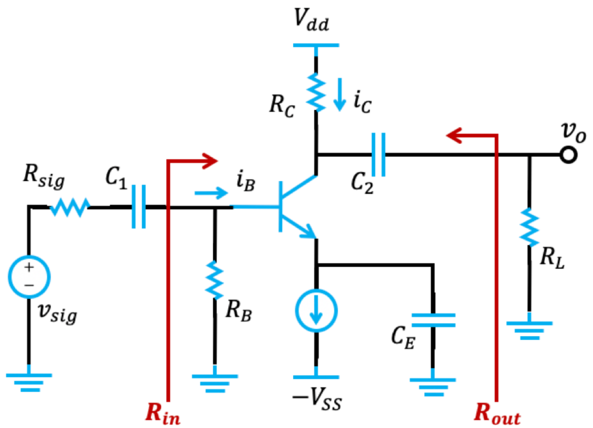
$$G_v = \frac{v_{out}}{v_{sig}} = \frac{v_b}{v_{sig}} \frac{v_{out}}{v_b}$$

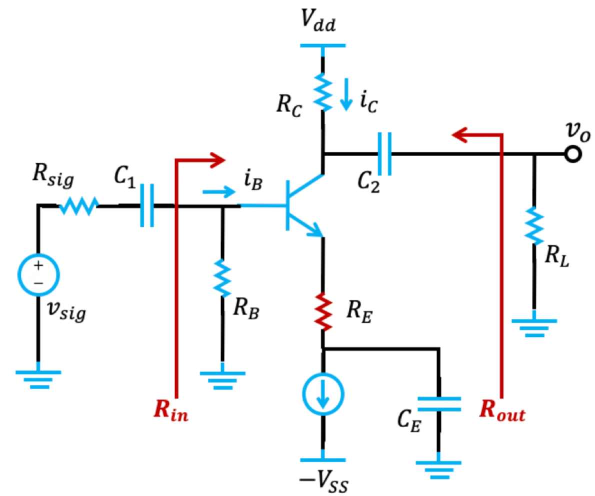
$$= -\frac{R_{in}}{R_{sig} + R_{in}} \frac{g_m r_\pi (R_C || R_L)}{(1 + g_m r_\pi) R_E + r_\pi}$$

Since $R_{in} \approx (1 + g_m r_\pi) R_E + r_\pi$

$$\approx -\frac{g_m r_\pi (R_C || R_L)}{R_{sig} + (1 + g_m r_\pi) R_E + r_\pi}$$

Example 6: CE Amp. w/ emitter res.

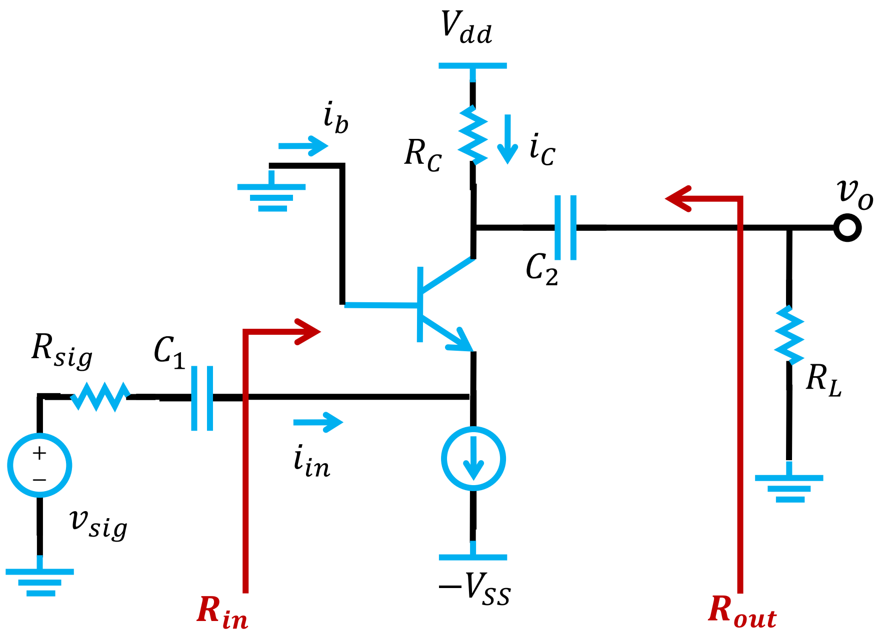




	CE w/o emitter res.	CE w/ emitter res.
R_{in}	r_π	$(1+g_m r_\pi)R_E + r_\pi$ ↑
G_v	$-\frac{R_B\|\|r_\pi}{R_{sig}+R_B\|\|r_\pi} g_m(R_C\|\|R_L)$	$-\frac{g_m r_\pi (R_C\|\|R_L)}{R_{sig}+(1+g_m r_\pi)R_E + r_\pi}$ ↓

Example 7: CB Amplifier

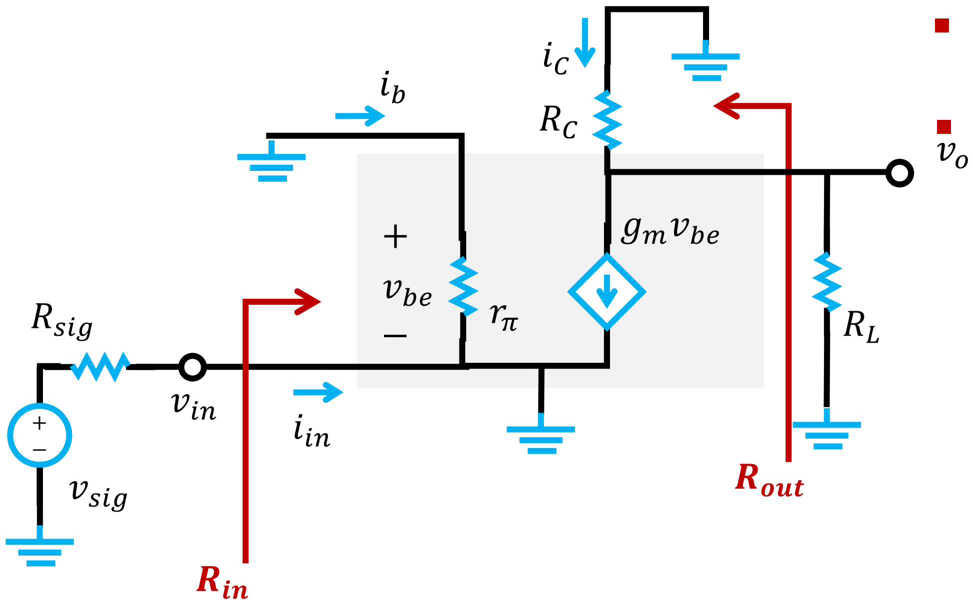
QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β. Ignore the Early effect.



- **Step 1: perform DC analysis (SKIP)**

Example 7: CB Amplifier

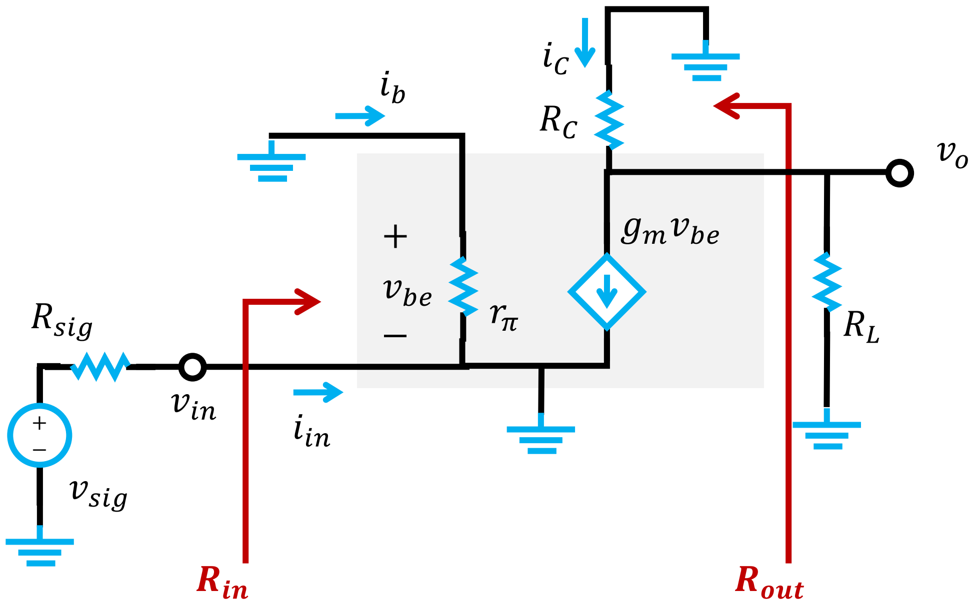
QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β. Ignore the Early effect.



- **Step 1: perform DC analysis (SKIP)**
- **Step 2: perform AC analysis**
  - Step 2.1: turn off DC sources **(SKIP)**
  - Step 2.2: Calculate small-signal model parameters, β and r_π **(SKIP)**
  - Step 2.3: replace the transistor with the small-signal model **(SKIP)**
  - **Step 2.4: Analyze the resulting circuit**

Example 7: CB Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2, C_E, V_{dd}, V_{SS}, and β. Ignore the Early effect.



- According to KVL

$$v_{out} = -g_m v_{be}(R_C||R_L)$$

$$v_{in} = -v_{be} = \frac{R_{in}}{R_{sig} + R_{in}} v_{sig}$$

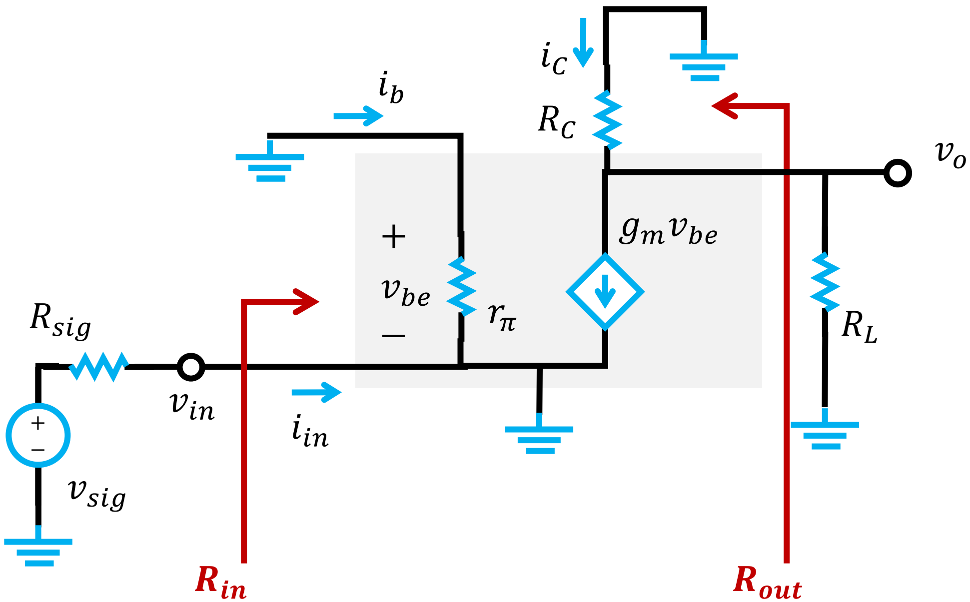
- The input resistance

$$R_{in} = \frac{v_{test}}{i_{test}} = \frac{v_{in}}{i_{in}}$$

$$= \frac{-i_b r_\pi}{-i_b - g_m i_b r_\pi} = \frac{r_\pi}{1 + \beta}$$

Example 7: CB Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β. Ignore the Early effect.



- The overall voltage gain

$$G_v = \frac{v_{out}}{v_{sig}} = \frac{v_{in}}{v_{sig}}\frac{v_{out}}{v_{in}}$$

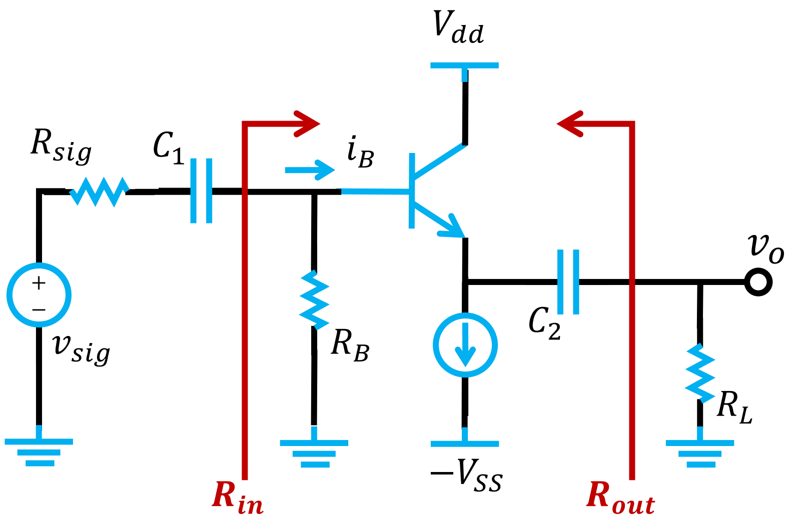
$$= \frac{R_{in}}{R_{sig} + R_{in}} g_m (R_C||R_L)$$

$$= \frac{1}{R_{sig}\dfrac{1+\beta}{\beta} + \dfrac{1}{g_m}}(R_C||R_L)$$

$$\approx \frac{1}{R_{sig} + \dfrac{1}{g_m}}(R_C||R_L)$$

Example 8: CC Amplifier

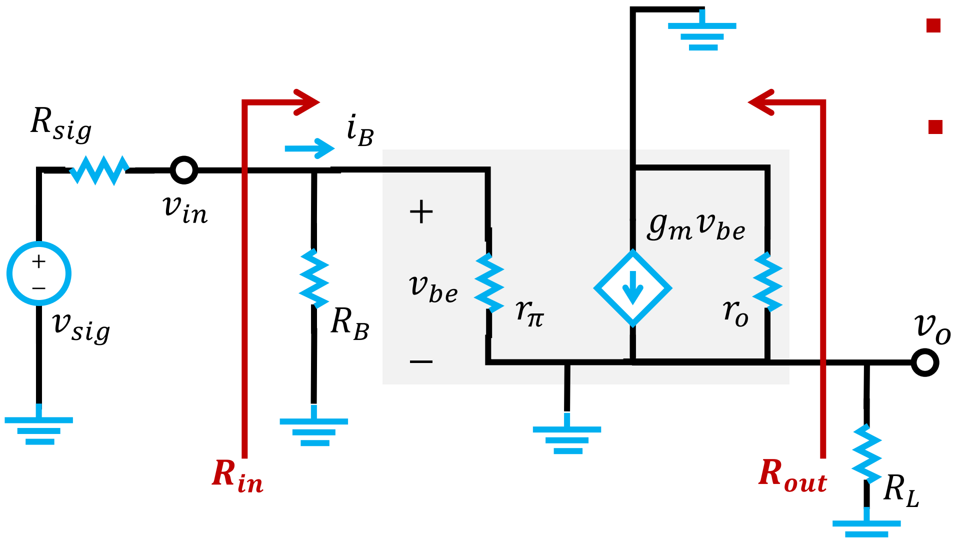
QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β.



- **Step 1: perform DC analysis (SKIP)**

Example 8: CC Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β.



- **Step 1: perform DC analysis (SKIP)**
- **Step 2: perform AC analysis**
  - Step 2.1: turn off DC sources **(SKIP)**
  - Step 2.2: Calculate small-signal model parameters, β and r_π **(SKIP)**
  - Step 2.3: replace the transistor with the small-signal model **(SKIP)**
  - **Step 2.4: Analyze the resulting circuit**

Example 8: CC Amplifier

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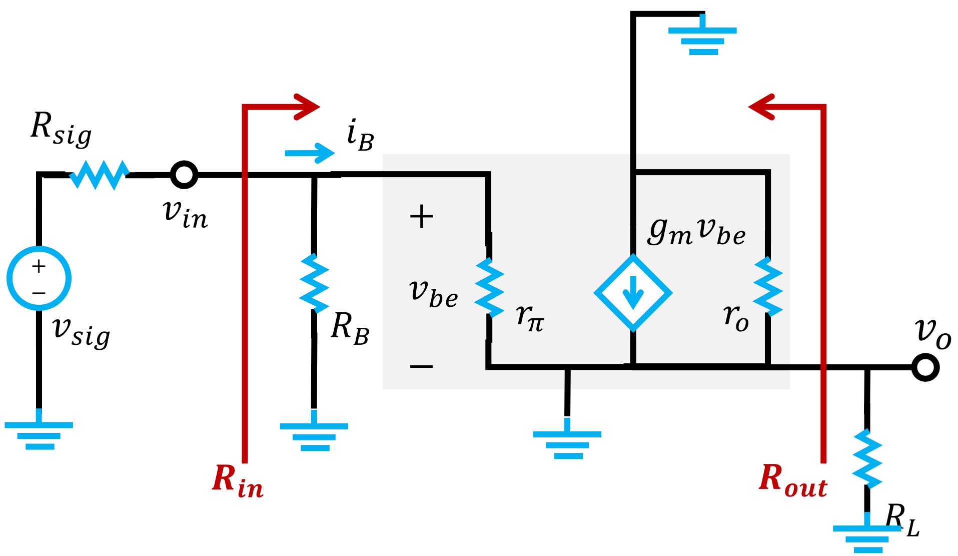
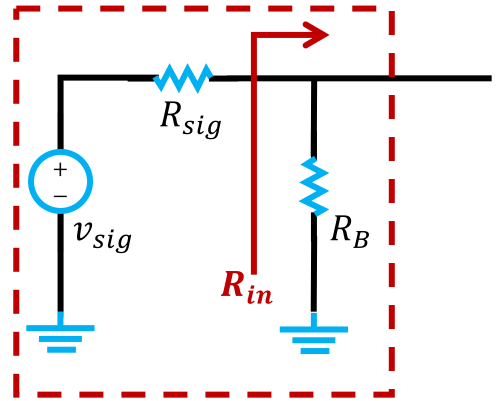
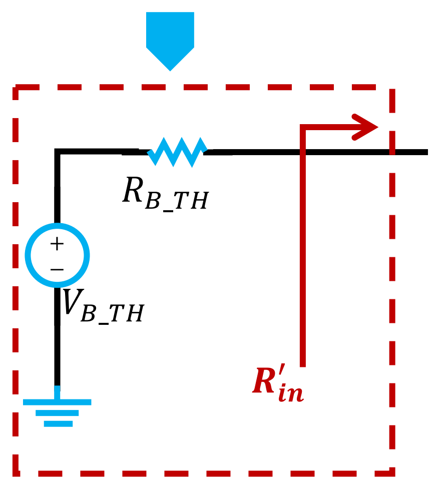






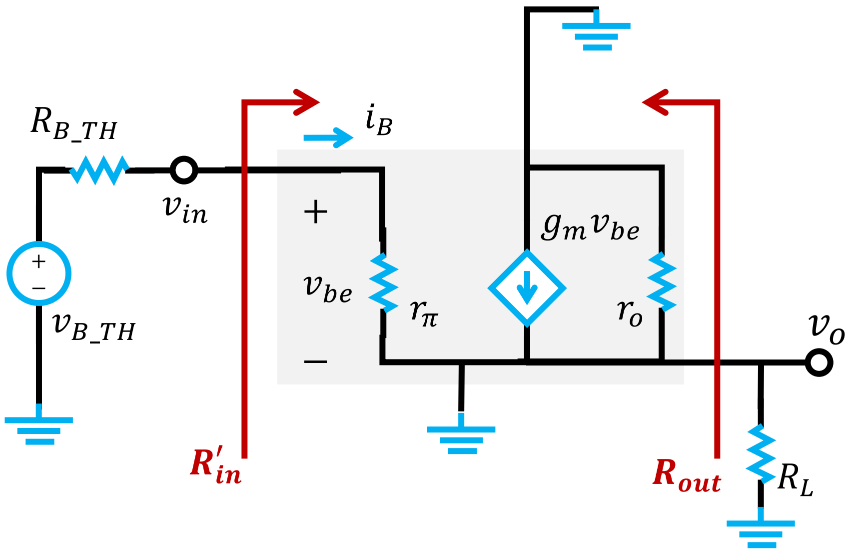
$$R_{B\_TH} = R_{sig}||R_B$$

$$v_{B\_TH} = \frac{R_B}{R_{sig} + R_B} v_{sig}$$

$$R_{in} = R'_{in}||R_B$$

Example 8: CC Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2, C_E, V_{dd}, V_{SS}, and β.



- The input resistance

$$R_{in} = R'_{in}||R_B = \frac{v_{be}}{i_b}||R_B$$

$$= \frac{i_B r_\pi + (i_B + g_m i_B r_\pi)(r_o||R_L)}{i_B}||R_B$$

$$= [r_\pi + (1+\beta)(r_o||R_L)]||R_B$$

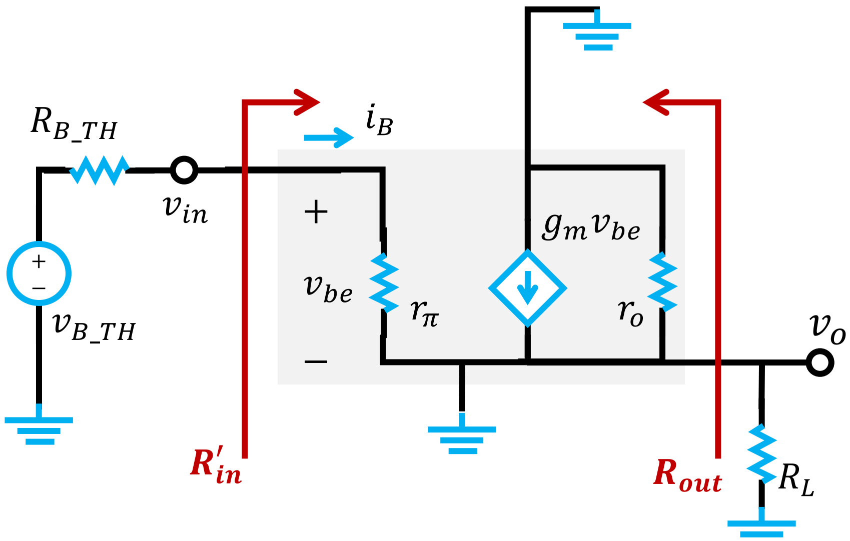
$$R_{B\_TH} = R_{sig}||R_B$$

$$v_{B\_TH} = \frac{R_B}{R_{sig}+R_B} v_{sig}$$

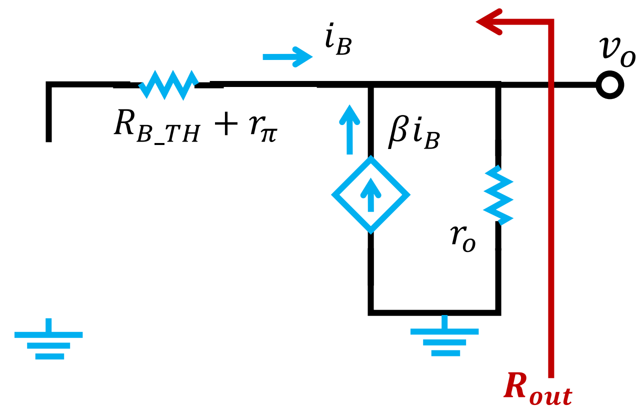
$$R_{in} = R'_{in}||R_B$$

Example 8: CC Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β.



- The output resistance



$$R_{B\_TH} = R_{sig}||R_B$$

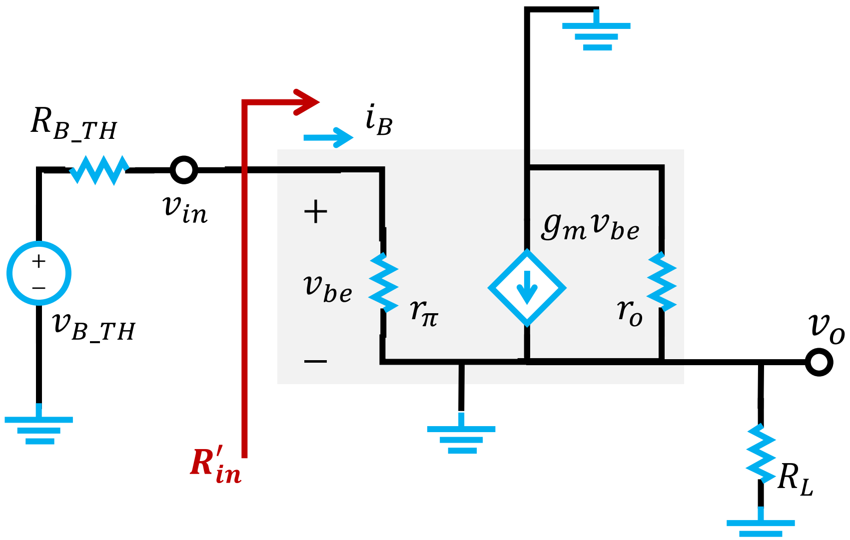
$$v_{B\_TH} = \frac{R_B}{R_{sig} + R_B} v_{sig}$$

$$R_{in} = R'_{in}||R_B$$

$$R_{out} = r_o|| \frac{R_{TH} + r_\pi}{1 + \beta} \approx \frac{R_{TH} + r_\pi}{1 + \beta}$$

Example 8: CC Amplifier

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β.



- The output voltage

$$v_o = i_B(R'_{in} - r_\pi) = \frac{v_{B_{TH}}}{R_{B_{TH}} + R'_{in}}(R'_{in} - r_\pi)$$

$$= \frac{R_B}{R_{sig} + R_B}\frac{(1+\beta)(r_o||R_L)}{R_{sig}||R_B + R'_{in}}v_{sig}$$

- The overall gain

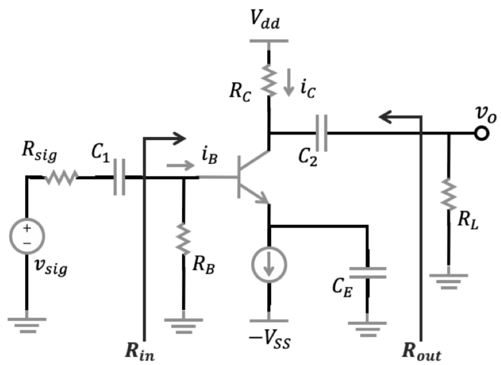
$$G_v = \frac{v_o}{v_{sig}} = \frac{R_B}{R_{sig} + R_B}\frac{(1+\beta)(r_o||R_L)}{R_{sig}||R_B + R'_{in}}v_{sig}$$

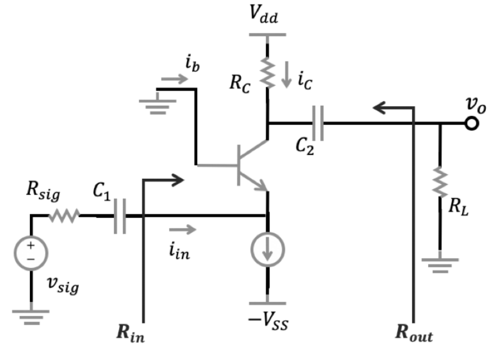
If $R_B \gg R_{sig}$, $R_{in} \gg R_{sig}||R_B$

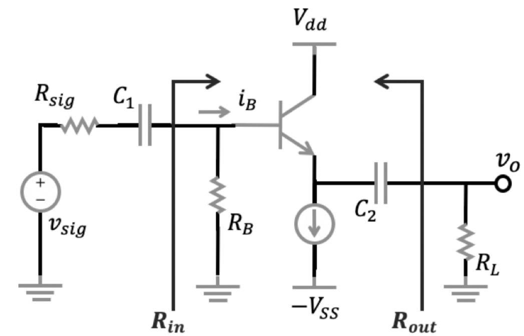
≈ 1

The CC amplifier is also called EMITTER FOLLOWER

Summary: 3 BJT Amp. Configurations



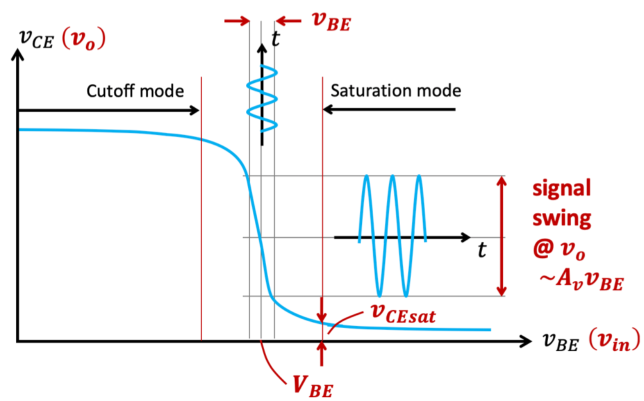




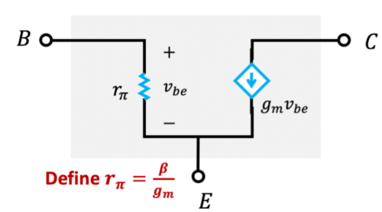
	CE Amplifier	CB Amplifier	CC Amplifier
R_{in}	$\approx r_\pi$	$\frac{r_\pi}{1+\beta}$	$[r_\pi + (1+\beta)(r_o \| R_L)] \| R_B$
R_{out}	$\approx R_C$	R_C	$\approx \frac{R_{TH} + r_\pi}{1+\beta}$
G_v	$-\frac{R_B \| r_\pi}{R_{sig} + R_B \| r_\pi} g_m (R_C \| R_L)$	$\approx \frac{1}{R_{sig} + \frac{1}{g_m}} (R_C \| R_L)$	≈ 1

Outline

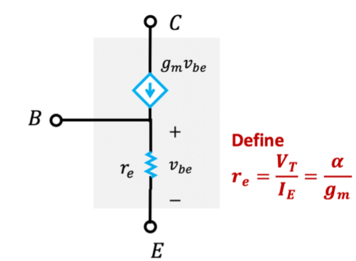
- Introduction to BJT
  - Device structure
  - How does it work?
    - Cutoff / Active / Reverse / Saturation mode
  - npn v.s. pnp
- The characteristic curves
  - $i - v$ characteristics
  - The transfer characteristic
- Circuit analysis techniques with BJT
  - DC analysis techniques
  - AC analysis techniques
    - Locate the bias point
    - Small-signal operation & model
    - Characterizing Amplifiers
    - Basic BJT Amplifier Configurations
  - **The frequency response**

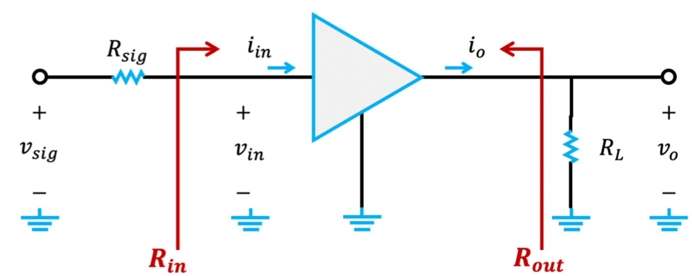


SIMPLIFIED HYBRID-π MODEL

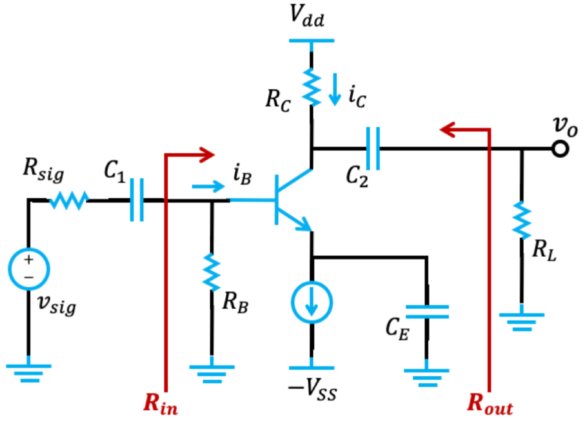


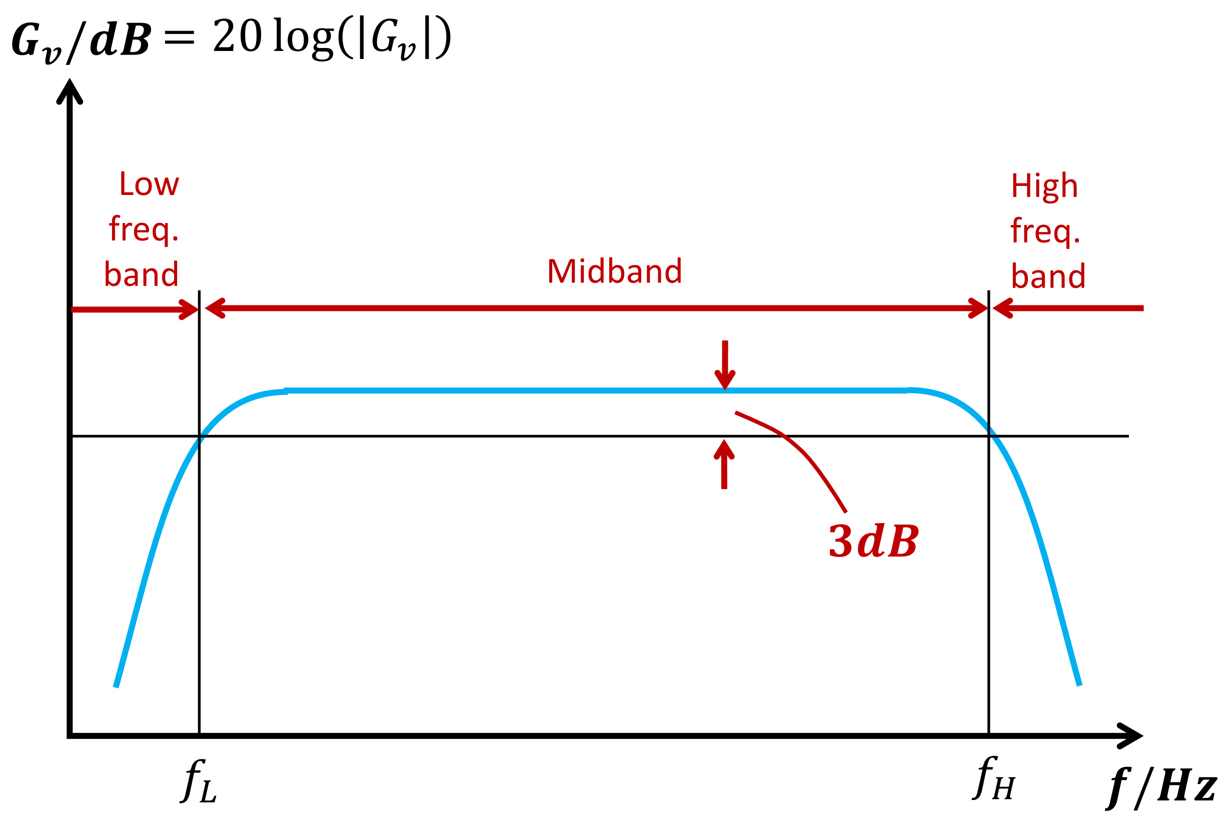
SIMPLIFIED T MODEL



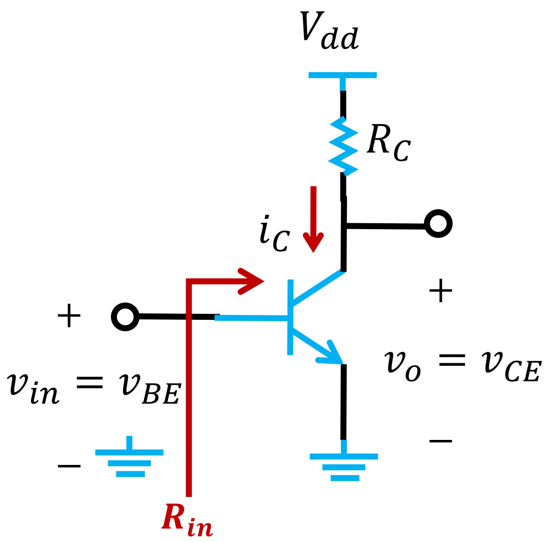


The Frequency Response

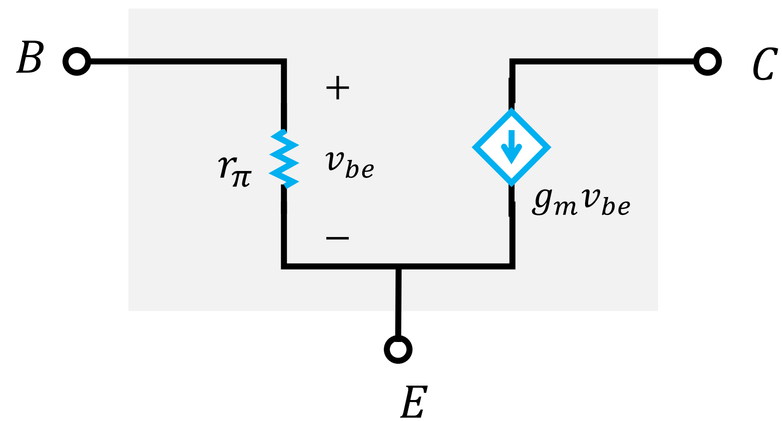




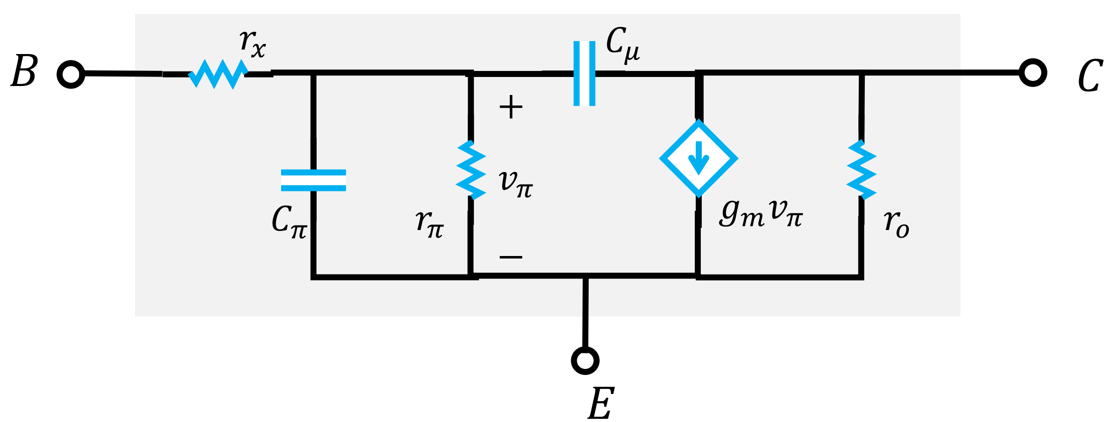
Small-Signal Model



SIMPLIFIED HYBRID-π MODEL

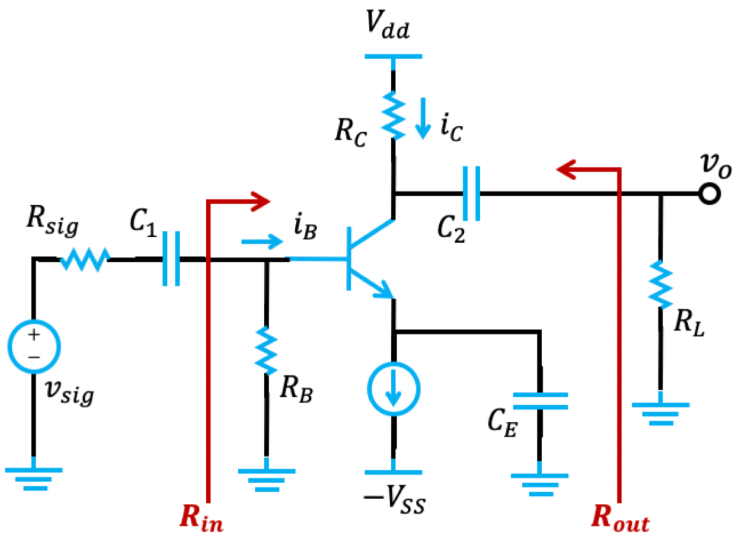


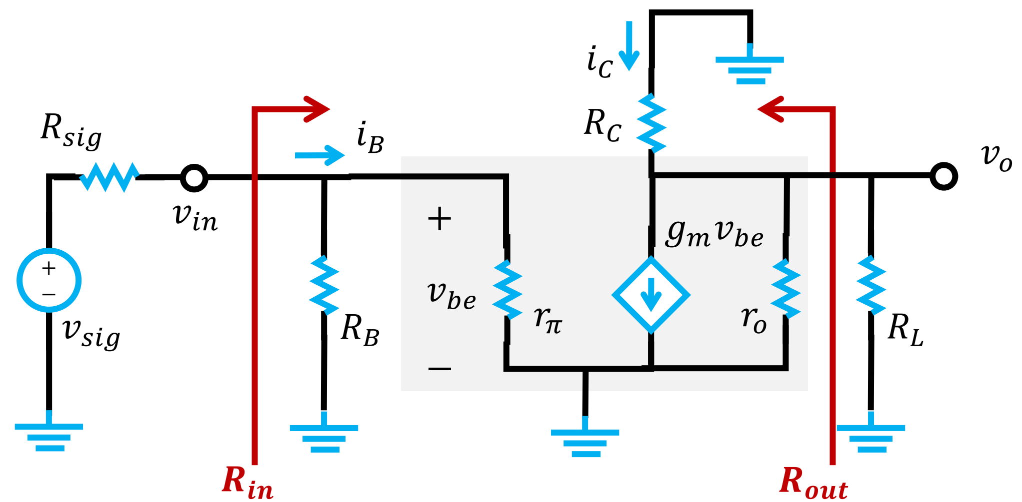
THE HIGH FREQUENCY MODEL



Recall: Example 5: CE Amplifier

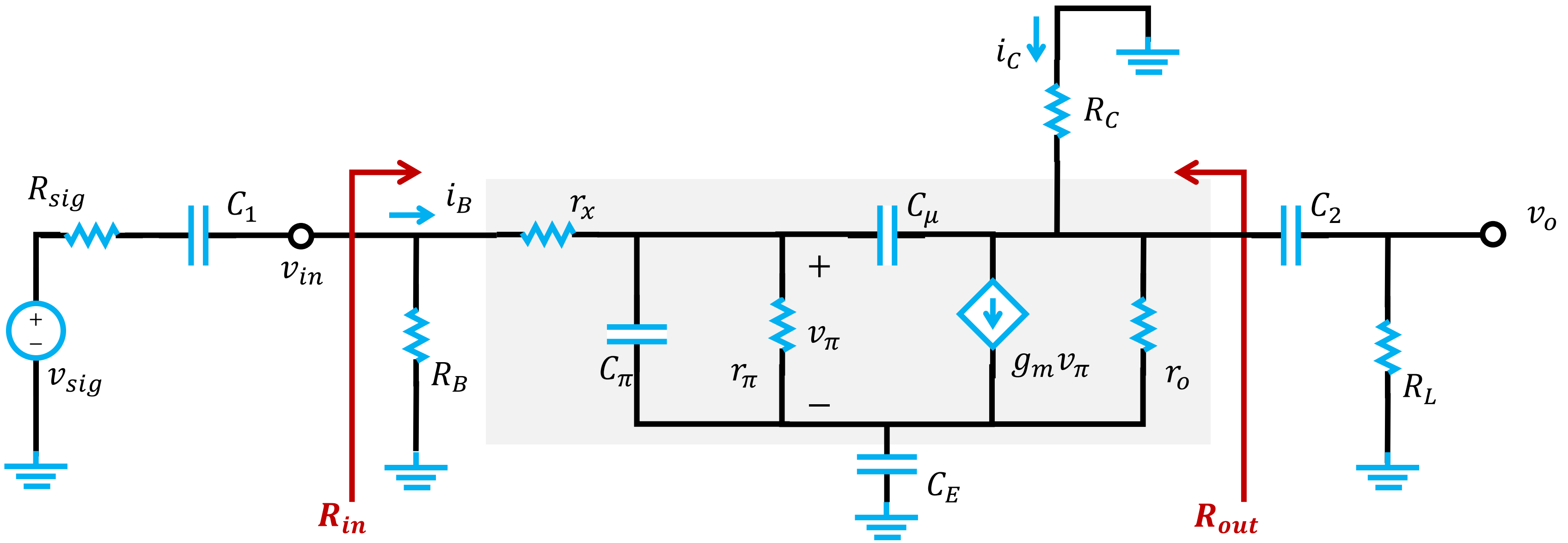
QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2,C_E, V_{dd}, V_{SS}, and β.





CE Amplifier w/ High Freq. Model

QUESTION: Find out the voltage gain, the input and output resistances with given v_{in}, R_{sig}, R_B, R_C, C_1, C_2, C_E, V_{dd}, V_{SS}, and β.



We will go through frequency response in details for MOS transistor circuits

Outline

- Introduction to BJT
  - Device structure
  - How does it work? – 4 modes
  - npn v.s. pnp
- The characteristic curves
  - $i - v$ characteristics
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- Circuit analysis techniques with BJT
  - DC analysis techniques
  - AC analysis techniques
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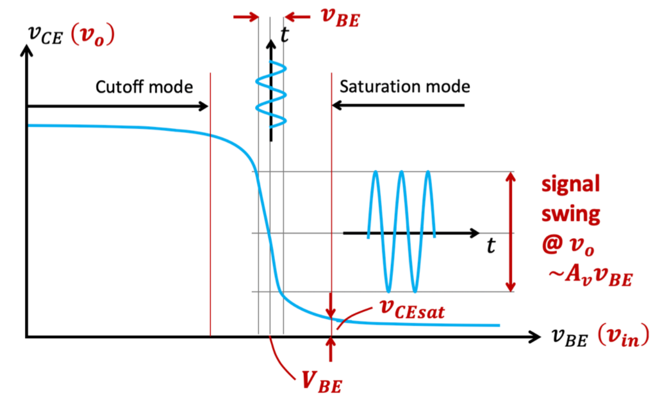

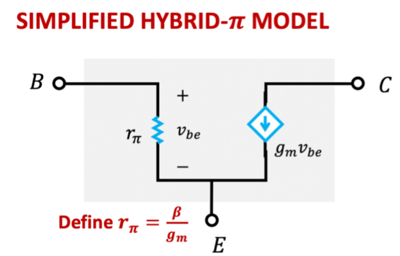

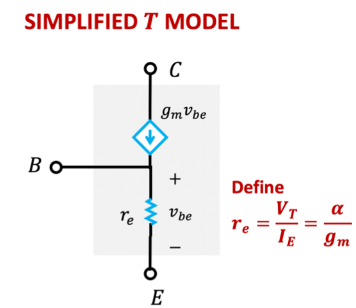

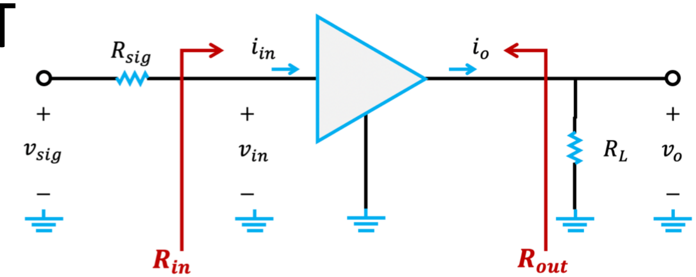

Reading tasks & learning goals

- Reading tasks
  - Microelectronic Circuits, 6^{th} edition
    - Chapter 6
- Learning goals
  - Know the **structure** of a BJT and **how it works**
  - Well understand the **characteristic** of BJT
  - Well understand how to locate the **bias point** of a BJT in circuit
  - Understand how to describe the performance of an amplifier
  - Well understand how to analyze a circuit with **BJT** in active mode
  - Know the BJT amplification circuit **configuration** and **analysis**