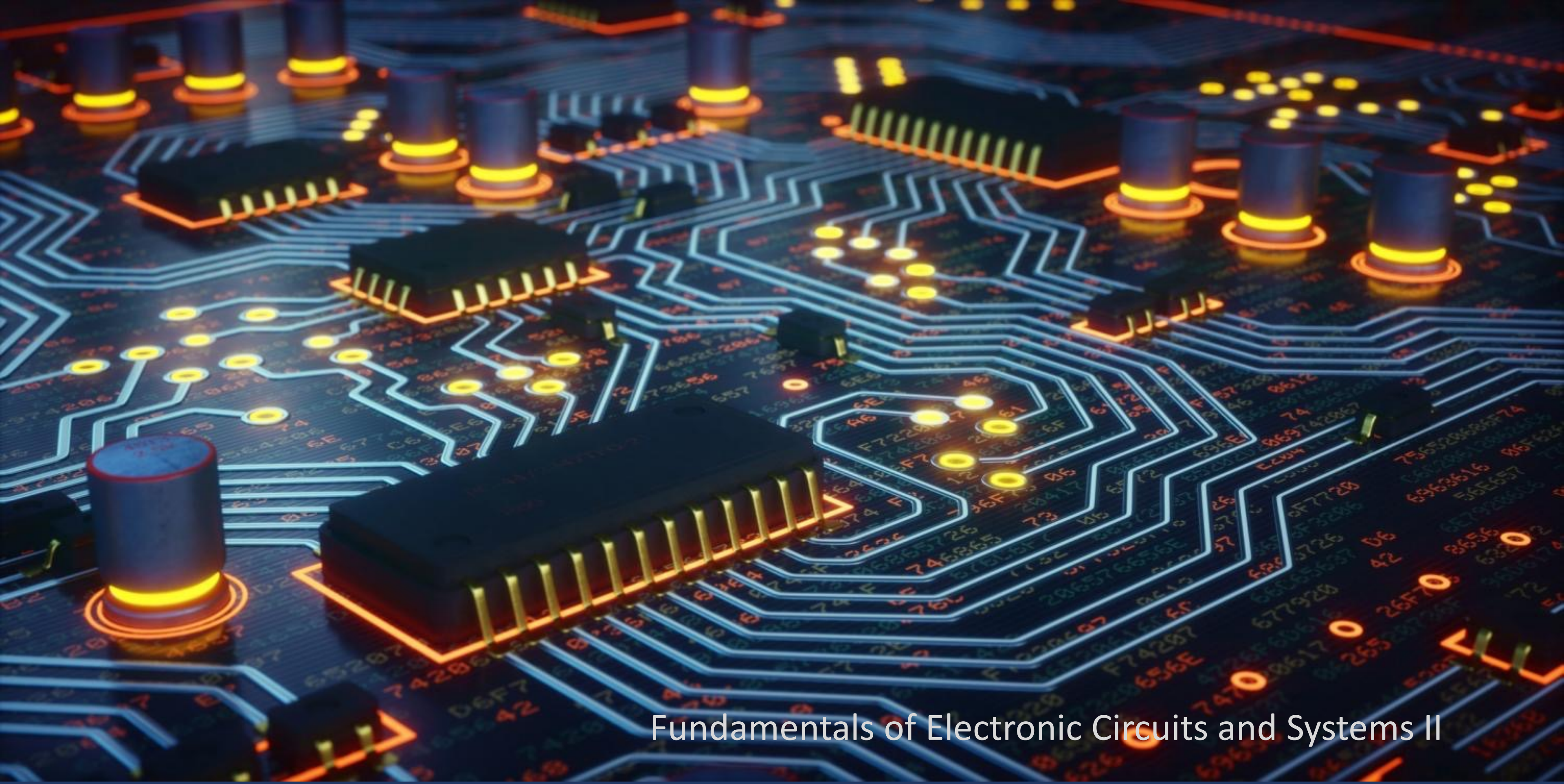

Fundamentals of Electronic Circuits and Systems II

pn Junction & Diodes

Milin Zhang

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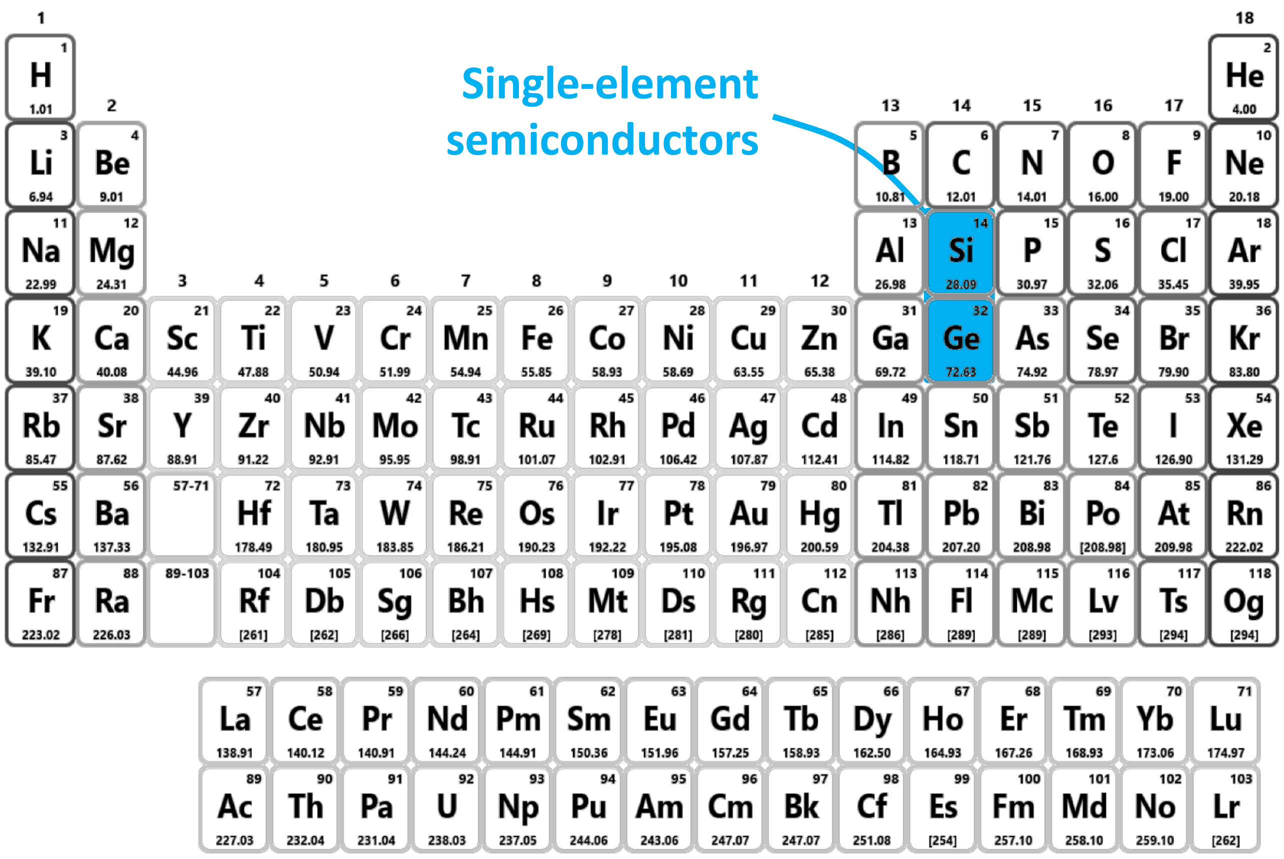
Outline

- Introduction to semiconductors
- Diodes

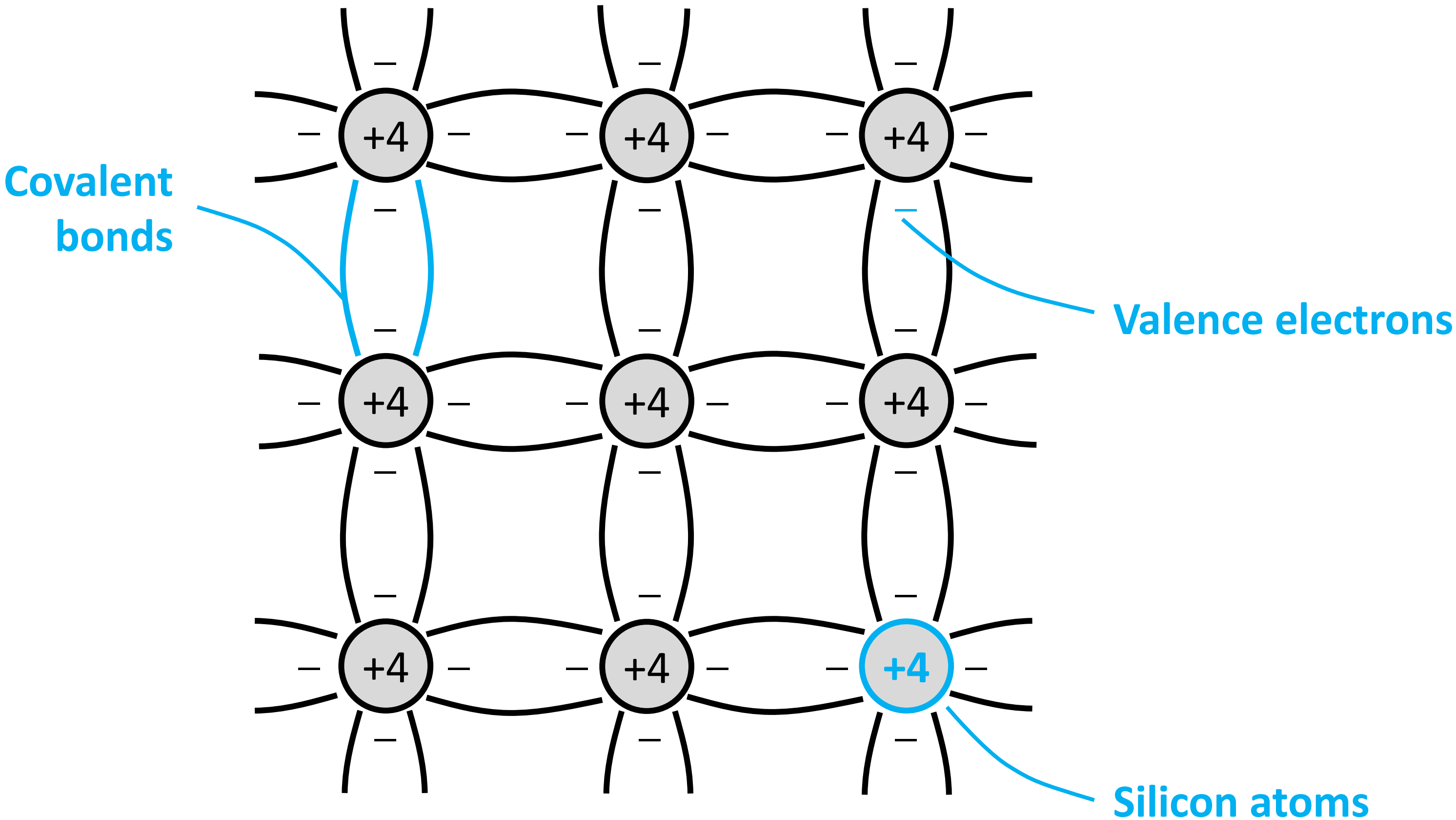
Semiconductor material

SEMICONDUCTORS are materials whose conductivity lies between that of conductors and insulators



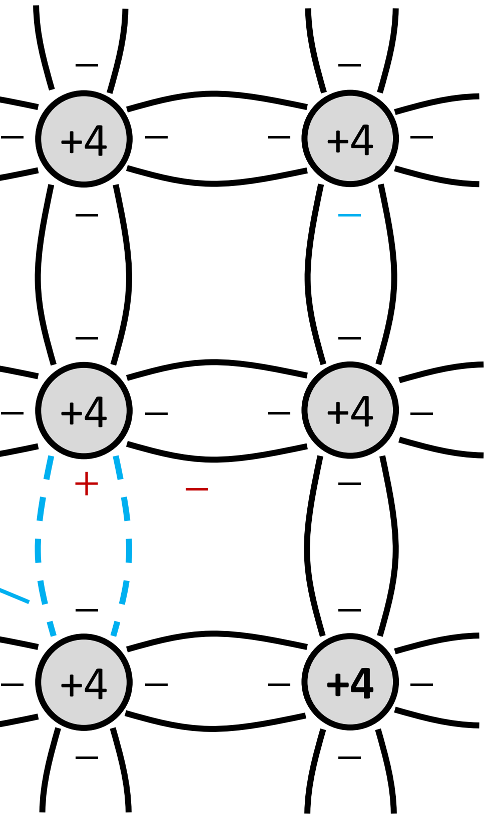
Silicon crystal

@0K all bonds are intact & no free electrons

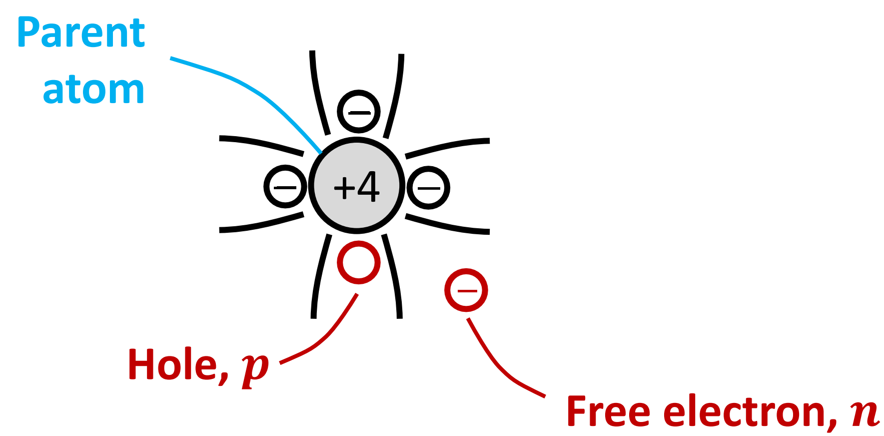


Silicon crystal

@room temperature

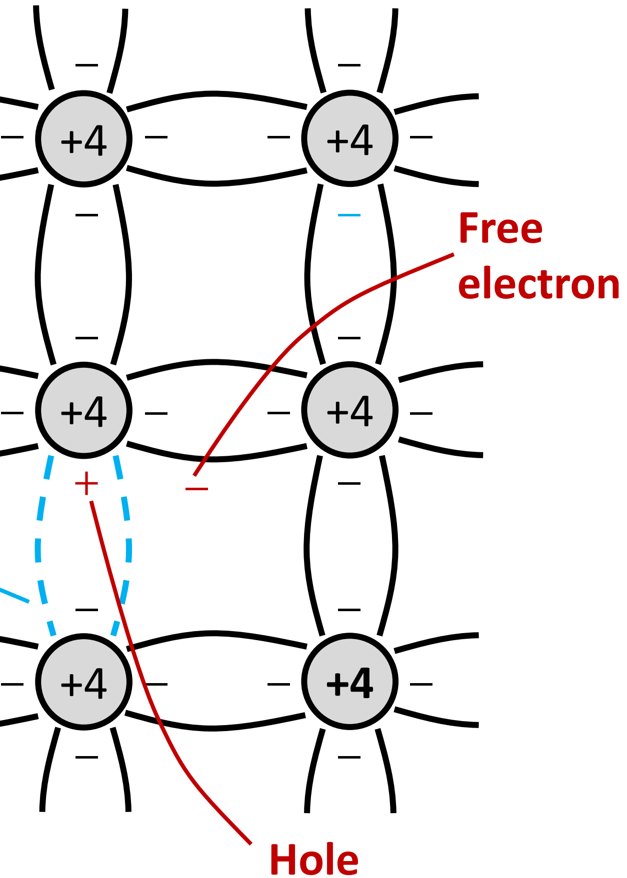


- **free electron** wanders away from its parent atom
- A net positive charge left at the parent atom
- A "**hole**" left at the parent atom

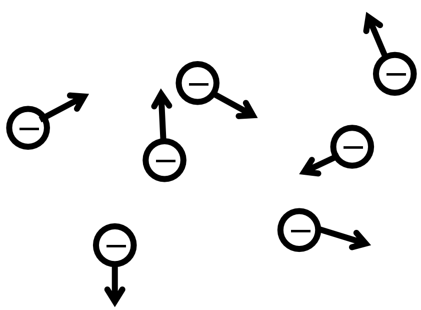


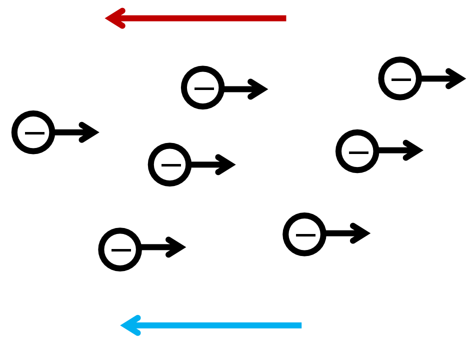
Silicon crystal

@room temperature



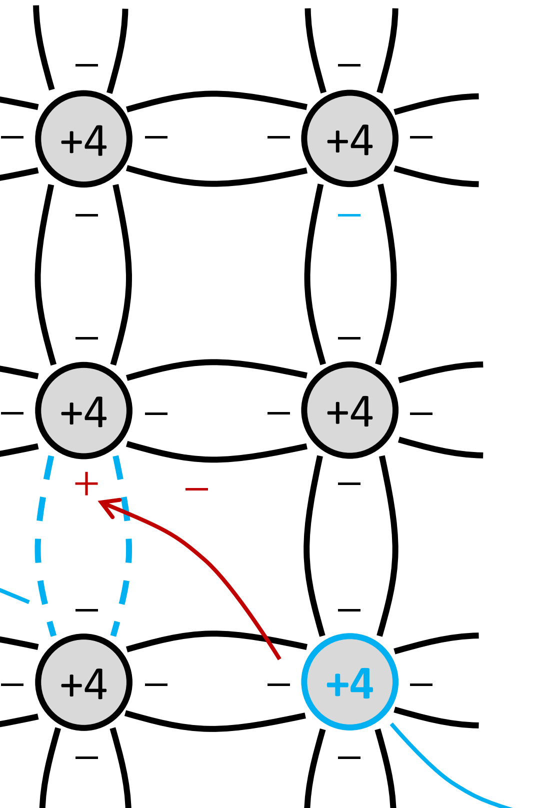
- free electron wanders away from its parent atom
- A net positive charge left at the parent atom
- A "hole" left at the parent atom
- **Current** generated when an **electric field** applied



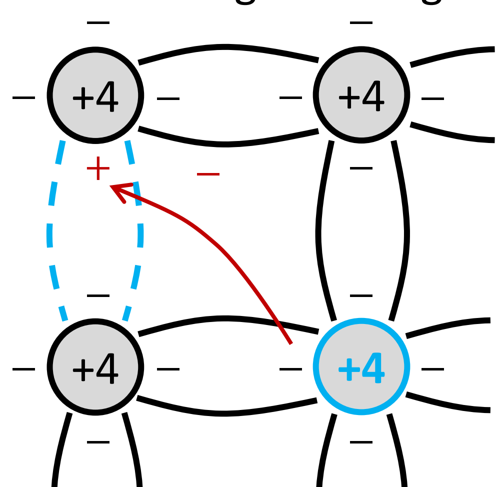


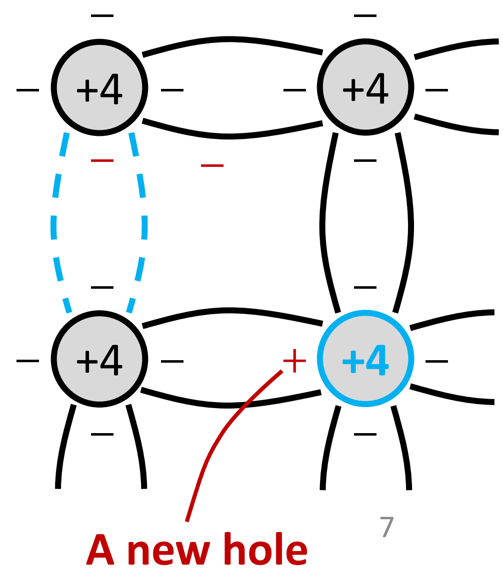
Silicon crystal

@room temperature



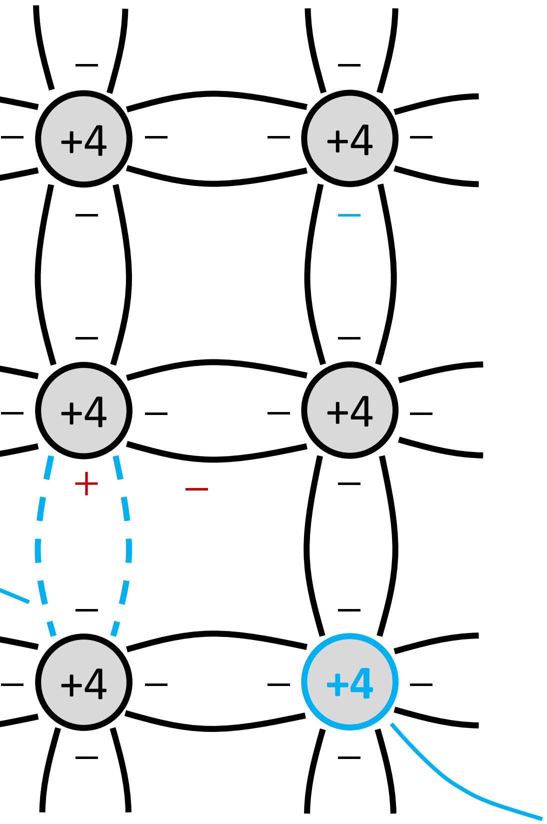
- free electron wanders away from its parent atom
- A net positive charge left at the parent atom
- A “hole” left at the parent atom
- Current generated when an electric field applied
- Electron from neighboring atom may fill up the hole, but generating a new hole – **RECOMBINATION**





Silicon crystal

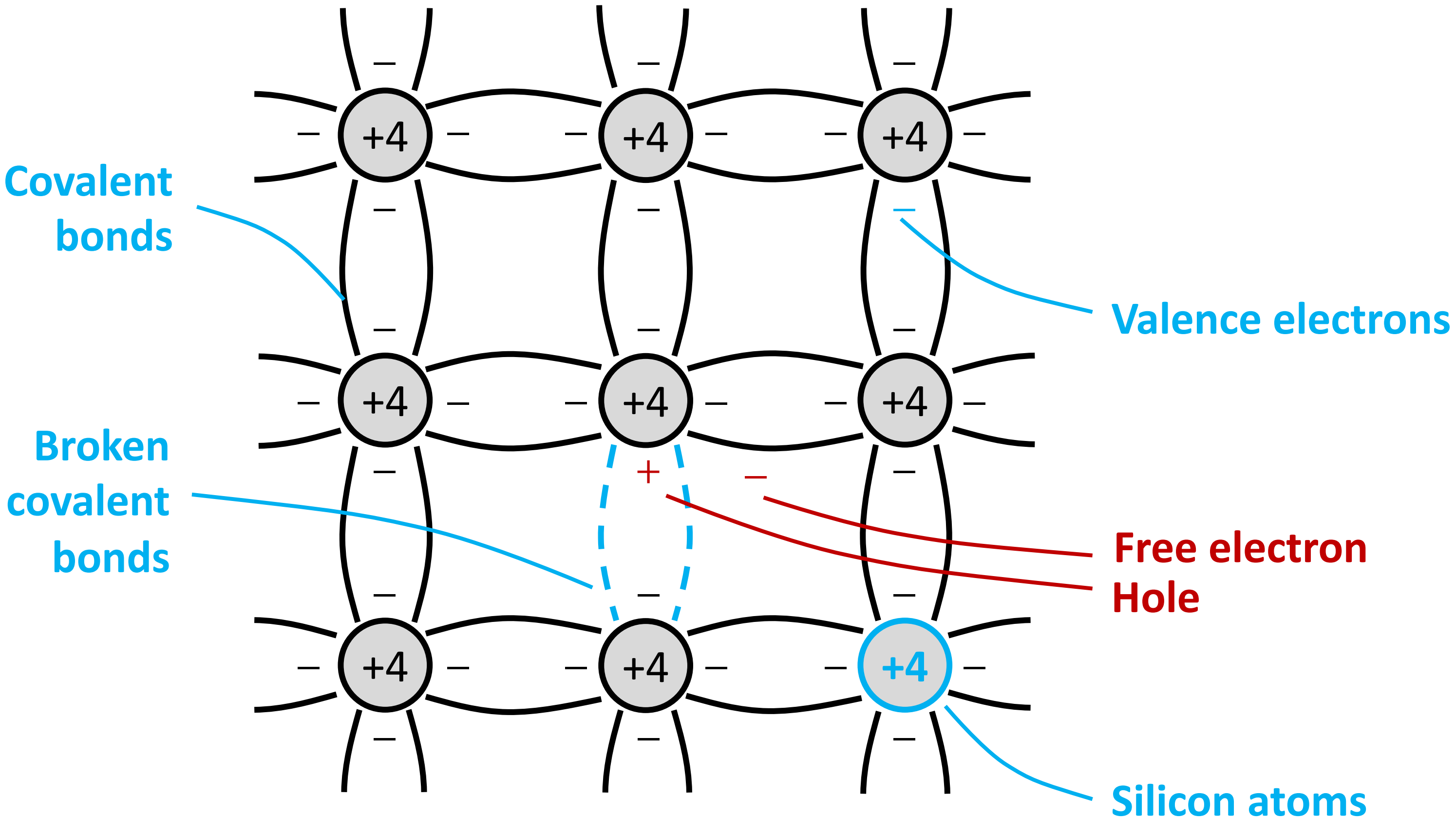
@room temperature



- free electron wanders away from its parent atom
- A net positive charge left at the parent atom
- A “hole” left at the parent atom
- Current generated when an electric field applied
- Electron from neighboring atom may fill up the hole, but generating a new hole - RECOMBINATION
- **Charge of a hole = −charge of an electron**

Silicon crystal

2 carriers in semiconductor: free electron & hole



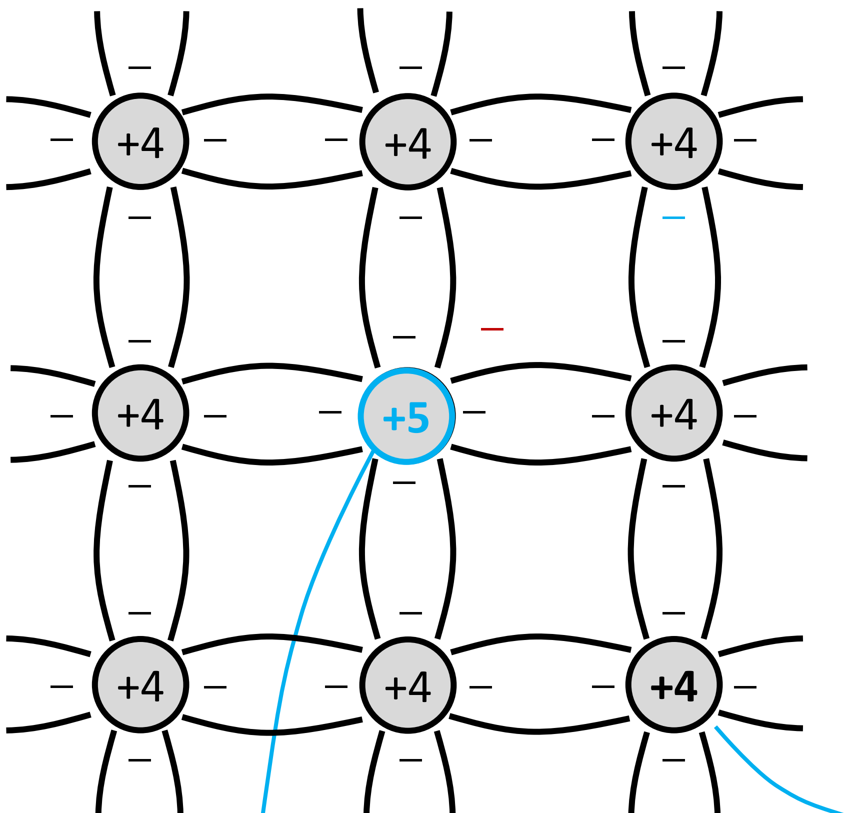
Outline

- Introduction to semiconductors
  - Semiconductor material & silicon crystal
  - **Doped semiconductors**

Doped semiconductors

A key issue of nature silicon crystal:

The **concentrations** of carriers are **too low** to conduct appreciable current



Solution: **DOPING**

- Introduce **impurity atoms**
- To increase concentration of n, doping element with a **valence of 5**, e.g. P

  → **n type doped silicon**
- Each phosphorus atom **donates** a free electron

  the P atom → **DONOR**
- For the **n type doped silicon**

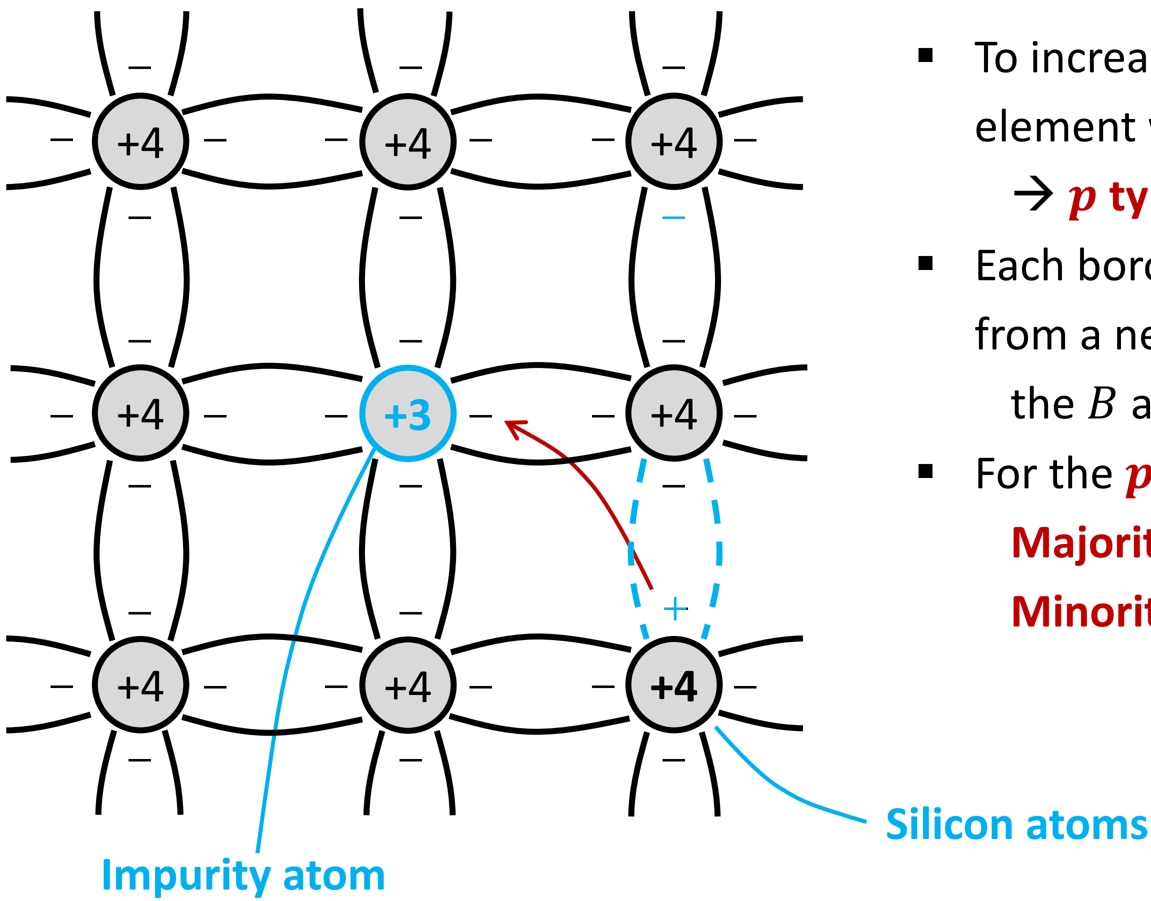
  Majority charge carriers – electrons

  Minority charge carriers - holes

Doped semiconductors

A key issue of nature silicon crystal:

The **concentrations** of carriers are **too low** to conduct appreciable current



- To increase concentration of p, doping element with a **valence of 3**, e.g. B

  → ***p* type doped silicon**
- Each boron atom **accepts** an electron from a neighboring atom

  the B atom → **ACCEPTOR**
- For the ***p* type doped silicon**

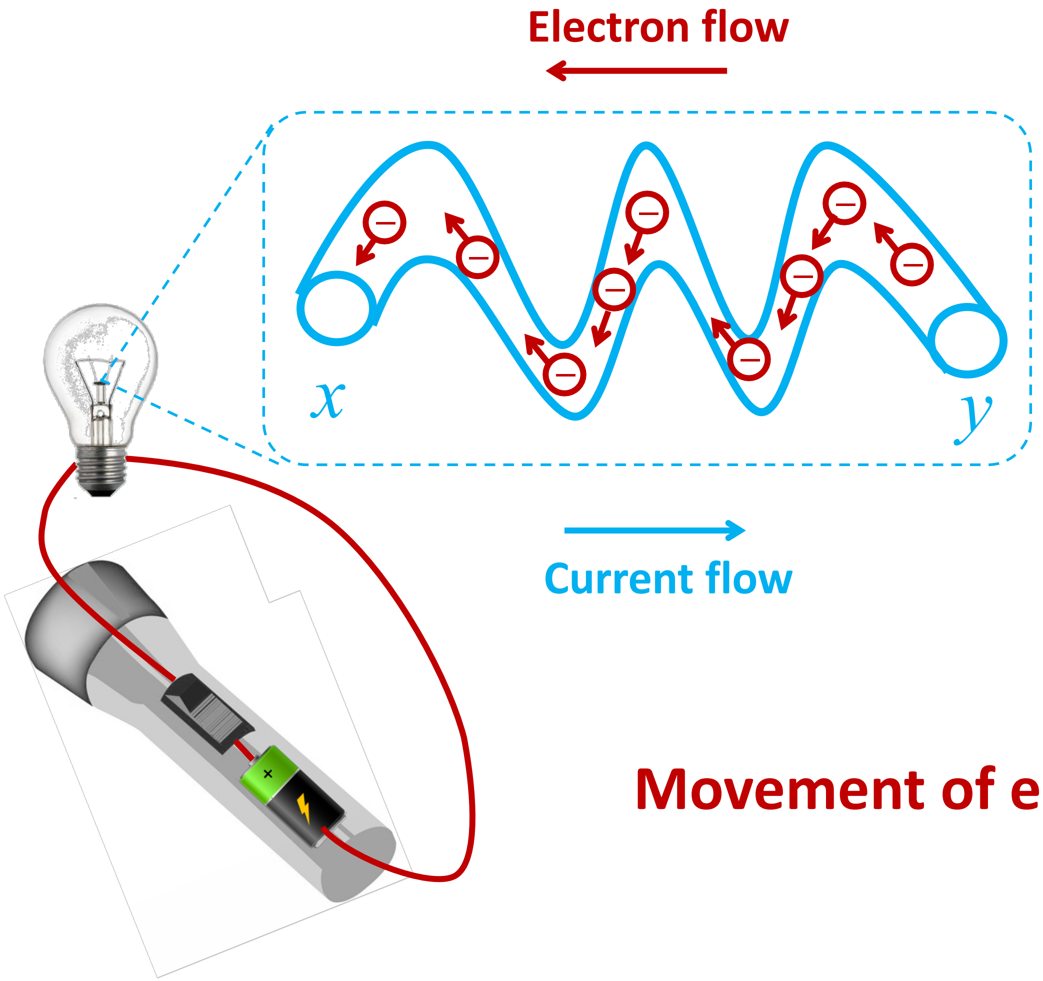
  Majority charge carriers – holes

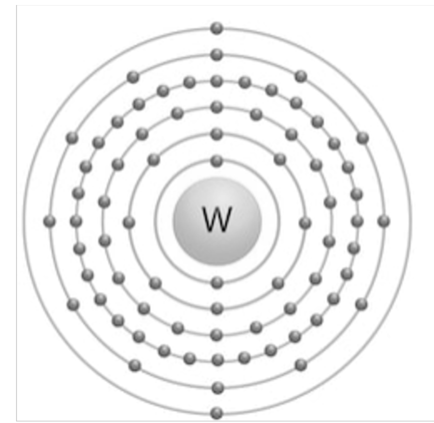
  Minority charge carriers - electrons

Outline

- Introduction to semiconductors
  - Semiconductor material & silicon crystal
  - Doped semiconductors
  - **Current flow in semiconductor**

Recall: where does current come from?





Tungsten

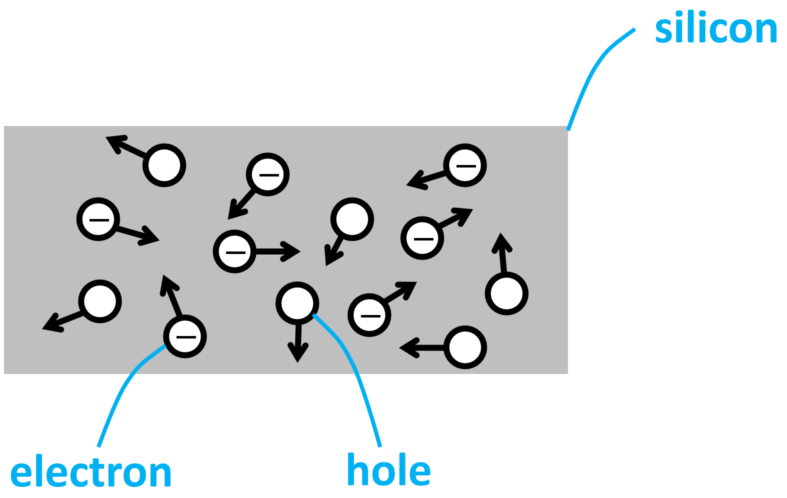
Movement of electrons generates current

The current flow in semiconductors

Movement of carriers (electrons/holes) in semiconductors generates current

Movement I – drift

- Free electrons and holes **randomly move** in a silicon
- An **electrical field $\boldsymbol{E}$** is applied in a semiconductor

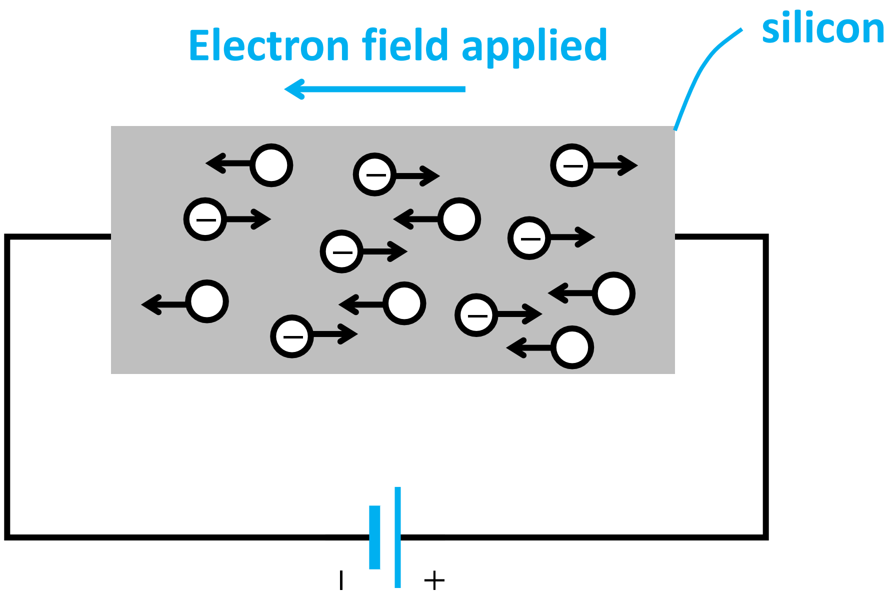


The current flow in semiconductors

Movement of carriers (electrons/holes) in semiconductors generates current

Movement I – drift

- Free electrons and holes **randomly move** in a silicon
- An **electrical field E** is applied in a semiconductor
- Holes moves in the direction of E
- Electrons moves in the opposite direction of E



The current flow in semiconductors

Movement of carriers (electrons/holes) in semiconductors generates current

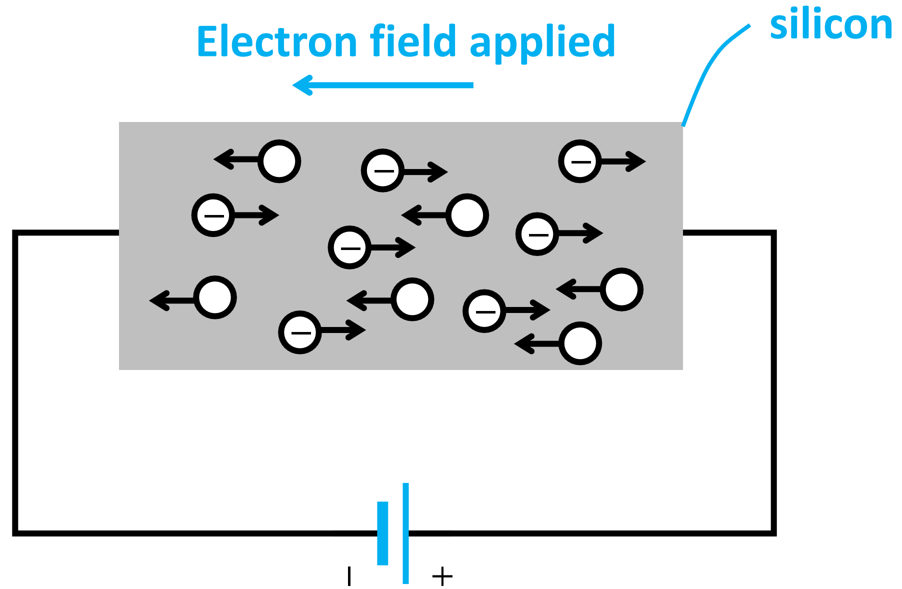
Movement I – drift

- Current density of the holes

$$J_p = qp\mu_p E$$

hole concentration (p)

hole mobility (μ_p)



- Current density of the electrons

$$J_n = qn\mu_n E$$

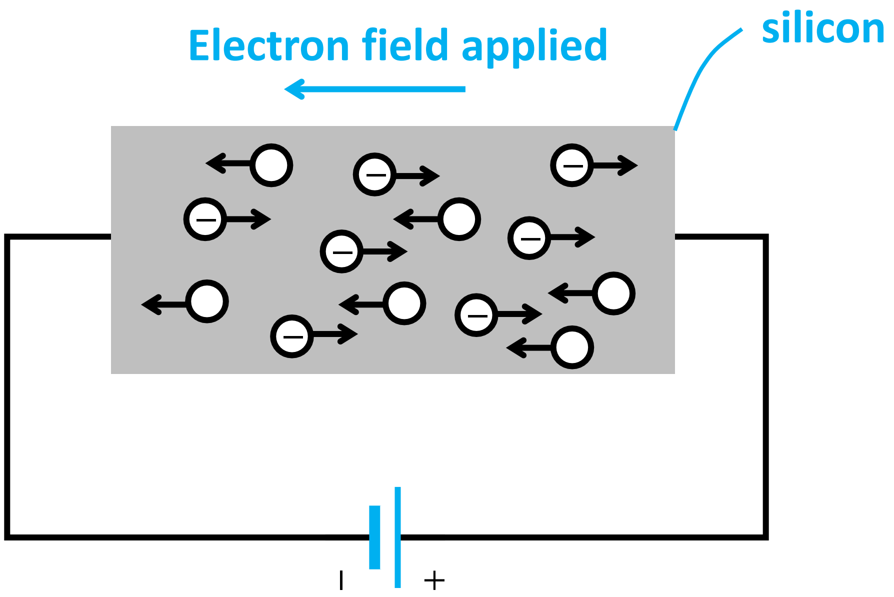
electron concentration (n)

electron mobility (μ_n)

The current flow in semiconductors

Movement of carriers (electrons/holes) in semiconductors generates current

Movement I – drift



- Current density of the holes

$$J_p = qp\mu_p E$$

- Current density of the electrons

$$J_n = qn\mu_n E$$

- Total drift current density

$$\boldsymbol{J = J_p + J_n = qp\mu_p E + qn\mu_n E}$$

Since $\boldsymbol{J = \sigma E}$

- The conductivity $\boldsymbol{\sigma = q(p\mu_p + n\mu_n)}$

The current flow in semiconductors

Movement of carriers (electrons/holes) in semiconductors generates current

Movement II – diffusion

- Why there is carrier diffusion? – carrier **density gradient**
- Current density of the holes

$$J_p = -qD_p \frac{dp(x)}{dx}$$

Diffusion constant of the holes (D_p); hole concentration gradient ($\frac{dp(x)}{dx}$)

- Current density of the electrons

$$J_n = -qD_n \frac{dn(x)}{dx}$$

Diffusion constant of the electron (D_n); electron concentration gradient ($\frac{dn(x)}{dx}$)

The current flow in semiconductors

Movement of carriers (electrons/holes) in semiconductors generates current

2 types of movement – DRIFT & DIFFUSION

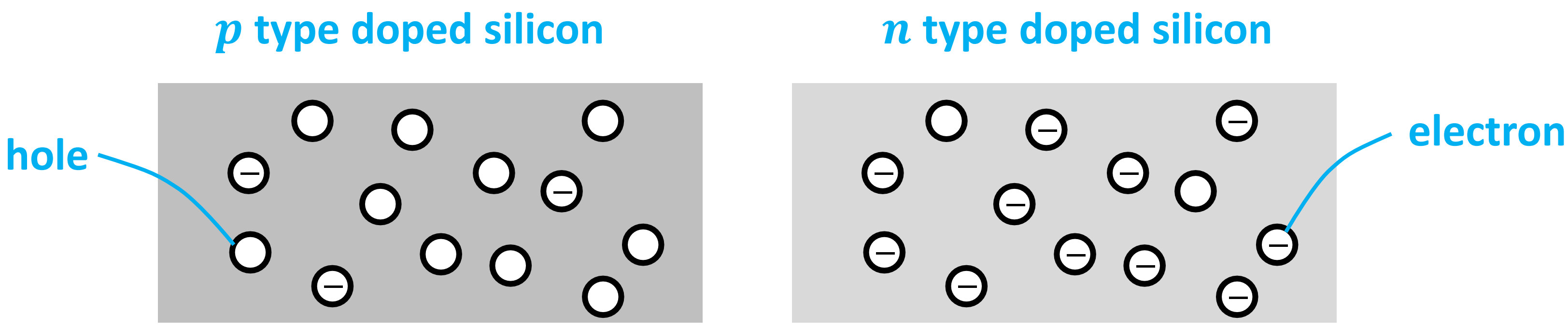
- Total drift current density $$J = J_p + J_n = qp\mu_p E + qn\mu_n E$$
- Total diffusion current density $$J = -q\left(D_p \frac{dp(x)}{dx} + D_n \frac{dn(x)}{dx}\right)$$
- A relationship between diffusion constant and mobility

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$$ **← thermal voltage** $$V_T = \frac{kT}{q}$$

Outline

- Introduction to semiconductors
  - Semiconductor material & silicon crystal
  - Doped semiconductors
  - Current flow in semiconductor
  - **The pn junction**

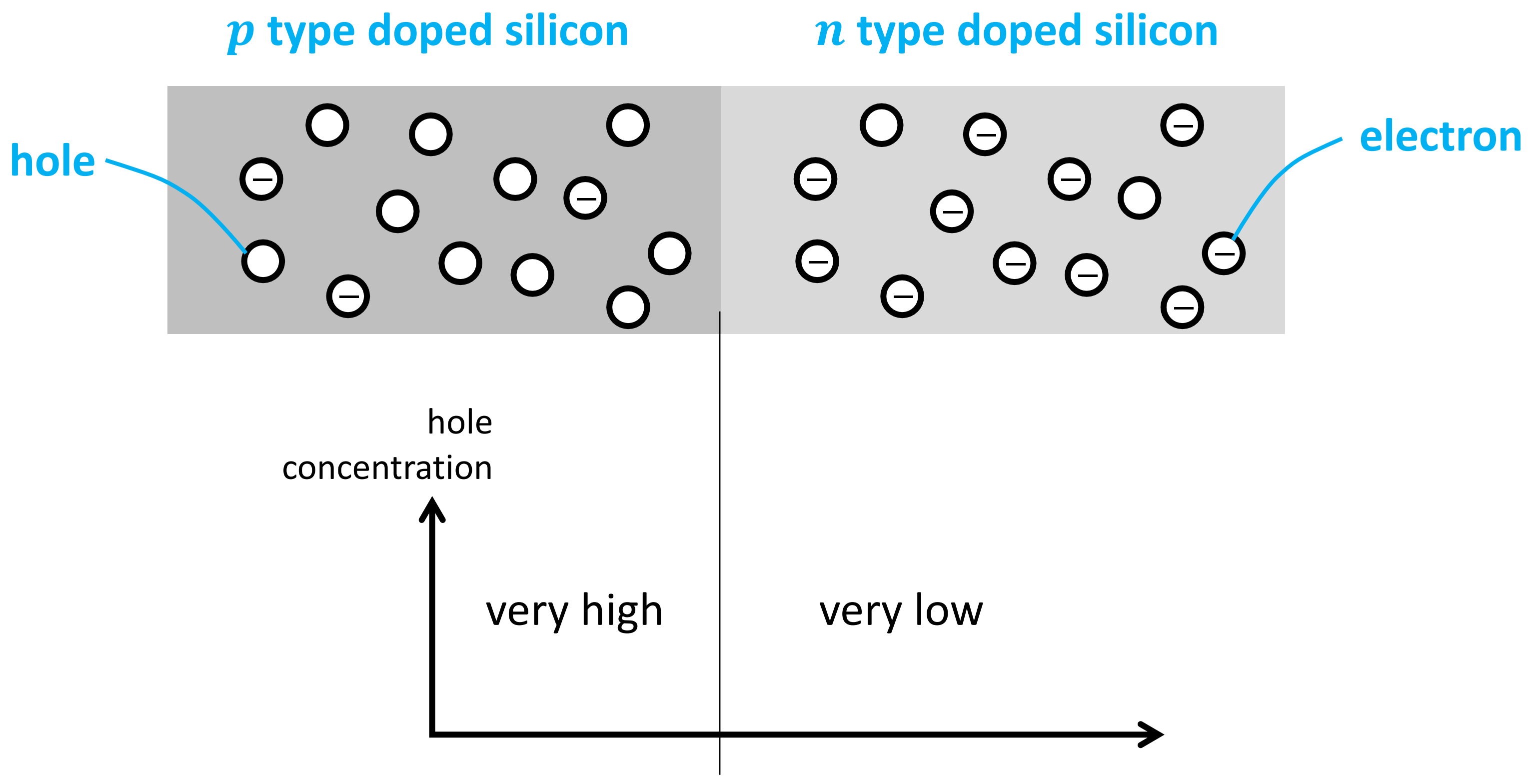
The two types doped silicon



	p type doped silicon	n type doped silicon
majority carrier	hole	electron
minority carrier	electron	hole

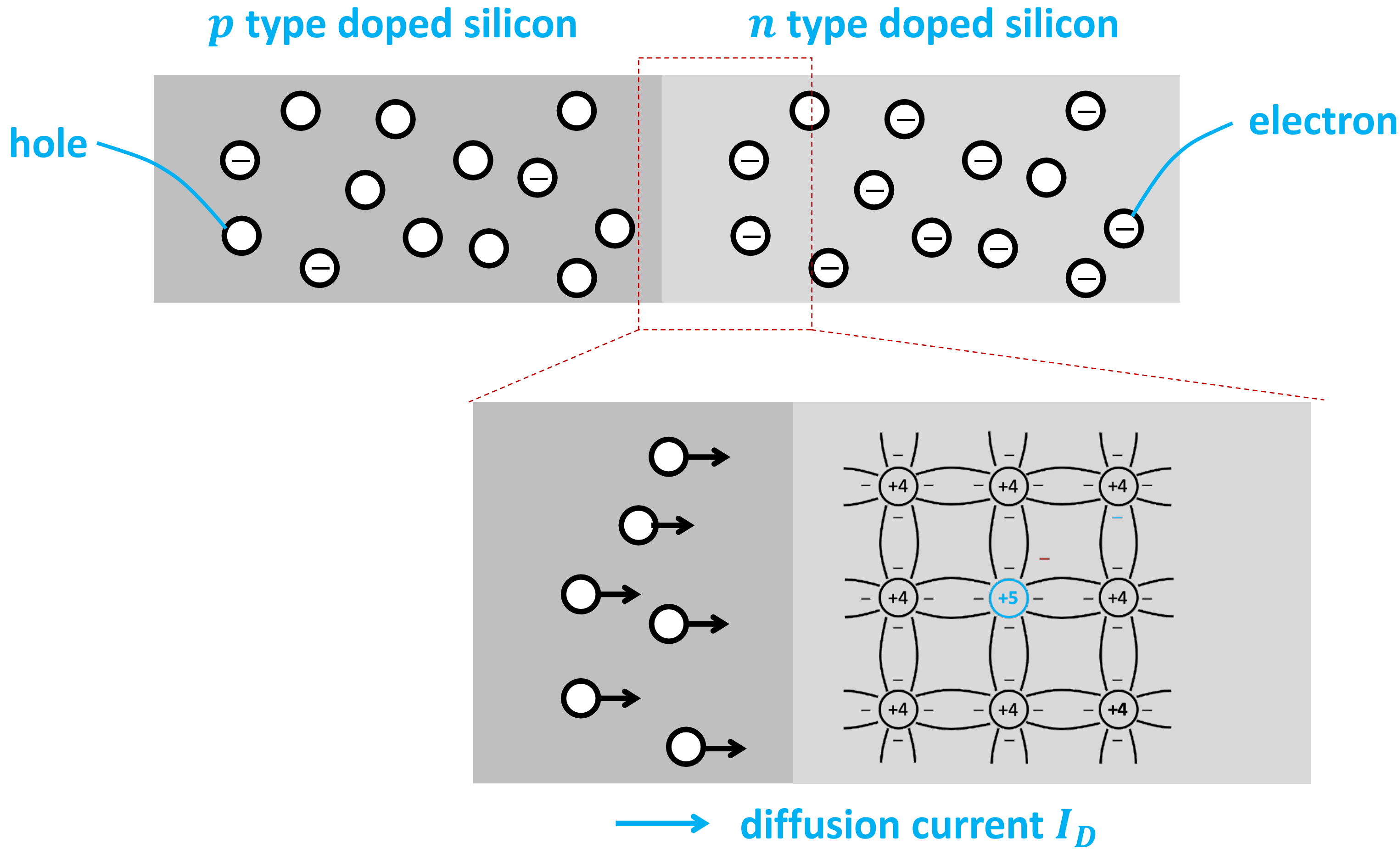
The pn junction

Step 1: a p-type & a n-type semiconductor brought into close contact with each other, a **pn junction** is generated



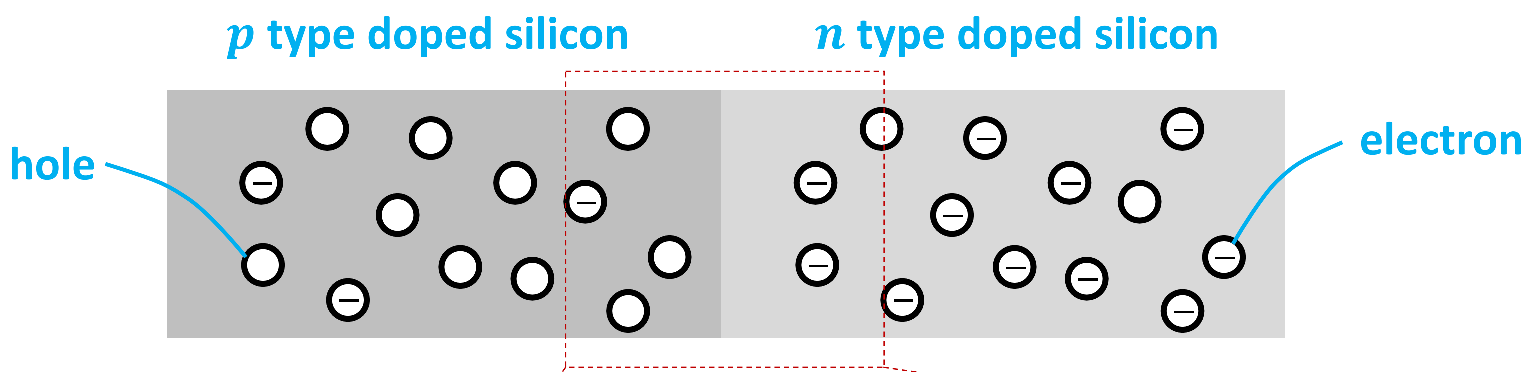
The pn junction

Step 2: diffusion current generated due to concentration gradient



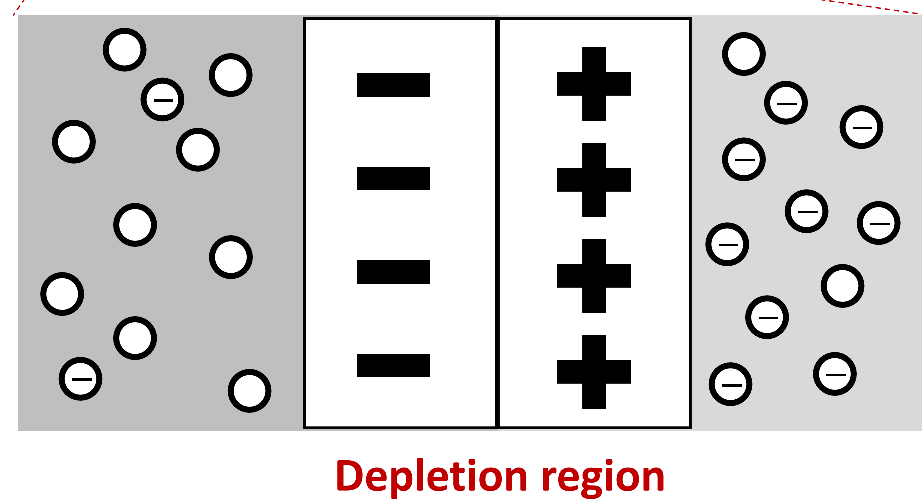
The pn junction

Step 3: the holes crossed the junction and recombine with the majority (electron) in the n type doped silicon



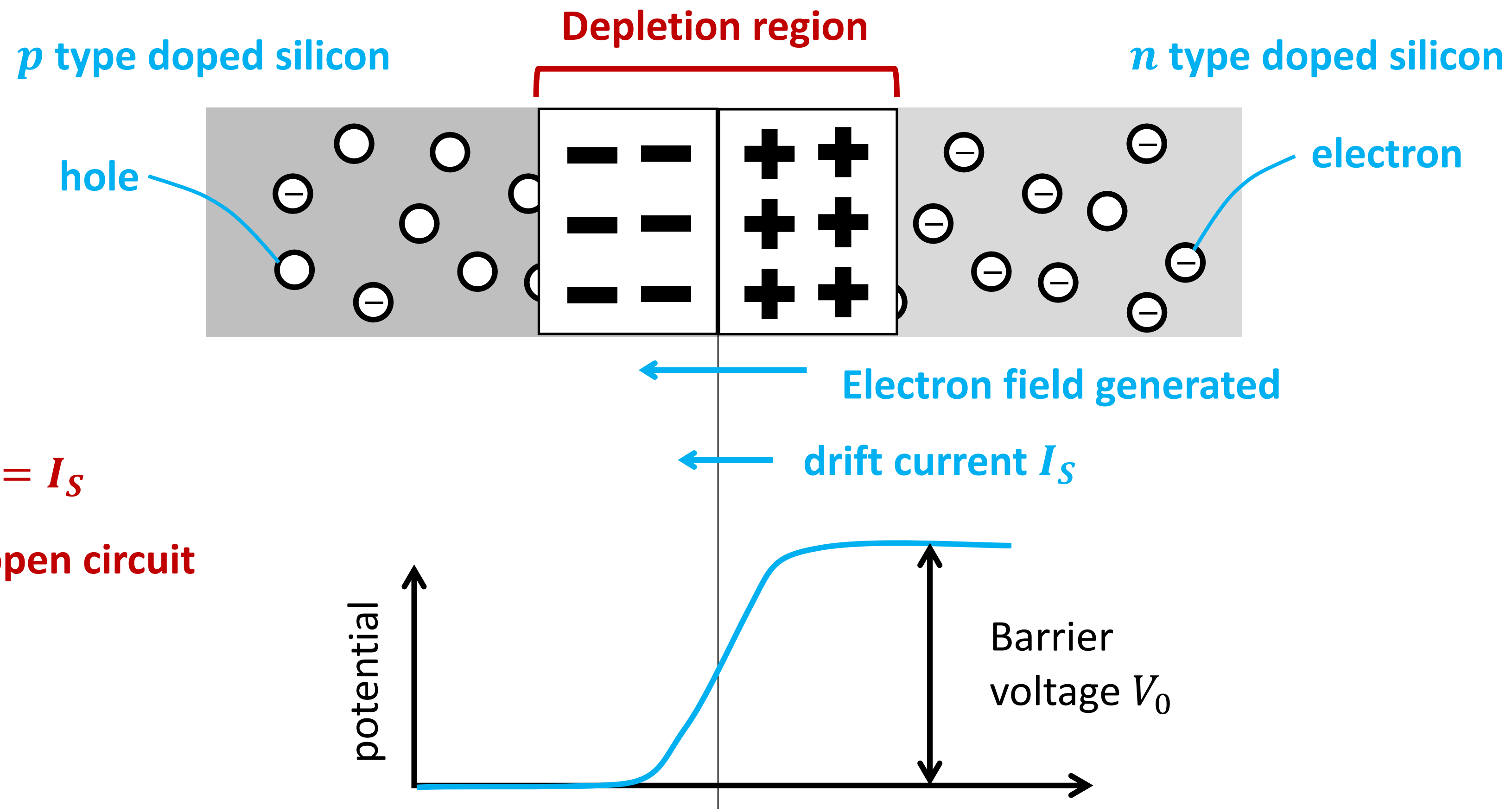
Carrier-depletion region

- **Exist in both sides**
- **Uncovered charges**
- **n is more positive than p**
- **Electronic field generated**



The pn junction

Step 4: an electronic field E is generated. Drift current is created due to minority carrier drift in this electronic field

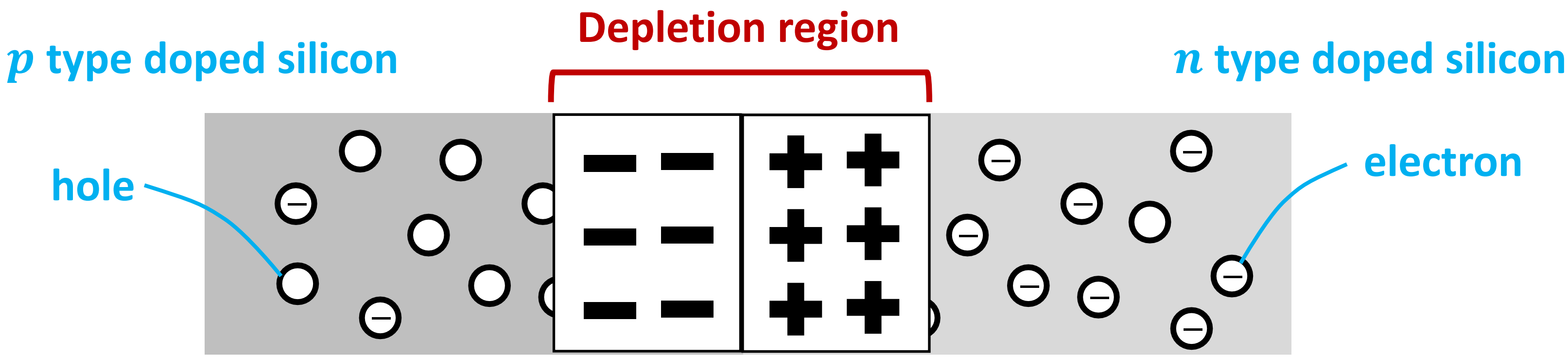


$I_D = I_S$

@open circuit

The pn junction

Step 4: an electronic field E is generated. Drift current is created due to minority carrier drift in this electronic field



doping concentration of p side

doping concentration of n side

Barrier voltage $$V_0 = V_T In\left(\frac{N_A N_D}{n_i^2}\right)$$

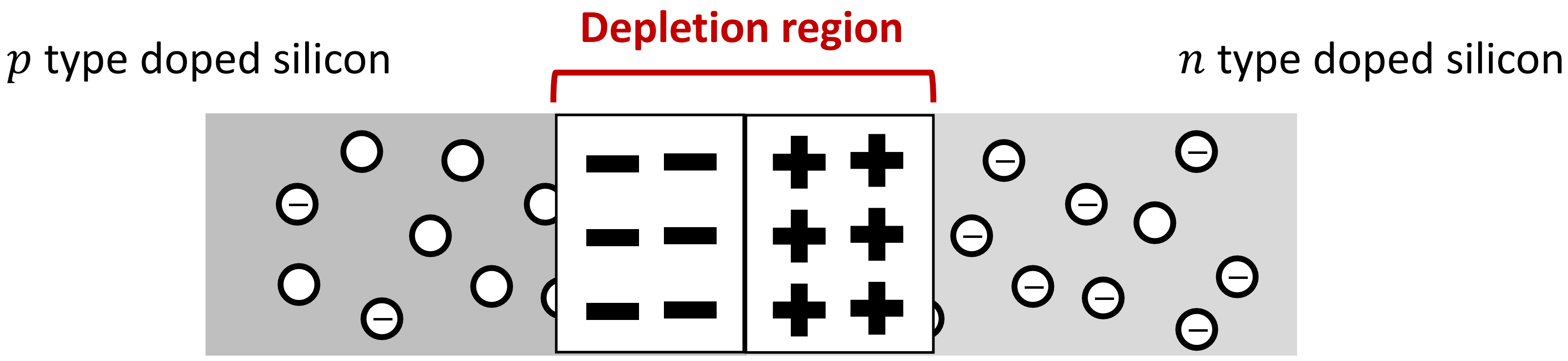
thermal voltage $V_T = \frac{kT}{q}$

The barrier voltage is known as the **junction build-in voltage**

Outline

- Introduction to semiconductors
  - Semiconductor material & silicon crystal
  - Doped semiconductors
  - Current flow in semiconductor
  - The $\boldsymbol{pn}$ junction
    - $\boldsymbol{pn}$ junction @open circuit
    - **$\boldsymbol{pn}$ junction with applied voltage**

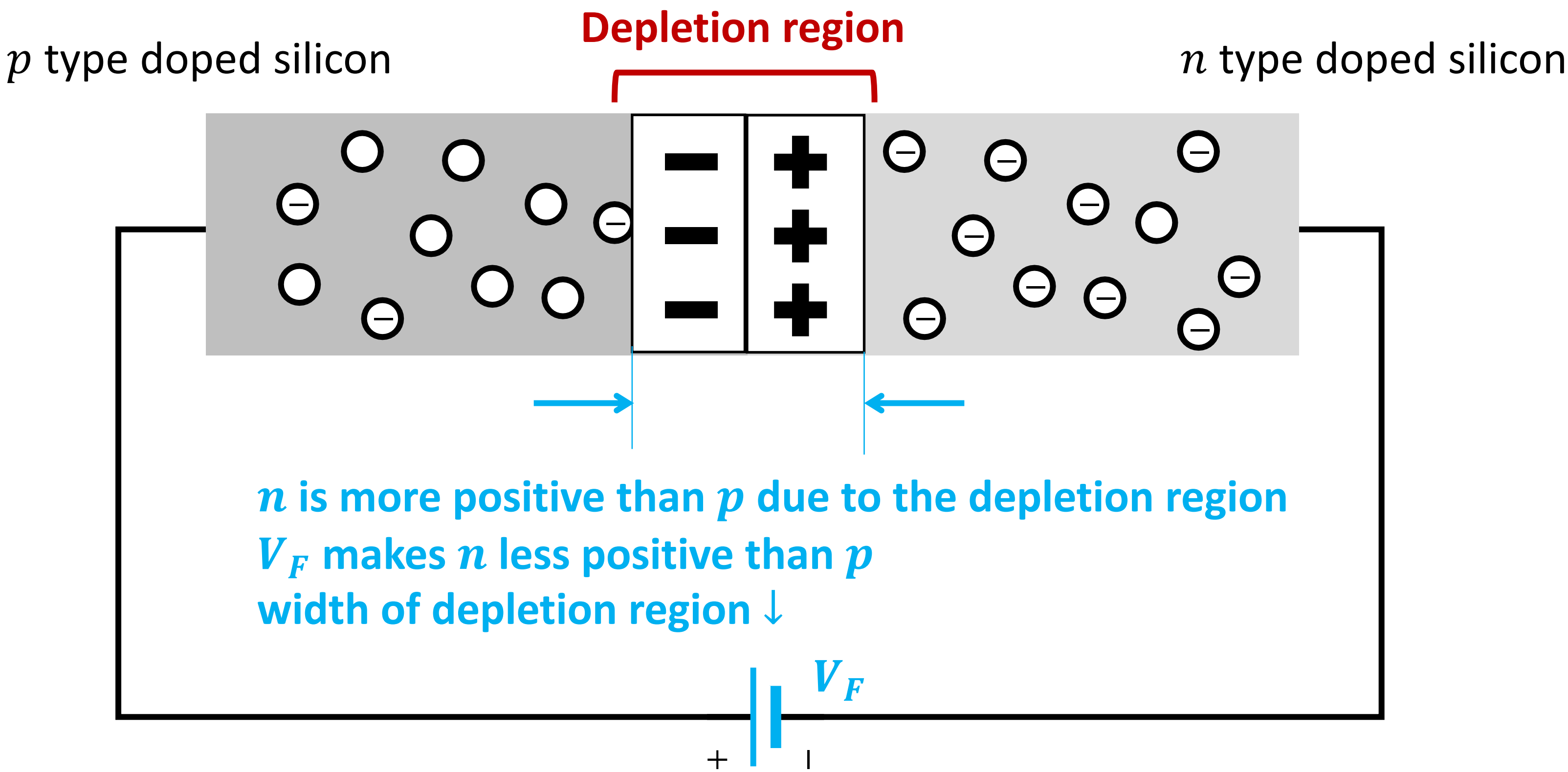
Recall: pn junction @open circuit



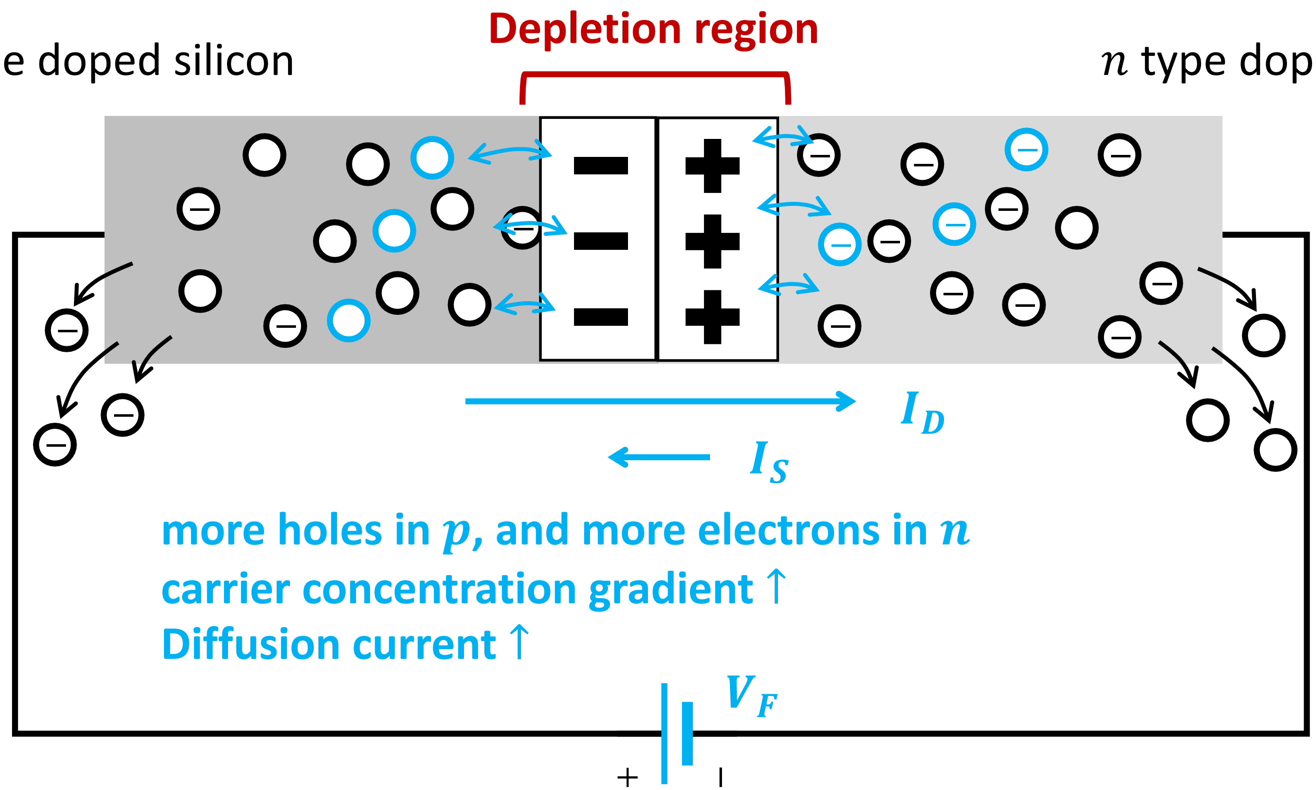
n is more positive than p due to the depletion region

$I_D = I_S$ due to the barrier voltage

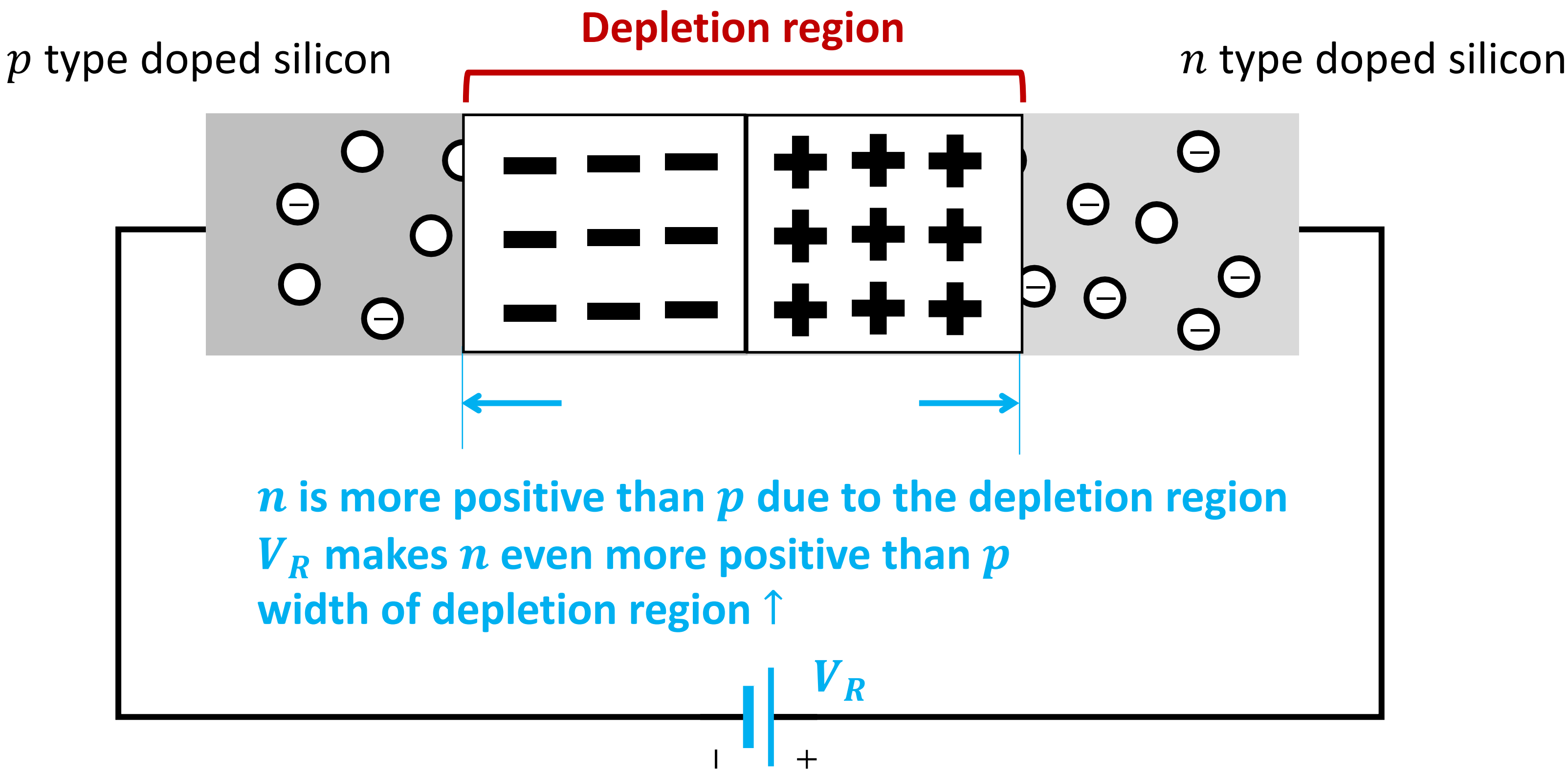
pn junction with forward-bias voltage



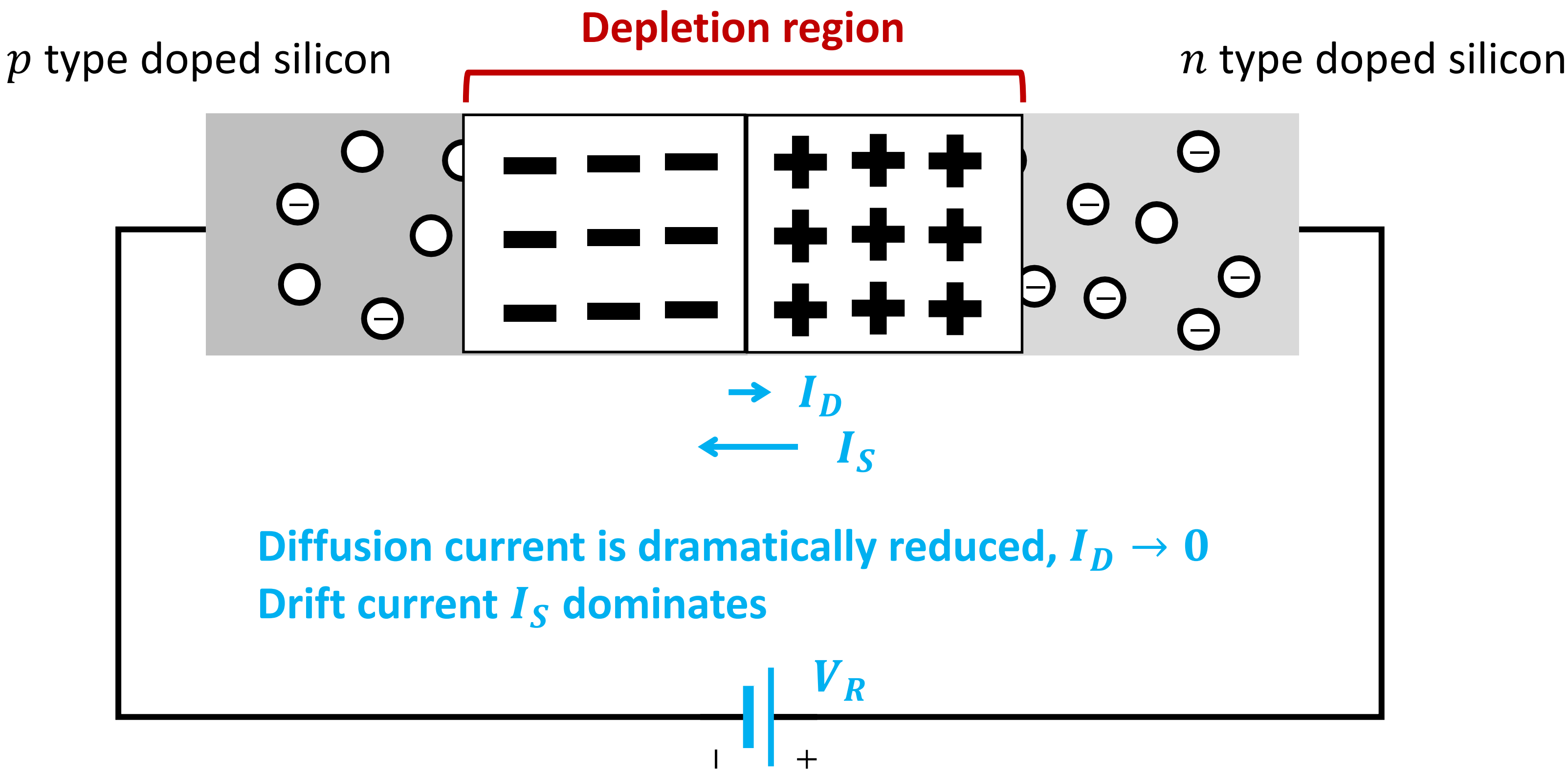
pn junction with forward-bias voltage



pn junction with reverse-bias voltage

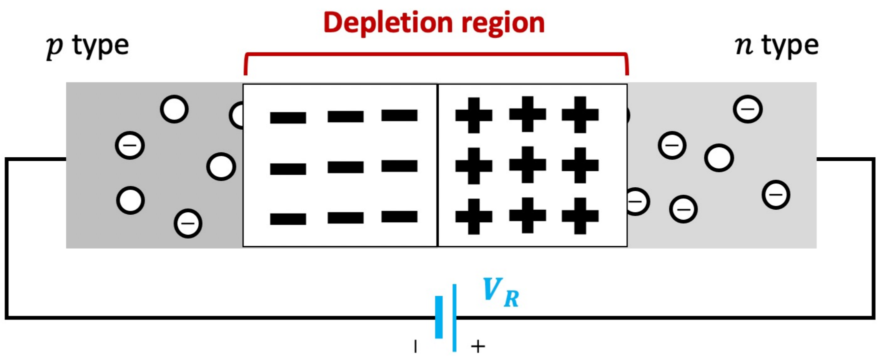


pn junction with reverse-bias voltage



Reverse breakdown

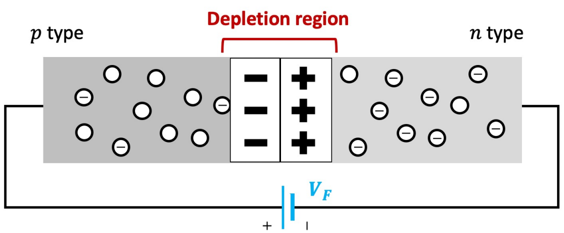
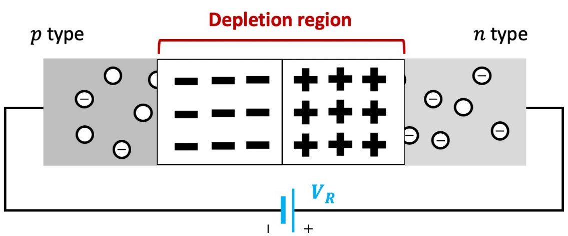
JUNCTION BREAKDOWN happens when the reverse voltage is very high



- **Zener effect**
  - Reverse voltage is usually less than $5V$
  - Breaks covalent bonds & generates electron-hole pairs

- **Avalanche effect**
  - Reverse voltage is usually $> 7V$
  - Breaks covalent bonds in atoms

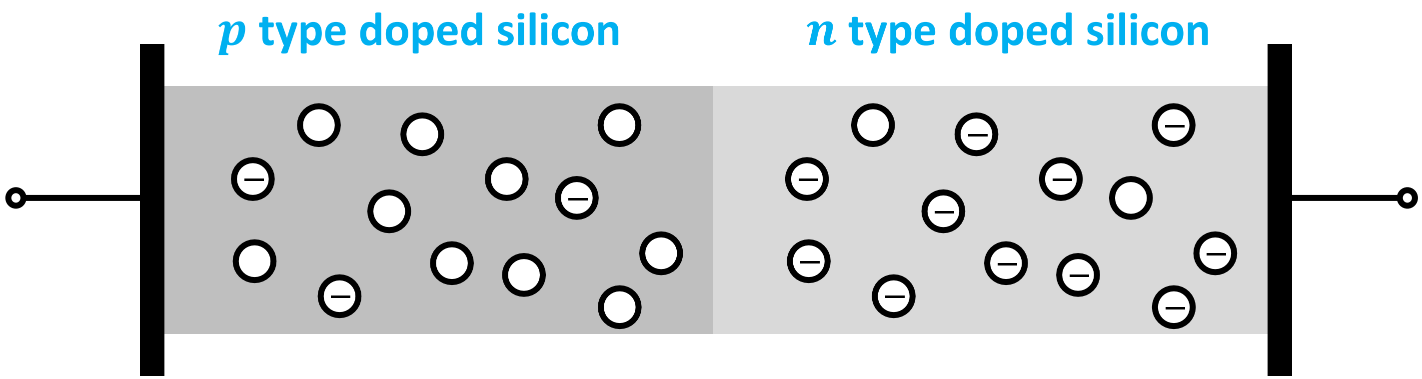
pn junction with applied voltage

	 forward-bias voltage	 reverse-bias voltage
Depletion region width	↓	↑
Barrier voltage	↓	↑
current	$I = I_D - I_S$	$I = I_D - I_S \approx -I_S$

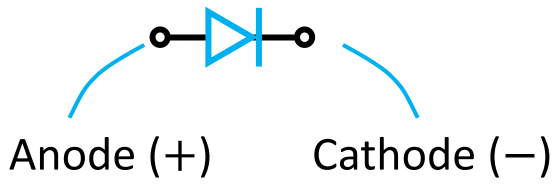
Outline

- Introduction to semiconductors
  - Semiconductor material & silicon crystal
  - Doped semiconductors
  - Current flow in semiconductor
  - The $\boldsymbol{pn}$ junction
    - pn junction @open circuit
    - pn junction with applied voltage
    - Reverse breakdown
- **Diodes**

Junction diodes



symbol

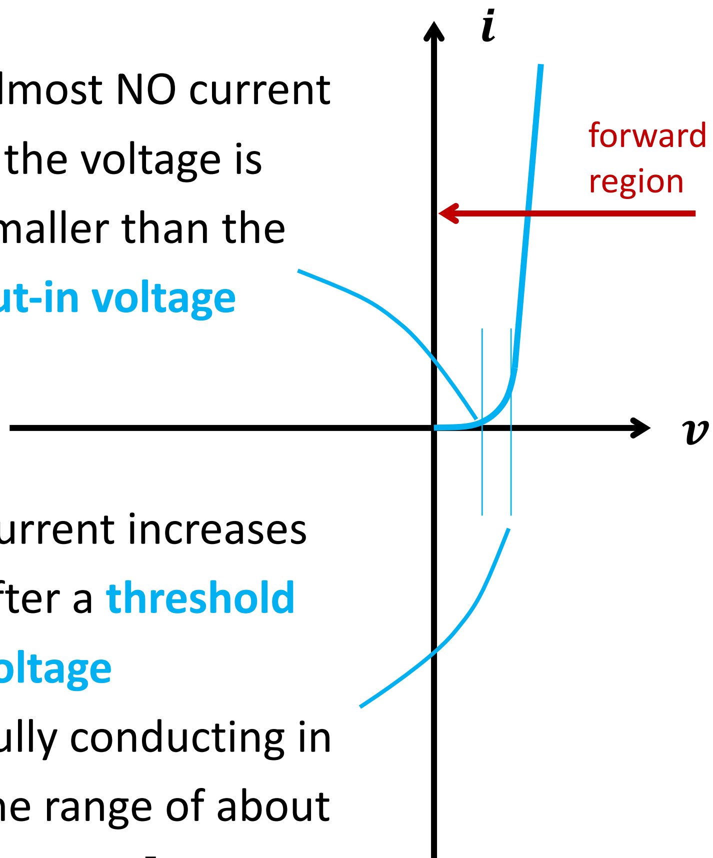


Diode

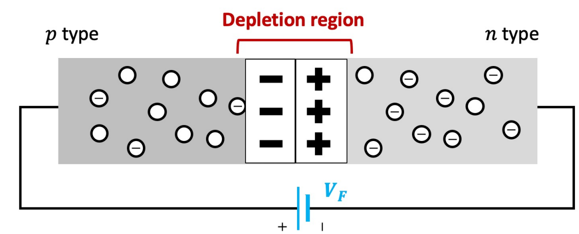
- A two-terminal device

$i - v$ characteristics of pn junction

- Almost NO current if the voltage is smaller than the **cut-in voltage**
- Current increases after a **threshold voltage**
- Fully conducting in the range of about $[0.6,\ 0.8]$



Recall: pn junction with forward-bias voltage



- n is more positive than p due to the depletion region
- V_F makes n less positive than p
- width of depletion region ↓
- more holes in $\boldsymbol{p}$, and more electrons in $\boldsymbol{n}$
- carrier concentration gradient ↑
- Diffusion current ↑

$i - v$ characteristics of pn junction

- Diode current in forward region

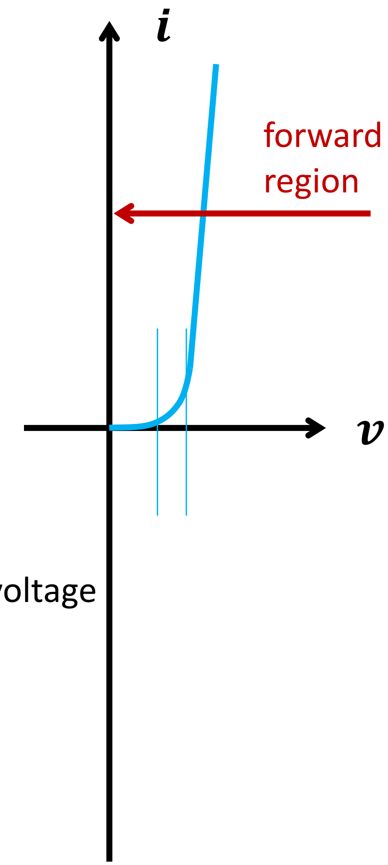
terminal voltage

$$i = I_S\left(e^{\frac{v}{V_T}} - 1\right)$$

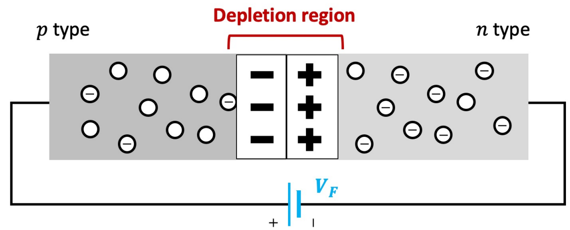
a constant @given temp.

Thermal voltage $V_T = \frac{kT}{q}$

$$v = V_T In \frac{i}{I_S}$$



Recall: pn junction with forward-bias voltage



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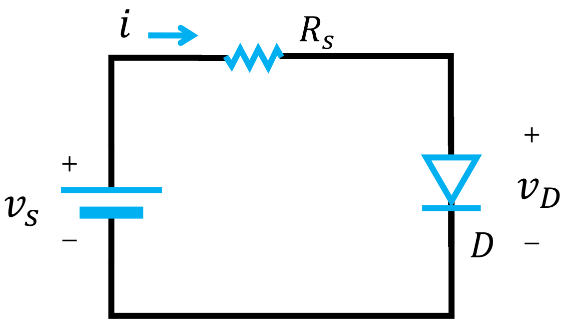
Example 1

QUESTION: Find the current through the resistor R_s



- According to KVL

$$v_s = iR_s + v_D$$

- According to $i - v$ characteristics of the diode

$$v_s = iR_s + V_T In \frac{i}{I_S}$$

[
$i - v$ characteristics of the diode

$$i = I_S\left(e^{\frac{v}{V_T}} - 1\right)$$

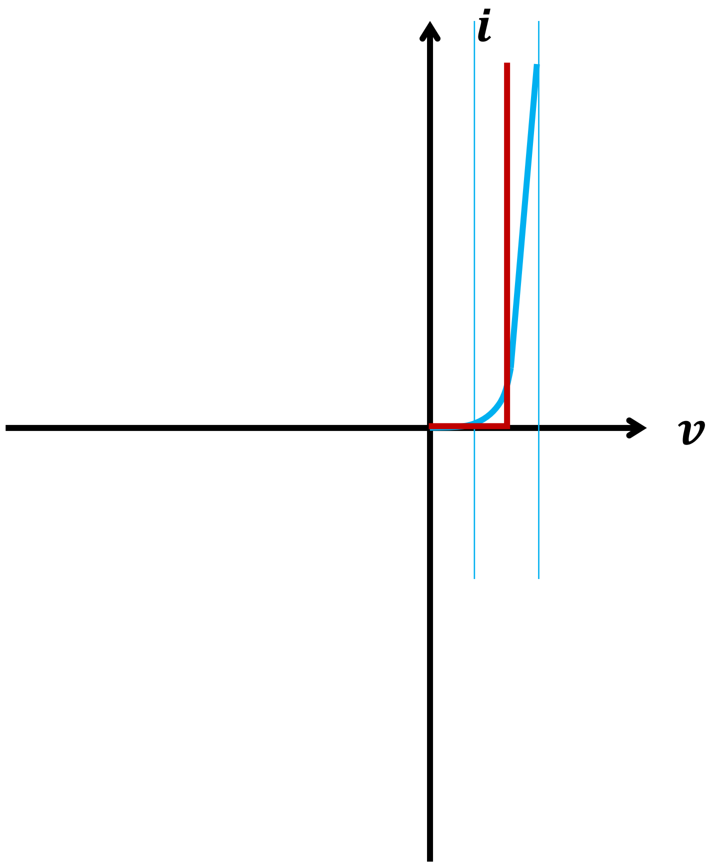
$$v = V_T In \frac{i}{I_S}$$
]

1 unknown in 1 equation

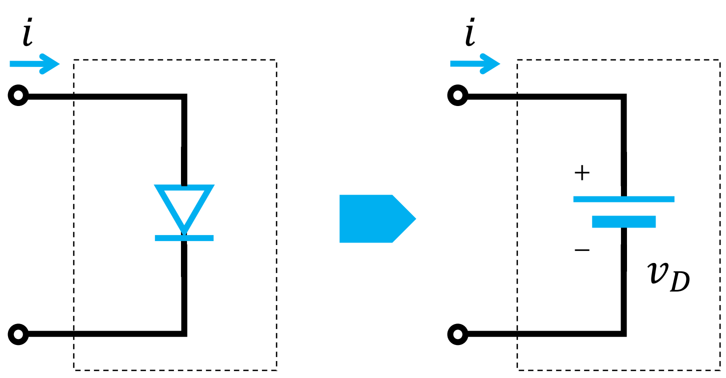
BUT WE CANNOT FIND AN ANALYTICAL SOLUTION

NUMERICAL SOLUTION AVAILABLE IN 20230253

The constant-voltage-drop model



- One of the most widely used diode **MODEL**
- Voltage drops in a narrow range, roughly $[0.6,\ 0.8]$
- A constant value of 0.7 is used

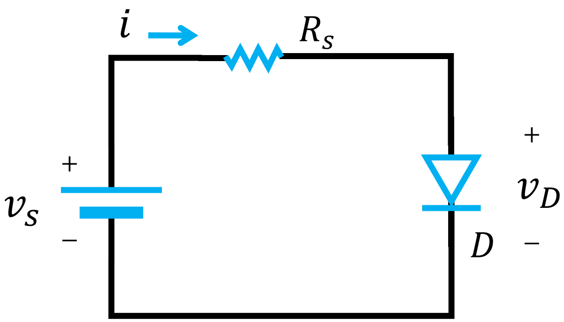


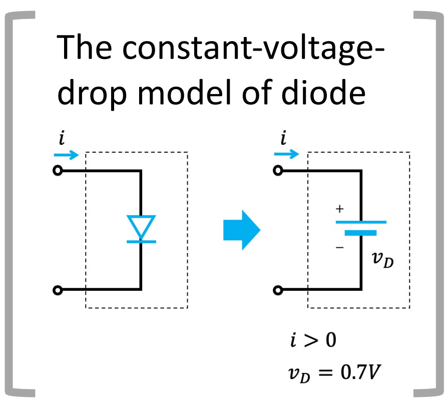
$i > 0$

$v_D = 0.7V$

Example 1

QUESTION: Find the current through the resistor R_s





- According to KVL

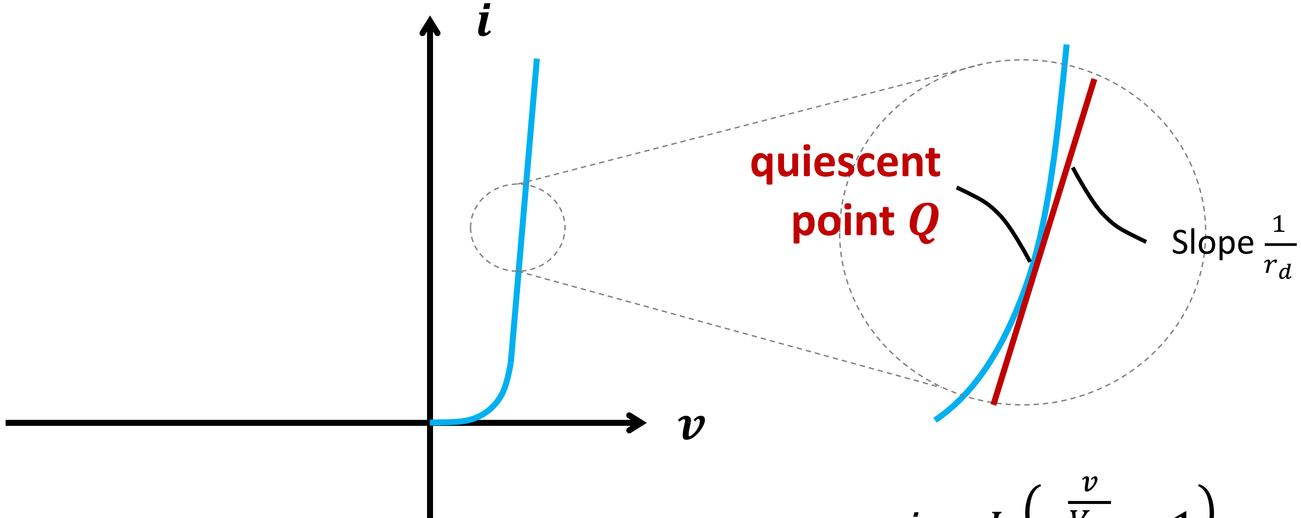
$$v_s = iR_s + v_D$$

- According to the constant-volt-drop model of diode

$$v_s = iR_s + 0.7$$

$$\Rightarrow \quad i = \frac{v_s - 0.7}{R_s}$$

The small-signal model



$$i = I_S\left(e^{\frac{v}{V_T}} - 1\right)$$

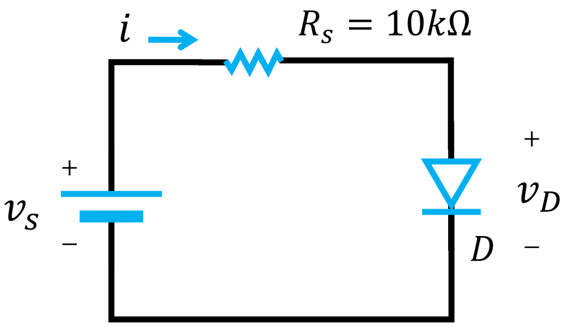
$$\left.\frac{\partial i}{\partial v}\right|_{@Q} = \frac{I_{@Q}}{V_T} = \frac{1}{r_d}$$

Incremental resistance

The small-signal model works ONLY when the signal amplitude is close enough to the quiescent point

Example 1

QUESTION: Find the current through the resistor R_s with $v_s = 10 + \sin(\omega t)\ (\mathrm{V})$



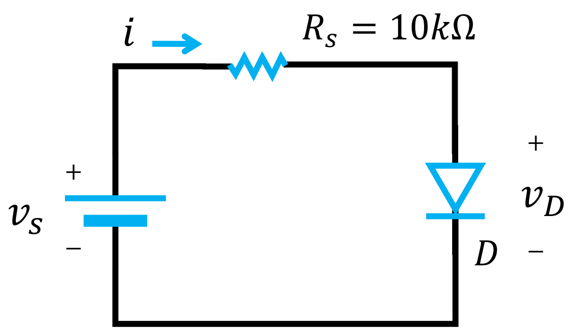
- There are 2 parts in v_s

$$v_s = 10 + \sin(\omega t)\ (\mathrm{V})$$

DC voltage　　AC voltage with a peak of 1V

Example 1

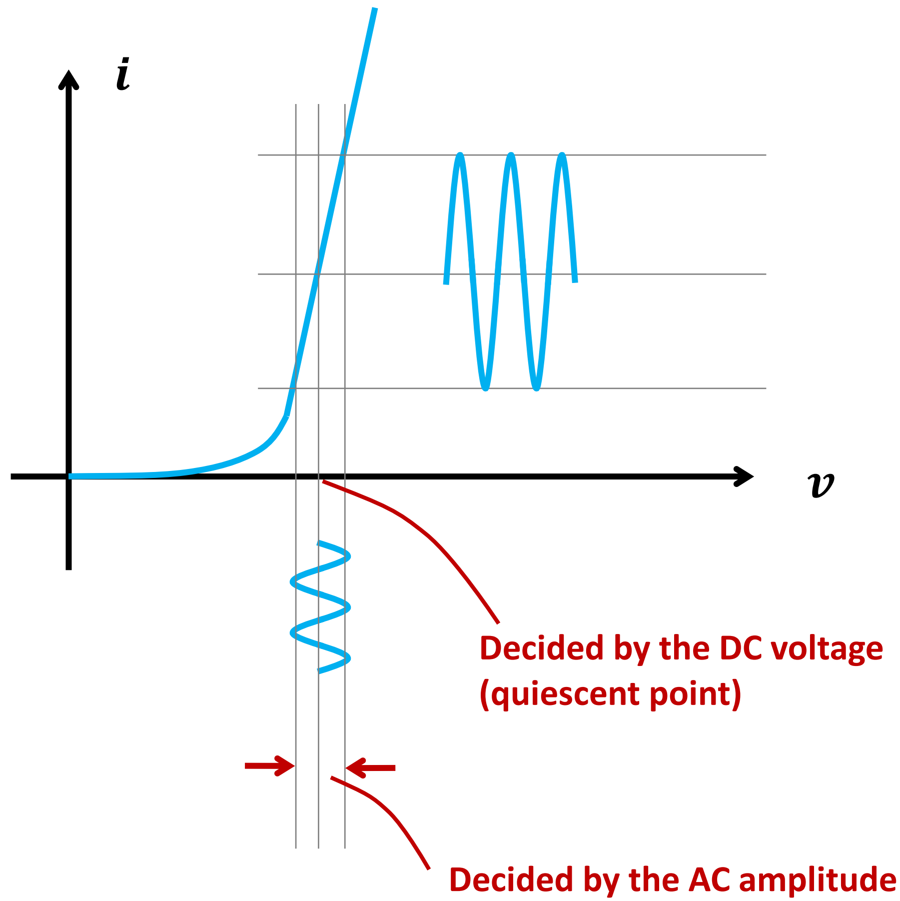
QUESTION: Find the current through the resistor R_s with $v_s = 10 + \sin(\omega t)$ (V)



$$v_s = \underline{10} + \underline{\sin(\omega t)} \text{ (V)}$$

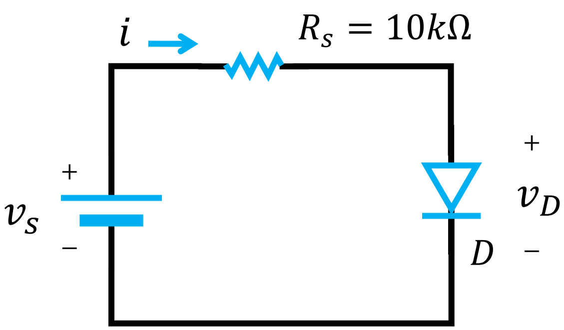
DC voltage

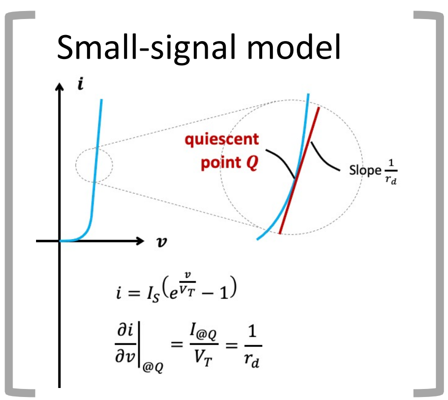
AC voltage with a peak of 1V



Example 1

QUESTION: Find the current through the resistor R_s with $v_s = 10\text{V} + \sin(\omega t)$





- According to KVL

$$v_s = iR_s + v_D$$

- Find the quiescent point by assuming $v_D|_{DC} = 0.7V$

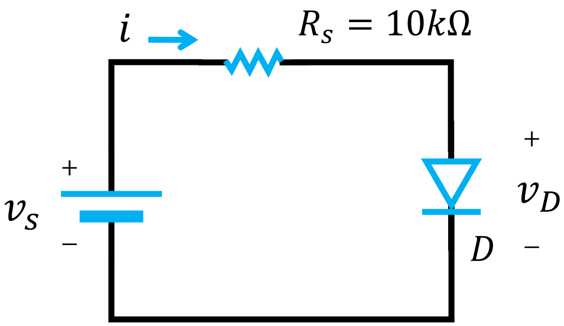
$$I_D = \frac{v_s|_{DC} - 0.7}{R_s} = 0.93mA$$

$$r_d = \frac{V_T}{I_{@Q}} = 26.9\Omega$$

- According to KVL

$$v_D\Big|_{AC} = \frac{r_d\, v_s|_{AC}}{R_s + r_d} = 2.68mV$$

Example 1

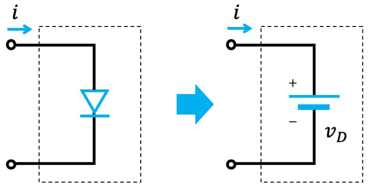


- Solution 1 – using **$i - v$ characteristics** of the diode

$$v_s = iR_s + V_T In \frac{i}{I_S}$$

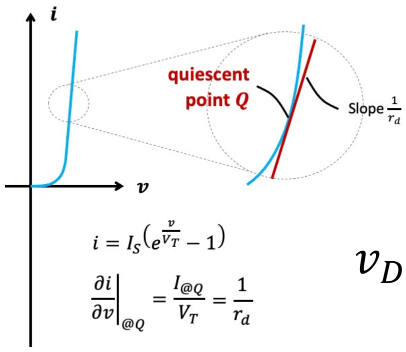
BUT WE CANNOT FIND AN ANALYTICAL SOLUTION

- Solution 2 – using **constant-voltage-drop model**



$$i = \frac{v_s - 0.7}{R_s}$$

- Solution 3 – using the **small-signal model**



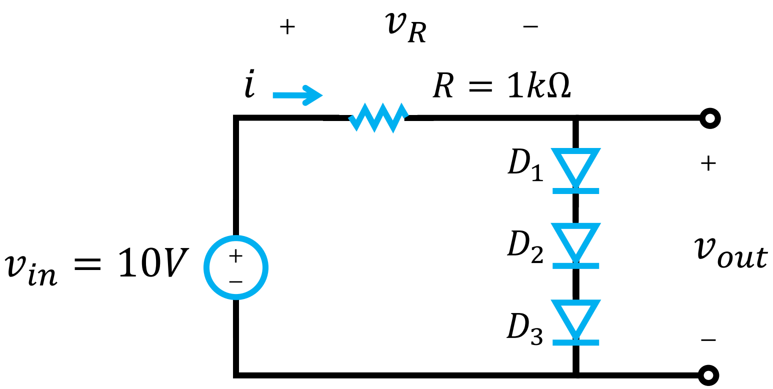
$$i = I_S\left(e^{\frac{v}{V_T}} - 1\right)$$

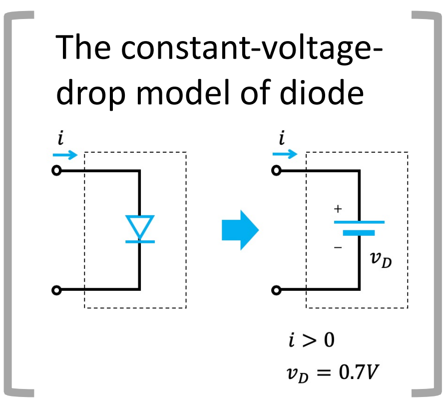
$$\left.\frac{\partial i}{\partial v}\right|_{@Q} = \frac{I_{@Q}}{V_T} = \frac{1}{r_d}$$

$$v_D\Big|_{AC} = \frac{r_d\ v_s|_{AC}}{R_s + r_d} = 2.68mV$$

Example 2

QUESTION: Find the current through the resistor R. Use the constant-voltage-drop model.





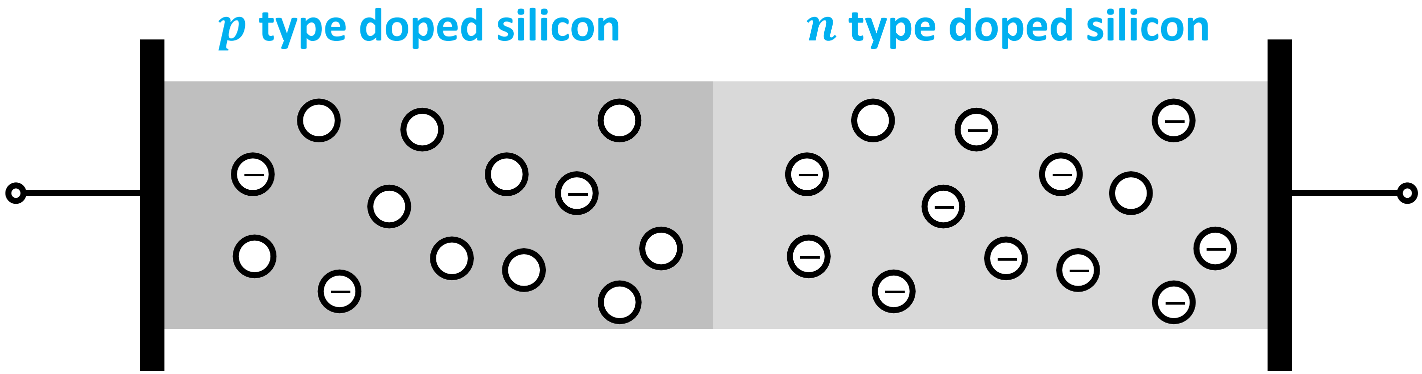
- Use the constant-voltage-drop model

$$v_{out} = 0.7V \times 3 = 2.1V$$

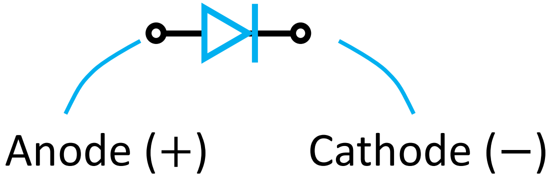
- According to KVL

$$I_R = \frac{v_{in} - v_{out}}{R} = 7.9mA$$

Junction diodes



symbol



Diode

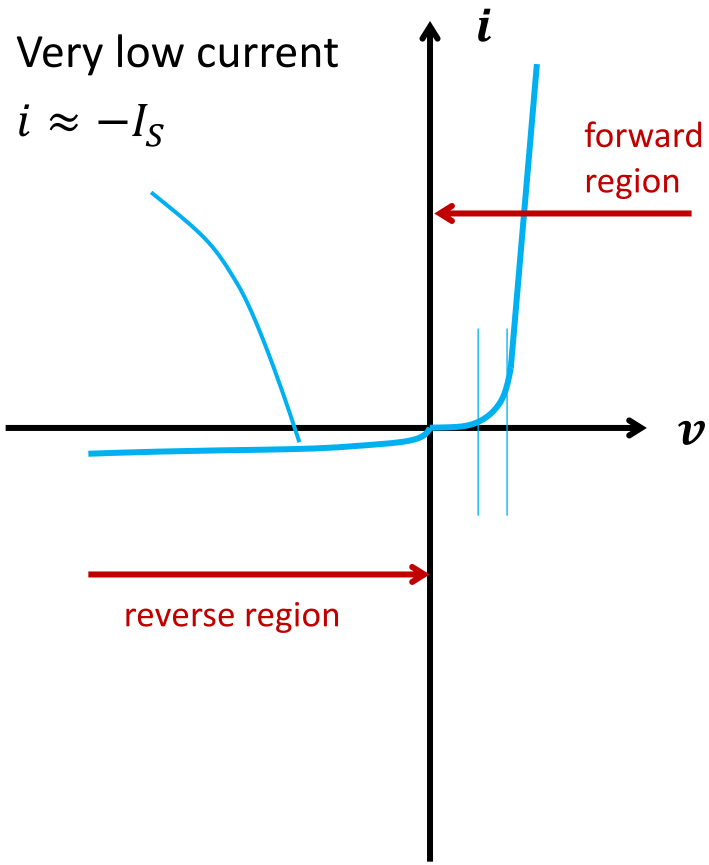
- A two-terminal device
- **Current flows from Anode to Cathode**

Outline

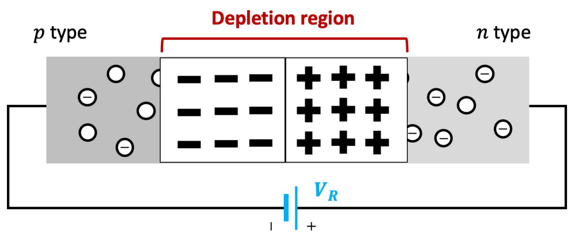
- Introduction to semiconductors
  - Semiconductor material & silicon crystal
  - Doped semiconductors
  - Current flow in semiconductor
  - The $\boldsymbol{pn}$ junction
    - pn junction @open circuit
    - pn junction with applied voltage
- Diodes
  - The forward region
  - **The reverse region**

$i - v$ characteristics of pn junction

- Very low current
  $i \approx -I_S$



Recall: pn junction with reverse-bias voltage

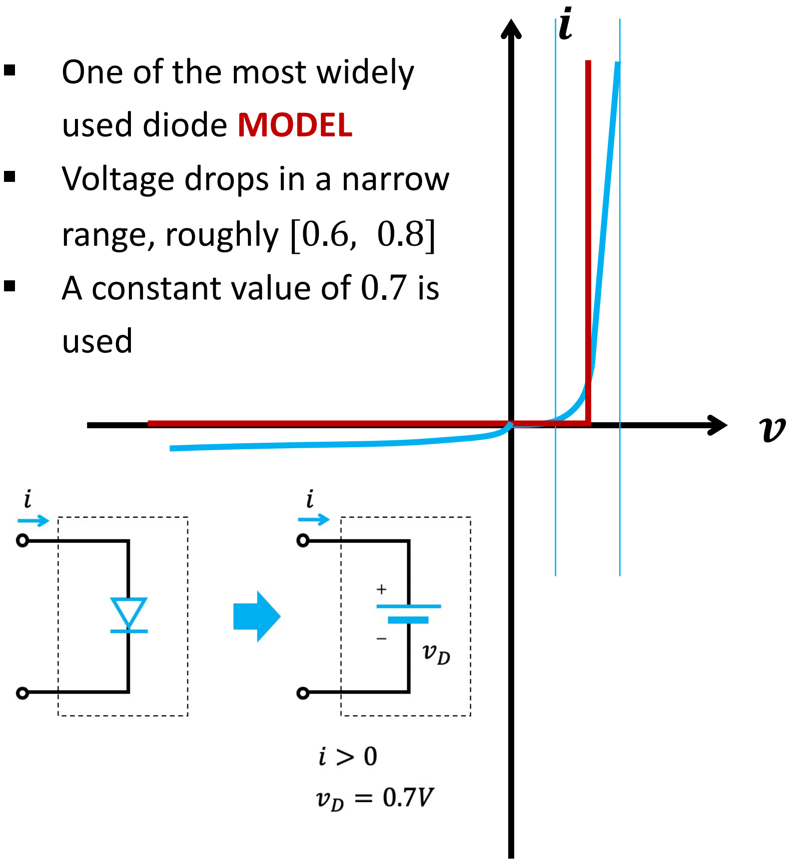


- n is more positive than p due to the depletion region
- V_F makes n even more positive than p
- width of depletion region ↑
- Diffusion current is dramatically reduced, $I_D \to 0$
- Drift current I_S dominates

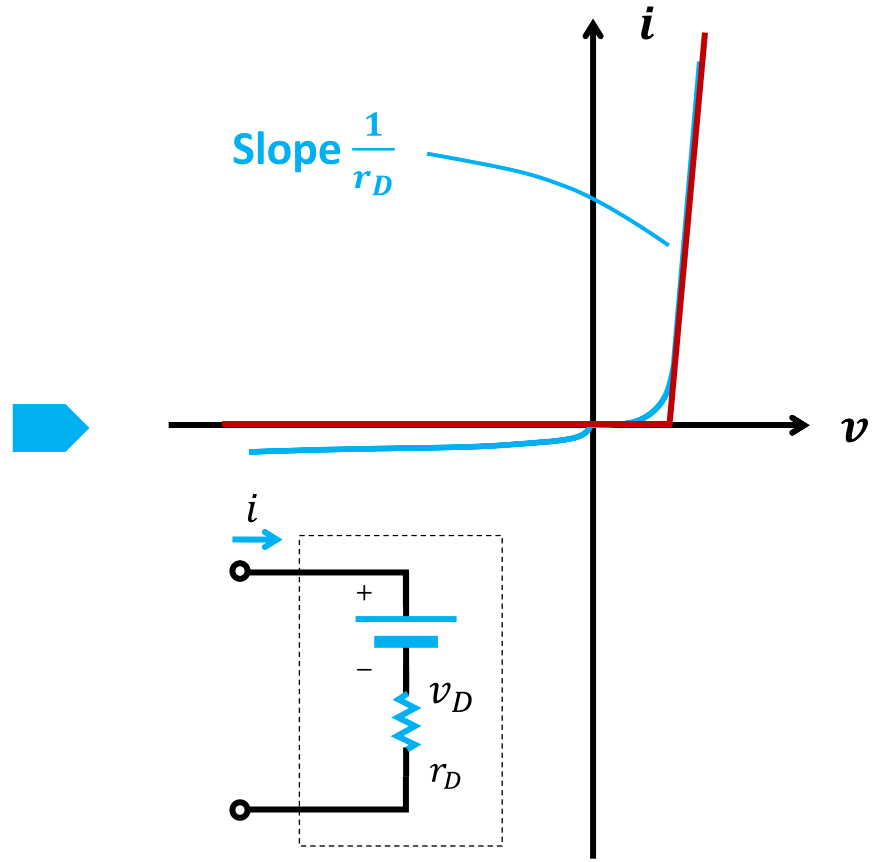
The models of diode

CONSTANT-VOLTAGE-DROP MODEL

- One of the most widely used diode **MODEL**
- Voltage drops in a narrow range, roughly $[0.6,\ 0.8]$
- A constant value of 0.7 is used

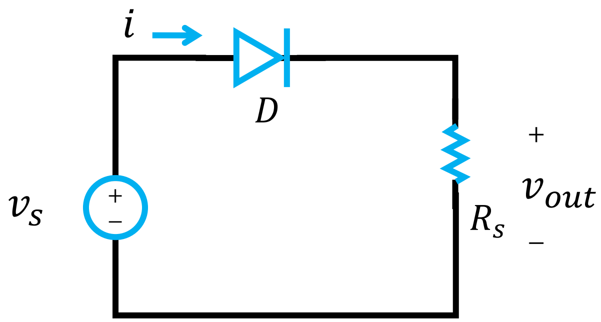


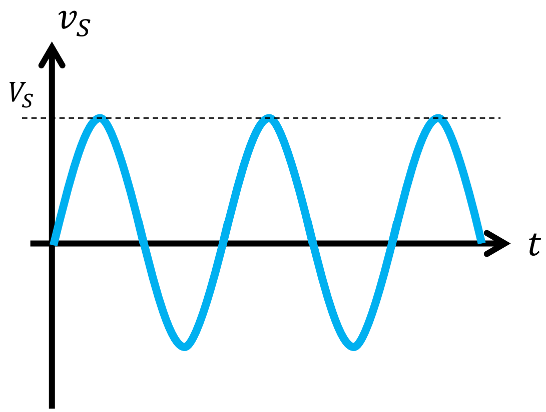
More realistic model



Example 4: half-wave rectifier

QUESTION: Find the output voltage with the given input. The resistance of the diode must be considered





- When $v_s < 0$

  The diode is reverse biased

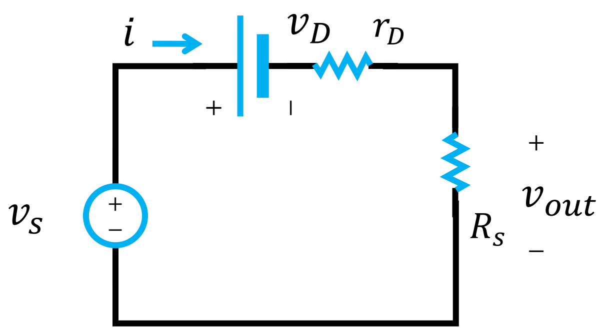
  $i = 0 \qquad v_{out} = 0$

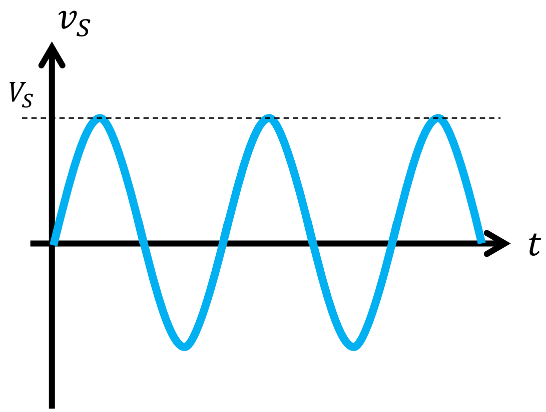
- When $v_s > 0$

  The diode is forward biased

Example 4: half-wave rectifier

QUESTION: Find the output voltage with the given input. The resistance of the diode must be considered





- When $v_s < 0$

  The diode is reverse biased

  $$i = 0 \qquad v_{out} = 0$$

- When $v_s > 0$

  The diode is forward biased

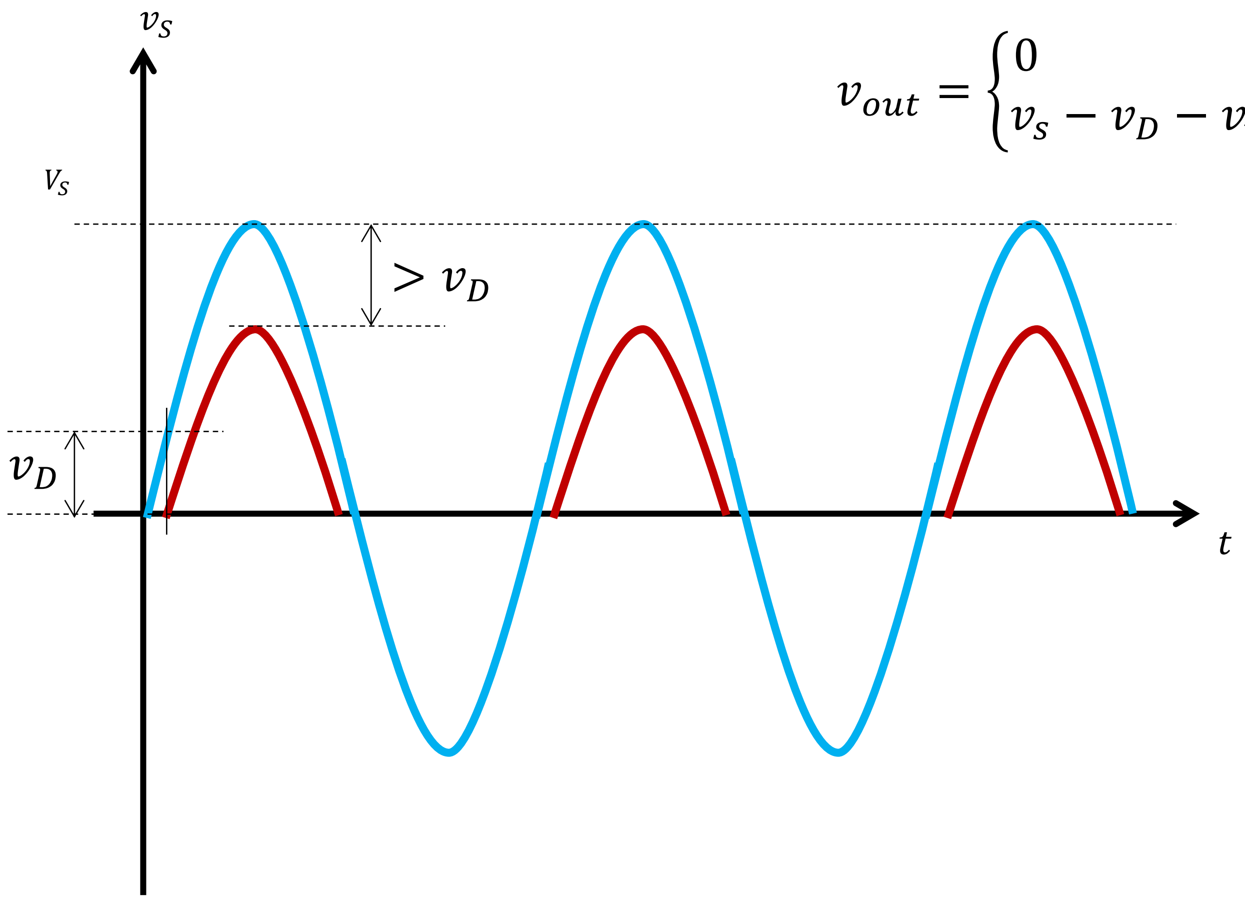
  $$i = \frac{v_s - v_D}{r_D + R_s}$$

  $$v_{out} = iR_s$$

Example 4: half-wave rectifier

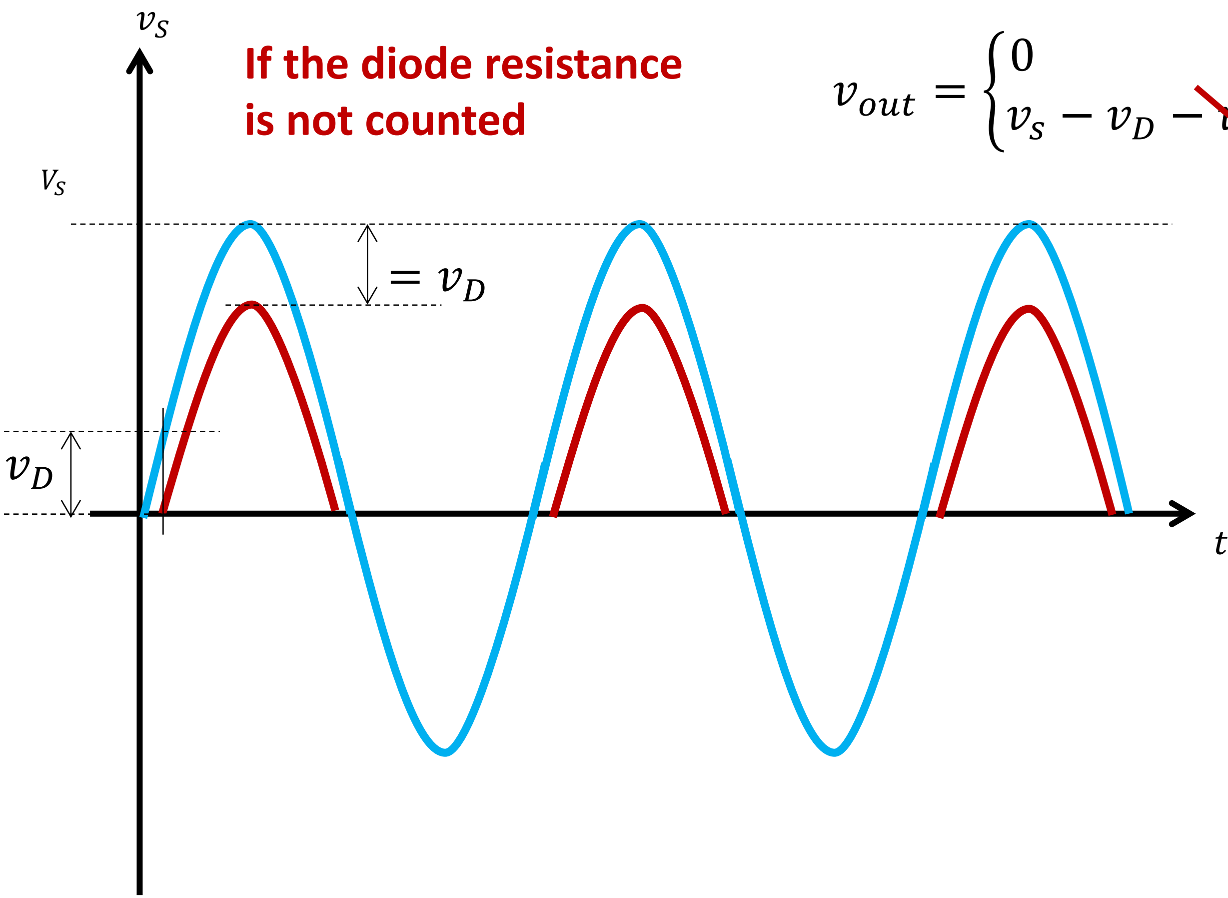
QUESTION: Find the output voltage with the given input. The resistance of the diode must be considered

$$v_{out} = \begin{cases} 0 & when \quad v_s < 0 \\ v_s - v_D - v_{r_D} & when \quad v_s > 0 \end{cases}$$



Example 4: half-wave rectifier

QUESTION: Find the output voltage with the given input. The resistance of the diode must be considered

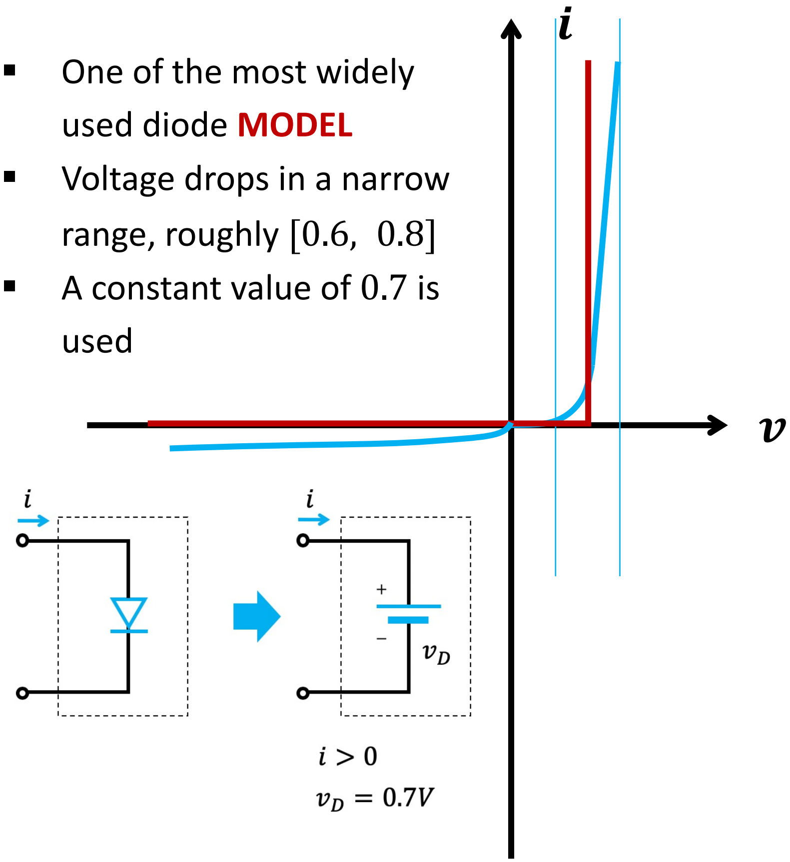


$$v_{out} = \begin{cases} 0 & when \quad v_s < 0 \\ v_s - v_D - \cancel{v_{rD}} & when \quad v_s > 0 \end{cases}$$

The models of diode

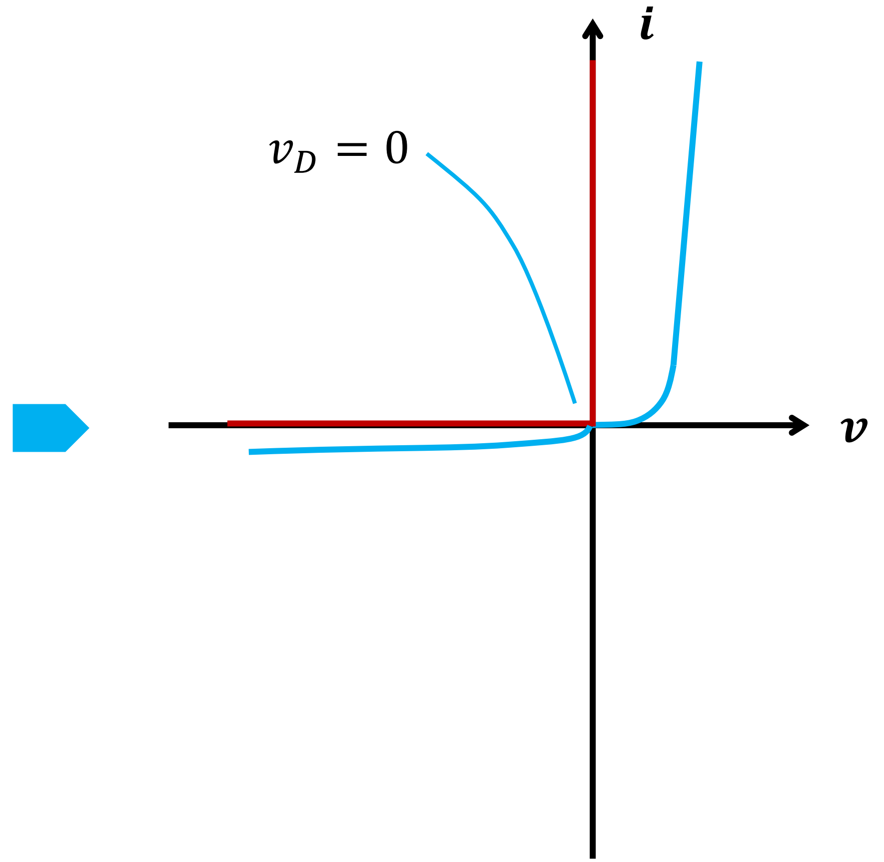
CONSTANT-VOLTAGE-DROP MODEL

- One of the most widely used diode **MODEL**
- Voltage drops in a narrow range, roughly $[0.6,\ 0.8]$
- A constant value of 0.7 is used

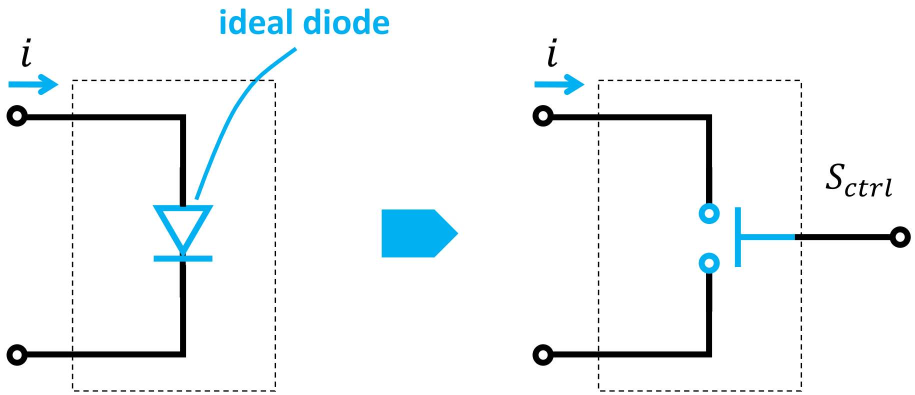


$i > 0$
$v_D = 0.7V$

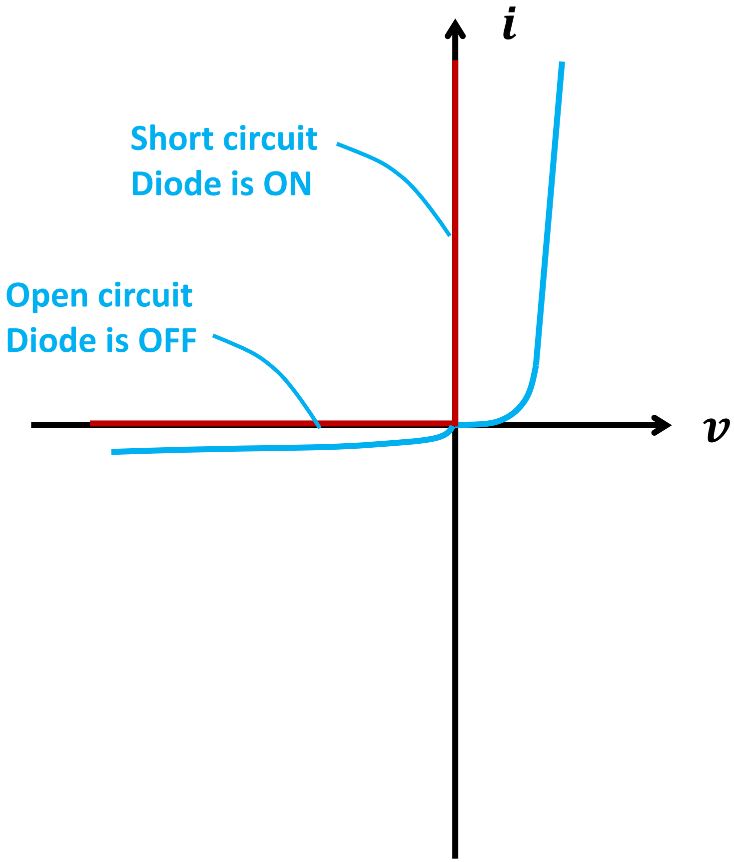
IDEAL MODEL



The models of ideal diode



$$i = \begin{cases} > 0 & if \quad S_{ctrl} > 0 \\ 0 & if \quad S_{ctrl} < 0 \end{cases}$$

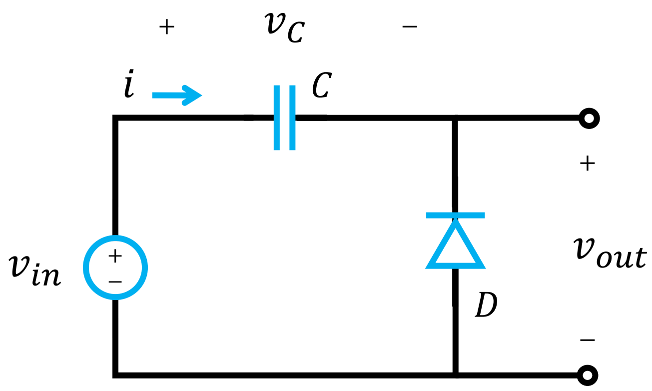


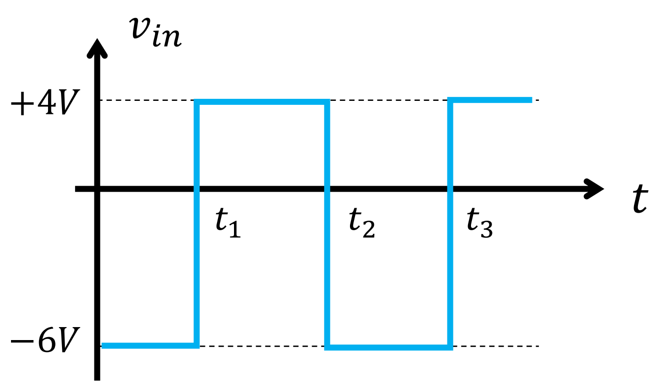
Ideal Diode

- A two-terminal device
- Current **ONLY** flows from Anode to Cathode
- An electronic 1-way valve

Example 3: ideal diode

QUESTION: Find the output voltage with the given input. The diode is ideal





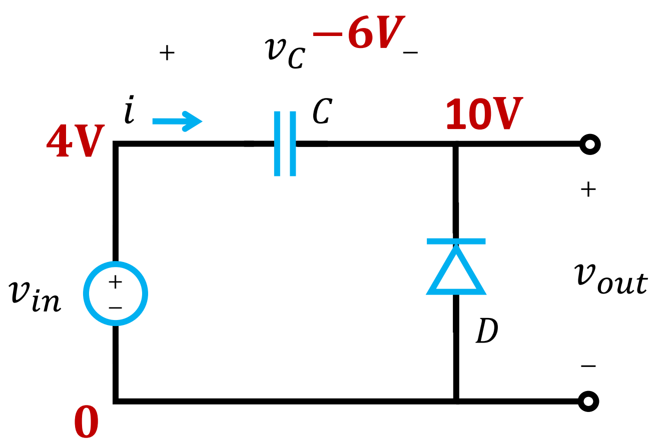
- When $t \in [0, t_1)$

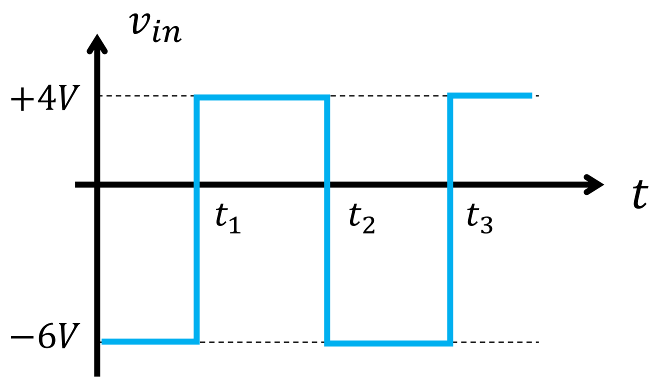
  D is forward-biased

  $\rightarrow v_{out} = 0 \qquad v_C = -6V$

Example 3: ideal diode

QUESTION: Find the output voltage with the given input. The diode is ideal





- When $t \in [0, t_1)$

  D is forward-biased

  $\rightarrow v_{out} = 0 \qquad v_C = -6V$

- When $t = t_1^+$

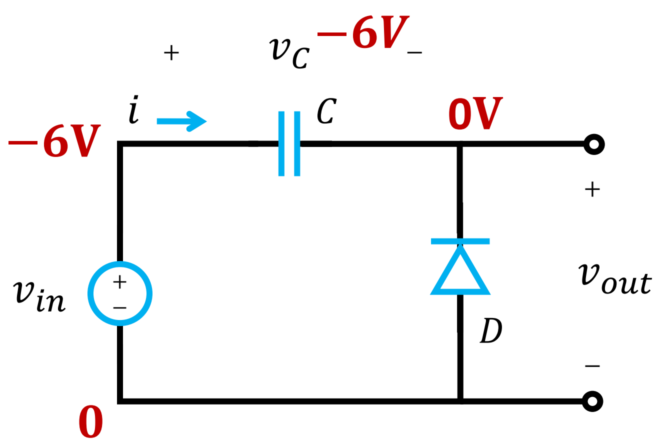
  Since voltage on capacitor CANNOT change abruptly

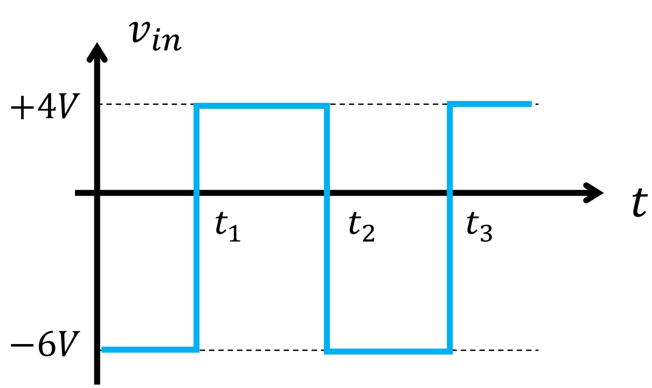
  $\rightarrow v_{out} = 10V \qquad v_C = -6V$

  D is reverse-biased

Example 3: ideal diode

QUESTION: Find the output voltage with the given input. The diode is ideal





- When $t = t_2^+$

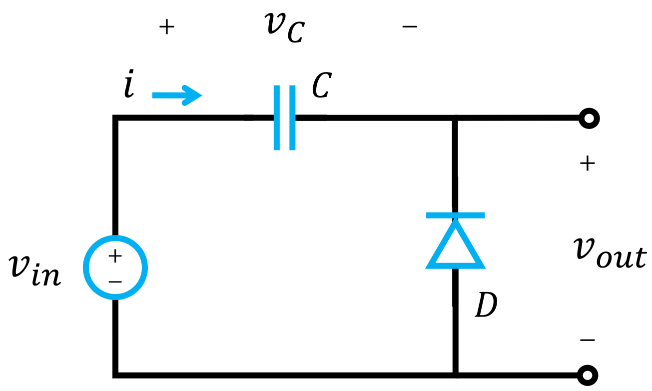
  D is forward-biased

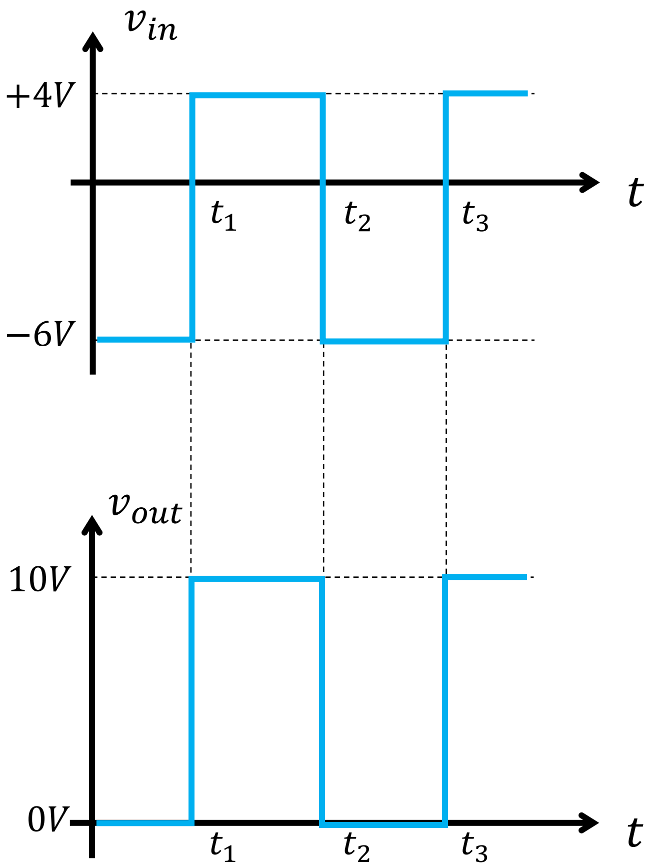
  Since voltage on capacitor CANNOT change abruptly

  $\rightarrow v_{out} = 0 \qquad v_C = -6V$

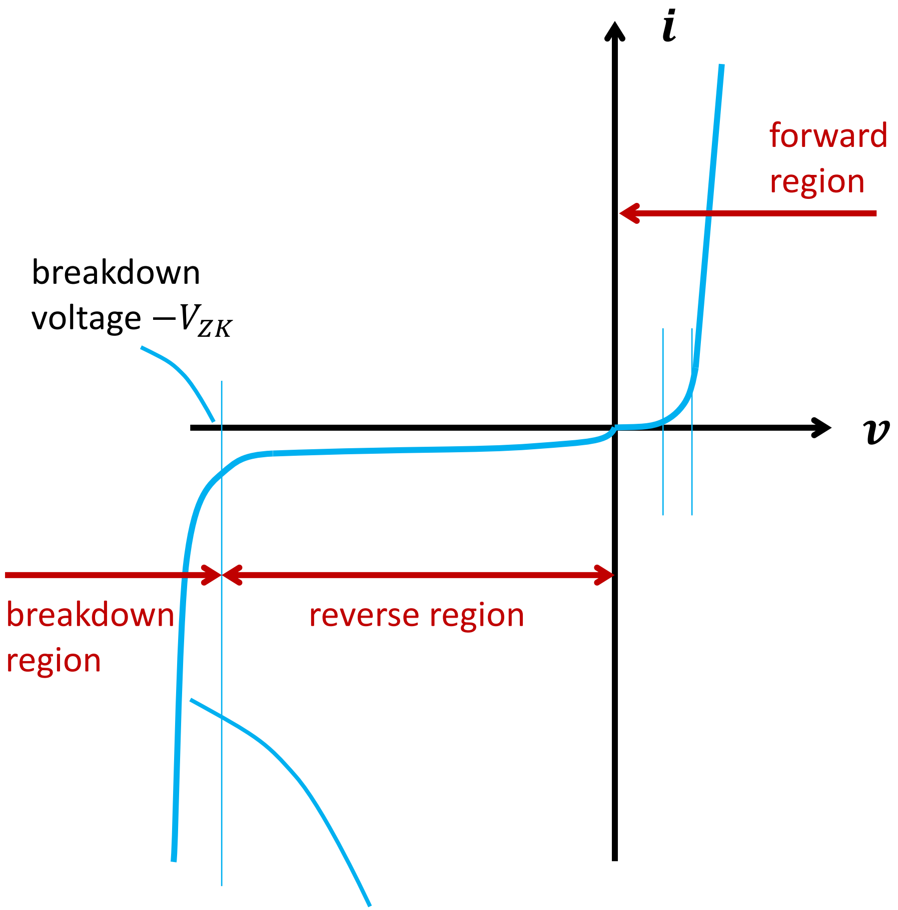
Example 3: ideal diode

QUESTION: Find the output voltage with the given input. The diode is ideal





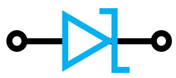
$i - v$ characteristics of pn junction



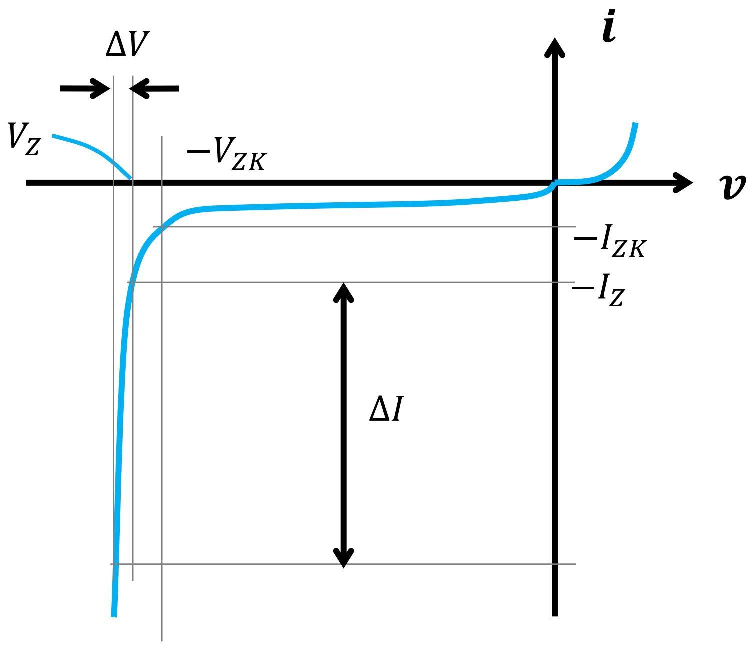
- Current increases rapidly
- Voltage drops very small

- Junction breakdown @$V = - -V_{ZK}$
- Case 1 – **Zener effect**
  - Current generated by breaking the electron-hole pairs
- Case 2 – **Avalanche effect**
  - Current generated by breaking the covalent bonds in atoms

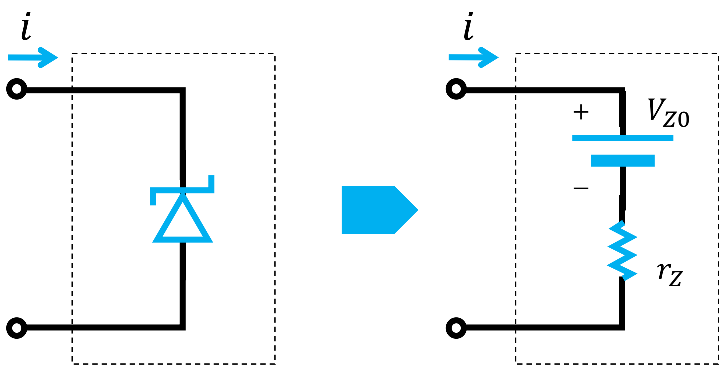
Zener diode

symbol 

ZENER DIODE is special diodes manufactured to operate in the breakdown region



Dynamic resistance $r_Z = \frac{\Delta V}{\Delta I}$

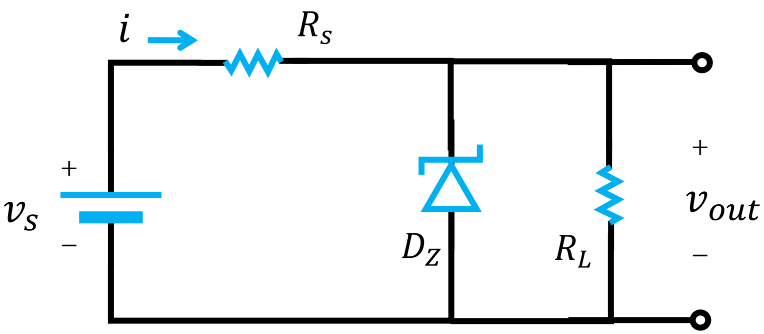


Model for Zener diode

$$V_Z = V_{Z0} + I_Z r_Z$$

Example 3: Zener diode

QUESTION: the zener diode D_Z is specified to have $V_Z = 6.8\text{V}$ at $I_Z = 5mA$, $r_Z = 20\Omega$. The supply voltage v_s is nominally $10V$ but can vary by $\pm 1V$. $R_S = 0.5k\Omega$. Find the output voltage with different load $R_L = \infty, 2k\Omega\ or\ 0.5k\Omega$.

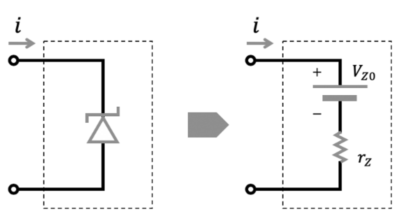


- For the Zener diode model, according to KVL

$$V_Z = V_{Z0} + I_Z r_Z$$

$$\rightarrow \quad V_{Z0} = 6.7V$$

Recall: model for Zener diode



Model for Zener diode

$V_Z = V_{Z0} + I_Z r_Z$

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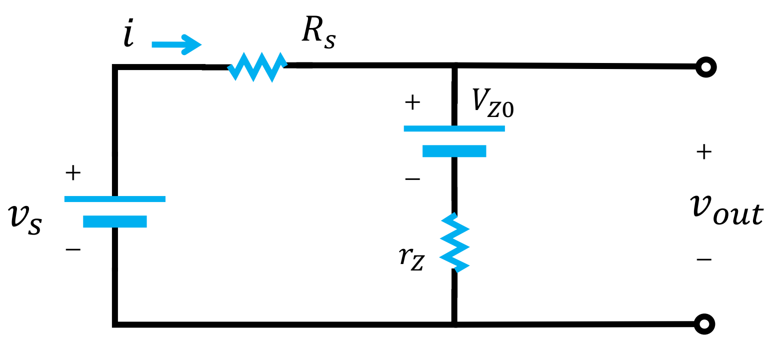
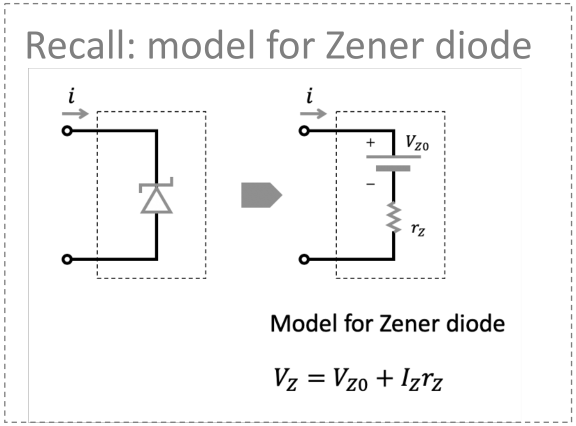




- For the zener diode model, according to KVL

$$V_Z = V_{Z0} + I_Z r_Z$$

$$\rightarrow \quad V_{Z0} = 6.7V$$

- If there is no load ($R_L = \infty$), according to KVL

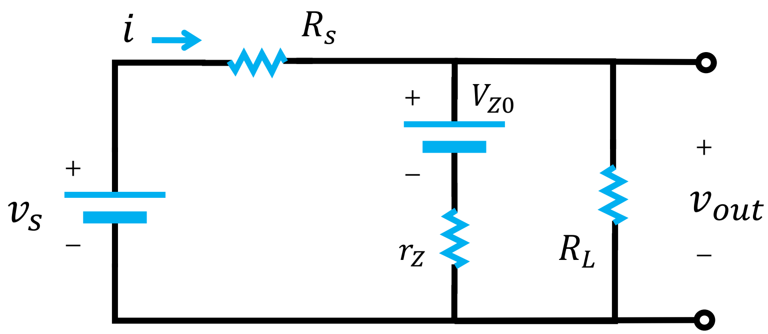
$$v_s = I_Z R_S + V_{Z0} + I_Z r_Z$$

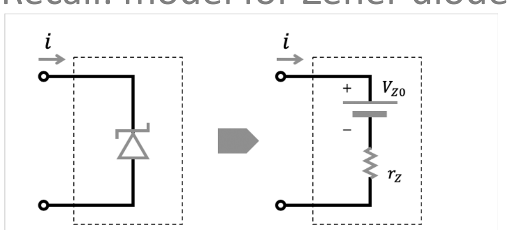
$$\rightarrow \quad I_Z = 6.35mA$$

$$v_{out} = V_{Z0} + I_Z r_Z = 6.83V$$

Example 3: Zener diode

QUESTION: the zener diode D_Z is specified to have $V_Z = 6.8\text{V}$ at $I_Z = 5mA$, $r_Z = 20\Omega$. The supply voltage v_s is nominally $10V$ but can vary by $\pm 1V$. $R_S = 0.5k\Omega$. Find the output voltage with different load $R_L = \infty, 2k\Omega\ or\ 0.5k\Omega$.





Model for Zener diode

$V_Z = V_{Z0} + I_Z r_Z$

- If $R_L = 2k\Omega$, since $R_L \gg r_Z$, approximately,

  $\rightarrow i$ does not change

  $\rightarrow i_{R_L} \uparrow$ compared to no load

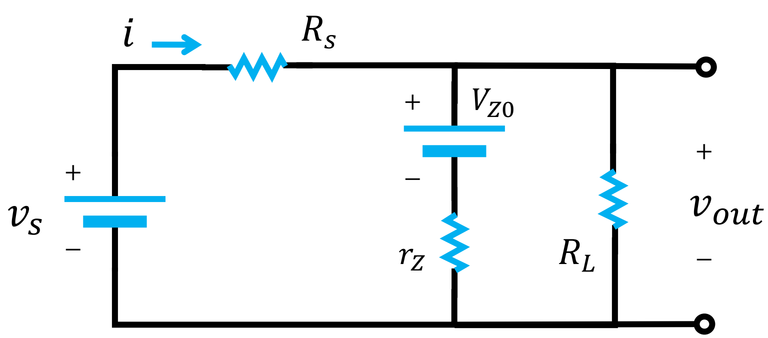
  $\rightarrow i_{r_Z} \downarrow$ compared to no load

$$i_{R_L} \approx \frac{V_Z}{R_L} = 3.4mA \quad = -\Delta i_{r_Z}$$

$$\Delta v_{out} \approx -\Delta i_{r_Z} r_Z = -68\text{mV}$$

Example 3: Zener diode

QUESTION: the zener diode D_Z is specified to have $V_Z = 6.8\text{V}$ at $I_Z = 5mA$, $r_Z = 20\Omega$. The supply voltage v_s is nominally $10V$ but can vary by $\pm 1V$. $R_S = 0.5k\Omega$. Find the output voltage with different load $R_L = \infty, 2k\Omega\ or\ 0.5k\Omega$.

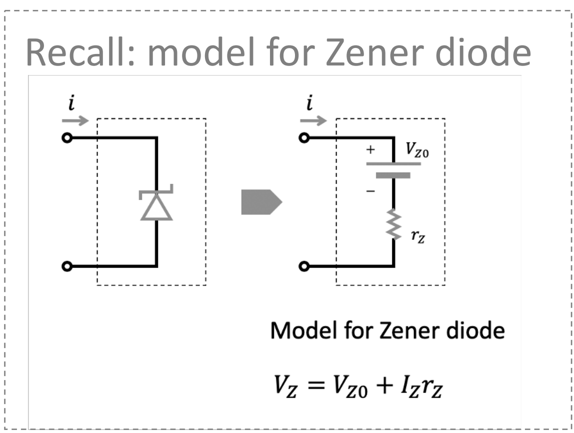


- If $R_L = 2k\Omega$, more precisely, according to KCL

$$\frac{v_s - v_{out}}{R_S} = \frac{v_{out} - V_{Z0}}{r_Z} + \frac{v_{out}}{R_L}$$

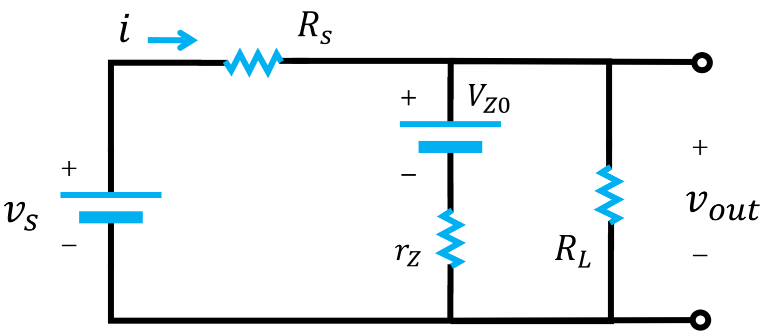
$$\rightarrow\ v_{out} = 6.87V$$

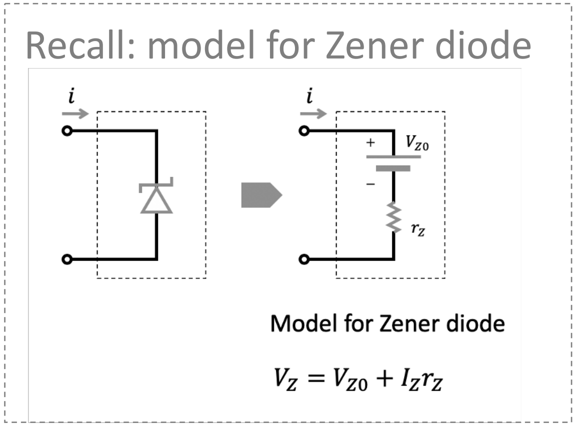
$$\Delta v_{out} = -70\text{mV}$$



Example 3: Zener diode

QUESTION: the zener diode D_Z is specified to have $V_Z = 6.8\text{V}$ at $I_Z = 5mA$, $r_Z = 20\Omega$. The supply voltage v_s is nominally $10V$ but can vary by $\pm 1V$. $R_S = 0.5k\Omega$. Find the output voltage with different load $R_L = \infty, 2k\Omega\ or\ 0.5k\Omega$.





- If $R_L = 0.5k\Omega$, if the zener diode is on

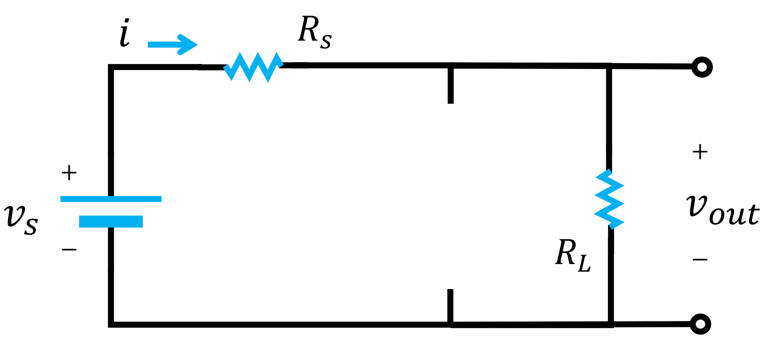
$$I_{R_L} \approx \frac{V_Z}{R_L} = 13.6mA > I_Z$$

It's impossible

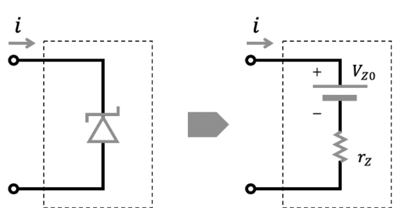
Thus, the Zener diode is disabled

Example 3: Zener diode

QUESTION: the zener diode D_Z is specified to have $V_Z = 6.8\text{V}$ at $I_Z = 5mA$, $r_Z = 20\Omega$. The supply voltage v_s is nominally $10V$ but can vary by $\pm 1V$. $R_S = 0.5k\Omega$. Find the output voltage with different load $R_L = \infty, 2k\Omega\ or\ 0.5k\Omega$.



Recall: model for Zener diode



Model for Zener diode

$V_Z = V_{Z0} + I_Z r_Z$

- If $R_L = 0.5k\Omega$, if the zener diode is on

$$I_{R_L} \approx \frac{V_Z}{R_L} = 13.6mA > I_Z$$

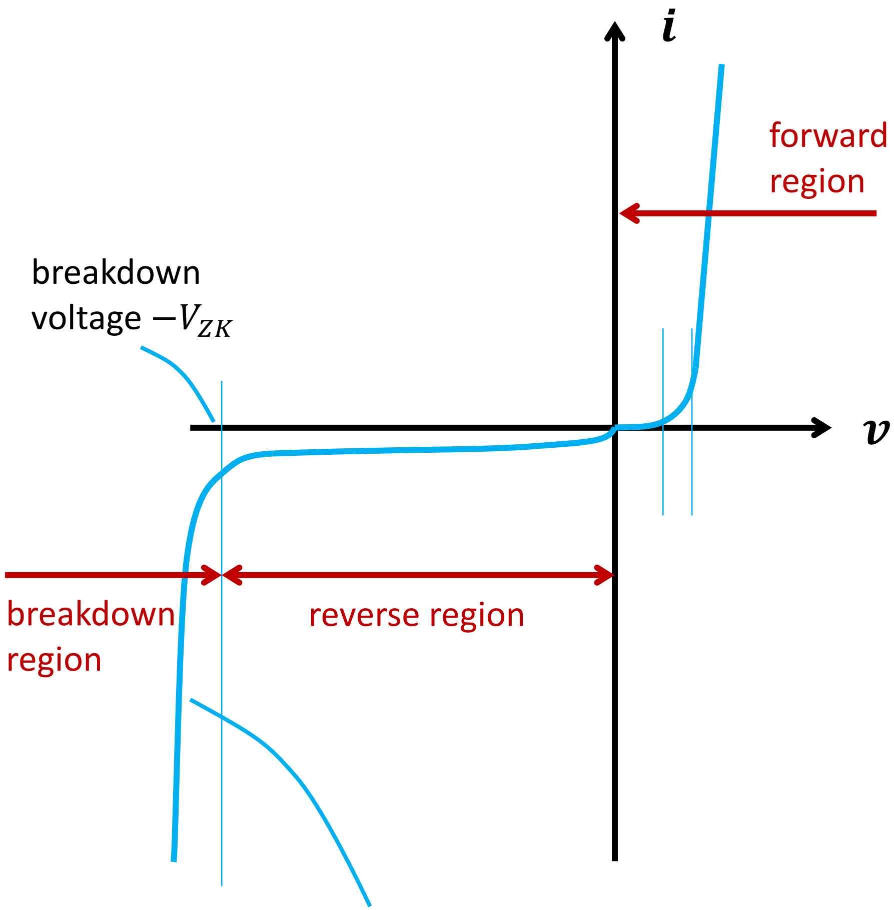
It's impossible

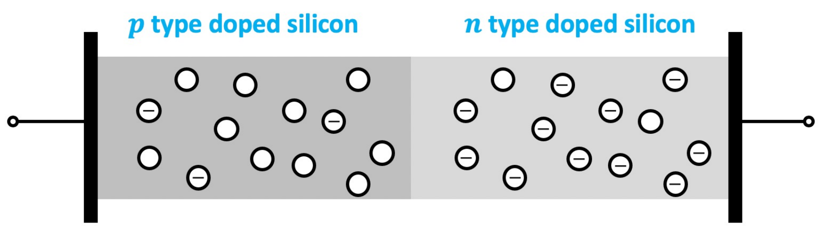
Thus, the Zener diode is disabled

- According to KVL

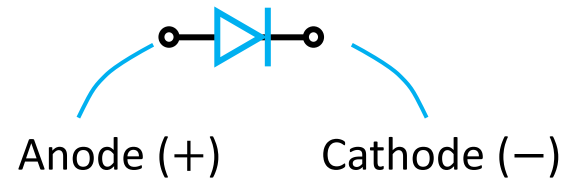
$$v_{out} = v_s \frac{R_L}{R_s + R_L} = 5V$$

Summary: the pn junction diode





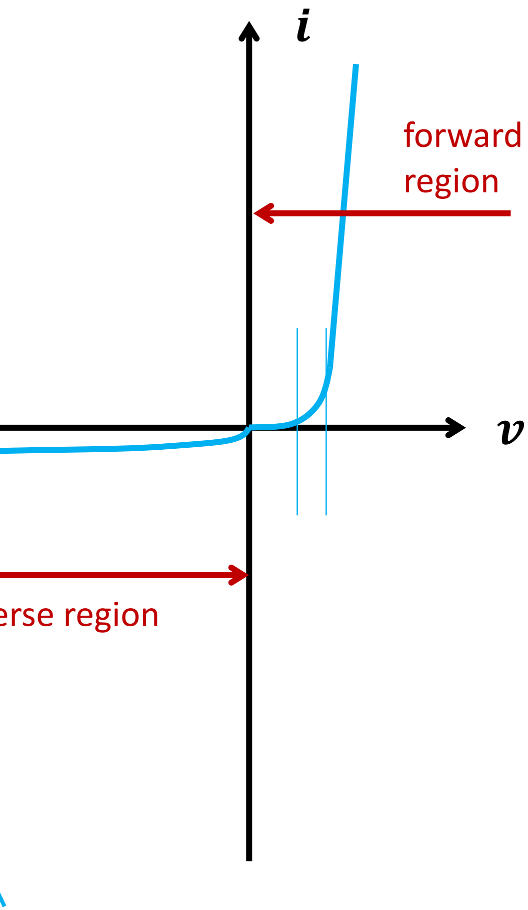
symbol



Diode

- A two-terminal device
- Current flows from Anode to Cathode

Summary: the pn junction diode



- In forward region

  $$i = I_S\left(e^{\frac{v}{V_T}} - 1\right)$$

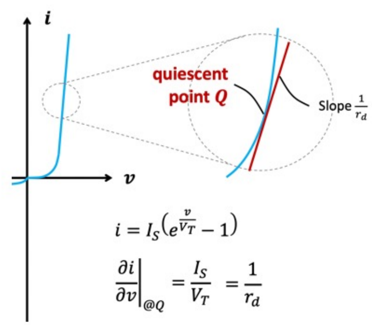
  Solution to circuit with diodes

  → KVL/KCL + $i - v$ characteristics

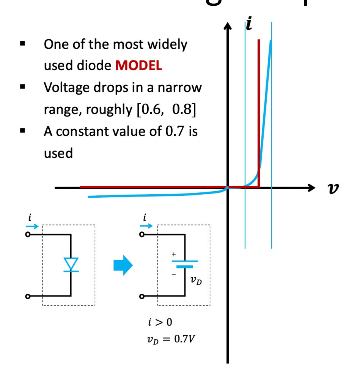
  → Analytical solution **UNAVAILABLE**

  Alternative practical solutions – MODELLING

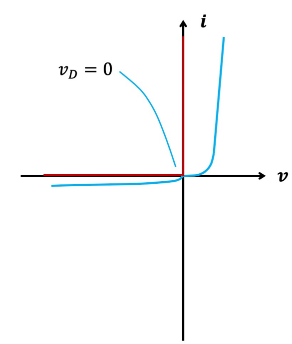
Small-signal model



Constant-voltage drop model



Ideal model

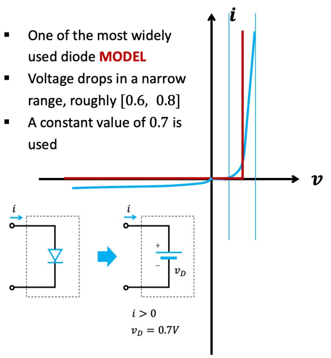


Summary: the pn junction diode

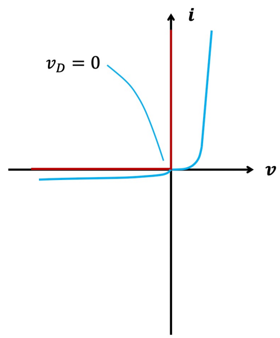
- In reverse region

  Modelling: no current pass through

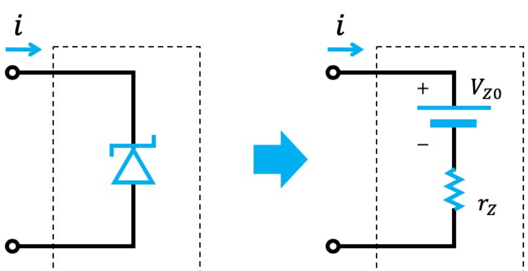
  Constant-voltage drop model



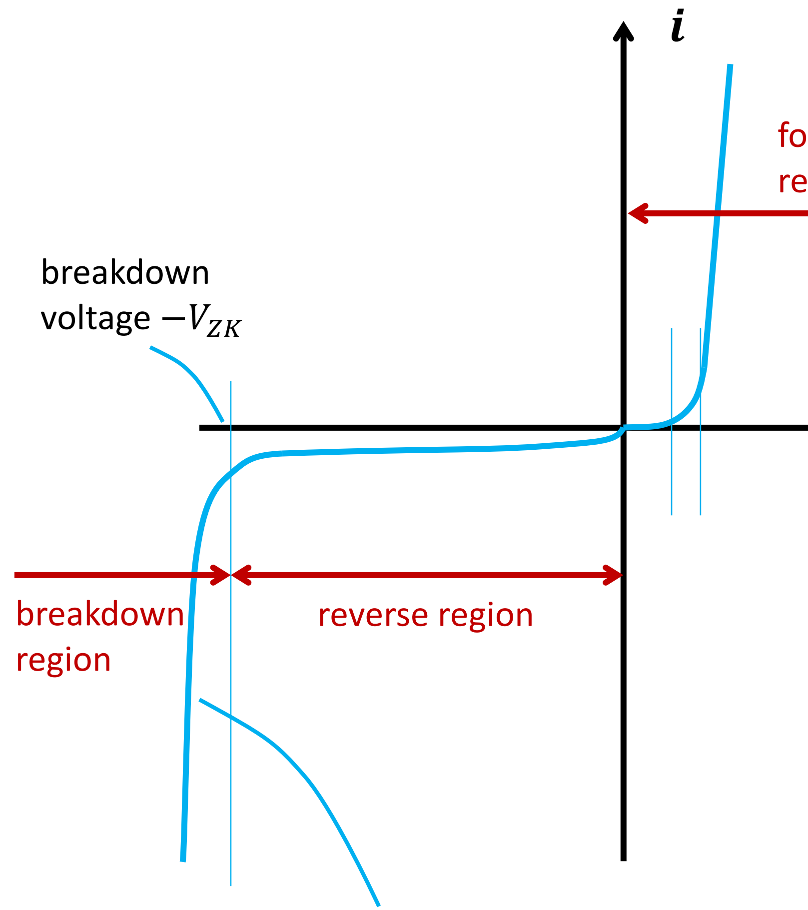
  Ideal model



- In breakdown region



Model for Zener diode

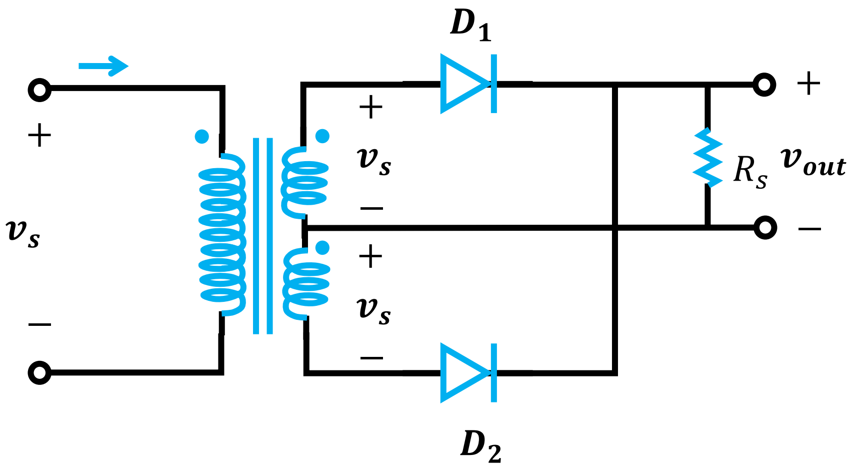


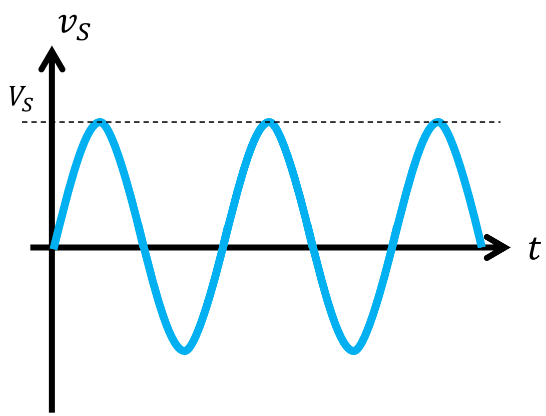
Outline

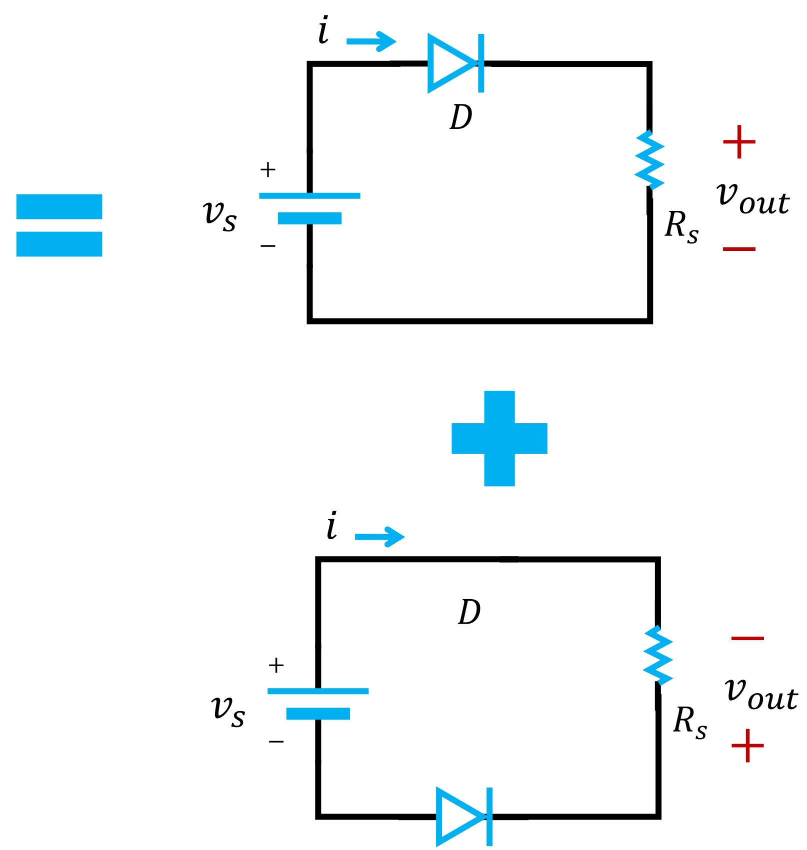
- Introduction to semiconductors
  - Semiconductor material & silicon crystal
  - Doped semiconductors
  - Current flow in semiconductor
  - The $\boldsymbol{pn}$ junction
- Diodes
  - The $i - v$ characteristics
  - The models: Constant-voltage-drop / ideal / Small-signal model
  - The 3 working regions: forward / reverse / reverse breakdown
  - **Applications of diodes**

Example 5: full-wave rectifier

QUESTION: Find the output voltage with the given input.

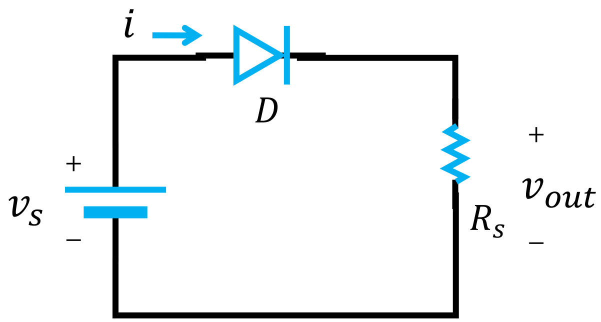


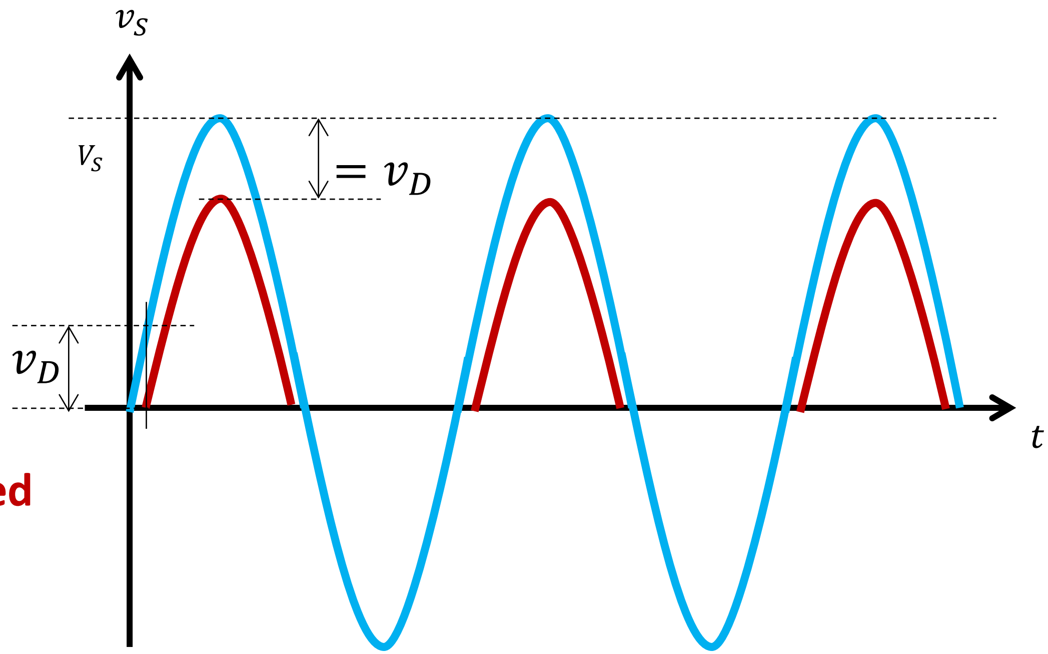




Recall: half-wave rectifier

QUESTION: Find the output voltage with the given input. The resistance of the diode must be considered



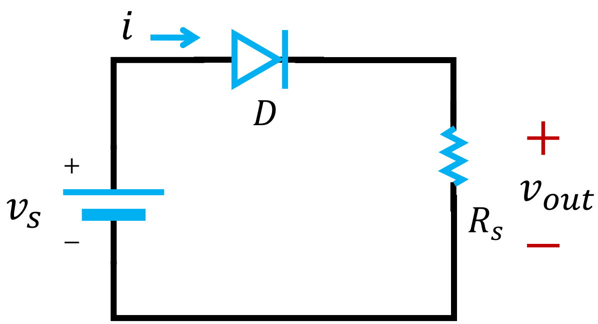


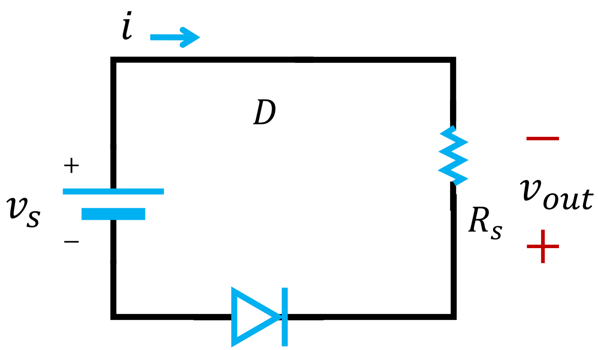
If the diode resistance is not counted

$$v_{out} = \begin{cases} 0 & when \quad v_s < 0 \\ v_s - v_D & when \quad v_s > 0 \end{cases}$$

Example 5: full-wave rectifier

QUESTION: Find the output voltage with the given input.





If the diode resistance is not counted

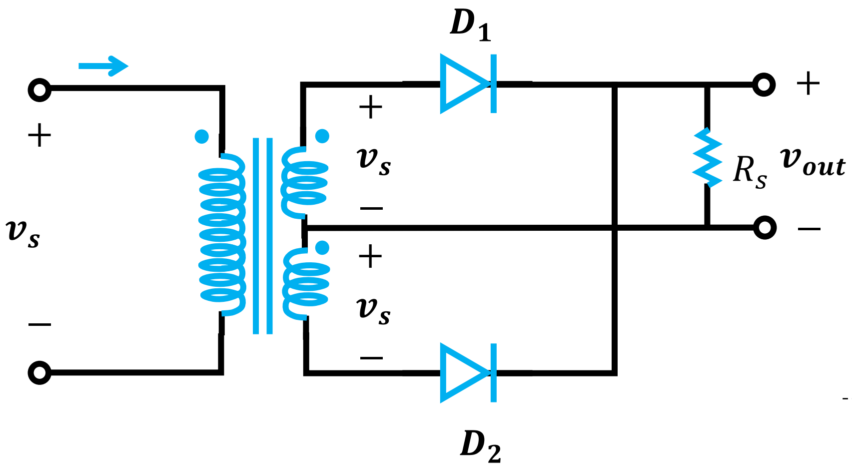
$$v_{out} = \begin{cases} 0 & when \quad v_s < 0 \\ v_s - v_D & when \quad v_s > 0 \end{cases}$$

If the diode resistance is not counted

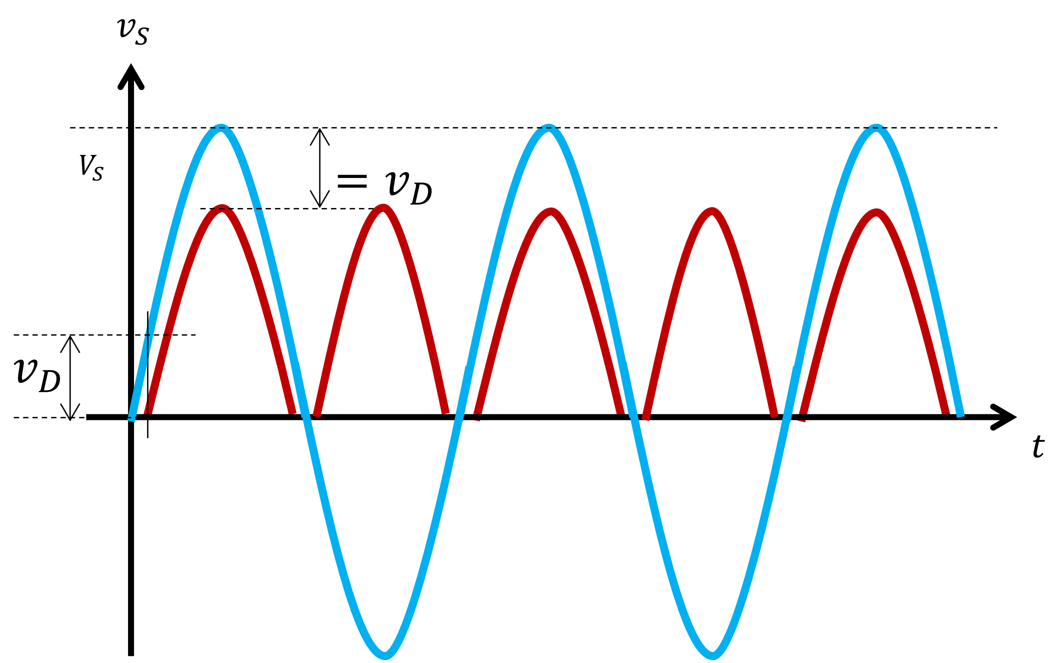
$$v_{out} = \begin{cases} -v_s - v_D & when \quad v_s < 0 \\ 0 & when \quad v_s > 0 \end{cases}$$

Example 5: full-wave rectifier

QUESTION: Find the output voltage with the given input.



Full-wave rectifier utilizes both halves of the input. It is more energetic and more useful.

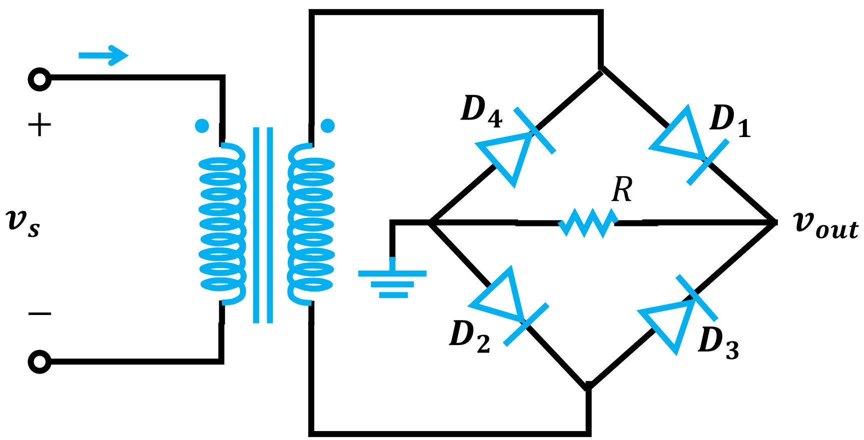


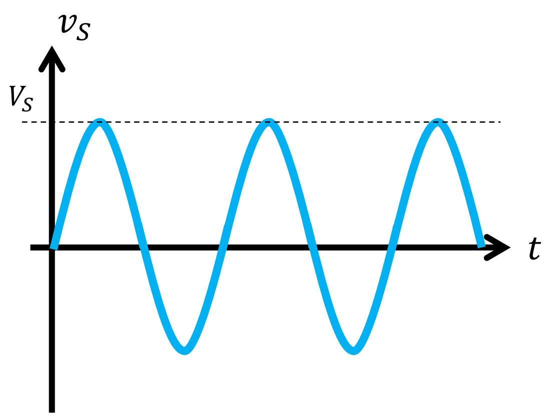
If the diode resistance is not counted

$$v_{out} = \begin{cases} -v_S - v_D & when \quad v_S < 0 \\ v_S - v_D & when \quad v_S > 0 \end{cases}$$

Example 6: bridge rectifier

QUESTION: Find the output voltage with the given input. The diodes are ideal.





- Each diode maybe ON/OFF
- How to solve it?

  Step 1a: assume D_1 is ON/OFF

  Step 1b: assume D_2 is ON/OFF

  Step 1c: assume D_3 is ON/OFF

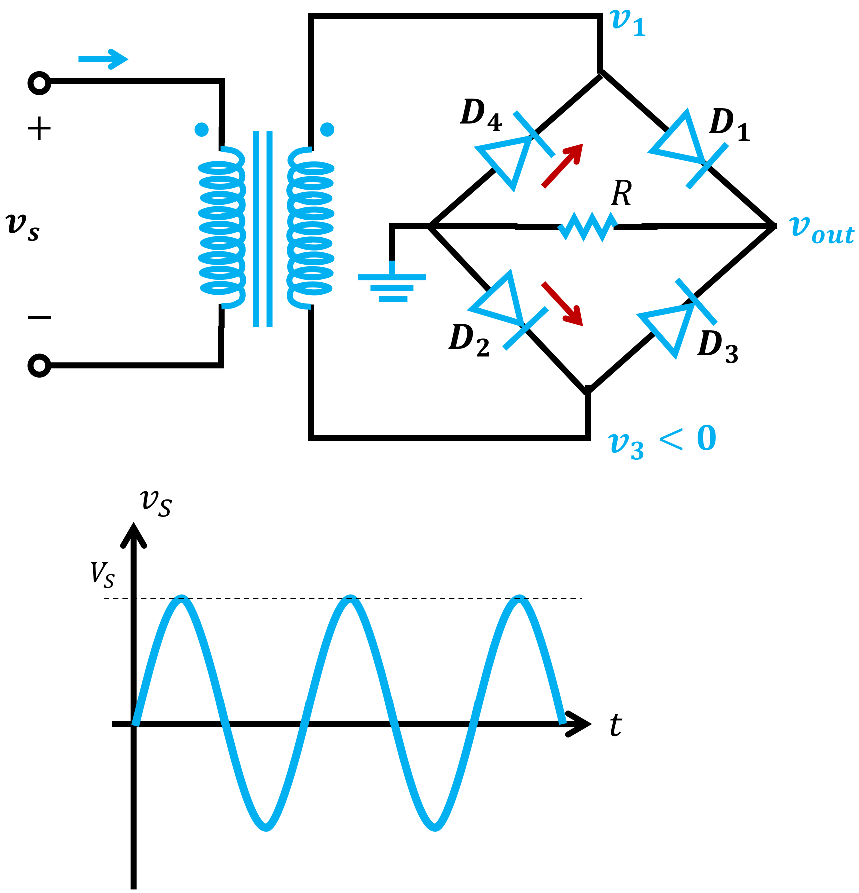
  Step 1d: assume D_4 is ON/OFF

A combination of 16 cases, a lot of work!!!

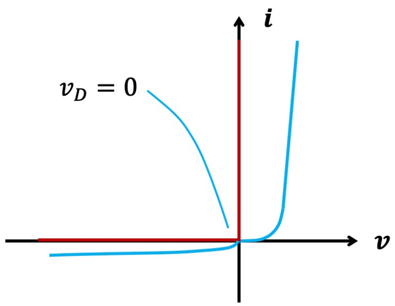
IS THERE A BETTER WAY?

Example 6: bridge rectifier

QUESTION: Find the output voltage with the given input. The diodes are ideal.



- Assume D_2 is ON
- According to the $i - v$ characteristics of ideal diode

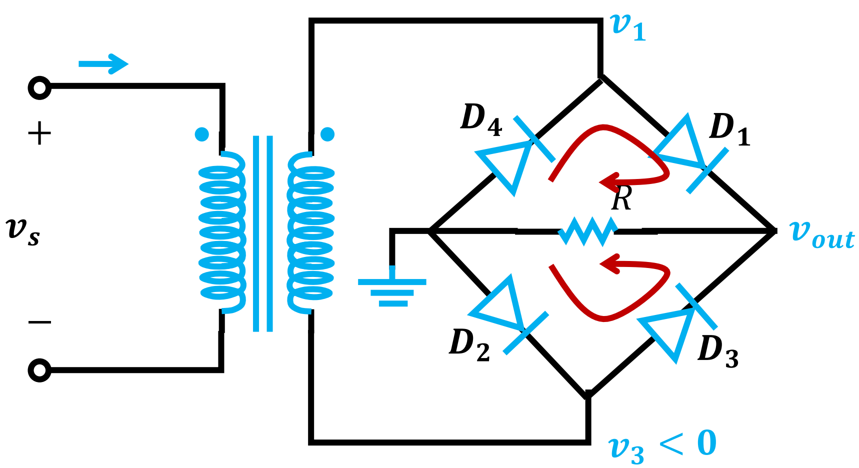


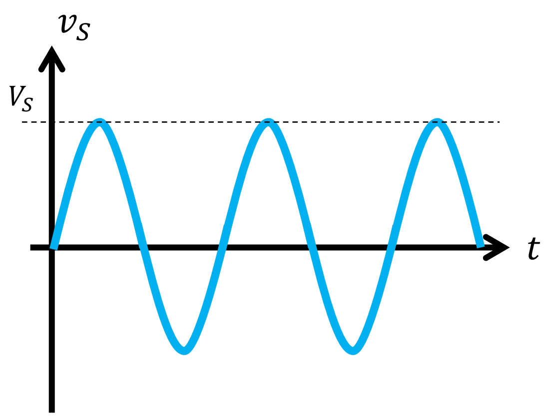
$\rightarrow \quad v_3 < 0$

- If D_4 is also ON $\quad \rightarrow \quad v_1 < 0$
- Only 1 possible path for current flow

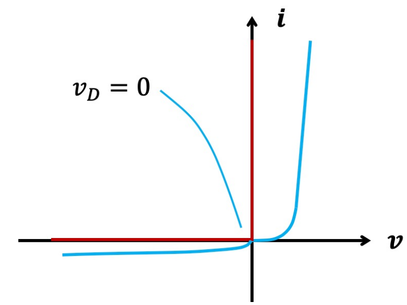
Example 6: bridge rectifier

QUESTION: Find the output voltage with the given input. The diodes are ideal.





- Assume D_2 is ON
- According to the $i - v$ relationship of ideal diode



$\rightarrow \quad v_3 < 0$

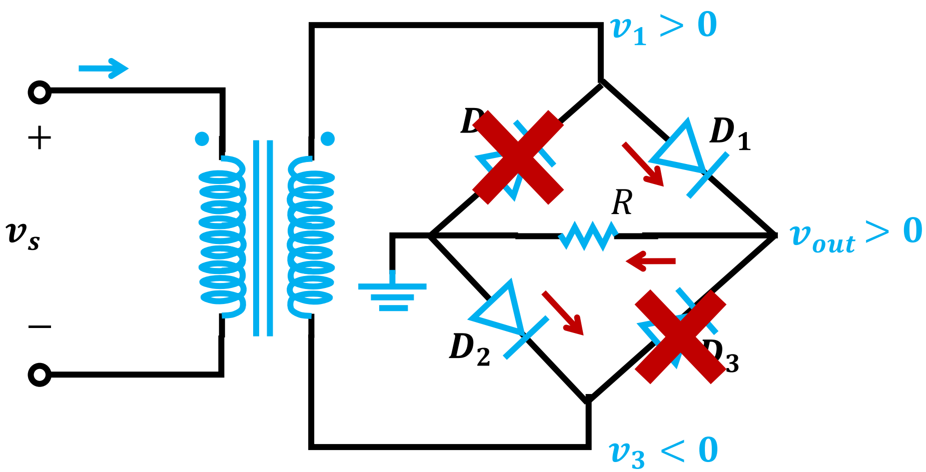
- If D_4 is also ON $\quad \rightarrow \quad v_1 < 0$ ✗
- Only 1 possible path for current flow

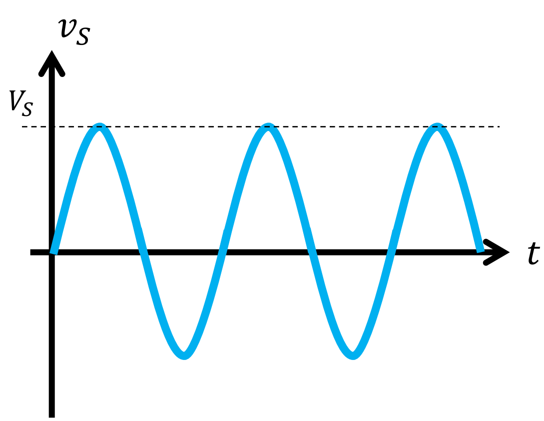
requiring $\begin{cases} D_1 \text{ is ON} \quad \rightarrow \quad v_{out} < 0 \\ v_{out} > 0 \end{cases}$

IMPOSSIBLE

Example 6: bridge rectifier

QUESTION: Find the output voltage with the given input. The diodes are ideal.

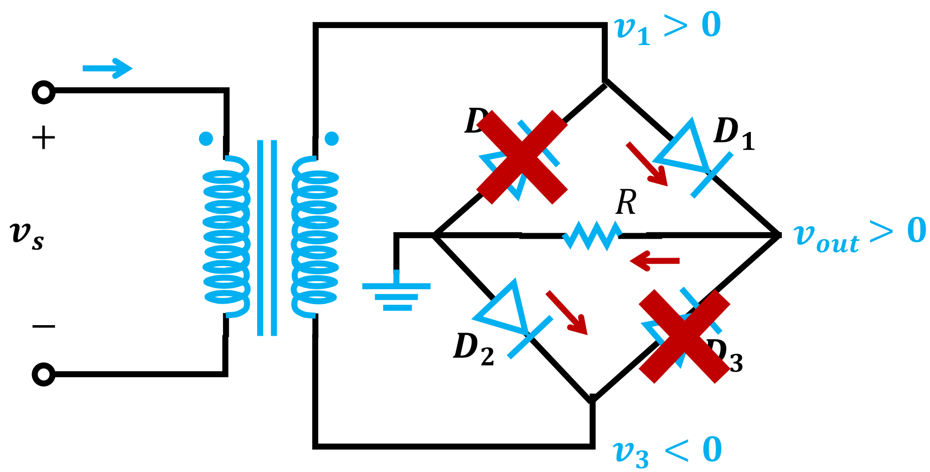


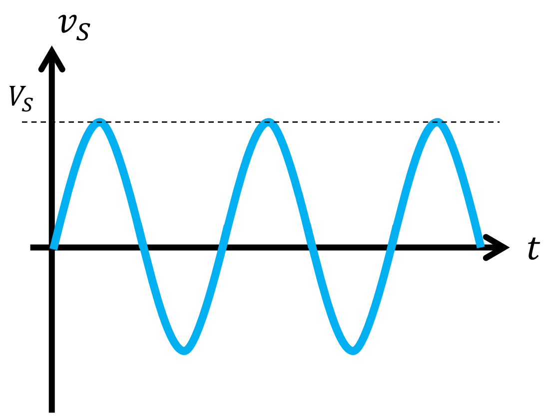


- Assume D_2 is ON
- According to the $i - v$ relationship of ideal diode $\rightarrow$ $v_3 < 0$
- D_4 must be OFF $\rightarrow$ $v_1 > 0$
- According to KCL, current goes through R $\rightarrow$ $v_{out} > 0$
- D_3 is OFF, since $v_3 < v_{out}$
- According to KCL, current goes through D_1 $\rightarrow$ $v_1 > v_{out}$

Example 6: bridge rectifier

QUESTION: Find the output voltage with the given input. The diodes are ideal.



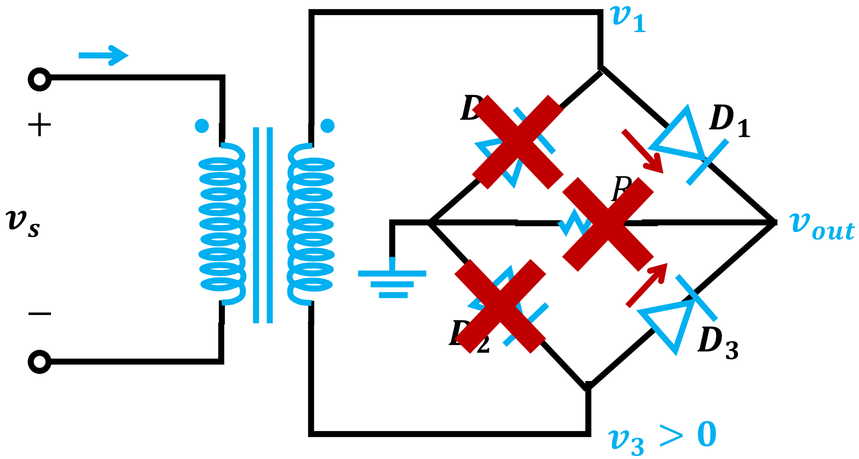


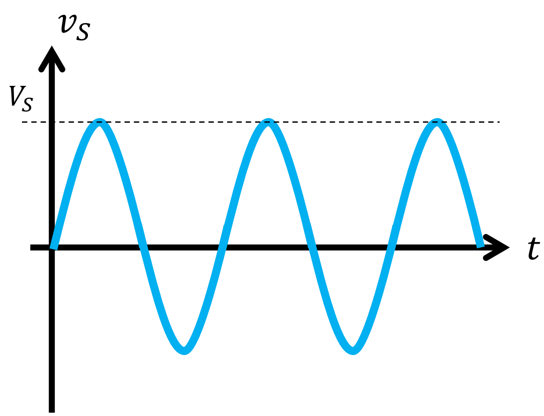
- Assume D_2 is ON
- D_1 must be ON
- D_3 must be OFF
- D_4 must be OFF

$$\begin{cases} v_1 > v_{out} > 0 \\ v_3 < 0 \end{cases} \rightarrow \quad \text{requiring } v_s > 0$$

Example 6: bridge rectifier

QUESTION: Find the output voltage with the given input. The diodes are ideal.





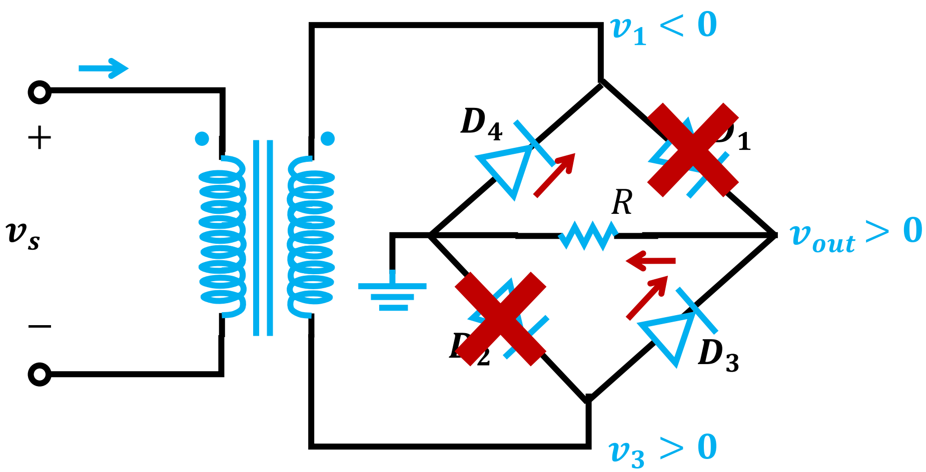
- Assume D_2 is OFF
- According to the $i - v$ characteristics of ideal diode $\rightarrow$ $v_3 > 0$
- If D_4 is also OFF
- According to KCL, no current through R
- Only 1 possible path for current flow

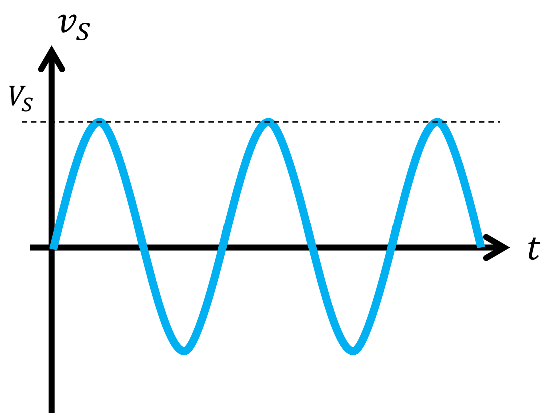
  D_1 and D_3 must be ON at the same time

  Conflict of current flow direction!!!

Example 6: bridge rectifier

QUESTION: Find the output voltage with the given input. The diodes are ideal.

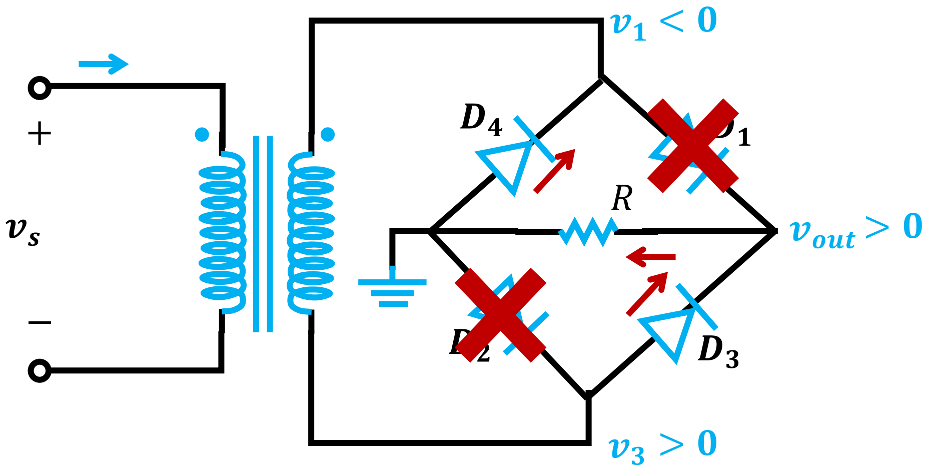


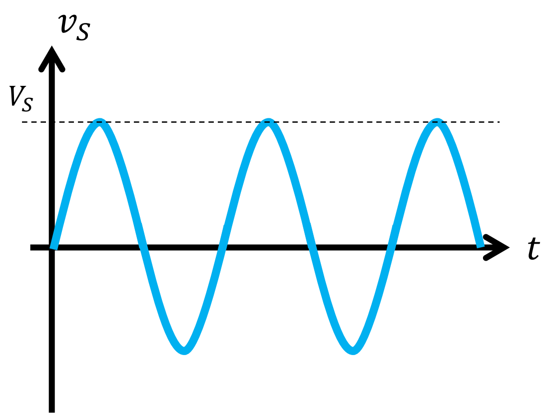


- Assume D_2 is OFF
- According to the $i - v$ relationship of ideal diode $\rightarrow$ $v_3 > 0$
- D_4 must be ON $\rightarrow$ $v_1 < 0$
- According to KCL, current goes through R $\rightarrow$ $v_{out} > 0$
- D_1 is OFF, since $v_1 < v_{out}$
- According to KCL, current goes through D_4 $\rightarrow$ $v_3 > v_{out}$

Example 6: bridge rectifier

QUESTION: Find the output voltage with the given input. The diodes are ideal.



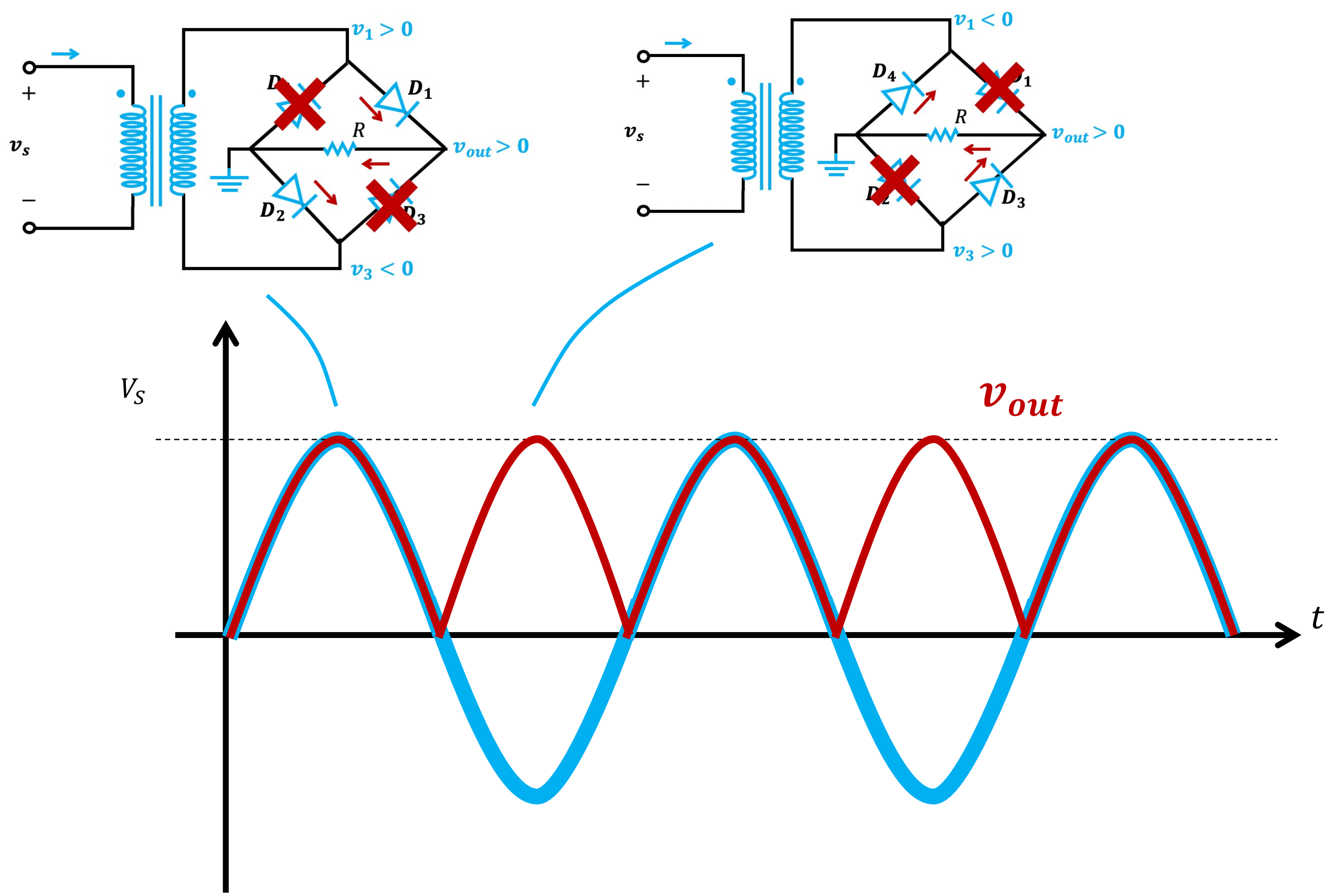


- Assume D_2 is OFF
- D_1 must be OFF
- D_3 must be ON
- D_4 must be ON

$$\begin{cases} v_3 > v_{out} > 0 \\ v_1 < 0 \end{cases} \rightarrow \quad \text{requiring } v_s < 0$$

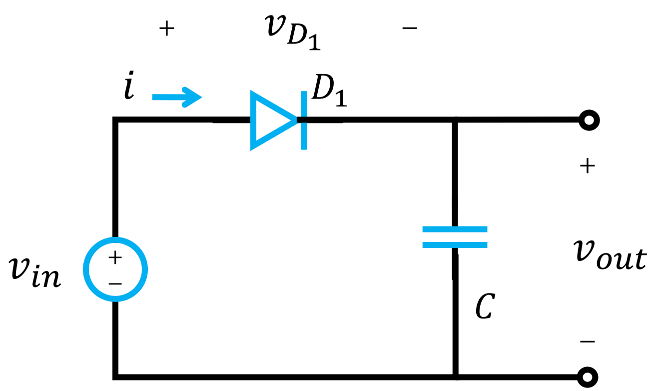
Example 6: bridge rectifier

QUESTION: Find the output voltage with the given input. The diodes are ideal.



Example 7: peak rectifier

QUESTION: Find the output voltage with the given input. Use the constant-voltage-drop model of the diode with $v_D = 0.7V$. $v_{out} = 0$ at $t = 0$.

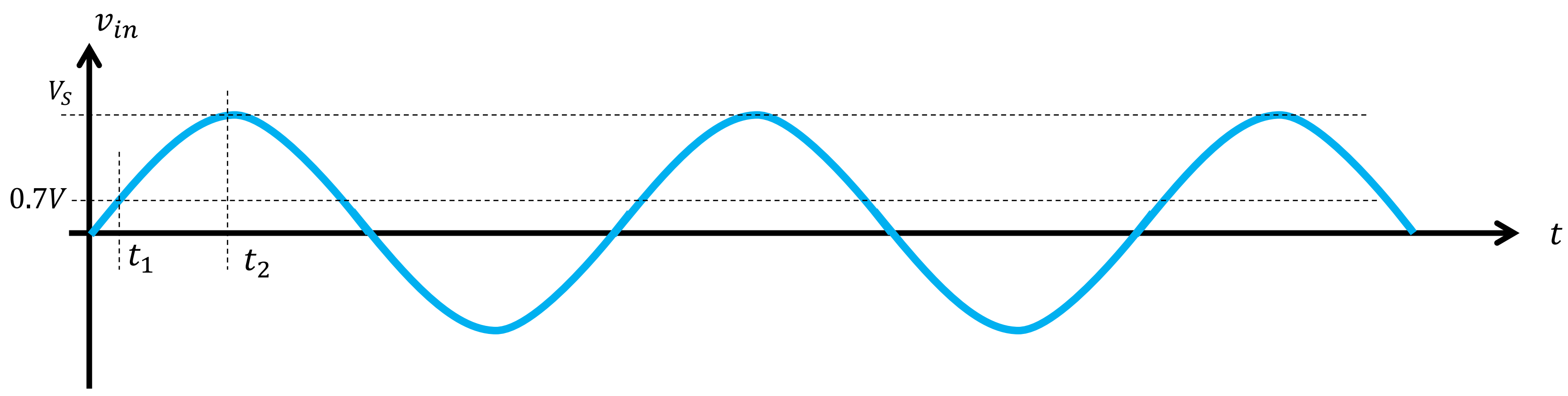


- When t $< t_1$

  $v_{D_1} = v_{in} - v_{out} < v_D$

  The diode is OFF

  $\rightarrow \quad v_{out} = 0$



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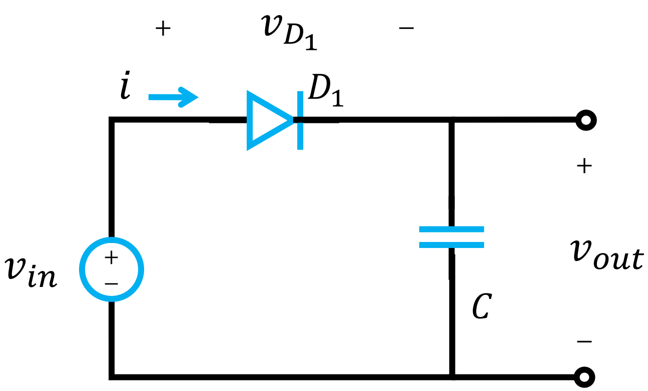
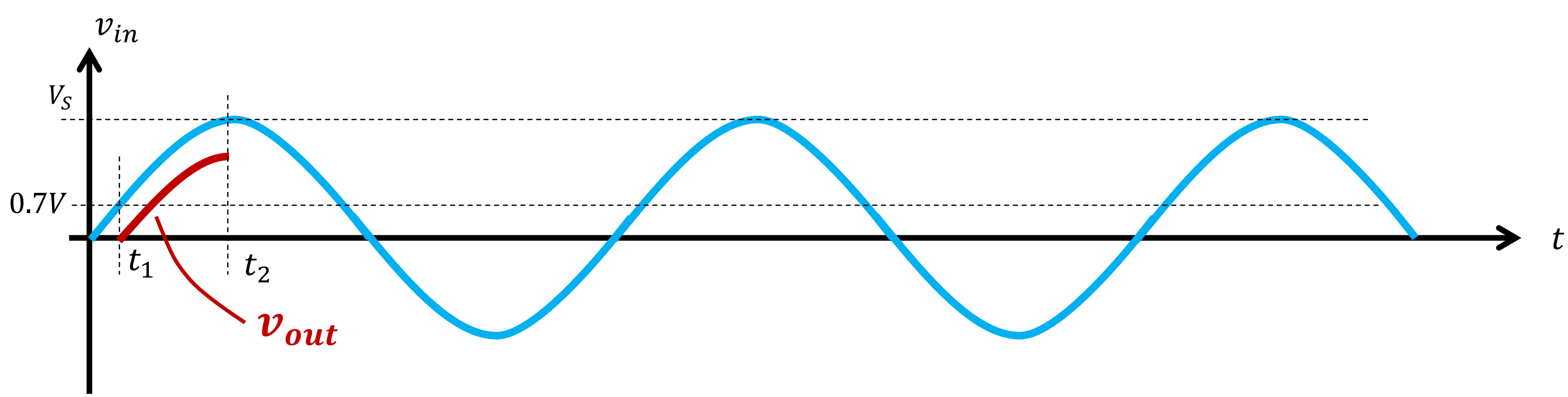


- When t ∈ $[t_1, t_2]$

  $v_{D_1} = v_{in} - v_{out} > v_D$

  The diode is ON. C charged to the peak.

  → $v_{out} = v_{in} - v_D$



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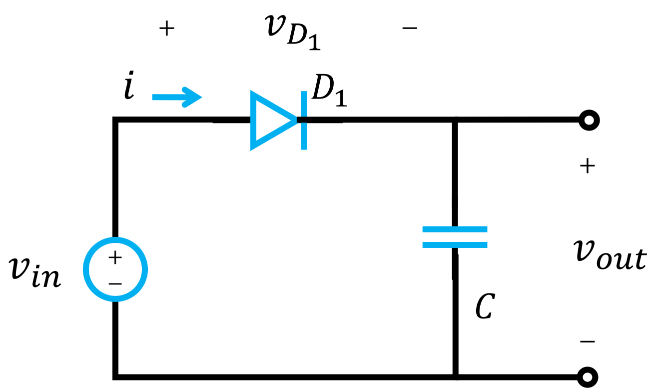


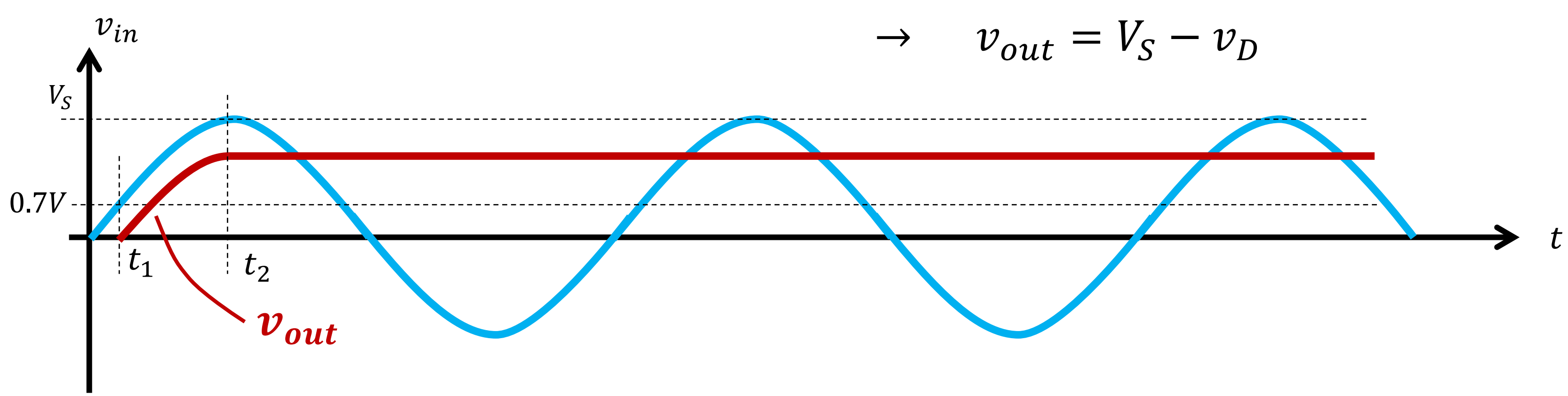
- When t > t_2

  $v_{D_1} = v_{in} - v_{out} < v_D$

  The diode is OFF.

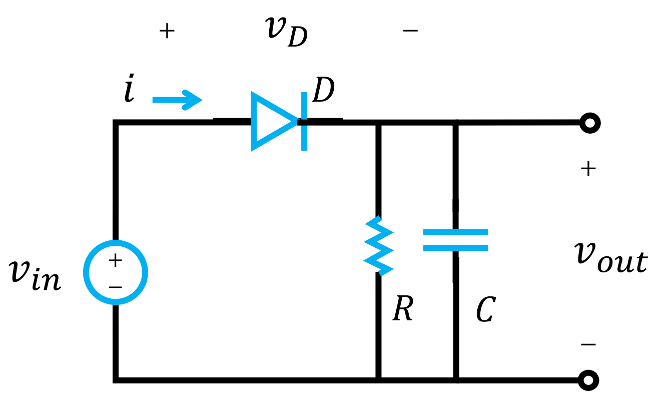
  There is no way to discharge C

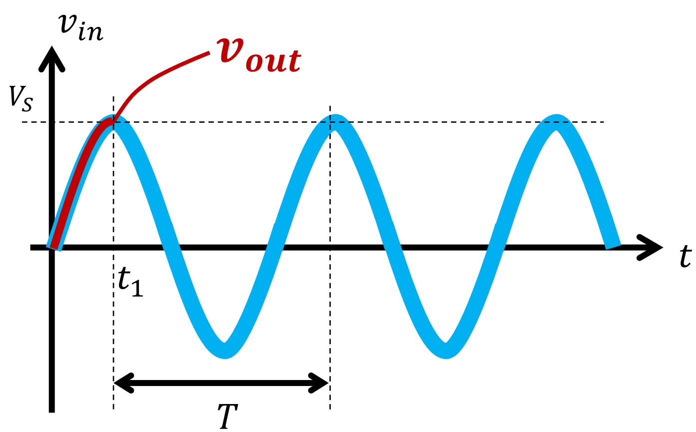
  $\rightarrow \quad v_{out} = V_S - v_D$



Example 8: peak rectifier

QUESTION: Find the output voltage with the given input. The diode is ideal. $v_{out} = 0$ at $t = 0$. RC is much larger than T





- When t $< t_1$

  $v_{D_1} = v_{in} - v_{out} > 0$

  The diode is ON. C charges to the peak V_S

  $\rightarrow \quad v_{out} = v_{in}$

- When t $> t_1$

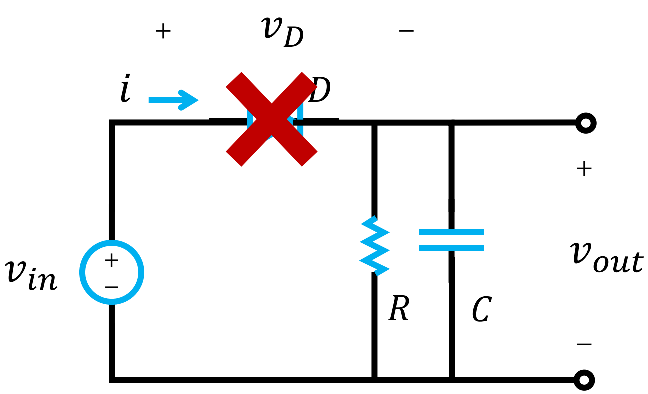
  $v_{D_1} = v_{in} - v_{out} < 0$

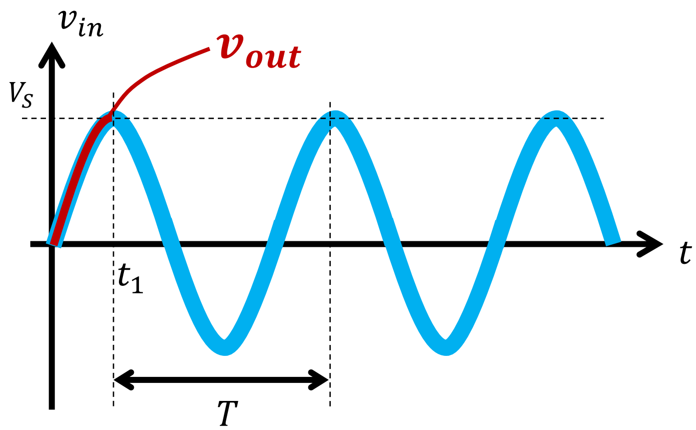
  The diode is OFF.

  C discharges through R

Example 8: peak rectifier

QUESTION: Find the output voltage with the given input. The diode is ideal. $v_{out} = 0$ at $t = 0$. RC is much larger than T



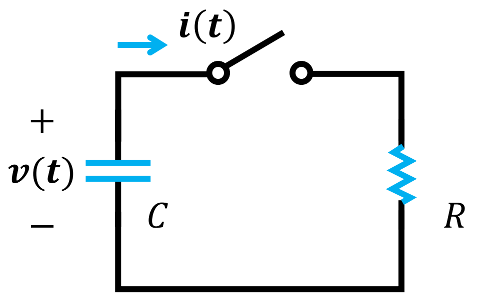


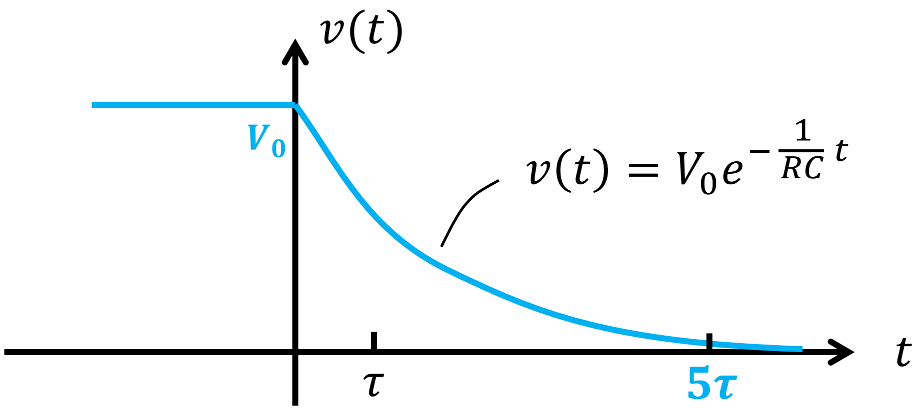
- According to KCL

$$\frac{v_{out}}{R} + C\frac{dv_{out}}{dt} = 0$$

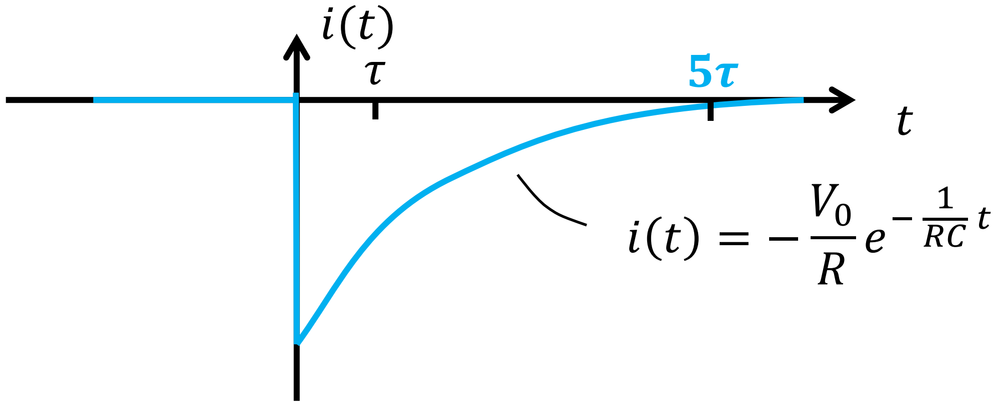
Recall: Source free RC circuit

QUESTION: Assume the capacitor C has been charged to V_0 before the switch is turned on. Find the response after the switch is turned on.



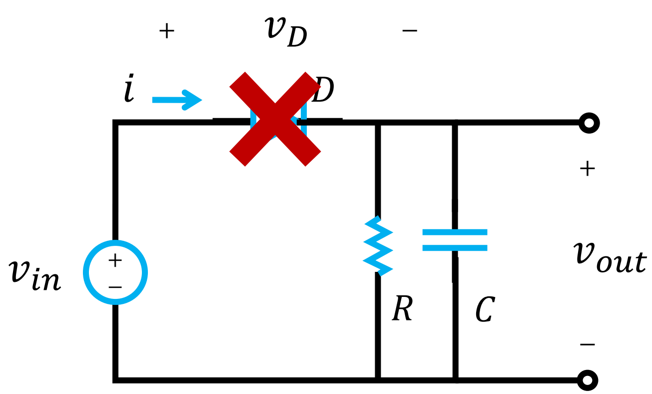


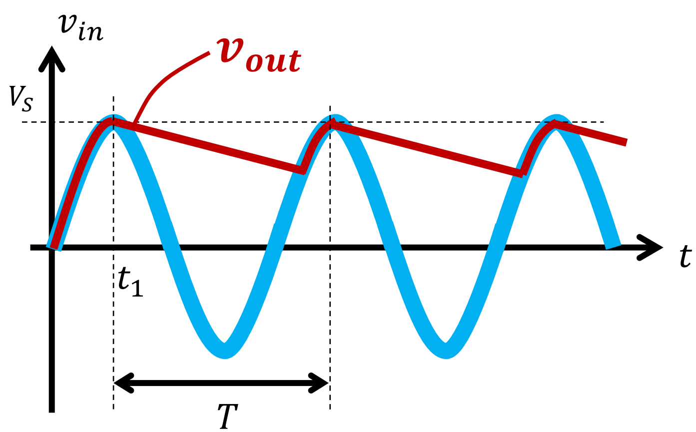
Voltage on capacitor CANNOT change abruptly, but current can.



Example 8: peak rectifier

QUESTION: Find the output voltage with the given input. The diode is ideal. $v_{out} = 0$ at $t = 0$. RC is much larger than T





- According to KCL

$$\frac{v_{out}}{R} + C\frac{dv_{out}}{dt} = 0$$

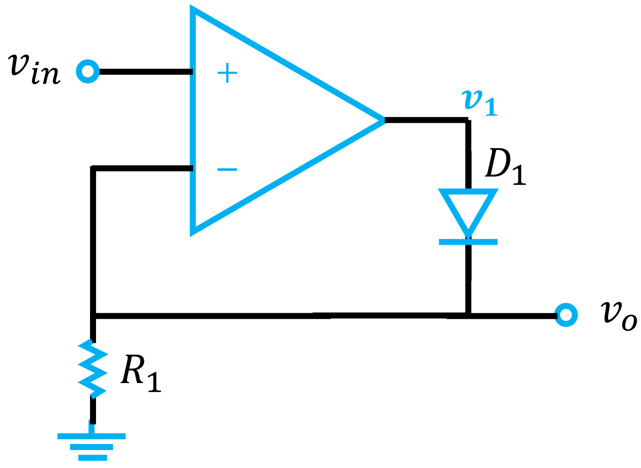
$$v(t) = V_S e^{-\frac{1}{RC}t}$$

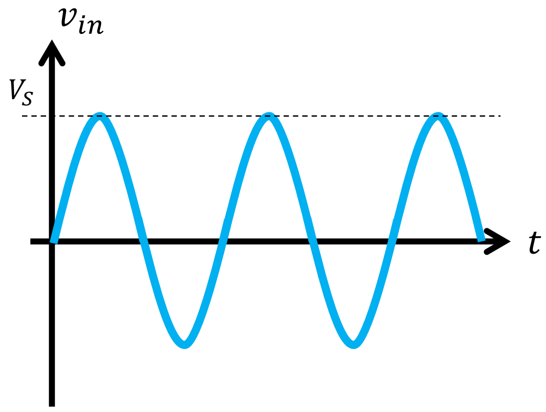
- Since $CR \gg T$

$$v(T) = V_S e^{-\frac{T}{RC}} \approx V_S\left(1 - \frac{T}{RC}\right)$$

Example 9: precision half-wave rectifier

QUESTION: Find the output voltage with the given input. The diode is not ideal.

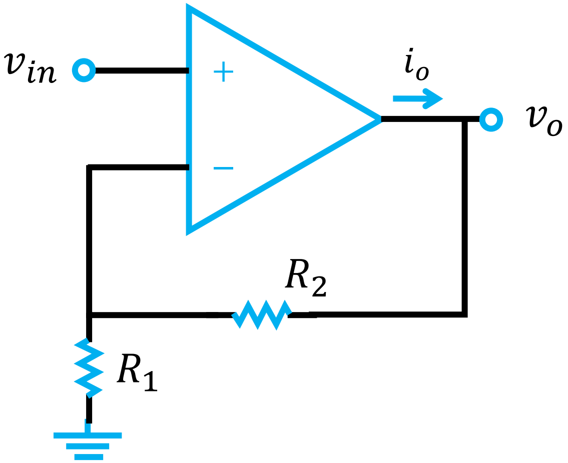




- Assume D_1 is ON

Recall: Op-Amp & Feedback

QUESTION: Find the output of the circuit, v_o, and the relationship between i_o and v_{in}.



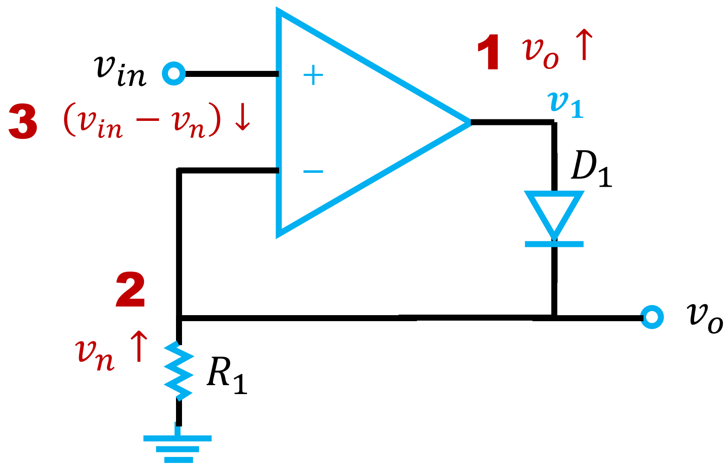
$$
\begin{cases}
i_{R_1} = \dfrac{v_n}{R_1} = \dfrac{v_{in}}{R_1} \\[2ex]
i_{R_2} = \dfrac{v_o - v_n}{R_2} = \dfrac{v_o - v_{in}}{R_2} \\[2ex]
i_{R_1} = i_{R_2}
\end{cases}
$$

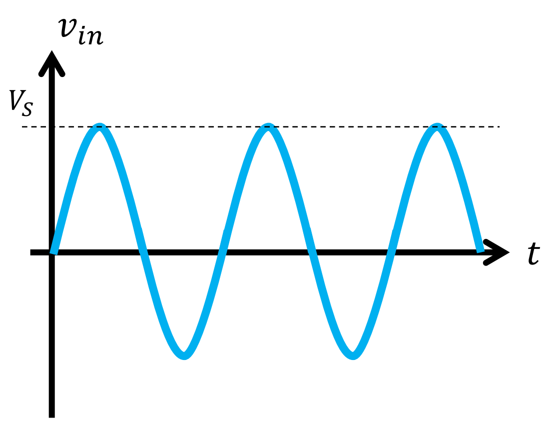
IDEAL OP-AMP with NEGATIVE FEEDBACK enables linear region biasing

$$\boldsymbol{v_o = \frac{R_1 + R_2}{R_1} v_{in}}$$

Example 9: precision half-wave rectifier

QUESTION: Find the output voltage with the given input. The diode is not ideal.



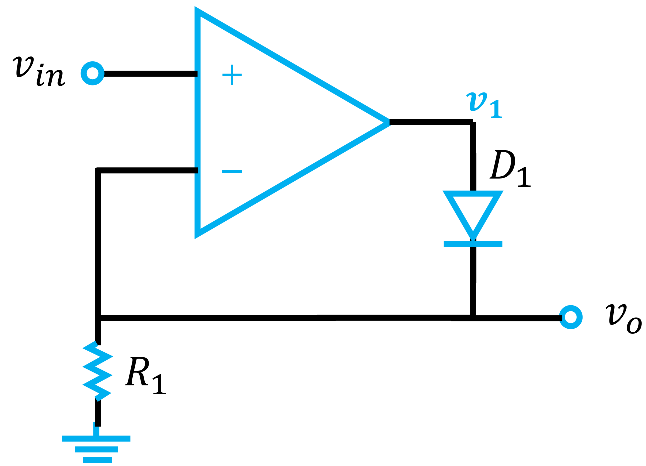


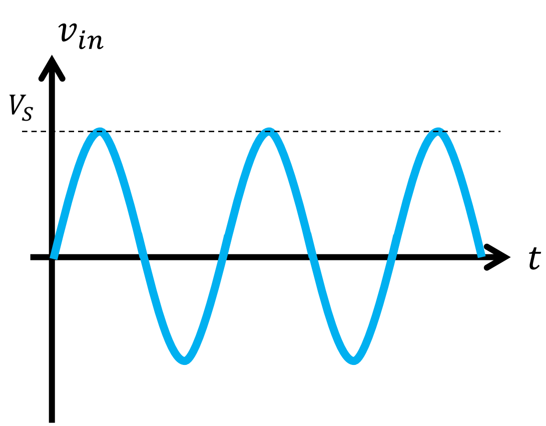
- Step 1: Assume D_1 is ON
- Step 2: check if negative feedback
  - If there is an increase @ v_o — $v_o \uparrow$
  - The inverting input v_n increases correspondingly
  - If the op-amp is biased in the linear region, $v_o = A(v_{in} - v_n)$ decreases — $v_o \downarrow$

NEGATIVE FEEDBACK is observed

Example 9: precision half-wave rectifier

QUESTION: Find the output voltage with the given input. The diode is not ideal.





- Step 1: Assume D_1 is ON
- Step 2: check if negative feedback
- Step 3a: ideal op-amp

  → open circuit @ inputs

  → $i_p = i_n = 0$
- Step 3b: "short" the inputs

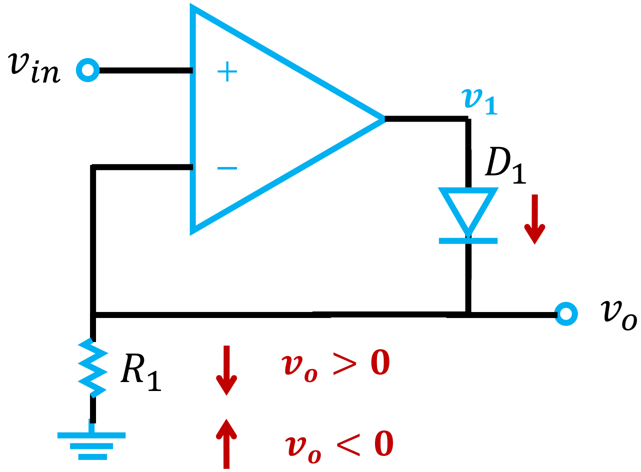
$$v_n = v_p = v_{in} = v_o$$

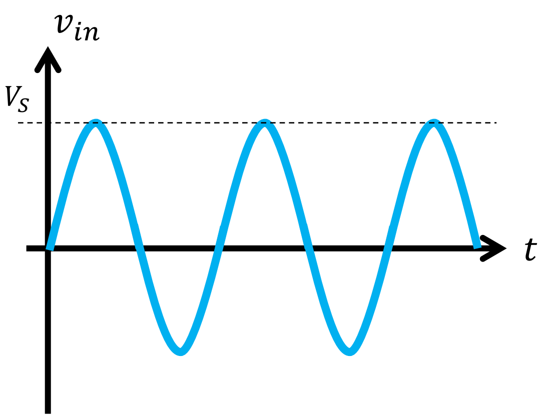
$$i_{R_1} = \frac{v_{in}}{R_1}$$

$$v_1 = v_o + v_{D_1} = v_{in} + v_{D_1}$$

Example 9: precision half-wave rectifier

QUESTION: Find the output voltage with the given input. The diode is not ideal.





- Step 1: Assume D_1 is ON
- Step 2: check if negative feedback
- Step 3: solve the circuit

$$v_n = v_p = v_{in} = v_o$$

$$v_1 = v_{in} + v_{D_1}$$

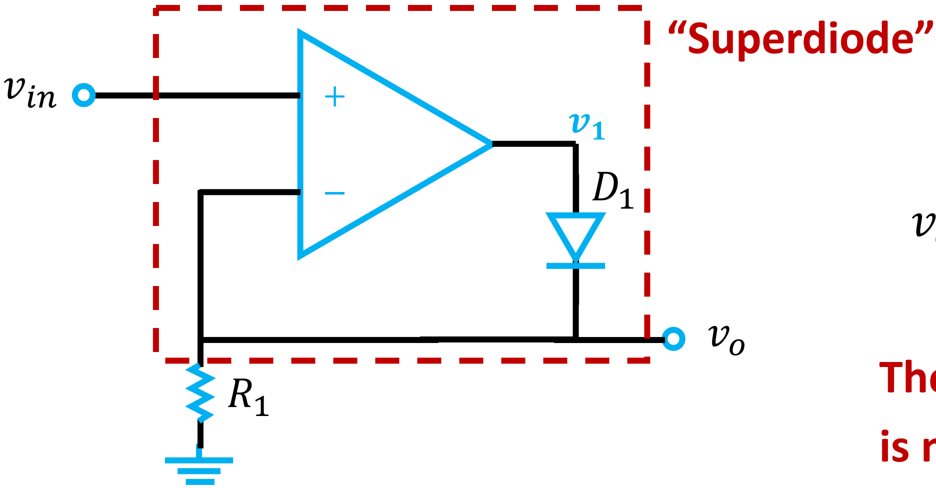
- Step 4: check the assumption

If $v_{in} > 0 \quad \rightarrow v_o > 0$

If $v_{in} < 0 \quad \rightarrow v_o < 0$ ✗

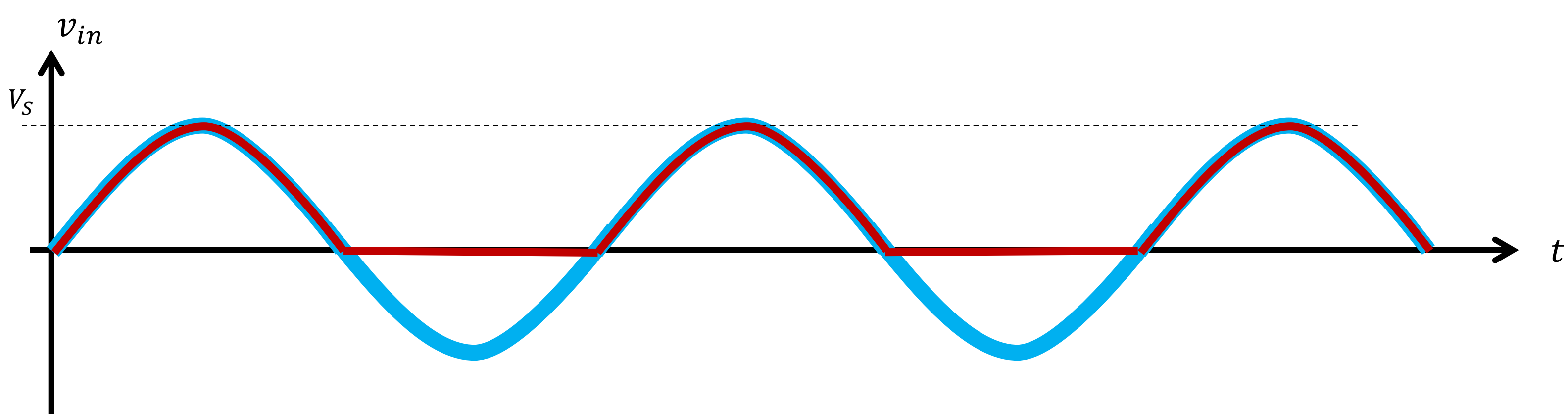
Example 9: precision half-wave rectifier

QUESTION: Find the output voltage with the given input. The diode is not ideal.

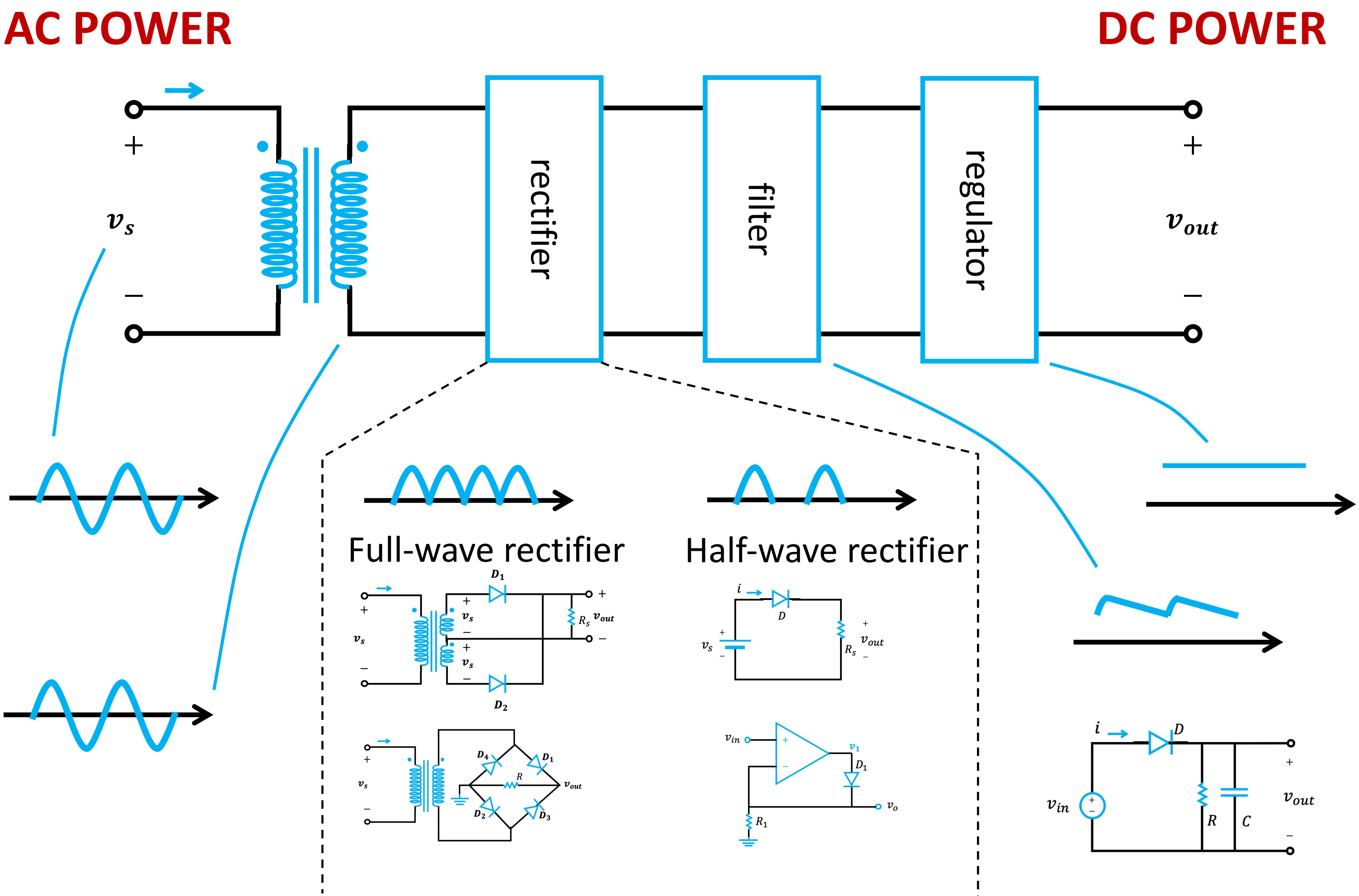


$$v_o = \begin{cases} v_{in} & \text{If } v_{in} > 0 \\ 0 & \text{If } v_{in} < 0 \end{cases}$$

The offset voltage due to the diode is no longer present in the output



Summary: rectifier

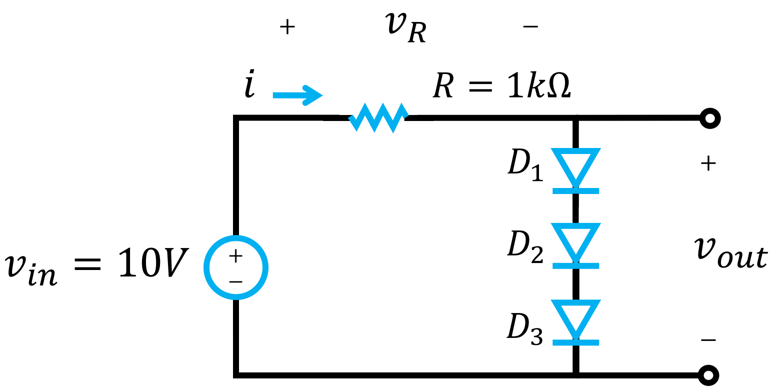


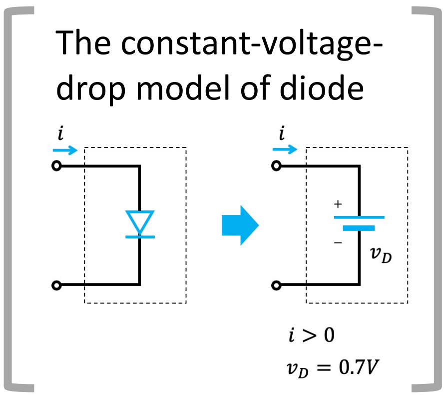
Outline

- Introduction to semiconductors
  - Semiconductor material & silicon crystal
  - Doped semiconductors
  - Current flow in semiconductor
  - The $\boldsymbol{pn}$ junction
- Diodes
  - The $i-v$ characteristics
  - The models: Constant-voltage-drop / ideal / Small-signal model
  - The 3 working regions: forward / reverse / reverse breakdown
  - Applications of diodes
    - Rectifiers
    - **Limiting & clamping circuits**

Recall: Example 2: limiting circuit

QUESTION: Find the current through the resistor R.





- Use the constant-voltage-drop model

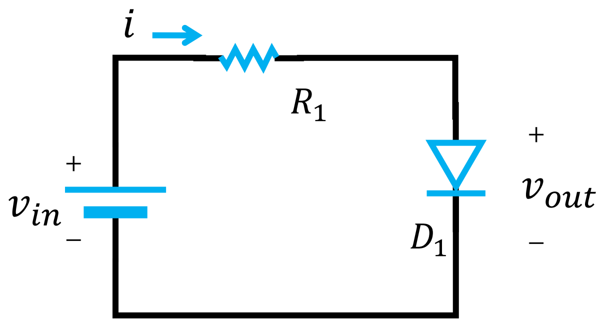
$$v_{out} = 0.7V \times 3 = 2.1V$$

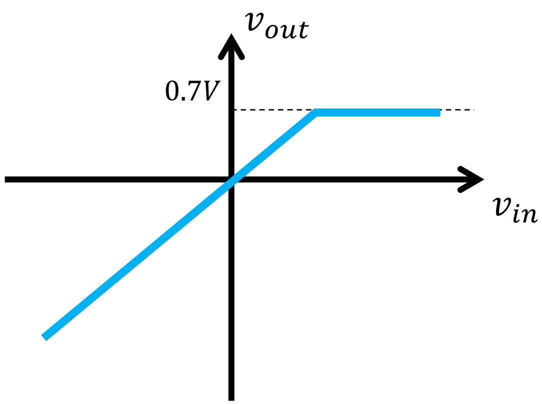
- According to KVL

$$I_R = \frac{v_{in} - v_{out}}{R} = 7.9mA$$

Example 10: limiting circuit

QUESTION: Find the transfer function between the input and output voltage





- Use the constant-voltage-drop model
- If $v_{in} > 0.7V$, according to KVL

$$v_{out} = 0.7V$$

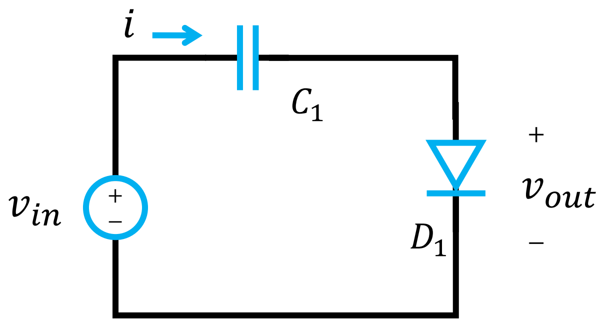
$$v_{R_1} = v_{in} - 0.7V$$

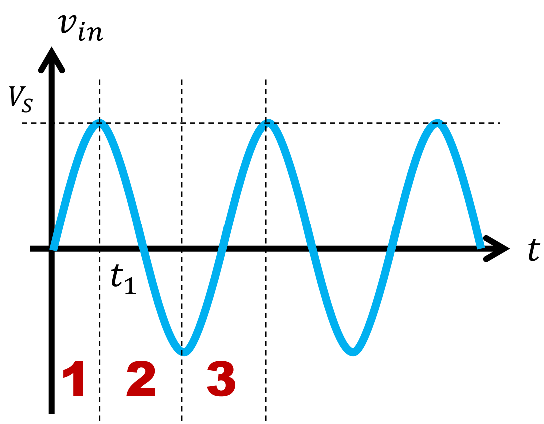
- If $v_{in} < 0.7V$, D_1 is OFF

$$v_{out} = v_{in}$$

Example 11: DC restorer

QUESTION: Find the output voltage with the given input. The diode is ideal. $v_{out} = 0$ at $t = 0$.





- In phase 1

  D_1 is ON

  According to KVL

  $v_{C_1} = v_{in} \quad v_{D_1} = 0$

  $v_{C_1}(t = t_1) = V_S$

- In phase 2

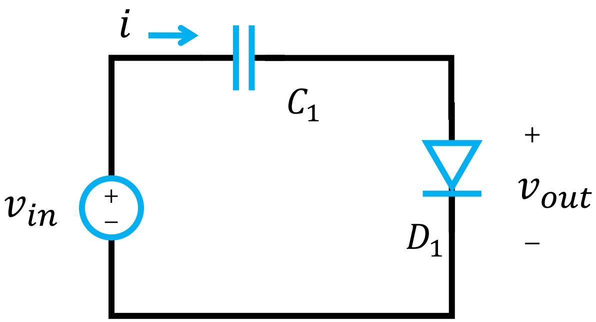
  D_1 is OFF

  Voltage on C_1 CANNOT change abruptly

  $v_{out} = v_{in} - V_S$

Example 11: DC restorer

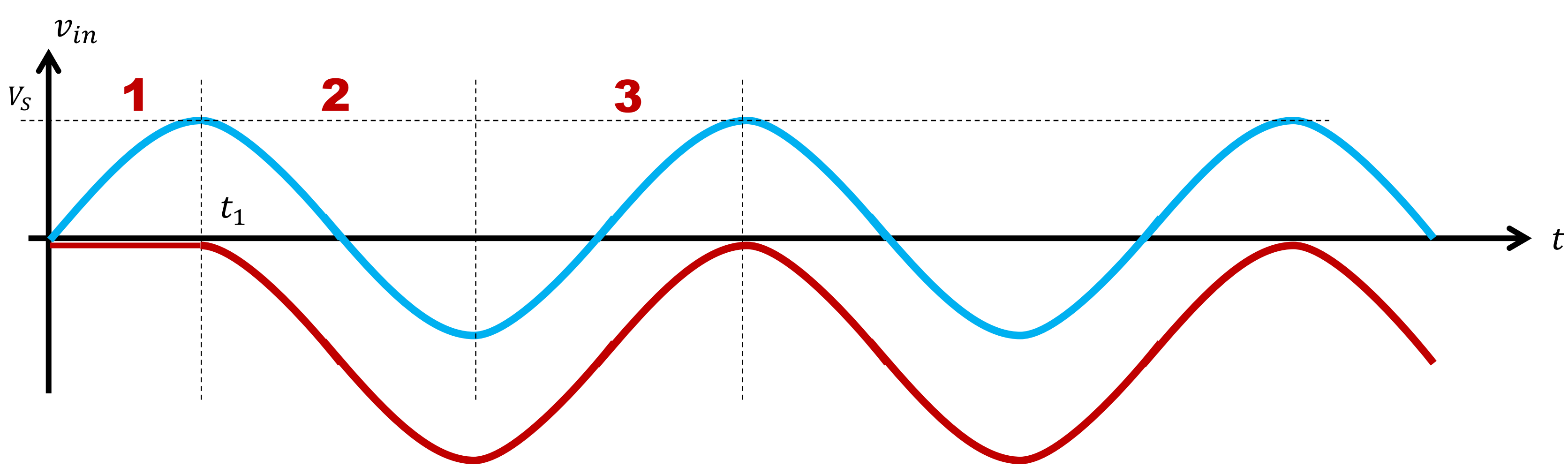
QUESTION: Find the output voltage with the given input. The diode is ideal. $v_{out} = 0$ at $t = 0$.



- In phase 3

  D_1 is OFF

  $v_{out} = v_{in} - V_S$



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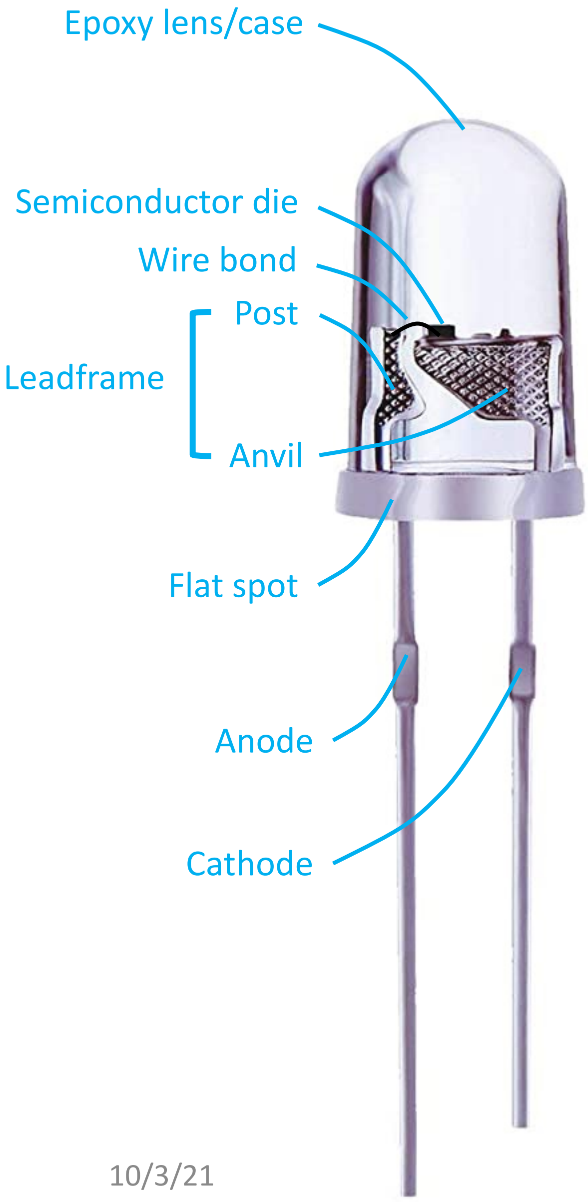
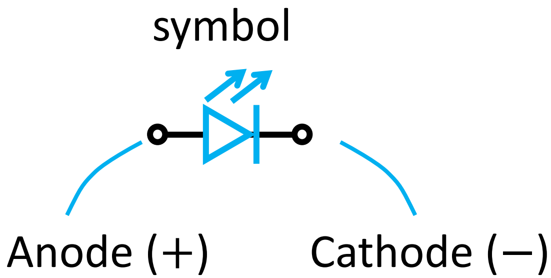
    - Limiting & clamping circuits
  - **Special diodes**

Light-Emitting Diodes (LEDs)

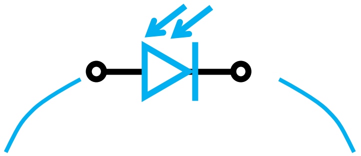




- Convert a **forward current** into **light**
- The light emitted by an LED is proportional to the forward current in the diode

Photodiodes

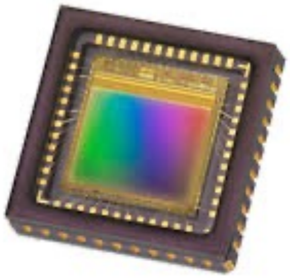
symbol



Anode (+) Cathode (−)

- Photodiode is a reverse-biased pn junction
- Convert incident light to a reverse current

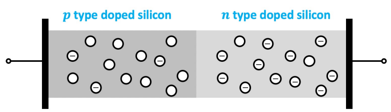
Application 1: Fiber-optic receiver

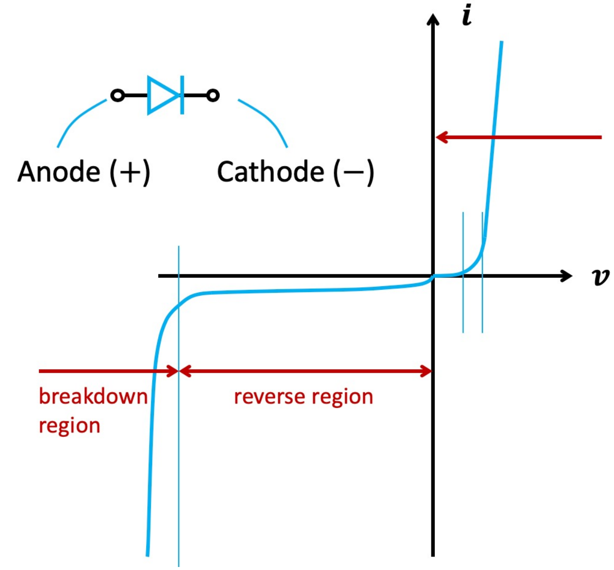


Application 2: Image sensor

Summary

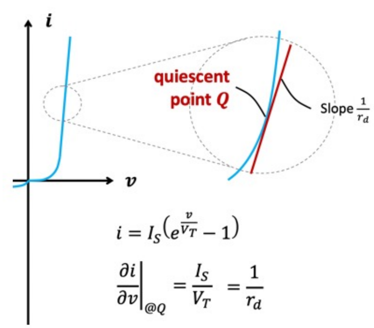
- Introduction to semiconductors
  - Semiconductor material & silicon crystal
  - Doped semiconductors
  - Current flow in semiconductor
  - The pn junction



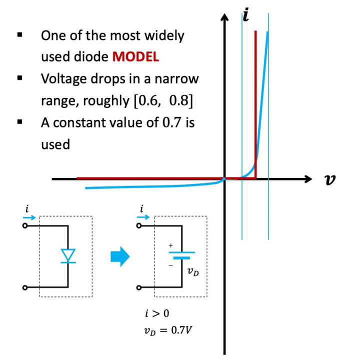


- Diodes
  - The $i - v$ characteristics
  - The models:

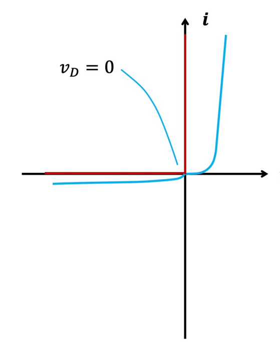
Small-signal model



Constant-voltage drop model



Ideal model



  - Applications of diodes / circuit analysis with diodes
  - Special diodes

Reading tasks & learning goals

- Reading tasks
  - Microelectronic Circuits, 6th edition
    - Chapter 3-4
- Learning goals
  - Know the two types of doped semiconductors
  - Know how pn **junction** works
  - Well understand how to analyze a circuit with **diode** using different **models**