电子电路与系统基础I

习题课第十三讲

第十一讲作业讲解
第十二讲作业讲解（部分）

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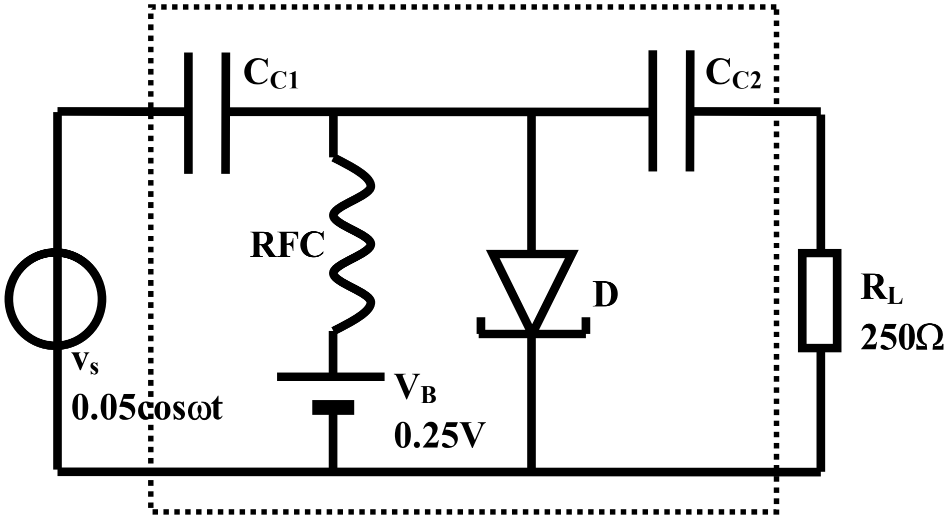
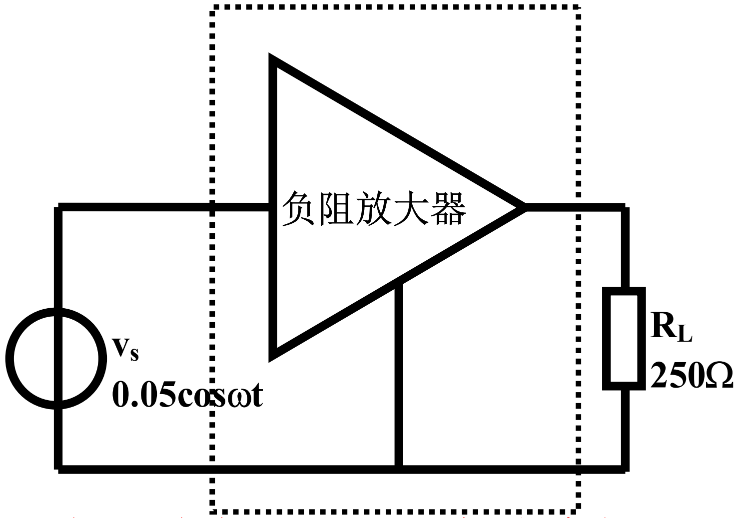
第十一讲作业　　作业1、2、3

交流小信号放大器网络参量及其对应等效电路

- （1）给出图示二端口网络的网络参量（交流小信号参量，自选zyhg和S参数）
  - 负阻放大器　　p32图
  - 反射型负阻放大器　　p33图
  - CE组态晶体管放大器　　p48图
- （2）给出对应参量的等效电路模型
- （3）求放大器输入阻抗和输出阻抗
  - 考虑信源内阻、负载电阻的影响
- （4）根据网络参量具体数值说明其有源性

负阻放大器抽象





负阻放大器就是一个并臂负阻

$$G_p = \frac{\overline{p_{out}}}{\overline{p_{in}}} = \frac{G_L}{G_L + g_d} > 1$$

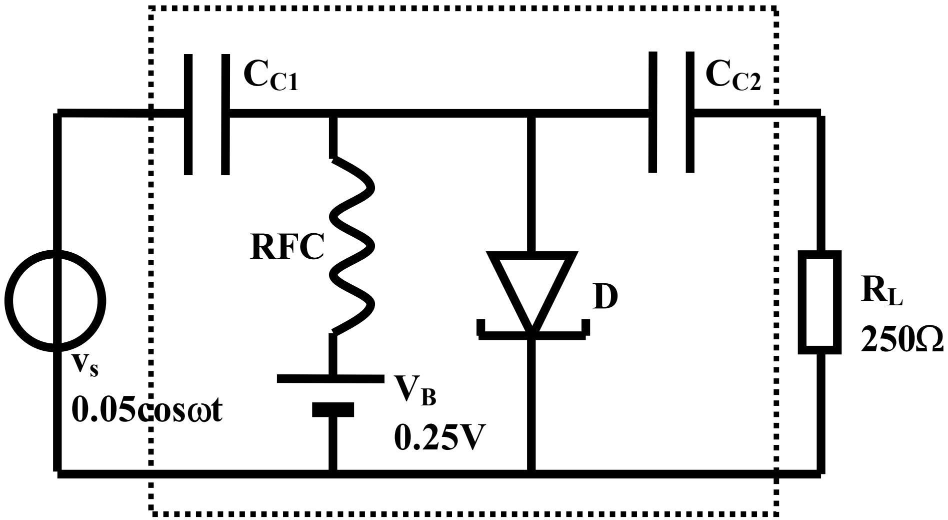
$$g_d < 0$$

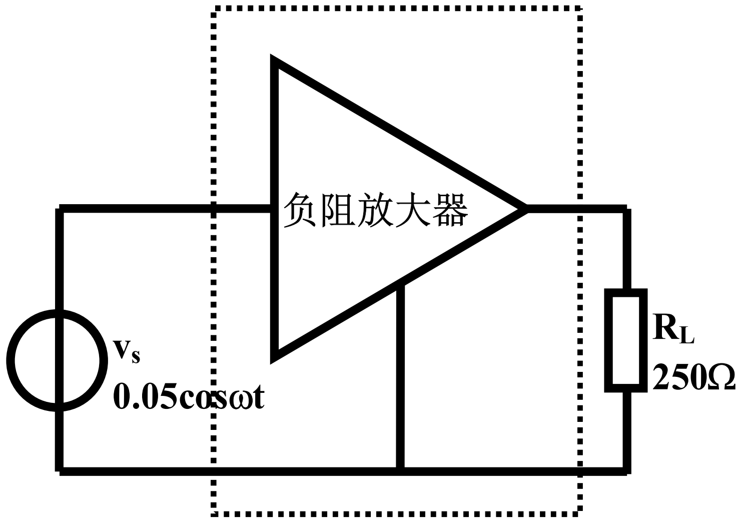
作业**1**：

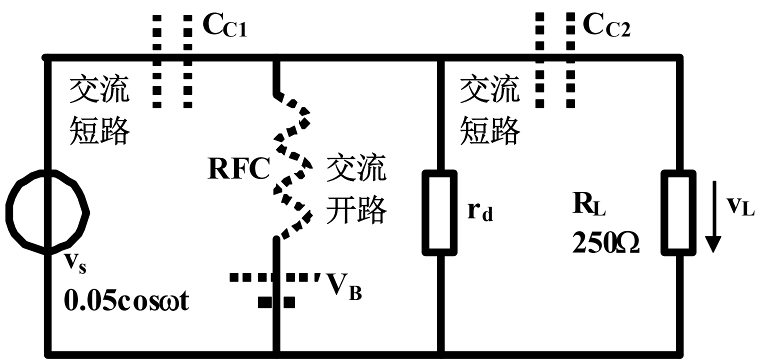
（**1**）给出图示虚框二端口网络的网络参量（自选**zyhg**）

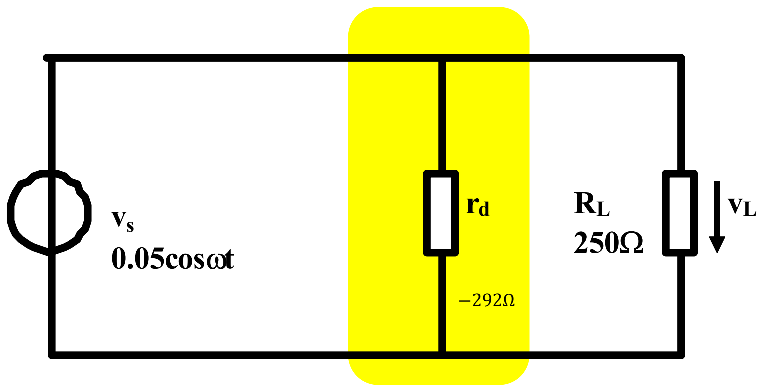
（**2**）给出对应参量的等效电路模型

（**3**）求放大器输入阻抗和输出阻抗

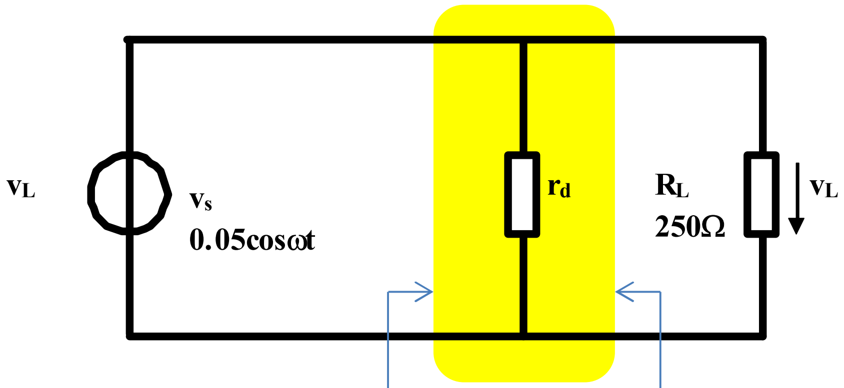








直流工作点位于负阻区，微分电阻为负阻



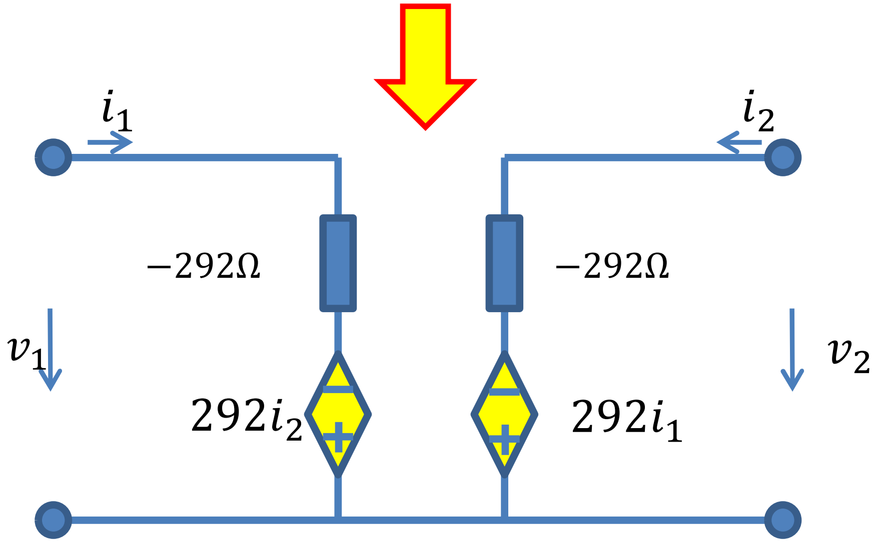
$$\mathbf{z} = \begin{bmatrix} r_d & r_d \\ r_d & r_d \end{bmatrix} = \begin{bmatrix} -292 & -292 \\ -292 & -292 \end{bmatrix} \Omega$$

并臂电阻，**z**参量

$$v_1 = (i_1 + i_2) r_d$$
$$v_2 = (i_1 + i_2) r_d$$

$$R_{in} = r_d || R_L = \frac{r_d R_L}{r_d + R_L} = \frac{-292 \times 250}{-292 + 250} = 1.74 k\Omega$$

$$R_{out} = r_d || R_S = 0$$

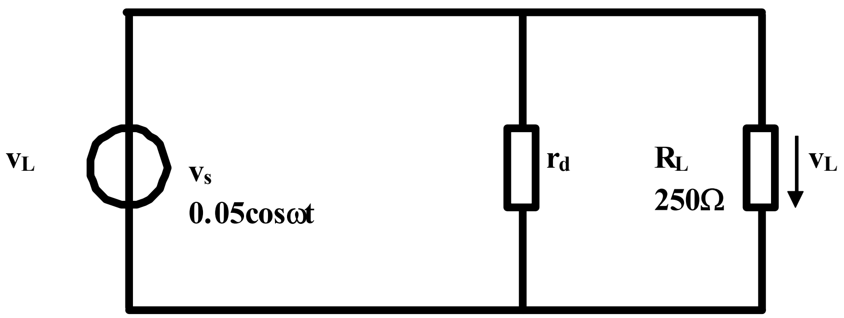


等效电路模型

$$R_{in} = z_{11} - \frac{z_{12} z_{21}}{R_L + z_{22}}$$

$$R_{out} = z_{22} - \frac{z_{12} z_{21}}{R_S + z_{11}}$$

有源性



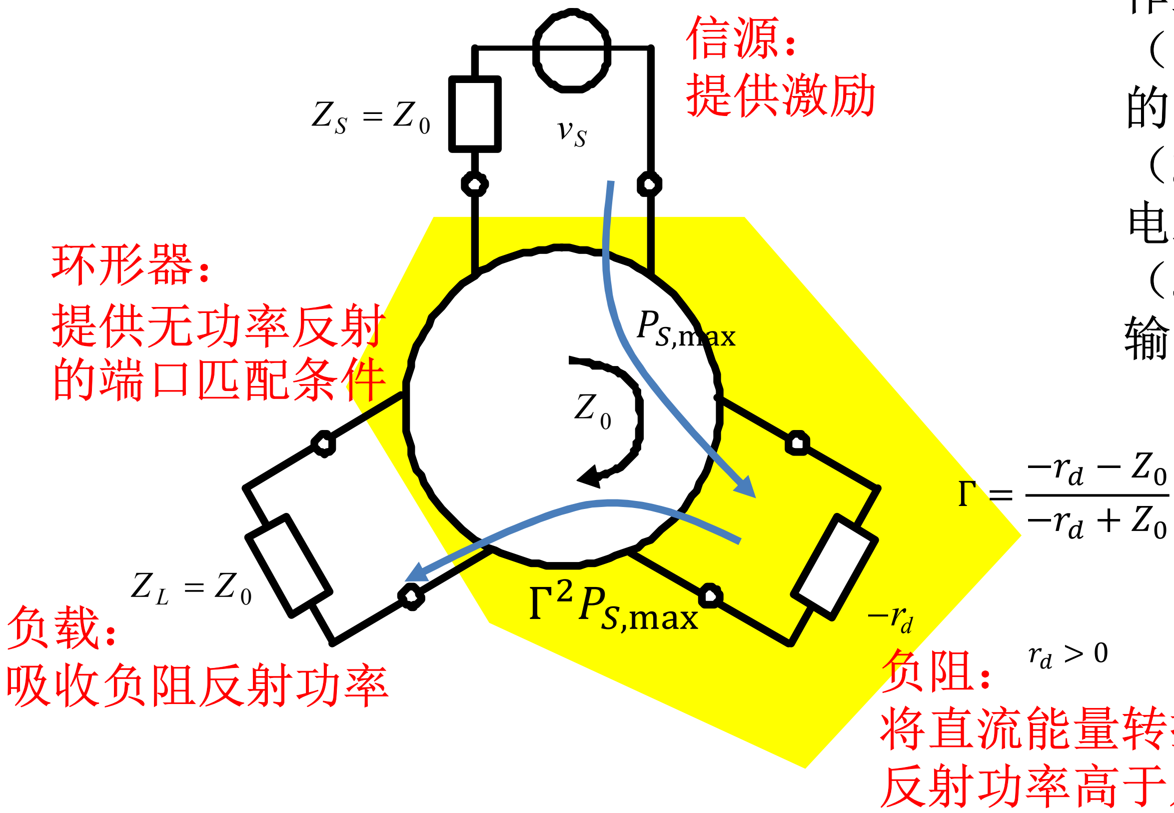
$$\mathbf{z}=\begin{bmatrix} r_d & r_d \\ r_d & r_d \end{bmatrix}=\begin{bmatrix} -292 & -292 \\ -292 & -292 \end{bmatrix}\Omega$$

只要端口电流之和不为**0**，
只要有电流流过负阻，
负阻即向外输出功率，
故而有源：具有向外输
出功率的能力则称之为
有源

$$\begin{aligned} p &= v_1 i_1 + v_2 i_2 \\ &= (r_d i_1 + r_d i_2) i_1 + (r_d i_1 + r_d i_2) i_2 \\ &= r_d (i_1^2 + 2 i_1 i_2 + i_2^2) \\ &= r_d (i_1 + i_2)^2 \\ &= -292 \cdot (i_1 + i_2)^2 < 0 \end{aligned}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 & Z_0 & -Z_0 \\ -Z_0 & 0 & Z_0 \\ Z_0 & -Z_0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

反射型负阻放大器



作业2：

（1）给出图示二端口网络的网络参量（自选zyhg）

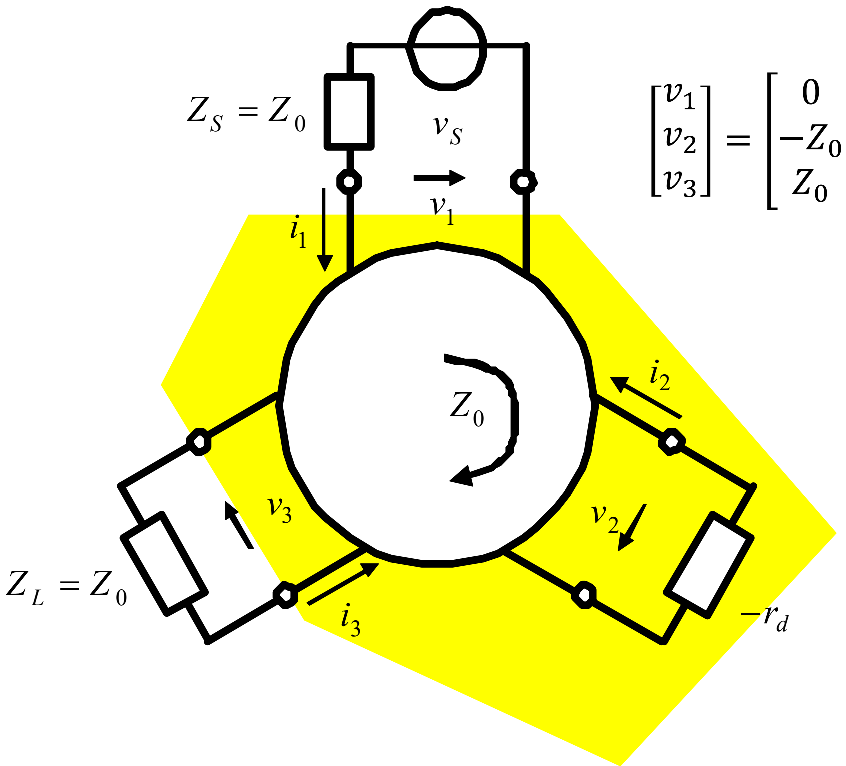
（2）给出对应参量的等效电路模型

（3）求放大器输入阻抗和输出阻抗（端接匹配情况）

$Z_0 = 250\Omega$

$$P_L = \Gamma^2 P_{Smax} = \left(\frac{Z_0 + r_d}{Z_0 - r_d}\right)^2 P_{Smax}$$

$$G_T = \frac{P_L}{P_{Smax}} = \Gamma^2 = \left(\frac{Z_0 + r_d}{Z_0 - r_d}\right)^2$$



$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 & Z_0 & -Z_0 \\ -Z_0 & 0 & Z_0 \\ Z_0 & -Z_0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$$v_1 = Z_0 i_2 - Z_0 i_3$$
$$v_2 = Z_0 i_3 - Z_0 i_1$$
$$v_3 = Z_0 i_1 - Z_0 i_2$$

环行器元件约束

$$v_2 = r_d i_2$$

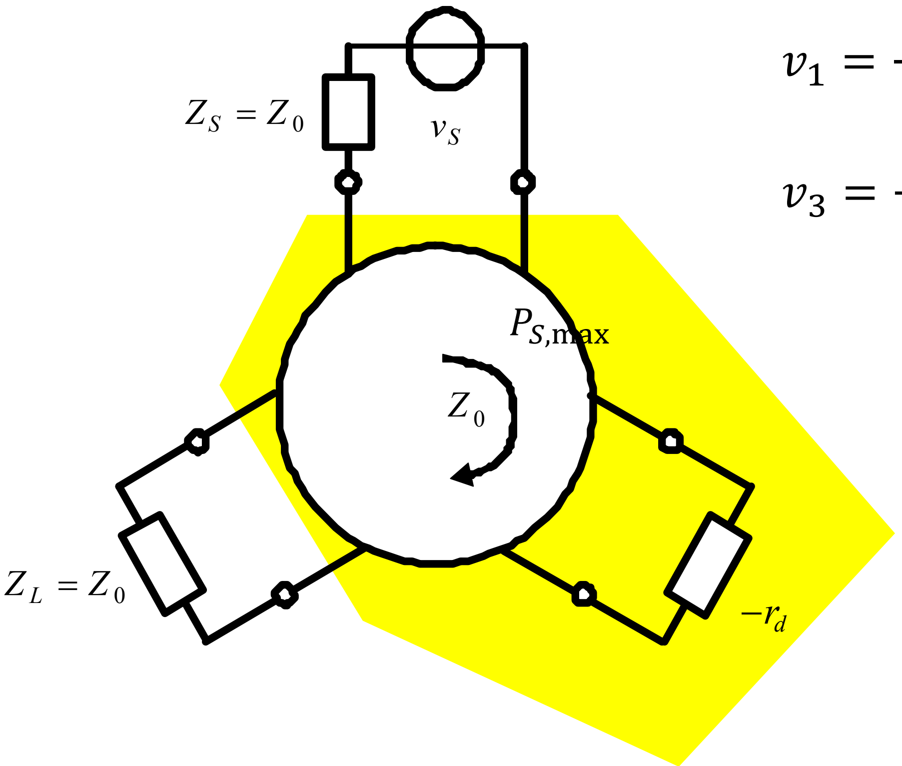
端口**2**接负阻：负阻约束

$$v_2 = Z_0 i_3 - Z_0 i_1 = r_d i_2$$

$$i_2 = \frac{Z_0}{r_d} i_3 - \frac{Z_0}{r_d} i_1$$

$$v_1 = Z_0 i_2 - Z_0 i_3 = Z_0 \left(\frac{Z_0}{r_d} i_3 - \frac{Z_0}{r_d} i_1 \right) - Z_0 i_3 = -\frac{Z_0^2}{r_d} i_1 + \left(\frac{Z_0^2}{r_d} - Z_0 \right) i_3$$

$$v_3 = Z_0 i_1 - Z_0 i_2 = Z_0 i_1 - Z_0 \left(\frac{Z_0}{r_d} i_3 - \frac{Z_0}{r_d} i_1 \right) = \left(\frac{Z_0^2}{r_d} + Z_0 \right) i_1 - \frac{Z_0^2}{r_d} i_3$$



$$v_1 = -\frac{Z_0^2}{r_d} i_1 + \left(\frac{Z_0^2}{r_d} - Z_0\right) i_3$$

$$v_3 = +\left(\frac{Z_0^2}{r_d} + Z_0\right) i_1 - \frac{Z_0^2}{r_d} i_3$$

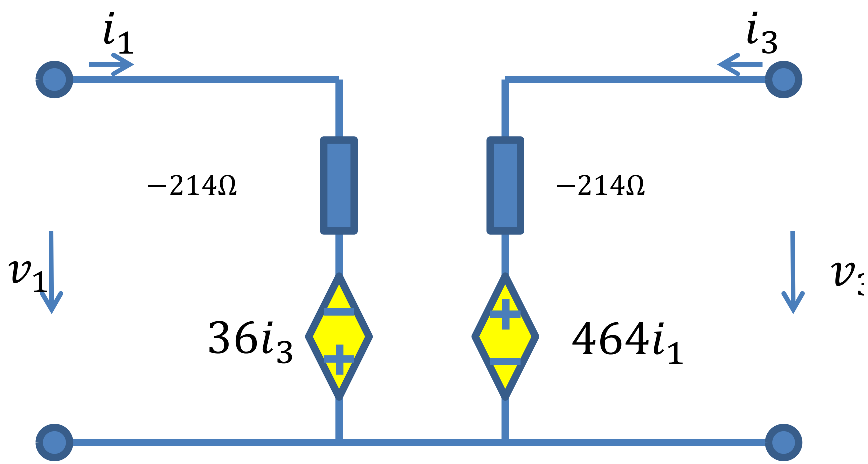
$$Z_0 = 250\Omega, r_d = 292\Omega$$

$$\mathbf{z} = \begin{bmatrix} -\frac{Z_0^2}{r_d} & \frac{Z_0^2}{r_d} - Z_0 \\ \frac{Z_0^2}{r_d} + Z_0 & -\frac{Z_0^2}{r_d} \end{bmatrix} = \begin{bmatrix} -214 & -36 \\ 464 & -214 \end{bmatrix} \Omega$$

非互易网络

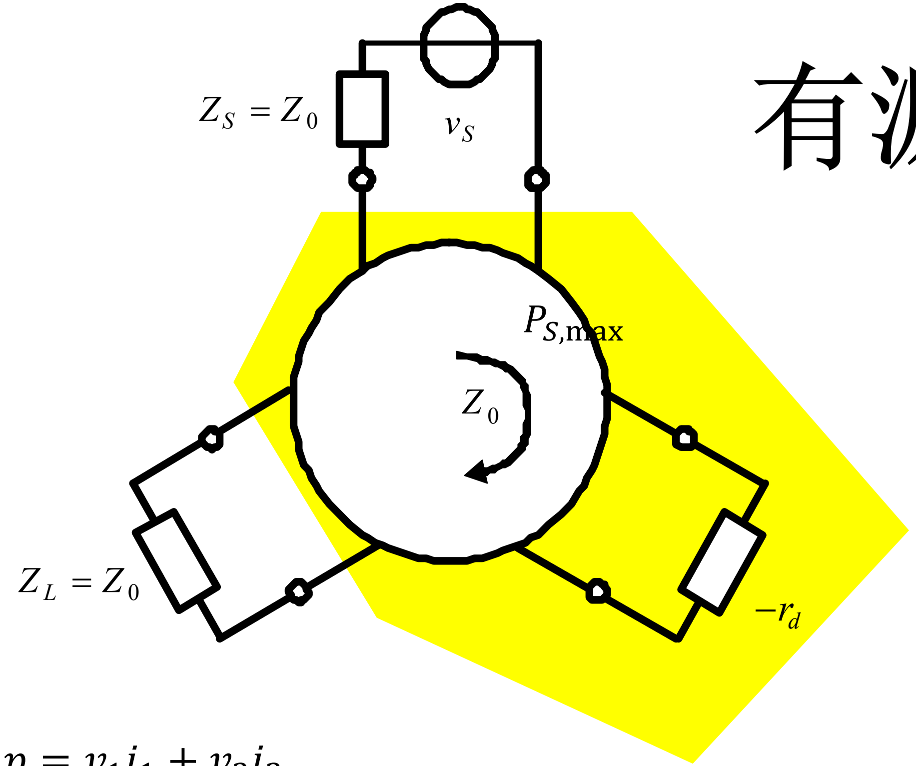
$$R_{in} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + R_L} = -\frac{Z_0^2}{r_d} - \frac{\left(\frac{Z_0^2}{r_d} - Z_0\right)\left(\frac{Z_0^2}{r_d} + Z_0\right)}{-\frac{Z_0^2}{r_d} + Z_0} = Z_0$$

$$R_{out} = z_{22} - \frac{z_{12} z_{21}}{z_{11} + R_S} = -\frac{Z_0^2}{r_d} - \frac{\left(\frac{Z_0^2}{r_d} - Z_0\right)\left(\frac{Z_0^2}{r_d} + Z_0\right)}{-\frac{Z_0^2}{r_d} + Z_0} = Z_0$$



通过环行器，可以实现双端同时匹配

有源性



$$\mathbf{z} = \begin{bmatrix} -\dfrac{Z_0^2}{r_d} & \dfrac{Z_0^2}{r_d} - Z_0 \\ \dfrac{Z_0^2}{r_d} + Z_0 & -\dfrac{Z_0^2}{r_d} \end{bmatrix}$$

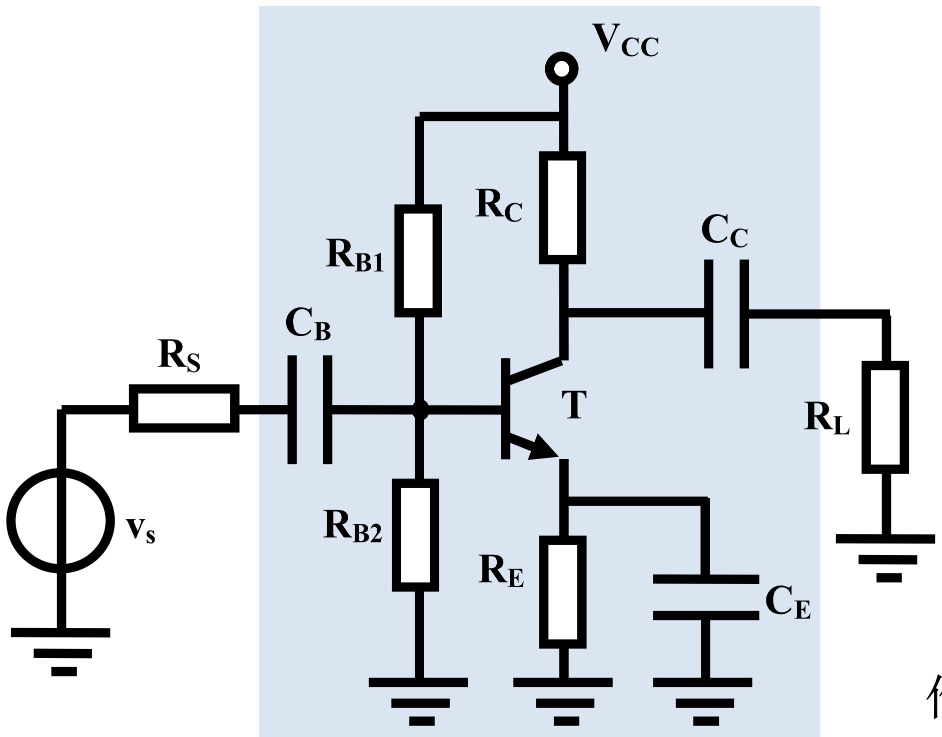
$$\begin{aligned} p &= v_1 i_1 + v_3 i_3 \\ &= \left(-\frac{Z_0^2}{r_d} i_1 + \left(\frac{Z_0^2}{r_d} - Z_0\right) i_3\right) i_1 + \left(+\left(\frac{Z_0^2}{r_d} + Z_0\right) i_1 - \frac{Z_0^2}{r_d} i_3\right) i_3 \\ &= -\frac{Z_0^2}{r_d} i_1^2 - \frac{Z_0^2}{r_d} i_3^2 + 2\frac{Z_0^2}{r_d} i_1 i_3 \\ &= -\frac{Z_0^2}{r_d}(i_1 - i_3)^2 < 0 \\ &= -\frac{v_2^2}{r_d} < 0 \end{aligned}$$

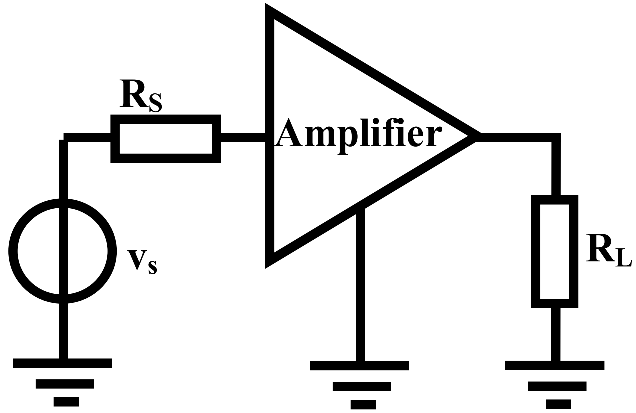
环形器+负阻：向外释放的纯功率全部是负阻释放的

只要端口电流之差不为**0**，只要有电压加载到负阻两端，负阻即向外输出功率，故有源

$$v_2 = Z_0 i_3 - Z_0 i_1$$

晶体管放大器抽象



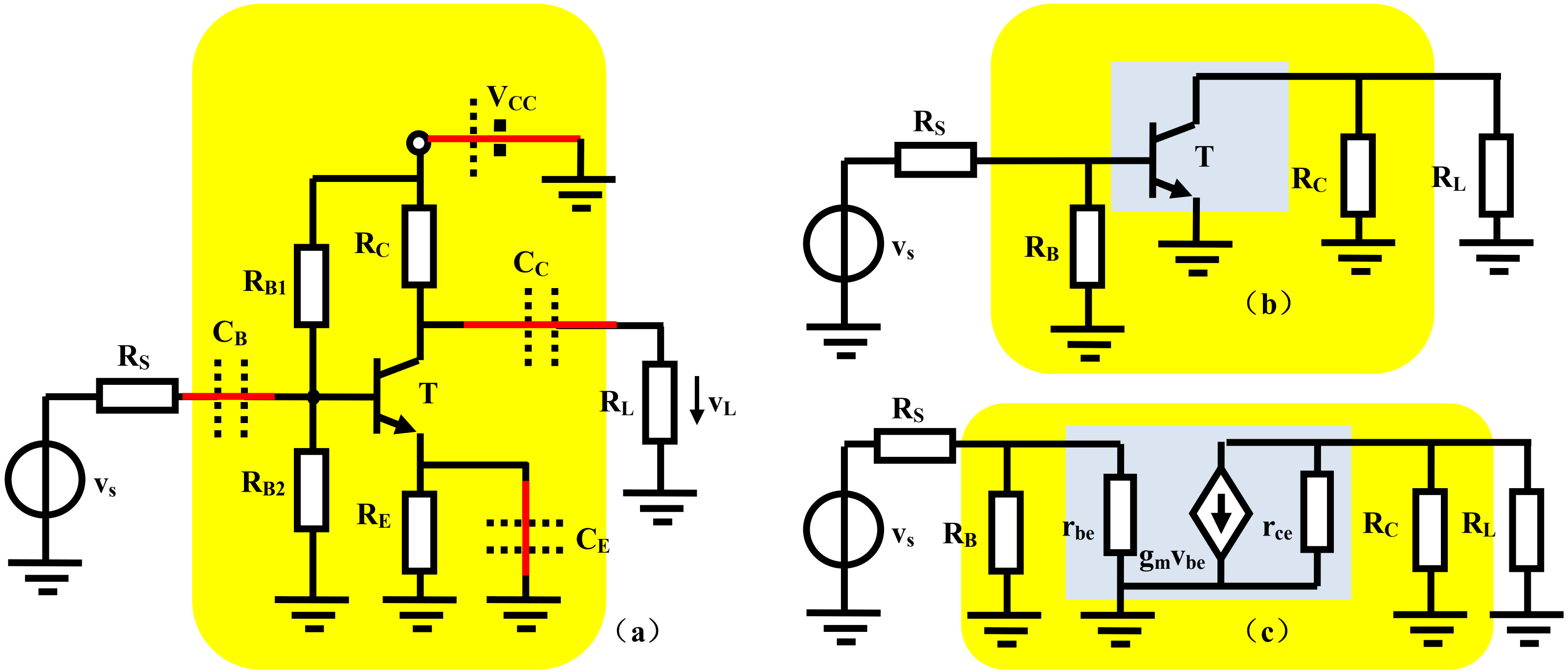


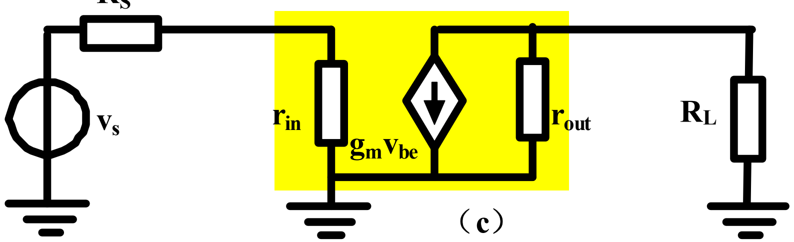
作业**3**：

（**1**）给出图示二端口网络的网络参量（自选**zyhg**）

（**2**）给出对应参量的等效电路模型

（**3**）求放大器输入阻抗和输出阻抗



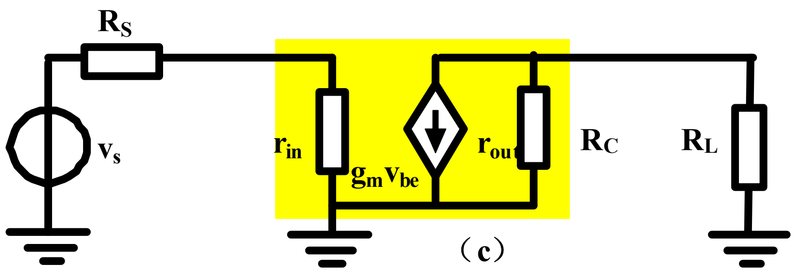


$$\mathbf{y} = \begin{bmatrix} g_{in} & 0 \\ g_m & g_{out} \end{bmatrix} = \begin{bmatrix} g_{be} + G_B & 0 \\ g_m & g_{ce} + G_C \end{bmatrix} = \begin{bmatrix} 0.256 & 0 \\ 41.5 & 0.189 \end{bmatrix} mS$$

$$r_{in} = \frac{1}{g_{be} + G_B} = r_{be} || R_B = 7.22k\Omega || 8.48k\Omega = 3.90k\Omega$$

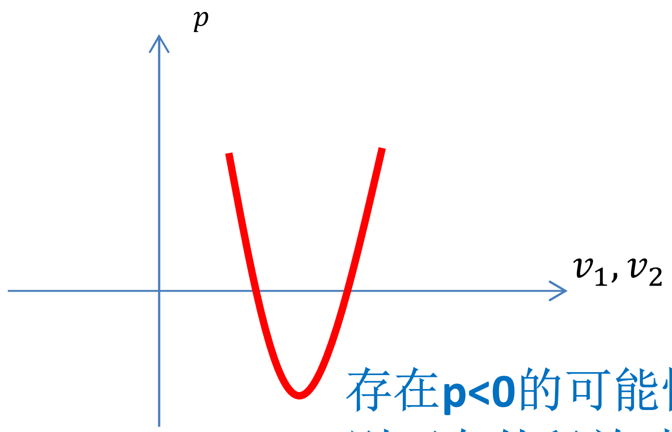
$$r_{out} = \frac{1}{g_{ce} + G_C} = r_{ce} || R_C = 92.6k\Omega || 5.6k\Omega = 5.28k\Omega$$

有源性



$$\mathbf{y} = \begin{bmatrix} g_{in} & 0 \\ g_m & g_{out} \end{bmatrix} = \begin{bmatrix} g_{be} + G_B & 0 \\ g_m & g_{ce} + G_C \end{bmatrix} = \begin{bmatrix} 0.256 & 0 \\ 41.5 & 0.189 \end{bmatrix} mS$$

$$p = v_1 i_1 + v_2 i_2$$
$$= v_1\big((g_{be} + G_B)v_1\big) + v_2\big(g_m v_1 + (g_{ce} + G_C)v_2\big)$$
$$= (g_{be} + G_B)v_1^2 + g_m v_1 v_2 + (g_{ce} + G_C)v_2^2$$

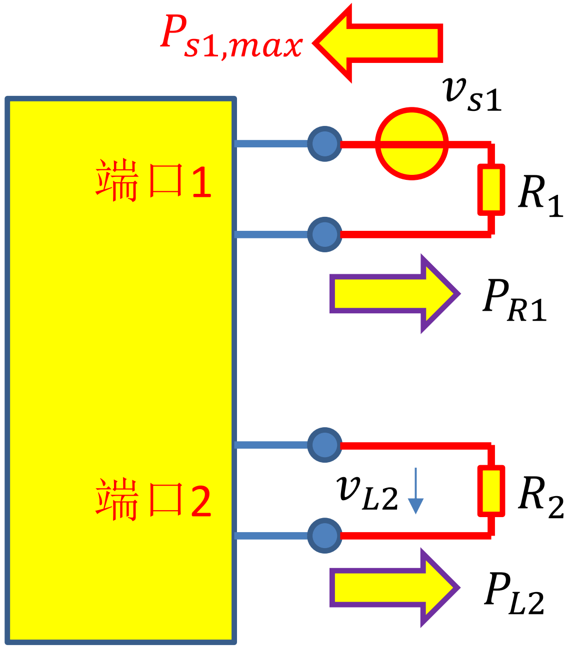


存在**p<0**的可能性，
则可向外释放功率，
故而有源

$$p < 0: \Delta = {g_m}^2 - 4(g_{be} + G_B)(g_{ce} + G_C) > 0$$

$$\Delta = {g_m}^2 - 4(g_{be} + G_B)(g_{ce} + G_C)$$
$$= 41.5^2 - 4 \times 0.256 \times 0.189 = 1722 > 0$$

S参量

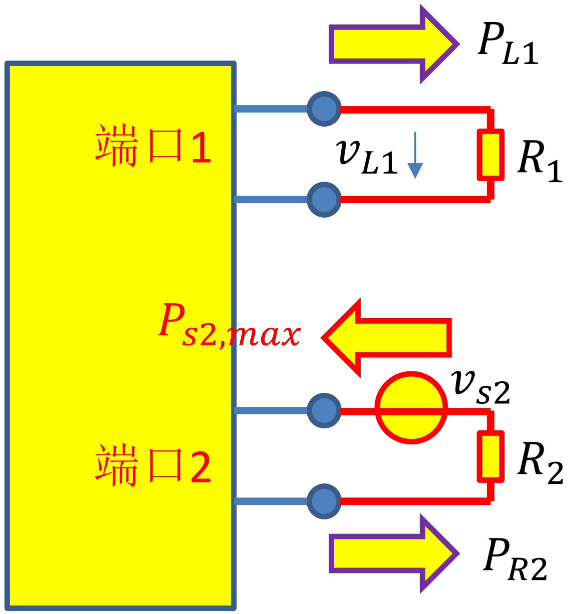


$$|s_{11}|^2 = \frac{P_{R1}}{P_{s1,max}}$$

$$|s_{21}|^2 = \frac{P_{L2}}{P_{s1,max}}$$

$$\boldsymbol{s}_{\boldsymbol{R_1,R_2}} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} = \begin{bmatrix} \frac{z_{in}-R_1}{z_{in}+R_1} & 2\sqrt{\frac{R_2}{R_1}}\frac{v_{L1}}{v_{S2}} \\ 2\sqrt{\frac{R_1}{R_2}}\frac{v_{L2}}{v_{S1}} & \frac{z_{out}-R_2}{z_{out}+R_2} \end{bmatrix}$$

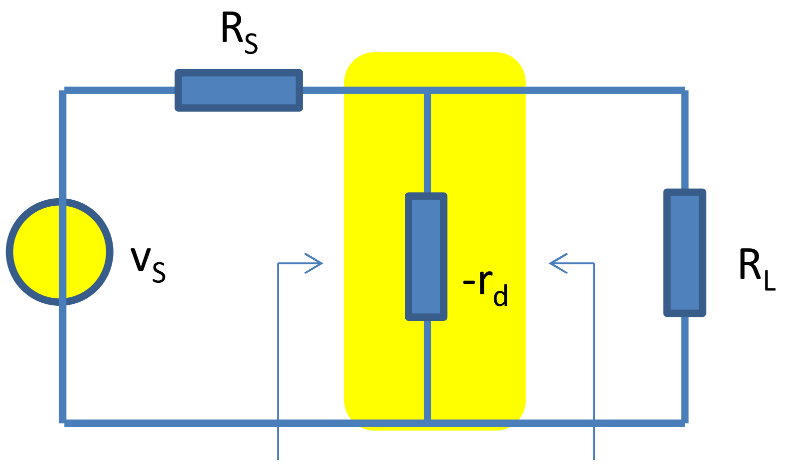
要求掌握S参量定义下的S参量计算，明确其物理意义



$$|s_{12}|^2 = \frac{P_{L1}}{P_{s2,max}}$$

$$|s_{22}|^2 = \frac{P_{R2}}{P_{s2,max}}$$

负阻放大器：直接型



$$s_{11}=\frac{z_{in}-R_S}{z_{in}+R_S}=\frac{R_L||(-r_d)-R_S}{R_L||(-r_d)+R_S}=\frac{\frac{-R_Lr_d}{R_L-r_d}-R_S}{\frac{-R_Lr_d}{R_L-r_d}+R_S}$$

$$=\frac{-R_Lr_d-R_SR_L+R_Sr_d}{-R_Lr_d+R_SR_L-R_Sr_d}\overset{R_S=R_L}{\triangleq}\frac{-R_SR_L}{-r_d(R_S+R_L)+R_SR_L}$$

$$=\frac{1}{\frac{r_d}{R_S||R_L}-1}=\frac{1}{\frac{292}{25}-1}=0.0936$$

二端口网络式对称，故而

$$s_{22}=s_{11}$$
$$s_{12}=s_{21}$$

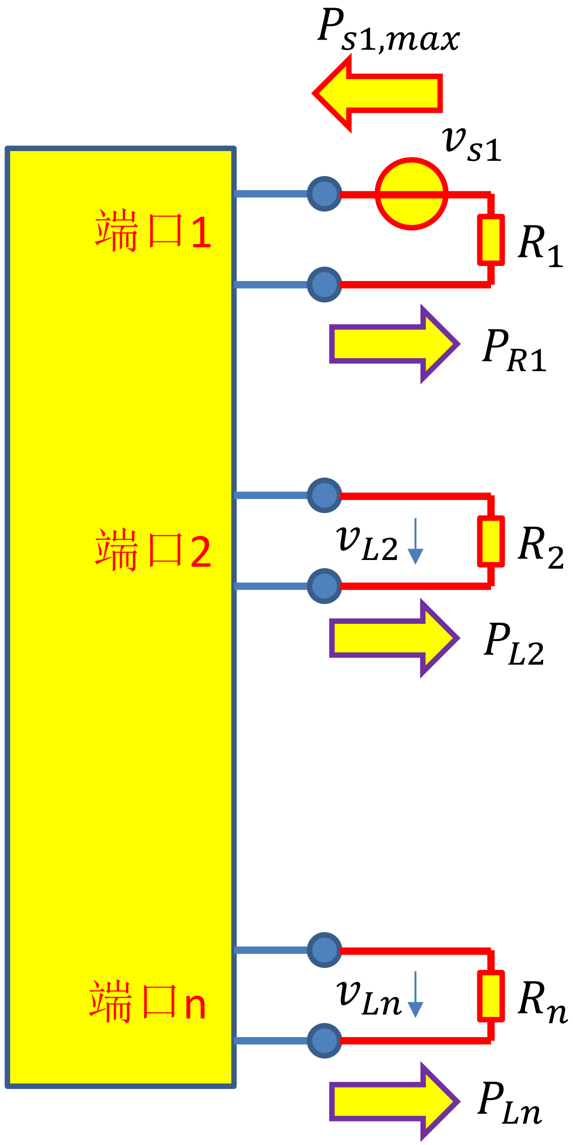
$$s_{21}=2\sqrt{\frac{R_S}{R_L}}\frac{v_{L2}}{v_{S1}}=2\sqrt{\frac{R_S}{R_L}}\frac{R_L||(-r_d)}{R_L||(-r_d)+R_S}$$

$$=2\sqrt{\frac{R_S}{R_L}}\frac{\frac{-R_Lr_d}{R_L-r_d}}{\frac{-R_Lr_d}{R_L-r_d}+R_S}=2\sqrt{\frac{R_S}{R_L}}\frac{-R_Lr_d}{-r_d(R_S+R_L)+R_SR_L}$$

$$\boldsymbol{s}_{R_S=50\Omega,R_L=50\Omega}=\begin{bmatrix}0.0936 & 1.0936\\ 1.0936 & 0.0936\end{bmatrix}$$

$$=2\frac{-50\times 292}{-292\times 100+50\times 50}=1.0936$$

散射参量和阻抗导纳参量之间可相互转换：n端口网络



归一化阻抗参量

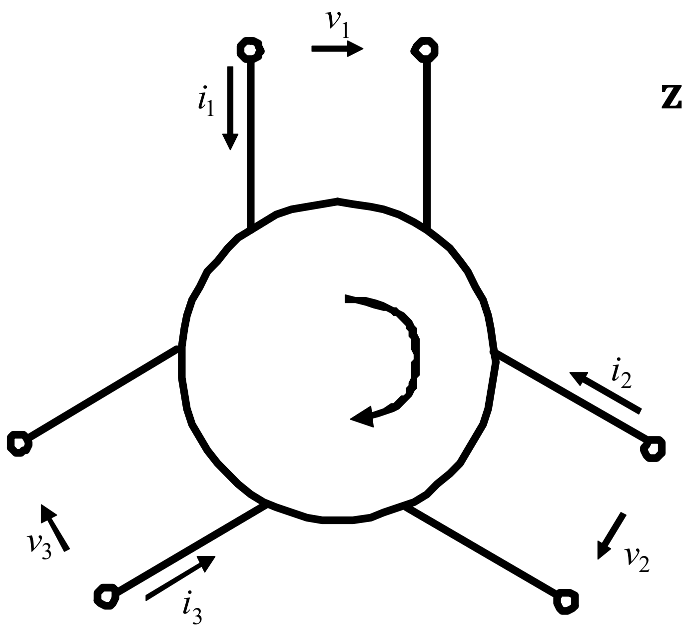
$$\hat{z}=\begin{bmatrix}\frac{1}{\sqrt{R_1}} & & \\ & \cdot & \\ & & \frac{1}{\sqrt{R_n}}\end{bmatrix} z \begin{bmatrix}\frac{1}{\sqrt{R_1}} & & \\ & \cdot & \\ & & \frac{1}{\sqrt{R_n}}\end{bmatrix}$$

$$s=(\hat{z}+I)^{-1}(\hat{z}-I) \qquad s=(I-\hat{y})(I+\hat{y})^{-1}$$

$$\hat{z}=(I+s)(I-s)^{-1} \qquad \hat{y}=(I-s)(I+s)^{-1}$$

$$z=\begin{bmatrix}\sqrt{R_1} & & \\ & \cdot & \\ & & \sqrt{R_n}\end{bmatrix} \hat{z} \begin{bmatrix}\sqrt{R_1} & & \\ & \cdot & \\ & & \sqrt{R_n}\end{bmatrix}$$

环行器例



$$\mathbf{z} = \begin{bmatrix} 0 & R & -R \\ -R & 0 & R \\ R & -R & 0 \end{bmatrix}$$

在$R_1 = R_2 = R_3 = Z_0 = R$条件下

$$\hat{z} = \begin{bmatrix} \frac{1}{\sqrt{R}} & & \\ & \frac{1}{\sqrt{R}} & \\ & & \frac{1}{\sqrt{R}} \end{bmatrix} \begin{bmatrix} 0 & R & -R \\ -R & 0 & R \\ R & -R & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{R}} & & \\ & \frac{1}{\sqrt{R}} & \\ & & \frac{1}{\sqrt{R}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$s = (\hat{z} + I)^{-1}(\hat{z} - I)$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 & -1 \\ -1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

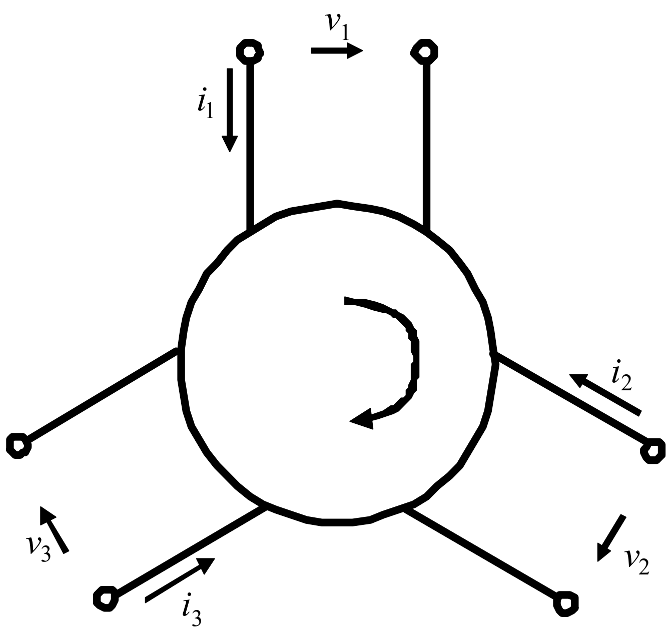
$$= \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

在三个端口均端接匹配时，端口**1**信号只能反相无损传输到端口**2**，端口**2**信号只能反相无损传输到端口**3**，端口**3**信号只能反相无损传输到端口**1**：环行

思考题

$$S_{R_1=R_2=R_3=Z_0} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$z = \begin{bmatrix} 0 & Z_0 & -Z_0 \\ -Z_0 & 0 & Z_0 \\ Z_0 & -Z_0 & 0 \end{bmatrix}$$



$$S_{R_1=R_2=R_3=Z_0} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

请给出该理想环行器的**zy**参量矩阵？

n端口线性网络的**S**参量一定存在，但**zy**参量可能不存在

二端口网络的散射参量和阻抗导纳参量之间的转换关系式

$$s=(\hat{z}+I)^{-1}(\hat{z}-I) \qquad s=(I-\hat{y})(I+\hat{y})^{-1}$$

$$s_{11}=\frac{\Delta\hat{z}+\hat{z}_{11}-\hat{z}_{22}-1}{\Delta\hat{z}+\hat{z}_{11}+\hat{z}_{22}+1} \qquad s_{11}=-\frac{\Delta\hat{y}+\hat{y}_{11}-\hat{y}_{22}-1}{\Delta\hat{y}+\hat{y}_{11}+\hat{y}_{22}+1}$$

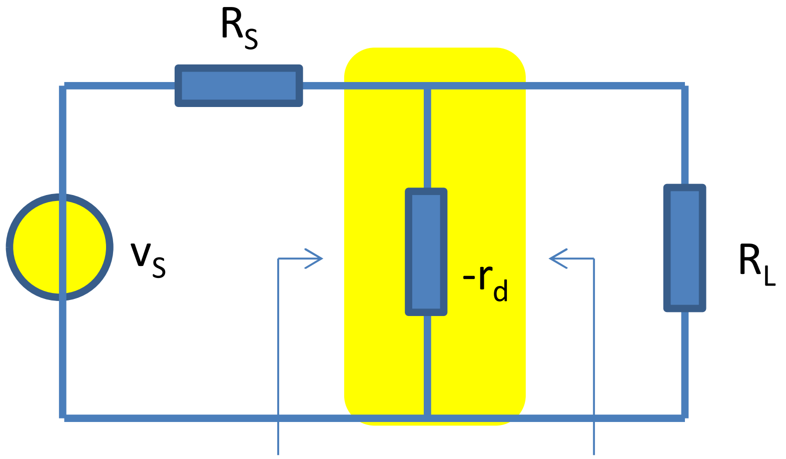
$$s_{12}=\frac{2\hat{z}_{12}}{\Delta\hat{z}+\hat{z}_{11}+\hat{z}_{22}+1} \qquad s_{12}=-\frac{2\hat{y}_{12}}{\Delta\hat{y}+\hat{y}_{11}+\hat{y}_{22}+1}$$

$$s_{21}=\frac{2\hat{z}_{21}}{\Delta\hat{z}+\hat{z}_{11}+\hat{z}_{22}+1} \qquad s_{21}=-\frac{2\hat{y}_{21}}{\Delta\hat{y}+\hat{y}_{11}+\hat{y}_{22}+1}$$

$$s_{22}=\frac{\Delta\hat{z}+\hat{z}_{22}-\hat{z}_{11}-1}{\Delta\hat{z}+\hat{z}_{11}+\hat{z}_{22}+1} \qquad s_{22}=-\frac{\Delta\hat{y}+\hat{y}_{22}-\hat{y}_{11}-1}{\Delta\hat{y}+\hat{y}_{11}+\hat{y}_{22}+1}$$

$$\hat{z}=(I+s)(I-s)^{-1} \qquad \hat{y}=(I-s)(I+s)^{-1}$$

套公式计算



$$z=\begin{bmatrix}-r_d & -r_d\\ -r_d & -r_d\end{bmatrix}$$

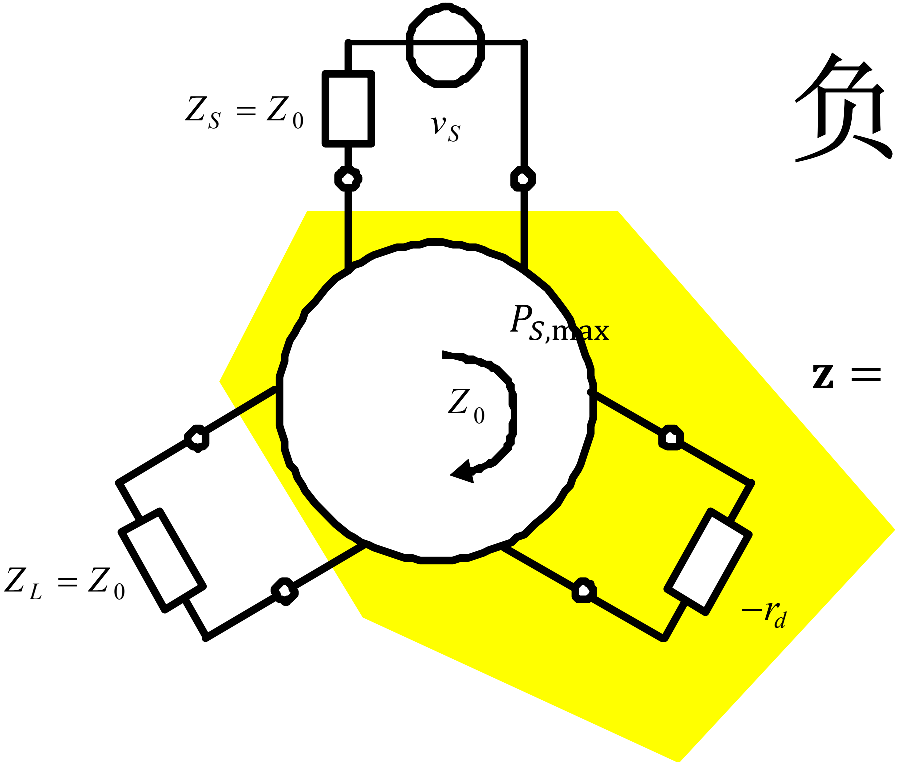
$$\hat{z}=\begin{bmatrix}\frac{1}{\sqrt{R_S}} & \\ & \frac{1}{\sqrt{R_L}}\end{bmatrix}\begin{bmatrix}-r_d & -r_d\\ -r_d & -r_d\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{R_S}} & \\ & \frac{1}{\sqrt{R_L}}\end{bmatrix}=\begin{bmatrix}-\frac{r_d}{R_S} & -\frac{r_d}{\sqrt{R_S R_L}}\\ -\frac{r_d}{\sqrt{R_S R_L}} & -\frac{r_d}{R_L}\end{bmatrix}$$

$$s_{11}=\frac{\Delta\hat{z}+\hat{z}_{11}-\hat{z}_{22}-1}{\Delta\hat{z}+\hat{z}_{11}+\hat{z}_{22}+1}=\frac{-1}{-\frac{r_d}{R_S}-\frac{r_d}{R_L}+1}=\frac{1}{\frac{r_d}{R_S||R_L}-1}=0.0936$$

$$\boldsymbol{s}_{\boldsymbol{R_S=50\Omega,R_L=50\Omega}}=\begin{bmatrix}0.0936 & 1.0936\\ 1.0936 & 0.0936\end{bmatrix}$$

$$s_{21}=\frac{2\hat{z}_{21}}{\Delta\hat{z}+\hat{z}_{11}+\hat{z}_{22}+1}=\frac{2\frac{-r_d}{\sqrt{R_S R_L}}}{-\frac{r_d}{R_S}-\frac{r_d}{R_L}+1}=\frac{2\frac{r_d}{\sqrt{R_S R_L}}}{\frac{r_d}{R_S||R_L}-1}=1.0936$$

负阻放大器：反射型



$$\mathbf{z} = \begin{bmatrix} -\frac{Z_0^2}{r_d} & \frac{Z_0^2}{r_d} - Z_0 \\ \frac{Z_0^2}{r_d} + Z_0 & -\frac{Z_0^2}{r_d} \end{bmatrix}$$

$$\hat{z}_{R_1=Z_0,R_3=Z_0} = \begin{bmatrix} -\frac{Z_0}{r_d} & \frac{Z_0}{r_d} - 1 \\ \frac{Z_0}{r_d} + 1 & -\frac{Z_0}{r_d} \end{bmatrix}$$

$$s_{11} = \frac{\Delta\hat{z} + \hat{z}_{11} - \hat{z}_{22} - 1}{\Delta\hat{z} + \hat{z}_{11} + \hat{z}_{22} + 1} = \frac{1 - 1}{1 - \frac{Z_0}{r_d} - \frac{Z_0}{r_d} + 1} = 0$$

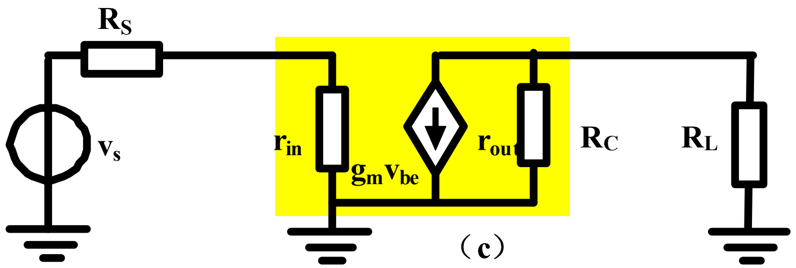
$$\mathbf{s}_{R_1=Z_0,R_2=Z_0} = \begin{bmatrix} 0 & -1 \\ \Gamma_d & 0 \end{bmatrix}$$

$$s_{12} = \frac{2\hat{z}_{12}}{\Delta\hat{z} + \hat{z}_{11} + \hat{z}_{22} + 1} = \frac{2\left(\frac{Z_0}{r_d} - 1\right)}{1 - \frac{Z_0}{r_d} - \frac{Z_0}{r_d} + 1} = -1$$

双向非互易网络
端口**1**到端口**3**功率增益**>1**
端口**3**到端口**1**功率增益为**1**

$$s_{21} = \frac{2\hat{z}_{21}}{\Delta\hat{z} + \hat{z}_{11} + \hat{z}_{22} + 1} = \frac{2\left(\frac{Z_0}{r_d} + 1\right)}{1 - \frac{Z_0}{r_d} - \frac{Z_0}{r_d} + 1} = \frac{Z_0 + r_d}{r_d - Z_0} = \Gamma_d$$

CE组态晶体管放大器



$$\mathbf{y}=\begin{bmatrix} g_{in} & 0 \\ g_m & g_{out}\end{bmatrix}=\begin{bmatrix} g_{be}+G_B & 0 \\ g_m & g_{ce}+G_C\end{bmatrix}=\begin{bmatrix} 0.256 & 0 \\ 41.5 & 0.189\end{bmatrix} mS$$

$$\hat{y}=\begin{bmatrix} \frac{1}{\sqrt{G_S}} & \\ & \frac{1}{\sqrt{G_L}}\end{bmatrix}\begin{bmatrix} g_{in} & 0 \\ g_m & g_{out}\end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{G_S}} & \\ & \frac{1}{\sqrt{G_L}}\end{bmatrix}$$

$$=\begin{bmatrix} \frac{g_{in}}{G_S} & 0 \\ \frac{g_m}{\sqrt{G_S G_L}} & \frac{g_{out}}{G_L}\end{bmatrix} \overset{G_S=g_{in},G_L=g_{out}}{\triangleq} \begin{bmatrix} 1 & 0 \\ \frac{g_m}{\sqrt{g_{in}g_{out}}} & 1\end{bmatrix}$$

$$s_{11}=-\frac{\Delta\hat{y}+\hat{y}_{11}-\hat{y}_{22}-1}{\Delta\hat{y}+\hat{y}_{11}+\hat{y}_{22}+1}=0$$

$$s_{21}=-\frac{2\hat{y}_{21}}{\Delta\hat{y}+\hat{y}_{11}+\hat{y}_{22}+1}=-\frac{2\frac{g_m}{\sqrt{g_{in}g_{out}}}}{1+1+1+1}=-\frac{1}{2}\frac{g_m}{\sqrt{g_{in}g_{out}}}$$

$$\mathbf{s}_{G_1=g_{in},G_2=g_{out}}=\begin{bmatrix} 0 & 0 \\ -\frac{1}{2}\frac{g_m}{\sqrt{g_{in}g_{out}}} & 0\end{bmatrix}=\begin{bmatrix} 0 & 0 \\ -94.33 & 0\end{bmatrix}$$

39.5dB反相

CE组态晶体管放大器

$$\mathbf{y}=\begin{bmatrix} g_{in} & 0 \\ g_m & g_{out} \end{bmatrix} = \begin{bmatrix} g_{be}+G_B & 0 \\ g_m & g_{ce}+G_C \end{bmatrix} = \begin{bmatrix} 0.256 & 0 \\ 41.5 & 0.189 \end{bmatrix} mS$$

$$s_{11}=-\frac{\Delta\hat{y}+\hat{y}_{11}-\hat{y}_{22}-1}{\Delta\hat{y}+\hat{y}_{11}+\hat{y}_{22}+1}=-\frac{\frac{Z_0^2}{r_{in}r_{out}}+\frac{Z_0}{r_{in}}-\frac{Z_0}{r_{out}}-1}{\frac{Z_0^2}{r_{in}r_{out}}+\frac{Z_0}{r_{in}}+\frac{Z_0}{r_{out}}+1}$$

$$=-\frac{\left(\frac{Z_0}{r_{in}}-1\right)\left(\frac{Z_0}{r_{out}}+1\right)}{\left(\frac{Z_0}{r_{in}}+1\right)\left(\frac{Z_0}{r_{out}}+1\right)}=\frac{r_{in}-Z_0}{r_{in}+Z_0}=\Gamma_{in}$$

$$\hat{y}=\begin{bmatrix} \frac{1}{\sqrt{G_S}} & \\ & \frac{1}{\sqrt{G_L}} \end{bmatrix}\begin{bmatrix} g_{in} & 0 \\ g_m & g_{out} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{G_S}} & \\ & \frac{1}{\sqrt{G_L}} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{g_{in}}{G_S} & 0 \\ \frac{g_m}{\sqrt{G_S G_L}} & \frac{g_{out}}{G_L} \end{bmatrix} \overset{R_S=R_L=Z_0=50\Omega}{\triangleq} \begin{bmatrix} g_{in}Z_0 & 0 \\ g_m Z_0 & g_{out}Z_0 \end{bmatrix}$$

$$s_{21}=-\frac{2\hat{y}_{21}}{\Delta\hat{y}+\hat{y}_{11}+\hat{y}_{22}+1}=-\frac{2g_m Z_0}{\left(\frac{Z_0}{r_{in}}+1\right)\left(\frac{Z_0}{r_{out}}+1\right)}$$

$$=-\frac{2g_m Z_0 r_{in} r_{out}}{(Z_0+r_{in})(Z_0+r_{out})}=2\sqrt{\frac{Z_0}{Z_0}}\frac{r_{out}Z_0}{Z_0+r_{out}}(-g_m)\frac{r_{in}}{Z_0+r_{in}}$$

单向网络电压增益

阻抗严重不匹配

$$\mathbf{s}_{R_S=R_L=Z_0=50\Omega}=\begin{bmatrix} \Gamma_{in} & 0 \\ -2\sqrt{\frac{R_S}{R_L}}\frac{v_L}{v_S} & \Gamma_{out} \end{bmatrix}=\begin{bmatrix} 0.9747 & 0 \\ -4.0592 & 0.9813 \end{bmatrix}$$

12.2dB反相增益

对S参量的要求

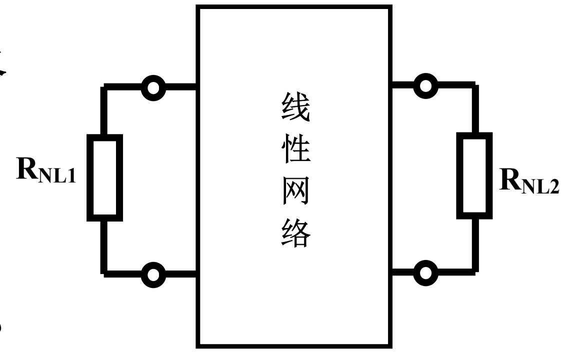
- 记住定义

$$\boldsymbol{s}_{R_1,R_2} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} = \begin{bmatrix} \frac{z_{in}-R_1}{z_{in}+R_1} & 2\sqrt{\frac{R_2}{R_1}}\frac{v_{L1}}{v_{S2}} \\ 2\sqrt{\frac{R_1}{R_2}}\frac{v_{L2}}{v_{S1}} & \frac{z_{out}-R_2}{z_{out}+R_2} \end{bmatrix}$$

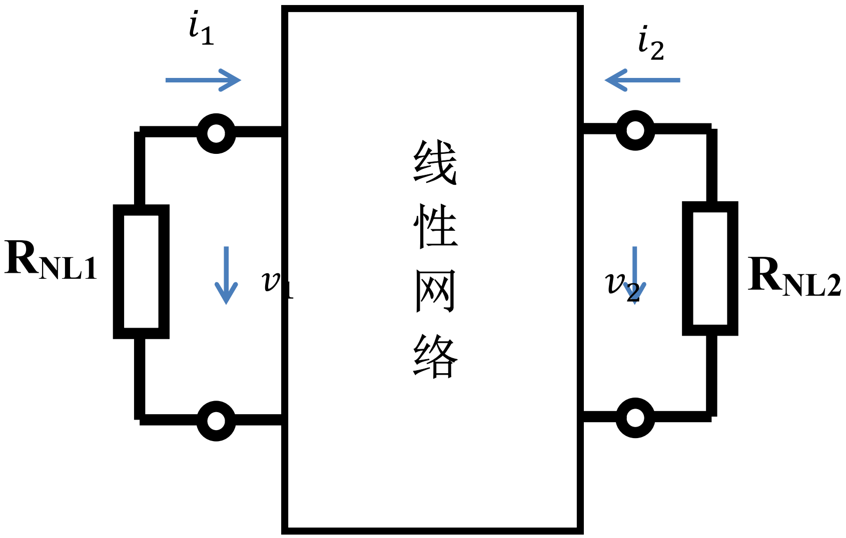
- 知道S参量物理含义
- 会根据定义计算S参量
- S参量和Z参量Y参量转换关系
  - 知道可以相互转换，需要时会查公式运用即可

作业4 非线性局部线性化

- 练习**4.30**：如图所示，假设某非线性电路中包含两个单端口的非线性电阻器件，剩余的电路则是线性电阻电路和理想电源构成的线性网络。
  - （**1**）假设两个非线性电阻器件都是压控器件，则二端口的线性网络应该采用什么参量描述比较适当？
  - （**2**）假设两个非线性电阻器件都是流控器件，则二端口的线性网络应该采用什么参量描述比较适当？
  - （**3**）假设两个非线性电阻器件一个是压控器件，一个是流控器件，则二端口的线性网络应该采用什么参量描述比较适当？
  - （**4**）不妨假设两个非线性电阻器件都是流控器件，并且假设线性网络中的源等效中包含直流分量和交流小信号分量，请描述该网络的交直流分析全过程。



两个非线性电阻都是压控器件



简单对接关系，每个端口只需定义一套端口电压、端口电流即可

压控器件

$$-i_1 = i_{NL1} = f_{iv,1}(v_{NL1}) = f_{iv,1}(v_1) \quad \mathbf{R_{NL1}}$$

$$-i_2 = i_{NL2} = f_{iv,2}(v_{NL2}) = f_{iv,2}(v_2) \quad \mathbf{R_{NL2}}$$

所谓压控，就是以电压作为自变量描述其他因变量，如电流：故而应首先获得电压，其次由电压获得电流

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \mathbf{y}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} i_{N1} \\ i_{N2} \end{bmatrix}$$

线性网络

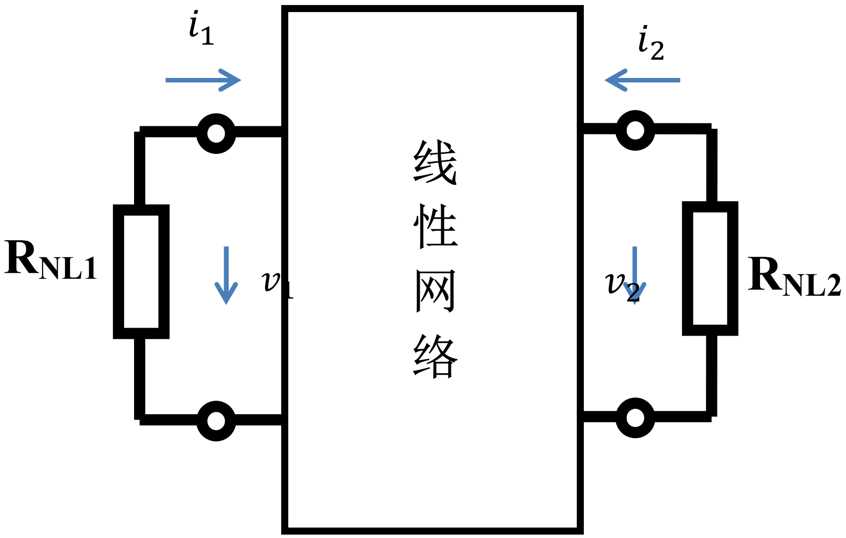
全部 采用压控形式，则最终电路方程中只有电压为未知量

$$\begin{bmatrix} f_{iv,1}(v_1) \\ f_{iv,2}(v_2) \end{bmatrix} + \mathbf{y}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} i_{N1} \\ i_{N2} \end{bmatrix} = 0$$

最适电路方程

由此方程求出$\mathbf{v_1}$，$\mathbf{v_2}$，再代回求$\mathbf{i_1}$，$\mathbf{i_2}$

两个非线性电阻都是流控器件



流控器件

$$v_1 = v_{NL1} = f_{vi,1}(i_{NL1}) = f_{vi,1}(-i_1) \quad R_{NL1}$$

$$v_2 = v_{NL2} = f_{vi,2}(i_{NL2}) = f_{vi,2}(-i_2) \quad R_{NL2}$$

所谓流控，就是以电流作为自变量描述其他因变量，如电压：故而应首先获得电流，其次由电流求取电压

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \mathbf{z}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_{TH1} \\ v_{TH2} \end{bmatrix}$$

线性网络

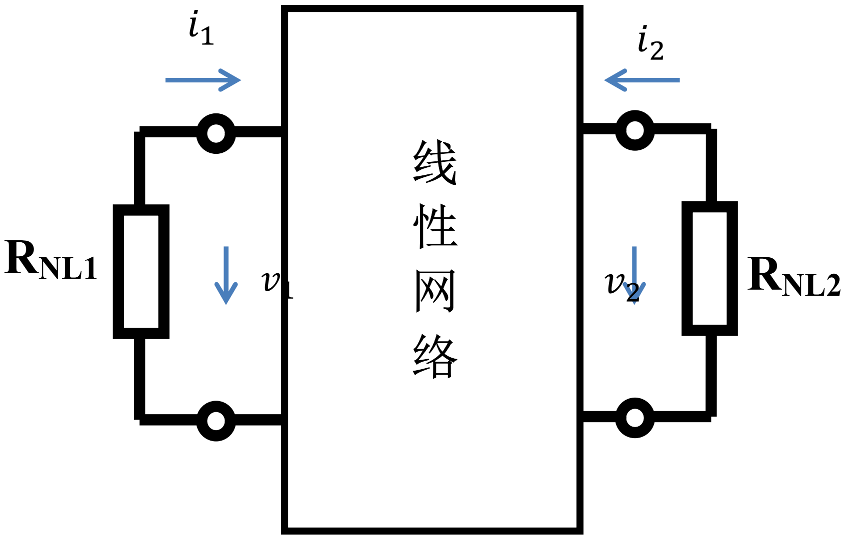
简单对接关系，每个端口只需定义一套端口电压、端口电流即可

$$\begin{bmatrix} f_{vi,1}(-i_1) \\ f_{vi,2}(-i_2) \end{bmatrix} = \mathbf{z}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_{TH1} \\ v_{TH2} \end{bmatrix}$$

最适电路方程

由此方程求出$\mathbf{i_1}$，$\mathbf{i_2}$，再代回求$\mathbf{v_1}$，$\mathbf{v_2}$

两个非线性电阻—压控—流控



压控器件

$$-i_1 = i_{NL1} = f_{iv,1}(v_{NL1}) = f_{iv,1}(v_1)$$

R_{NL1}

流控器件

$$v_2 = v_{NL2} = f_{vi,2}(i_{NL2}) = f_{vi,2}(-i_2)$$

R_{NL2}

流控器件需要先确认其电流，压控器件需要先确认其电压

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \mathbf{g}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} i_{N1} \\ v_{TH2} \end{bmatrix}$$

线性网络

R_{NL1}压控R_{NL2}流控，g参量描述线性网络最适当：获得最适电路方程

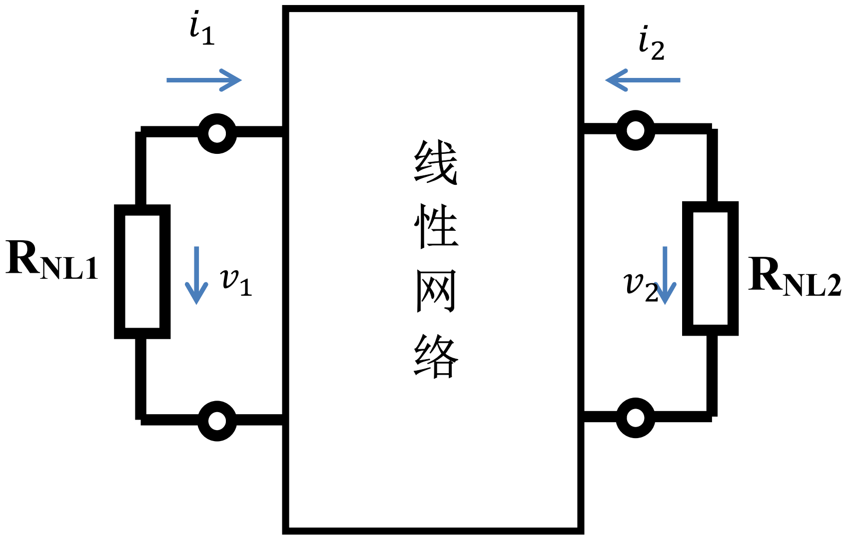
$$\begin{bmatrix} -f_{iv,1}(v_1) \\ f_{vi,2}(-i_2) \end{bmatrix} = \mathbf{g}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} i_{N1} \\ v_{TH2} \end{bmatrix}$$

最适电路方程

R_{NL1}流控R_{NL2}压控，h参量描述线性网络最适当：获得最适电路方程

由此方程求出$\mathbf{v_1}$，$\mathbf{i_2}$，再代回求$\mathbf{i_1}$，$\mathbf{v_2}$

两个非线性电阻都是流控器件



$$\begin{bmatrix} f_{vi,1}(-i_1) \\ f_{vi,2}(-i_2) \end{bmatrix} = \mathbf{z}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_{TH1} \\ v_{TH2} \end{bmatrix}$$

$$\begin{bmatrix} f_1(-i_1) \\ f_2(-i_2) \end{bmatrix} = \mathbf{z}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_{TH1} \\ v_{TH2} \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} I_{10} \\ I_{20} \end{bmatrix} + \begin{bmatrix} \Delta i_1 \\ \Delta i_2 \end{bmatrix} \qquad \begin{bmatrix} v_{TH1} \\ v_{TH2} \end{bmatrix} = \begin{bmatrix} V_{TH10} + \Delta v_{TH1} \\ V_{TH20} + \Delta v_{TH2} \end{bmatrix}$$

$$\begin{bmatrix} f_1(-i_1) \\ f_2(-i_2) \end{bmatrix} = \begin{bmatrix} f_1(-I_{10} - \Delta i_1) \\ f_2(-I_{20} - \Delta i_2) \end{bmatrix}$$

$$= \begin{bmatrix} f_1(-I_{10}) - \dfrac{\partial f_1}{\partial i_{NL1}} \Delta i_1 + \ldots \\ f_2(-I_{20}) - \dfrac{\partial f_2}{\partial i_{NL2}} \Delta i_2 + \ldots \end{bmatrix} \approx \begin{bmatrix} f_1(-I_{10}) \\ f_2(-I_{20}) \end{bmatrix} - \begin{bmatrix} \dfrac{\partial f_1}{\partial i_{NL}} \Delta i_1 \\ \dfrac{\partial f_2}{\partial i_{NL}} \Delta i_2 \end{bmatrix}$$

$$\begin{bmatrix} f_1(-i_1) \\ f_2(-i_2) \end{bmatrix} = \mathbf{z}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_{TH1} \\ v_{TH2} \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} I_{10} \\ I_{20} \end{bmatrix} + \begin{bmatrix} \Delta i_1 \\ \Delta i_2 \end{bmatrix}$$

$$\mathbf{z}\begin{bmatrix} I_{10} + \Delta i_1 \\ I_{20} + \Delta i_2 \end{bmatrix} + \begin{bmatrix} V_{TH10} + \Delta v_{TH1} \\ V_{TH20} + \Delta v_{TH2} \end{bmatrix} = \begin{bmatrix} f_1(-I_{10} - \Delta i_1) \\ f_2(-I_{20} - \Delta i_2) \end{bmatrix} \approx \begin{bmatrix} f_1(-I_{10}) \\ f_2(-I_{20}) \end{bmatrix} - \begin{bmatrix} \frac{\partial f_1}{\partial i_{NL1}} \Delta i_1 \\ \frac{\partial f_2}{\partial i_{NL2}} \Delta i_2 \end{bmatrix}$$

$$\mathbf{Z}\begin{bmatrix} I_{10} \\ I_{20} \end{bmatrix} + \begin{bmatrix} V_{TH10} \\ V_{TH20} \end{bmatrix} = \begin{bmatrix} f_1(-I_{10}) \\ f_2(-I_{20}) \end{bmatrix}$$

直流非线性分析

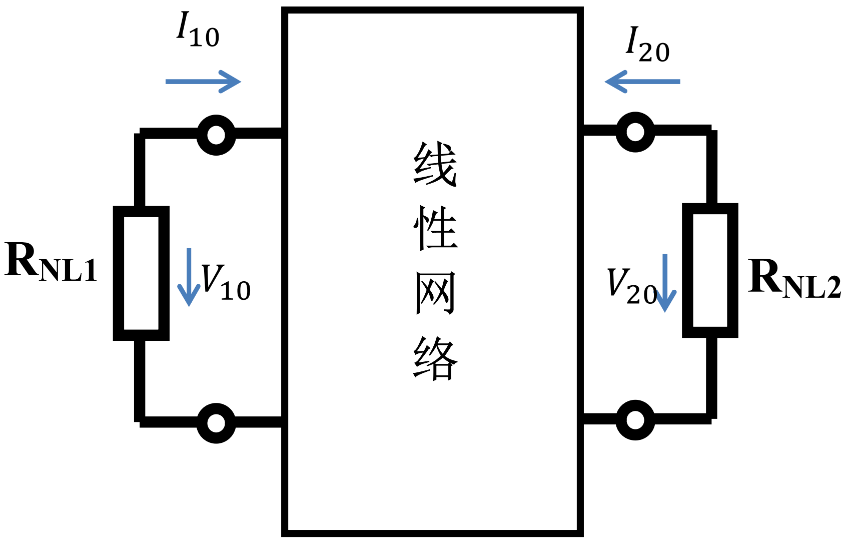
$$\mathbf{z}\begin{bmatrix} \Delta i_1 \\ \Delta i_2 \end{bmatrix} + \begin{bmatrix} \Delta v_{TH1} \\ \Delta v_{TH2} \end{bmatrix} = -\begin{bmatrix} \frac{\partial f_1}{\partial i_{NL1}} \Delta i_1 \\ \frac{\partial f_2}{\partial i_{NL}} \Delta i_2 \end{bmatrix} = -\begin{bmatrix} r_{d1}\Delta i_1 \\ r_{d2}\Delta i_2 \end{bmatrix}$$

交流线性分析

如果线性网络中存在耦合电容、高频扼流圈，
直流分析和交流分析不同，体现在**Z**参量不同

直流分析和交流小信号分析

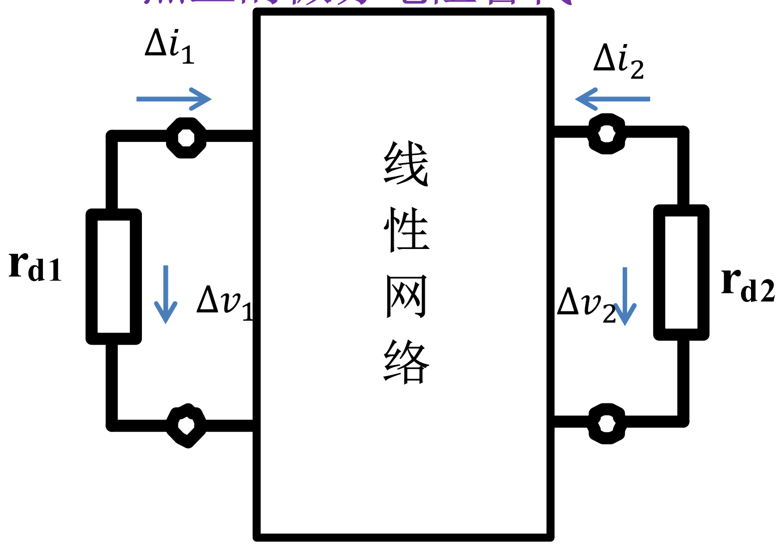
直流分析时，两个单端口
非线性电阻无变动



线性网络中只有直流电源起作用，交流电源不起作用（交流电压源短路，交流电流源开路），耦合电容开路，高频扼流圈短路

$$\mathbf{z}_{DC}\begin{bmatrix} I_{10} \\ I_{20} \end{bmatrix} + \begin{bmatrix} V_{TH10} \\ V_{TH20} \end{bmatrix} = \begin{bmatrix} V_{10} \\ V_{20} \end{bmatrix} = \begin{bmatrix} f_1(-I_{10}) \\ f_2(-I_{20}) \end{bmatrix}$$

交流分析时，两个单端口
非线性电阻用其直流工作
点上的微分电阻替代



线性网络中只有交流电源起作用，直流电源不起作用（直流电压源短路，直流电流源开路），耦合电容短路，高频扼流圈开路

$$\mathbf{z}_{AC}\begin{bmatrix} \Delta i_1 \\ \Delta i_2 \end{bmatrix} + \begin{bmatrix} \Delta v_{TH1} \\ \Delta v_{TH2} \end{bmatrix} = \begin{bmatrix} \Delta v_1 \\ \Delta v_2 \end{bmatrix} = -\begin{bmatrix} r_{d1}\Delta i_1 \\ r_{d2}\Delta i_2 \end{bmatrix}$$

作业5 线性范围

- 已知某非线性电阻器件的伏安特性曲线具有如下特性，

（**1**）　$i = I_0 \tanh \frac{v}{2v_T}$　**v**为输入，**i**为输出

（**2**）　$y = K_d \sin x$　**x**为输入，**y**为输出

请给出线性范围最大的直流工作点位置，以及**1dB**线性范围大小。

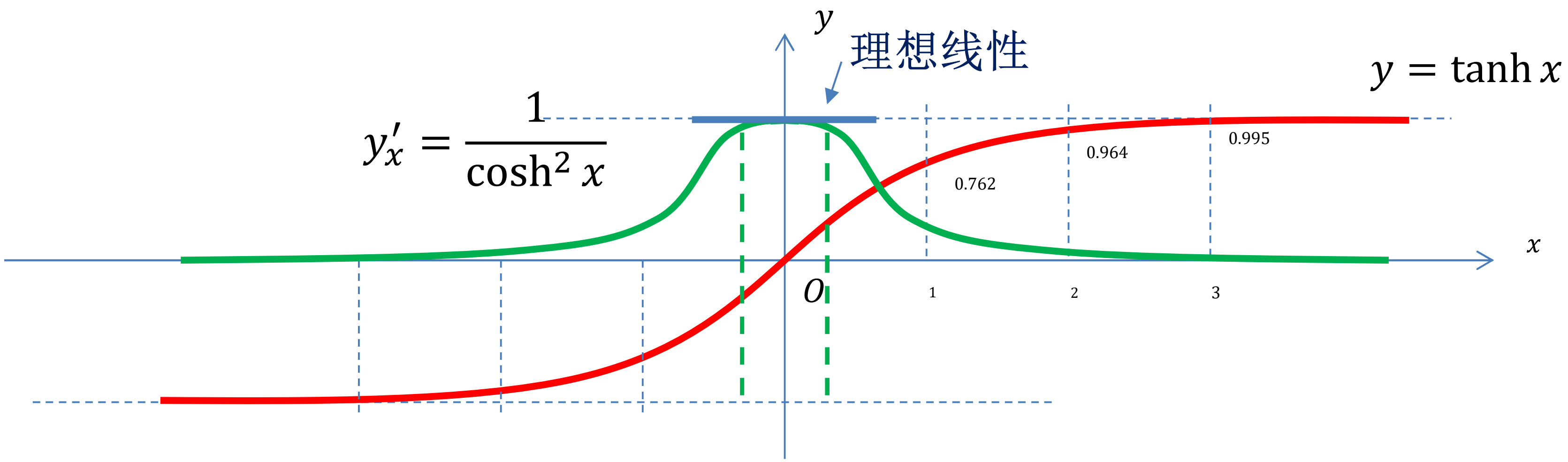
双曲正切函数

$$i = I_0 \tanh\frac{v}{2v_T}$$

$$y = \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

BJT差分对管非线性跨导转移特性

$$y'_x = \frac{1}{\cosh^2 x}$$



最大的线性区在x=0位置

$$i_o = I_0 \tanh \frac{v_{id}}{2v_T}$$

$$20\log_{10}\left(\cosh^2 \frac{v_{id,1dB}}{2v_T}\right) = 1dB$$

$$g_m = \frac{di_o}{dv_{id}} = \frac{I_0}{2v_T}\frac{1}{\cosh^2 \frac{v_{id}}{2v_T}}$$

$$\cosh^2 \frac{v_{id,1dB}}{2v_T} = 10^{\frac{1}{20}}$$

$$\cosh \frac{v_{id,1dB}}{2v_T} = 10^{\frac{1}{40}} = 1.059$$

$$g_{m0} = g_m(v_{id} = 0) = \frac{I_0}{2v_T} = \frac{0.5I_{EE}}{v_T} = \frac{I_{C0}}{v_T}$$

I_{EE}，差分对尾电流
I_{C0}，差分对管直流电流

$$\frac{g_{m0}}{g_m} = \frac{\frac{I_0}{2v_T}}{\frac{I_0}{2v_T}\frac{1}{\cosh^2 \frac{v_{id}}{2v_T}}} = \cosh^2 \frac{v_{id}}{2v_T}$$

$$v_{id,1dB} = 2v_T \cdot \cosh^{-1} 1.059$$
$$= \pm 0.685 v_T = \pm 17.8mV$$
$$\approx \pm 18mV$$

±18mV作为BJT差分对的线性范围

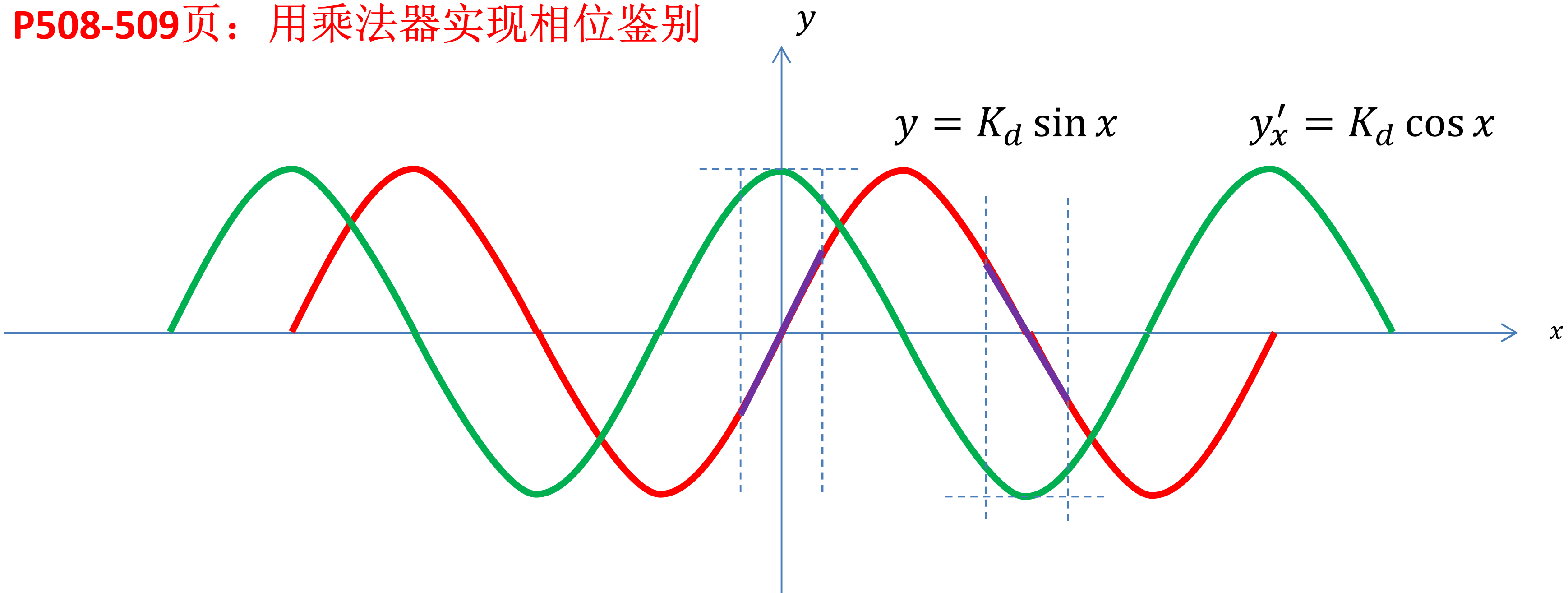
1dB线性范围内： $i_o = I_0 \tanh \frac{v_{id}}{2v_T} \approx I_0 \frac{v_{id}}{2v_T} = \frac{I_0}{2v_T} v_{id} = g_{m0} v_{id}$

正弦函数

$y = K_d \sin x$　　　　$y'_x = K_d \cos x$

来源：正弦鉴相特性

P508-509页：用乘法器实现相位鉴别



最大的线性区在**x=0**，π位置

$$y = K_d \sin x$$

$$y'_x = K_d \cos x$$

$$(y'_x)_{max} = K_d \cos 0 = K_d$$

$$\frac{(y'_x)_{\max}}{y'_x} = \frac{K_d}{K_d \cos x} = \frac{1}{\cos x}$$

$$20 \log_{10} \frac{1}{\cos x_{1dB}} = 1dB$$

$$\frac{1}{\cos x_{1dB}} = 10^{\frac{1}{20}}$$

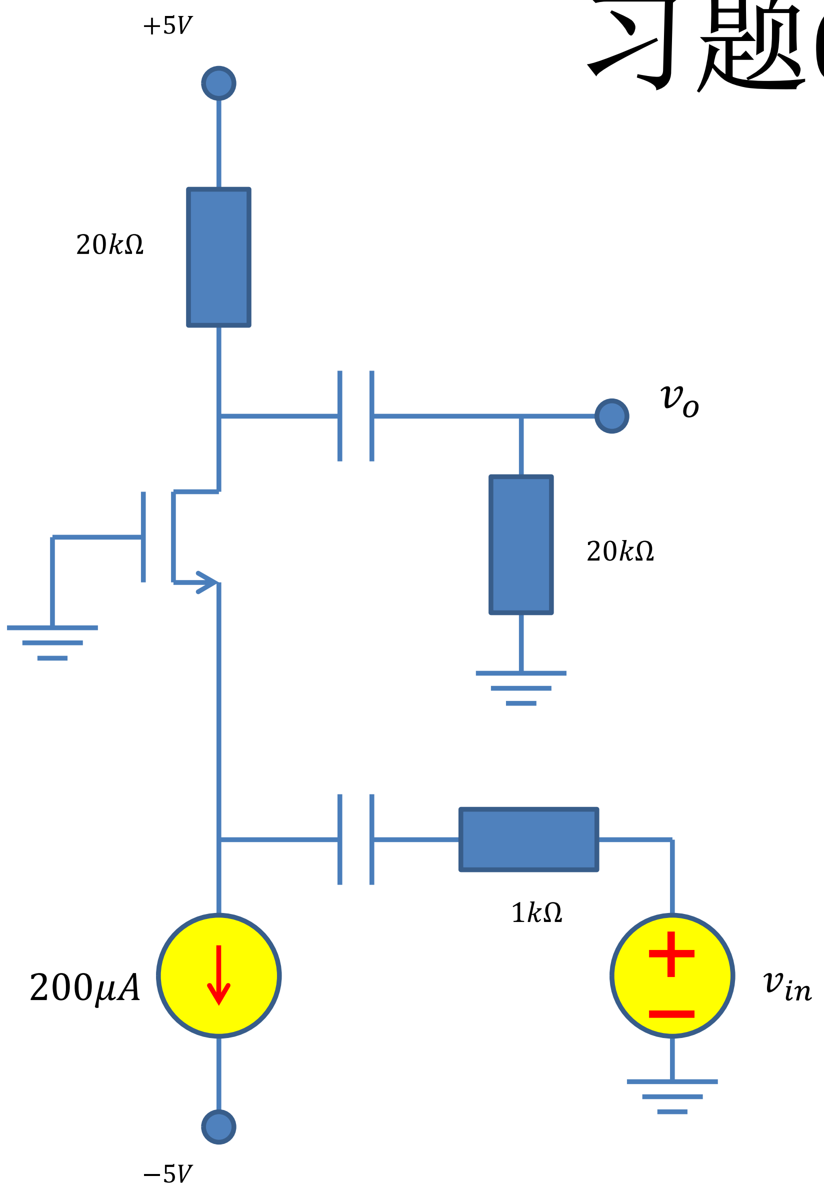
$$x_{1dB} = \arccos 10^{-\frac{1}{20}}$$
$$= \pm 0.471 = \pm 27°$$

方便记忆：±**30**°作为**sinx**的线性范围

1dB线性范围内： $y = K_d \sin x \approx K_d x$

输入信号幅度**1dB**线性范围内，非线性用线性替代，线性分析产生的误差一定程度上可以接受

习题6：CG组态放大器



$$i_D = \beta(v_{GS} - V_{TH})^2\left(1 + \frac{v_{DS}}{V_E}\right)$$

$\beta = 1mA/V^2$

$V_{TH} = 1V$

$V_E = 20V$

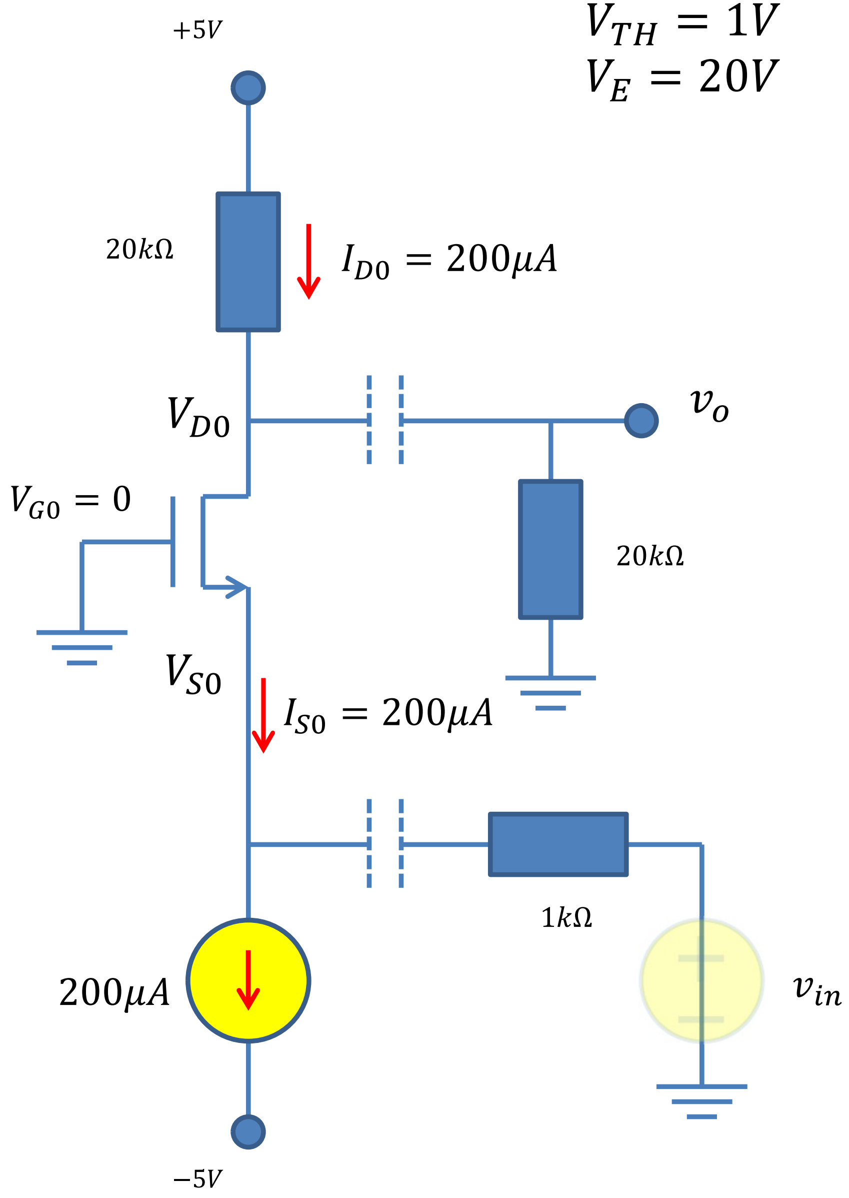
- 确认直流工作点在恒流区
- 求电压放大倍数和功率放大倍数

$$A_v = \frac{v_o}{v_s} \qquad G_p = \frac{P_L}{P_{S,\max}}$$

- 选作：分析说明**MOSFET**将直流电能转换为交流电能
  - （**1**）将电容抽象为直流电压源，分析每个部件上的电压电流，说明无交流小信号激励时晶体管消耗的能量多，有交流小信号激励时，晶体管消耗的能量降低。可以理解为晶体管将吸收的直流能量转换为交流能量送出去，被负载电阻吸收
  - （**2**）说明晶体管微分元件**y**参量电路为有源电路

直流分析

$$\beta_n = 1mA/V^2$$
$$V_{TH} = 1V$$
$$V_E = 20V$$



$$V_{D0} = V_{DD} - I_{D0}R_D = 5 - 4 = 1V$$

$$V_{GD} = -1V < V_{TH}$$

$$I_{D0} = \beta_n (V_{GS0} - V_{TH})^2 \left(1 + \frac{V_{DS0}}{V_E}\right)$$
$$= \beta_n (V_{G0} - V_{S0} - V_{TH})^2 \left(1 + \frac{V_{D0} - V_{S0}}{V_E}\right)$$
$$= \beta_n (V_{S0} + V_{TH})^2 \left(1 + \frac{1 - V_{S0}}{V_E}\right)$$

$$0.2 = (V_{S0} + 1)^2 \left(1 + \frac{1 - V_{S0}}{20}\right)$$

方程化简后，只有一个未知量V_{S0}待求

直流非线性分析：非线性方程求解
简单迭代法

$$0.2=(V_{S0}+1)^2\left(1+\frac{1-V_{S0}}{20}\right)$$

$$V_{S0}=-1-\sqrt{\frac{0.2}{1+\frac{1-V_{S0}}{20}}}$$

有意义解

无意义解？自己分析

$$V_{S0}{}^{(0)}=-1-\sqrt{\frac{0.2}{1}}=-1.447$$

不考虑厄利效应的解作为初始值

$$V_{S0}{}^{(1)}=-1-\sqrt{\frac{0.2}{1+\frac{1-V_{S0}{}^{(0)}}{20}}}=-1-\sqrt{\frac{0.2}{1+\frac{1+1.447}{20}}}=-1.422$$

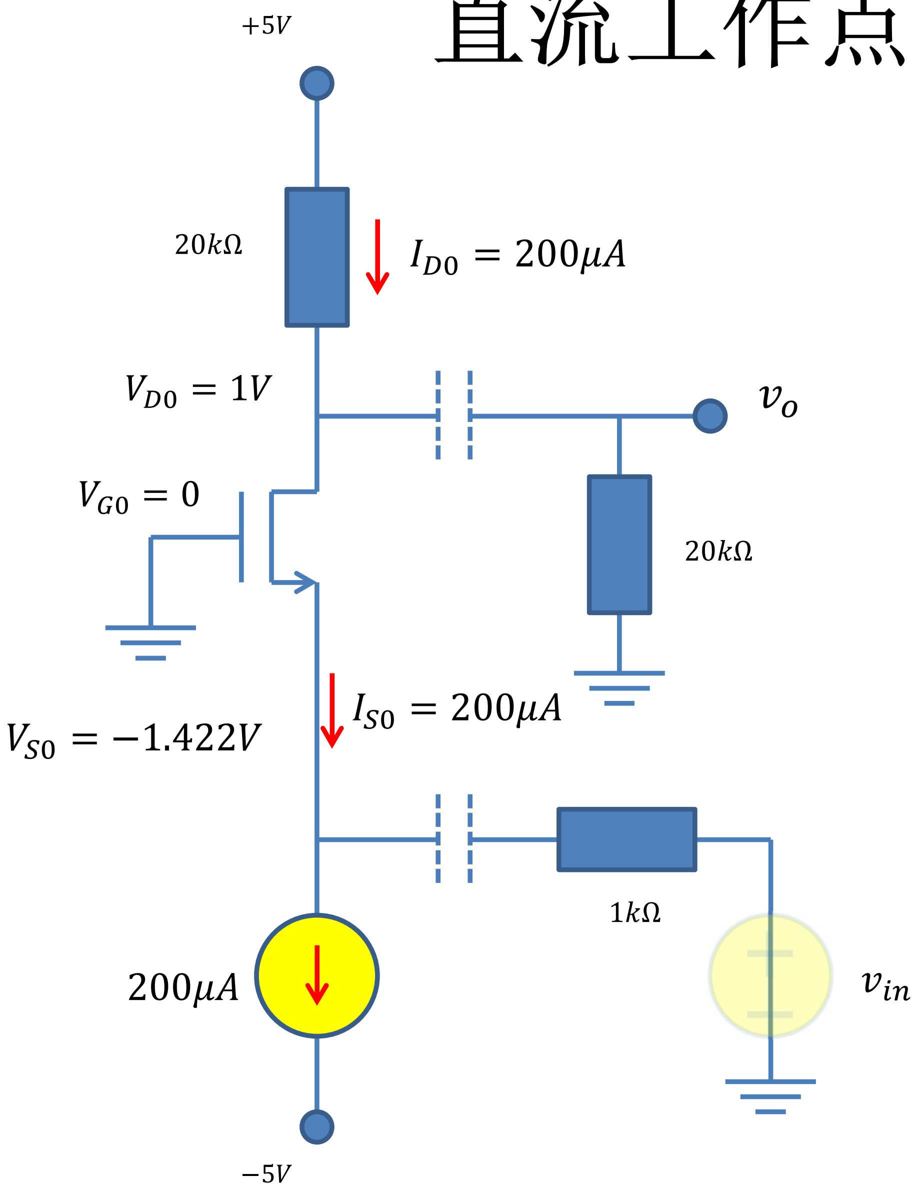
考虑厄利效应和 不考虑厄利效应微有差别，差别不大

$$V_{S0}{}^{(2)}=-1-\sqrt{\frac{0.2}{1+\frac{1-V_{S0}{}^{(1)}}{20}}}=-1-\sqrt{\frac{0.2}{1+\frac{1+1.422}{20}}}=-1.422$$

迭代结果在有效位数内看已无差别，迭代结束

直流工作点决定微分元件

$$\beta_n = 1mA/V^2$$
$$V_{TH} = 1V$$
$$V_E = 20V$$



$$i_D = \beta_n(v_{GS} - V_{TH})^2\left(1 + \frac{v_{DS}}{V_E}\right)$$

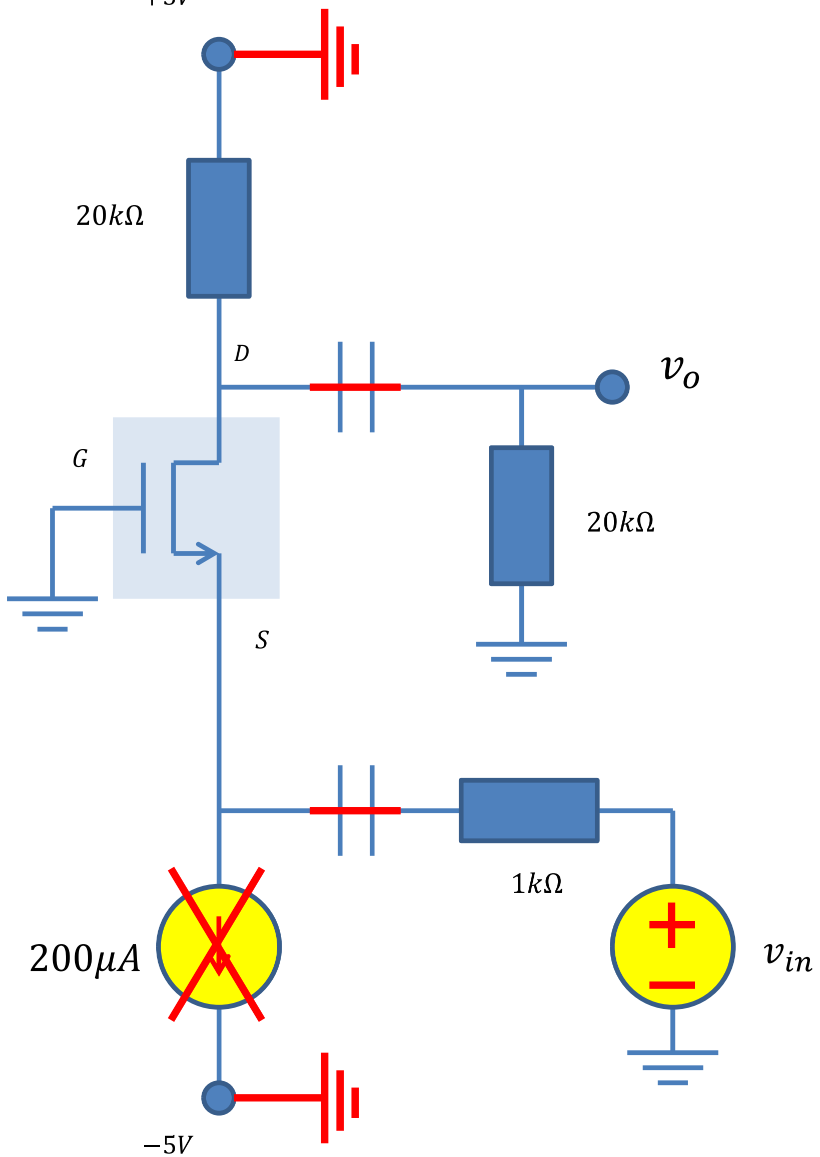
$$g_m = \frac{\partial i_D}{\partial v_{GS}}|Q$$
$$= 2\beta_n(V_{GS0} - V_{TH})\left(1 + \frac{V_{DS0}}{V_E}\right)$$
$$= \frac{2I_{D0}}{V_{GS0} - V_{TH}} = \frac{2 \times 200\mu}{1.422 - 1} = 0.947mS$$

$$g_{ds} = \frac{\partial i_D}{\partial v_{DS}}|Q = \beta_n(V_{GS0} - V_{TH})^2\frac{1}{V_E}$$
$$= 1 \times (0.422)^2 \times \frac{1}{20} = 8.92\mu S$$

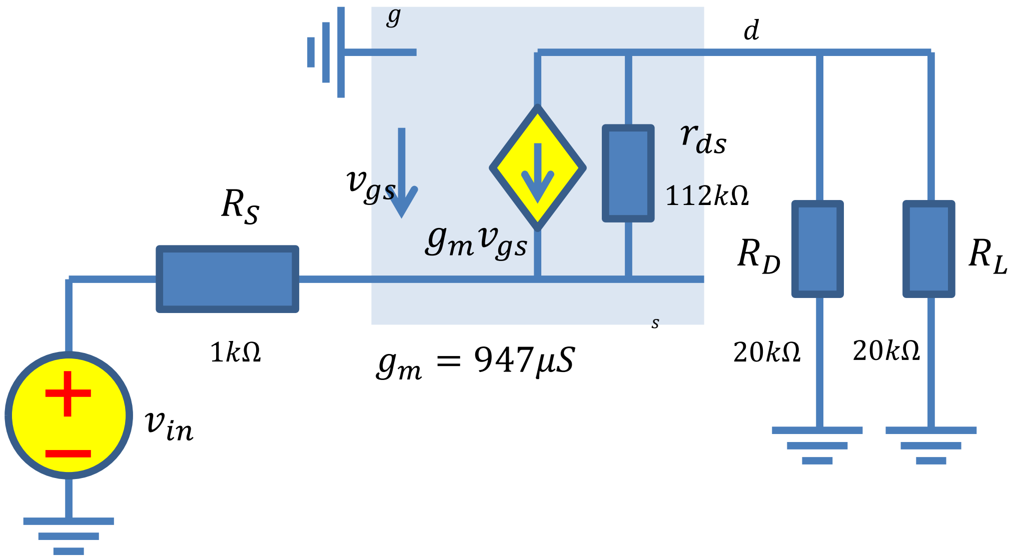
$$r_{ds} = 112k\Omega$$

$$g_{ds} \approx \frac{I_{D0}}{V_E} = 10\mu S \qquad r_{ds} \approx 100k\Omega$$

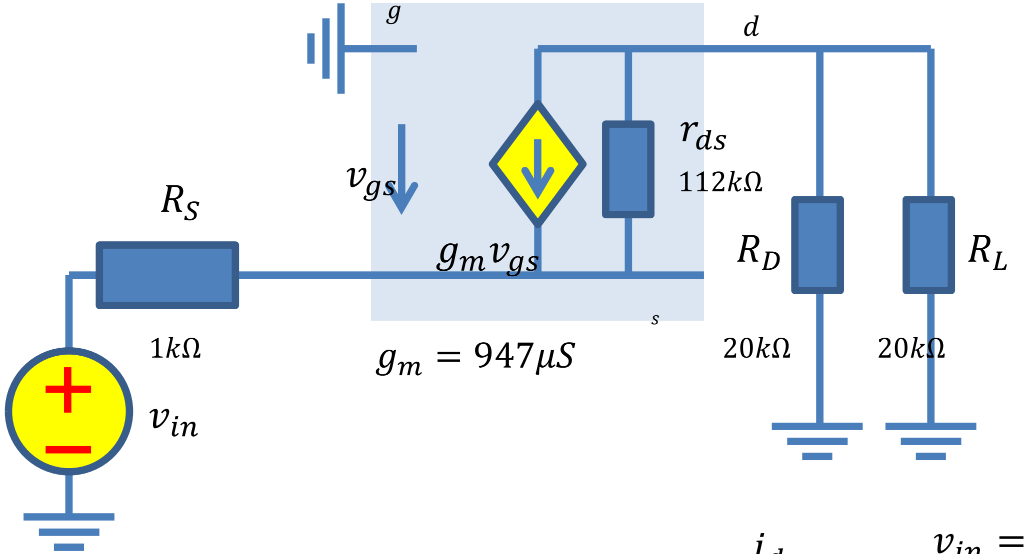
交流小信号分析



+5V
20kΩ
D
G
S
20kΩ
200μA
1kΩ
-5V



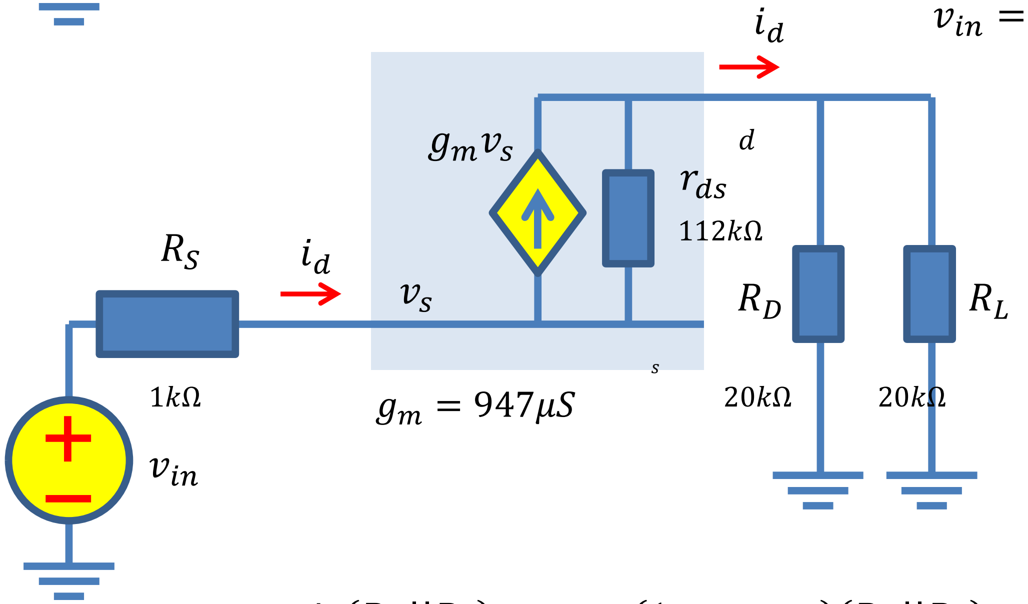
g
d
s
r_{ds}
112kΩ
v_{gs}
$g_m v_{gs}$
R_S
1kΩ
R_D
R_L
20kΩ
20kΩ
v_{in}
$g_m = 947\mu S$



$$v_{in} = i_d R_s + (i_d - g_m v_s) r_{ds} + i_d (R_D || R_L)$$

$$v_{in} = i_d R_s + v_s$$

$$v_s = v_{in} - i_d R_s$$



$$v_{in} = i_d R_s + \left(i_d - g_m (v_{in} - i_d R_s)\right) r_{ds} + i_d (R_D || R_L)$$

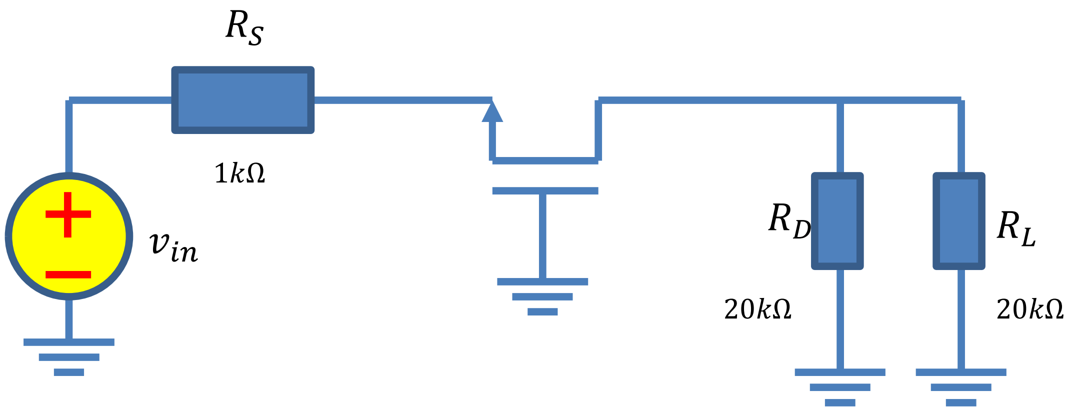
$$i_d = \frac{1 + g_m r_{ds}}{R_s + r_{ds} + g_m r_{ds} R_s + (R_D || R_L)} v_{in}$$

$$G_p = \frac{V_{L,rms}^2 / R_L}{V_{S,rms}^2 / 4R_S} = 4 \frac{R_S}{R_L} \left(\frac{V_{L,rms}}{V_{S,rms}} \right)^2$$
$$= 4 \times \frac{1k}{20k} \times (4.674)^2 = 4.369 = 6.4dB$$

$$A_v = \frac{v_L}{v_{in}} = \frac{i_d (R_D || R_L)}{v_{in}} = \frac{(1 + g_m r_{ds})(R_D || R_L)}{R_s + r_{ds} + g_m r_{ds} R_s + (R_D || R_L)}$$
$$= \frac{(1 + 0.947m \times 112k) \cdot 10k}{1k + 112k + 0.947m \times 112k \times 1k + 10k} = \frac{1070.64k}{229.064k} = 4.674 = 13.4dB$$

功率增益和电压增益不同

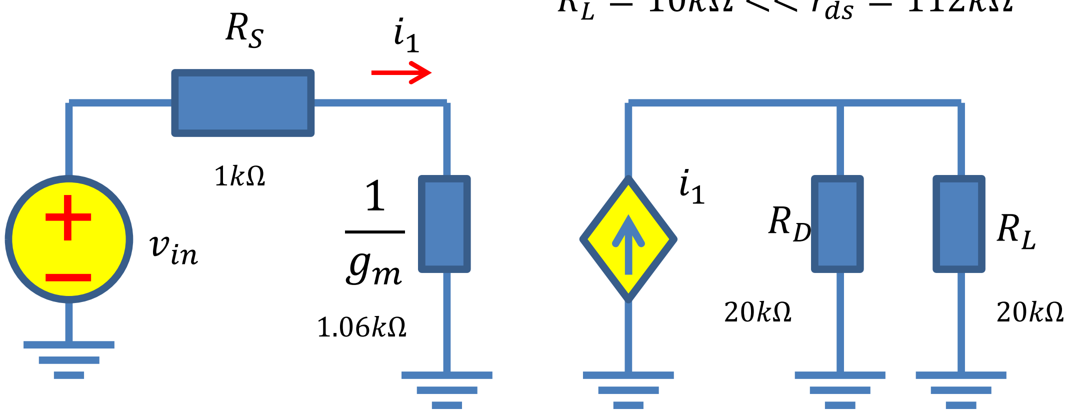
电流缓冲器模型



$$i_1 = \frac{1}{R_s + \frac{1}{g_m}} v_{in}$$
$$= \frac{g_m}{1 + g_m R_s} v_{in} = g_{mf} v_{in}$$

$$v_L = i_1 R_L' = g_{mf} R_L' v_{in}$$

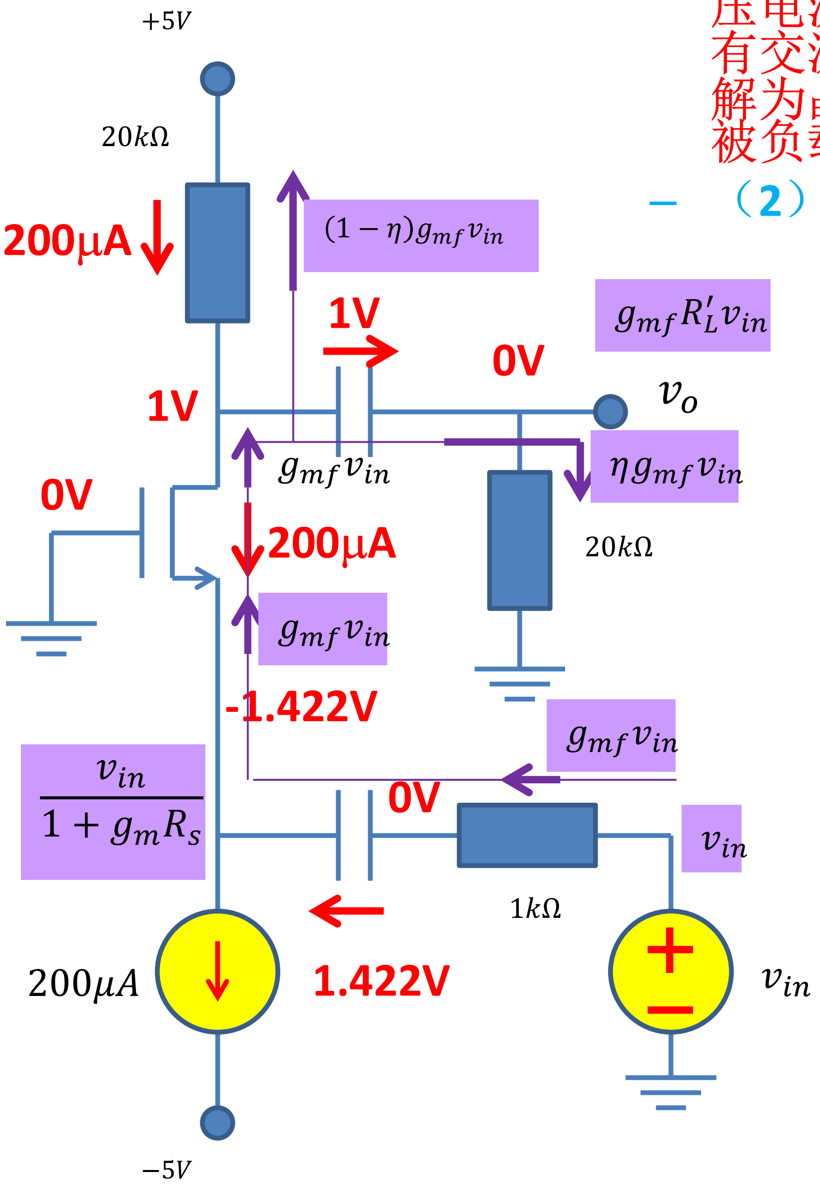
$R_L' = 10k\Omega << r_{ds} = 112k\Omega$



$$\frac{v_L}{v_{in}} = g_{mf} R_L'$$
$$= \frac{0.947m}{1 + 0.947m \times 1k} \times 10k$$
$$= 0.4864m \times 10k = 4.864$$
$$= 13.7dB$$

差**0.3dB**：误差可以容忍

- 选作：分析说明MOSFET将直流电能转换为交流电能
  - （1）将电容抽象为直流电压源，分析每个部件上的电压电流，说明无交流小信号激励时晶体管消耗的能量多，有交流小信号激励时，晶体管消耗的能量降低。可以理解为晶体管将吸收的直流能量转换为交流能量送出去，被负载电阻吸收
  - （2）说明晶体管微分元件y参量电路为有源电路



如果没有激励信号 $v_{in}=0$

直流功率

+5V电源提供 $5V\times 200\mu A=1mW$

-5V电源提供 $5V\times 200\mu A=1mW$

偏置电阻消耗 $4V\times 200\mu A=0.8mW$

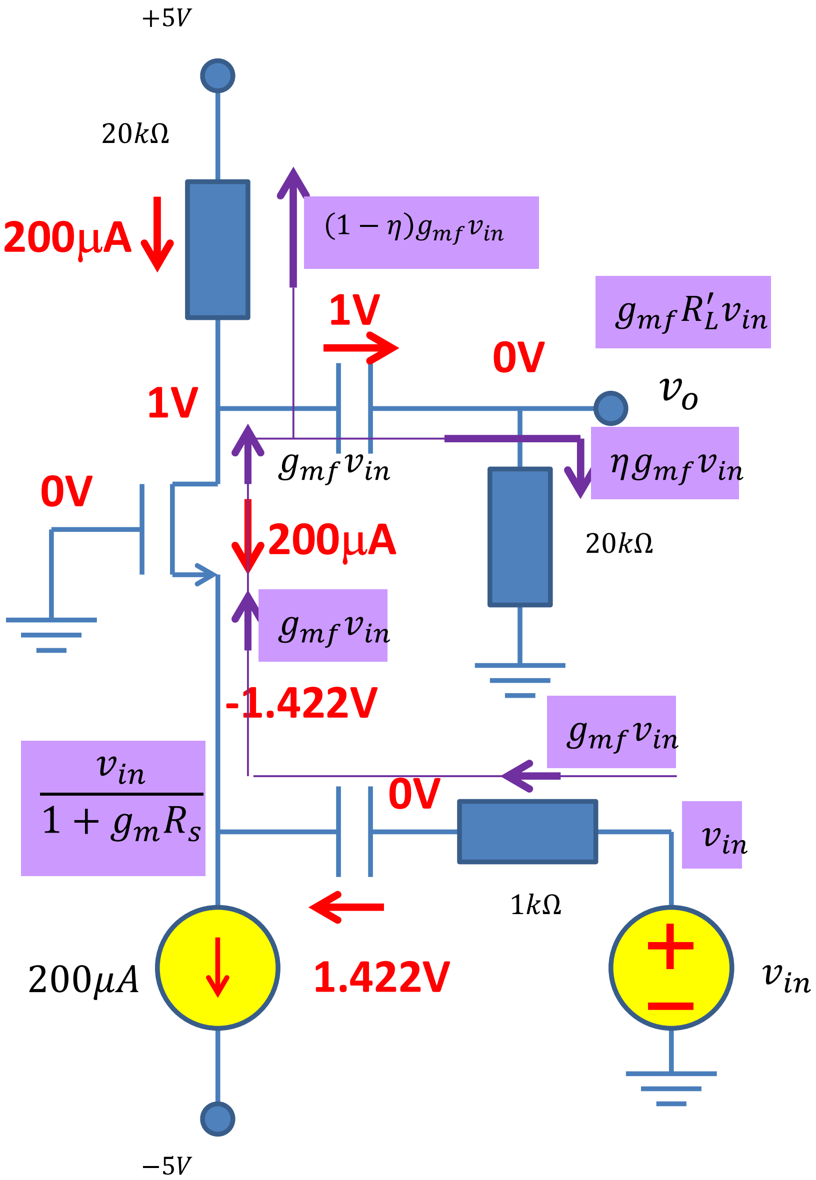
晶体管消耗 $2.422V\times 200\mu A=0.48mW$

偏置电流源消耗 $3.578V\times 200\mu A=0.72mW$

由工作在恒流区的晶体管等效

本质上仍然是电阻，仅提供恒流特性而已

供能



现加入激励信号 $v_{in}=V_m\cos\omega t$

直流功率

+5V电源提供 $\overline{5V\times(0.2mA-(1-\eta)g_{mf}v_{in})}=1mW$

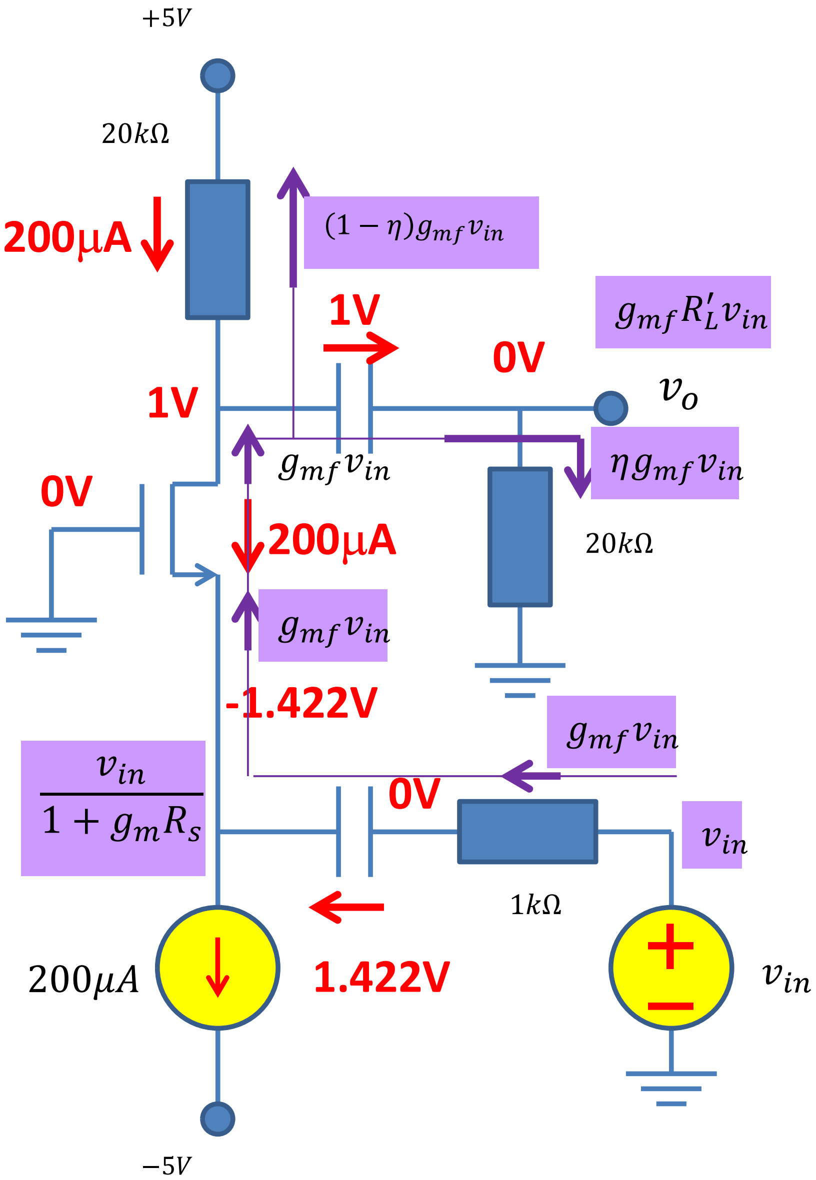
-5V电源提供 $5V\times200\mu A=1mW$

小信号激励功率 **v_{in}**提供 $\overline{v_{in}\cdot g_{mf}v_{in}}=g_{mf}\overline{v_{in}^2}=0.5g_{mf}V_m^2=0.24V_m^2$

小信号源内阻耗能 $\overline{(g_{mf}v_{in})^2}R_s=g_{mf}{}^2R_s\overline{v_{in}^2}=0.5g_{mf}{}^2R_sV_m^2=0.12V_m^2$

偏置电流源消耗 $\overline{\left(3.578V+\frac{v_{in}}{1+g_mR_s}\right)\times200\mu A}=0.72mW$

耗能



现加入激励信号 $v_{in} = V_m \cos\omega t$

偏置电阻消耗

$$\overline{(4 - g_{mf}R_L'v_{in}) \cdot (0.2 - (1-\eta)g_{mf}v_{in})}$$
$$= 0.8 + (1-\eta){g_{mf}}^2R_L'\overline{v_{in}^2}$$
$$= 0.8 + 0.5(1-\eta){g_{mf}}^2R_L'V_m^2$$
$$= (0.8 + 0.59V_m^2)mW$$

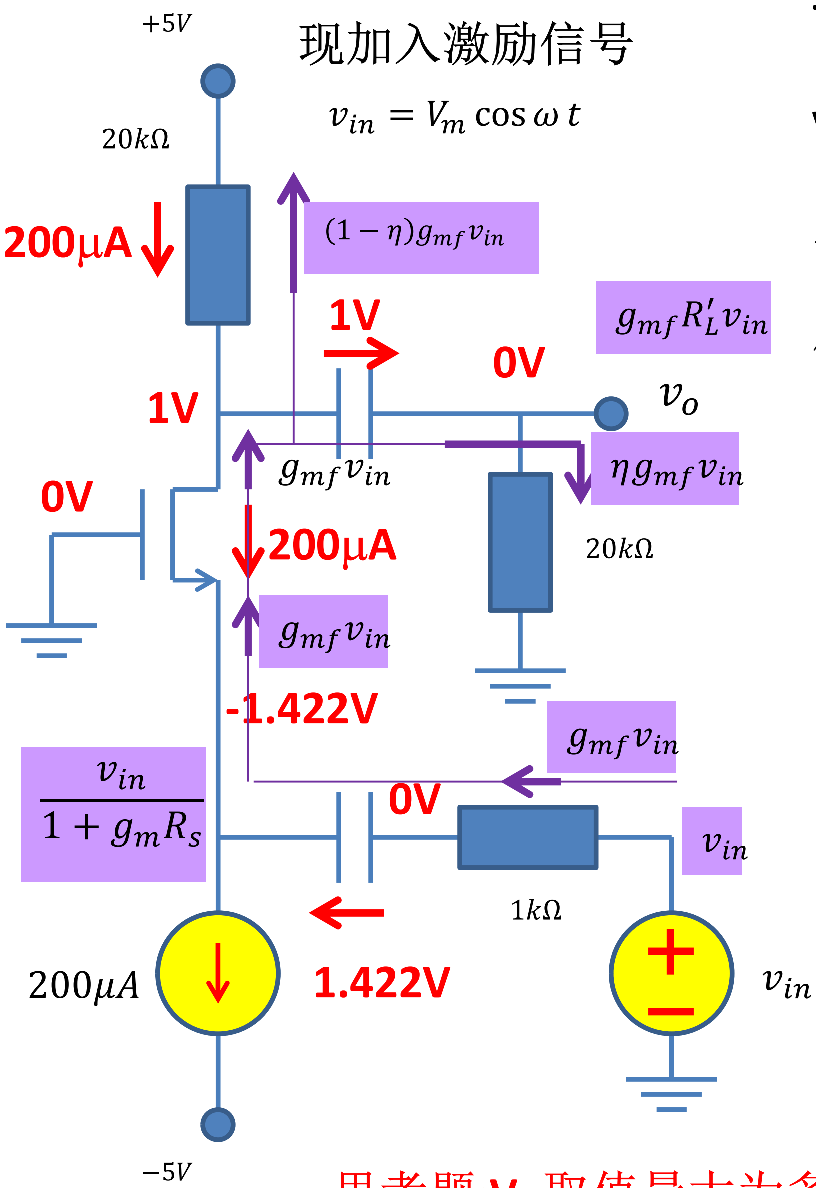
负载电阻消耗

$$\overline{g_{mf}R_L'v_{in} \cdot \eta g_{mf}v_{in}} = \eta {g_{mf}}^2R_L'\overline{v_{in}^2}$$
$$= 0.5\eta {g_{mf}}^2R_L'V_m^2 = 0.59V_m^2$$

晶体管消耗

$$\overline{(2.422 + g_{mf}(R_L' - r_e)v_{in}) \cdot (0.2 - g_{mf}v_{in})}$$
$$= 0.48 - {g_{mf}}^2(R_L' - r_e)\overline{v_{in}^2}$$
$$= 0.48 - 0.5{g_{mf}}^2(R_L' - r_e)V_m^2$$
$$= (0.48 - 1.06V_m^2)mW$$

供能与耗能



+5V电源供能 $\overline{5V\times\left(0.2mA-(1-\eta)g_{mf}v_{in}\right)} = 1mW$

-5V电源供能 $5V\times 200\mu A = 1mW$

v$_{in}$供能 $\overline{v_{in}\cdot g_{mf}v_{in}} = 0.24V_m^2$

小信号源内阻耗能 $\overline{\left(g_{mf}v_{in}\right)^2 R_s} = 0.12V_m^2$

偏置电流源耗能 $\overline{(3.578V+v)\times 200\mu A} = 0.72mW$

偏置电阻消耗 $\overline{\left(4-g_{mf}R_L' v_{in}\right)\cdot\left(0.2-(1-\eta)g_{mf}v_{in}\right)} = (0.8+0.59V_m^2)mW$

负载电阻消耗 $\overline{g_{mf}R_L' v_{in}\cdot \eta g_{mf}v_{in}} = 0.59V_m^2$

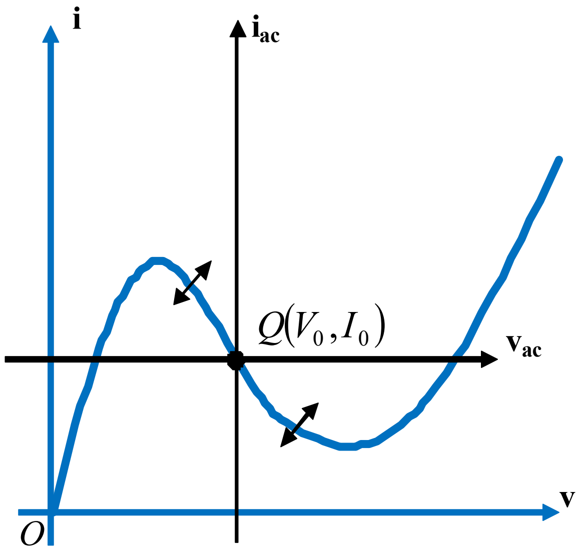
晶体管消耗 $\overline{\left(2.422+g_{mf}(R_L'-r_e)v_{in}\right)\cdot\left(0.2-g_{mf}v_{in}\right)} = (0.48-1.06V_m^2)mW$

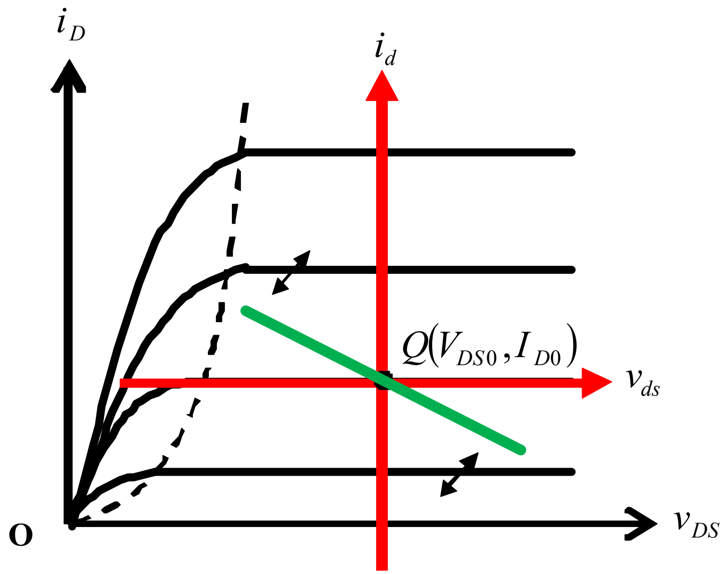
思考题:V$_m$取值最大为多少?

晶体管是换能器件

晶体管为换能器

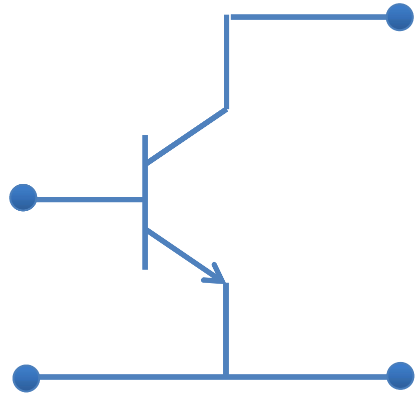
- 工作在负阻区的负阻器件，工作在有源区的晶体管，具有将直流能量转换为交流能量的能力，它们都是换能器件，和直流偏置源组合后，可形成向外端口输出交流能量的有源器件

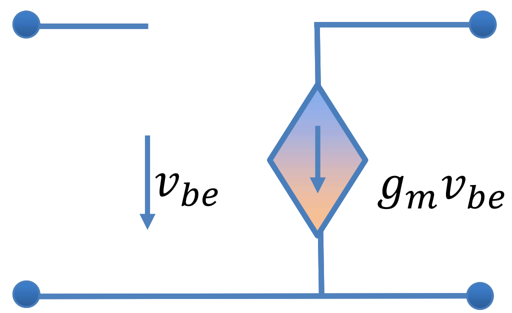




第十二讲作业

作业1 理想晶体管





电流增益$\beta \to \infty$，厄利电压$V_A \to \infty$

理想晶体管模型为理想压控流源。

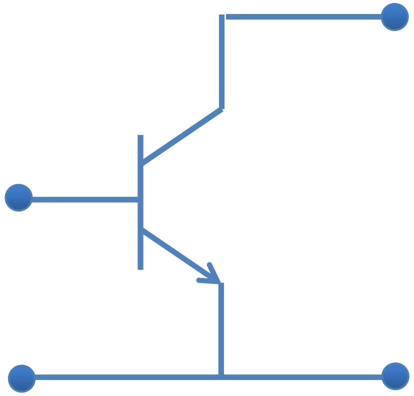
1）列写含有串串负反馈电阻的**CE**组态理想晶体管的端口约束方程，并将其转化为二端口等效电路

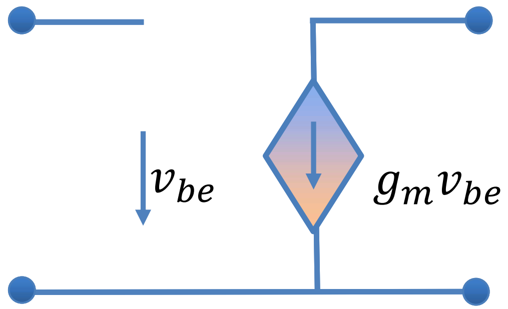
2）列写**CB**组态理想晶体管的端口约束方程，并将其转化为二端口等效电路

3）列写**CC**组态理想晶体管的端口约束方程，并将其转化为二端口等效电路

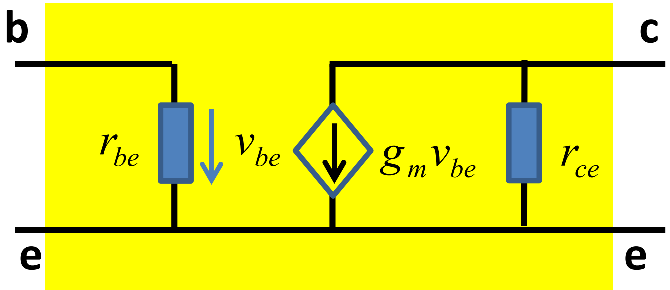
4）前述三个二端口网络，端口**1**对接戴维南源（v_s，R_s），端口**2**对接负载电阻R_L，分析电压增益$A_v=v_L/v_s$

晶体管是接近理想的压控流源





电流增益β→∞，厄利电压$\mathbf{V_A}$→∞



$$g_m = 40mS$$

$$r_{be} = 10k\Omega$$

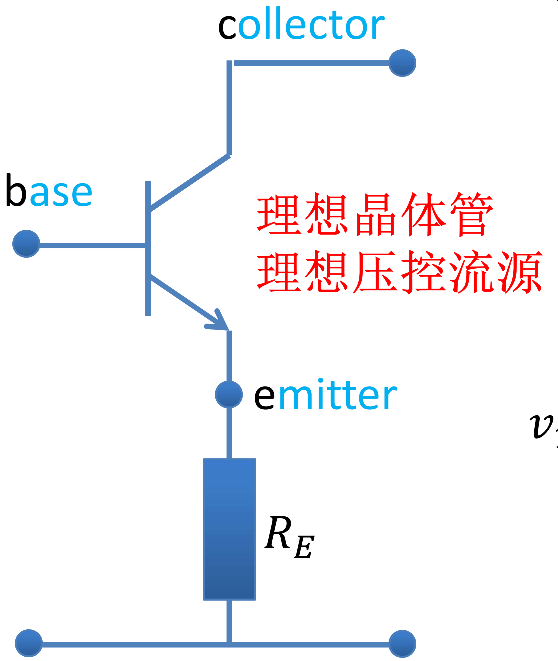
$$r_{ce} = 100k\Omega$$

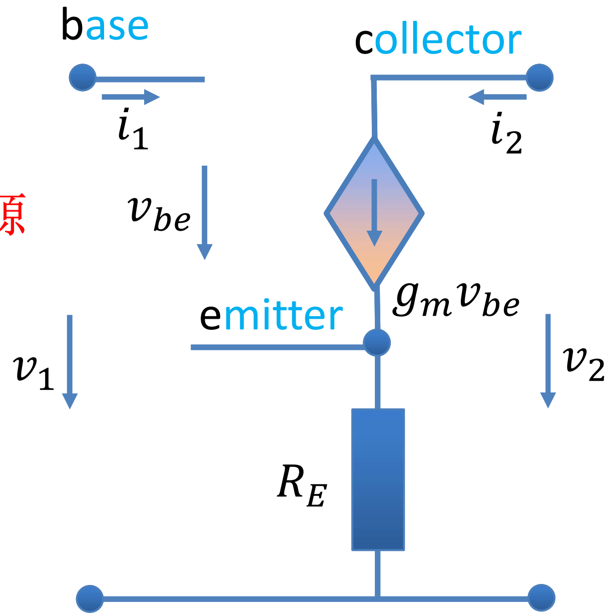
$$g_m = \frac{I_C}{v_T}$$

$$r_{be} = \beta\frac{1}{g_m} \xrightarrow{\beta\to\infty} \infty$$

$$r_{ce} = \frac{V_A}{I_{C0}} \xrightarrow{V_A\to\infty} \infty$$

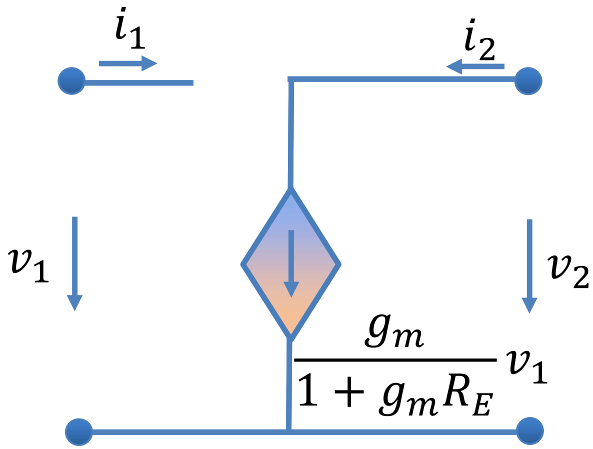
理想压控流源的串串负反馈仍然是理想压控流源





$$i_1 = 0$$

$$i_2 = \frac{g_m}{1 + g_m R_E} v_1$$



$$i_2 = g_m v_{be} = g_m (v_b - v_e)$$

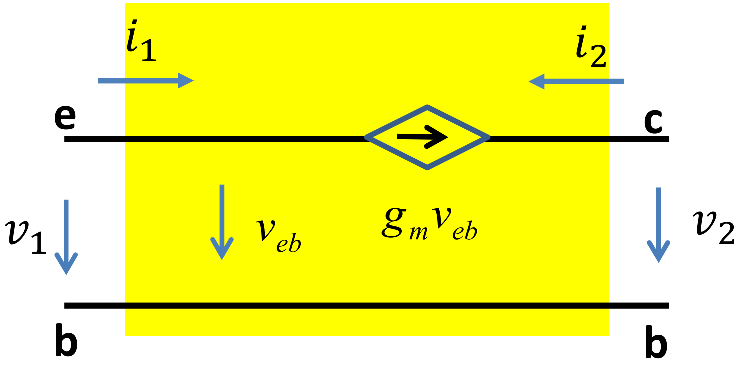
$$= g_m (v_1 - i_2 R_E) = g_m v_1 - i_2 g_m R_E$$

$$i_2 = \frac{g_m}{1 + g_m R_E} v_1$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{g_m}{1 + g_m R_E} & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

结论：理想晶体管加串联负反馈**RE**后仍然是理想晶体管，只不过跨导增益发生改变而已

CB组态电路模型



$$i_1 = g_m v_1$$

$$i_2 = -i_1 = -g_m v_1$$

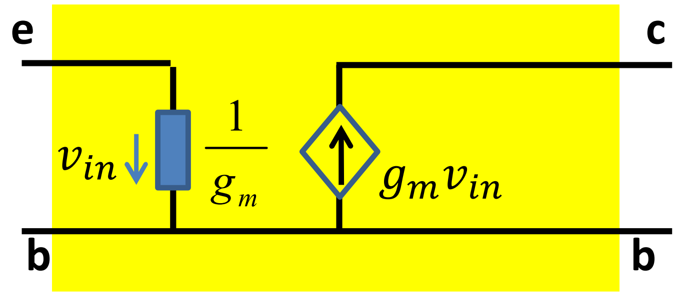
端口伏安特性方程：y参量表述

$$v_1 = \frac{1}{g_m} i_1$$

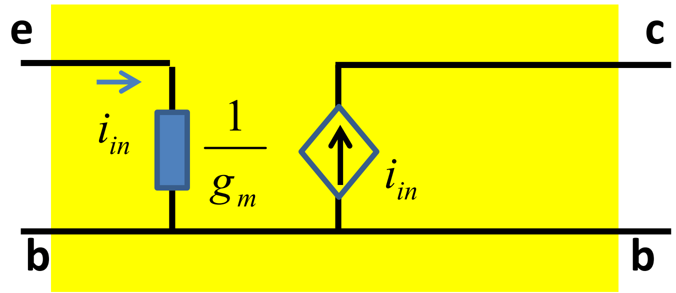
$$i_2 = -i_1$$

h参量表述

$$R_L \ll r_{ce} \to \infty$$

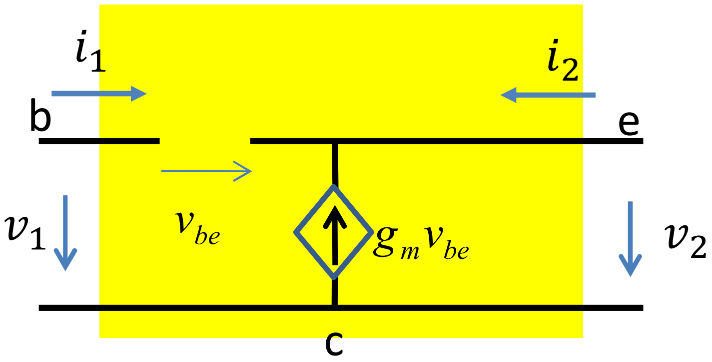


理想晶体管**CB**组态**y**参量电路模型



理想晶体管**CB**组态**h**参量电路模型
电流缓冲器模型

CC组态电路模型

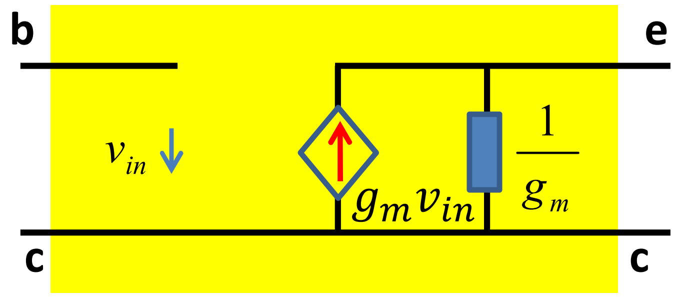


$R_S \ll r_{be} \to \infty$

$$i_1 = 0$$

$$i_2 = -g_m v_{be} = -g_m v_1 + g_m v_2$$

端口伏安特性方程：y参量表述

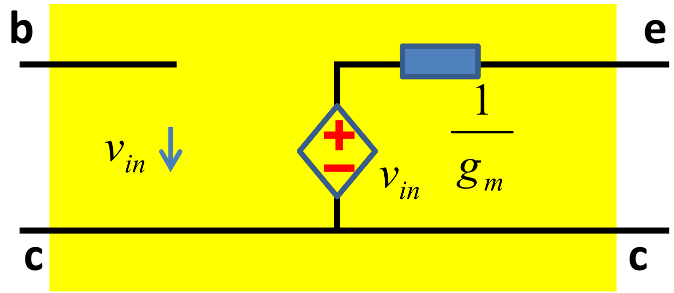


理想晶体管CC组态y参量电路模型

$$i_1 = 0$$

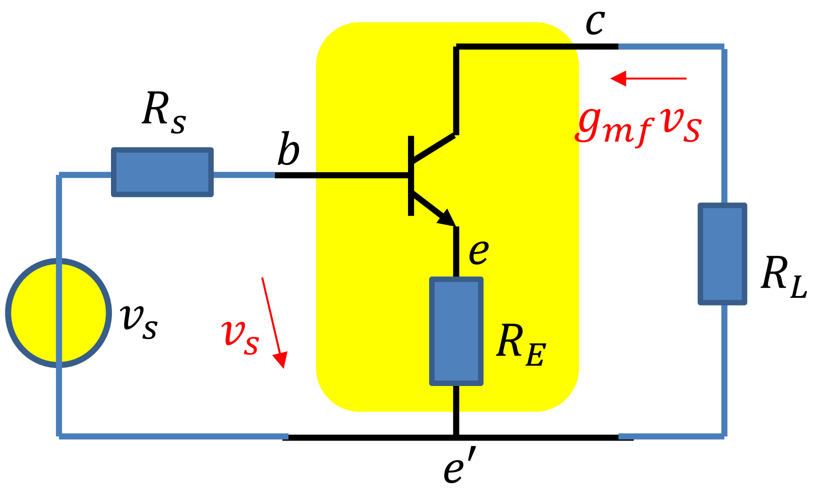
$$v_2 = v_1 + \frac{1}{g_m} i_2$$

g参量表述



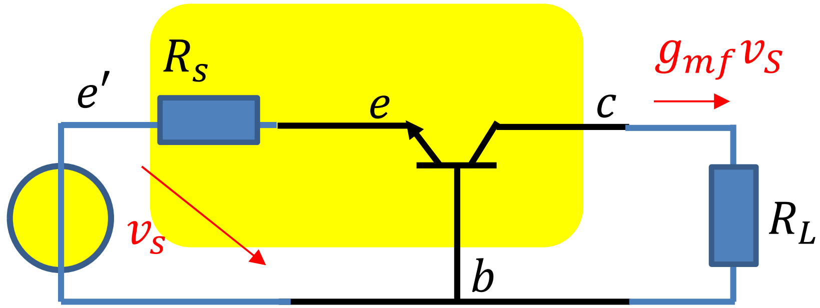
理想晶体管CC组态g参量电路模型
电压缓冲器模型：单向网络

三种组态放大器放大倍数



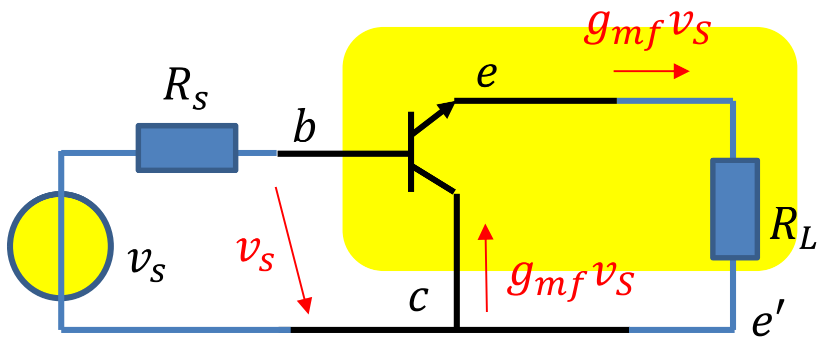
$$A_{v,CE} = -\boldsymbol{g_{mf}R_L} = -\frac{g_m}{1+g_m R_E}R_L$$

$R_S \ll r_{be}$ 或 $R_L \ll r_{ce}$



$$A_{v,CB} = +\boldsymbol{g_{mf}R_L} = \frac{g_m}{1+g_m R_S}R_L$$

$R_L \ll r_{ce}$

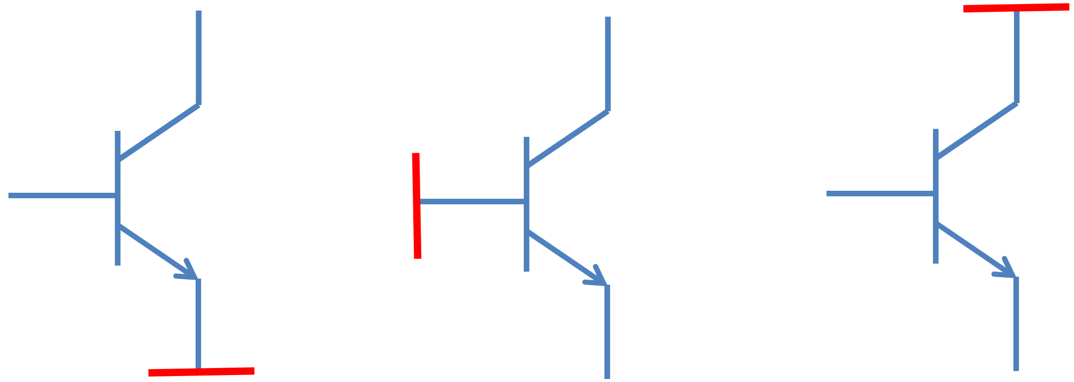


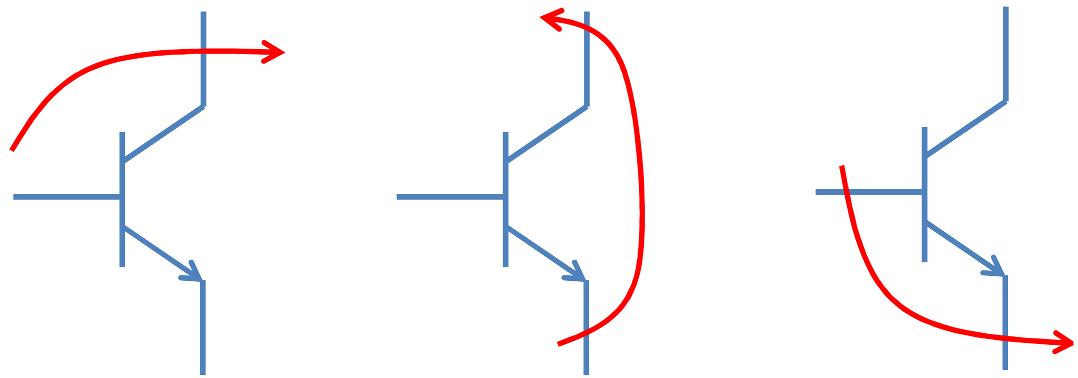
$$A_{v,CC} = +\boldsymbol{g_{mf}R_L} = \frac{g_m}{1+g_m R_L}R_L$$

$R_S \ll r_{be}$

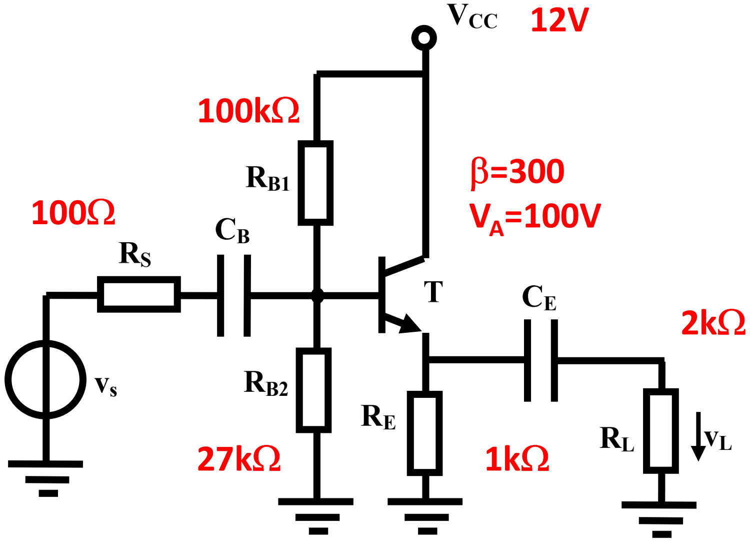
晶体管组态判定

- 看哪个端点交流接地
  - 谁接地，该端就是公共端
- 看信号放大路径，信号如何流动
  - 信号从B到C，就是共E
  - 信号从B到E，就是共C
  - 信号从E到C，就是共B





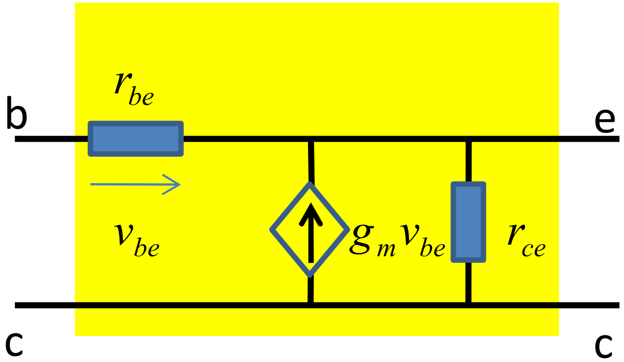
作业2 集电极交流地，故而CC组态
CC组态放大器

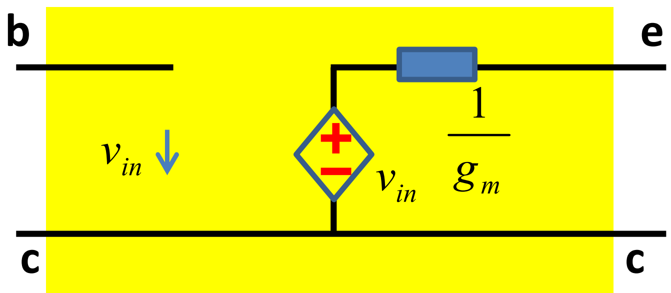


- （1）直流分析
- （2）交流分析
  - 采用y参量跨导器模型分析
  - 采用CC电压缓冲器模型分析

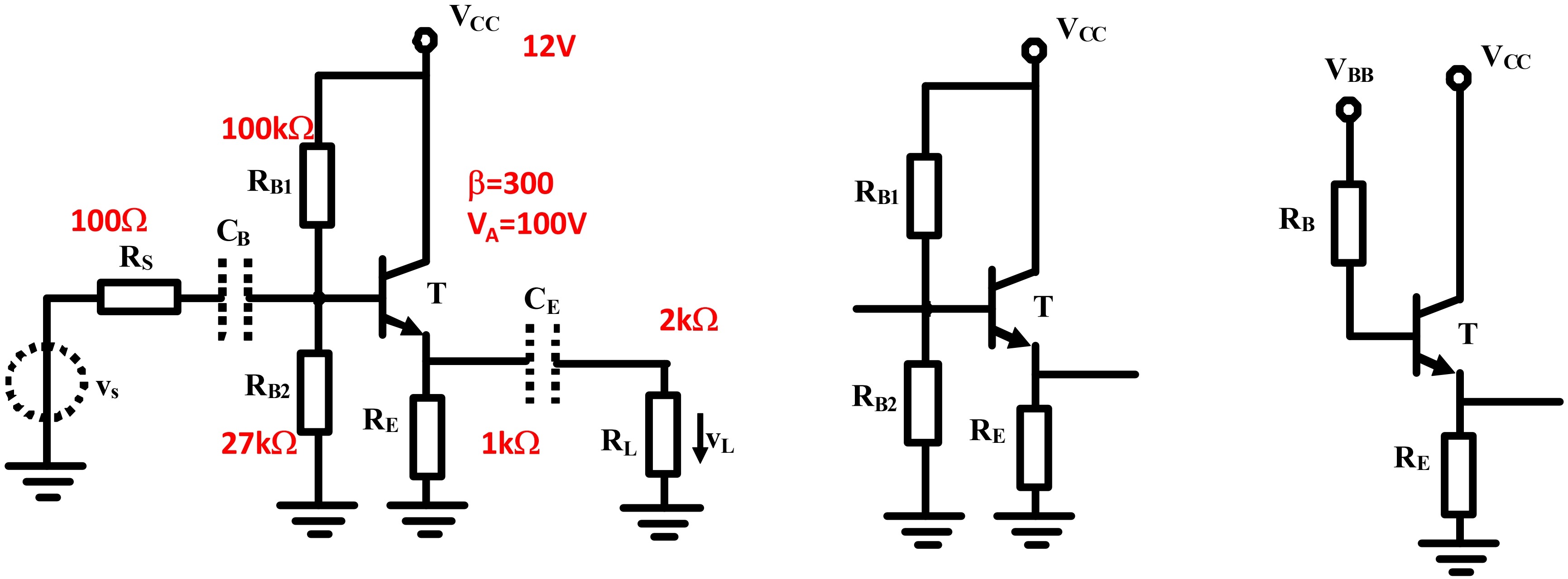
$$A_v=\frac{v_L}{v_S}=?$$

$$A_i=\frac{i_L}{i_S}=\frac{i_L}{G_S v_S}=?$$





直流分析

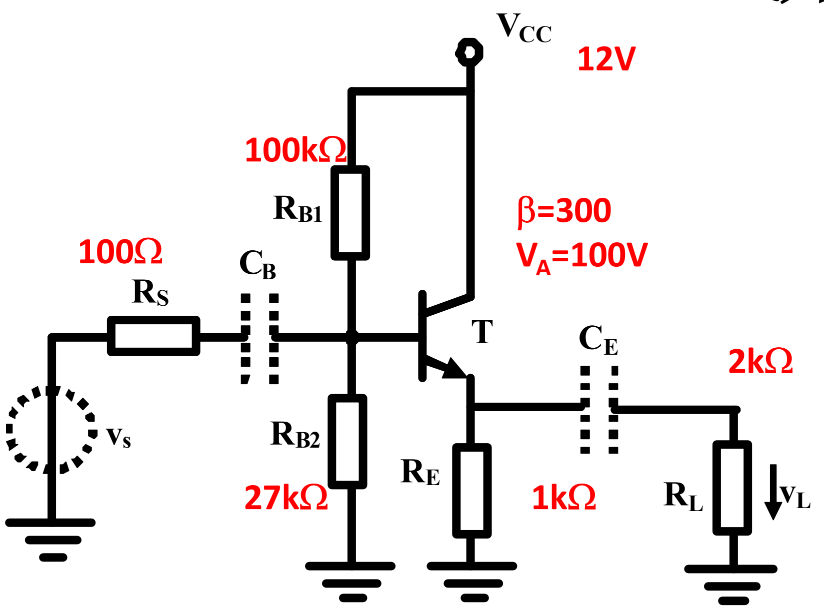


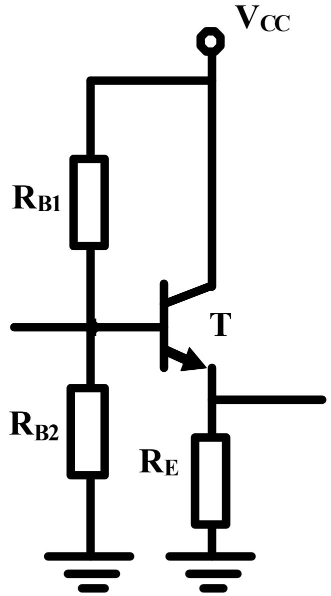
戴维南等效

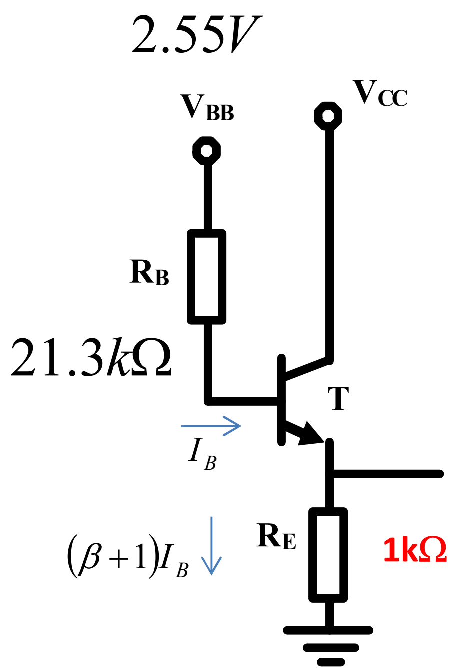
$$V_{BB}=\frac{R_{B2}}{R_{B1}+R_{B2}}V_{CC}=\frac{27k}{100k+27k}\times 12=2.55V$$

$$R_B=R_{B1}\parallel R_{B2}=\frac{R_{B1}R_{B2}}{R_{B1}+R_{B2}}=\frac{100k\times 27k}{100k+27k}=21.3k\Omega$$

直流分析







$$I_B R_B + V_{BE} + (\beta + 1) I_B R_E = V_{BB}$$

戴维南等效

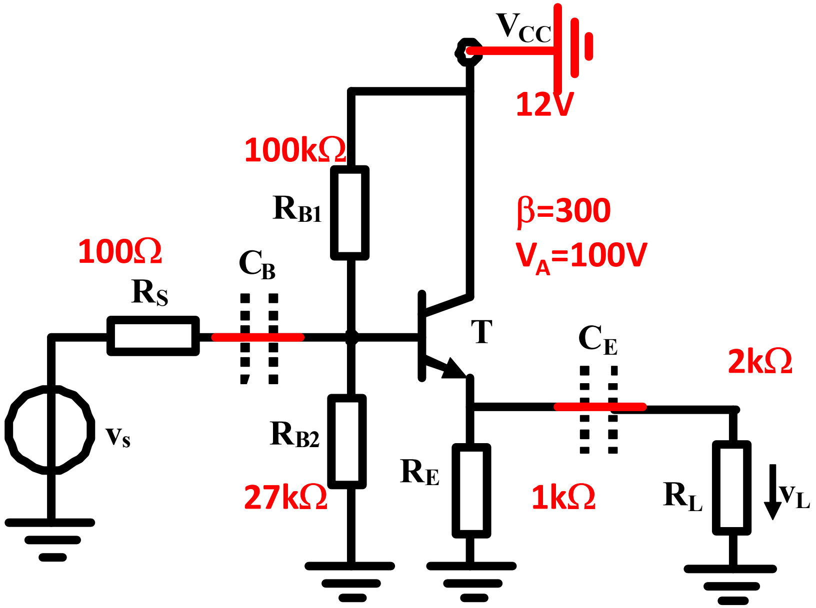
$$I_B = \frac{V_{BB} - V_{BE}}{R_B + (\beta + 1) R_E} = \frac{2.55 - 0.7}{21.3k + 301 \times 1} = 5.74\mu A$$

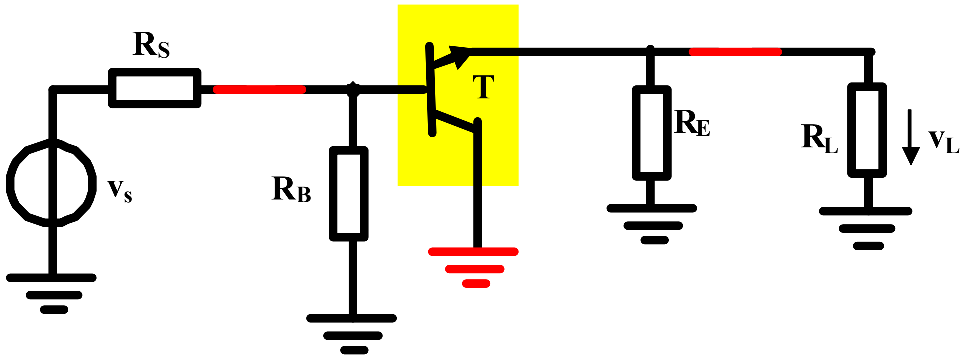
$$I_C = \beta I_B = 1.72mA$$

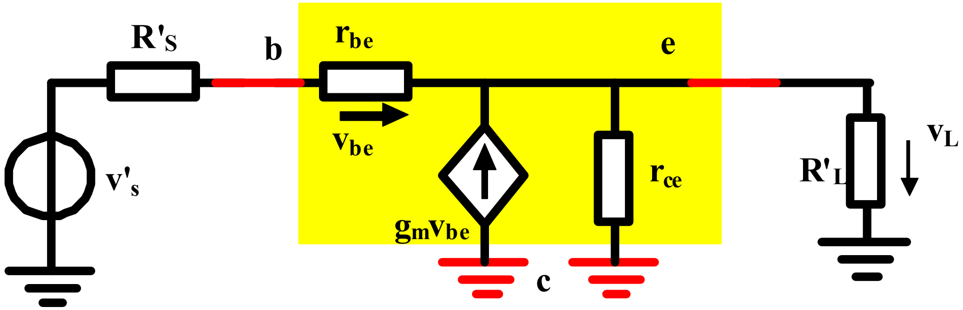
$$V_{CE} = V_{CC} - R_E I_E = 12 - 301 \times 5.74\mu \times 1k = 10.27V > 0.2 = V_{CE,sat}$$

确认在恒流区

交流小信号分析







$$v'_S = \frac{R_B}{R_S + R_B} v_S = \frac{21.3k}{0.1k + 21.3k} v_S = 0.995 v_S$$

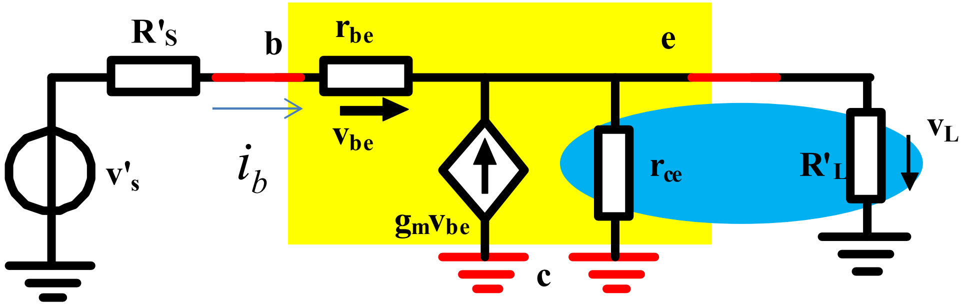
$$g_m = \frac{I_C}{v_T} = 66.2mS$$

$$R'_S = R_S \,||\, R_B = \frac{R_S R_B}{R_S + R_B} = 99.5\Omega$$

$$r_{be} = \beta \frac{1}{g_m} = 4.53k\Omega$$

$$R'_L = R_L \,||\, R_E = 1k\Omega \,||\, 2k\Omega = 667\Omega$$

$$r_{ce} = \frac{V_A}{I_C} = 58.1k\Omega$$



$$R_L'' = R_L' \parallel r_{ce} = 667 \parallel 58.1k = 659\Omega$$

$$v_S' = i_b\left(R_S' + r_{be}\right) + \left(i_b + g_m r_{be} i_b\right) R_L''$$

$$v_L = \left(i_b + g_m r_{be} i_b\right) R_L''$$

$$A_v = \frac{v_L}{v_S} = \frac{\left(i_b + g_m r_{be} i_b\right) R_L''}{i_b\left(R_S' + r_{be}\right) + \left(i_b + g_m r_{be} i_b\right) R_L''} \frac{v_S'}{v_S}$$

$$= \frac{\left(1 + g_m r_{be}\right) R_L''}{R_S' + r_{be} + R_L'' + g_m r_{be} R_L''} \frac{v_S'}{v_S}$$

$$= \frac{\left(1 + 66.2m \times 4.53k\right) \times 0.659}{0.0995 + 4.53 + \left(1 + 66.2m \times 4.53k\right) \times 0.659} \times 0.995$$

$$= 0.977 \times 0.995 = 0.972$$ 预期之中的结果

$$A_i = \frac{i_L}{G_S v_S} = \frac{v_L / R_L}{G_S v_S}$$

$$= \frac{v_L}{v_S} \frac{R_S}{R_L}$$

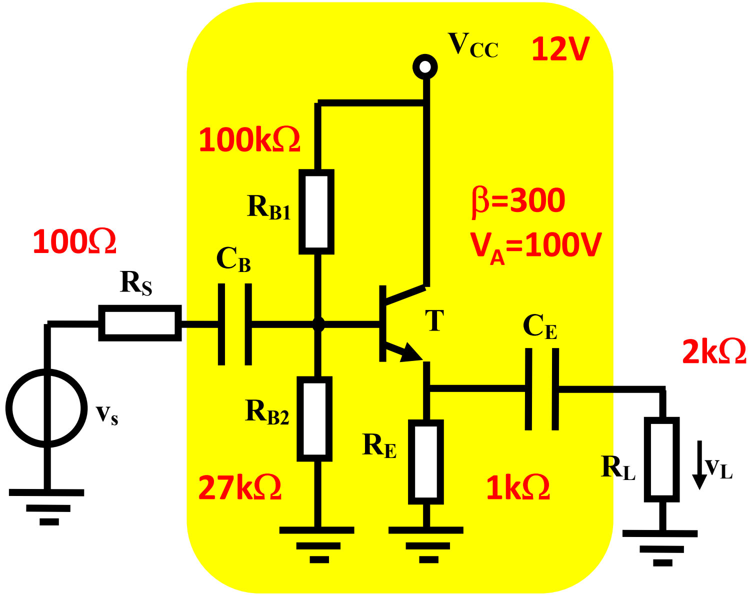
$$= 0.972 \times \frac{100}{2000} = 0.0486$$

太小了：源内阻太小，源内阻分流过大，导致电流增益很小

换一种定义：输出端口电流比输入端口电流

$$A_i = \frac{i_{out}}{i_{in}}$$

电流增益



$$A_i = \frac{i_{out}}{i_{in}} = \frac{i_{out}R_L}{i_{in}(R_S + R_B||r_{bc})}\frac{(R_S + R_B||r_{bc})}{R_L}$$

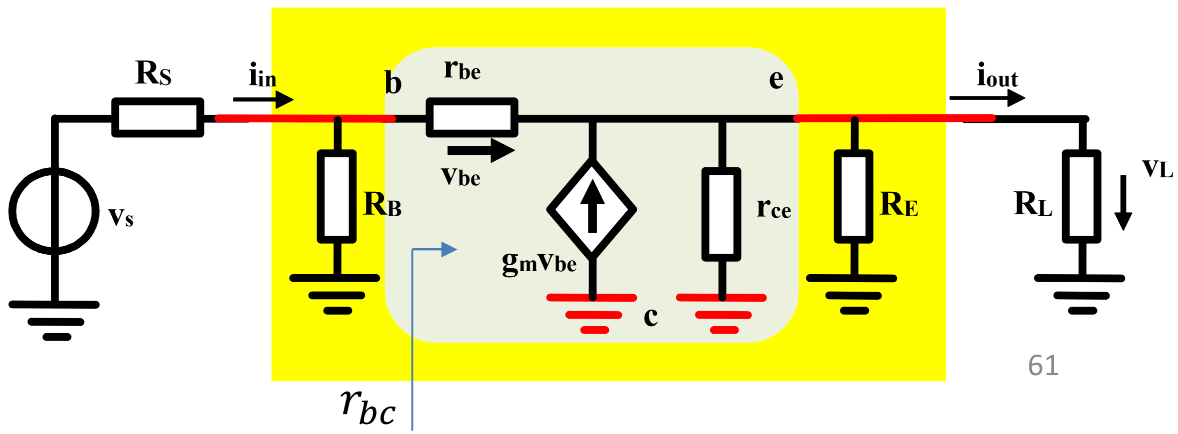
$$= \frac{v_L}{v_S}\frac{(R_S + R_B||r_{bc})}{R_L}$$

$$= A_v\frac{\left(R_S + R_B||\left(r_{be} + (r_{ce}||R_E||R_C) + g_m r_{be}(r_{ce}||R_E||R_C)\right)\right)}{R_L}$$

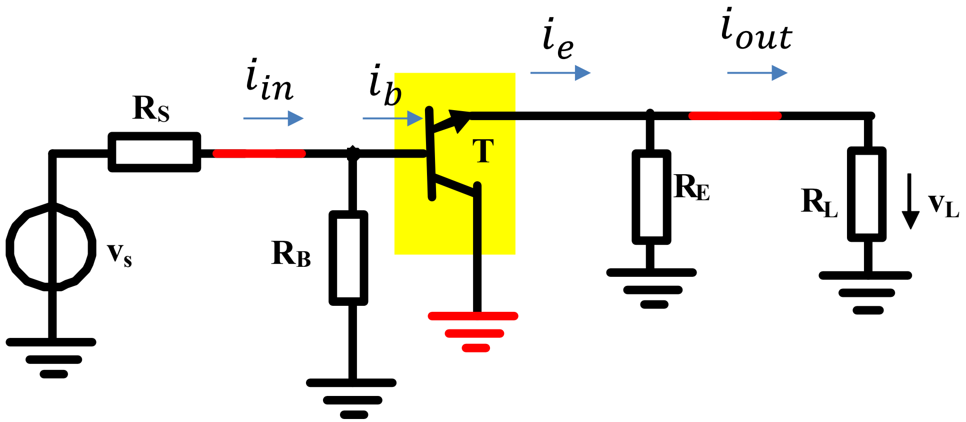
$$= 0.972 \times \frac{\left(100 + 21.3k||(4.53k + (58.1k||1k||2k) + 66.2m \times 4.53k \times 659)\right)}{2k}$$

$$= 0.972 \times \frac{(100 + 21.3k||203k)}{2k}$$

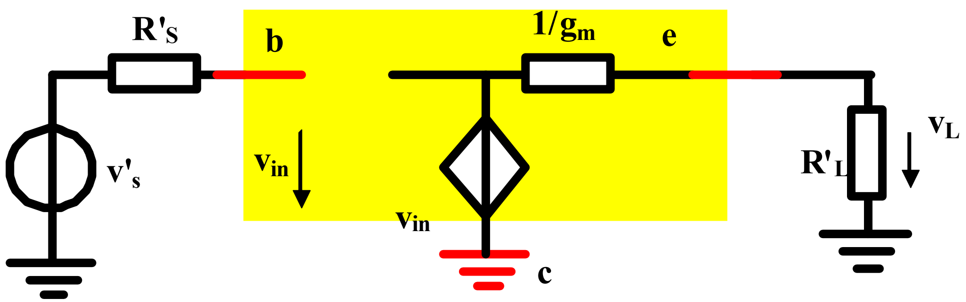
$$= 0.972 \times \frac{19.3k}{2k} = 9.4$$



电压缓冲器模型分析



$$99.5\Omega = R_S' << r_{be} = 4.53k$$



$$r_{bc} = r_{be} + (r_{ce}||R_E||R_C) + g_m r_{be}(r_{ce}||R_E||R_C)$$
$$= 4.53k + (58.1k||1k||2k) + 66.2m \times 4.53k \times 659$$
$$= 203k\Omega$$

$$v_{in} = v_S' = 0.995 v_S$$

$$v_L = \frac{R_L'}{R_L' + \frac{1}{g_m}} v_S'$$

$$= \frac{g_m}{1 + g_m R_L'} R_L' v_S' = g_{mf} R_L' v_S'$$

$$= \frac{66.2m \times 0.667k}{1 + 66.2m \times 0.667k} \times 0.995 v_S$$

$$= \frac{44.1}{45.1} \times 0.995 v_S = 0.973 v_S$$

$$A_i = \frac{i_{out}}{i_{in}} = \frac{i_{out}}{i_e}\frac{i_e}{i_b}\frac{i_b}{i_{in}}$$

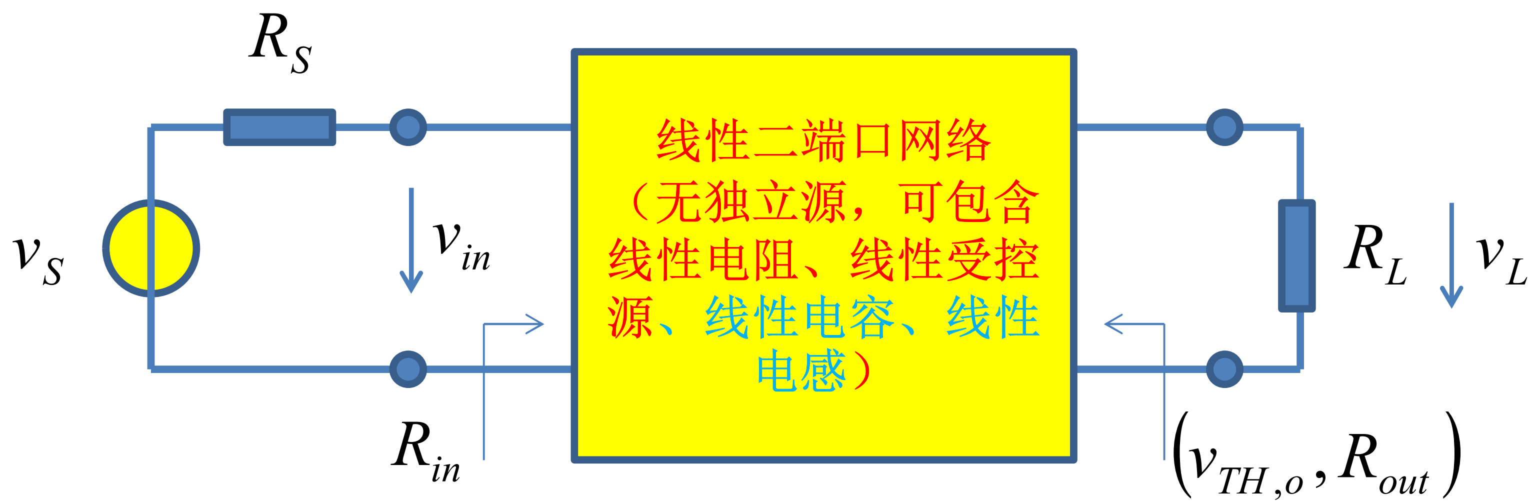
$$= \frac{G_L}{G_L + G_E}(\beta + 1)\frac{g_{bc}}{g_{bc} + G_B}$$

$$= \frac{R_E}{R_L + R_E}(\beta + 1)\frac{R_B}{r_{bc} + R_B}$$

$$= \frac{1k}{1k + 2k} \times 301 \times \frac{21.3k}{203k + 21.3k}$$

$$= 9.5$$

增益定义



$$G_T = \frac{P_L}{P_{si,\max}} \qquad A_v = 2\sqrt{\frac{R_S}{R_L}}\frac{v_L}{v_S} \qquad A_i = 2\sqrt{\frac{G_S}{G_L}}\frac{i_L}{i_S}$$

射频电路常用定义

$$G_A = \frac{P_{so,\max}}{P_{si,\max}} \qquad A_v = \frac{v_L}{v_S} \qquad A_i = \frac{i_L}{i_S}$$

一般性定义，不要求电阻负载

$$G_p = \frac{P_L}{P_{in}} \qquad A_v = \frac{v_L}{v_{in}} \qquad A_i = \frac{i_L}{i_{in}}$$

低频电路常用定义

$$P_L = \frac{V_{L,rms}^2}{R_L} \qquad P_{si,\max} = \frac{V_{S,rms}^2}{4R_S} \qquad P_{so,\max} = \frac{V_{TH,o,rms}^2}{4R_{out}} \qquad P_{in} = \frac{V_{in,rms}^2}{R_{in}}$$