电子电路与系统基础I

理论课第六讲　等效电路法：线性二端口网络

（二端口网络的加压-加流测量法与戴维南-诺顿定理）

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二端口网络 Two-Port Network

- 二端口网络是电路中最常见的网络
  - 单入单出信号处理系统的基本模型
    - 一个输入端口，一个输出端口：激励信号或能量自输入端口进入，经二端口网络处理后自输出端口输出，形成对后级电路的激励

不做特别说明时，一般默认端口1为输入端口，端口2为输出端口

二端口线性网络等效电路法　　大纲

- 加压求流/加流求压法
  - 从单端口网络等效到二端口网络等效

- 二端口网络参量
  - 阻抗参量**z**
  - 导纳参量**y**
  - 混合参量**h**
  - 逆混参量**g**
  - 传输参量**ABCD**
    - 逆传参量**abcd**

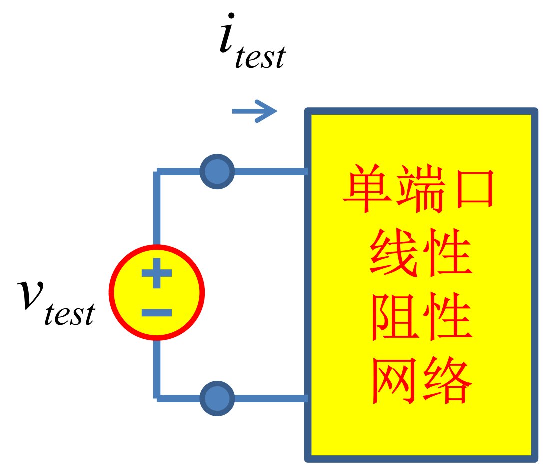
用加压-加流测量方法求线性二端口网络的等效电路

获得二端口线性网络的戴维南-诺顿定理

- 二端口网络连接分析

- 传递函数分析

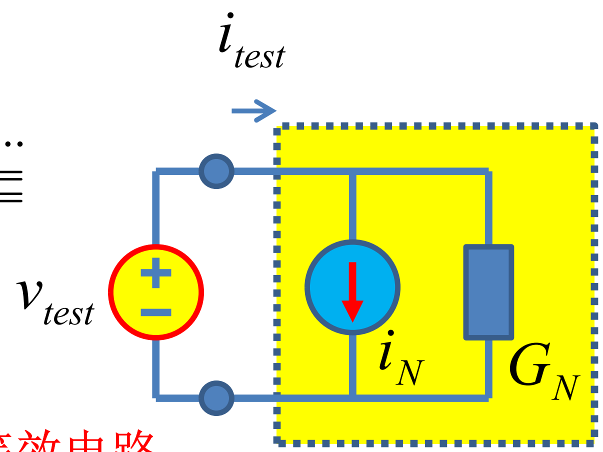
一、单端口网络等效基本方法



加压求流

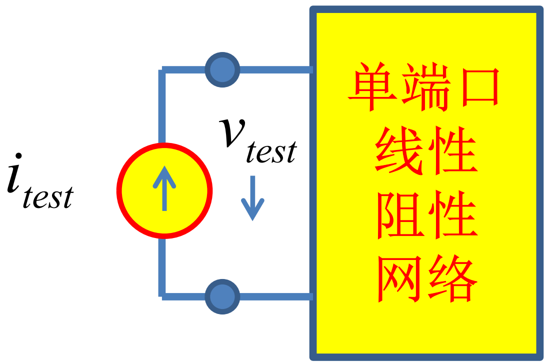
$$i_{test} = \alpha_v \cdot v_{test} + \beta_v \cdot v_{s1} + \gamma_v \cdot i_{s2} + \dots$$

$$= G_N \cdot v_{test} + i_N$$



加压测试获得压控形式的等效电路

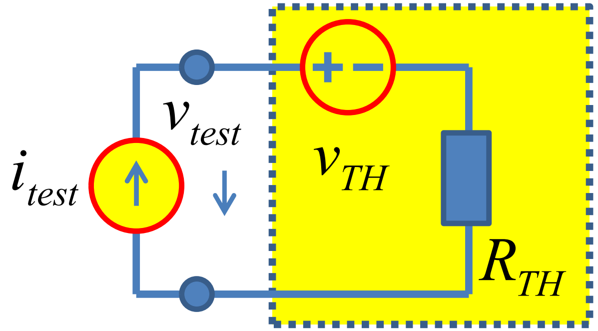
加流测试获得流控形式的等效电路



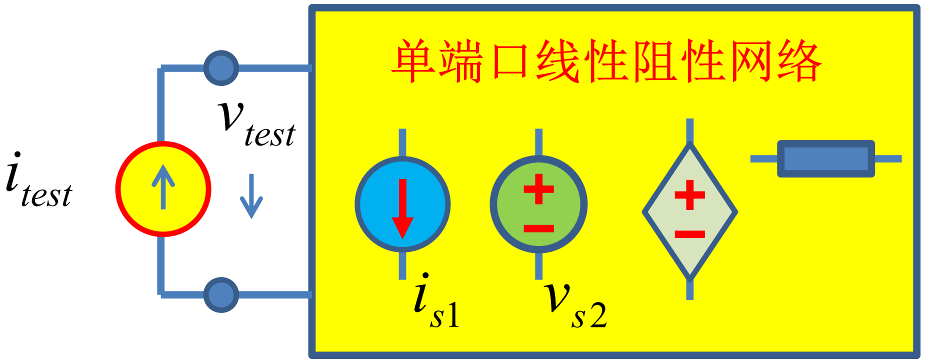
加流求压

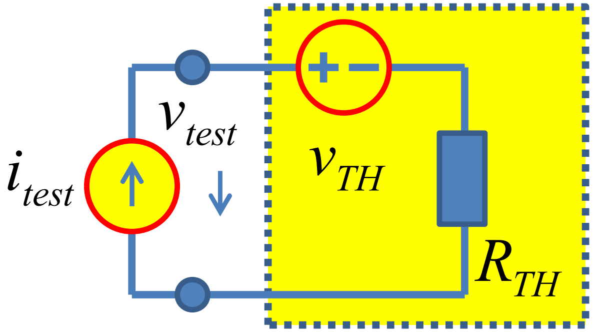
$$v_{test} = \alpha_i \cdot i_{test} + \beta_i \cdot v_{s1} + \gamma_i \cdot i_{s2} + \dots$$

$$= R_{TH} \cdot i_{test} + v_{TH}$$

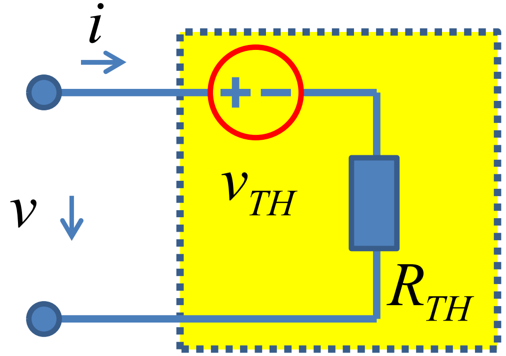


戴维南等效参量的含义





$$v_{test} = \alpha \cdot i_{test} + \lambda_1 \cdot i_{s1} + \lambda_2 \cdot v_{s2} + \ldots$$
$$= R_{TH} \cdot i_{test} + v_{TH}$$



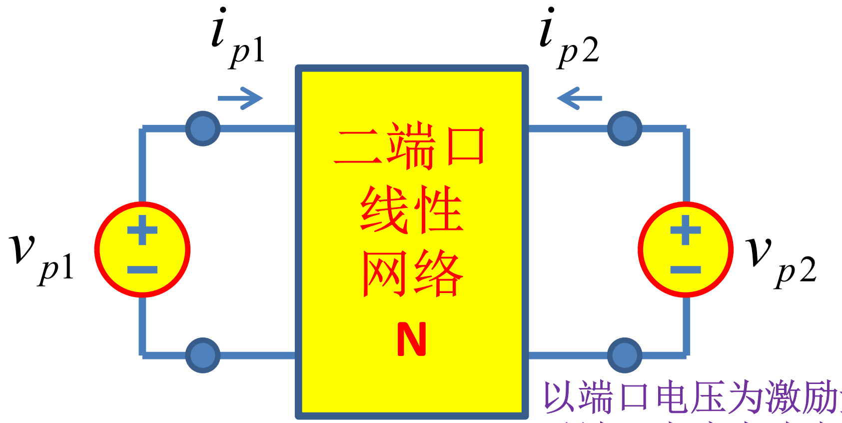
戴维南定理：
（1）端口开路电压v_{TH}为网络内部独立源在端口的表现
（2）R_{TH}为网络内部独立源置零时的输入电阻

$$v_{TH} = v_{test}\Big|_{i_{test}=0}$$

$$R_{TH} = \frac{v_{test}}{i_{test}}\Bigg|_{v_{TH}=0}$$

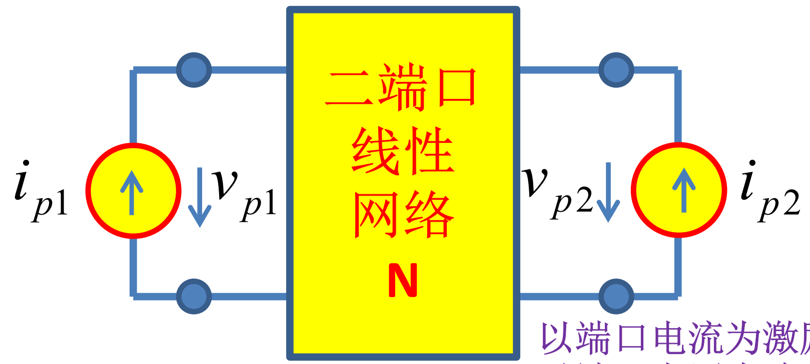
$$v = R_{TH} \cdot i + v_{TH}$$

端口加压、加流方法
对二端口网络的测量：4种基本测量手段



以端口电压为激励量，
以端口电流为响应量

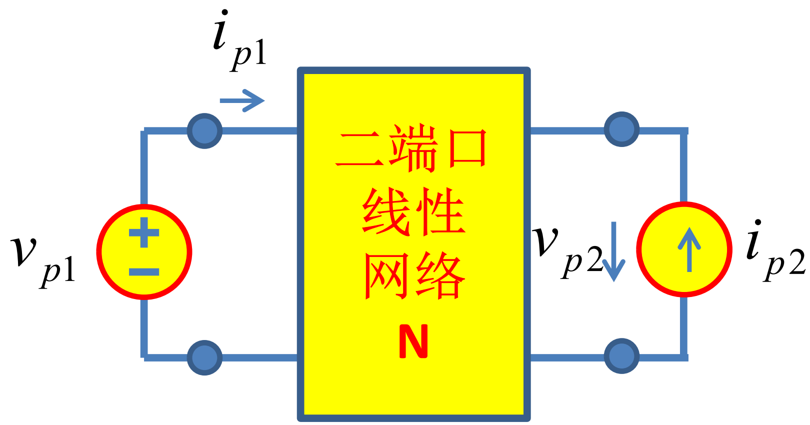
两个端口同时加独立变化的测试电压



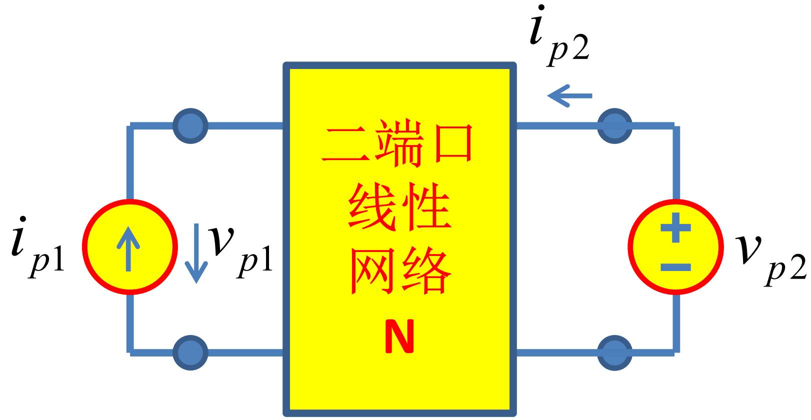
以端口电流为激励量，
以端口电压为响应量

两个端口同时加独立变化的测试电流

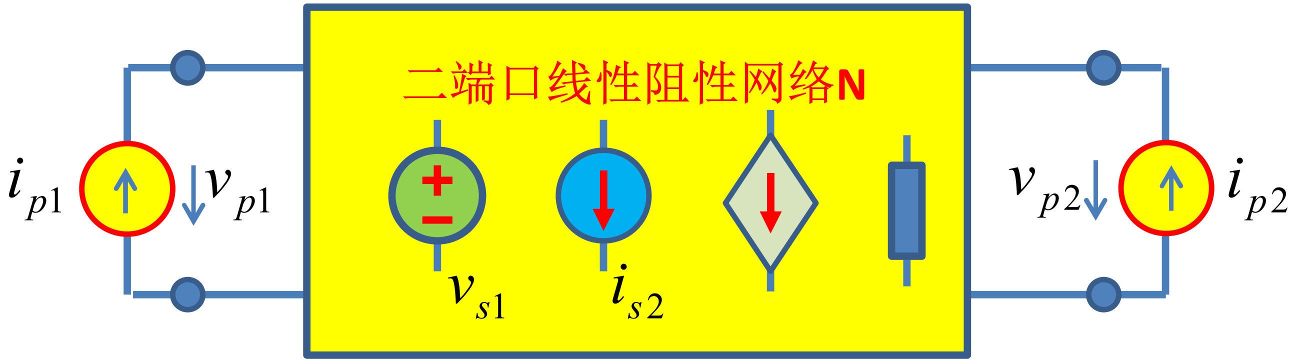
端口1加测试电压同时端口2加测试电流



端口1加测试电流同时端口2加测试电压



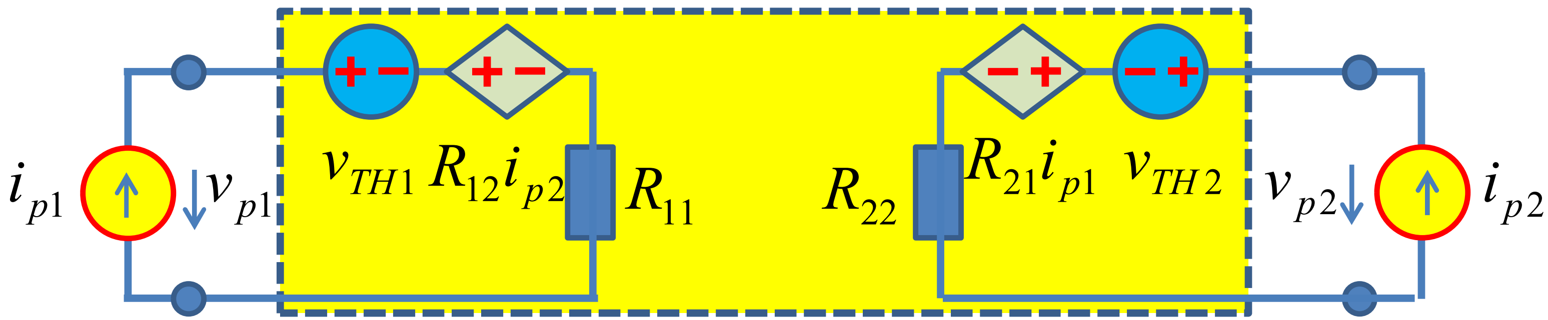
2.1 两个端口同时加流测试



$$v_{p1}=\alpha_{11}i_{p1}+\alpha_{12}i_{p2}+\lambda_{11}v_{s1}+\lambda_{12}i_{s2}+\ldots=R_{11}i_{p1}+R_{12}i_{p2}+v_{TH1}$$

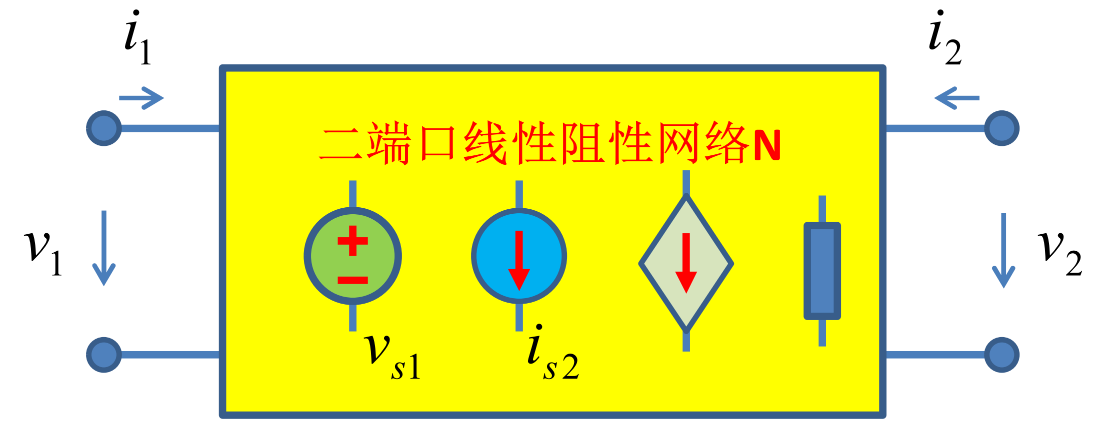
叠加定理

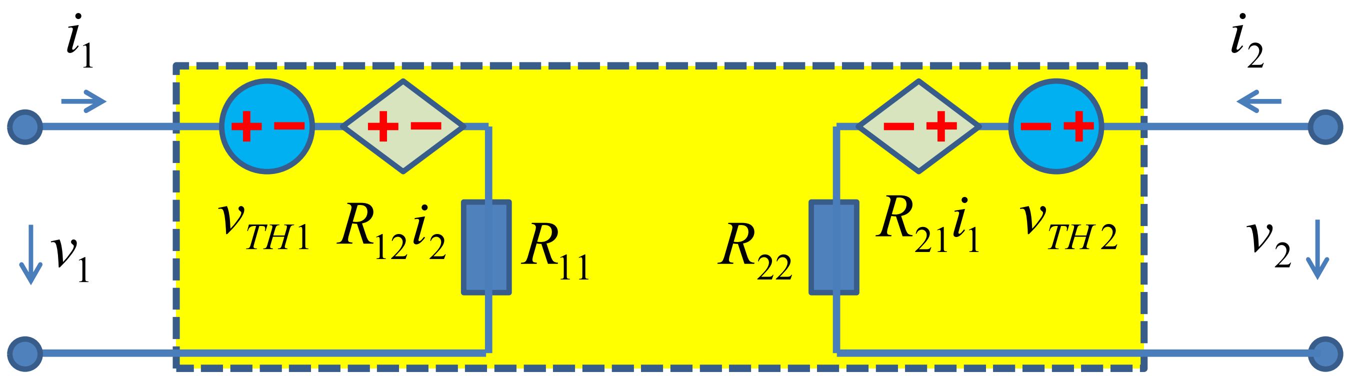
$$v_{p2}=\alpha_{21}i_{p1}+\alpha_{22}i_{p2}+\lambda_{21}v_{s1}+\lambda_{22}i_{s2}+\ldots=R_{21}i_{p1}+R_{22}i_{p2}+v_{TH2}$$



两个端口同时加流测量：阻抗参量，电阻参量

二端口线性网络的戴维南等效电路





$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_{TH1} \\ v_{TH2} \end{bmatrix}$$

阻抗参量：**impedance parameters**

$\mathbf{v} = \mathbf{z} \cdot \mathbf{i} + \mathbf{v}_{TH}$ 二端口网络的戴维南定理

$v = R_{TH} \cdot i + v_{TH}$ 单端口网络的戴维南定理

二端口线性网络的戴维南等效参量

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_{TH1} \\ v_{TH2} \end{bmatrix}$$

$$v_1 = R_{11} i_1 + R_{12} i_2 + v_{TH1}$$
$$v_2 = R_{21} i_1 + R_{22} i_2 + v_{TH2}$$

$$v_{TH1} = v_1 \Big|_{i_1=0, i_2=0}$$ 端口1开路，端口2开路，端口1的开路电压

$$v_{TH2} = v_2 \Big|_{i_1=0, i_2=0}$$ 端口1开路，端口2开路，端口2的开路电压

$$R_{11} = \frac{v_1}{i_1} \Big|_{i_2=0, v_{TH1}=0}$$

内部独立源置零
端口2开路
端口1看入电阻

$$R_{12} = \frac{v_1}{i_2} \Big|_{i_1=0, v_{TH1}=0}$$

内部独立源置零
端口2电流对端口1开路电压的线性跨阻控制系数

输出端口对输入端口的反馈，有时可能起关键作用

$$R_{21} = \frac{v_2}{i_1} \Big|_{i_2=0, v_{TH2}=0}$$

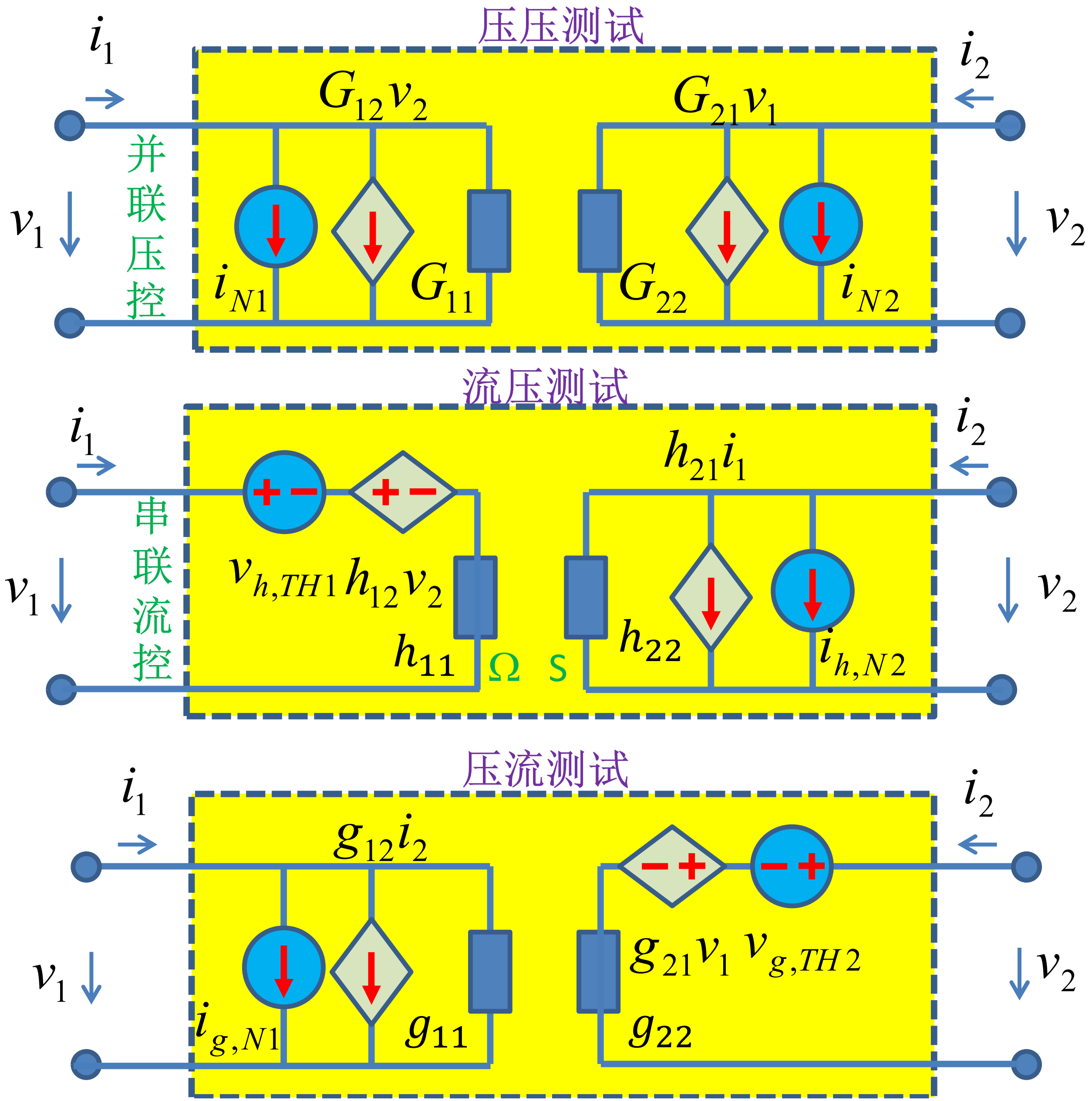
内部独立源置零
端口1电流对端口2开路电压的线性跨阻控制系数

$$R_{22} = \frac{v_2}{i_2} \Big|_{i_1=0, v_{TH2}=0}$$

内部独立源置零
端口1开路
端口2看入电阻

1为输入端口，2为输出端口，最感兴趣的参量：代表了二端口网络处理器的功能：将输入电流转换为输出开路电压：跨阻传递系数

导纳、混合、逆混参量



导纳参量：**admittance parameters**

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} i_{N1} \\ i_{N2} \end{bmatrix}$$

$$\mathbf{i} = \mathbf{y} \cdot \mathbf{v} + \mathbf{i}_N$$

二端口网络的诺顿定理

$$i = G_N \cdot v + i_N$$

单端口网络的诺顿定理

混合参量：**hybrid parameters**

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} v_{h,TH1} \\ i_{h,N2} \end{bmatrix}$$

逆混参量

inverse hybrid parameters

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} i_{g,N1} \\ v_{g,TH2} \end{bmatrix}$$

两端口同时加压、加流测量参量

- 单端口线性网络
  - 加流：流控表述：戴维南等效参量（v_{TH}，R_{TH}）
  - 加压：压控表述：诺顿等效参量（i_N，G_N）

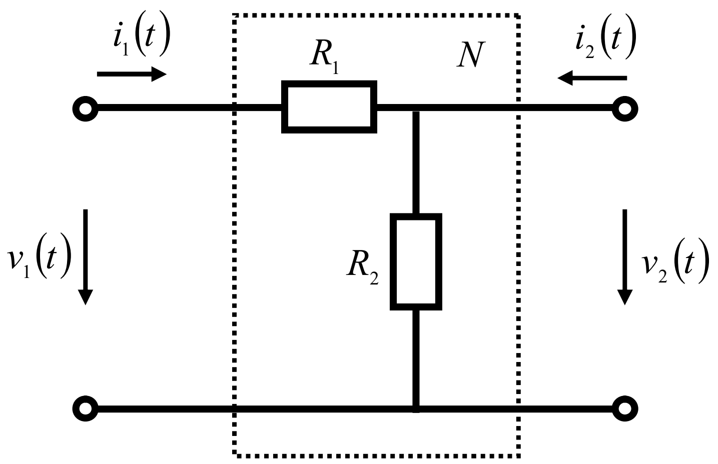
$$R_{TH} = G_N^{-1}$$

- 二端口线性网络
  - 1端口加流同时2端口加流：阻抗参量**z**
  - 1端口加压同时2端口加压：导纳参量**y**
  - 1端口加流同时2端口加压：混合参量**h**
  - 1端口加压同时2端口加流：逆混参量**g**

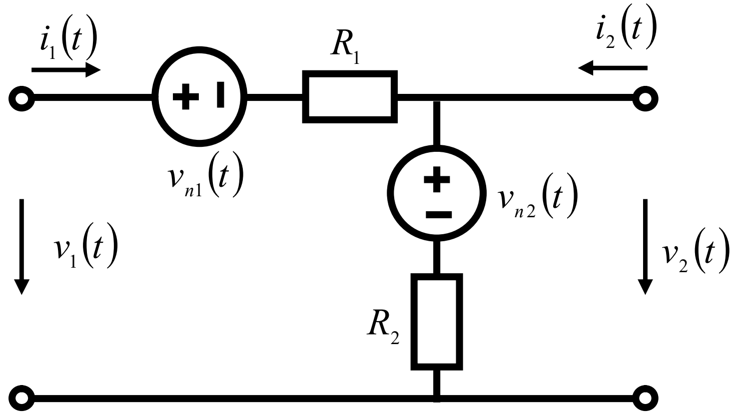
$$\mathbf{z} = \mathbf{y}^{-1}$$

$$\mathbf{h} = \mathbf{g}^{-1}$$

线性二端口网络等效电路例



电阻网络

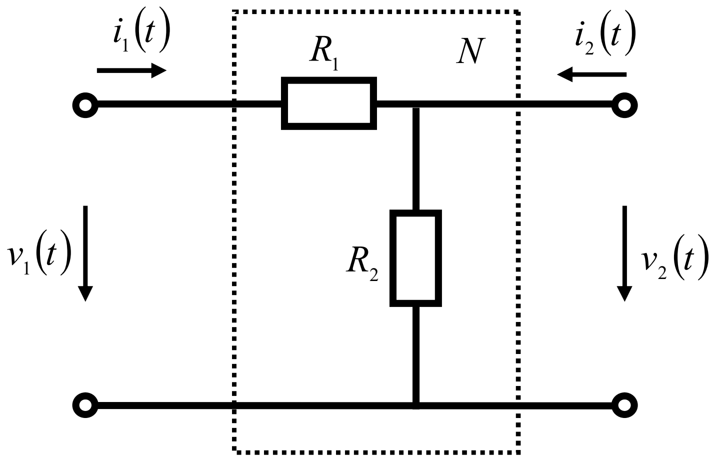


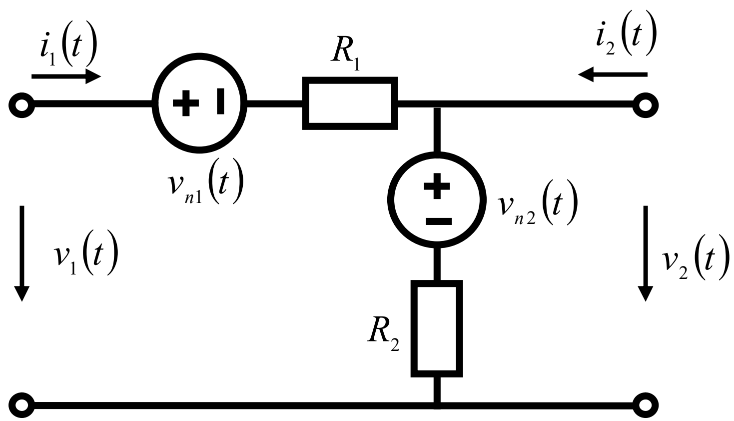
如果处理的信号比较微弱，
则需考虑噪声影响

z参量：戴维南等效：两个端口电流表述两个端口电压，测试电流为0则开路

$$v_{TH1} = v_1\big|_{i_1=0,i_2=0} = v_{n1} + v_{n2} \qquad \overline{v_{n1}^2} = 4kTR_1\Delta f \qquad \overline{v_{TH1}^2} = 4kT(R_1 + R_2)\Delta f$$

$$v_{TH2} = v_2\big|_{i_1=0,i_2=0} = v_{n2} \qquad \overline{v_{n2}^2} = 4kTR_2\Delta f \qquad \overline{v_{TH2}^2} = 4kTR_2\Delta f$$





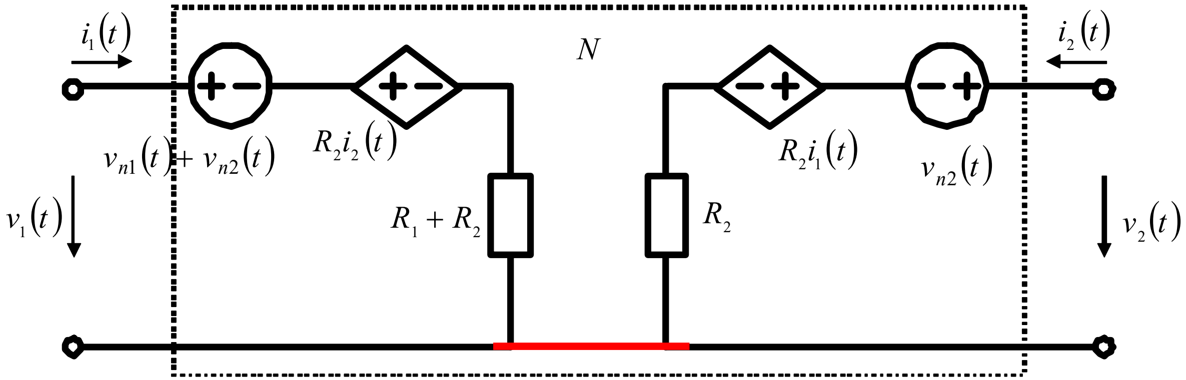
$$R_{11} = \frac{v_1}{i_1}\Big|_{i_2=0, v_{TH1}=0} = R_1 + R_2$$

$$R_{12} = \frac{v_1}{i_2}\Big|_{i_1=0, v_{TH1}=0} = R_2$$

$$R_{21} = \frac{v_2}{i_1}\Big|_{i_2=0, v_{TH2}=0} = R_2$$

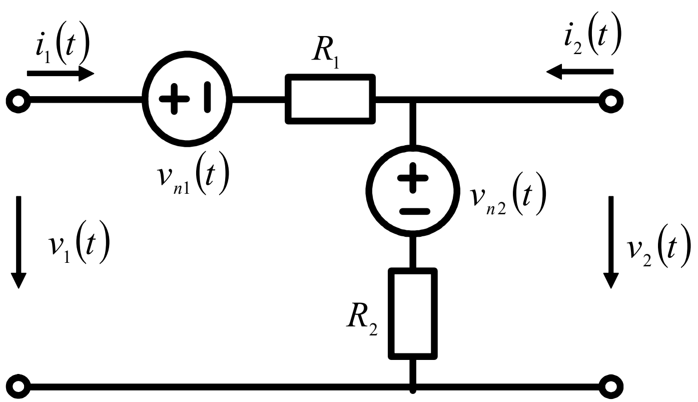
$$R_{22} = \frac{v_2}{i_2}\Big|_{i_1=0, v_{TH2}=0} = R_2$$

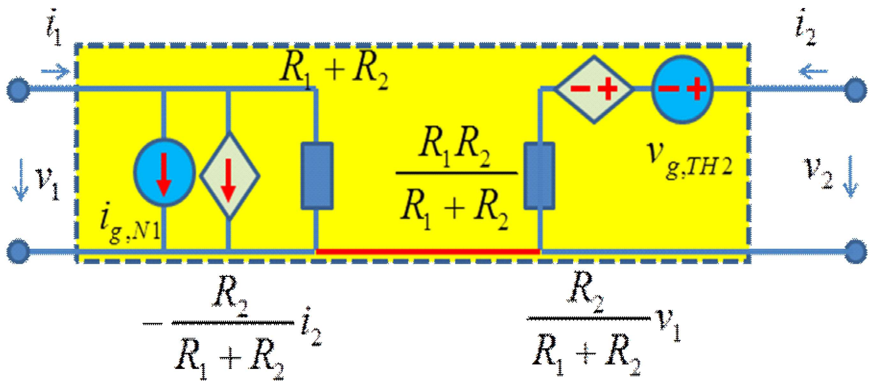
端口1等效噪声电压为端口1输入电阻产生的热噪声电压



端口2等效噪声电压为端口2输出电阻产生的热噪声电压：无源网络的热噪声分析均有此结论

诺顿戴维南等效物理意义更明确





端口1为输出则为对端口2电流的分流：分流器

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1+R_2} & -\frac{R_2}{R_1+R_2} \\ \frac{R_2}{R_1+R_2} & \frac{R_1R_2}{R_1+R_2} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} -\frac{v_{n1}+v_{n2}}{R_1+R_2} \\ \frac{R_1}{R_1+R_2}v_{n2} - \frac{R_2}{R_1+R_2}v_{n1} \end{bmatrix}$$

端口2为输出则为对端口1电压的分压：分压器

端口1等效噪声电流为端口1输入电导产生热噪声电流

$$\overline{i_{n,g,N}^{\,2}} = \overline{\left(-\frac{v_{n1}+v_{n2}}{R_1+R_2}\right)^2} = \frac{\overline{v_{n1}^2}+\overline{v_{n2}^2}}{(R_1+R_2)^2} = \frac{4kTR_1\Delta f + 4kTR_2\Delta f}{(R_1+R_2)^2} = 4kT\frac{1}{R_1+R_2}\Delta f = 4kTG_{in}\Delta f$$

端口2等效噪声电压为端口2看入电阻产生的热噪声电压

$$\overline{v_{n,g,TH}^{\,2}} = \overline{\left(\frac{R_1}{R_1+R_2}v_{n2} - \frac{R_2}{R_1+R_2}v_{n1}\right)^2} = \left(\frac{R_1}{R_1+R_2}\right)^2 4kTR_2\Delta f + \left(\frac{R_2}{R_1+R_2}\right)^2 4kTR_1\Delta f = 4kT\frac{R_1R_2}{R_1+R_2}\Delta f = 4kTR_{out}\Delta f$$

假设网络内部无独立源可画出等效电路的四个网络参量

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

跨阻增益

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

跨导增益

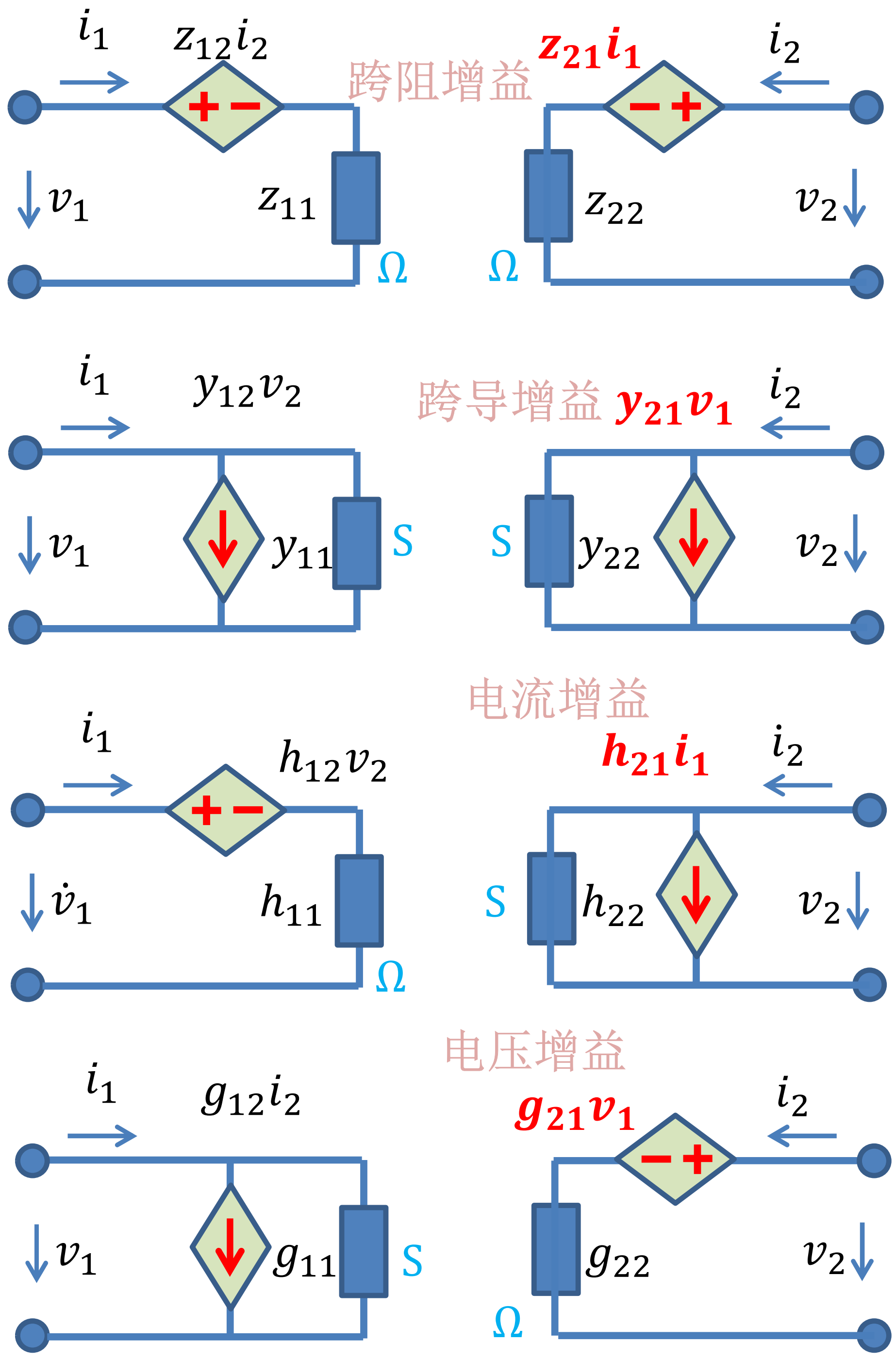
$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

电流增益

记忆定义

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

电压增益



网络参量的6种表述方式

- 由于端口电压、端口电流可以任取两个作为自变量，剩下两个作为因变量，因而线性二端口网络有6种表述，**z**、**y**、**g**、**h**只是其中端口同时加压、加流获得的测量参量，还有两种不是端口同时加压、加流测量参量，而是一个端口加压加流获得的传输参量

v_1

v_2

i_1

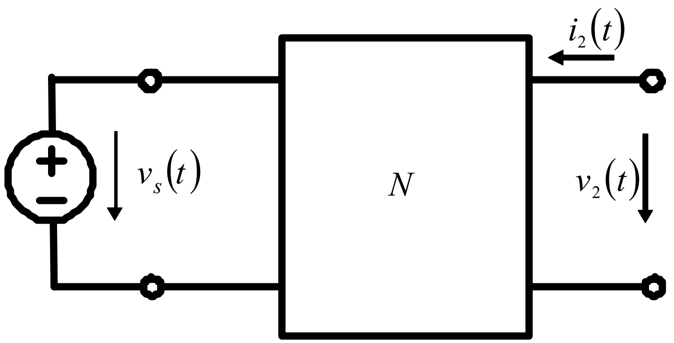
i_2

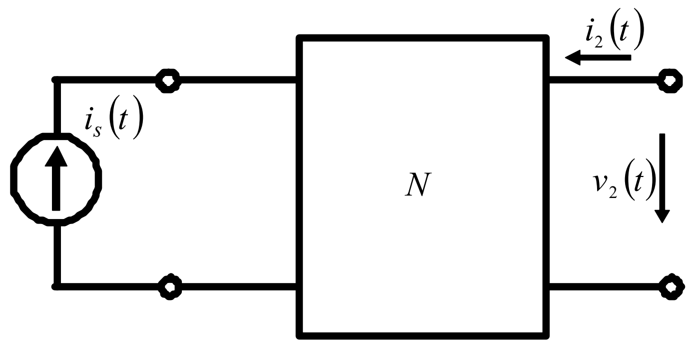
$$C_4^2 = 6$$

因变量	自变量	二端口网络参量
v_1,v_2	i_1,i_2	**z**
i_1,i_2	v_1,v_2	**y**
v_1,i_2	i_1,v_2	**h**
i_1,v_2	v_1,i_2	**g**
v_1,i_1	v_2,i_2	**ABCD T**
v_2,i_2	v_1,i_1	**abcd t**

2.5 传输参量：单端加压加流测试

先假设内部独立源都置零：内部独立电压源短路，内部独立电流源开路





端口1加测试电压
对端口2进行测量

端口1加测试电流
对端口2进行测量

端口2开路电压：电压传递系数

$$g_{21}=\frac{v_2}{v_1}\bigg|_{i_2=0,\text{内部独立源置零}}$$

端口2短路电流：跨导传递系数

$$y_{21}=\frac{i_2}{v_1}\bigg|_{v_2=0,\text{内部独立源置零}}$$

端口2开路电压：跨阻 传递系数

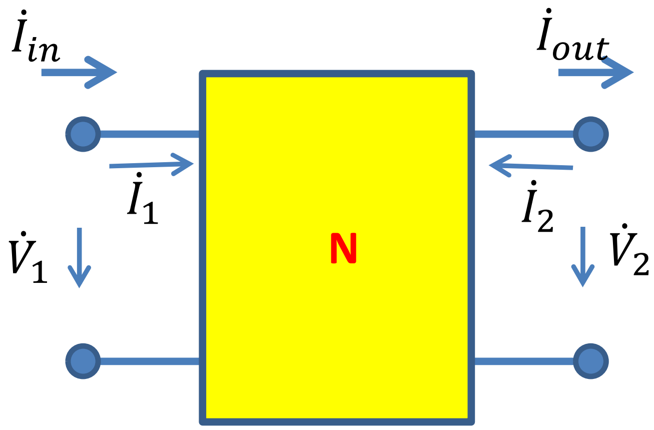
$$z_{21}=\frac{v_2}{i_1}\bigg|_{i_2=0,\text{内部独立源置零}}$$

端口2短路电流：电流传递系数

$$h_{21}=\frac{i_2}{i_1}\bigg|_{v_2=0,\text{内部独立源置零}}$$

传输参量

内部独立源已置零



$$A_{v0} = \left.\frac{v_{out}}{v_{in}}\right|_{i_{out}=0} = g_{21}$$

本征电压增益

$$G_{m0} = \left.\frac{i_{out}}{v_{in}}\right|_{v_{out}=0} = -y_{21}$$

本征跨导增益

$$R_{m0} = \left.\frac{v_{out}}{i_{in}}\right|_{i_{out}=0} = z_{21}$$

本征跨阻增益

$$A_{i0} = \left.\frac{i_{out}}{i_{in}}\right|_{v_{out}=0} = -h_{21}$$

本征电流增益

$$v_1 = Av_2 - Bi_2$$

$$A = \left.\frac{v_1}{v_2}\right|_{i_2=0} = \left.\frac{1}{\frac{v_2}{v_1}}\right|_{i_2=0} = \frac{1}{g_{21}}$$

端口2已经开路，激励源只能在端口1

$$B = \left.\frac{v_1}{-i_2}\right|_{v_2=0} = \left.\frac{1}{-\frac{i_2}{v_1}}\right|_{v_2=0} = \frac{1}{-y_{21}}$$

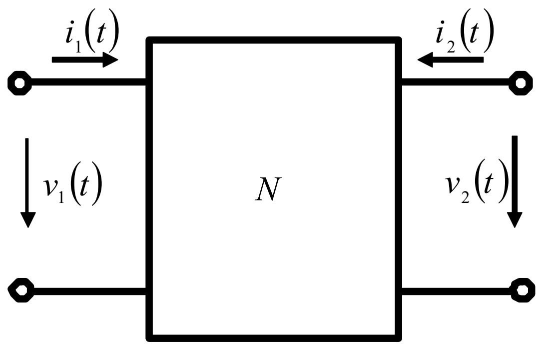
端口2已经短路，激励源只能在端口1

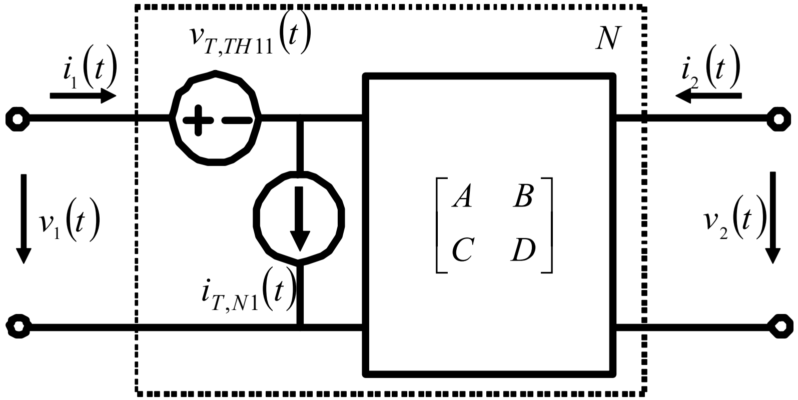
$$\begin{bmatrix} v_{in} \\ i_{in} \end{bmatrix} = \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{g_{21}} & \frac{1}{-y_{21}} \\ \frac{1}{z_{21}} & \frac{1}{-h_{21}} \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{A_{v0}} & \frac{1}{G_{m0}} \\ \frac{1}{R_{m0}} & \frac{1}{A_{i0}} \end{bmatrix} \begin{bmatrix} v_{out} \\ i_{out} \end{bmatrix}$$

形式上是用v_2，i_2表述v_1，i_1，传输参量表述的其实是端口1到端口2的传输系数或本征增益

自变量在端口2，但激励量却是在端口1的测量方式：传输参量测量

传输参量的输入端口源等效





$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} + \begin{bmatrix} v_{T,TH1} \\ i_{T,N1} \end{bmatrix}$$

$$v_1 = Av_2 - Bi_2 + v_{T,TH1}$$

$$i_1 = Cv_2 - Di_2 + i_{T,N1}$$

Transmission Parameters

无法用电路元件描述，但**ABCD**参量包含了该二端口网络的所有端口信息，和**z**、**y**、**h**、**g**可以相互转换

$$v_{T,TH1} = -Av_2\Big|_{v_1=0,i_2=0} = \frac{v_2\Big|_{v_1=0,i_2=0}}{-A_{v0}}$$

端口**1**短路，端口**2**开路电压除以本征电压增益，折合到输入端的等效源电压

$$i_{N1} = -Cv_2\Big|_{i_1=0,i_2=0} = \frac{v_2\Big|_{i_1=0,i_2=0}}{-R_{m0}}$$

端口**1**开路，端口**2**开路电压除以本征跨阻增益，折合到输入端的等效源电流

传输参量
逆传参量

假设内部无独立源

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

$$\begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} v_1 \\ -i_1 \end{bmatrix}$$

abcd参量几乎没用

$$A = \frac{v_1}{v_2}\bigg|_{i_2=0} \qquad B = \frac{v_1}{-i_2}\bigg|_{v_2=0}$$

$$C = \frac{i_1}{v_2}\bigg|_{i_2=0} \qquad D = \frac{i_1}{-i_2}\bigg|_{v_2=0}$$

$$a = \frac{v_2}{v_1}\bigg|_{i_1=0} \qquad b = \frac{v_2}{-i_1}\bigg|_{v_1=0}$$

$$c = \frac{i_2}{v_1}\bigg|_{i_1=0} \qquad d = \frac{i_2}{-i_1}\bigg|_{v_1=0}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} A & -B \\ -C & D \end{bmatrix}^{-1}$$

测出任意一个网络参量，就代表了网络的端口特性，其他网络参量均可知

	z	**y**	**h**	**g**	**ABCD**	**abcd**
z	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\frac{1}{\Delta_y}\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$	$\frac{1}{h_{22}}\begin{bmatrix} \Delta_h & h_{12} \\ -h_{21} & 1 \end{bmatrix}$	$\frac{1}{g_{11}}\begin{bmatrix} 1 & -g_{12} \\ g_{21} & \Delta_g \end{bmatrix}$	$\frac{1}{C}\begin{bmatrix} A & \Delta_T \\ 1 & D \end{bmatrix}$	$\frac{1}{c}\begin{bmatrix} d & 1 \\ \Delta_t & a \end{bmatrix}$
y	$\frac{1}{\Delta_z}\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\frac{1}{h_{11}}\begin{bmatrix} 1 & -h_{12} \\ h_{21} & \Delta_h \end{bmatrix}$	$\frac{1}{g_{22}}\begin{bmatrix} \Delta_g & g_{12} \\ -g_{21} & 1 \end{bmatrix}$	$\frac{1}{B}\begin{bmatrix} D & -\Delta_T \\ -1 & A \end{bmatrix}$	$\frac{1}{b}\begin{bmatrix} a & -1 \\ -\Delta_t & d \end{bmatrix}$
h	$\frac{1}{z_{22}}\begin{bmatrix} \Delta_z & z_{12} \\ -z_{21} & 1 \end{bmatrix}$	$\frac{1}{y_{11}}\begin{bmatrix} 1 & -y_{12} \\ y_{21} & \Delta_y \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\frac{1}{\Delta_g}\begin{bmatrix} g_{22} & -g_{12} \\ -g_{21} & g_{11} \end{bmatrix}$	$\frac{1}{D}\begin{bmatrix} B & \Delta_T \\ -1 & C \end{bmatrix}$	$\frac{1}{a}\begin{bmatrix} b & 1 \\ -\Delta_t & c \end{bmatrix}$
g	$\frac{1}{z_{11}}\begin{bmatrix} 1 & -z_{12} \\ z_{21} & \Delta_z \end{bmatrix}$	$\frac{1}{y_{22}}\begin{bmatrix} \Delta_y & y_{12} \\ -y_{21} & 1 \end{bmatrix}$	$\frac{1}{\Delta_h}\begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}$	$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$	$\frac{1}{A}\begin{bmatrix} C & -\Delta_T \\ 1 & B \end{bmatrix}$	$\frac{1}{d}\begin{bmatrix} c & -1 \\ \Delta_t & b \end{bmatrix}$
ABCD	$\frac{1}{z_{21}}\begin{bmatrix} z_{11} & \Delta_z \\ 1 & z_{22} \end{bmatrix}$	$-\frac{1}{y_{21}}\begin{bmatrix} y_{22} & 1 \\ \Delta_y & y_{11} \end{bmatrix}$	$-\frac{1}{h_{21}}\begin{bmatrix} \Delta_h & h_{11} \\ h_{22} & 1 \end{bmatrix}$	$\frac{1}{g_{21}}\begin{bmatrix} 1 & g_{22} \\ g_{11} & \Delta_g \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\frac{1}{\Delta_t}\begin{bmatrix} d & b \\ c & a \end{bmatrix}$
abcd	$\frac{1}{z_{12}}\begin{bmatrix} z_{22} & \Delta_z \\ 1 & z_{11} \end{bmatrix}$	$-\frac{1}{y_{12}}\begin{bmatrix} y_{11} & 1 \\ \Delta_y & y_{22} \end{bmatrix}$	$\frac{1}{h_{12}}\begin{bmatrix} 1 & h_{11} \\ h_{22} & \Delta_h \end{bmatrix}$	$-\frac{1}{g_{12}}\begin{bmatrix} \Delta_g & g_{22} \\ g_{11} & 1 \end{bmatrix}$	$\frac{1}{\Delta_T}\begin{bmatrix} D & B \\ C & A \end{bmatrix}$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\quad \Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\quad \Delta_T = AD - BC$

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\quad \Delta_g = g_{11}g_{22} - g_{12}g_{21}$ $\quad \Delta_t = ad - bc$

只需记定义，随时可推导

$\boldsymbol{g} = \boldsymbol{h}^{-1}$

$\boldsymbol{h} = \boldsymbol{g}^{-1}$

$\boldsymbol{y} = \boldsymbol{z}^{-1}$

$\boldsymbol{z} = \boldsymbol{y}^{-1}$

$abcd = A\bar{B}\bar{C}D^{-1}$　　$ABCD = a\bar{b}\bar{c}d^{-1}$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

$$i_1 = y_{11}v_1 + y_{12}v_2 \Rightarrow \frac{1}{y_{11}} i_1 = v_1 + \frac{y_{12}}{y_{11}} v_2$$

$$v_1 = \frac{1}{y_{11}} i_1 - \frac{y_{12}}{y_{11}} v_2$$

$$i_2 = y_{21}v_1 + y_{22}v_2$$

$$i_2 = y_{21}\left(\frac{1}{y_{11}} i_1 - \frac{y_{12}}{y_{11}} v_2\right) + y_{22}v_2$$

$$= \frac{y_{21}}{y_{11}} i_1 + \left(y_{22} - \frac{y_{21}y_{12}}{y_{11}}\right) v_2$$

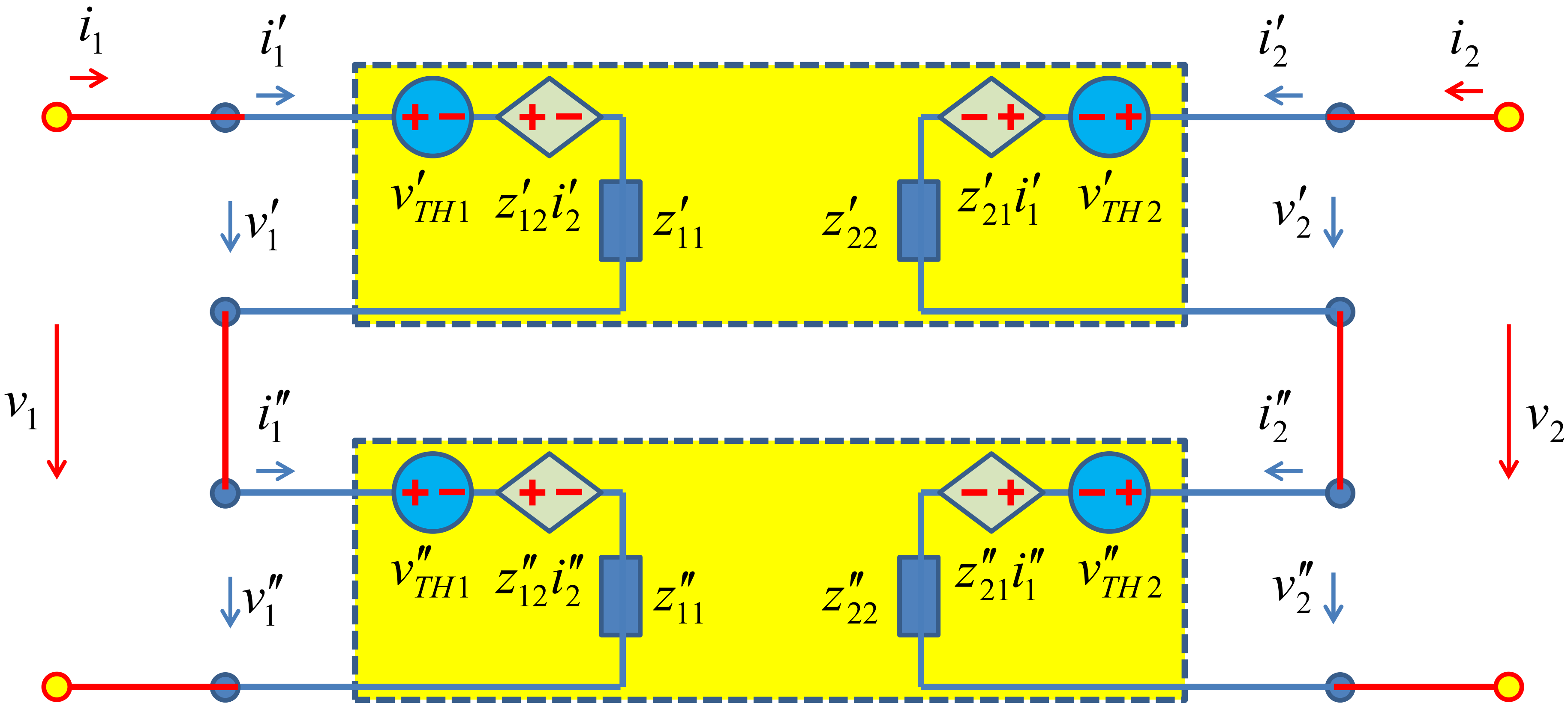
有些网络参量可能不存在

二端口线性网络等效电路法　　大纲

- 加压求流/加流求压法
  - 从单端口网络等效到二端口网络等效
- 二端口网络参量
  - **z、y、h、g、ABCD、abcd**
- 网络连接
  - 串串连接
  - 并并连接
  - 串并连接
  - 并串连接
  - 级联
- 传递函数

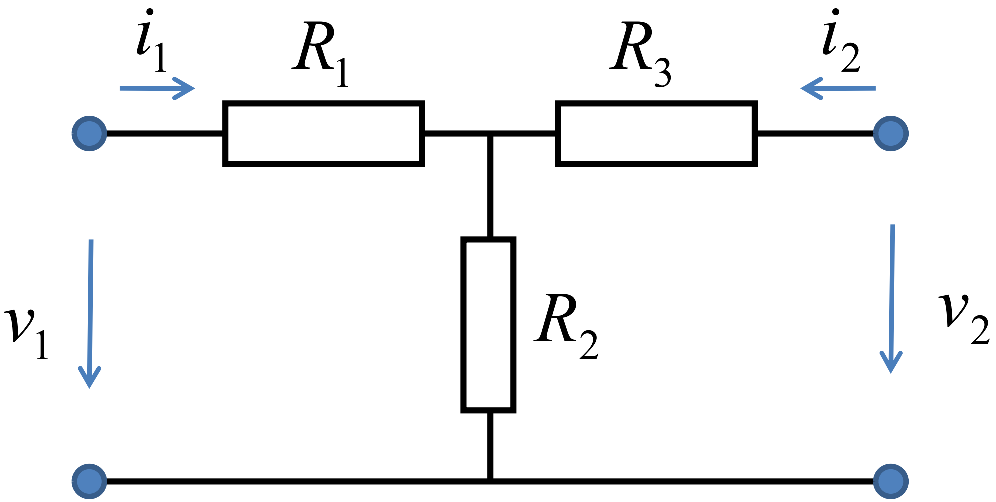
网络串联

Series-series connection



$$\mathbf{v} = \mathbf{v}' + \mathbf{v}'' = \mathbf{z}'\mathbf{i}' + \mathbf{v}_{TH}' + \mathbf{z}''\mathbf{i}'' + \mathbf{v}_{TH}'' = \left(\underline{\underline{\mathbf{z}' + \mathbf{z}''}}\right)\mathbf{i} + \left(\mathbf{v}_{TH}' + \mathbf{v}_{TH}''\right) = \underline{\underline{\mathbf{z}}}\mathbf{i} + \mathbf{v}_{TH}$$

例：根据定义求z参量



$$z_{11} = \frac{v_1}{i_1}\Big|_{i_2=0} = R_1 + R_2$$

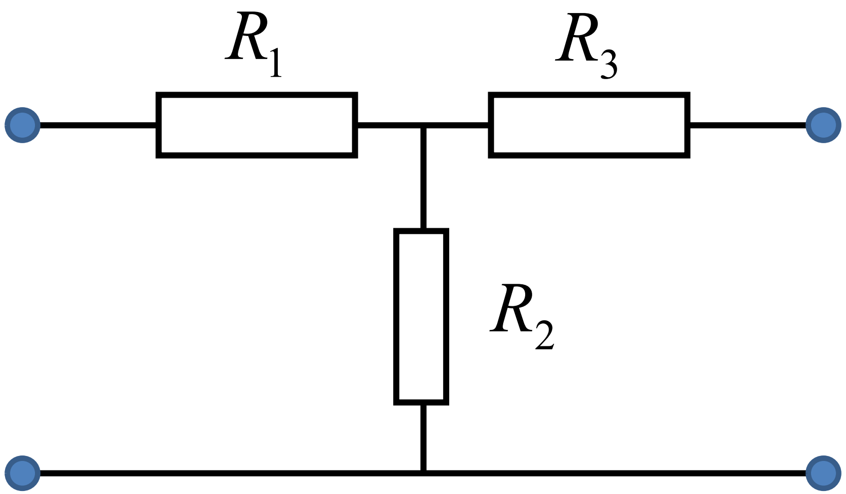
$$z_{12} = \frac{v_1}{i_2}\Big|_{i_1=0} = R_2$$

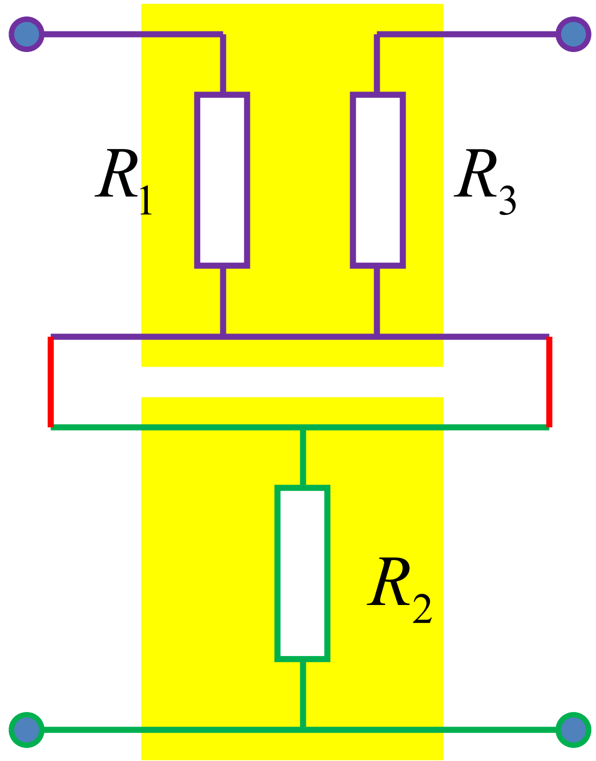
$$z_{21} = \frac{v_2}{i_1}\Big|_{i_2=0} = R_2$$

$$z_{22} = \frac{v_2}{i_2}\Big|_{i_1=0} = R_2 + R_3$$

$$\mathbf{z} = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix}$$

串串连接z相加

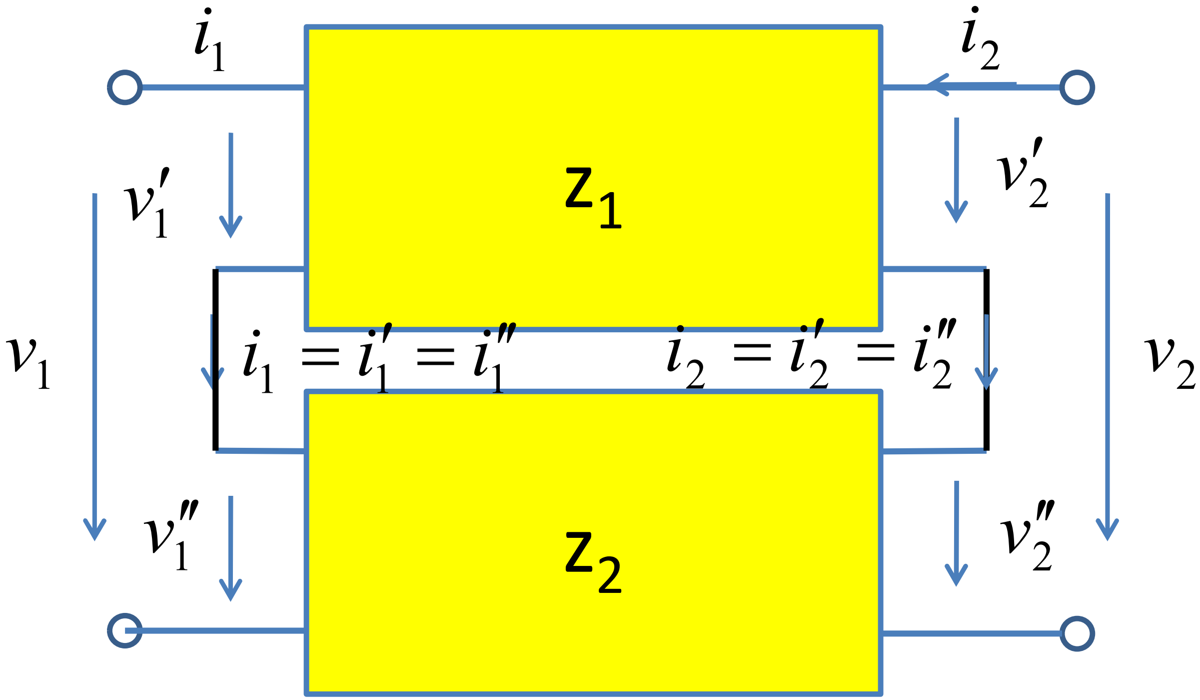




$$z_1 = \begin{bmatrix} R_1 & 0 \\ 0 & R_3 \end{bmatrix} \qquad z_2 = \begin{bmatrix} R_2 & R_2 \\ R_2 & R_2 \end{bmatrix}$$

$$\mathbf{z} = \mathbf{z_1} + \mathbf{z_2} = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix}$$

网络连接不能破坏端口条件



$$\mathbf{v} = \mathbf{v}' + \mathbf{v}'' = \mathbf{z}_1\mathbf{i}' + \mathbf{z}_2\mathbf{i}''$$
$$= \mathbf{z}_1\mathbf{i} + \mathbf{z}_2\mathbf{i} = (\mathbf{z}_1 + \mathbf{z}_2)\mathbf{i} = \mathbf{z}\mathbf{i}$$

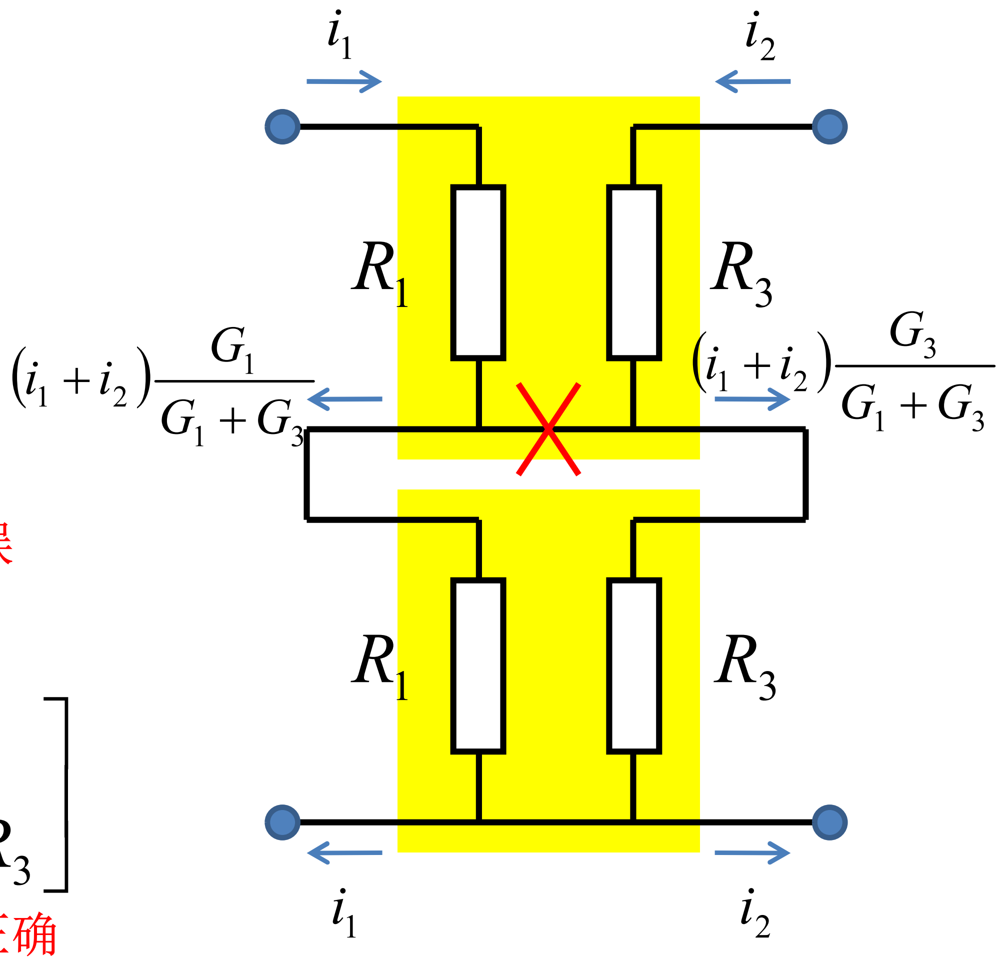
$\mathbf{i}' = \mathbf{i}'' = \mathbf{i}$ ⇒ $\mathbf{z} = \mathbf{z}_1 + \mathbf{z}_2$

端口条件破坏，则不能随意运用网络参量连接公式计算

$$\mathbf{z}_1=\mathbf{z}_2=\begin{bmatrix}R_1 & 0\\ 0 & R_3\end{bmatrix}$$

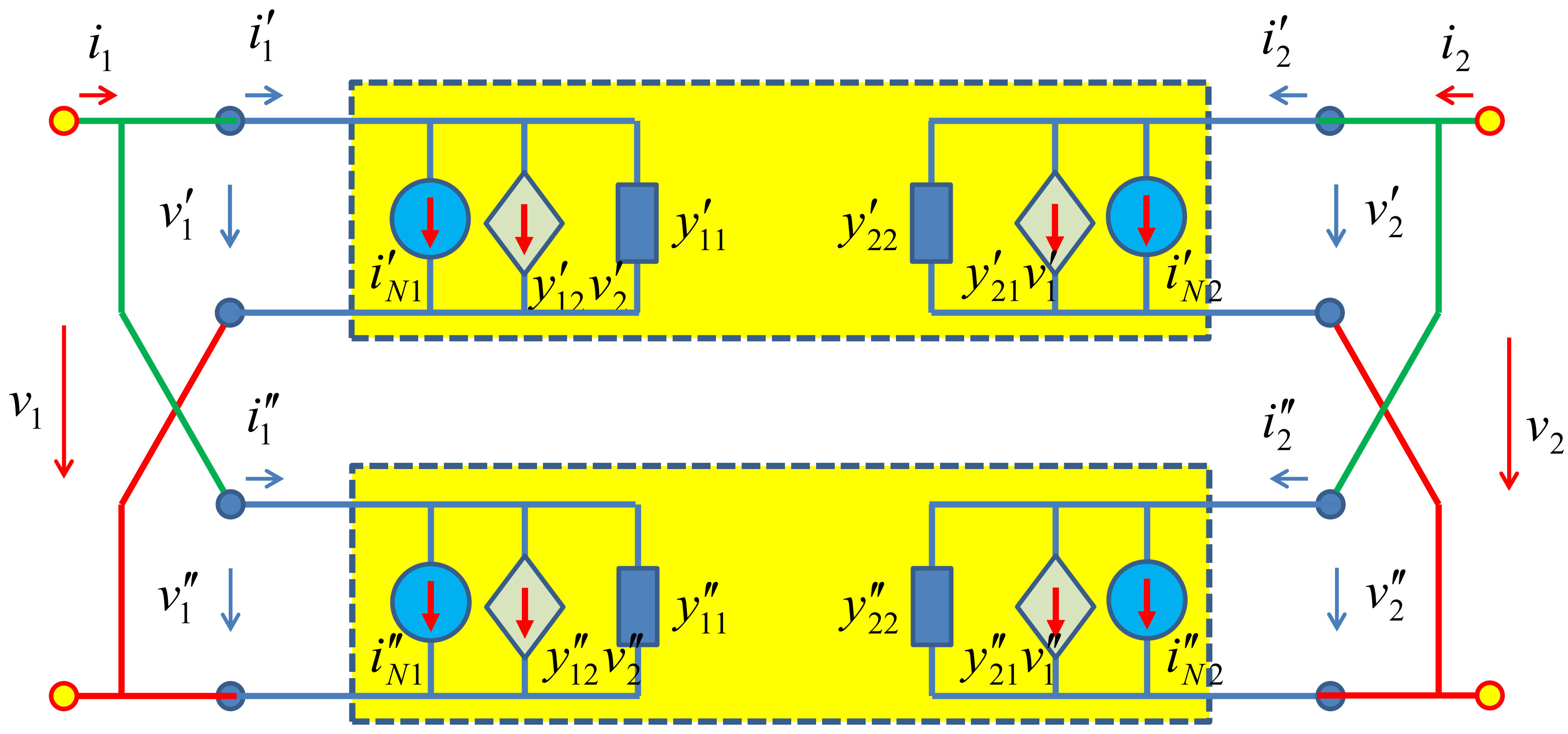
$$\mathbf{z}=\mathbf{z}_1+\mathbf{z}_2=\begin{bmatrix}2R_1 & 0\\ 0 & 2R_3\end{bmatrix}$$ 错误

$$\mathbf{z}=\begin{bmatrix}R_1+R_1\parallel R_3 & R_1\parallel R_3\\ R_1\parallel R_3 & R_3+R_1\parallel R_3\end{bmatrix}$$ 正确



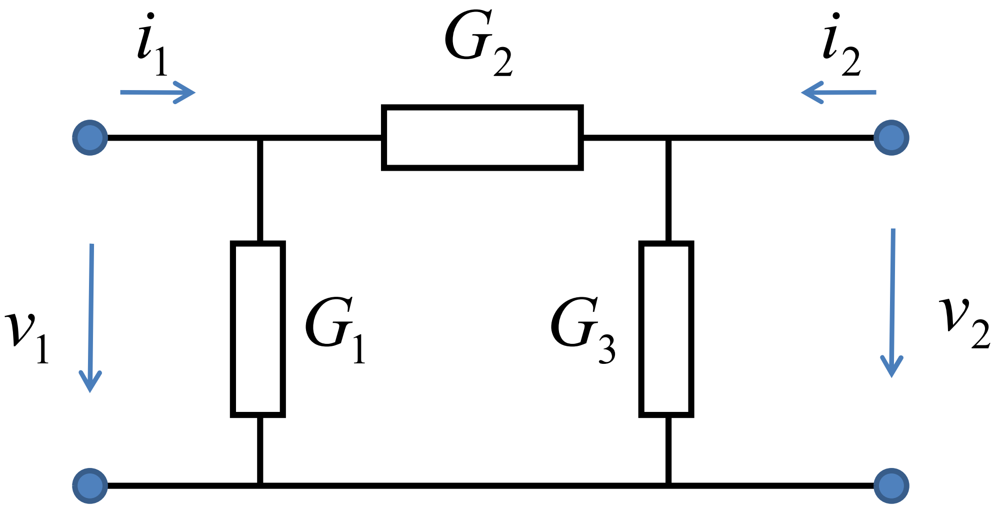
网络并联

Parallel-parallel connection



$$\mathbf{i} = \mathbf{i}' + \mathbf{i}'' = \mathbf{y}'\mathbf{v}' + \mathbf{i}_N' + \mathbf{y}''\mathbf{v}'' + \mathbf{i}_N'' = (\mathbf{y}' + \mathbf{y}'')\mathbf{v} + (\mathbf{i}_N' + \mathbf{i}_N'') = \mathbf{y}\mathbf{v} + \mathbf{i}_N$$

例：根据定义求y参量



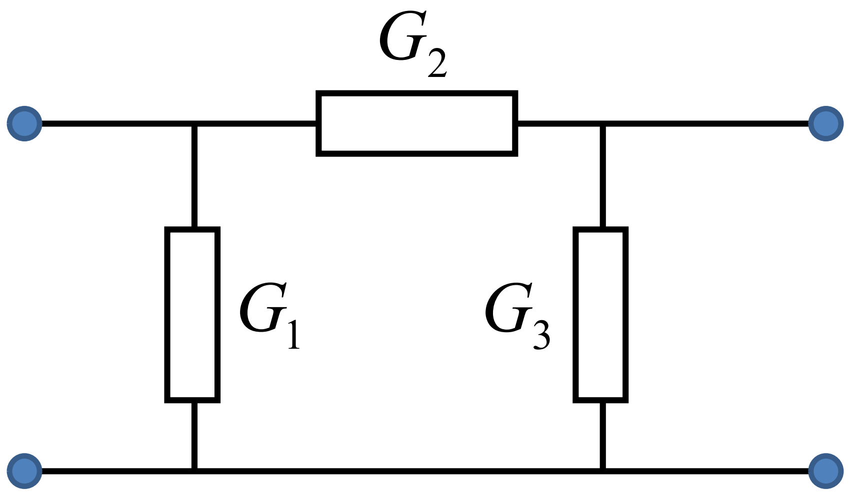
$$y_{11} = \frac{i_1}{v_1}\bigg|_{v_2=0} = G_1 + G_2$$

$$y_{12} = \frac{i_1}{v_2}\bigg|_{v_1=0} = -G_2$$

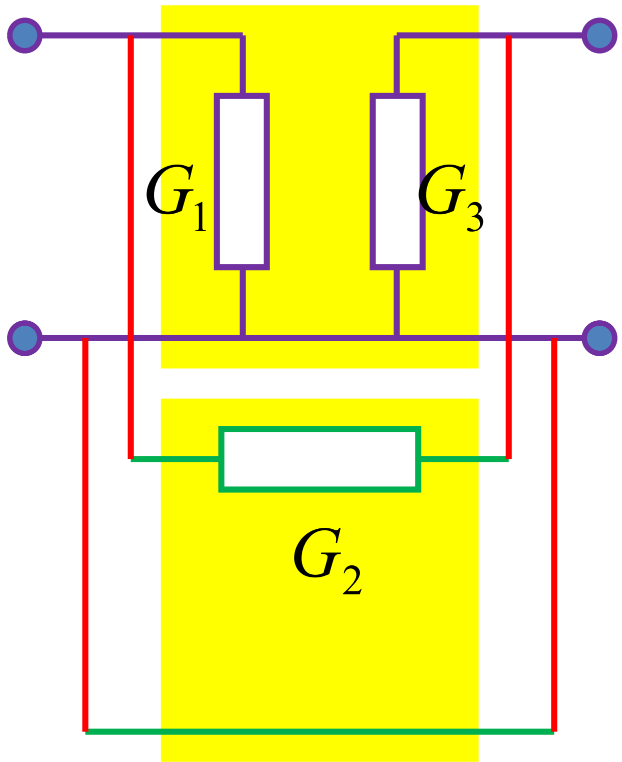
$$y_{21} = \frac{i_2}{v_1}\bigg|_{v_2=0} = -G_2$$

$$y_{22} = \frac{i_2}{v_2}\bigg|_{v_1=0} = G_2 + G_3$$

$$\mathbf{y} = \begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix}$$



并并连接y相加

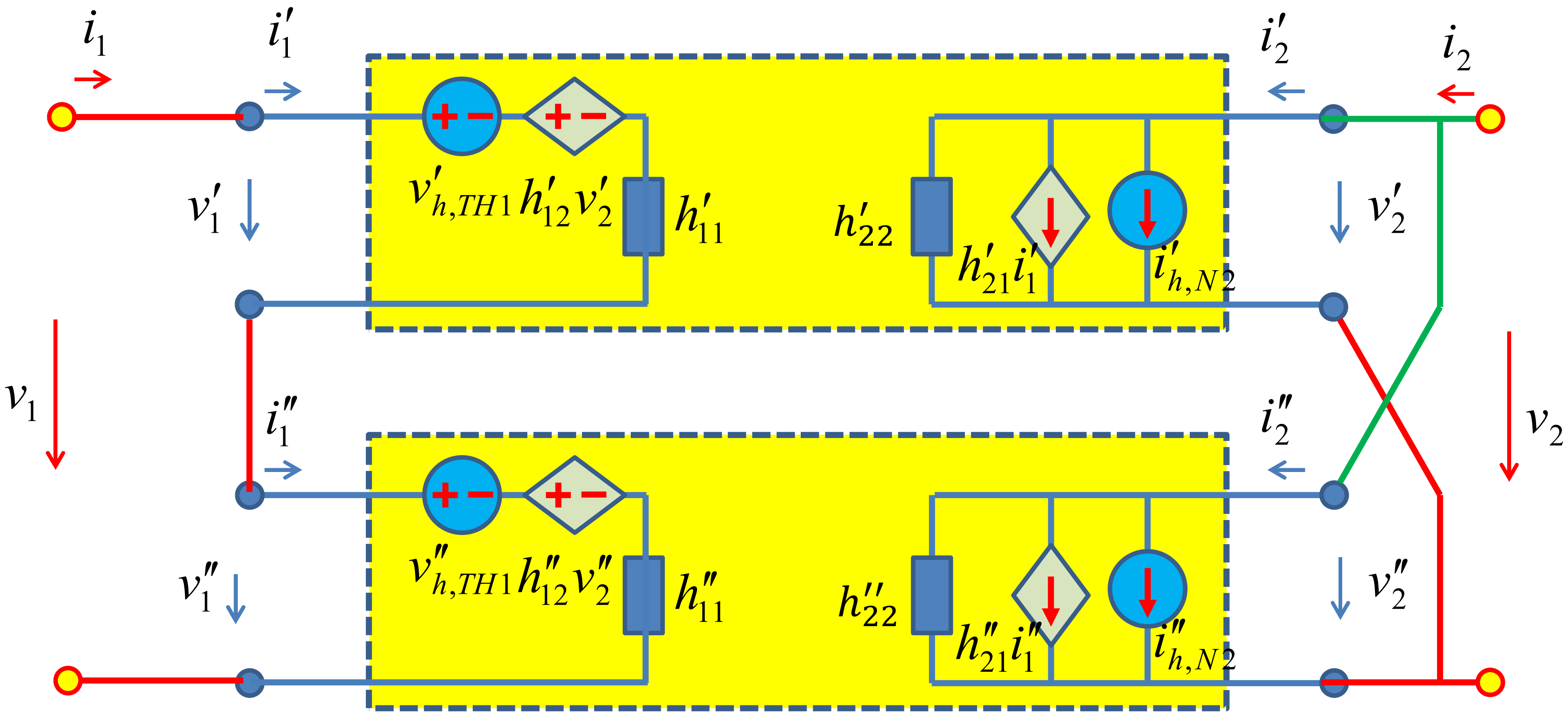


$$y_1=\begin{bmatrix} G_1 & 0 \\ 0 & G_3 \end{bmatrix} \qquad y_2=\begin{bmatrix} G_2 & -G_2 \\ -G_2 & G_2 \end{bmatrix}$$

$$\mathbf{y}=\mathbf{y_1}+\mathbf{y_2}=\begin{bmatrix} G_1+G_2 & -G_2 \\ -G_2 & G_2+G_3 \end{bmatrix}$$

串并连接

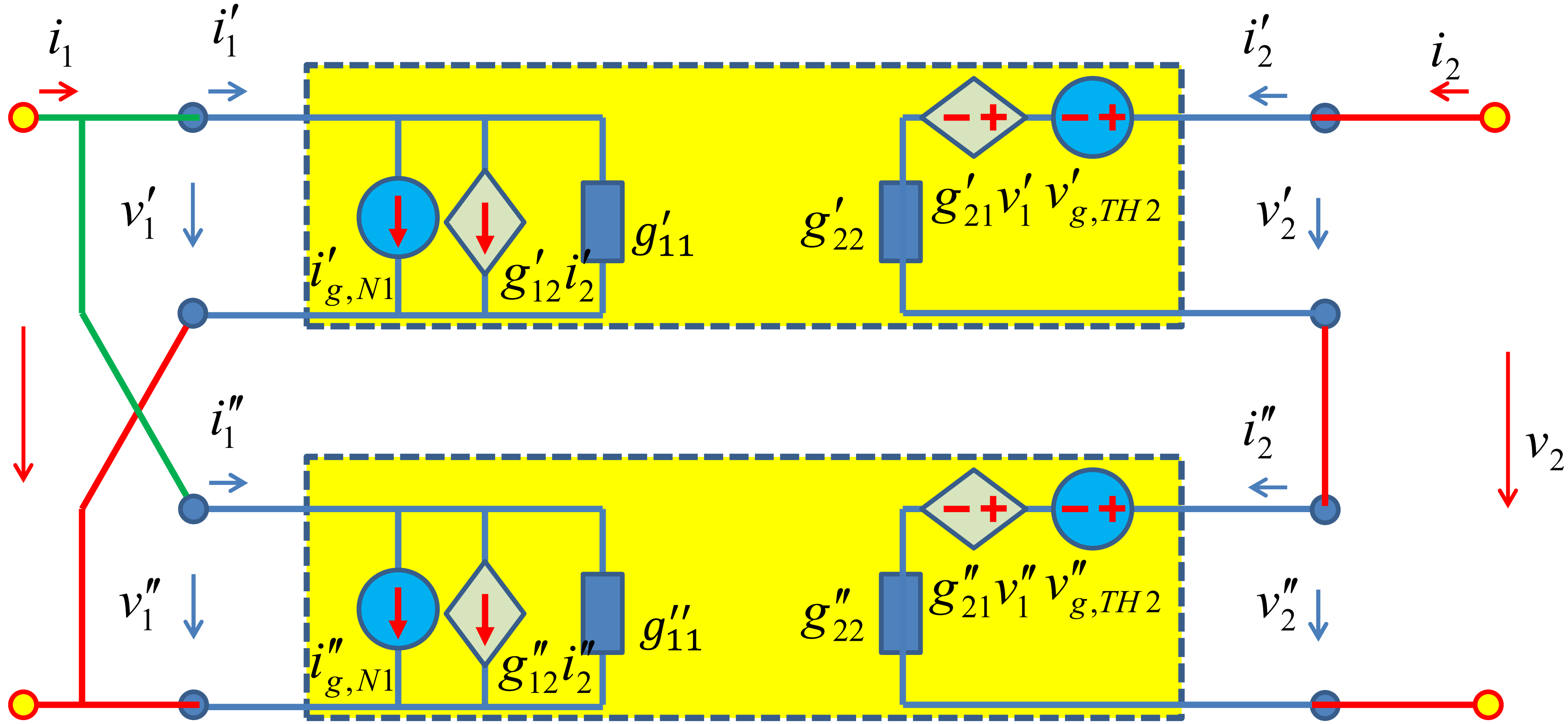
Series-parallel connection



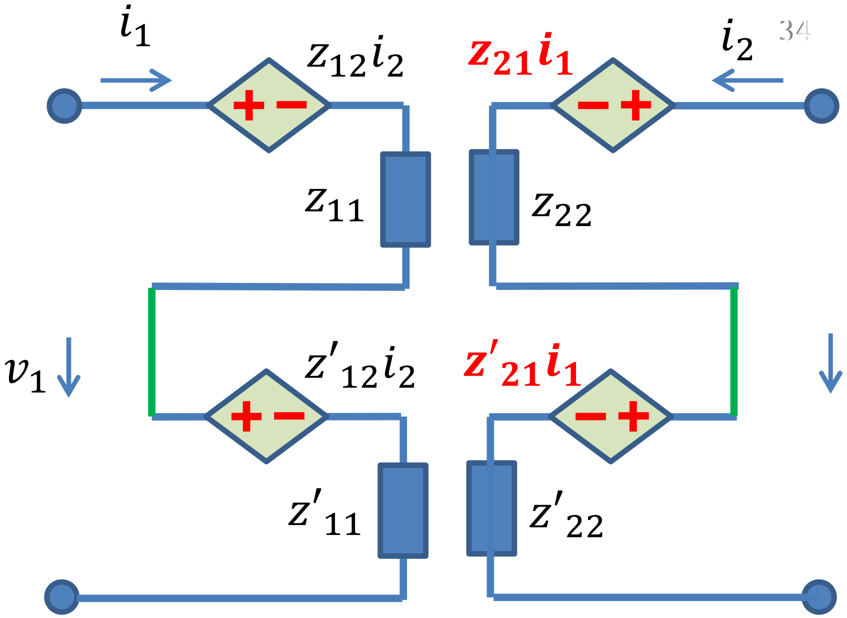
$$\mathbf{h} = \mathbf{h}' + \mathbf{h}''$$

并串连接

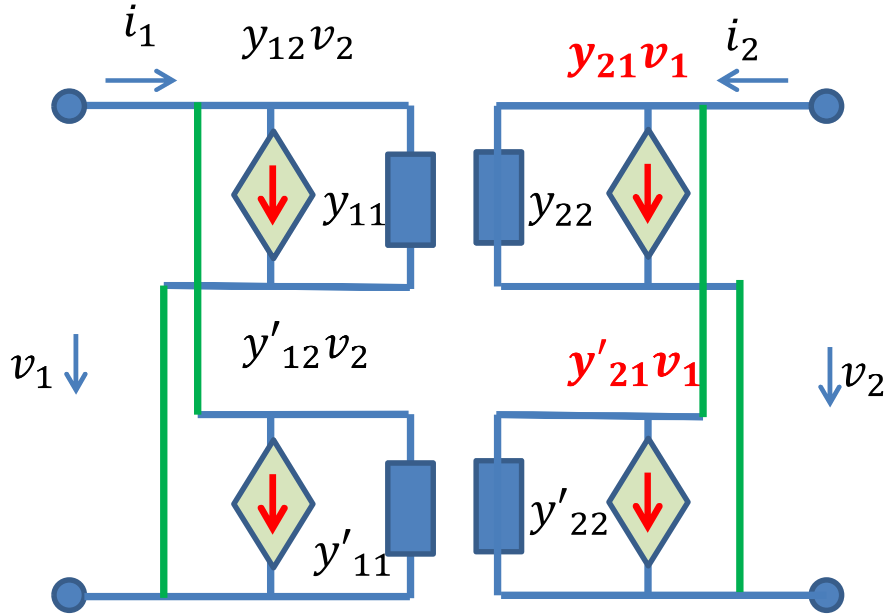
Parallel-series connection



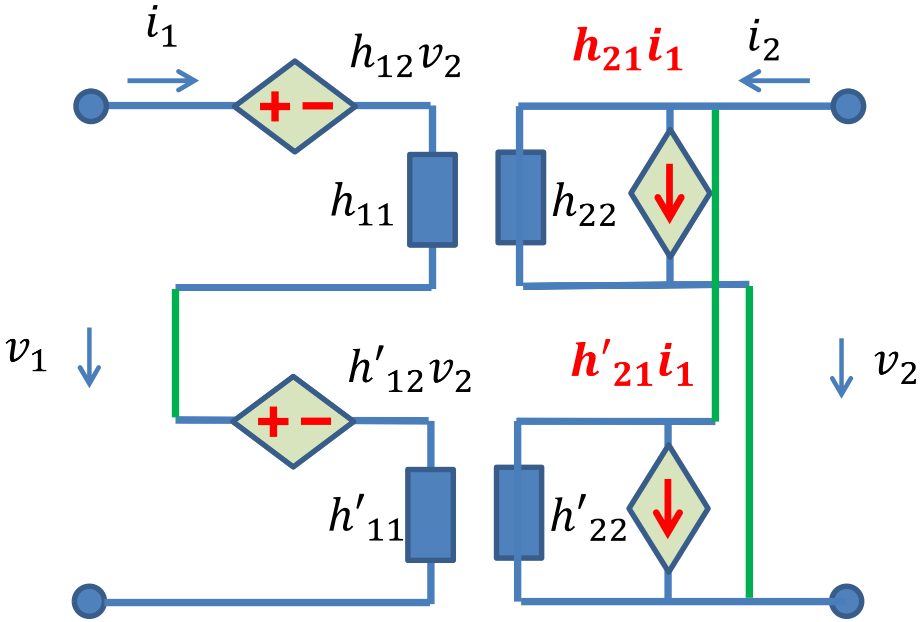
$$\mathbf{g} = \mathbf{g}' + \mathbf{g}''$$



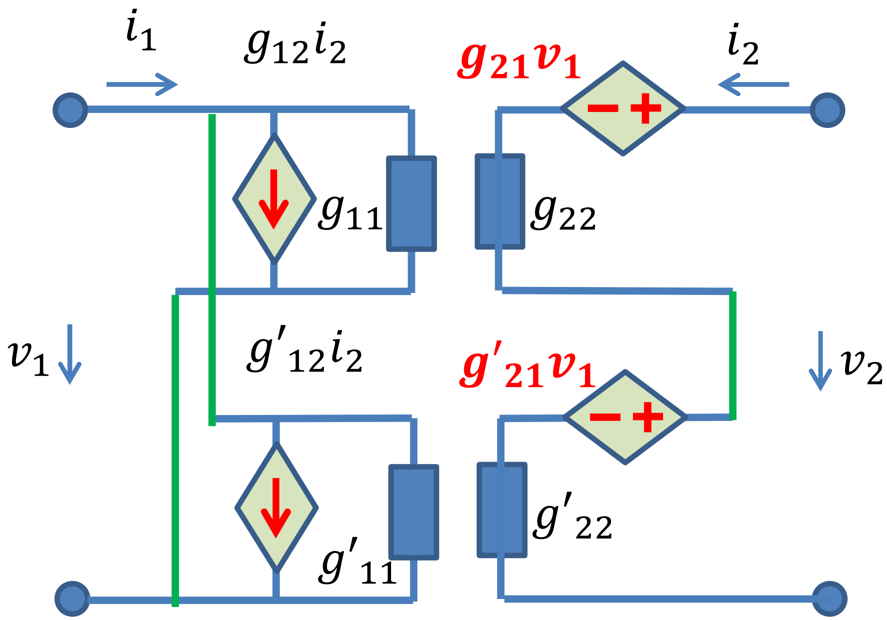
串串连接z相加



并并连接y相加

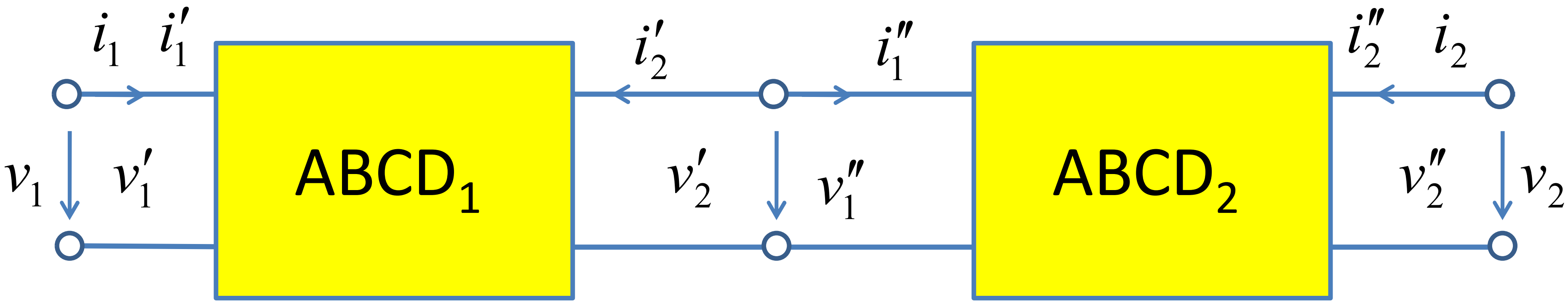


串并连接h相加



并串连接g相加

网络级联：cascade



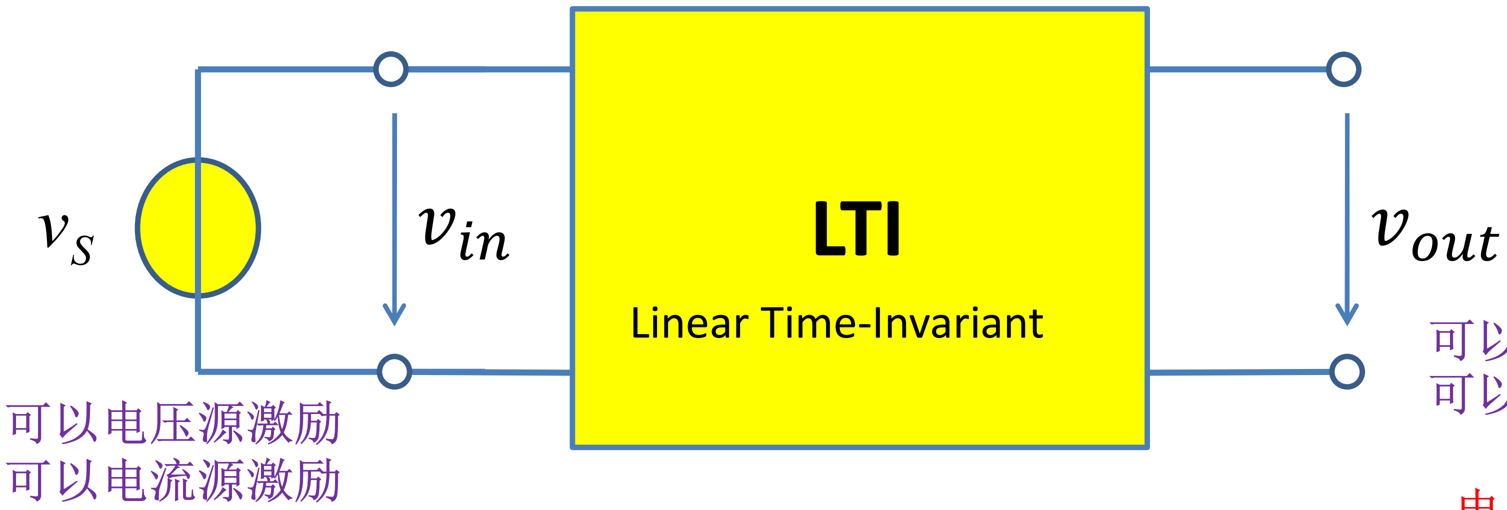
$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} v_1' \\ i_1' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \begin{bmatrix} v_2' \\ -i_2' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \begin{bmatrix} v_1'' \\ i_1'' \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \begin{bmatrix} A & B \\ C & D \end{bmatrix}_2 \begin{bmatrix} v_2'' \\ -i_2'' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \begin{bmatrix} A & B \\ C & D \end{bmatrix}_2 \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \begin{bmatrix} A & B \\ C & D \end{bmatrix}_2$$

四、 传递函数 Transfer Function

$$H = \frac{v_{out}}{v_{in}}$$



可以测量开路电压，
可以测量短路电流

电压传递函数是最常见的传递函数，以其为例

$$H = \frac{v_L}{v_S}$$

传递函数的获得

- 列写电路方程，其中，v_S为激励源视为已知量、v_L为响应视为未知量，求解电路方程即可获得电压传递函数
  - 基本方法：结点电压法、回路电流法
    - 上节课最后求放大器放大倍数例：放大倍数就是传递函数
  - 简单电路结构：利用简单串并联的分压分流
    - 上节课的电阻衰减器衰减系数例：衰减系数为放大倍数的倒数，也是传递函数例
    - 可以用结点电压法，回路电流法，串并联，戴维南等效等
  - 网络参量已测得或容易获得，则可利用网络参量获取传递函数

简单对接关系，一套端口电压、电流，**KVL**、**KCL**自动满足
只需列写元件约束方程即可

$$v_1 + R_S i_1 = v_S$$

激励源元件约束方程

$$y_{11}v_1 + y_{12}v_2 - i_1 = 0$$

$$y_{21}v_1 + y_{22}v_2 - i_2 = 0$$

二端口网络元件约束方程

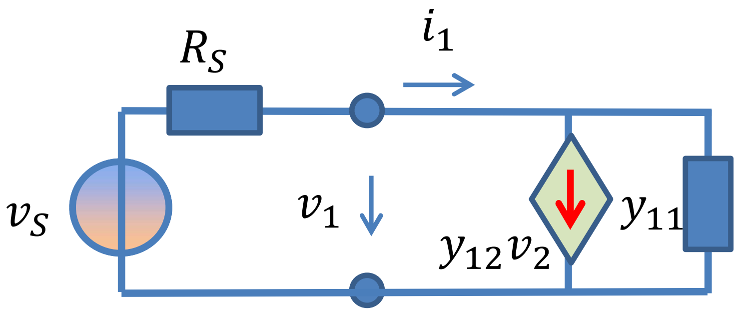
4个未知数
v_1, i_1, v_2, i_2

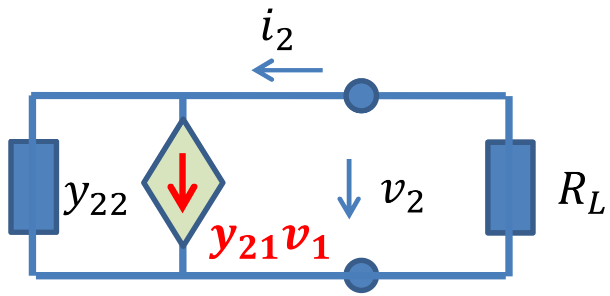
4个方程

$$v_2 + R_L i_2 = 0$$

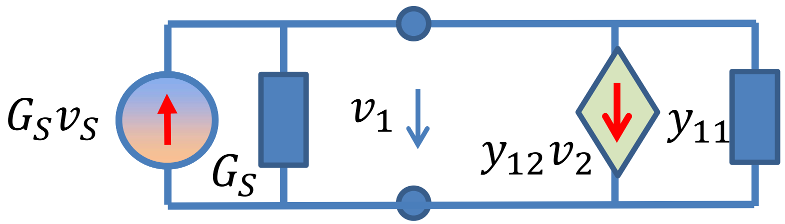
负载电阻元件约束方程

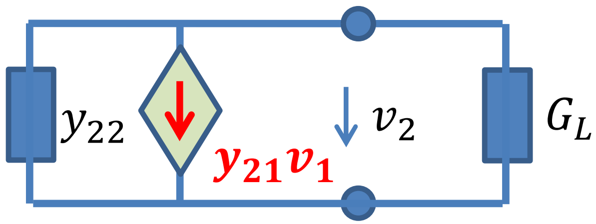
不仅仅数学语言可求解，用电路语言进行分析更是本课程基本功要求





y参量为导纳参量，将戴维南电压源转换为诺顿电流源处理是适当的





$$v_1 = \frac{G_Sv_S - y_{12}v_2}{G_S + y_{11}} \qquad v_2 = -\frac{y_{21}v_1}{G_L + y_{22}} = -\frac{y_{21}}{G_L + y_{22}}\frac{G_Sv_S - y_{12}v_2}{G_S + y_{11}}$$

$$(G_S + y_{11})(G_L + y_{22})v_2 = -y_{21}G_Sv_S + y_{21}y_{12}v_2$$

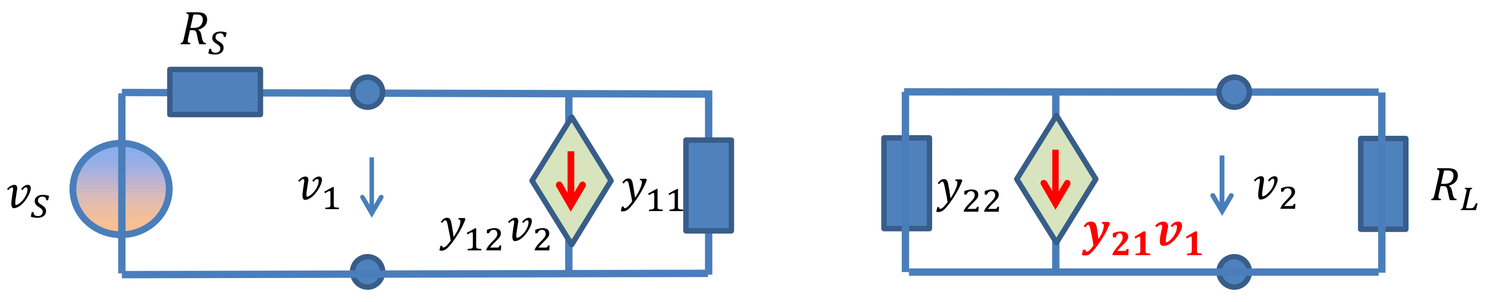
$$y_{21}G_Sv_S = \big(y_{21}y_{12} - (G_S + y_{11})(G_L + y_{22})\big)v_2$$

电路和数学是一体的

$$H = \frac{v_L}{v_S} = \frac{v_2}{v_S} = \frac{y_{21}G_S}{y_{21}y_{12} - (G_S + y_{11})(G_L + y_{22})}$$

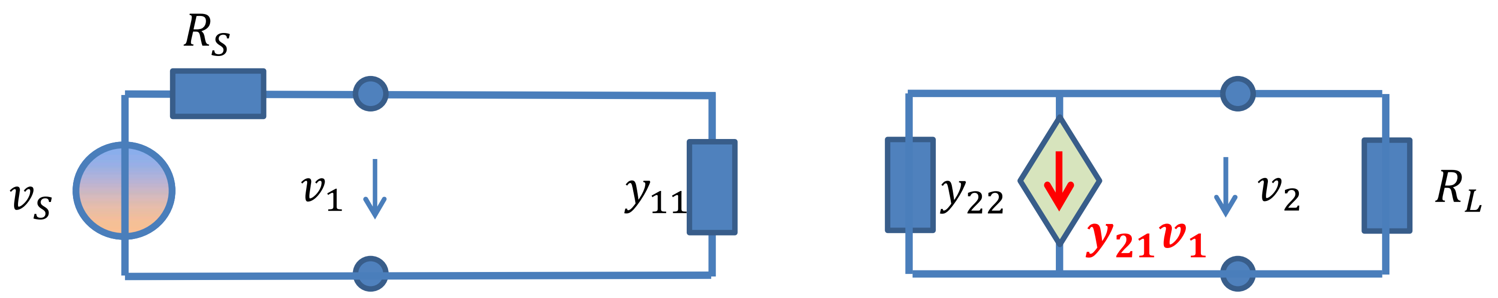
凡是复杂一点的公式，都应做量纲检查，极端检查，确保基本无误

极端检查通过：假设是单向网络



$$H=\frac{v_L}{v_S}=\frac{v_2}{v_S}=\frac{y_{21}G_S}{y_{21}y_{12}-(G_S+y_{11})(G_L+y_{22})}\overset{y_{12}=0}{\cong}\frac{-y_{21}G_S}{(G_S+y_{11})(G_L+y_{22})}$$

单向网络比较简单



$$v_1=\frac{\frac{1}{y_{11}}}{R_S+\frac{1}{y_{11}}}v_S=\frac{G_S}{G_S+y_{11}}v_S$$

$$v_2=-\frac{y_{21}v_1}{y_{22}+G_L}=\frac{1}{y_{22}+G_L}(-y_{21})\frac{G_S}{G_S+y_{11}}v_S$$

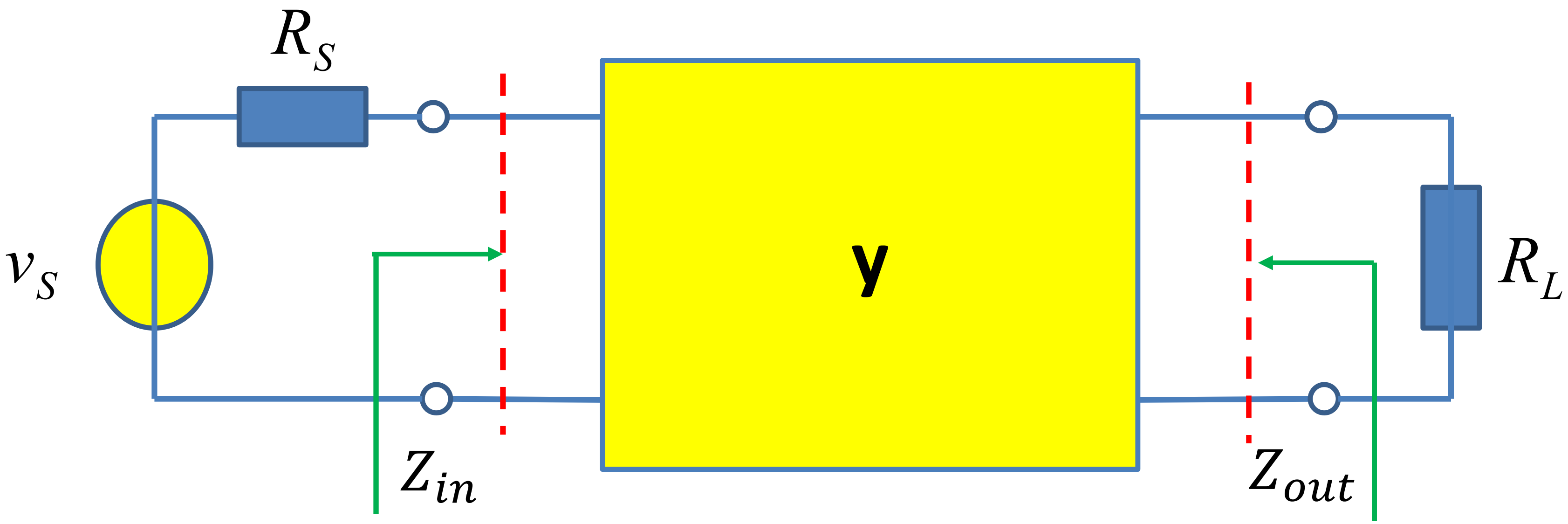
输出电流流过输出回路总电阻形成输出电压

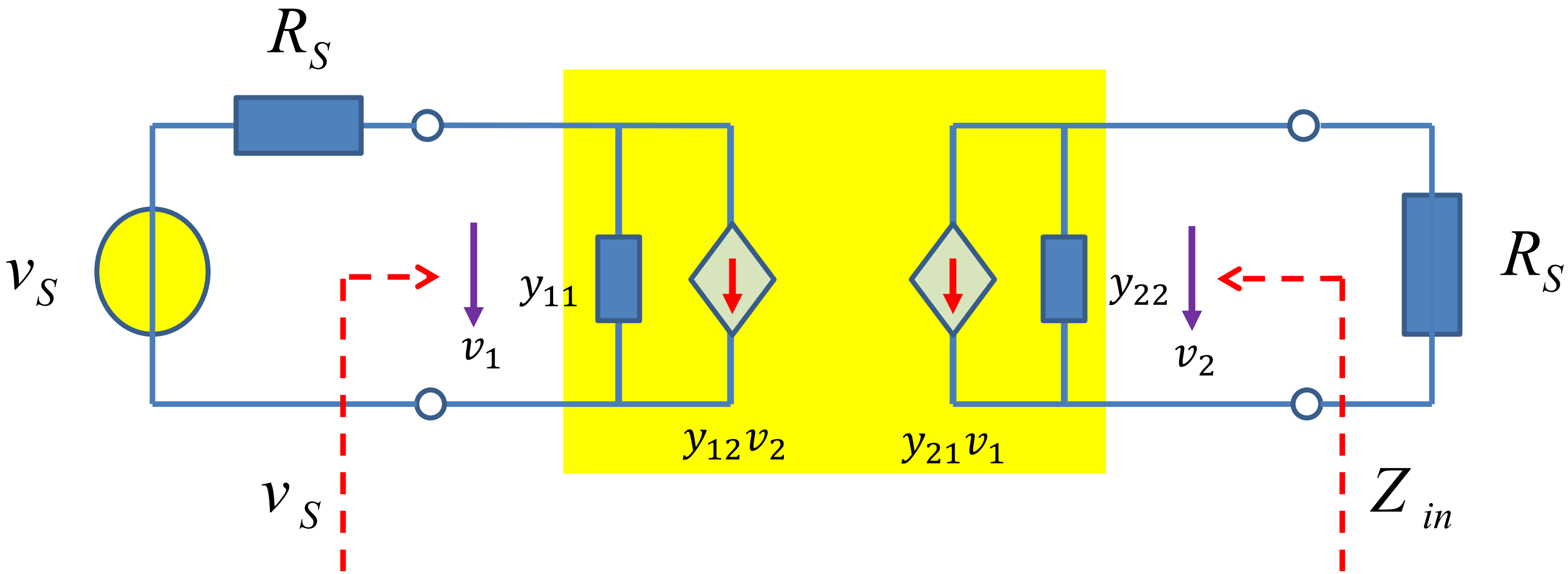
被本征跨导增益转换为输出电流

输入回路分压

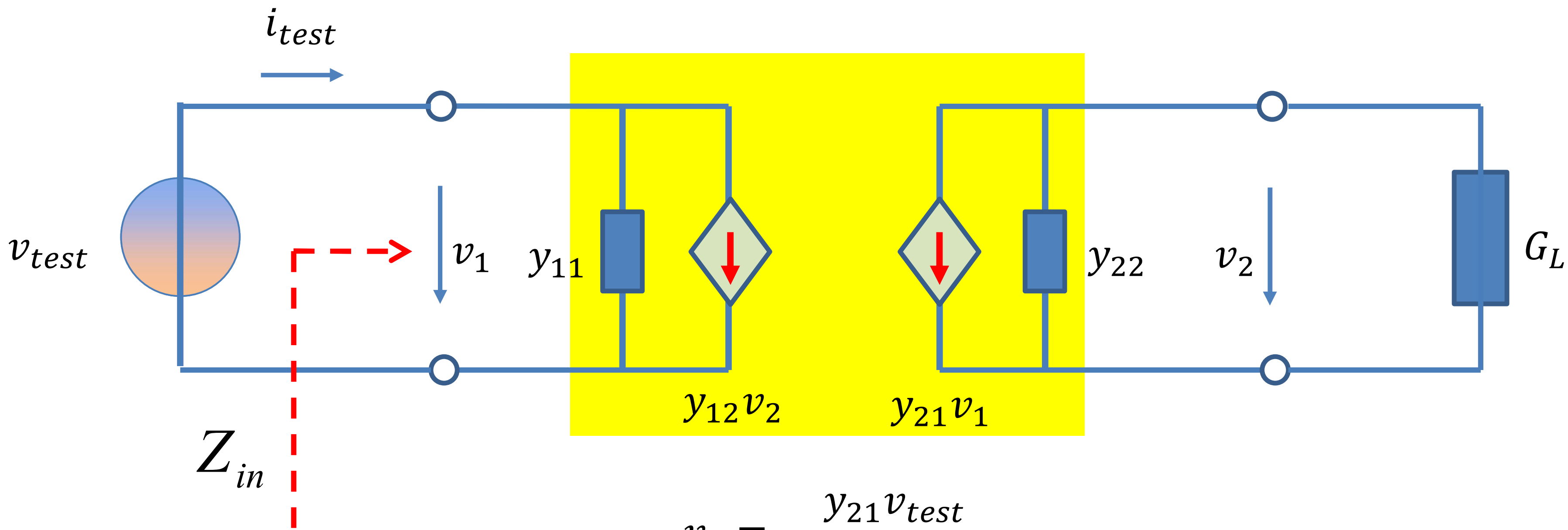
单向网络的传递函数为分传递函数之积，这是我们喜好单向网络的重要原因：随手写答案

输入阻抗，输出阻抗





输入阻抗，输入导纳



$$v_2 = -\frac{y_{21}v_{test}}{y_{22}+G_L}$$

$$i_{test} = y_{11}v_1 + y_{12}v_2 = y_{11}v_{test} - \frac{y_{12}y_{21}v_{test}}{y_{22}+G_L}$$

$$Y_{in} = \frac{i_{test}}{v_{test}} = y_{11} - \frac{y_{12}y_{21}}{y_{22}+G_L}$$

输入/输出阻抗/导纳

$$Y_{in}=y_{11}-\frac{y_{12}y_{21}}{y_{22}+G_L}\qquad Y_{out}=y_{22}-\frac{y_{21}y_{12}}{y_{11}+G_S}$$

$$Y_{in}=\frac{1}{Z_{in}}$$

$$Z_{in}=z_{11}-\frac{z_{12}z_{21}}{z_{22}+R_L}\qquad Z_{out}=z_{22}-\frac{z_{21}z_{12}}{z_{11}+Z_S}$$

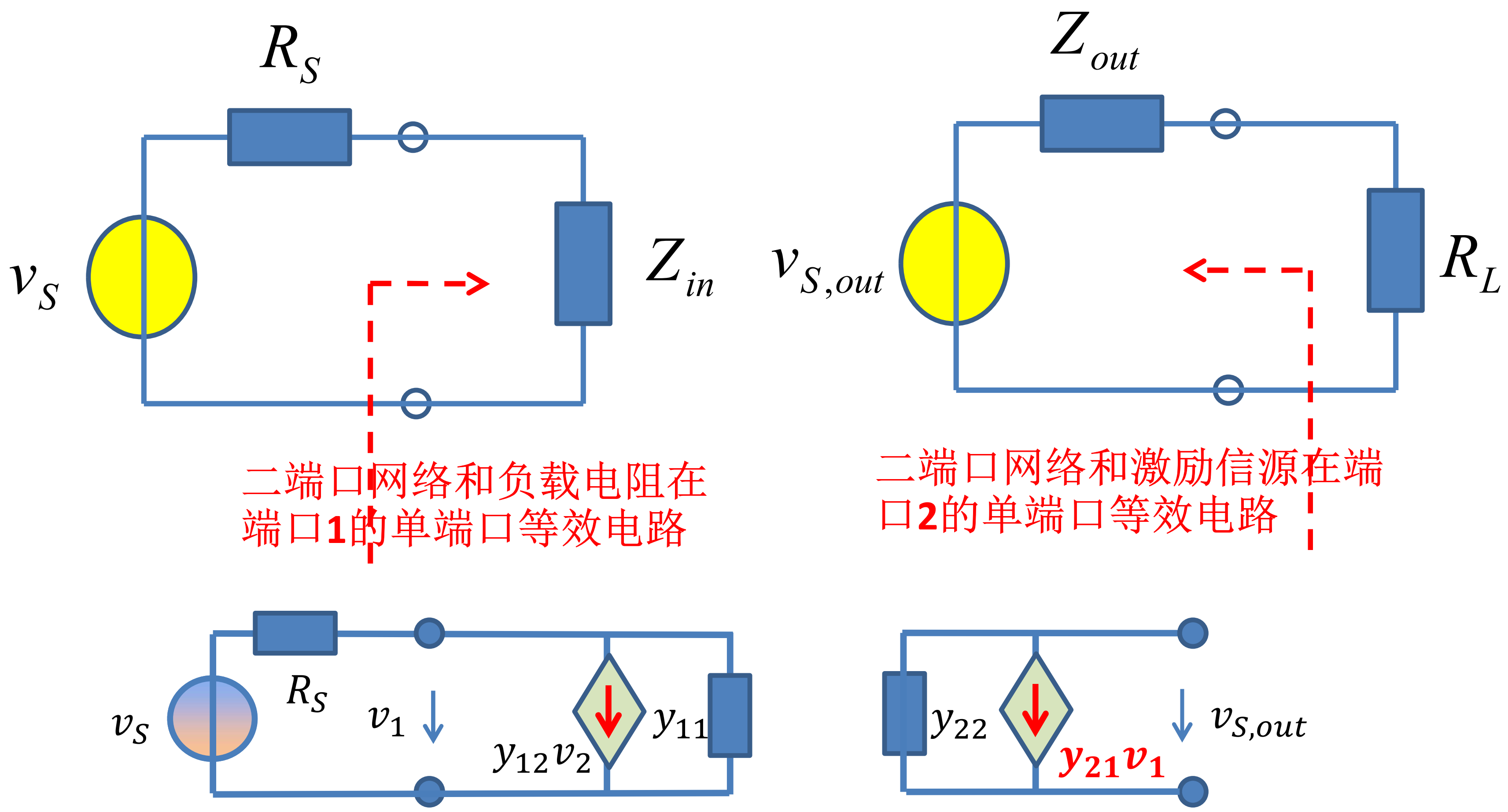
$$Y_{out}=\frac{1}{Z_{out}}$$

$$Z_{in}=h_{11}-\frac{h_{12}h_{21}}{h_{22}+G_L}\qquad Y_{out}=h_{22}-\frac{h_{21}h_{12}}{h_{11}+R_S}$$

$$Y_{in}=g_{11}-\frac{g_{12}g_{21}}{g_{22}+R_L}\qquad Z_{out}=g_{22}-\frac{g_{21}g_{12}}{g_{11}+G_S}$$

格式规范一致，记忆十分方便简单

输入等效和输出等效



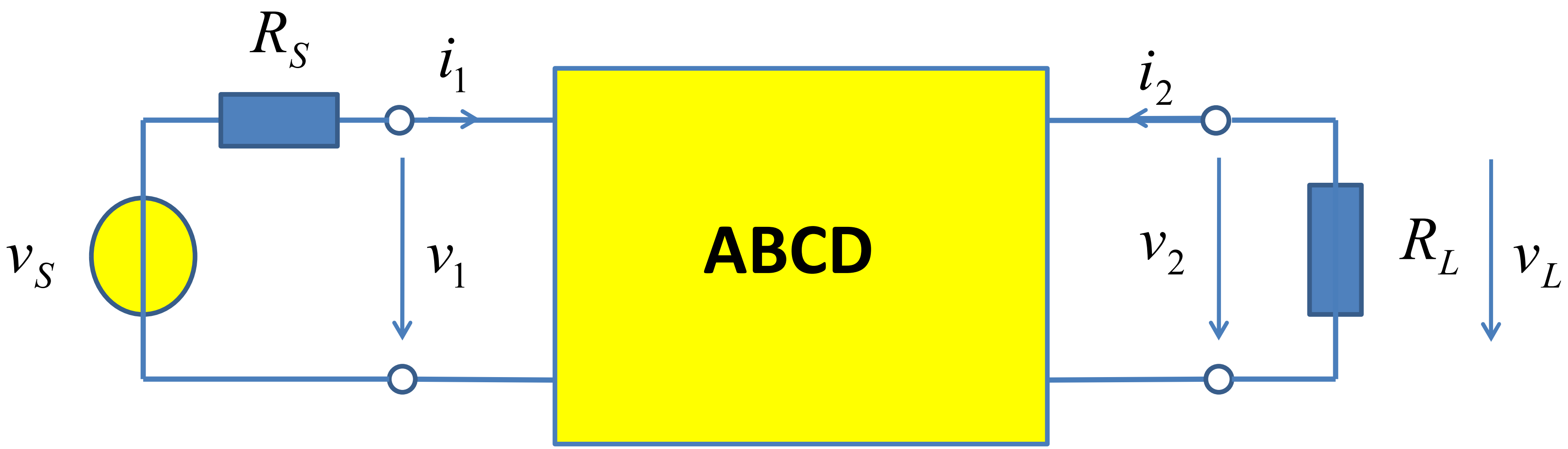
$$v_1=\frac{G_s v_s-y_{12}v_2}{G_s+y_{11}}$$

$$v_2=-\frac{y_{21}v_1}{y_{22}}=-\frac{y_{21}}{y_{22}}\frac{G_s v_s-y_{12}v_2}{G_s+y_{11}}$$

$$-y_{22}(G_s+y_{11})v_2=y_{21}G_s v_s-y_{21}y_{12}v_2$$

$$v_{S,out}=v_2=\frac{y_{21}G_s}{y_{21}y_{12}-y_{22}(G_s+y_{11})}v_s$$

ABCD参量在传递函数中的应用例



简单对接关系，一套端口电压、电流，**KVL**、**KCL**自动满足
只需列写元件约束方程即可

$$v_1 + R_S i_1 = v_S$$

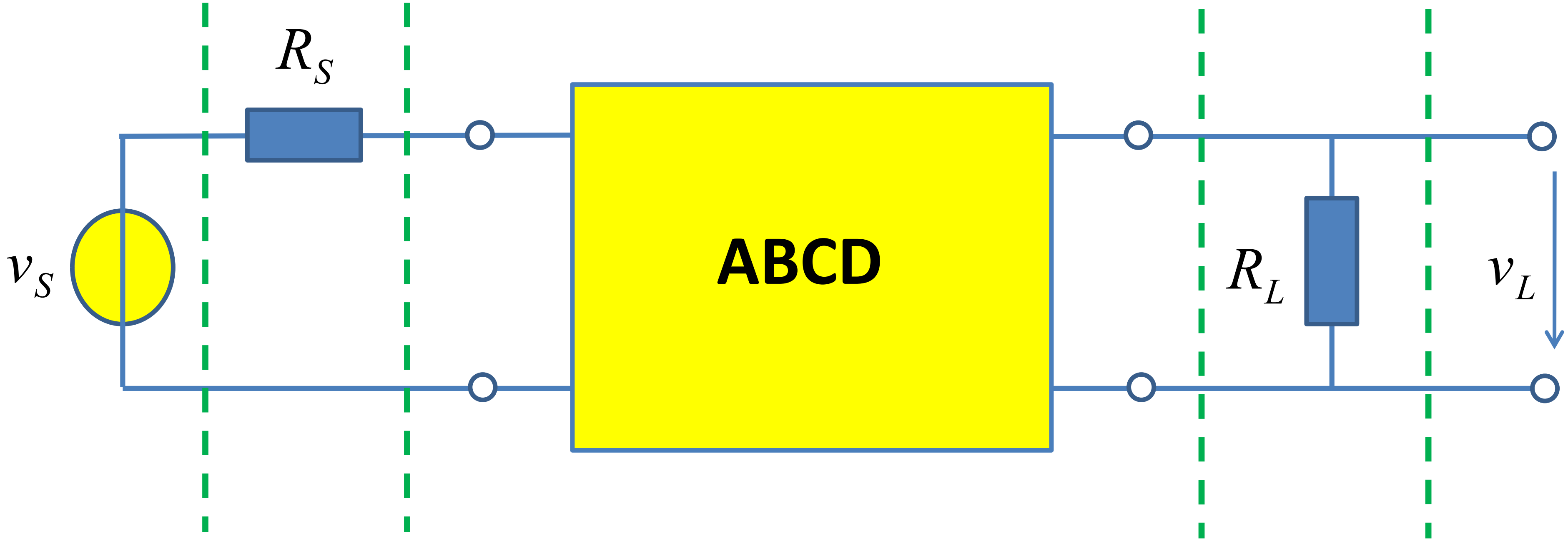
$$v_1 - A v_2 + B i_2 = 0$$

$$i_1 - C v_2 + D i_2 = 0$$

$$v_2 + R_L i_2 = 0$$

直接求解电路方程是数学基本功，
但没有掌握电路语言分析的基本功

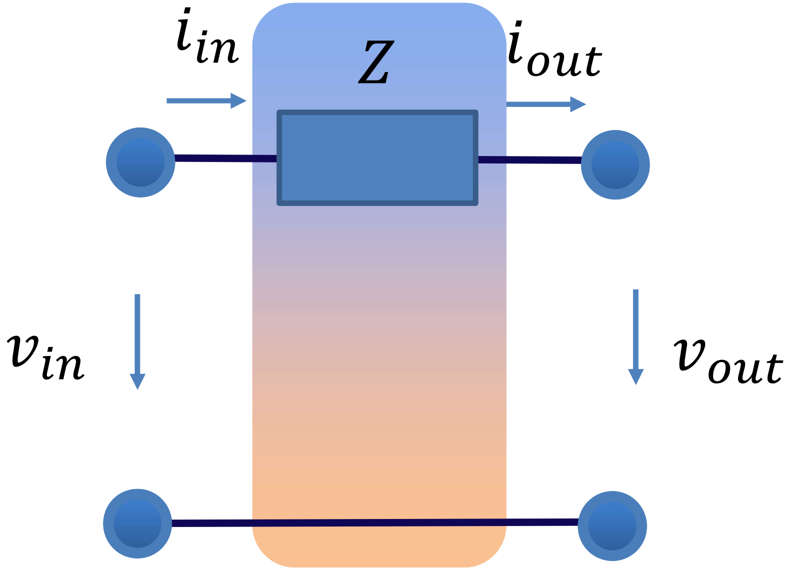
电路语言基本功



$$\begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} = \begin{bmatrix} A_S & B_S \\ C_S & D_S \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} A_L & B_L \\ C_L & D_L \end{bmatrix}$$

$$H = \frac{v_L}{v_S} = \frac{1}{A_c}$$

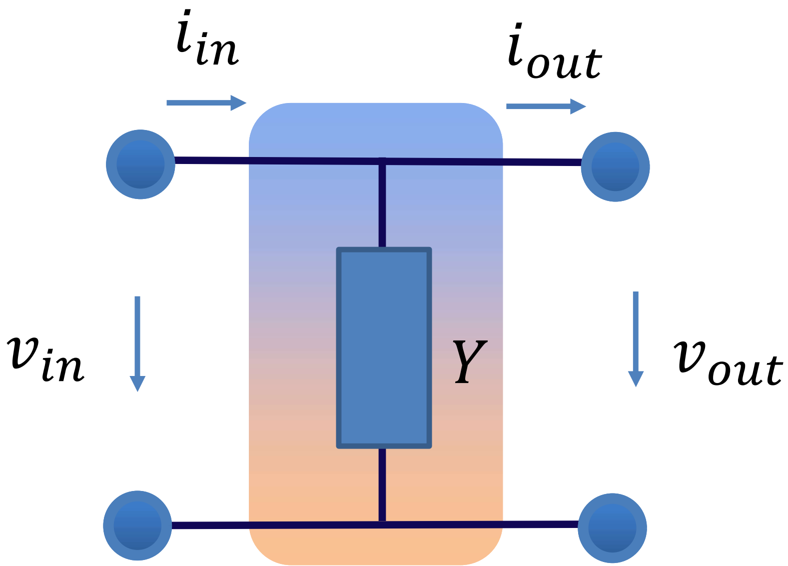
串臂阻抗和并臂导纳的传输参量矩阵



$$i_{in} = i_{out}$$

$$v_{in} = Z\, i_{in} + v_{out}$$

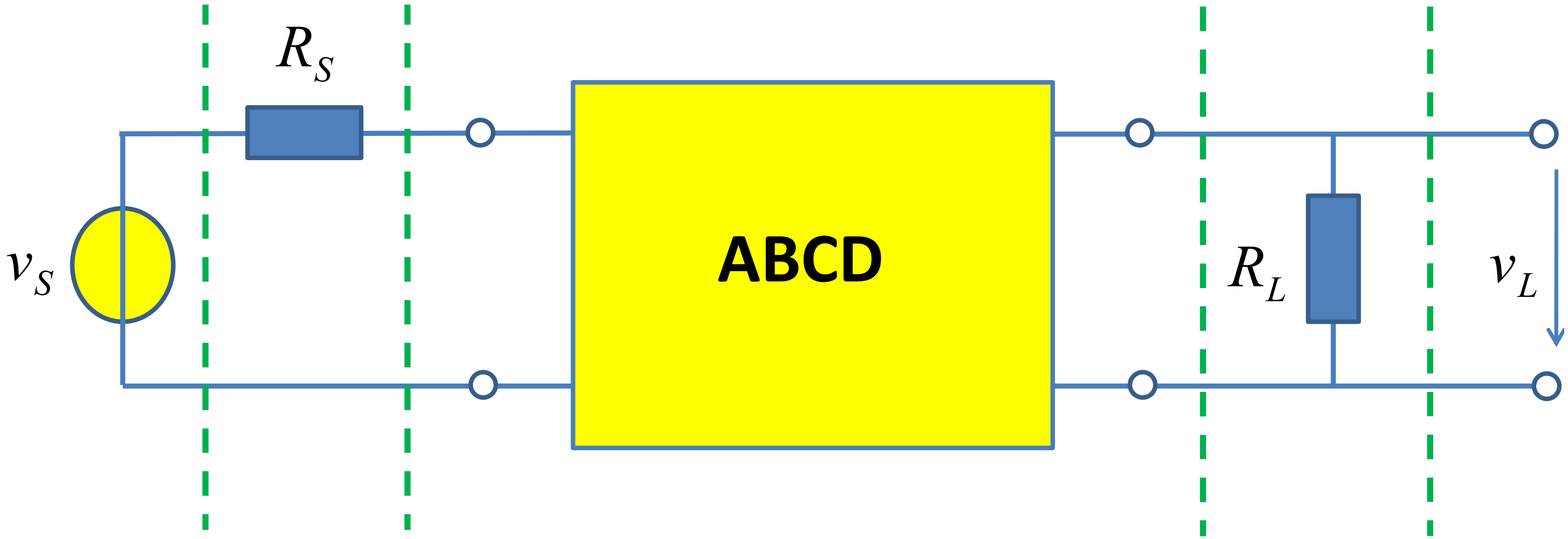
$$\begin{bmatrix} v_{in} \\ i_{in} \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{out} \\ i_{out} \end{bmatrix}$$



$$v_{in} = v_{out}$$

$$i_{in} = Y\, v_{in} + i_{out}$$

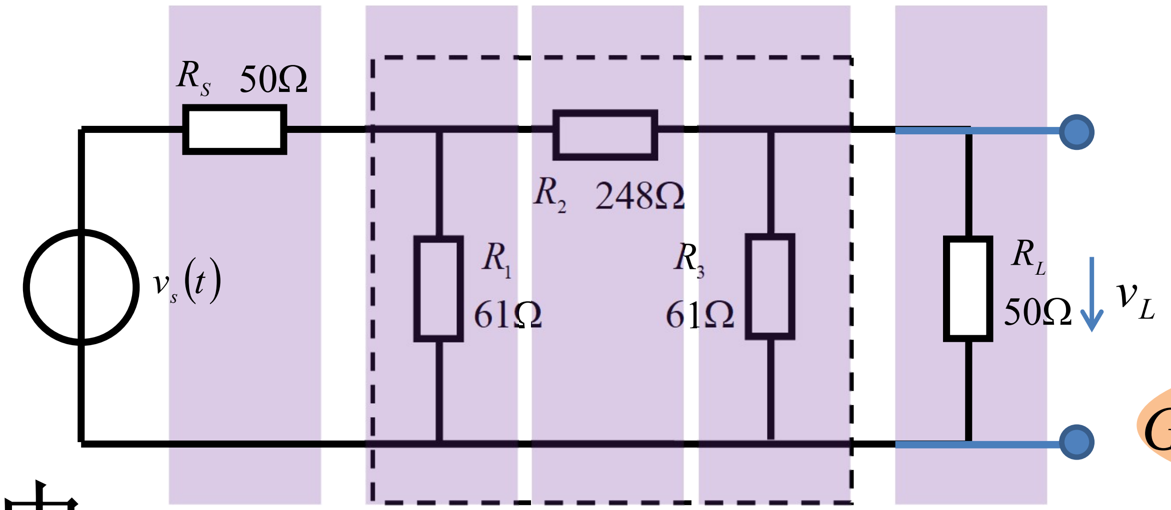
$$\begin{bmatrix} v_{in} \\ i_{in} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} v_{out} \\ i_{out} \end{bmatrix}$$



$$\begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} = \begin{bmatrix} A_S & B_S \\ C_S & D_S \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} A_L & B_L \\ C_L & D_L \end{bmatrix} = \begin{bmatrix} 1 & R_S \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_L} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} A + CR_S & B + DR_S \\ C & D \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_L} & 1 \end{bmatrix} = \begin{bmatrix} A + CR_S + \frac{B + DR_S}{R_L} & \cdots \\ \cdots & \cdots \end{bmatrix}$$

$$H = \frac{v_L}{v_S} = \frac{1}{A_c} = \frac{1}{A + CR_S + \frac{B + DR_S}{R_L}} = \frac{R_L}{AR_L + B + CR_S R_L + DR_S}$$



$$A_{v0}=\frac{v_L}{v_S}=\frac{1}{A}=0.0498$$

$$G_T=20\log(2A_{v0})=-20dB$$

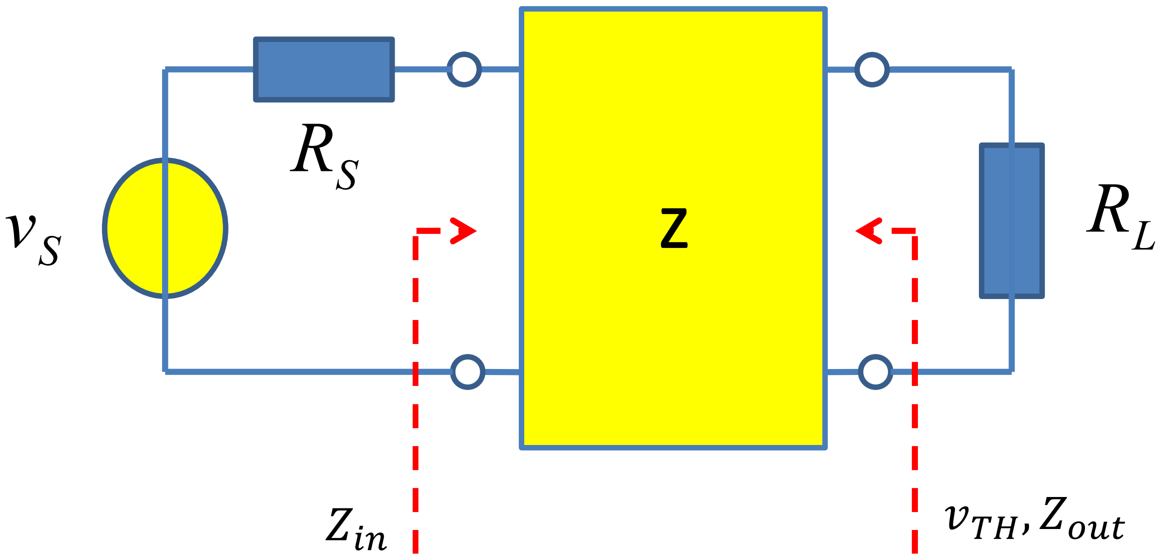
电阻衰减器分析

$$\mathbf{ABCD}=\begin{bmatrix}1 & R_S\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\G_1 & 1\end{bmatrix}\begin{bmatrix}1 & R_2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\G_3 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\G_L & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 50\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0.0164 & 1\end{bmatrix}\begin{bmatrix}1 & 248\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0.0164 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0.020 & 1\end{bmatrix}$$

$$=\begin{bmatrix}20.063 & 501.28\\0.2007 & 5.0656\end{bmatrix}=\begin{bmatrix}\frac{1}{0.0498} & \frac{1}{0.0020}\\\frac{1}{4.9814} & \frac{1}{0.1974}\end{bmatrix}=\begin{bmatrix}\frac{1}{A_{v0}} & \frac{1}{G_{m0}}\\\frac{1}{R_{m0}} & \frac{1}{A_{i0}}\end{bmatrix}$$

作业1

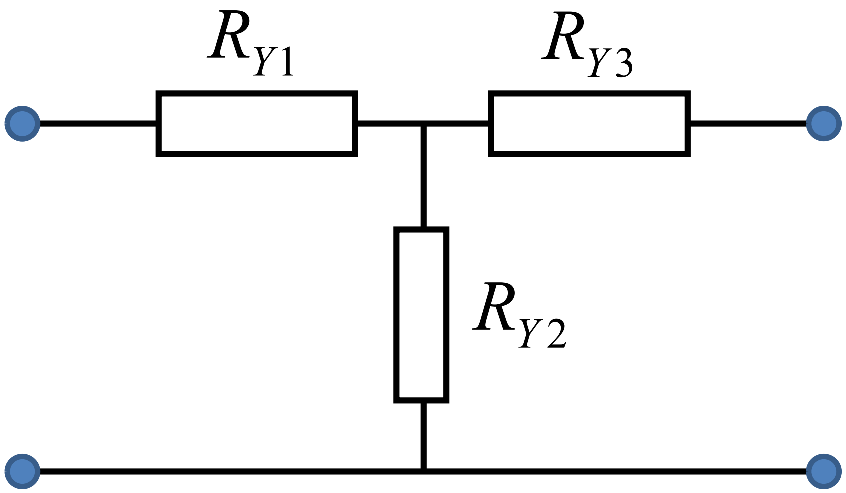


- 已知二端口网络的z参量，1端口接信源（v_s,R_s），2端口接负载R_L
  - 求输入阻抗Z_{in}
  - 求输出端戴维南等效v_{TH}，Z_{out}
    - 要求有详细的推导步骤：要求用电路语言分析
  - 在此基础上，考察单向网络的表达式与等效电路之间的关系
    - z参量单向网络：将$z_{12}=0$，$z_{21}=R_m$代入表达式即可
    - 通过等效电路图分析，比对解表达式，理解对电路中的分压、分流关系

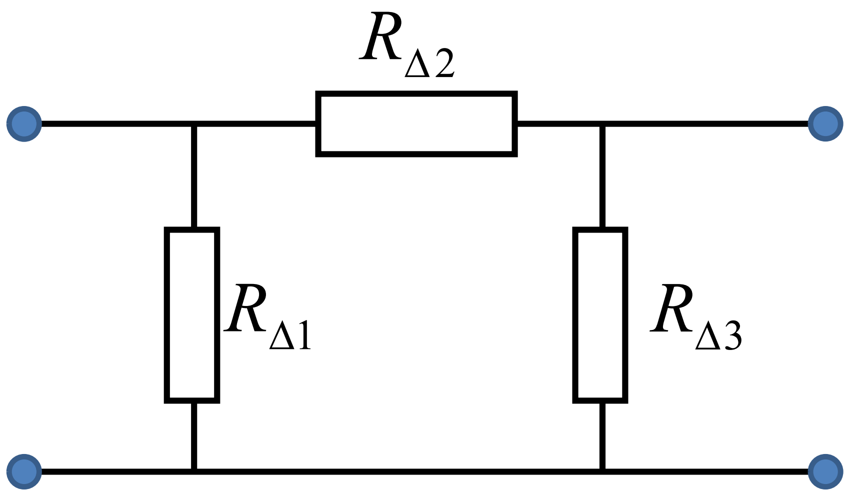
作业2：Y-Δ转换关系的推导

- 如果两个二端口网络具有相同的网络参量矩阵，这两个二端口网络则可认为是等效的
  - Y形网络和Δ形网络等价，显然它们的电阻必须满足某种关系
    - 求Y形网络的z矩阵，求逆获得其y矩阵
    - 求Δ形网络的y矩阵
    - 两者相等，求出Y-Δ转换关系：R_{Δ}如何用R_Y表示？
    - 反之，R_Y 如何用R_{Δ}表示？

ABCD参量是否更简单？

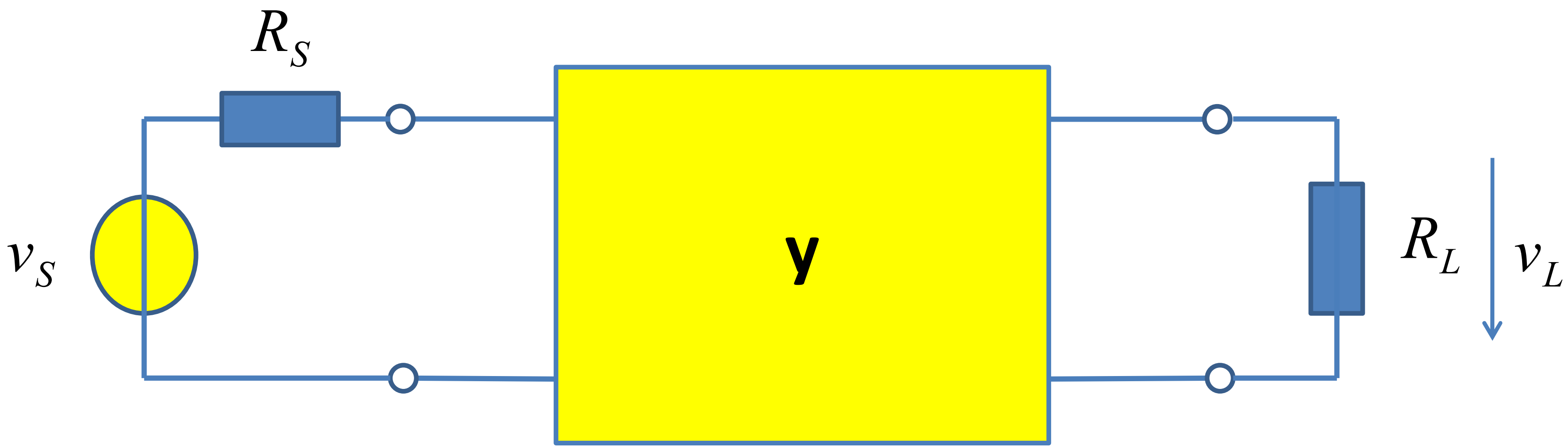


T形网络，Y形网络



π形网络，Δ形网络

作业3 单向化条件



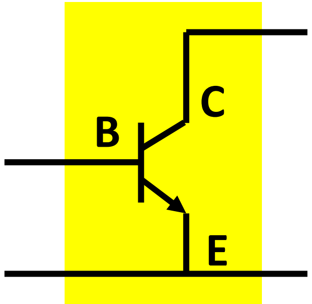
$$H_{双向网络}=\frac{v_L}{v_S}=\frac{v_2}{v_S}=\frac{y_{21}G_S}{y_{21}y_{12}-(y_{11}+G_S)(y_{22}+G_L)}\qquad H_{单向网络}\overset{y_{12}=0}{=}\frac{y_{21}R_L}{-(1+R_Sy_{11})(1+R_Ly_{22})}$$

如果满足单向化条件： $|y_{21}y_{12}| << |(y_{11}+G_S)(y_{22}+G_L)|$

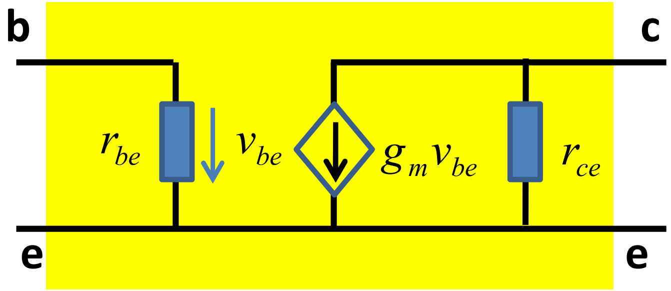
双向网络则可等视为单向网络 $H_{双向网络}\approx H_{单向网络}$ 单向网络可以直接写传递函数

给出用z参量、h参量、g参量表述的线性二端口网络的单向化条件

BJT 交流小信号电路模型



Common Emitter

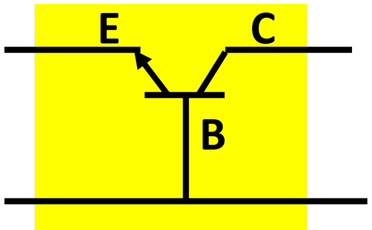


CE：共射组态

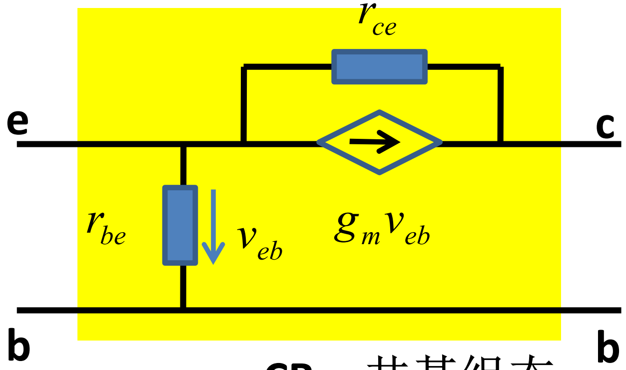
$$g_m = 40mS$$

$$r_{be} = 10k\Omega$$

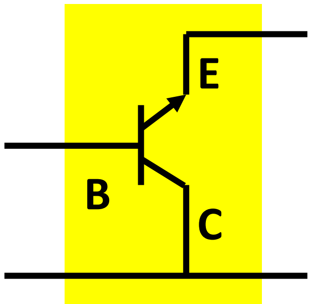
$$r_{ce} = 100k\Omega$$



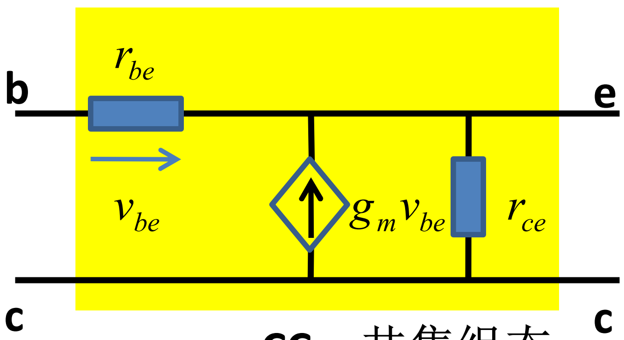
Common Base



CB：共基组态

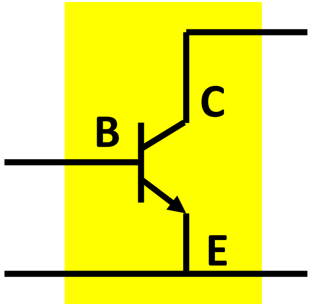


Common Collector

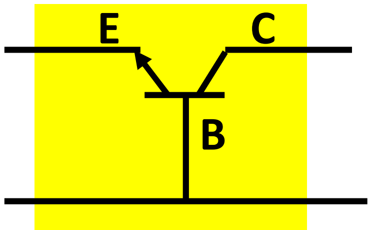


CC：共集组态

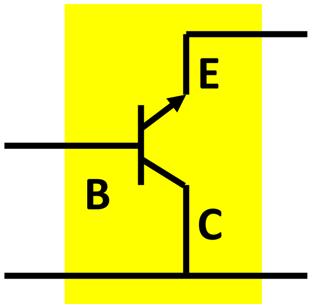
作业4 求电压放大倍数



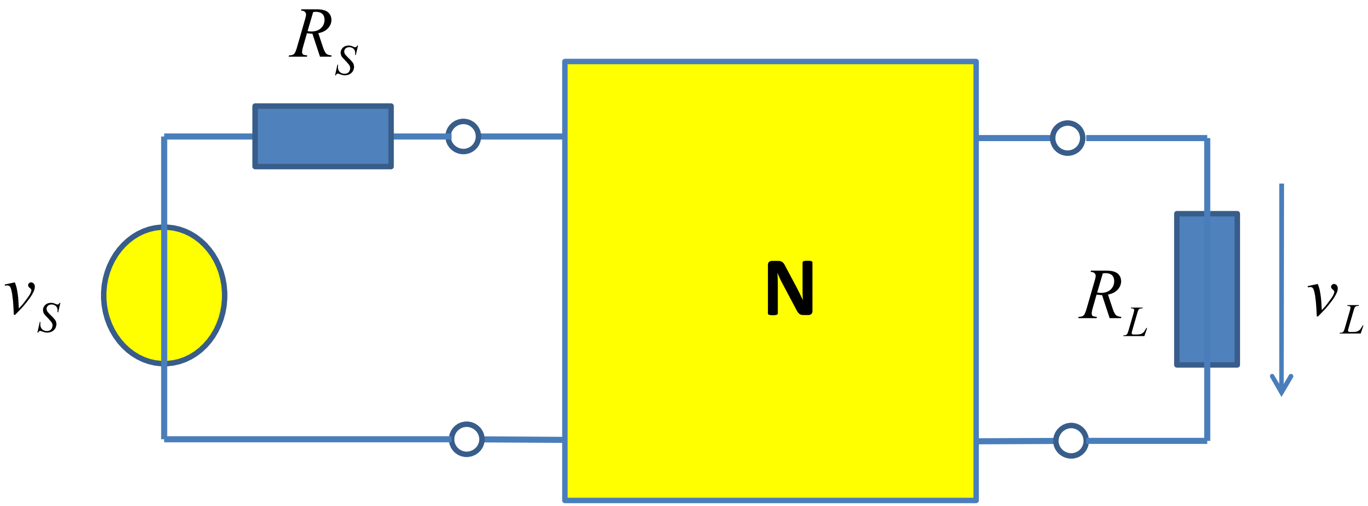
Common Emitter



Common Base



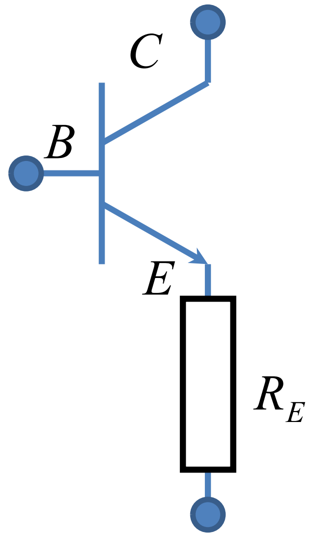
Common Collector



求三种组态晶体管放大器的输入电阻，输出电阻，电压传递函数表达式（符号表达式），代入具体数值求其电压放大倍数（R_S=50Ω,R_L=1kΩ ）

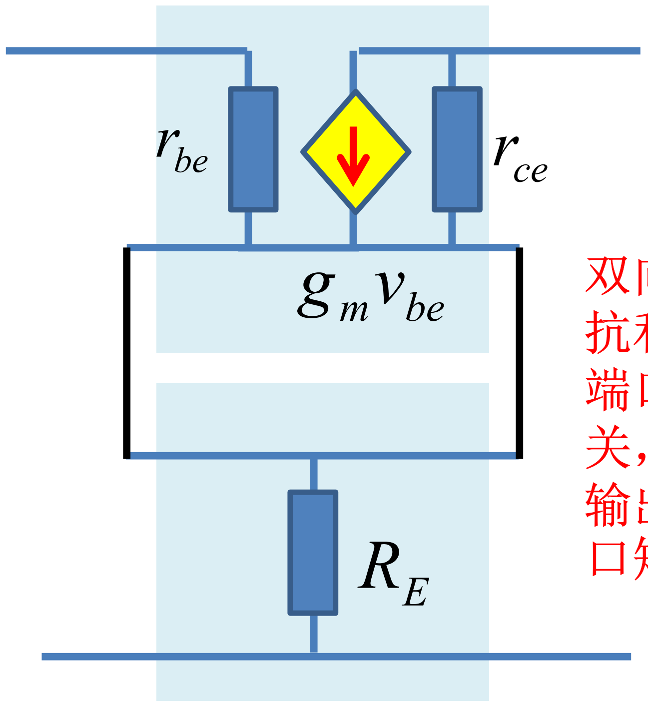
方法不限：可以用回路电流法，结点电压法，二端口网络参量法

作业5：串联负反馈



$$g_m = 40mS$$
$$r_{be} = 10k\Omega$$
$$r_{ce} = 100k\Omega$$
$$R_E = 100\Omega, 1k\Omega$$

- 负反馈电阻R_E和BJT是串串连接关系，求
  - 总导纳参量**y**
    - 先求总阻抗参量**z**，再求逆
    - 先符号运算，再代入具体数值
  - 思考：如果负反馈电阻很大，串串负反馈形成的跨导放大器的输入电阻(1/y_{11})、输出电阻(1/y_{22})、跨导增益(y_{21})有无简单表达式？
  - 加串串负反馈电阻后，形成的跨导放大器和原始晶体管跨导放大器有何好处？

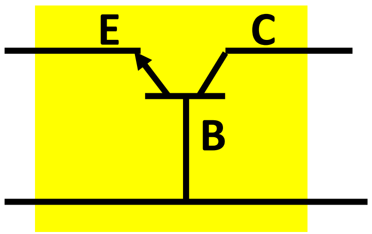


双向网络输入阻抗和输出阻抗和端口外接负载有关，这里的输入、输出阻抗特指端口短路阻抗

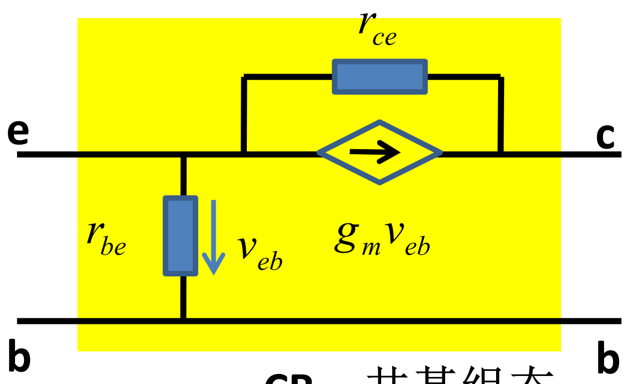
$$g_m r_{be} >> 1; g_m r_{ce} >> 1;$$
$$r_{be}, r_{ce} >> R_E; g_m R_E \overset{??}{>>} 1$$

CAD

- 画出右侧**CB**组态等效电路，用**CAD**工具获得该二端口网络的**y**参量
- 画出**y**参量等效电路，确认确实是等效电路
  - 一样的输入，导致一样的输出



Common Base



CB：共基组态