第10讲：串联RLC时频分析

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B班课程内容安排

第一学期：线性	序号	第二学期：非线性
电路定律	1	器件基础
电阻电源	2	二极管
电容电感	3	**MOSFET**
信号分析	4	**BJT**
分压分流	5	反相电路
正弦稳态	6	数字门
时频分析	7	放大器
期中复习	8	期中复习
RLC二阶	9	负反馈
二阶时频	**10**	差分放大
受控源	11	频率特性
网络参量	12	正反馈
典型网络	13	振荡器
作业选讲	14	作业选讲
期末复习	15	期末复习

二阶滤波器时频分析 内容

- 阻容感分压电路和分流电路
  - 串联RLC分压分析：并联RLC分流分析属对偶分析
    - 时域分析：一般性分析
      - 五要素法
    - 理想滤波特性与实际滤波特性
      - 理想滤波特性
        - 通带内信号无失真传输，通带外信号完全滤除
      - 实际滤波特性
        - 通带内信号传输存在幅度失真和相位失真，通带外无法完全滤除
        - 实际滤波特性应尽可能逼近理想滤波特性
    - 二阶滤波器时频分析：以串联RLC电路为例
      - 低通/高通/带通/带阻

      - 幅频特性与相频特性伯特图画法

LTI系统频域分析

$$V_{m0}\cos(\omega_0 t+\varphi_0)$$

线性时不变系统

$$H(j\omega)=A(\omega)e^{j\varphi(\omega)}$$

$$V_{m0}A(\omega_0)\cos(\omega_0 t+\varphi_0+\varphi(\omega_0))$$

正弦稳态响应

对线性时不变系统，用相量法很容易地直接获得正弦信号激励下的稳态响应

$$H(j\omega)=A(\omega)e^{j\varphi(\omega)}$$

传递函数　　幅频特性　　相频特性

理想低通滤波特性

理想低通系统：通带内传递函数

$$H(j\omega)=A_0 e^{-j\omega\tau_0}$$

假设输入信号频谱都在通带内

$$in=x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)e^{j\omega t}d\omega$$

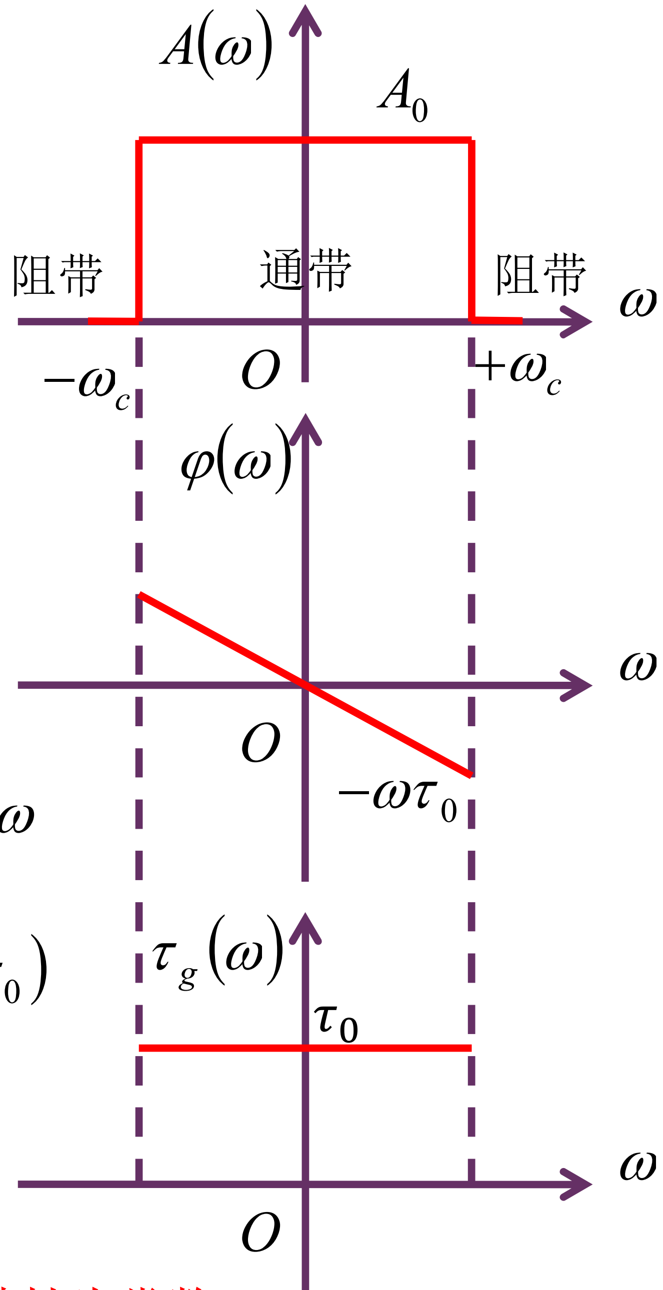
那么输出将仅是输入的比例放大和整体延时

$$out=y(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}A(\omega)X(\omega)e^{j(\omega t+\varphi(\omega))}d\omega$$

$$=A_0\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)e^{j(\omega(t-\tau_0))}d\omega=A_0 x(t-\tau_0)$$

称之为无失真传输 $\tau_g(\omega)=-\frac{d\varphi(\omega)}{d\omega}=\tau_0$

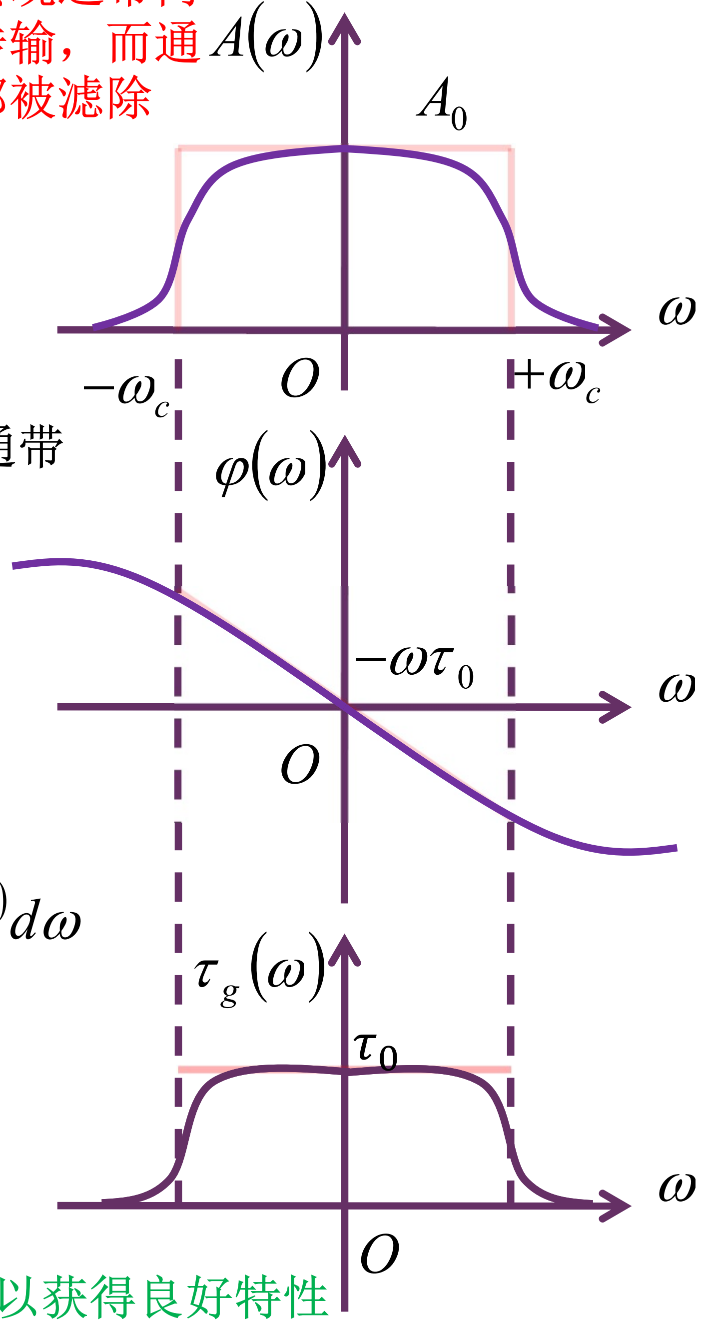
群延时



理想滤波器通带内的幅频特性为常数，群延时特性为常数

实际滤波特性

理想滤波器可实现通带内信号的无失真传输，而通带外信号则全部被滤除



但理想低通系统不可实现，实际的低通系统为：

$$H(j\omega)=A(\omega)e^{j\varphi(\omega)}$$

其通带内幅度不是常数，群延时也不是常数；通带外信号也无法全部被衰减清零

$$in=x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)e^{j\omega t}d\omega$$

即使输入所包含的频率分量全部都在通带内

$$out=y(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}A(\omega)X(\omega)e^{j(\omega t+\varphi(\omega))}d\omega$$

输出也会产生线性失真：幅度失真和相位失真

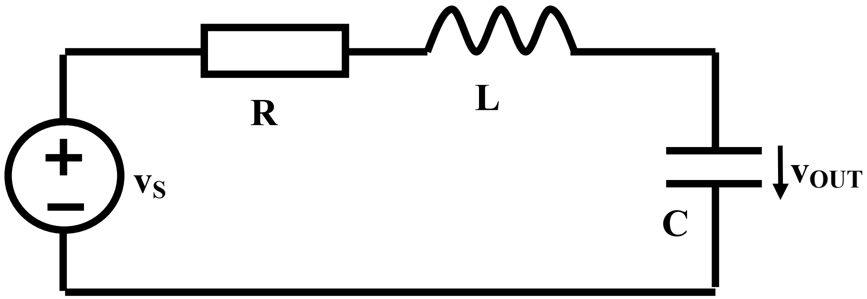
幅度失真：不同的频率分量有不同的增益

相位失真：不同的频率分量有不同的延时

失真：输出波形和输入波形形状不同

实际实现的滤波器应尽可能接近理想滤波器，以获得良好特性

二阶低通：电容分压



直观理解：

低频：电容开路，电感短路，信号通过

高频：电容短路，电感开路，信号不能通过

$$H(j\omega)=\frac{\dot{V}_C}{\dot{V}_S}=\frac{\frac{1}{j\omega C}}{R+j\omega L+\frac{1}{j\omega C}}=\frac{1}{(j\omega)^2LC+j\omega RC+1}$$

$$\overset{j\omega\to s}{=}\frac{1}{s^2LC+sRC+1}=\frac{\frac{1}{LC}}{s^2+s\frac{R}{L}+\frac{1}{LC}}=H_0\frac{\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}$$

二阶低通传函典型形式

$$H_0=1 \quad \omega_0=\frac{1}{\sqrt{LC}} \quad \xi=\frac{R}{2Z_0}=\frac{R}{2}\sqrt{\frac{C}{L}}$$

幅频特性、相频特性、群延时特性

$$H(s)=\frac{\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}\overset{s=j\omega}{=}\frac{\omega_0^2}{\left(\omega_0^2-\omega^2\right)+j2\xi\omega_0\omega}=A(\omega)e^{j\varphi(\omega)}$$

$$A(\omega)=\frac{\omega_0^2}{\sqrt{\left(\omega_0^2-\omega^2\right)^2+\left(2\xi\omega_0\omega\right)^2}}$$

幅频特性

$$\varphi(\omega)=-\arctan\frac{2\xi\omega_0\omega}{\omega_0^2-\omega^2}$$

相频特性

$$\tau_g(\omega)=-\frac{d\varphi(\omega)}{d\omega}$$

群延时：相频特性曲线的斜率

一群信号通过该系统的延时大小

群延时特性

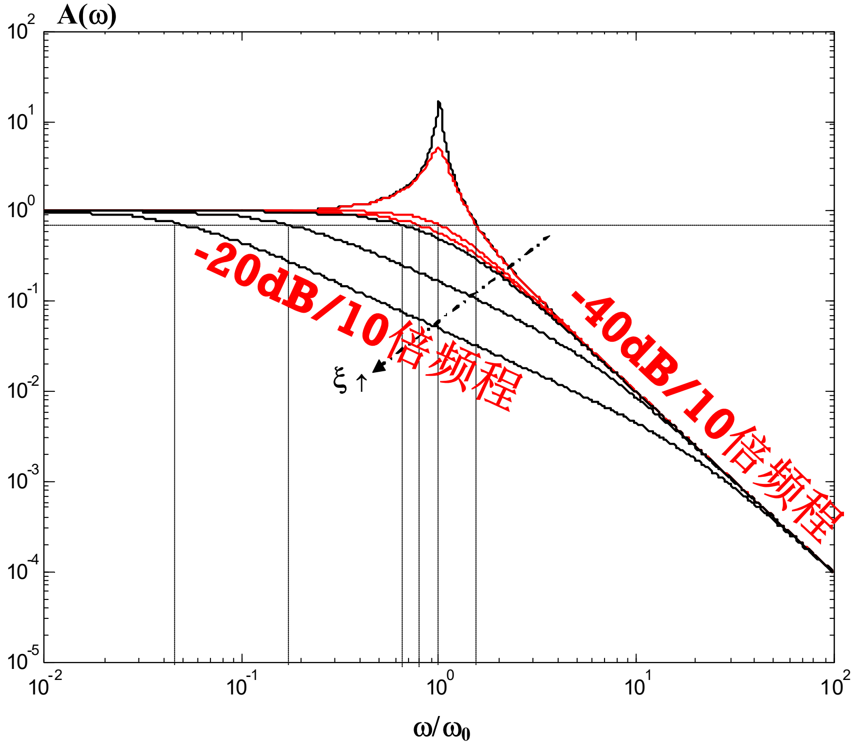
$$\tau_g(\omega)=\frac{2\xi\omega_0\left(\omega^2+\omega_0^2\right)}{\omega^4+2\left(2\xi^2-1\right)\omega_0^2\omega^2+\omega_0^4}$$

幅频特性

$$H(s)=\frac{\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}=\frac{\omega_0^2}{(s-\lambda_1)(s-\lambda_2)}$$

$$A(\omega)=\frac{\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2+(2\xi\omega_0\omega)^2}}$$

$$\lambda_{1,2}=\left(-\xi\pm\sqrt{\xi^2-1}\right)\omega_0$$



ξ=0.03, 0.1, 0.707, 0.866,1, 3, 10

$\xi>1$过阻尼，幅频特性明显可三段折线处理

$0.707\le\xi\le1$，幅频特性平坦，可两段折线处理

$\xi\ll0.707$，自由振荡频点附近有谐振峰出现

过阻尼：幅频特性可三段折线

$\xi > 1$

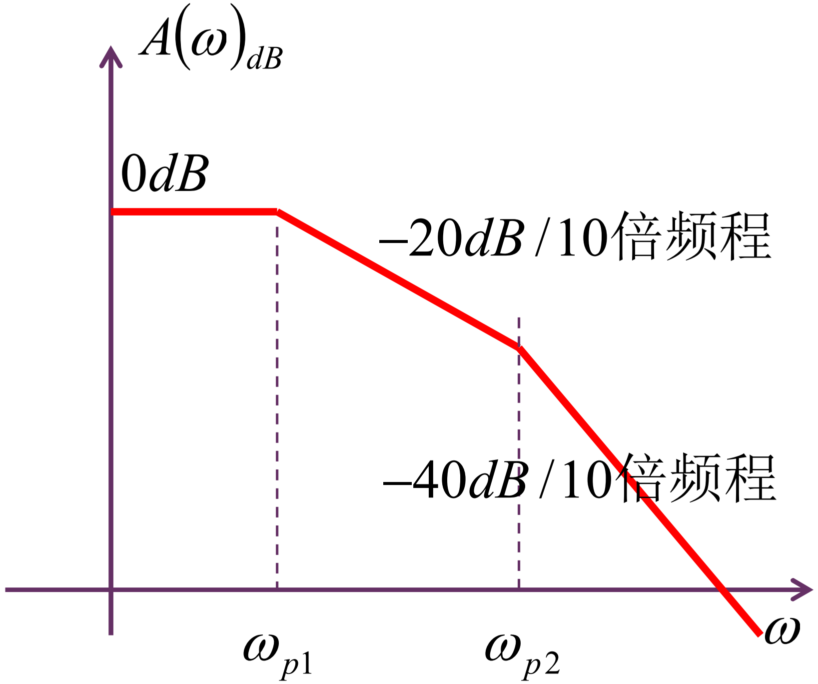
$$H(s) = \frac{\omega_0^2}{(s-\lambda_1)(s-\lambda_2)}$$

$$H(j\omega) = \frac{\omega_0^2}{\left(j\omega+\omega_{p1}\right)\left(j\omega+\omega_{p2}\right)}$$

$$= \frac{1}{\left(\frac{j\omega}{\omega_{p1}}+1\right)\left(\frac{j\omega}{\omega_{p2}}+1\right)}$$

$$\approx \begin{cases} 1 & \omega < \omega_{p1} \\ \frac{1}{\frac{j\omega}{\omega_{p1}}} = \frac{\omega_{p1}}{j\omega} & \omega_{p1} < \omega < \omega_{p2} \\ \frac{1}{\frac{j\omega}{\omega_{p1}}\cdot\frac{j\omega}{\omega_{p2}}} = \frac{\omega_0^2}{-\omega^2} & \omega > \omega_{p2} \end{cases}$$

$$\lambda_{1,2} = \left(-\xi \pm \sqrt{\xi^2-1}\right)\omega_0 = \begin{cases} -\left(\xi - \sqrt{\xi^2-1}\right)\omega_0 = -\omega_{p1} \\ -\left(\xi + \sqrt{\xi^2-1}\right)\omega_0 = -\omega_{p2} \end{cases}$$



幅频特性伯特图

欠阻尼存在谐振峰的可能性

$$\left(0<\xi<1\right) \qquad \lambda_{1,2}=\left(-\xi\pm j\sqrt{1-\xi^2}\right)\omega_0$$

$$H(s)=\frac{\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}=\frac{\omega_0^2}{(s-\lambda_1)(s-\lambda_2)}$$

也可分解，但为复根分解

$$H(j0)=1 \qquad H(j\infty)=0$$

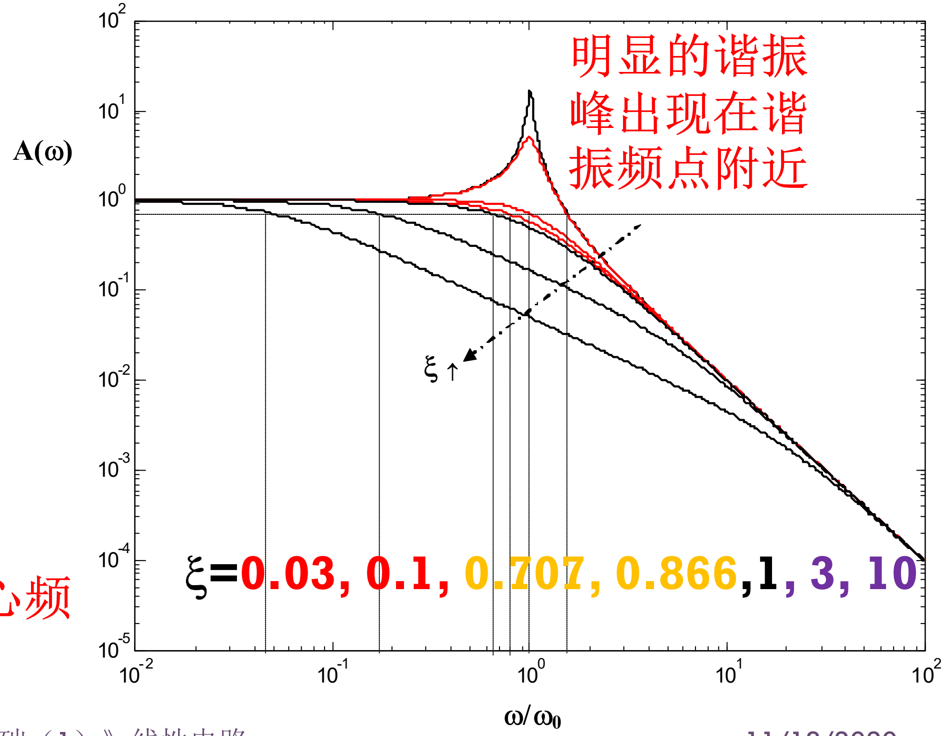
低通特性不会改变

$$H(j\omega_0)=\frac{\omega_0^2}{(j\omega_0)^2+2\xi\omega_0(j\omega_0)+\omega_0^2}$$

$$=-j\frac{1}{2\xi}=\frac{1}{2\xi}e^{-j\frac{\pi}{2}}$$

$$A(\omega_0)=\frac{1}{2\xi}\overset{\xi<0.5}{>}1=A(0)$$

自由振荡频点ω_0附近幅值可以高于中心频点（零频点），这就是谐振现象



谐振峰在哪里?

$$A(\omega)=\frac{\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2+(2\xi\omega_0\omega)^2}}$$

$$\frac{dA(\omega)}{d\omega}=0$$

$$=-\frac{1}{2}\frac{\omega_0^2}{\left((\omega_0^2-\omega^2)^2+(2\xi\omega_0\omega)^2\right)^{\frac{3}{2}}}\left[2(\omega_0^2-\omega^2)(-2\omega)+2(2\xi\omega_0\omega)2\xi\omega_0\right]$$

$$=-\frac{\omega_0^2}{\left((\omega_0^2-\omega^2)^2+(2\xi\omega_0\omega)^2\right)^{\frac{3}{2}}}2\omega\left[\omega^2+(2\xi^2-1)\omega_0^2\right]$$

阻尼系数很小时，谐振峰确实出现在自由振荡频点，谐振峰高度近似为**Q**

$$\omega_e=\sqrt{1-2\xi^2}\,\omega_0 \qquad (\xi<0.707)$$

谐振峰频点

$$A(\omega_e)=\frac{1}{2\xi\sqrt{1-\xi^2}}\overset{\xi<0.707}{>}1=A(0)$$

谐振峰高度

$$\omega_e=\sqrt{1-2\xi^2}\omega_0\overset{\xi<<0.707}{\approx}\omega_0$$

$$A(\omega_e)=\frac{1}{2\xi\sqrt{1-\xi^2}}\overset{\xi<<0.707}{\approx}\frac{1}{2\xi}=Q=A(\omega_0)$$

幅度最大平坦 $\xi = 0.707$

$$A(\omega) = \frac{\omega_0^2}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\xi\omega_0\omega\right)^2}}$$

$$\frac{dA(\omega)}{d\omega}\Big|_{\omega=0} = 0$$

$$\frac{d^2A(\omega)}{d\omega^2}\Big|_{\omega=0} = 0$$

Passband的中心频点最大平坦

$$\xi = \frac{\sqrt{2}}{2} = 0.707$$

二阶低通系统的幅频特性具有最大平坦特性

此为最接近理想传输系统幅频特性的二阶低通系统：最优

$$A_2(\omega) \overset{\xi=0.707}{=} \frac{\omega_0^2}{\sqrt{\omega_0^4 + \omega^4}}$$

理想传输系统幅频特性

$A(\omega) = A_0$ **Passband**绝对平坦

$$\frac{dA(\omega)}{d\omega}\Big|_{\omega=0} = 0$$

$$\frac{d^2A(\omega)}{d\omega^2}\Big|_{\omega=0} = 0$$

…

$$\frac{d^nA(\omega)}{d\omega^n}\Big|_{\omega=0} = 0$$

…

$$A_n(\omega) \overset{\text{幅度最大平坦n阶滤波器}}{=} \frac{\omega_0^n}{\sqrt{\omega_0^{2n} + \omega^{2n}}}$$

3dB带宽

$$A(\omega_{3dB}) = \frac{\omega_0^2}{\sqrt{\left(\omega_0^2 - \omega_{3dB}^2\right)^2 + \left(2\xi\omega_0\omega_{3dB}\right)^2}} = \frac{A(0)}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\omega_{3dB}^4 + \left(4\xi^2\omega_0^2 - 2\omega_0^2\right)\omega_{3dB}^2 - \omega_0^4 = 0$$

$$\omega_{3dB}^2 = \frac{-4\xi^2\omega_0^2 + 2\omega_0^2 + \sqrt{\left(4\xi^2\omega_0^2 - 2\omega_0^2\right)^2 + 4\omega_0^4}}{2} = \left(-2\xi^2 + 1 + \sqrt{\left(2\xi^2 - 1\right)^2 + 1}\right)\omega_0^2$$

$$\omega_{3dB} = \omega_0\sqrt{-2\xi^2 + 1 + \sqrt{\left(2\xi^2 - 1\right)^2 + 1}}$$

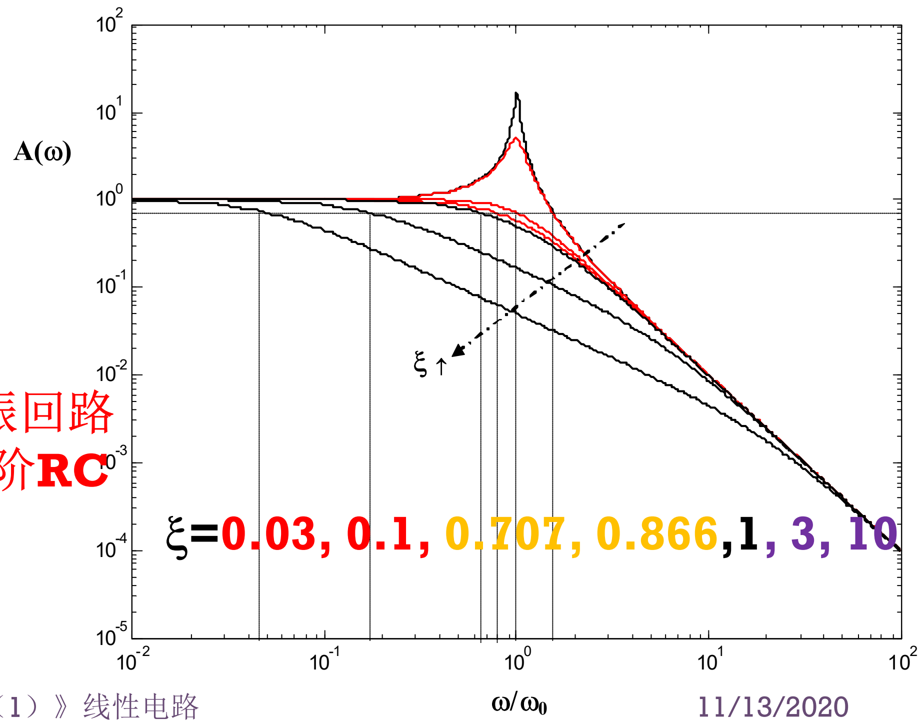
$$\boldsymbol{\omega_{3dB} \overset{\xi=0.707}{=} \omega_0}$$

$$\omega_{3dB} \overset{\xi>>1}{\approx} \frac{1}{2\xi}\omega_0$$

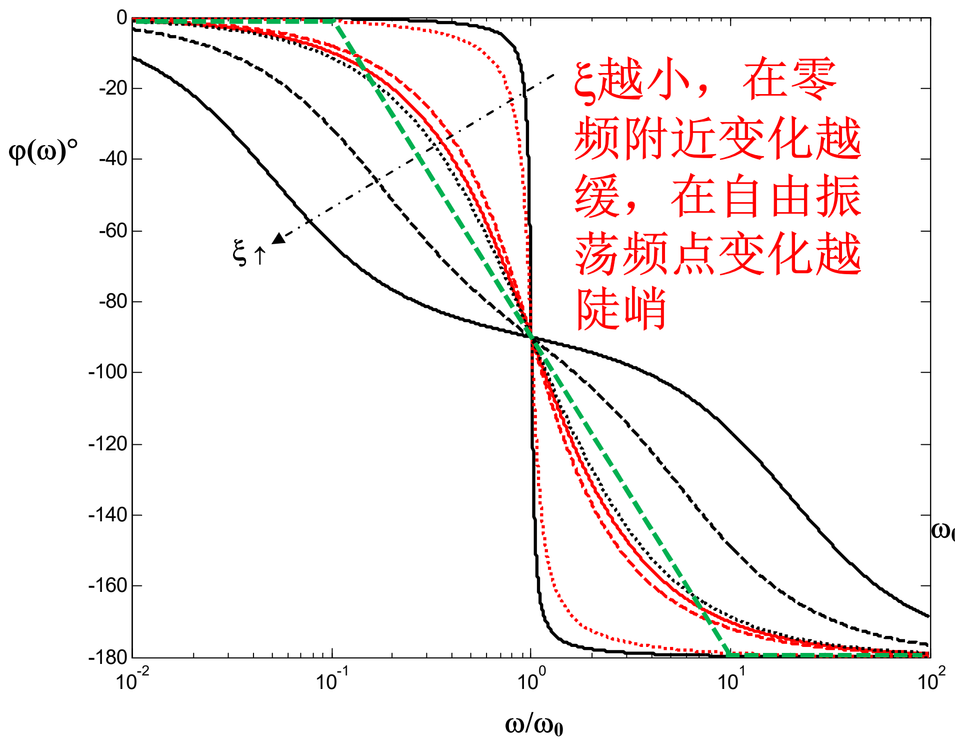
=1/RC：RLC串联谐振回路

R很大时，行为犹如一阶**RC**

$$\omega_{3dB} \overset{\xi<<1}{\approx} \omega_0\sqrt{1+\sqrt{2}} = 1.554\omega_0$$



相频特性和群延时特性

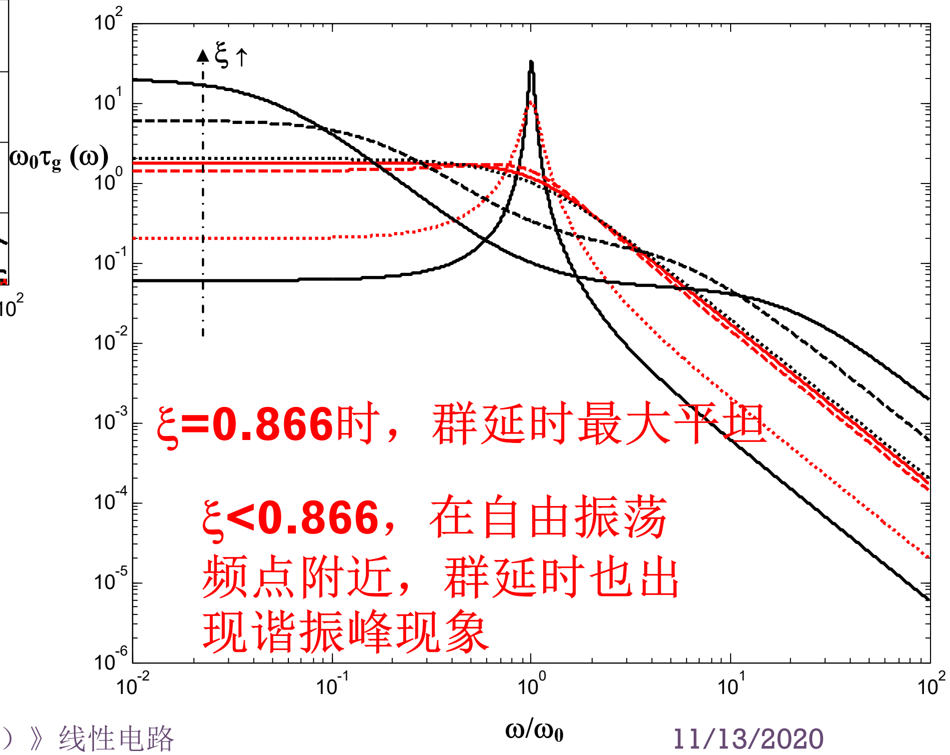


$$\varphi(\omega)=-\arctan\frac{2\xi\omega_0\omega}{\omega_0^2-\omega^2}$$

ξ=0.03, 0.1, 0.707, 0.866,1, 3, 10

$$\tau_g(\omega)=-\frac{d\varphi(\omega)}{d\omega}$$

$$=\frac{2\xi\omega_0\left(\omega^2+\omega_0^2\right)}{\omega^4+2\left(2\xi^2-1\right)\omega_0^2\omega^2+\omega_0^4}$$



群延时最大平坦 $\xi = 0.866$

理想传输系统群延时特性

$$\tau_g(\omega) = \tau_0 \quad \textbf{passband}$$

$$\frac{d\tau_g(\omega)}{d\omega}\bigg|_{\omega=0} = 0$$

$$\frac{d^2\tau_g(\omega)}{d\omega^2}\bigg|_{\omega=0} = 0$$

$$\cdots$$

$$\frac{d^n\tau_g(\omega)}{d\omega^n}\bigg|_{\omega=0} = 0$$

$$\cdots$$

$$\tau_g(\omega) = \frac{2\xi\omega_0\left(\omega^2 + \omega_0^2\right)}{\omega^4 + 2\left(2\xi^2 - 1\right)\omega_0^2\omega^2 + \omega_0^4}$$

$$\frac{d\tau_g(\omega)}{d\omega}\bigg|_{\omega=0} = 0$$

$$\frac{d^2\tau_g(\omega)}{d\omega^2}\bigg|_{\omega=0} = 0$$

$$\xi = \frac{\sqrt{3}}{2} = 0.866$$

二阶低通系统的群延时特性具有最大平坦特性

此为最接近理想传输系统群延时特性的二阶低通系统：最优

最接近理想滤波器的二阶低通系统

$$\xi = \frac{\sqrt{2}}{2} = 0.707$$

二阶低通系统的幅频特性具有最大平坦特性（巴特沃思滤波器）
此为最接近理想传输系统幅频特性的二阶低通系统：最优

$$\xi = \frac{\sqrt{3}}{2} = 0.866$$

二阶低通系统的群延时特性具有最大平坦特性（贝塞尔滤波器）
此为最接近理想传输系统群延时特性的二阶低通系统：最优

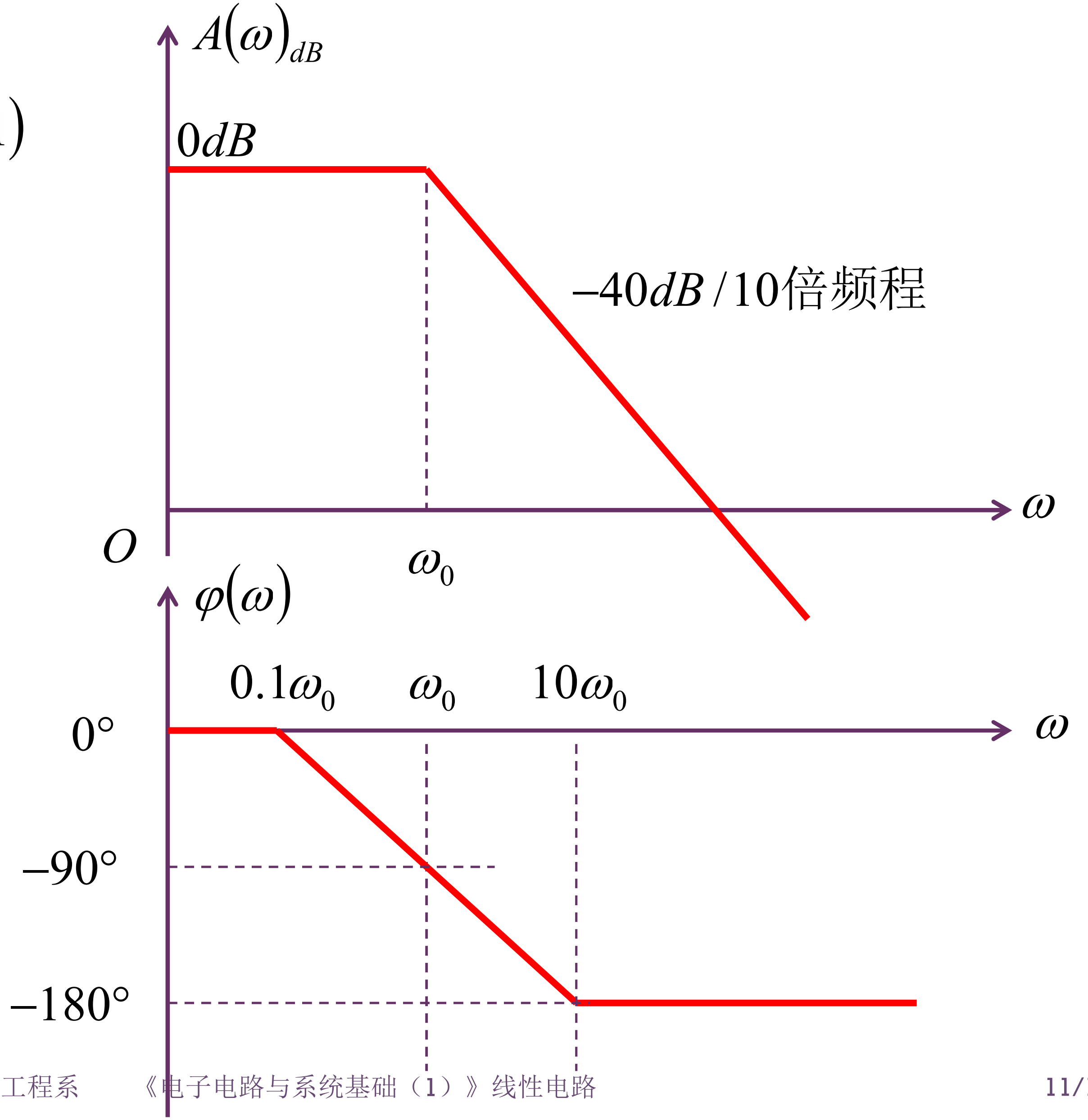
$$\xi \in (0.707,1)$$

最优二阶低通系统：

频域：幅频特性、群延时特性相对平坦
时域：具有最快的阶跃响应（时域波形具有最小的线性失真）

最优二阶低通的伯特图

$\xi \in (0.707,1)$



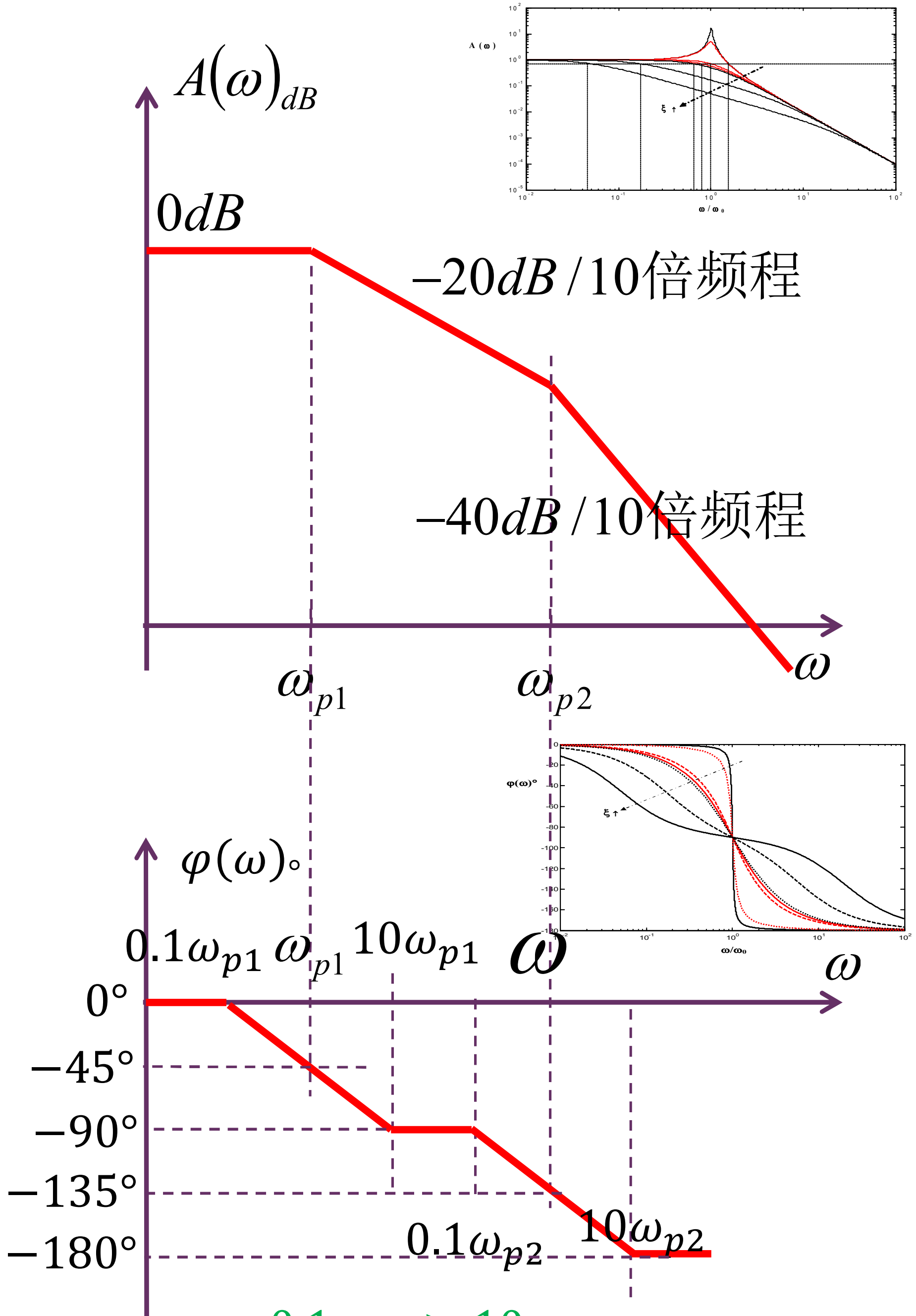
过阻尼的伯特图

$$H(s)=\frac{\omega_0^2}{(s-\lambda_1)(s-\lambda_2)}$$

$$H(j\omega)=\frac{\omega_0^2}{(j\omega+\omega_{p1})(j\omega+\omega_{p2})}$$

$$=\frac{1}{\left(\frac{j\omega}{\omega_{p1}}+1\right)\left(\frac{j\omega}{\omega_{p2}}+1\right)}$$

$$\approx\begin{cases}1 & \omega<\omega_{p1}\\ \dfrac{1}{\frac{j\omega}{\omega_{p1}}}=\dfrac{\omega_{p1}}{j\omega} & \omega_{p1}<\omega<\omega_{p2}\\ \dfrac{1}{\frac{j\omega}{\omega_{p1}}\cdot\frac{j\omega}{\omega_{p2}}}=\dfrac{\omega_0^2}{-\omega^2} & \omega>\omega_{p2}\end{cases}$$

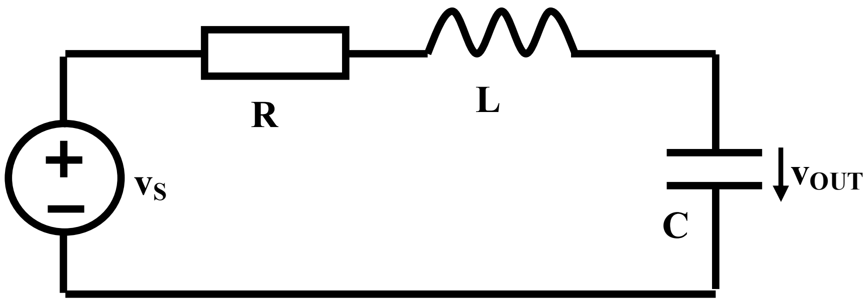


$0.1\omega_{p2}>10\omega_{p1}$

时域特性：冲激响应

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{L}{C}}$$

$$\xi = \frac{R}{2Z_0} = \frac{R}{2}\sqrt{\frac{C}{L}}$$



$$v_S(t) = \frac{V_{S0}}{\omega_0}\cdot\delta(t)$$

$$i_L(0^-) = 0 \qquad v_C(0^-) = 0$$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v_L(t)dt$$

$$= \frac{1}{L}\int_{0^-}^{0^+} \frac{V_{S0}}{\omega_0}\delta(t)dt = \frac{V_{S0}}{\omega_0 L} = \frac{V_{S0}}{Z_0}$$

$$v_C(0^+) = v_C(0^-) = 0$$

$$\frac{d}{dt}v_C(0^+) = \frac{i_C(0^+)}{C} = \frac{i_L(0^+)}{C} = \frac{V_{S0}}{Z_0 C} = \omega_0 V_{S0}$$

$$v_{C\infty}(t) = 0$$

$$x(t) = x_\infty(t) + (X_0 - X_{\infty 0})e^{-\xi\omega_0 t}\cos\sqrt{1-\xi^2}\,\omega_0 t$$

$$+\left(\frac{\dot{X}_0 - \dot{X}_{\infty 0}}{\xi\omega_0} + X_0 - X_{\infty 0}\right)\frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_0 t}\sin\sqrt{1-\xi^2}\,\omega_0 t \qquad (t \ge 0)$$

$$v_{OUT}(t) = \frac{V_{S0}}{\sqrt{1-\xi^2}}e^{-\xi\omega_0 t}\sin\left(\sqrt{1-\xi^2}\,\omega_0 t\right)\cdot U(t)$$

冲激响应时域波形

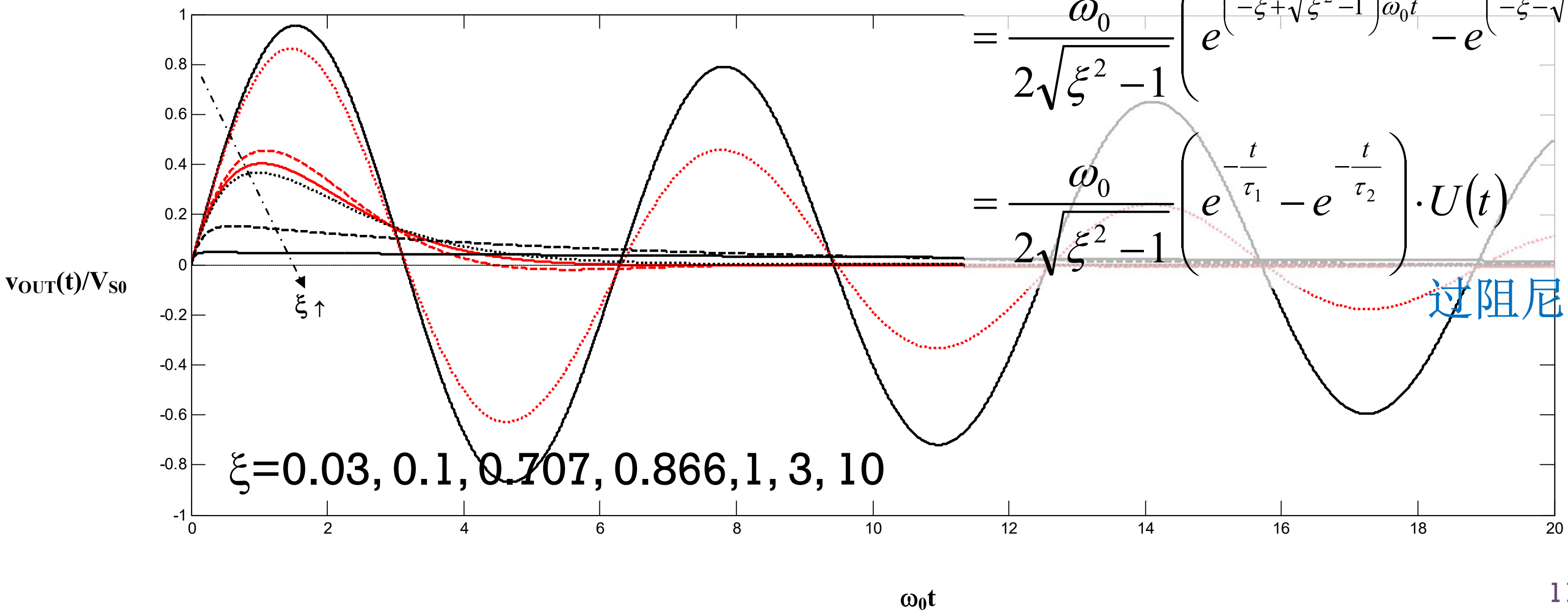
$$v_S(t)=\frac{V_{S0}}{\omega_0}\cdot\delta(t)$$

$$v_{OUT}(t)=\frac{V_{S0}}{\sqrt{1-\xi^2}}e^{-\xi\omega_0 t}\sin\left(\sqrt{1-\xi^2}\,\omega_0 t\right)\cdot U(t)$$

$$h(t)=\frac{\omega_0}{\sqrt{1-\xi^2}}e^{-\xi\omega_0 t}\sin\left(\sqrt{1-\xi^2}\,\omega_0 t\right)\cdot U(t)$$

欠阻尼：幅度指数衰减的正弦振荡

$$h(t)\overset{\xi=0}{=}\omega_0\sin(\omega_0 t)\cdot U(t)$$ 无阻尼：正弦振荡



$$h(t)\overset{\xi>1}{=}\frac{\omega_0}{\sqrt{\xi^2-1}}e^{-\xi\omega_0 t}\sinh\left(\sqrt{\xi^2-1}\,\omega_0 t\right)\cdot U(t)$$

$$=\frac{\omega_0}{\sqrt{\xi^2-1}}e^{-\xi\omega_0 t}\frac{e^{\sqrt{\xi^2-1}\omega_0 t}-e^{-\sqrt{\xi^2-1}\omega_0 t}}{2}\cdot U(t)$$

$$=\frac{\omega_0}{2\sqrt{\xi^2-1}}\left(e^{\left(-\xi+\sqrt{\xi^2-1}\right)\omega_0 t}-e^{\left(-\xi-\sqrt{\xi^2-1}\right)\omega_0 t}\right)\cdot U(t)$$

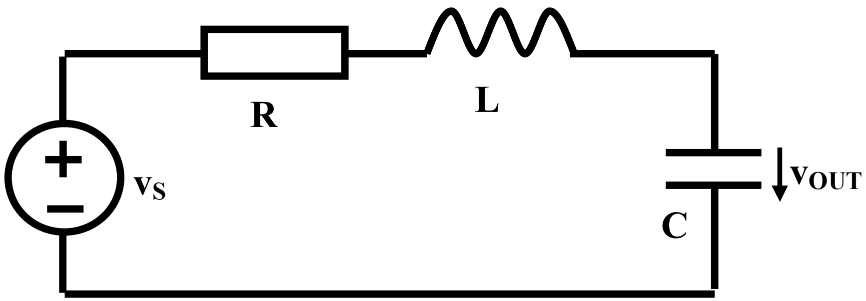
$$=\frac{\omega_0}{2\sqrt{\xi^2-1}}\left(e^{-\frac{t}{\tau_1}}-e^{-\frac{t}{\tau_2}}\right)\cdot U(t)$$

过阻尼：指数衰减

阶跃响应

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{L}{C}}$$

$$\xi = \frac{R}{2Z_0} = \frac{R}{2}\sqrt{\frac{C}{L}}$$



$$v_S(t) = V_{S0} \cdot U(t)$$

$$i_L(0^-) = 0 \qquad v_C(0^-) = 0$$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v_L(t)dt$$

$$= i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} V_{S0}U(t)dt = i_L(0^-) = 0$$

$$v_C(0^+) = v_C(0^-) = 0$$

$$\frac{d}{dt}v_C(0^+) = \frac{i_C(0^+)}{C} = \frac{i_L(0^+)}{C} = 0$$

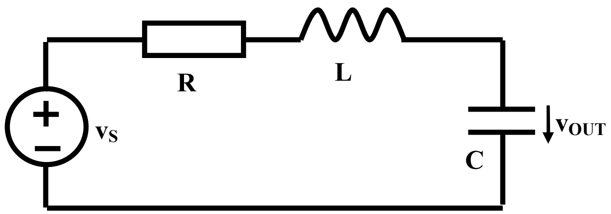
$$v_{C\infty}(t) = V_{S0}$$

$$x(t) = x_\infty(t) + (X_0 - X_{\infty 0})e^{-\xi\omega_0 t}\cos\sqrt{1-\xi^2}\,\omega_0 t$$
$$+\left(\frac{\dot{X}_0 - \dot{X}_{\infty 0}}{\xi\omega_0} + X_0 - X_{\infty 0}\right)\frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_0 t}\sin\sqrt{1-\xi^2}\,\omega_0 t \qquad (t>0)$$

$$v_{OUT}(t) = V_{S0}\left(1 - e^{-\xi\omega_0 t}\left(\cos\sqrt{1-\xi^2}\,\omega_0 t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\sqrt{1-\xi^2}\,\omega_0 t\right)\right)\cdot U(t)$$

阶跃响应时域波形

$$v_S(t)=V_{S0}\cdot U(t)$$

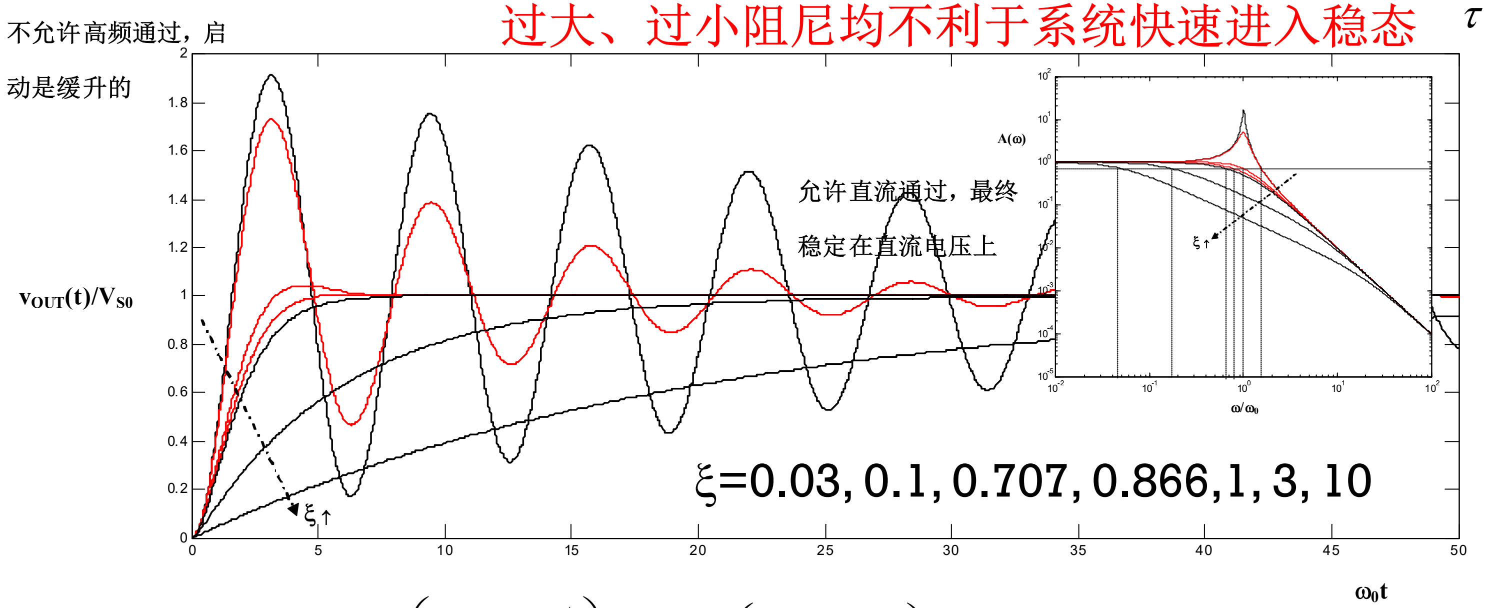


$$v_{OUT}(t)=V_{S0}\left(1-e^{-\xi\omega_0 t}\left(\cos\sqrt{1-\xi^2}\,\omega_0 t+\frac{\xi}{\sqrt{1-\xi^2}}\sin\sqrt{1-\xi^2}\,\omega_0 t\right)\right)\cdot U(t)$$

$$g(t)=\left(1-e^{-\xi\omega_0 t}\left(\cos\sqrt{1-\xi^2}\,\omega_0 t+\frac{\xi}{\sqrt{1-\xi^2}}\sin\sqrt{1-\xi^2}\,\omega_0 t\right)\right)\cdot U(t)$$

过大、过小阻尼均不利于系统快速进入稳态

$$\tau=\frac{1}{\xi\omega_0}\qquad (0<\xi<1)$$



$$(\xi>1)$$

$$\tau_1=\frac{\xi+\sqrt{\xi^2-1}}{\omega_0}$$

长寿命

$$\tau_2=\frac{1}{\left(\xi+\sqrt{\xi^2-1}\right)\omega_0}$$

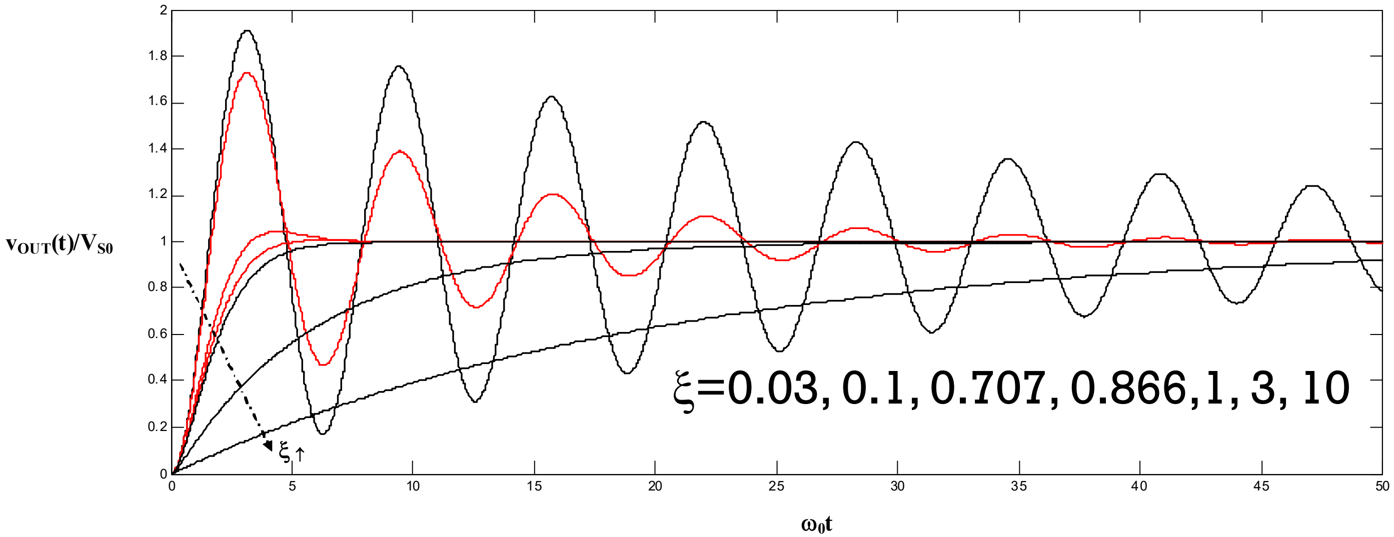
短寿命

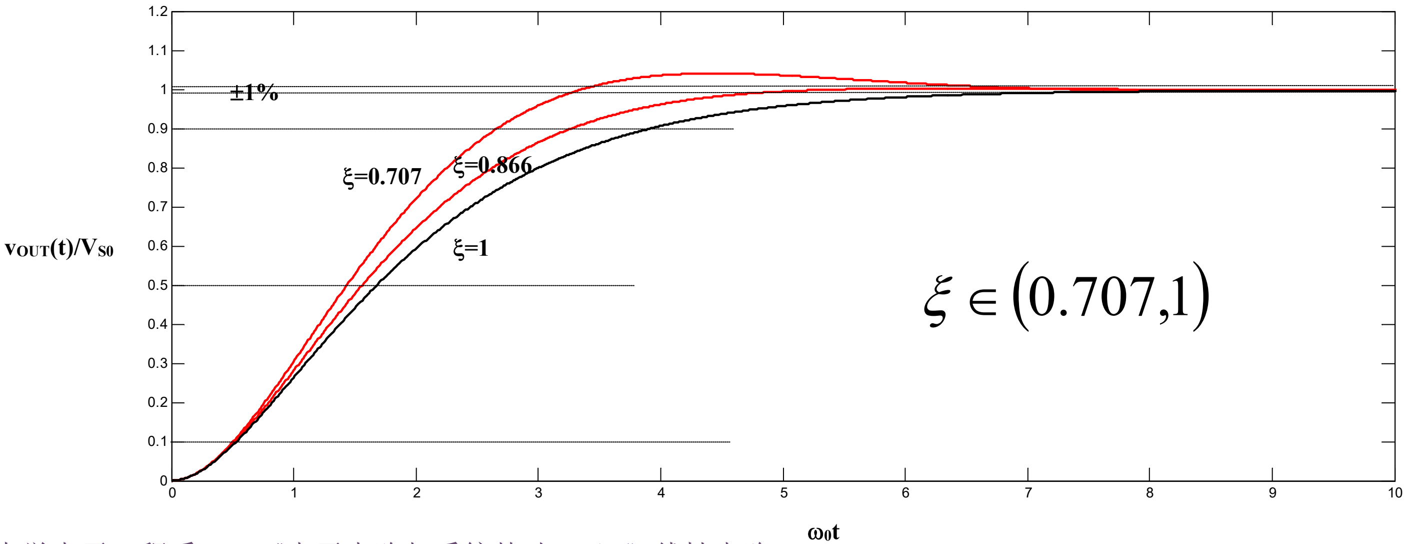
$$g(t)\overset{\xi极大时}{\approx}\left(1-e^{-\frac{\omega_0 t}{2\xi}}\right)U(t)=\left(1-e^{-\frac{t}{RC}}\right)U(t)$$

RLC在R很大时，犹如一阶RC

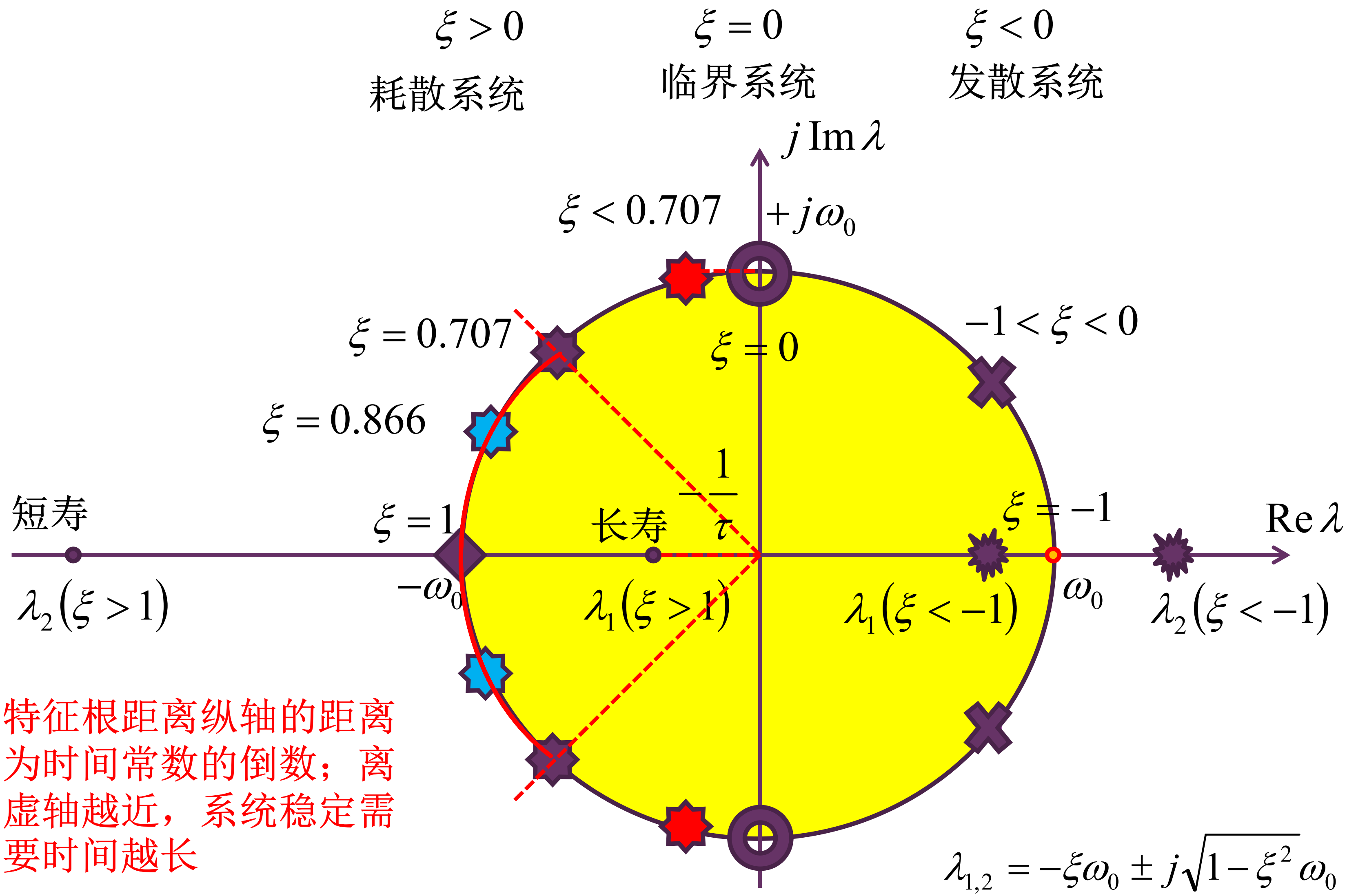
最优二阶低通的阶跃响应

阻尼系数在**0.707-1**之间时，系统进入稳态需要的时间最短：最优二阶低通系统，和一阶系统大体相当，大约**5τ (1.5QT,4.6τ)**时间进入偏离稳态**1%**误差范围，大约**7τ**（**2.2QT, 6.9τ**）进入偏离稳态**0.1%**误差范围





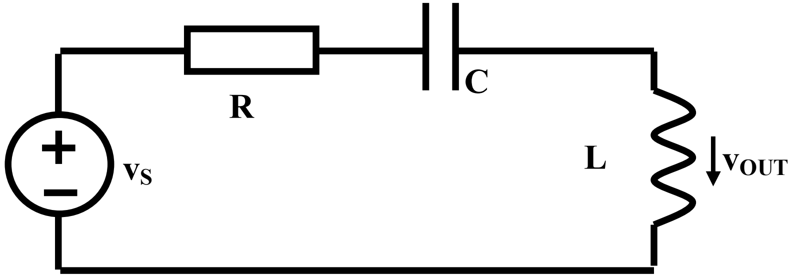
从特征根位置看最优响应为何是最快的



特征根距离纵轴的距离为时间常数的倒数；离虚轴越近，系统稳定需要时间越长

$$\lambda_{1,2} = -\xi\omega_0 \pm j\sqrt{1-\xi^2}\,\omega_0$$

二阶高通：电感分压



直观理解：

低频：电容开路，电感短路，信号通不过
高频：电容短路，电感开路，信号全部通过

$$H(j\omega)=\frac{\dot{V}_L}{\dot{V}_S}=\frac{j\omega L}{R+j\omega L+\frac{1}{j\omega C}}=\frac{(j\omega)^2LC}{(j\omega)^2LC+j\omega RC+1}$$

$$\overset{j\omega\to s}{=}\frac{s^2LC}{s^2LC+sRC+1}=\frac{s^2}{s^2+s\frac{R}{L}+\frac{1}{LC}}=H_0\frac{s^2}{s^2+2\xi\omega_0 s+\omega_0^2}$$

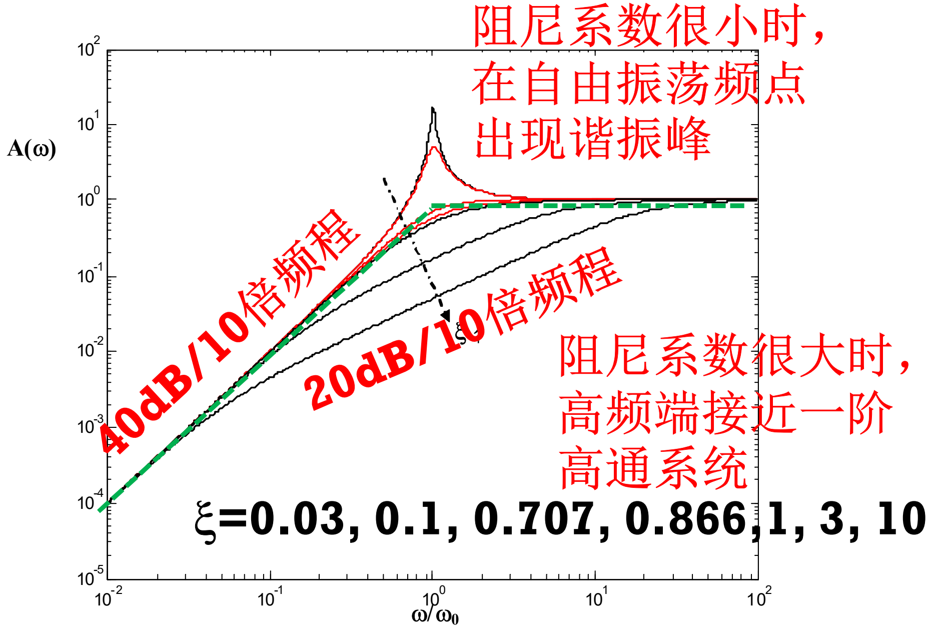
二阶高通传函典型形式

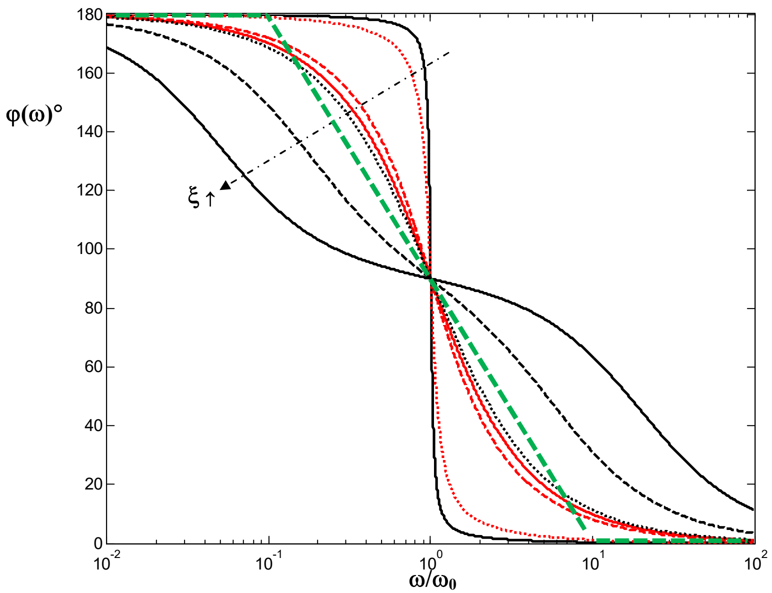
$$H_0=1\qquad \omega_0=\frac{1}{\sqrt{LC}}\qquad \xi=\frac{R}{2Z_0}=\frac{R}{2}\sqrt{\frac{C}{L}}$$

幅频特性、相频特性

$$H(s)=\frac{s^2}{s^2+2\xi\omega_0 s+\omega_0^2}\overset{s=j\omega}{=}\frac{-\omega^2}{\omega_0^2-\omega^2+j2\xi\omega_0\omega}=A(\omega)e^{j\varphi(\omega)}$$

$$A(\omega)=\frac{\omega^2}{\sqrt{\left(\omega_0^2-\omega^2\right)^2+\left(2\xi\omega_0\omega\right)^2}}\qquad \varphi(\omega)=\pi-\arctan\frac{2\xi\omega_0\omega}{\omega_0^2-\omega^2}$$





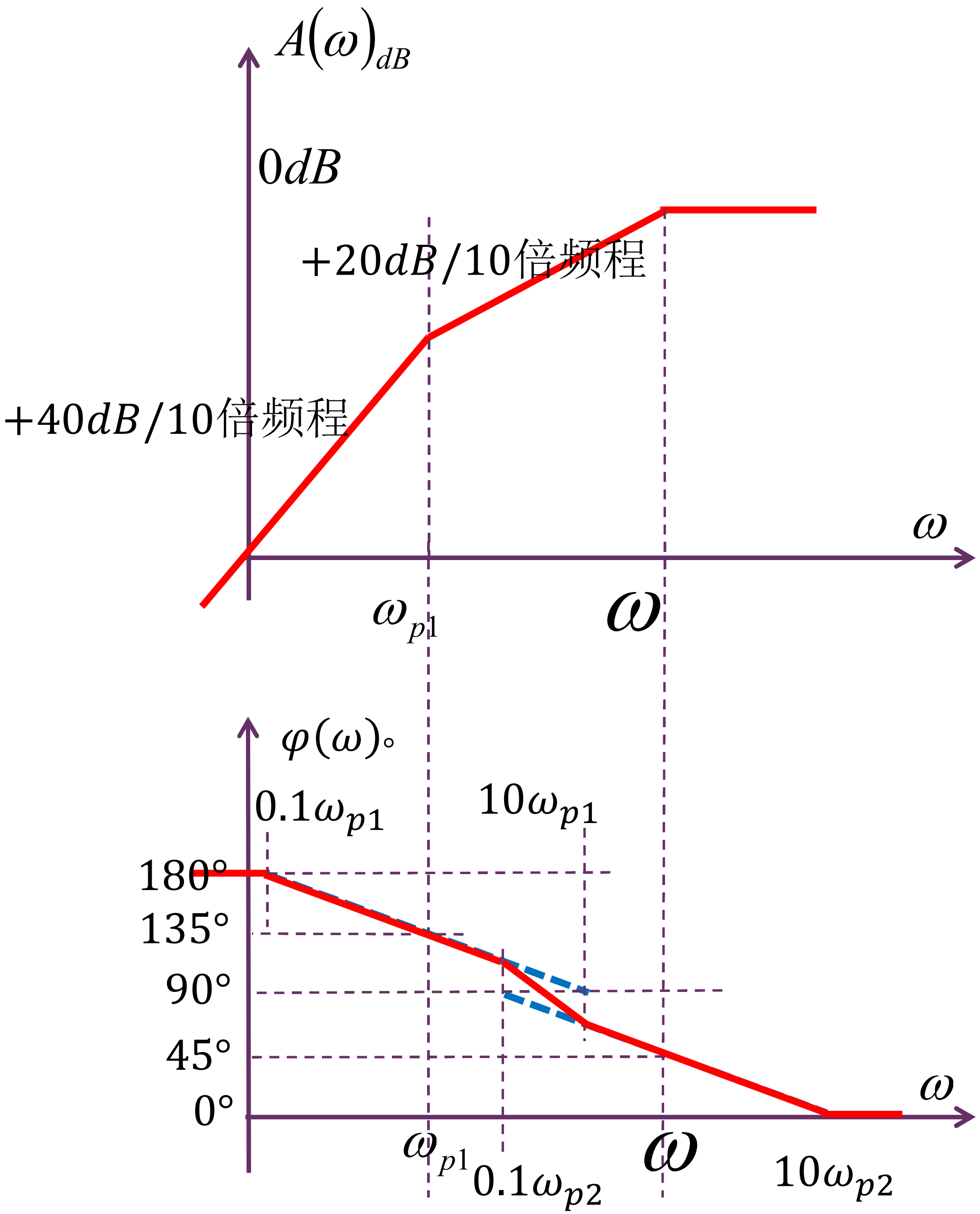
过阻尼的伯特图

$$H(s)=\frac{s^2}{(s-\lambda_1)(s-\lambda_2)}$$

$$H(j\omega)=\frac{-\omega^2}{(j\omega+\omega_{p1})(j\omega+\omega_{p2})}$$

$$=\frac{1}{\left(1+\frac{\omega_{p1}}{j\omega}\right)\left(1+\frac{\omega_{p2}}{j\omega}\right)}$$

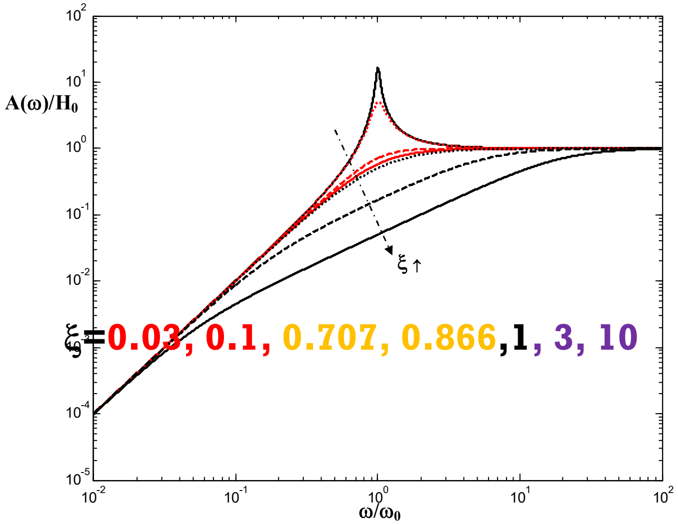
$$\approx\begin{cases}1 & \omega>\omega_{p2}\\ \dfrac{1}{\frac{\omega_{p2}}{j\omega}}=\dfrac{j\omega}{\omega_{p2}} & \omega_{p2}>\omega>\omega_{p1}\\ \dfrac{1}{\frac{\omega_{p1}}{j\omega}\cdot\frac{\omega_{p2}}{j\omega}}=\dfrac{-\omega^2}{\omega_0^2} & \omega<\omega_{p1}\end{cases}$$



$0.1\omega_{p2}<10\omega_{p1}$

欠阻尼的谐振峰

$$A(\omega)=\frac{\omega^2}{\sqrt{\left(\omega_0^2-\omega^2\right)^2+\left(2\xi\omega_0\omega\right)^2}}$$



$$\frac{dA(\omega)}{d\omega}=0$$

$$=\frac{2\omega\omega_0^2}{\left(\left(\omega_0^2-\omega^2\right)^2+\left(2\xi\omega_0\omega\right)^2\right)^{\frac{3}{2}}}\left[\omega_0^2+\left(2\xi^2-1\right)\omega^2\right]$$

$$\omega_e=\frac{\omega_0}{\sqrt{1-2\xi^2}}\qquad(\xi<0.707)$$

$$A(\omega_e)=\frac{1}{2\xi\sqrt{1-\xi^2}}\overset{\xi<0.707}{>}1=A(\infty)$$

$$\omega_e=\frac{\omega_0}{\sqrt{1-2\xi^2}}\overset{\xi<<0.707}{\approx}\omega_0$$

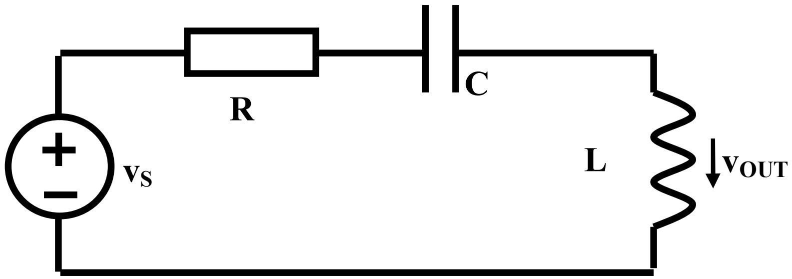
阻尼系数很小时，谐振峰近似出现在自由振荡频点位置，谐振峰高度近似为**Q**

$$A(\omega_e)=\frac{1}{2\xi\sqrt{1-\xi^2}}\overset{\xi<<0.707}{\approx}\frac{1}{2\xi}=Q=A(\omega_0)$$

时域特性：冲激响应

$$\omega_0=\frac{1}{\sqrt{LC}}\qquad Z_0=\sqrt{\frac{L}{C}}$$

$$\xi=\frac{R}{2Z_0}=\frac{R}{2}\sqrt{\frac{C}{L}}$$



$$v_S(t)=\frac{V_{S0}}{\omega_0}\cdot\delta(t)$$

$$i_L(0^-)=0\qquad v_C(0^-)=0$$

$$i_L(0^+)=i_L(0^-)+\frac{1}{L}\int_{0^-}^{0^+}v_L(t)dt$$

$$=\frac{1}{L}\int_{0^-}^{0^+}\frac{V_{S0}}{\omega_0}\delta(t)dt=\frac{V_{S0}}{\omega_0 L}=\frac{V_{S0}}{Z_0}$$

$$v_C(0^+)=v_C(0^-)=0$$

$$v_L(0^+)=v_S(0^+)-v_R(0^+)-v_C(0^+)$$
$$=0-i_L(0^+)R-0=-2\xi V_{S0}$$

$$\frac{d}{dt}v_L(0^+)=\frac{d}{dt}v_S(0^+)-\frac{d}{dt}v_R(0^+)-\frac{d}{dt}v_C(0^+)$$
$$=0-R\frac{d}{dt}i_L(0^+)-\frac{i_C(0^+)}{C}=-\frac{R}{L}v_L(0^+)-\frac{i_L(0^+)}{C}$$
$$=-2\xi\omega_0(-2\xi V_{S0})-\frac{V_{S0}}{Z_0C}=(4\xi^2-1)\omega_0V_{S0}$$

$$v_{L\infty}(t)=0$$

单位冲激响应

$$v_L(0^+) = -2\xi V_{S0} \qquad \frac{d}{dt}v_L(0^+) = (4\xi^2-1)\omega_0 V_{S0}$$

$$v_L(0) = \frac{V_{S0}}{\omega_0}\cdot\delta(t) \qquad v_{L\infty}(t) = 0 \qquad v_{L\infty}(t) = \frac{V_{S0}}{\omega_0}\delta(t)$$

t=0瞬间冲激电压加载到电感上

$$x(t) = x_\infty(t) + (X_0 - X_{\infty 0})e^{-\xi\omega_0 t}\cos\sqrt{1-\xi^2}\,\omega_0 t$$
$$+\left(\frac{\dot{X}_0 - \dot{X}_{\infty 0}}{\xi\omega_0} + X_0 - X_{\infty 0}\right)\frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_0 t}\sin\sqrt{1-\xi^2}\,\omega_0 t \qquad (t>0)$$

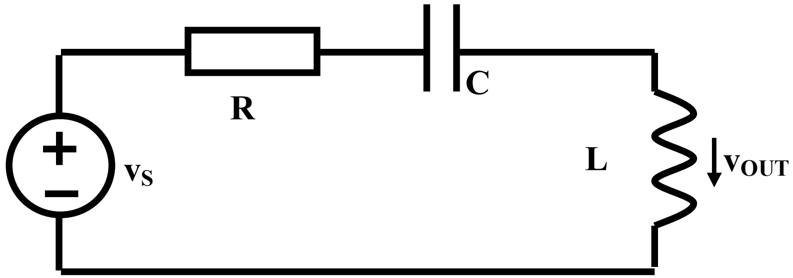
$$v_{OUT}(t) = \frac{V_{S0}}{\omega_0}\delta(t) + \left[\begin{matrix} -2\xi V_{S0}e^{-\xi\omega_0 t}\cos\sqrt{1-\xi^2}\,\omega_0 t \\ +\left(\frac{4\xi^2-1}{\xi} - 2\xi\right)V_{S0}\frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_0 t}\sin\left(\sqrt{1-\xi^2}\omega_0 t\right)\end{matrix}\right]\cdot U(t)$$
$$= \frac{V_{S0}}{\omega_0}\left[\delta(t) - 2\xi\omega_0 e^{-\xi\omega_0 t}\cos\sqrt{1-\xi^2}\,\omega_0 t\cdot U(t) + \frac{2\xi^2-1}{\sqrt{1-\xi^2}}\omega_0 e^{-\xi\omega_0 t}\sin\left(\sqrt{1-\xi^2}\omega_0 t\right)\cdot U(t)\right]$$

$$v_S(t) = \frac{V_{S0}}{\omega_0}\cdot\delta(t)$$

$$h(t) = \delta(t) + \omega_0 e^{-\xi\omega_0 t}\left[-2\xi\cos\sqrt{1-\xi^2}\,\omega_0 t + \frac{2\xi^2-1}{\sqrt{1-\xi^2}}\sin\left(\sqrt{1-\xi^2}\omega_0 t\right)\right]\cdot U(t)$$

阶跃响应

$$\omega_0=\frac{1}{\sqrt{LC}}\qquad \xi=\frac{R}{2Z_0}=\frac{R}{2}\sqrt{\frac{C}{L}}\qquad Z_0=\sqrt{\frac{L}{C}}$$



$$v_S(t)=V_{S0}\cdot U(t)$$

$$i_L(0^-)=0\qquad v_C(0^-)=0$$

$$i_L(0^+)=i_L(0^-)+\frac{1}{L}\int_{0^-}^{0^+}v_L(t)dt$$

$$=i_L(0^-)+\frac{1}{L}\int_{0^-}^{0^+}V_{S0}U(t)dt$$

$$=i_L(0^-)=0$$

$$v_L(0^+)=V_{S0}$$

$$\frac{d}{dt}v_L(0^+)=\frac{d}{dt}v_S(0^+)-\frac{d}{dt}v_R(0^+)-\frac{d}{dt}v_C(0^+)$$

$$=0-R\frac{d}{dt}i_L(0^+)-\frac{i_C(0^+)}{C}=-\frac{R}{L}v_L(0^+)-\frac{i_L(0^+)}{C}$$

$$=-2\xi\omega_0V_{S0}-0=-2\xi\omega_0V_{S0}$$

$$v_{L\infty}(t)=0$$

$$x(t)=x_\infty(t)+(X_0-X_{\infty 0})e^{-\xi\omega_0t}\cos\sqrt{1-\xi^2}\omega_0t$$

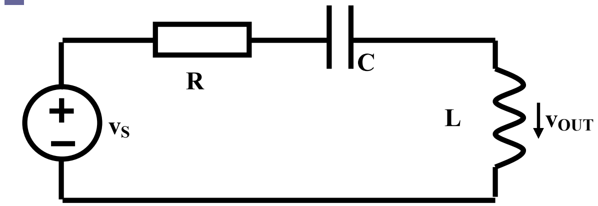
$$+\left(\frac{\dot{X}_0-\dot{X}_{\infty 0}}{\xi\omega_0}+X_0-X_{\infty 0}\right)\frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_0t}\sin\sqrt{1-\xi^2}\omega_0t$$

$$(t\geq 0)$$

$$v_{OUT}(t)=V_{S0}e^{-\xi\omega_0t}\left(\cos\sqrt{1-\xi^2}\omega_0t-\frac{\xi}{\sqrt{1-\xi^2}}\sin\sqrt{1-\xi^2}\omega_0t\right)\cdot U(t)$$

阶跃响应时域波形

$$v_S(t) = V_{S0} \cdot U(t)$$

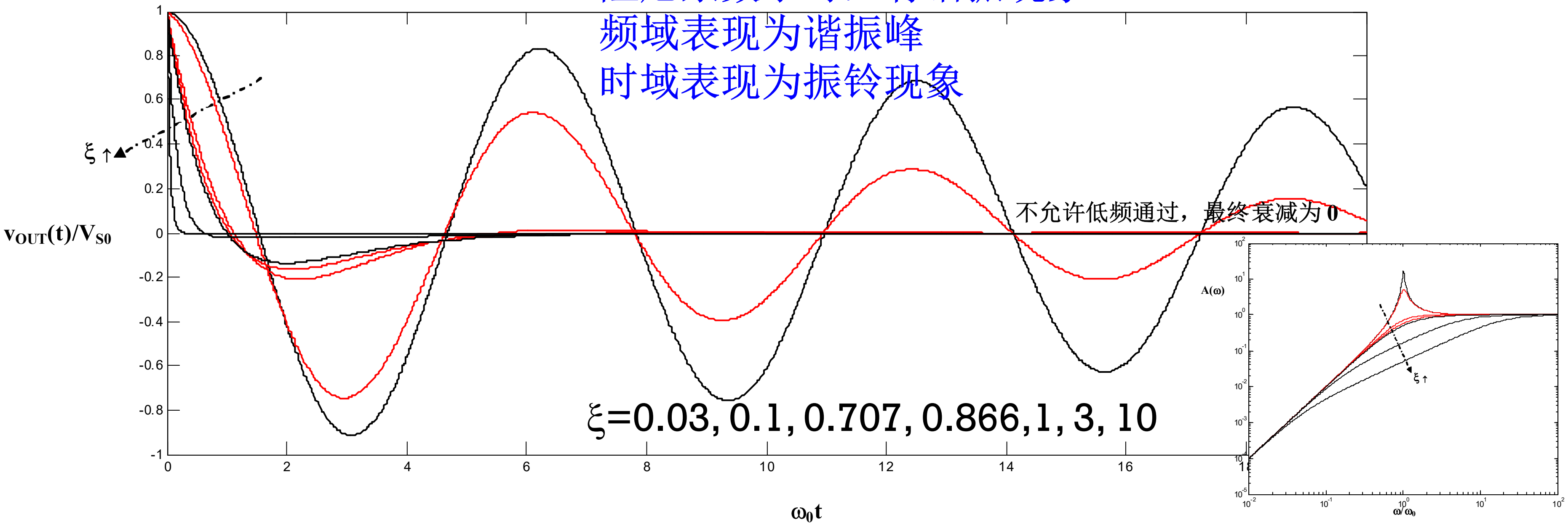


$$v_{OUT}(t) = V_{S0} e^{-\xi\omega_0 t}\left(\cos\sqrt{1-\xi^2}\,\omega_0 t - \frac{\xi}{\sqrt{1-\xi^2}}\sin\sqrt{1-\xi^2}\,\omega_0 t\right)\cdot U(t)$$

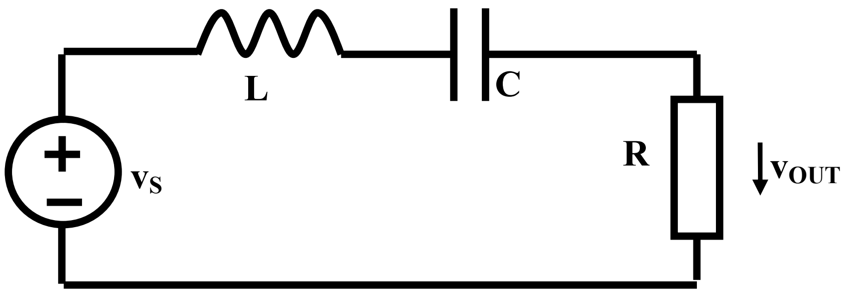
$$g(t) = e^{-\xi\omega_0 t}\left(\cos\sqrt{1-\xi^2}\,\omega_0 t - \frac{\xi}{\sqrt{1-\xi^2}}\sin\sqrt{1-\xi^2}\,\omega_0 t\right)\cdot U(t)$$

允许高频通过，瞬间跳变

阻尼系数小时，有谐振现象
频域表现为谐振峰
时域表现为振铃现象



二阶带通：电阻分压



直观理解：

低频：电容开路，电感短路，信号通不过
高频：电容短路，电感开路，信号通不过
自由振荡频点：电容电抗**-jZ$_0$**和电感电抗**+jZ$_0$**抵消，犹如短路，信号通过

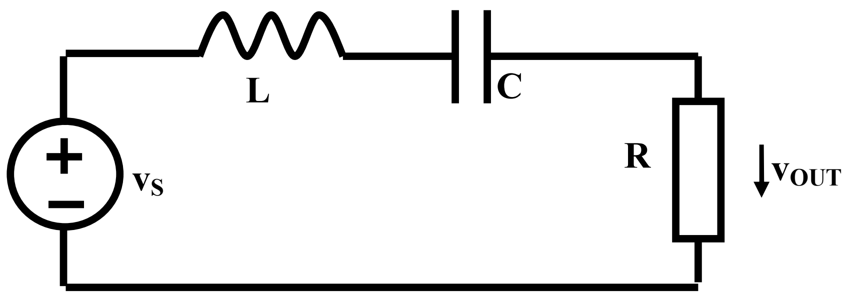
$$H(j\omega)=\frac{\dot{V}_R}{\dot{V}_S}=\frac{R}{R+j\omega L+\frac{1}{j\omega C}}=\frac{j\omega RC}{(j\omega)^2LC+j\omega RC+1}$$

$$\overset{j\omega\to s}{=}\frac{sRC}{s^2LC+sRC+1}=\frac{s\frac{R}{L}}{s^2+s\frac{R}{L}+\frac{1}{LC}}=H_0\frac{2\xi\omega_0 s}{s^2+2\xi\omega_0 s+\omega_0^2}$$

二阶带通传函典型形式

$$H_0=1 \qquad \omega_0=\frac{1}{\sqrt{LC}} \qquad \xi=\frac{R}{2Z_0}=\frac{R}{2}\sqrt{\frac{C}{L}}$$

考察带通习惯用Q值



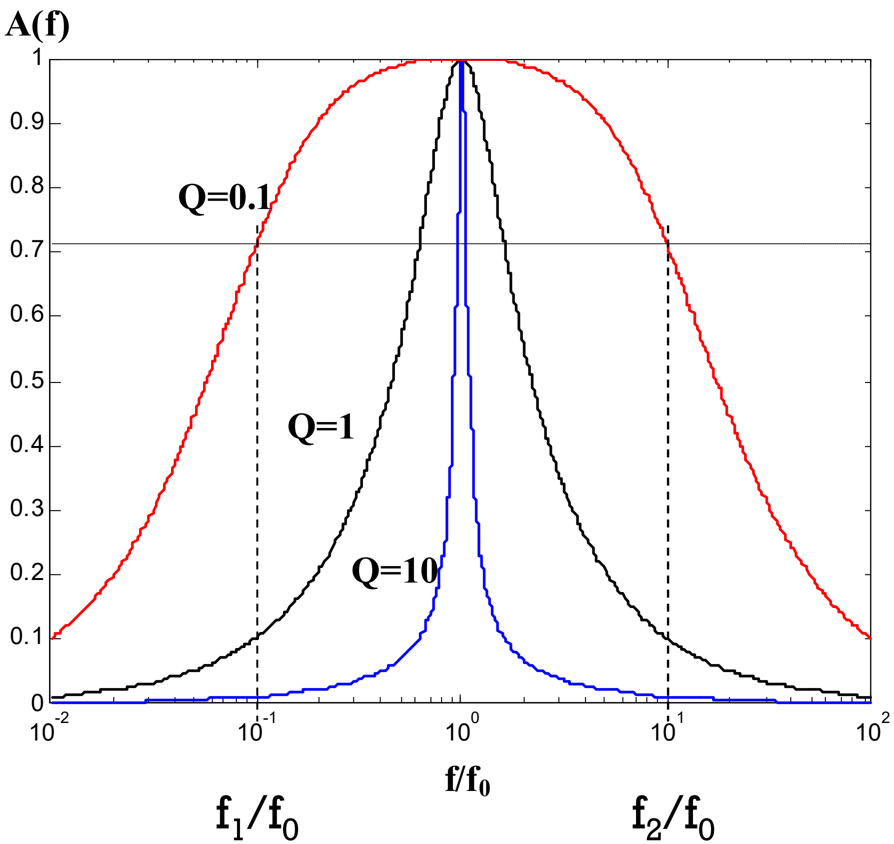
$$Q=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{Z_0}{R}=\frac{\omega_0 L}{R}=\frac{1}{2\xi}$$

品质因数：系统储能与耗能之比

$$H(j\omega)=\frac{2\xi\omega_0 s}{s^2+2\xi\omega_0 s+\omega_0^2}=\frac{\frac{\omega_0}{Q}s}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}\overset{s=j\omega}{=}\frac{j\frac{\omega_0}{Q}\omega}{-\omega^2+j\frac{\omega_0}{Q}\omega+\omega_0^2}$$

$$=\frac{1}{1+jQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}=\frac{1}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}e^{-j\arctan Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}=A(\omega)e^{j\varphi(\omega)}$$

幅频特性：带通选频特性



$$f_0=\frac{1}{2\pi\sqrt{LC}}$$

$$A(f)=\frac{1}{\sqrt{1+Q^2\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2}}$$

3dB通频带：$Q\left(\frac{f}{f_0}-\frac{f_0}{f}\right)=\pm1$

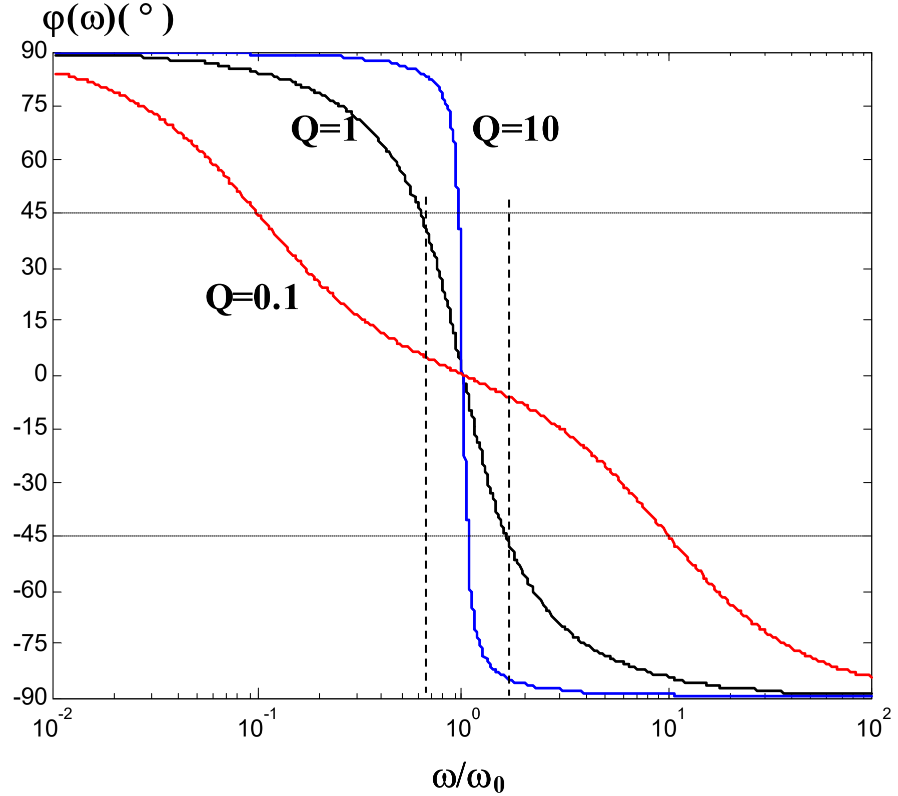
$\Rightarrow f_{1,2}=\ldots$

$\Rightarrow BW_{3dB}=f_2-f_1=\frac{f_0}{Q}$

$f_0=\sqrt{f_1 f_2}$

相频特性

$$\varphi(\omega) = -\arctan Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$



$$\tau_{g0} = -\frac{d\varphi_i}{d\omega}\bigg|_{\omega=\omega_0} = \frac{2Q}{\omega_0}$$

$$\varphi(\omega) \approx -Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

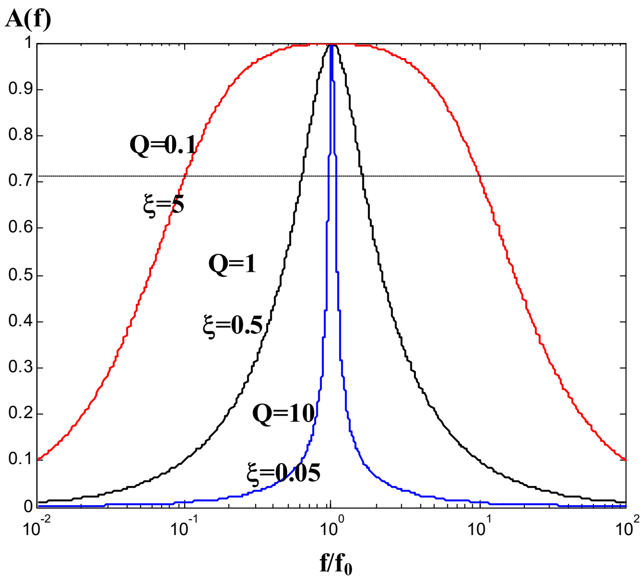
$$= -Q\left(\frac{(\omega-\omega_0)(\omega+\omega_0)}{\omega_0\omega}\right)$$

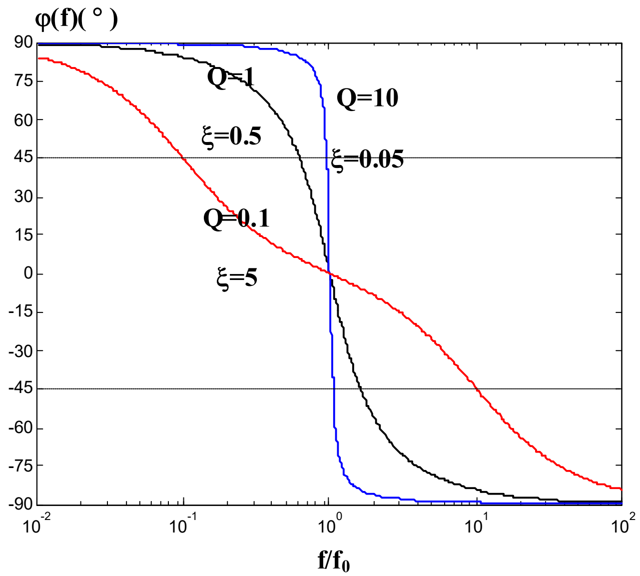
$$\approx -2Q\frac{(\omega-\omega_0)}{\omega_0} = -(\omega-\omega_0)\tau_{g0}$$

$$BW_{3dB}\cdot\tau_{g0} = \frac{f_0}{Q}\cdot\frac{2Q}{2\pi f_0} = \frac{1}{\pi} = 0.32$$

带宽越窄，信号延时越大：带通、低通均成立的结论

谐振频点的疑问?





$$H(j\omega)=\frac{\dot{V}_R}{\dot{V}_S}=A(\omega)e^{j\varphi(\omega)}$$

$$v_S(t)=V_{Sm}\cos\omega t$$

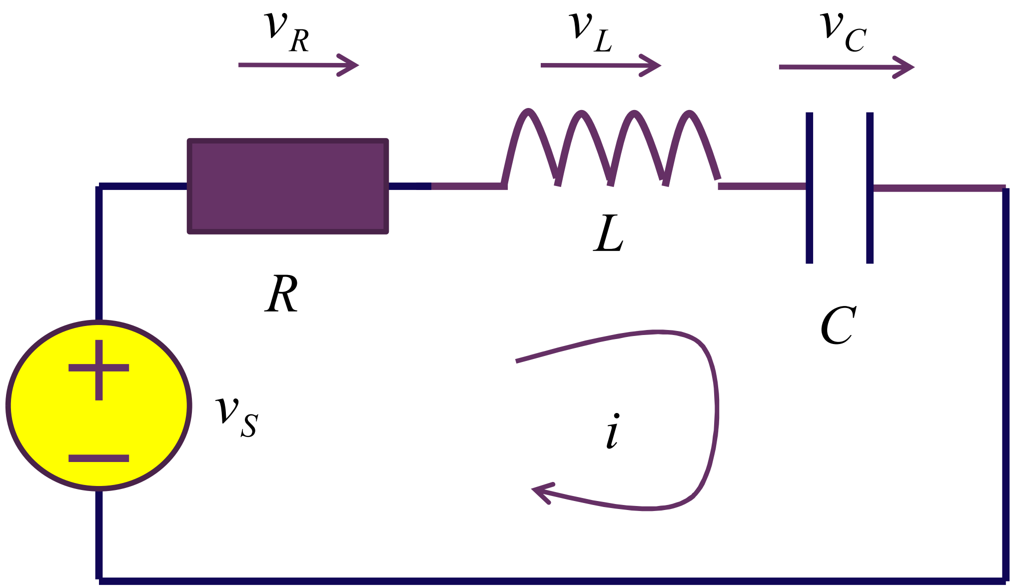
$$V_{R\infty}(t)=V_{Sm}A(\omega)\cos(\omega t+\varphi(\omega))$$

$$v_S(t)=V_{Sm}\cos\omega_0 t$$

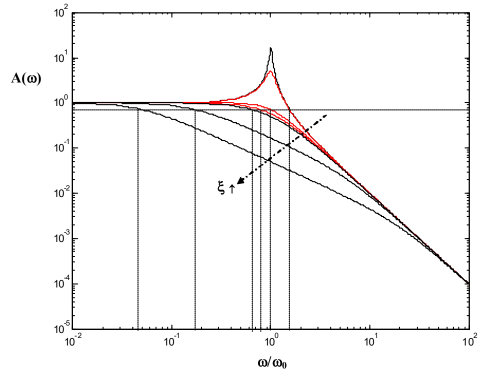
$$v_R(t)=V_{Sm}\cos\omega_0 t$$

显然：在谐振频点位置，电阻获得全部输入电压？？？难道电容、电感没有分压？

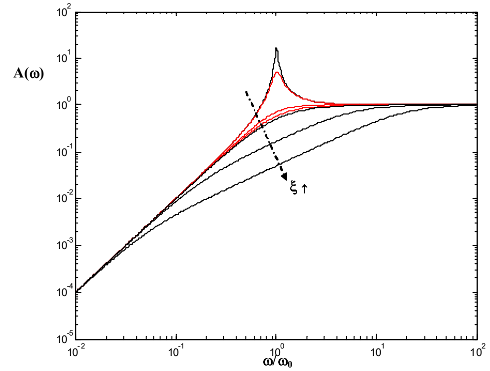
串联谐振为电压谐振



$\omega = \omega_0$



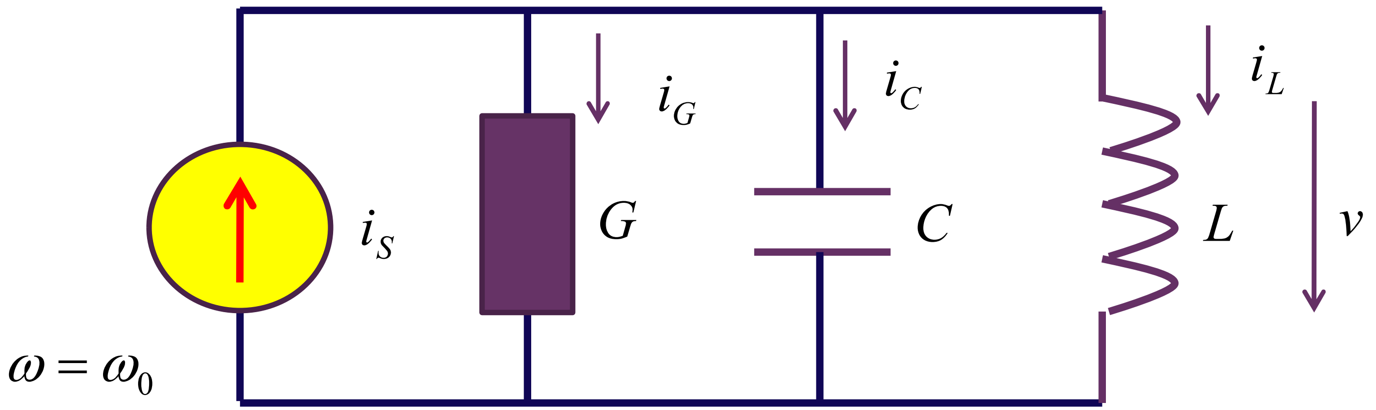
$$A(\omega_0) = \frac{1}{2\xi} = Q$$



$$\dot{V}_R = \dot{V}_S \qquad \dot{V}_L = j\omega_0 L\dot{I} = j\frac{\omega_0 L}{R}\dot{V}_S = j\frac{Z_0}{R}\dot{V}_S = jQ\dot{V}_S$$

$$\dot{I} = \frac{\dot{V}_R}{R} = \frac{\dot{V}_S}{R} \qquad \dot{V}_C = \frac{\dot{I}}{j\omega_0 C} = -j\frac{1}{\omega_0 CR}\dot{V}_S = -j\frac{Z_0}{R}\dot{V}_S = -jQ\dot{V}_S$$

并联谐振为电流谐振



$$\dot{I}_G = \dot{I}_S$$

$$\dot{I}_C = j\omega_0 C\dot{V} = j\frac{\omega_0 C}{G}\dot{I}_S = j\frac{Y_0}{G}\dot{I}_S = jQ\dot{I}_S$$

$$\dot{V} = \frac{\dot{I}_G}{G} = \frac{\dot{I}_S}{G}$$

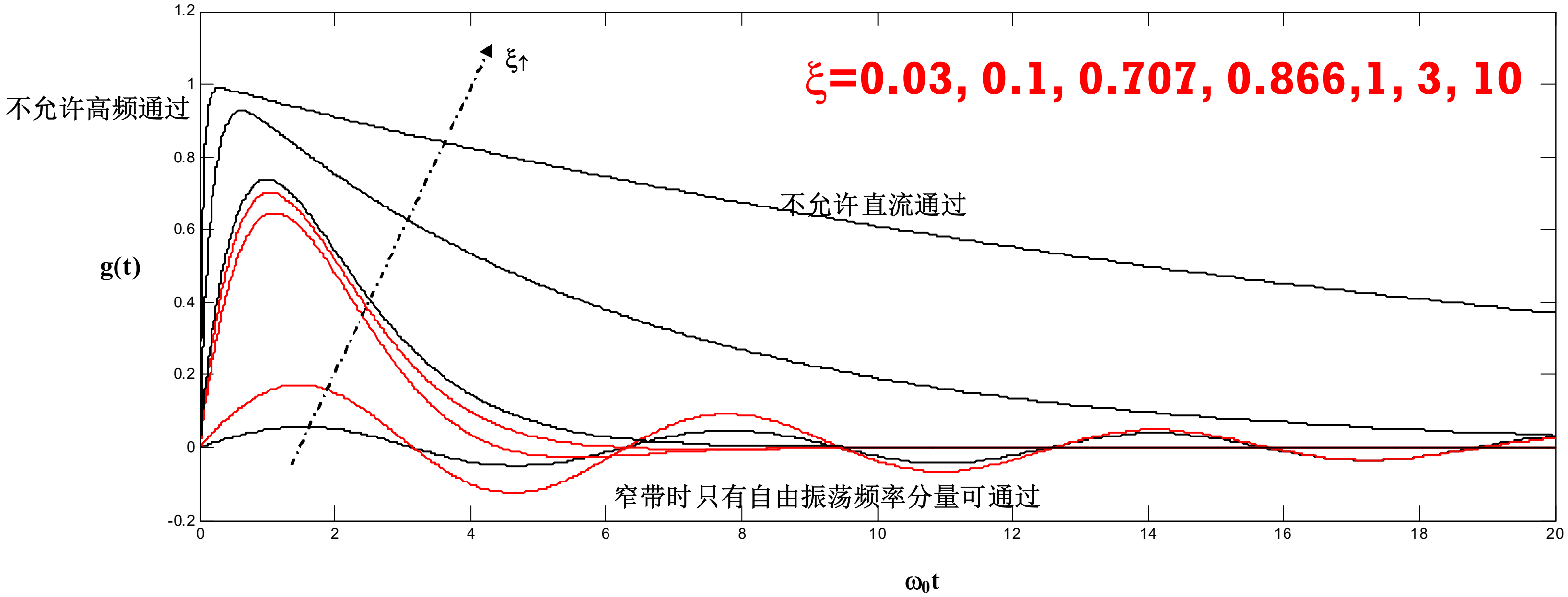
$$\dot{I}_L = \frac{\dot{V}}{j\omega_0 L} = -j\frac{1}{\omega_0 LG}\dot{I}_S = -j\frac{Y_0}{G}\dot{I}_S = -jQ\dot{I}_S$$

冲激响应和阶跃响应

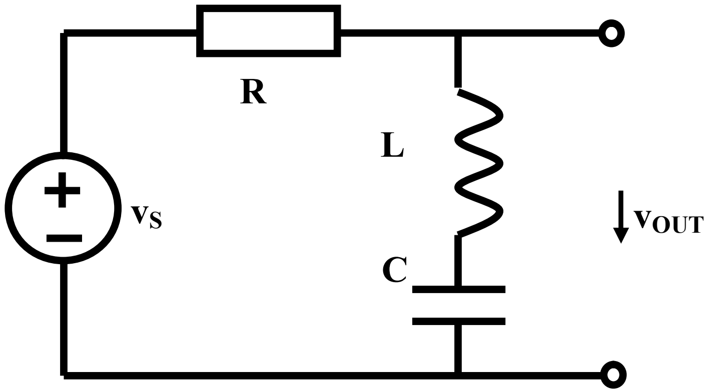
留作作业：用五要素法证明

$$h(t)=2\xi\omega_0 e^{-\xi\omega_0 t}\left(\cos\sqrt{1-\xi^2}\,\omega_0 t-\frac{\xi}{\sqrt{1-\xi^2}}\sin\sqrt{1-\xi^2}\,\omega_0 t\right)\cdot U(t)$$

$$g(t)=\frac{2\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_0 t}\sin\sqrt{1-\xi^2}\,\omega_0 t\cdot U(t)$$



二阶带阻：LC分压



直观理解：

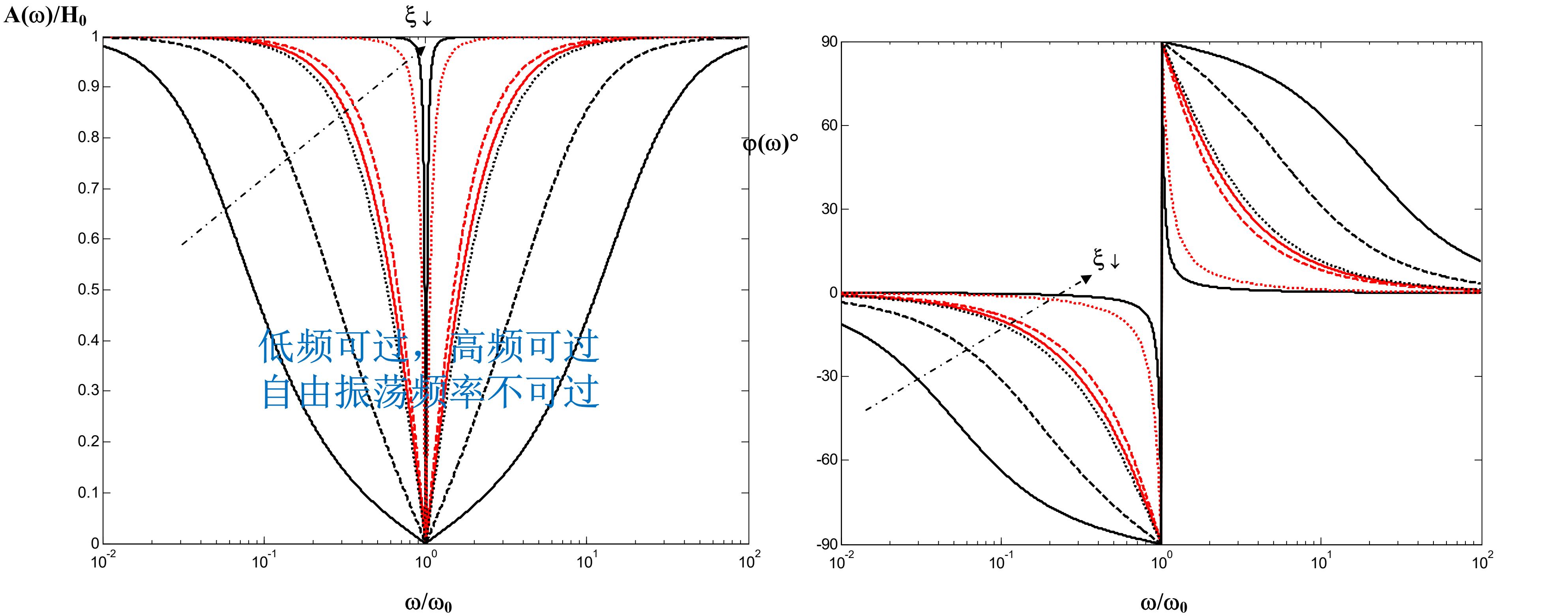
低频：电容开路，电感短路，信号全过
高频：电容短路，电感开路，信号全过
自由振荡频点：电容电抗和电感电抗抵消，犹如短路，输出为**0**，信号通不过

$$H(j\omega) = \frac{\dot{V}_L + \dot{V}_C}{\dot{V}_S} = H_0 \frac{s^2 + \omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}|s = j\omega$$

二阶带阻滤波器传函的一般形式

$$A(\omega) = H_0 \frac{\left|\omega_0^2 - \omega^2\right|}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\xi\omega_0\omega\right)^2}} \qquad \varphi(\omega) = \begin{cases} -\arctan\dfrac{2\xi\omega_0\omega}{\omega_0^2 - \omega^2} & \omega < \omega_0 \\ \pi - \arctan\dfrac{2\xi\omega_0\omega}{\omega_0^2 - \omega^2} & \omega > \omega_0 \end{cases}$$

幅频特性、相频特性



在谐振频点，信号通不过，传函为**0**，该位置出现相位**180°**跳变

只有传函为**0**的点允许相位出现**180°**跳变，其他位置相位均连续

时域特性：冲激响应

电容电压

$$h_{LP}(t)=\frac{\omega_0}{\sqrt{1-\xi^2}}e^{-\xi\omega_0 t}\sin\left(\sqrt{1-\xi^2}\omega_0 t\right)\cdot U(t)$$

电感电压

$$h_{HP}(t)=\delta(t)+\omega_0 e^{-\xi\omega_0 t}\left[-2\xi\cos\sqrt{1-\xi^2}\,\omega_0 t+\frac{2\xi^2-1}{\sqrt{1-\xi^2}}\sin\left(\sqrt{1-\xi^2}\omega_0 t\right)\right]\cdot U(t)$$

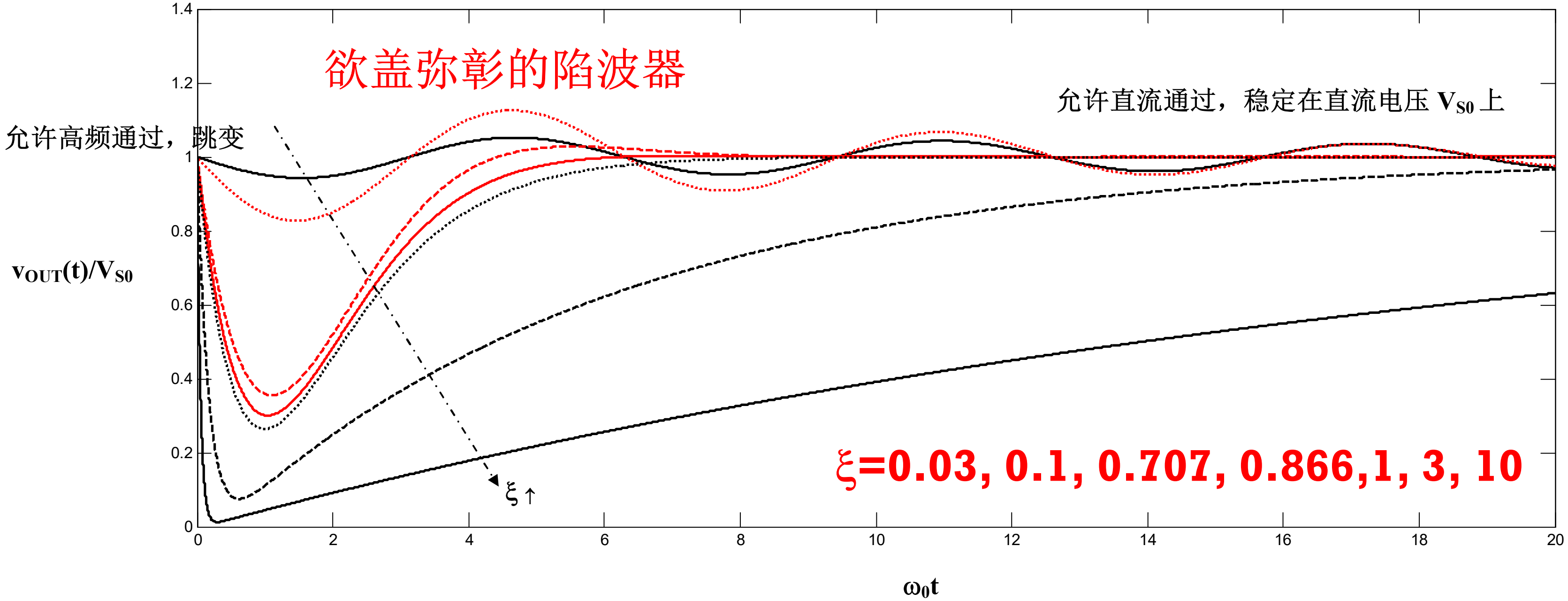
电感电压+电容电压

$$h_{BS}(t)=\delta(t)-2\xi\omega_0 e^{-\xi\omega_0 t}\left[\cos\sqrt{1-\xi^2}\,\omega_0 t-\frac{\xi}{\sqrt{1-\xi^2}}\sin\left(\sqrt{1-\xi^2}\omega_0 t\right)\right]\cdot U(t)$$

$$=\delta(t)-h_{BP}(t)$$

阶跃响应波形

$$g_{BS,2}(t)=U(t)-\frac{2\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_0 t}\sin\sqrt{1-\xi^2}\,\omega_0 t\cdot U(t)$$



以单频正弦波为输入，ω_0频率分量无法通过

本节内容小结

- RLC串联谐振电路，电容分压具有典型的二阶低通特性，电感分压具有典型的二阶高通特性，电阻分压具有典型的二阶带通特性，LC总分压具有典型的二阶带阻特性
- 当阻尼系数$\xi \in [\mathbf{0.707, 1}]$时，二阶低通系统具有接近理想低通系统的系统特性
  - 频域看：幅频特性足够平坦，群延时特性足够平坦
  - 时域看：具有最快的响应速度，最快进入稳态
- 过阻尼系数很大时，串联RLC电路行为犹如一阶RC电路行为
  - 过阻尼系数很大时，并联RLC电路行为犹如一阶RL电路行为
- 欠阻尼系数很小时，振铃十分严重，需要1.5Q个振铃周期后，振铃才会消失
  - 幅度指数衰减为初始值的1%以内

伯特图画法小结

- 1）记jω为s，以s为自变量，重新表述传递函数为实系数有理多项式

$$H(s)=A_0\frac{s^m+\beta_{m-1}s^{m-1}+\ldots+\beta_0}{s^n+\alpha_{n-1}s^{n-1}+\ldots+\alpha_0}$$

- （2）因式分解

$$H(s)=A_0\frac{(s+\omega_{z1})(s+\omega_{z2})\ldots(s+\omega_{zm})}{(s+\omega_{p1})(s+\omega_{p2})\ldots(s+\omega_{pn})}$$

这里假设只有实根
共轭复根暂不考虑

极点和零点

$$H(s)=A_0\frac{(s+\omega_{z1})(s+\omega_{z2})\ldots(s+\omega_{zm})}{(s+\omega_{p1})(s+\omega_{p2})\ldots(s+\omega_{pn})}$$

- 传递函数分母多项式的根称为极点，分子多项式的根称为零点
  - 稳定系统（滤波器，放大器，…）的极点（特征根）一定位于左半平面
  - 零点可正可负，可左可右

$$s_p=-\omega_{p1},-\omega_{p2},\ldots,-\omega_{pn}<0$$

出现右半平面极点（特征根），系统则不稳定，或者趋于无穷（进入非线性饱和区），或者自激振荡（变成振荡器）

不稳定系统没有传递函数，也没有伯特图

$$s_z=-\omega_{z1},-\omega_{z2},\ldots,-\omega_{zm}$$

伯特图画法规则

$$H(s)=A_0\frac{(s+\omega_{z1})(s+\omega_{z2})\ldots(s+\omega_{zm})}{(s+\omega_{p1})(s+\omega_{p2})\ldots(s+\omega_{pn})}$$

- 零极点按大小排序，其数值和频率比，随着频率的上升，⋯

$$H(s)=\mathrm{H}_0\frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\ldots\left(1+\frac{s}{\omega_{zm}}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\ldots\left(1+\frac{s}{\omega_{pn}}\right)}$$

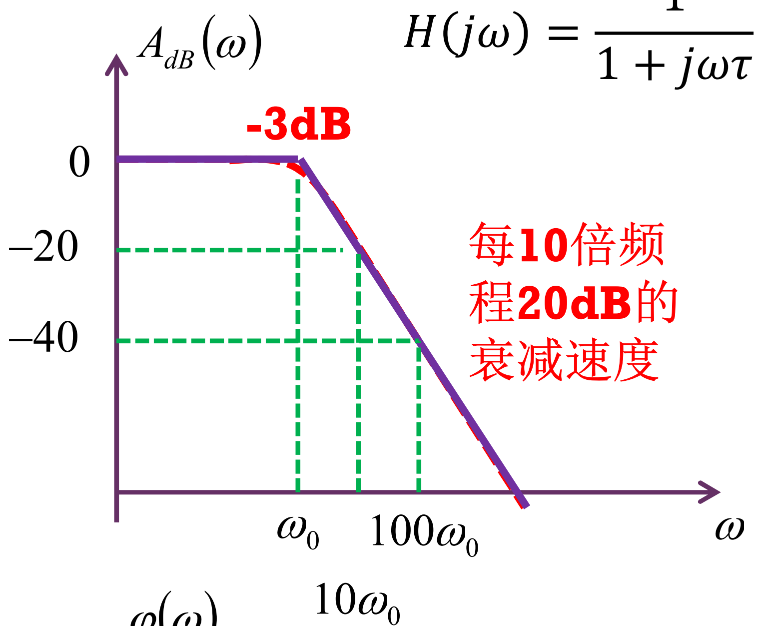
- 幅频特性
  - 碰到极点-20，碰到零点+20；
    - 每个极点都将导致20dB/10倍频程的幅频特性的下降，每个零点都将导致20dB/10倍频程的幅频特性的上升

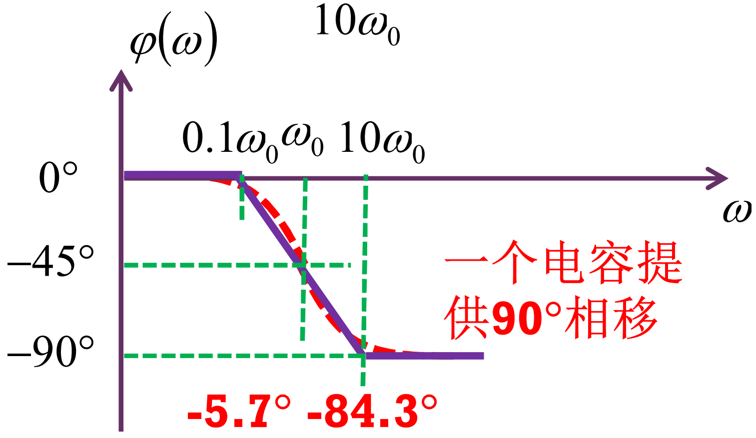
$$\overset{s=j\omega}{=}\mathrm{H}_0\frac{\left(1+\frac{j\omega}{\omega_{z1}}\right)\left(1+\frac{j\omega}{\omega_{z2}}\right)\ldots\left(1+\frac{j\omega}{\omega_{zm}}\right)}{\left(1+\frac{j\omega}{\omega_{p1}}\right)\left(1+\frac{j\omega}{\omega_{p2}}\right)\ldots\left(1+\frac{j\omega}{\omega_{pn}}\right)}$$

- 相频特性
  - 极点滞后90°，零点看左右，左超右滞90°
    - 极点只能是左半平面极点，每个极点将导致一个90°相位滞后；左半平面零点导致一个90°相位超前，右半平面零点导致一个90°相位滞后

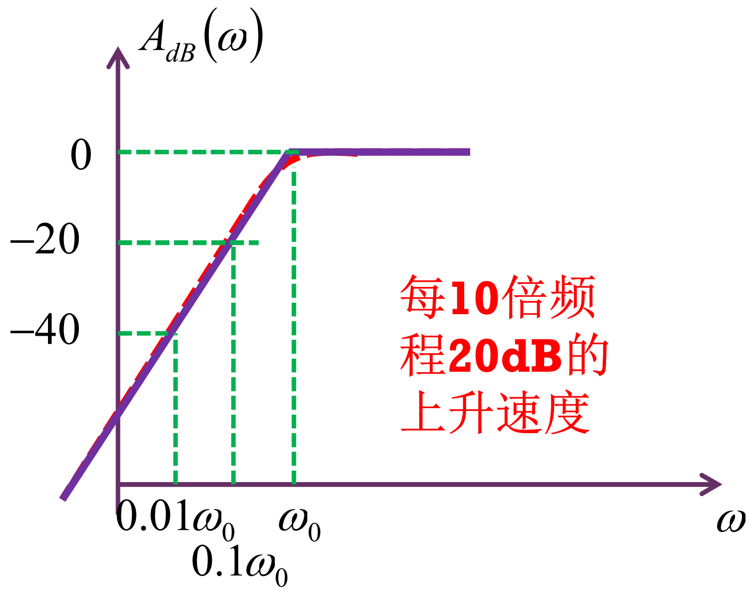
一阶低通和一阶高通

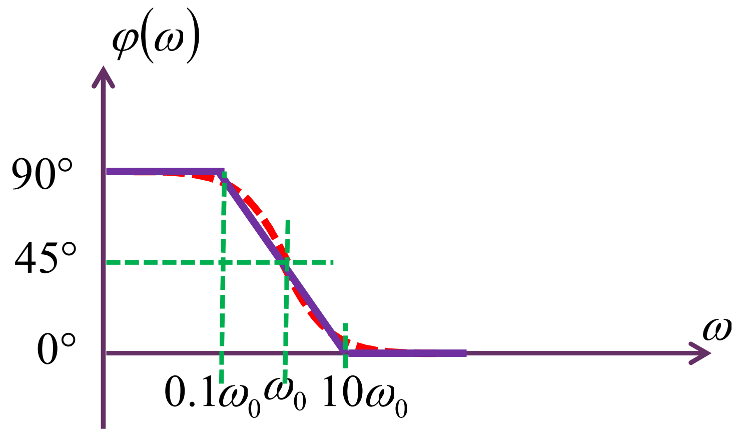
$$H(j\omega)=\frac{1}{1+j\omega\tau}=\frac{\omega_0}{s+\omega_0}$$





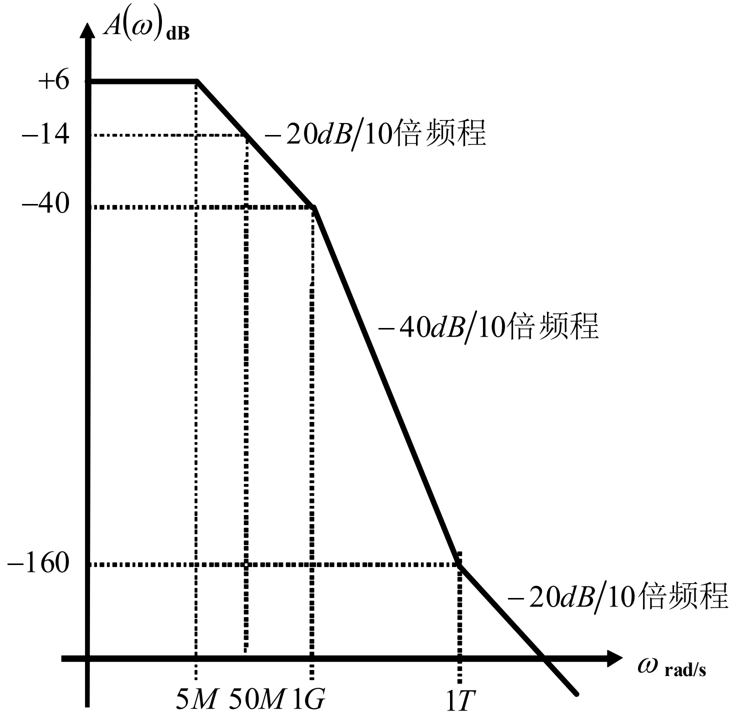
$$H(j\omega)=\frac{j\omega\tau}{1+j\omega\tau}=\frac{s}{s+\omega_0}$$





$$H(j\omega)=10000\frac{\left(j\omega+1\times10^{12}\right)}{\left(j\omega+5\times10^{6}\right)\left(j\omega+1\times10^{9}\right)}$$

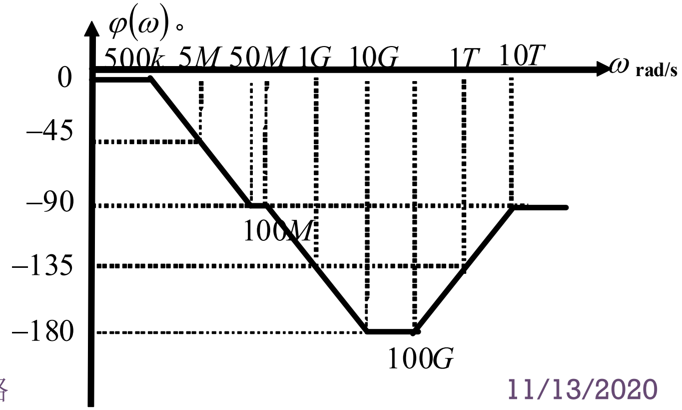
$$=2\frac{\left(1+\dfrac{j\omega}{1\times10^{12}}\right)}{\left(1+\dfrac{j\omega}{5\times10^{6}}\right)\left(1+\dfrac{j\omega}{1\times10^{9}}\right)}$$



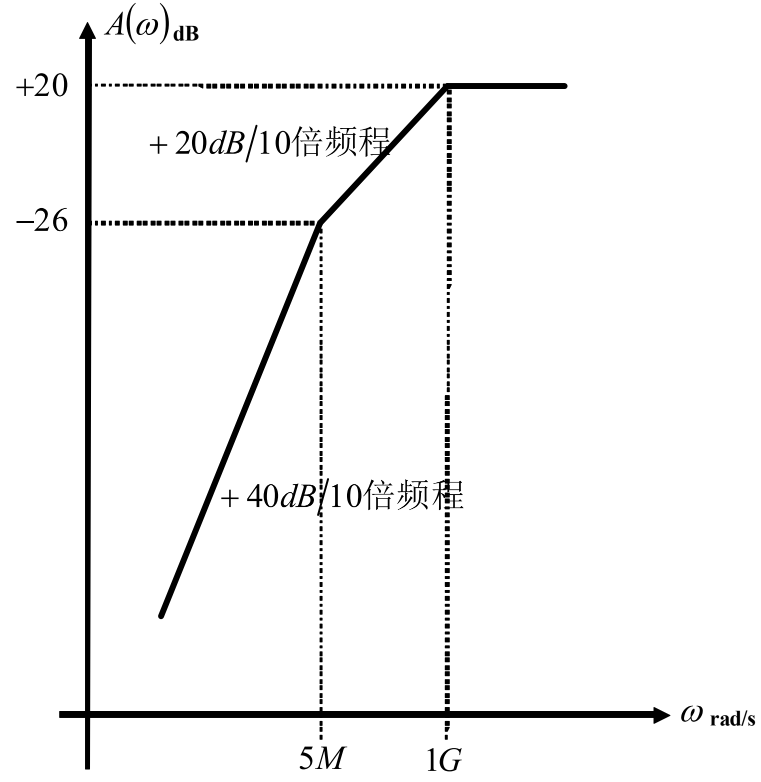
5×10^{6}

1×10^{9}

1×10^{12}



$$H(j\omega)=10\frac{(j\omega)^2}{(j\omega+5\times10^6)(j\omega+1\times10^9)}$$

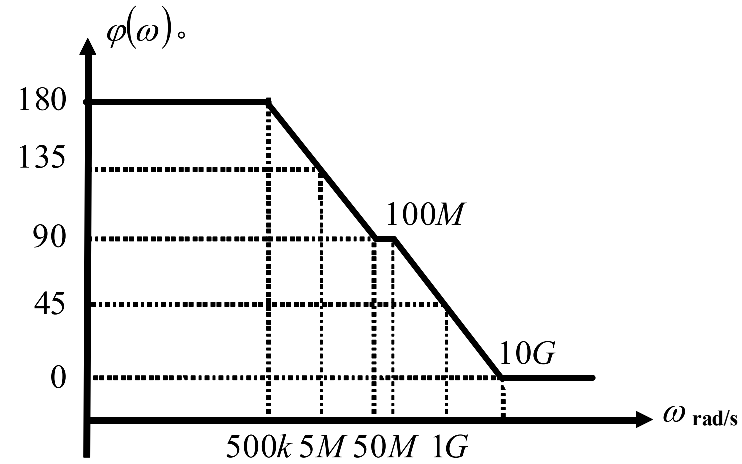


0

0

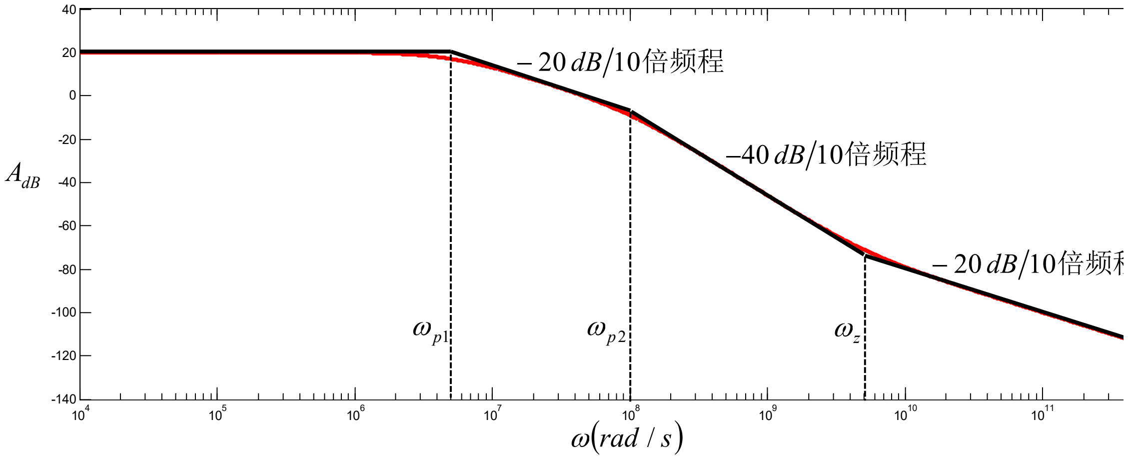
5×10^6

1×10^9

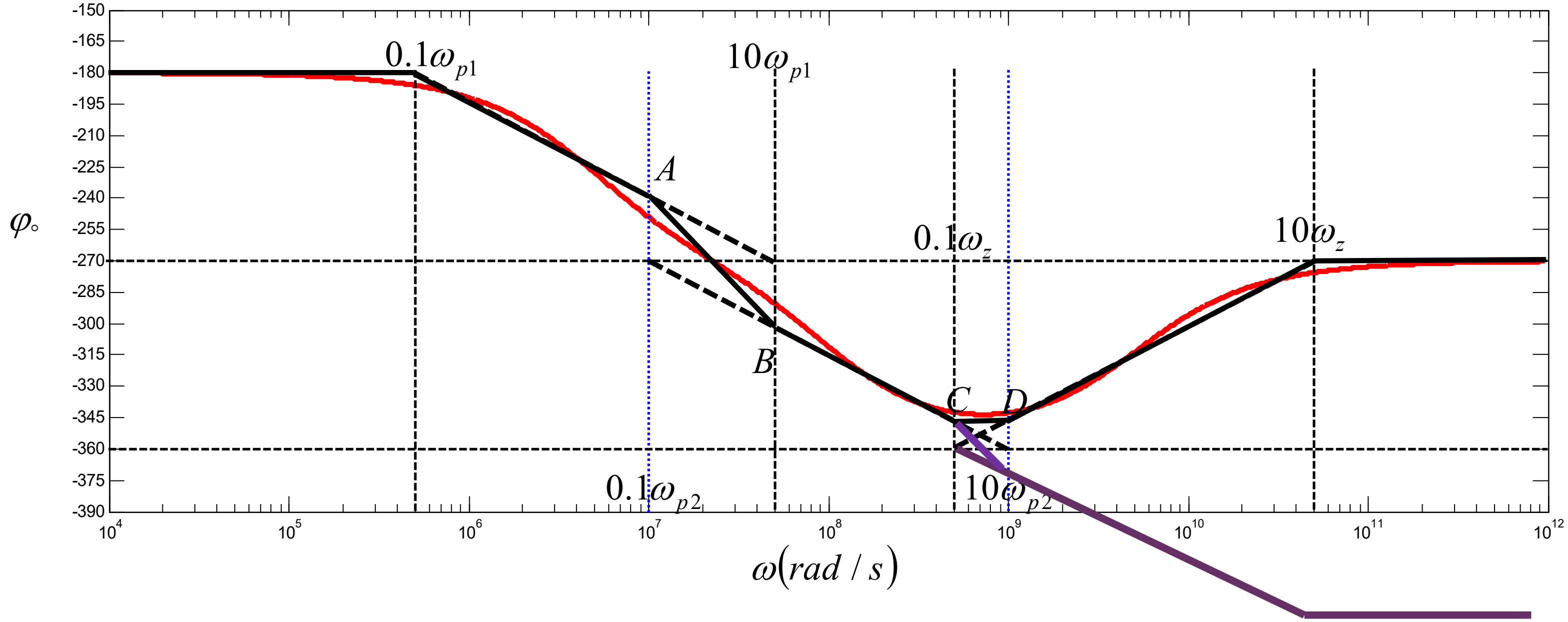


$$H(j\omega)=-10^{6}\frac{j\omega+5\times10^{9}}{(j\omega+5\times10^{6})(j\omega+1\times10^{8})}$$

$$=-10\frac{1+\frac{j\omega}{5\times10^{9}}}{\left(1+\frac{j\omega}{5\times10^{6}}\right)\left(1+\frac{j\omega}{1\times10^{8}}\right)}$$



5×10^{6}　　1×10^{8}　　5×10^{9}

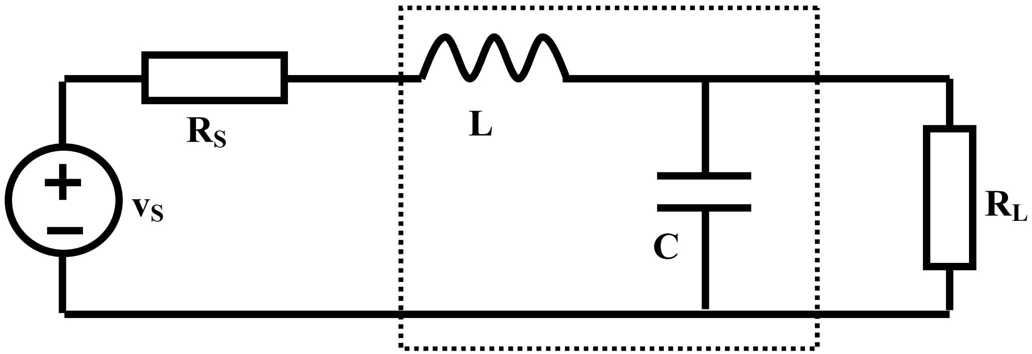


作业1 画伯特图

- 自学P640-648内容，学会画伯特图
  - 伯特图：幅频特性和相频特性的分段折线描述
- 练习8.3.20　画出如下传递函数的伯特图

$$H(j\omega)=-10\frac{1+\frac{j\omega}{5\times10^{9}}}{\left(1+\frac{j\omega}{5\times10^{6}}\right)\left(1+\frac{j\omega}{1\times10^{8}}\right)\left(1+\frac{j\omega}{5\times10^{10}}\right)}=A(\omega)e^{j\varphi(\omega)}$$

作业2 二阶低通滤波器设计



$\xi = \sqrt{3}/2$

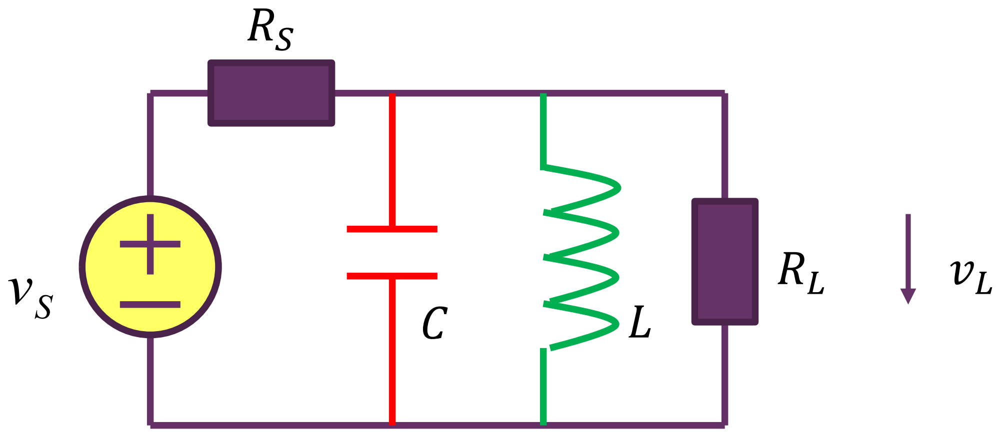
相频特性最接近理想直线，
相位失真最小

$\xi = \sqrt{2}/2$

幅频特性最接近理想平直，
幅度失真最小

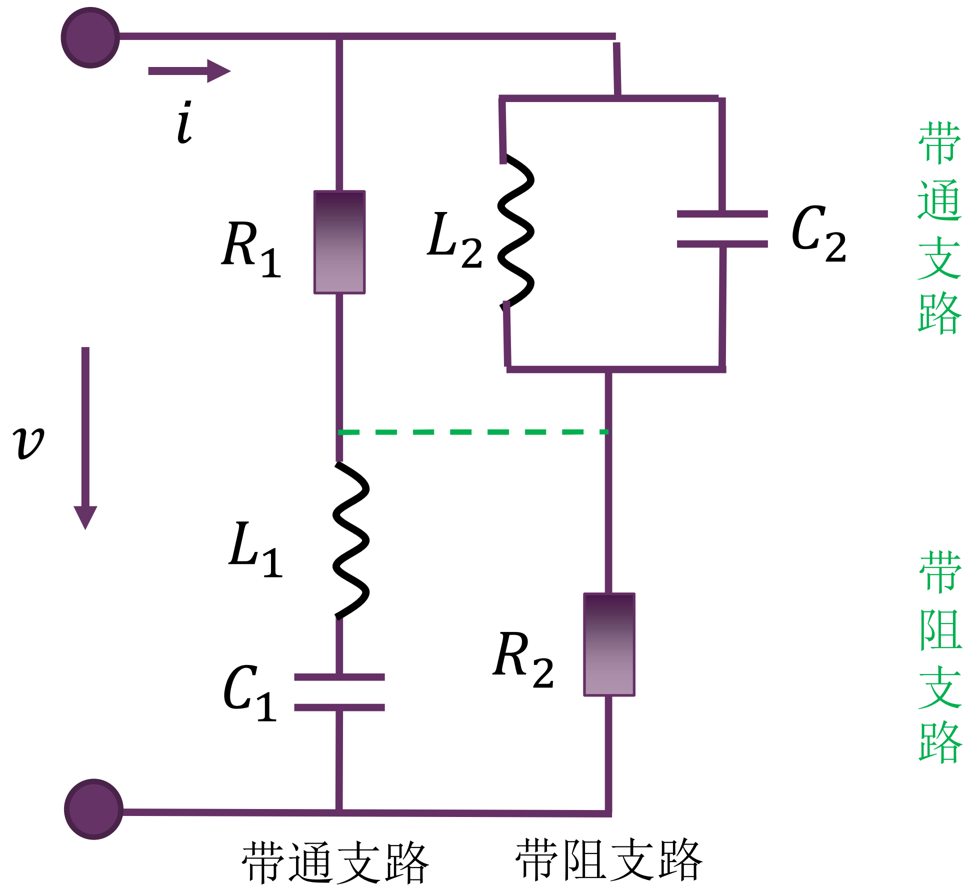
- 如图所示，已知信源内阻为50Ω，负载电阻也是50Ω，请设计一个阻尼系数为0.866($=\sqrt{3}/2$)的二阶低通LC滤波器，其3dB带宽为1MHz，请给出虚框表示的LC低通滤波器中电感和电容的具体数值（选作：仿真确认带宽设计正确）

作业3 带通滤波



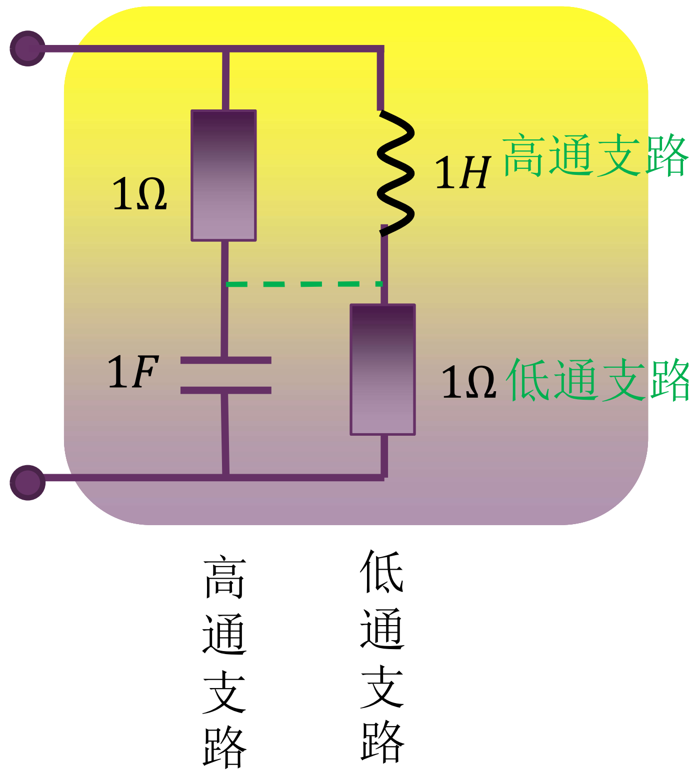
- 频域分析
  - 给出传递函数表达式
  - 说明这是一个典型带通滤波器传递函数，并给出相应的Q值、自由振荡频率$\boldsymbol{\omega_0}$，3dB带宽BW_{3dB}
- 时域分析
  - 用五要素法给出该系统的单位冲激响应和单位阶跃响应

作业4 带通带阻互补为直通



$$R\frac{2\xi\omega_0 s}{s^2+2\xi\omega_0 s+\omega_0^2}+R\frac{s^2+\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}=R$$

二阶带通和二阶带阻互补



低通+高通=直通

$$R\frac{\omega_0}{s+\omega_0}+R\frac{s}{s+\omega_0}=R$$

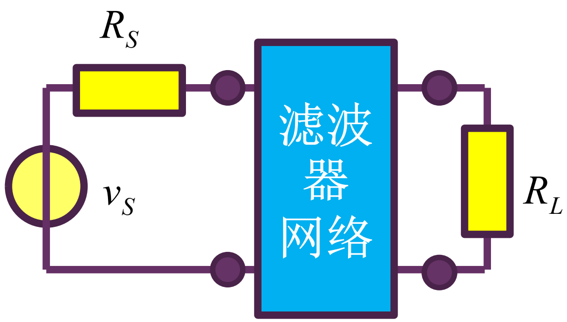
一阶低通和一阶高通互补

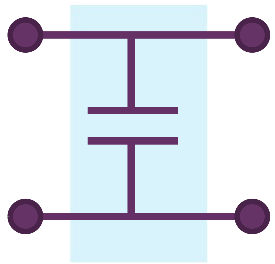
- 求电路中6个元件值满足什么关系时，总端口看入阻抗为纯阻？

作业5 滤波器类型判定

- 电容和电感的记忆能力或者积分效应，导致时域上的延时和频域上的选频特性

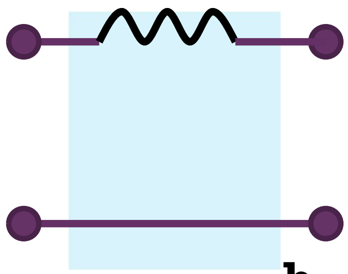
- 常见滤波器分类
  - 低通、高通、带通、带阻
  - 请给出正确的滤波器分类



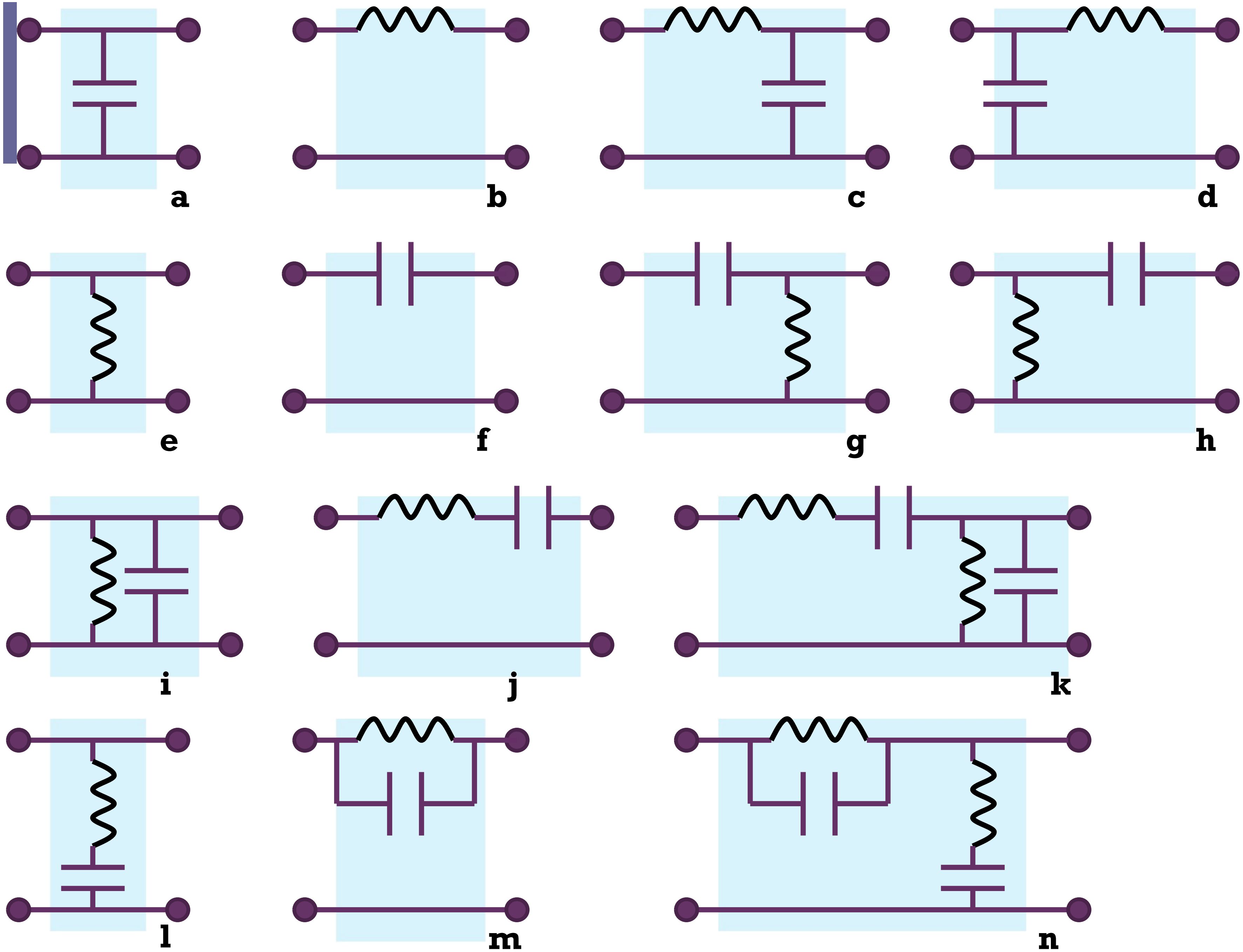


a

一阶低通



b


a
b
c
d
e
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i
j
k
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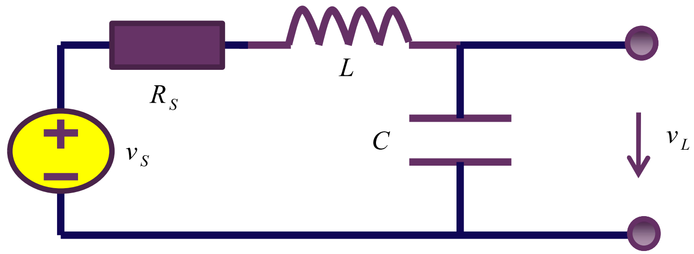
m
n

CAD选作

- 用串联RLC电路，设计$\xi = 0.1$，0.866，10三种情况下的低通滤波器，仿真其阶跃响应，说明$\xi = 0.866$的阶跃响应最优



- 提供matlab代码，供同学运行考察不同阻尼系数滤波器的优劣

```
RS=50;     %信源内阻为50欧姆
f3dB=1E6;  %3dB带宽为1MHz

kesai=[0.1 sqrt(3)/2 10];  %三种阻尼系数情况

Dt=1E-9;
timenum=20000;
f0=f3dB/5;          方波的基波频率
w0=2*pi*f0;
T=1/f0;
for j=1:timenum
   t(j)=(j-1000)*Dt;
   if j<1000
      vs1(j)=0;
      vs2(j)=0;
   else
      vs1(j)=1;         信号1为阶跃信号
      vs2(j)=0.5+2/pi*cos(w0*(t(j)-0.3*T))-2/3/pi*cos(3*w0*(t(j)-
0.3*T))+2/5/pi*cos(5*w0*(t(j)-0.3*T));
                信号2为方波信号取傅立叶展开前4项，设定为数字信号
      vs2n(j)=vs2(j)+0.3*sin(50*w0*t(j)+0.5*randn)+0.6*sin(52*w
0*t(j)+0.5*randn)+0.3*sin(54*w0*t(j)+0.5*randn)+0.5*randn;

                数字信号+噪声：通带外10倍位置
   end          的干扰+随机噪声
end
```

```
figure(1)
hold on
plot(t,vs1,'k')

figure(2)
hold on
plot(t,vs2,'k')
plot(t,vs2n,'b')
```

```
for k=1:3
   %串联RLC取值
   L(k)=RS*sqrt(-2*kesai(k)^2+1+sqrt((1-2*kesai(k)^2)^2+1))/(2*kesai(k)*2*pi*f3dB);
   C(k)=2*kesai(k)*sqrt(-2*kesai(k)^2+1+sqrt((1-2*kesai(k)^2)^2+1))/(RS*2*pi*f3dB);

   RCD=Dt/C(k);
   GLD=Dt/L(k);

   A=[1 -RCD; GLD 1+GLD*RS];
   invA=inv(A);
```

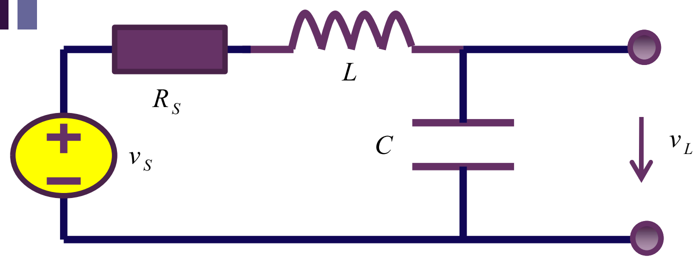
根据**3dB**带宽、阻尼系数、信源内阻计算电容、电感

$$L=\frac{R_S\sqrt{-2\xi^2+1+\sqrt{(-2\xi^2+1)^2+1}}}{2\xi\omega_{3dB}}$$

$$C=\frac{2\xi\sqrt{-2\xi^2+1+\sqrt{(-2\xi^2+1)^2+1}}}{R_S\omega_{3dB}}$$

$$\begin{bmatrix} v_C(t_{k+1}) \\ i_L(t_{k+1}) \end{bmatrix} = \begin{bmatrix} 1 & -R_{C\Delta} \\ G_{L\Delta} & 1+G_{L\Delta}R_S \end{bmatrix}^{-1} \left(\begin{bmatrix} v_C(t_k) \\ i_L(t_k) \end{bmatrix} + \begin{bmatrix} 0 \\ G_{L\Delta} \end{bmatrix} v_s(t_{k+1}) \right)$$

时域特性数值仿真



$$v_s = v_R + v_L + v_C = i_L R_S + L\frac{di_L}{dt} + v_C$$

$$i_L = i_C = C\frac{dv_C}{dt}$$

$$\frac{dv_C}{dt} = \frac{1}{C} i_L$$

$$\frac{di_L}{dt} = \frac{1}{L} v_s - \frac{1}{L} v_C - \frac{R_S}{L} i_L$$

$$\frac{d}{dt}\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_S}{L} \end{bmatrix}\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v_s(t)$$

状态方程

$$\frac{d}{dt}\boldsymbol{x}(t) = \mathbf{f}(\boldsymbol{x}(t), t)$$

$$\begin{bmatrix} v_C(t_{k+1}) \\ i_L(t_{k+1}) \end{bmatrix} - \begin{bmatrix} v_C(t_k) \\ i_L(t_k) \end{bmatrix} \approx \left(\begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_S}{L} \end{bmatrix}\begin{bmatrix} v_C(t_{k+1}) \\ i_L(t_{k+1}) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v_s(t_{k+1}) \right) \Delta t$$

$$\boldsymbol{x}(t_{k+1}) - \boldsymbol{x}(t_k) = \int_{t_k}^{t_{k+1}} \mathbf{f}(\boldsymbol{x}(t), t)dt \approx \mathbf{f}(\boldsymbol{x}(t_{k+1}), t_{k+1})\Delta t$$

(t_k, t_{k+1})区间内积分

积分面积用时间离散化后的矩形面积替代

矩形高度取**k+1**时刻，为后向欧拉法，收敛算法

取**k**时刻则为前向欧拉法，迭代可能不收敛

$$\begin{bmatrix} v_C(t_{k+1}) \\ i_L(t_{k+1}) \end{bmatrix} - \begin{bmatrix} v_C(t_k) \\ i_L(t_k) \end{bmatrix} \approx \begin{bmatrix} 0 & \frac{\Delta t}{C} \\ -\frac{\Delta t}{L} & -\frac{\Delta t R_S}{L} \end{bmatrix}\begin{bmatrix} v_C(t_{k+1}) \\ i_L(t_{k+1}) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\Delta t}{L} \end{bmatrix} v_s(t_{k+1})$$

```
%时域特性1：阶跃响应
  vC(1)=0;
  iL(1)=0;

  x=[vC(1);iL(1)];

  for j=2:timenum
     x=invA*(x+[0; GLD*vs1(j)]);
     vC(j)=x(1);
     iL(j)=x(2);
  end

  figure(1)
  hold on
  plot(t,vC)
```

阶跃激励产生的阶跃响应

```
%时域特性2：噪声滤波
  vC(1)=0;
  iL(1)=0;

  x=[vC(1);iL(1)];

  for j=2:timenum
     x=invA*(x+[0; GLD*vs2n(j)]);
     vC(j)=x(1);
     iL(j)=x(2);
  end

  figure(5+k)
  hold on
  plot(t,vs2,'k')
  plot(t,vC,'b')
```

数字信号带噪声，滤波器应当将噪声滤除

$$\begin{bmatrix} v_C(t_{k+1}) \\ i_L(t_{k+1}) \end{bmatrix} = \begin{bmatrix} 1 & -R_{C\Delta} \\ G_{L\Delta} & 1+G_{L\Delta}R_S \end{bmatrix}^{-1} \left(\begin{bmatrix} v_C(t_k) \\ i_L(t_k) \end{bmatrix} + \begin{bmatrix} 0 \\ G_{L\Delta} \end{bmatrix} v_s(t_{k+1}) \right)$$

```
%频率特性
   freqstart=f3dB/100;
   freqstop=f3dB*10000;
   freqnum=10000;
   freqstep=10^(log10(freqstop/freqstart)/(freqnum-1));
   freq=freqstart/freqstep;
   taog(1)=0;
   for j=1:freqnum
      freq=freq*freqstep;
      f(j)=freq;
      s=i*2*pi*freq;
      ks=0.5*RS*sqrt(C(k)/L(k));
      w0=1/sqrt(L(k)*C(k));
      H=w0^2/(s^2+2*ks*w0*s+w0^2);
      absH(j)=20*log10(abs(H));
      angH(j)=angle(H)/pi*180;
      if j>1
         taog(j)=-(angH(j)-angH(j-1))/(f(j)-f(j-1))/360;
      end
   end
   taog(1)=taog(2);
end
```

```
figure(3)
hold on
plot(f,absH)       %幅频特性

figure(4)
hold on
plot(f,angH)       %相频特性

figure(5)
hold on
plot(f,taog)       %群延时特性
```

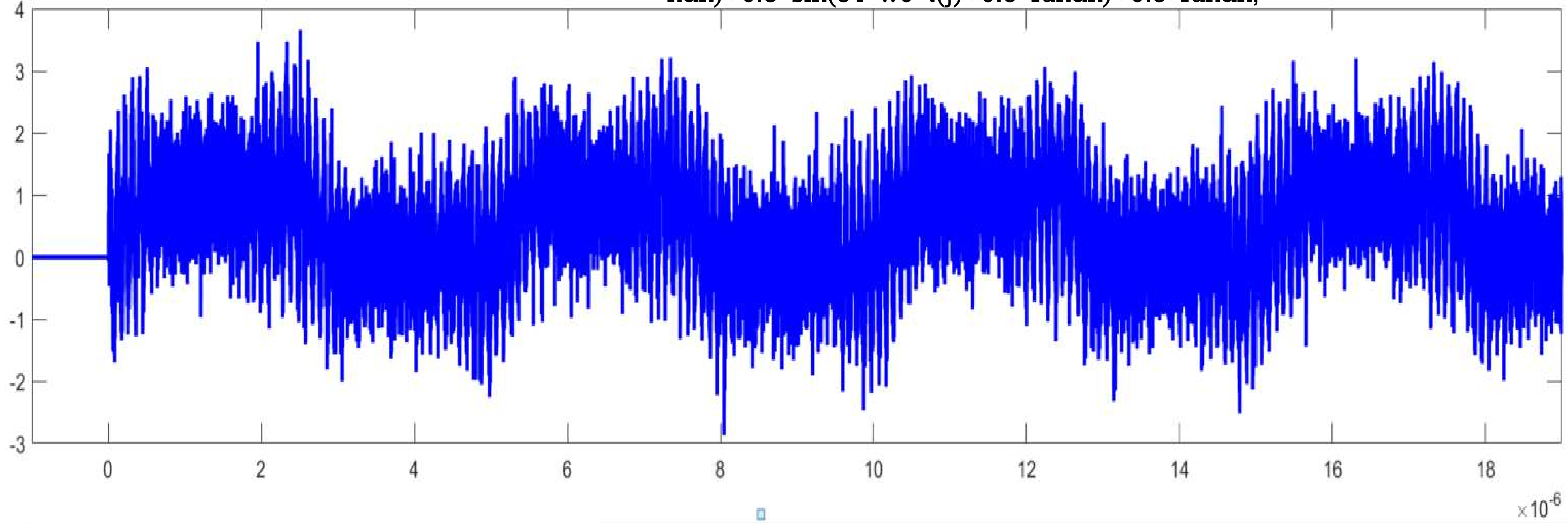
$$H(s)=\frac{\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}$$

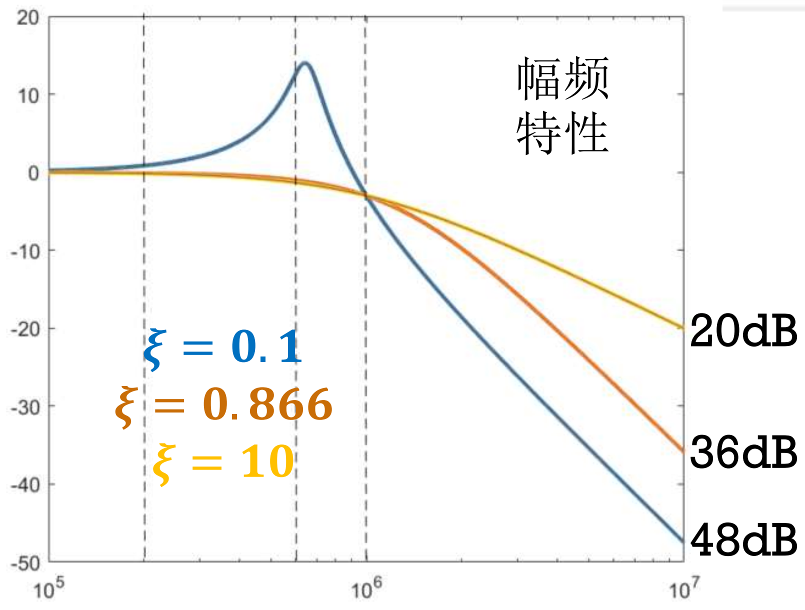
$$\xi=\frac{R_S}{2}\sqrt{\frac{C}{L}}\qquad \omega_0=\frac{1}{\sqrt{LC}}$$

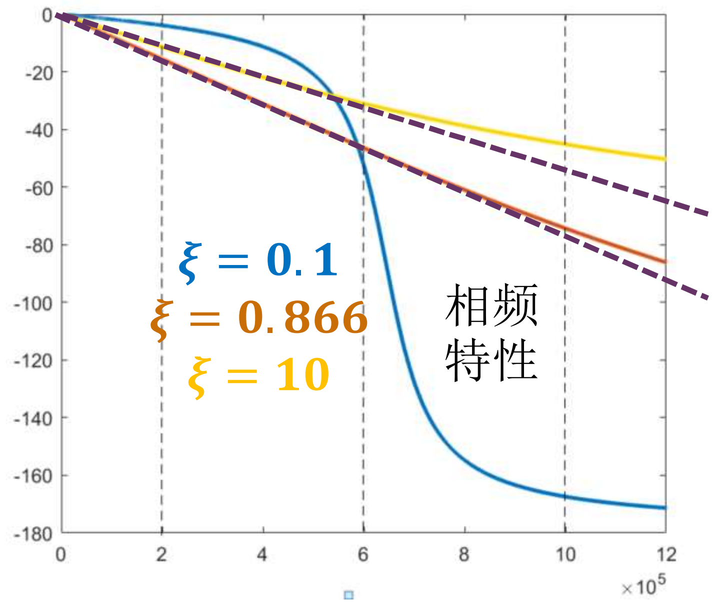
滤波特性

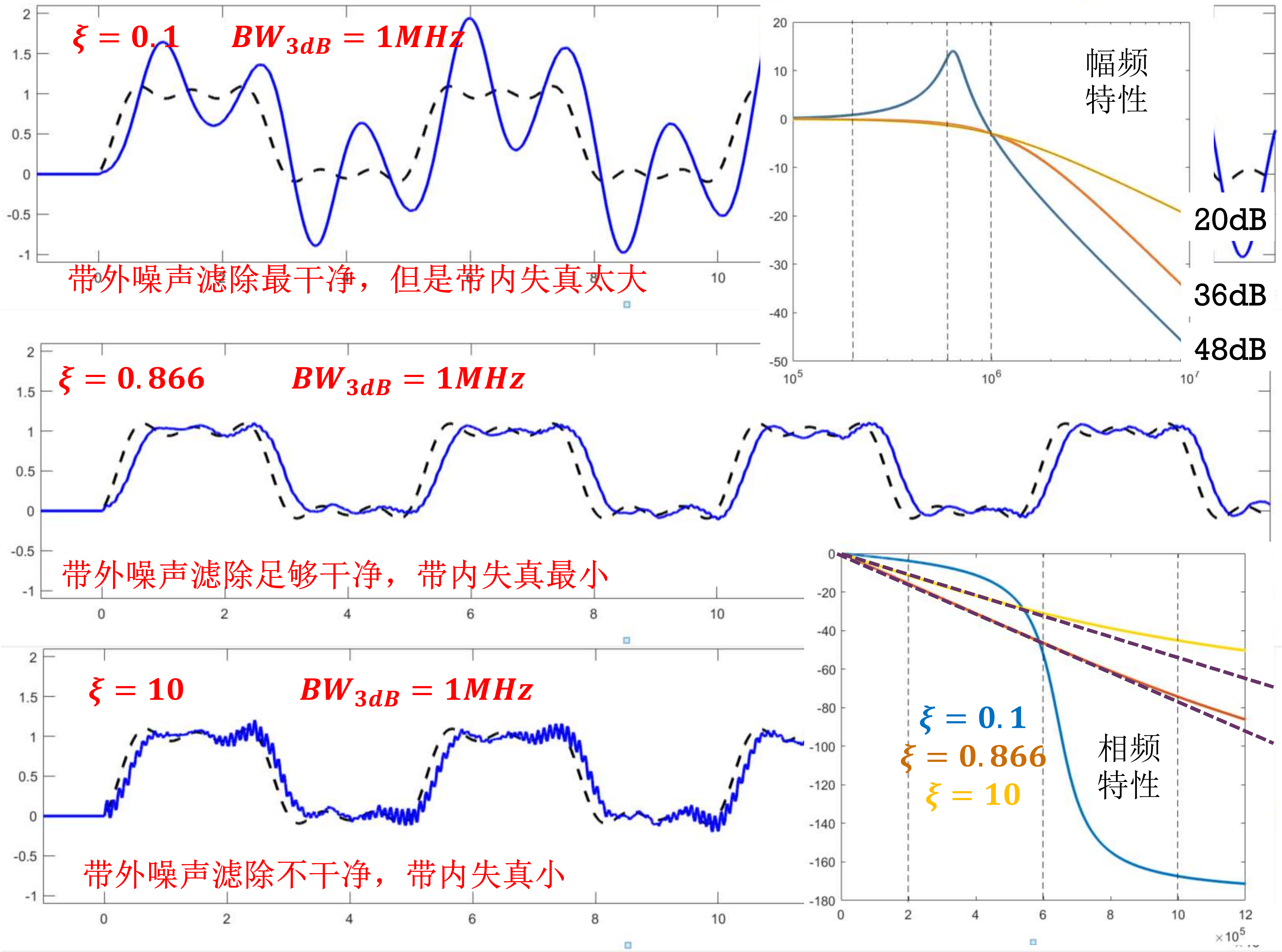
```
vs2(j)=0.5+2/pi*cos(w0*(t(j)-0.3*T))-2/3/pi*cos(3*w0*(t(j)-0.3*T))+2/5/pi*cos(5*w0*(t(j)-0.3*T));

vs2n(j)=vs2(j)+0.3*sin(50*w0*t(j)+0.5*randn)+0.6*sin(52*w0*t(j)+0.5*randn)+0.3*sin(54*w0*t(j)+0.5*randn)+0.5*randn;
```









ξ = 0.1 BW3dB = 1MHz
带外噪声滤除最干净，但是带内失真太大
ξ = 0.866 BW3dB = 1MHz
带外噪声滤除足够干净，带内失真最小
ξ = 10 BW3dB = 1MHz
带外噪声滤除不干净，带内失真小
幅频特性
20dB
36dB
48dB
ξ = 0.1
ξ = 0.866
ξ = 10
相频特性

本节课内容在教材中的章节对应

- P780-807: 二阶滤波器