电子电路与系统基础I

习题课第十讲

第**7**次作业讲解（部分）
第**8**次作业讲解（部分）

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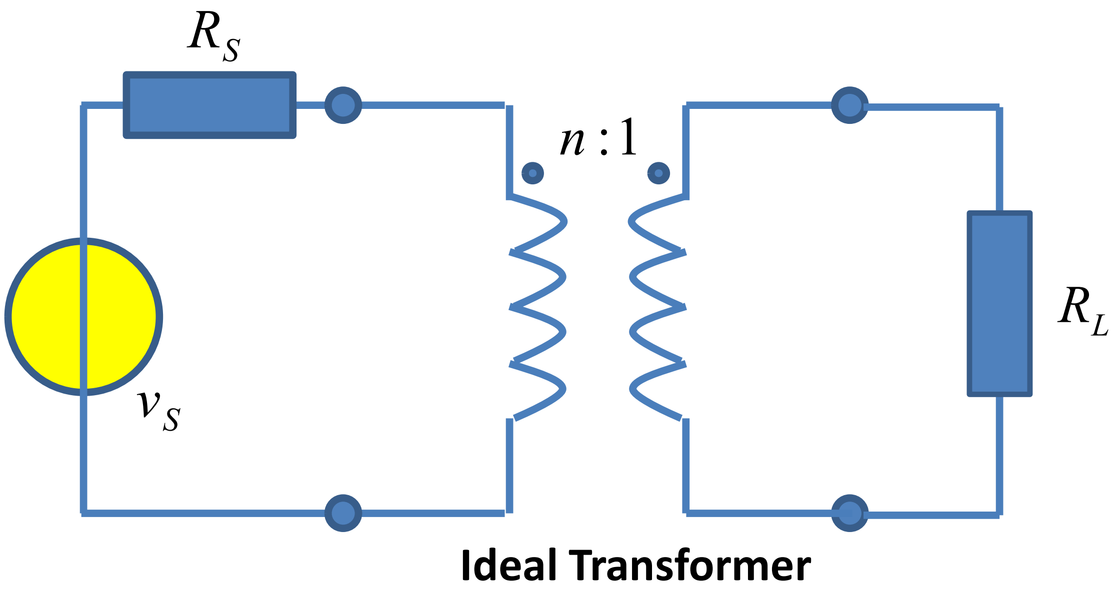
第7周作业

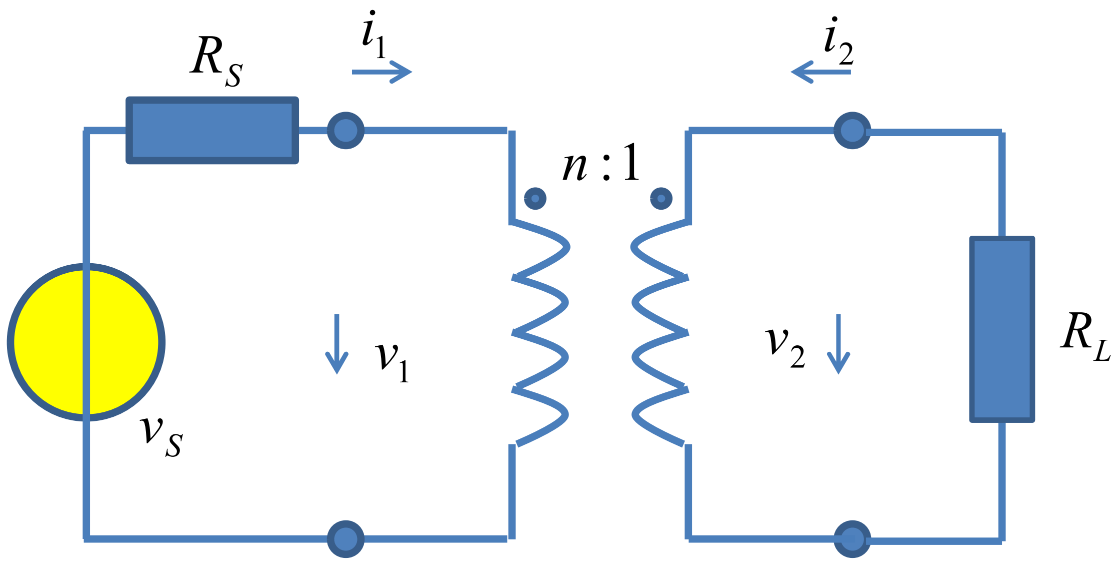
作业2 理想变压器实现阻抗匹配



负载电阻和信源内阻具有什么关系时，负载电阻可获得最大功率？此时信源输出多少功率？变压器消耗多少功率？负载消耗多少功率？

不考虑物理意义分析
不从电路角度理解
纯粹从数学方程求解角度分析

对不熟悉的第一次碰到的电路



端口对接关系：定义一套端口电压电流，**KVL**、**KCL**自动满足，只需列写元件约束方程即可

$$v_1 + i_1 R_S = v_S$$ 戴维南源约束 ⟹ $$v_2 = \frac{n^2 R_L}{n^2 R_L + R_S}\frac{v_S}{n}$$

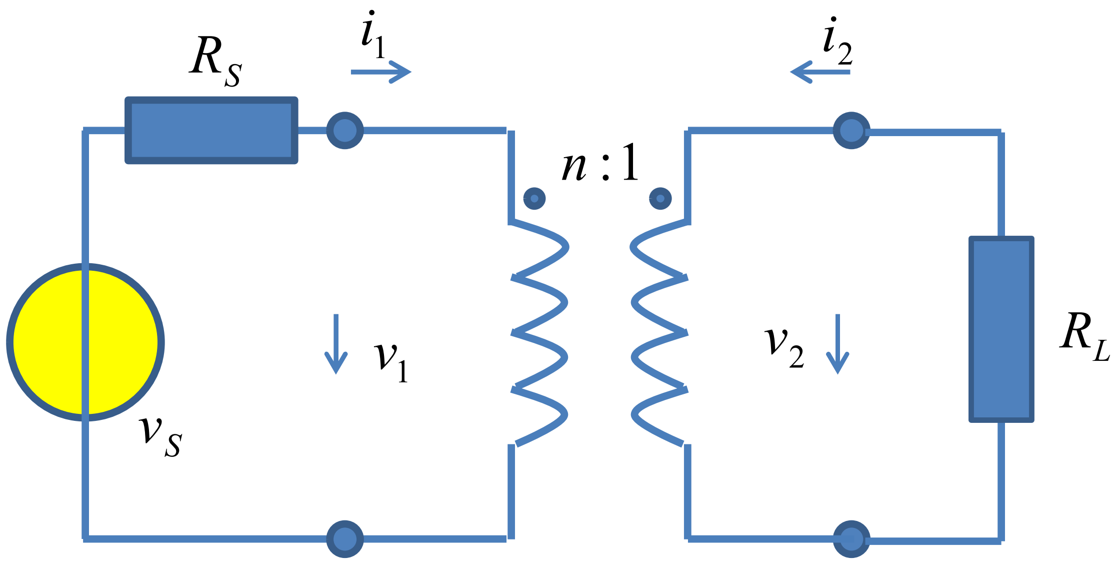
$$v_1 - n v_2 = 0$$ ⟹ $$v_1 = n v_2$$ ⟹ $$v_1 = \frac{n^2 R_L}{n^2 R_L + R_S} v_S$$

理想变压器约束

$$i_1 + \frac{1}{n} i_2 = 0$$ ⟹ $$i_1 = -\frac{1}{n} i_2 = \frac{v_2}{n R_L}$$ ⟹ $$i_1 = \frac{v_S}{n^2 R_L + R_S}$$

$$v_2 + i_2 R_L = 0$$ 负载约束 ⟹ $$i_2 = -\frac{v_2}{R_L}$$ ⟹ $$i_2 = -\frac{n}{n^2 R_L + R_S} v_S$$

对**v2**感兴趣，全部以**v2**为参变量表述其他中间变量



$$v_2 = \frac{n^2 R_L}{n^2 R_L + R_S}\frac{v_S}{n}$$

$$v_1 = \frac{n^2 R_L}{n^2 R_L + R_S} v_S$$

$$i_1 = \frac{v_S}{n^2 R_L + R_S}$$

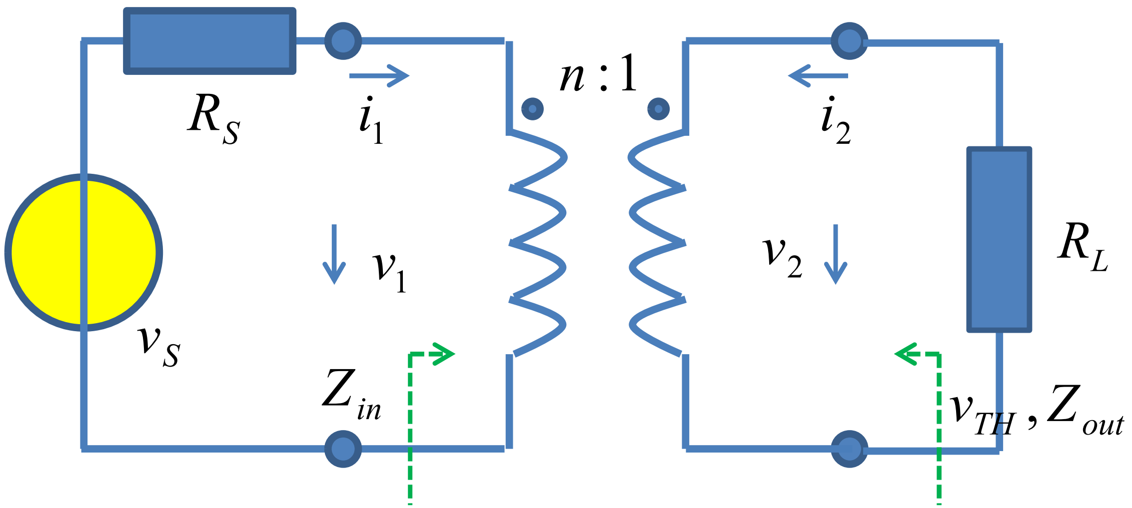
$$i_2 = -\frac{n}{n^2 R_L + R_S} v_S$$

$$P_L = -\overline{v_2 i_2} = \frac{n^2 R_L}{\left(n^2 R_L + R_S\right)^2} V_{s,rms}^2 \le \frac{V_{s,rms}^2}{4R_S} = P_{S,\max}$$

仅当 $R_S = n^2 R_L$ 等号成立

故而 $R_S = n^2 R_L$ 时，负载可获得信源额定功率

用等效电路解析表达式



$$v_1 = \frac{n^2 R_L}{n^2 R_L + R_S} v_S$$

$$i_1 = \frac{v_S}{n^2 R_L + R_S}$$

$$v_2 = \frac{n^2 R_L}{n^2 R_L + R_S} \frac{v_S}{n}$$

$$i_2 = -\frac{n}{n^2 R_L + R_S} v_S$$

$$Z_{in} = n^2 R_L$$

$$Z_{out} = \frac{R_S}{n^2}$$

$$v_{TH} = \frac{1}{n} v_S$$

对电路分析结果给出合理的解释（等效电路）：用等效电路理解则无需列大方程求解：足够积累后，应能够直接给结果

$$v_1 = \frac{Z_{in}}{R_S + Z_{in}} v_S$$

$$i_1 = \frac{v_S}{Z_{in} + R_S}$$

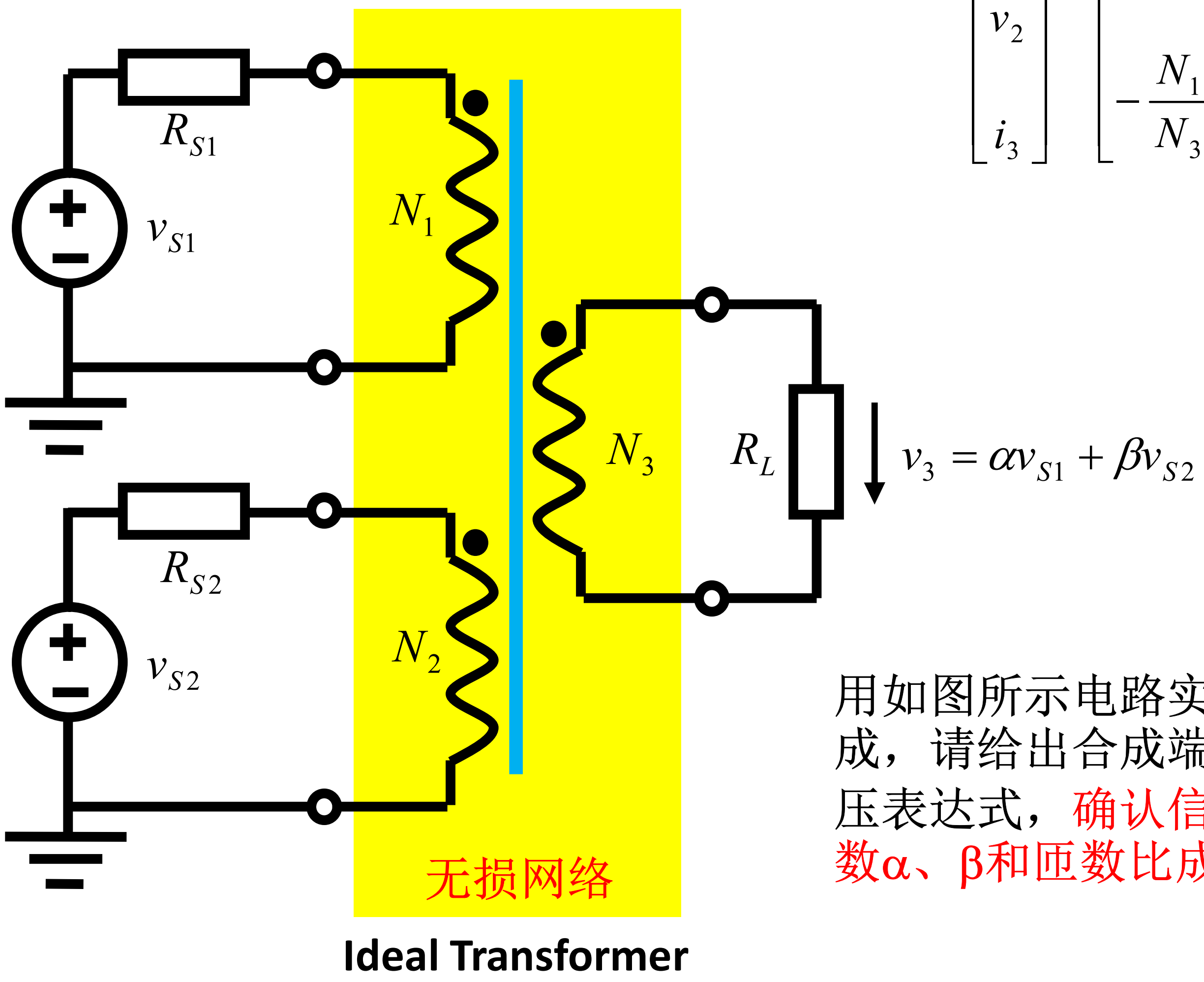
$$v_2 = \frac{R_L}{R_L + Z_{out}} v_{TH}$$

$$i_2 = -\frac{v_{TH}}{Z_{out} + R_L}$$

第7周作业

作业3　信号合成

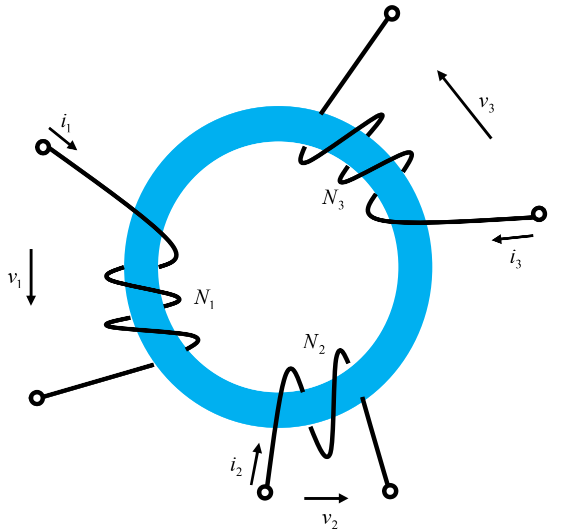
$$\begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{N_1}{N_3} \\ 0 & 0 & \frac{N_2}{N_3} \\ -\frac{N_1}{N_3} & -\frac{N_2}{N_3} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_3 \end{bmatrix}$$



用如图所示电路实现信号合成，请给出合成端口$\mathbf{v_3}$的电压表达式，确认信号合成系数α、β和匝数比成正比关系

三个端口
没有本质区别

$$\begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{N_1}{N_3} \\ 0 & 0 & \frac{N_2}{N_3} \\ -\frac{N_1}{N_3} & -\frac{N_2}{N_3} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_3 \end{bmatrix}$$

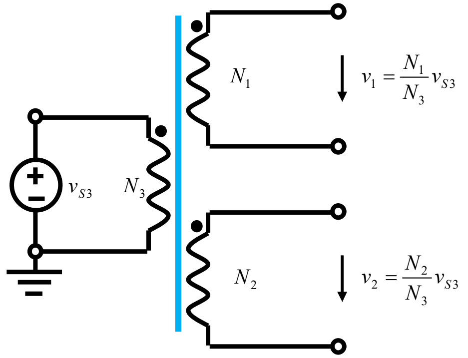


（a）三端口电感绕制方式

$$v_1 - \frac{N_1}{N_3} v_3 = 0$$

$$v_2 - \frac{N_2}{N_3} v_3 = 0$$

$$i_3 + \frac{N_1}{N_3} i_1 + \frac{N_2}{N_3} i_2 = 0$$



（b）三端口理想变压器信号分解

$$\frac{v_1}{N_1} = \frac{v_2}{N_2} = \frac{v_3}{N_3} = \Im_0$$

$$N_1 i_1 + N_2 i_2 + N_3 i_3 = 0$$

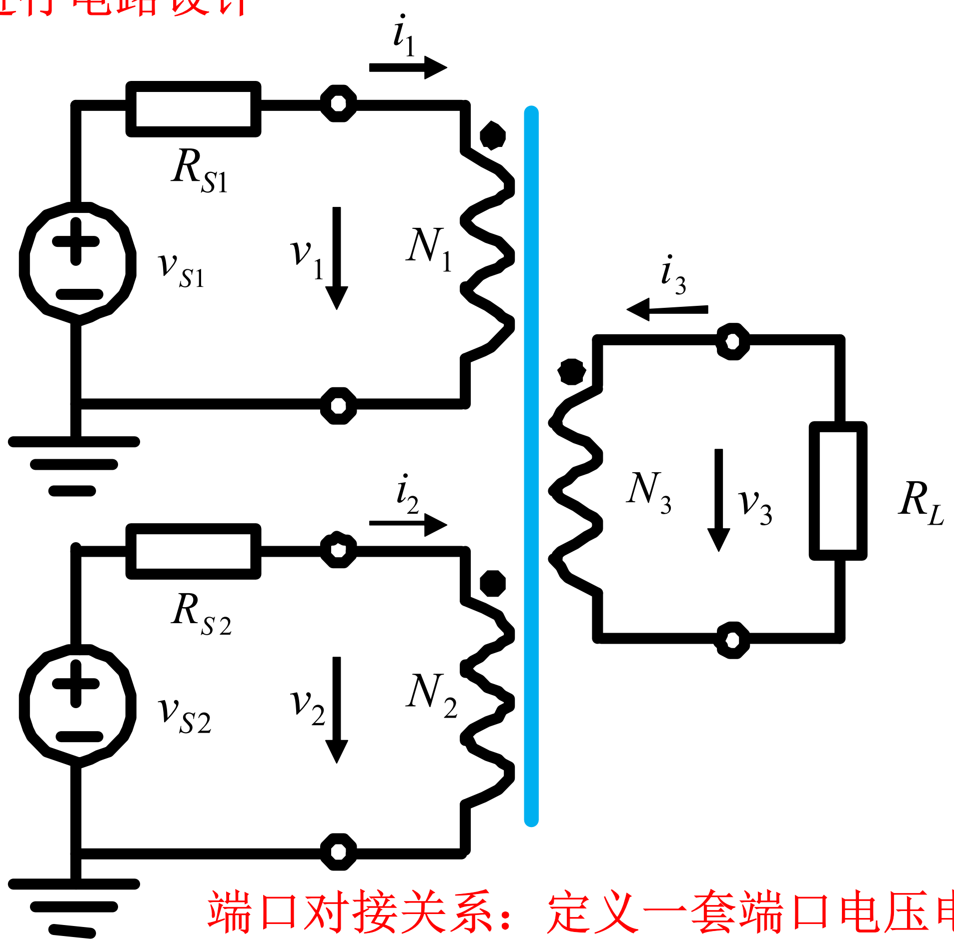
$$P_\Sigma = v_1 i_1 + v_2 i_2 + v_3 i_3 \equiv 0$$

无损网络

第一次碰到的电路，先列数学方程求解

之后对解进行解析，赋予明确的物理意义或等效电路，方便记忆

再后则可根据对电路的理解、物理解释（等效电路）进行电路设计



$$v_1 + i_1 R_{S1} = v_{S1}$$

$$v_2 + i_2 R_{S2} = v_{S2}$$

$$v_3 + i_3 R_L = 0$$

$$v_1 - \frac{N_1}{N_3} v_3 = 0$$

$$v_2 - \frac{N_2}{N_3} v_3 = 0$$

$$i_3 + \frac{N_1}{N_3} i_1 + \frac{N_2}{N_3} i_2 = 0$$

$$\begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{N_1}{N_3} \\ 0 & 0 & \frac{N_2}{N_3} \\ -\frac{N_1}{N_3} & -\frac{N_2}{N_3} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_3 \end{bmatrix}$$

端口对接关系：定义一套端口电压电流，**KVL**、**KCL**自动满足，只需列写元件约束方程即可

MATLAB辅助符号运算：可用手工推，可用借助计算机

$$v_1 + i_1 R_{S1} = v_{S1}$$

$$v_2 + i_2 R_{S2} = v_{S2}$$

$$v_3 + i_3 R_L = 0$$

$$v_1 - \frac{N_1}{N_3} v_3 = 0$$

$$v_2 - \frac{N_2}{N_3} v_3 = 0$$

$$i_3 + \frac{N_1}{N_3} i_1 + \frac{N_2}{N_3} i_2 = 0$$

```
>> syms RS1 RS2 RL vS1 vS2 N1 N2 N3
>> A=[1 0 0 RS1 0 0;0 1 0 0 RS2 0;0 0 1 0 0 RL;1 0 -N1/N3 0 0 0;0 1 -N2/N3 0 0 0;0 0 0 N1/N3 N2/N3 1]

A =

[      1,      0,      0,    RS1,      0,      0]
[      0,      1,      0,      0,    RS2,      0]
[      0,      0,      1,      0,      0,     RL]
[      1,      0, -N1/N3,      0,      0,      0]
[      0,      1, -N2/N3,      0,      0,      0]
[      0,      0,      0,  N1/N3,  N2/N3,      1]
```

$$\mathbf{A}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_{S1} \\ v_{S2} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

对v_3感兴趣，以v_3为参变量表述其他中间变量，即可最终只剩下关于v_3的方程

```
>> inv(A)

ans =

[           N1^2*RS2*RL/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),           N1*RS1*RL*N2/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),
[          N2*N1*RS2*RL/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),             N2^2*RS1*RL/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),
[          N1*N3*RS2*RL/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),           RS1*RL*N2*N3/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),
[   (N2^2*RL+RS2*N3^2)/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),              -N2*RL*N1/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),
[             -N2*RL*N1/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),    (N1^2*RL+RS1*N3^2)/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),
[            -RS2*N1*N3/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),             -RS1*N2*N3/(N1^2*RS2*RL+N2^2*RS1*RL+RS2*RS1*N3^2),    (N1^2*

>>
```

$$v_3 = \frac{N_1 N_3 R_L R_{S2}}{N_1^2 R_{S2} R_L + N_2^2 R_{S1} R_L + N_3^2 R_{S1} R_{S2}} v_{S1} + \frac{N_2 N_3 R_L R_{S1}}{N_1^2 R_{S2} R_L + N_2^2 R_{S1} R_L + N_3^2 R_{S1} R_{S2}} v_{S2}$$

计算机求解的最大问题：很难看出物理意义

$$v_1 + i_1 R_{S1} = v_{S1} \xrightarrow{③} i_1 = \frac{v_{S1}}{R_{S1}} - \frac{v_1}{R_{S1}} = \frac{v_{S1}}{R_{S1}} - \frac{N_1}{N_3}\frac{v_3}{R_{S1}}$$

$$v_2 + i_2 R_{S2} = v_{S2} \xrightarrow{④} i_2 = \frac{v_{S2}}{R_{S2}} - \frac{v_2}{R_{S2}} = \frac{v_{S2}}{R_{S2}} - \frac{N_2}{N_3}\frac{v_3}{R_{S2}}$$

$$v_3 + i_3 R_L = 0 \xrightarrow{⑤} i_3 = -\frac{v_3}{R_L}$$

$$v_1 - \frac{N_1}{N_3} v_3 = 0 \xrightarrow{①} v_1 = \frac{N_1}{N_3} v_3$$

$$v_2 - \frac{N_2}{N_3} v_3 = 0 \xrightarrow{②} v_2 = \frac{N_2}{N_3} v_3$$

$$i_3 + \frac{N_1}{N_3} i_1 + \frac{N_2}{N_3} i_2 = 0 \xrightarrow{⑦} -\frac{v_3}{R_L} + \frac{N_1}{N_3}\frac{v_{S1}}{R_{S1}} - \left(\frac{N_1}{N_3}\right)^2 \frac{v_3}{R_{S1}} + \frac{N_2}{N_3}\frac{v_{S2}}{R_{S2}} - \left(\frac{N_2}{N_3}\right)^2 \frac{v_3}{R_{S2}} = 0$$

$$\frac{N_1}{N_3}\frac{v_{S1}}{R_{S1}} + \frac{N_2}{N_3}\frac{v_{S2}}{R_{S2}} = \frac{v_3}{R_L} + \left(\frac{N_1}{N_3}\right)^2 \frac{v_3}{R_{S1}} + \left(\frac{N_2}{N_3}\right)^2 \frac{v_3}{R_{S2}}$$

$$v_3 = \frac{\frac{N_1}{N_3}\frac{v_{S1}}{R_{S1}} + \frac{N_2}{N_3}\frac{v_{S2}}{R_{S2}}}{\frac{1}{R_L} + \left(\frac{N_1}{N_3}\right)^2 \frac{1}{R_{S1}} + \left(\frac{N_2}{N_3}\right)^2 \frac{1}{R_{S2}}}$$

从表达式给出等效电路解释

②被**N1/N3**的变流比变换到端口**3**，形成端口**3**的源电流

①端口**1**所接电源的诺顿源电流

⑤端口**3**的两个源电流相加，形成总源电流（并联）

④被**N2/N3**的变流比变换到端口**3**，形成端口**3**的源电流

③端口**2**所接电源的诺顿源电流

$$v_3 = \frac{\frac{N_1}{N_3}\frac{v_{S1}}{R_{S1}} + \frac{N_2}{N_3}\frac{v_{S2}}{R_{S2}}}{\frac{1}{R_L} + \left(\frac{N_1}{N_3}\right)^2 \frac{1}{R_{S1}} + \left(\frac{N_2}{N_3}\right)^2 \frac{1}{R_{S2}}}$$

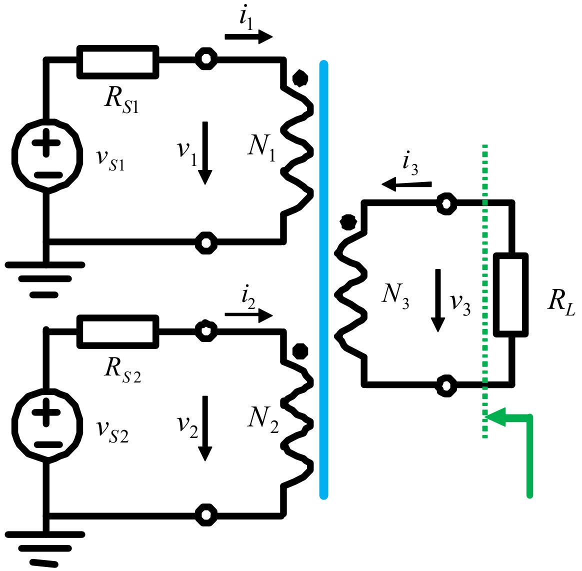
⑨端口**3**电压为源电流和总阻抗之积

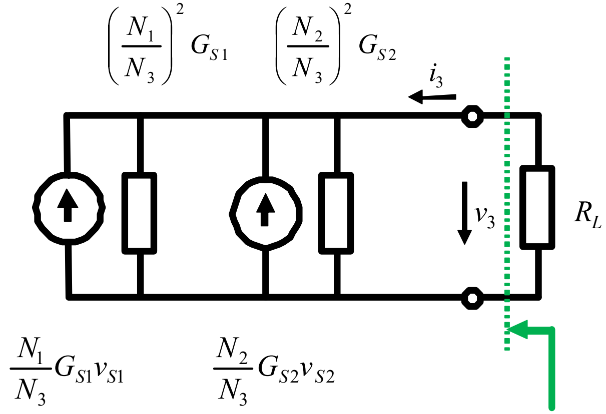
⑦端口**2**的源内导被变换到端口**3**

⑥端口**1**的源内导被变换到端口**3**

⑧端口**3**的总电导是两个等效源内导和负载电导之和（并联结构）

等效电路

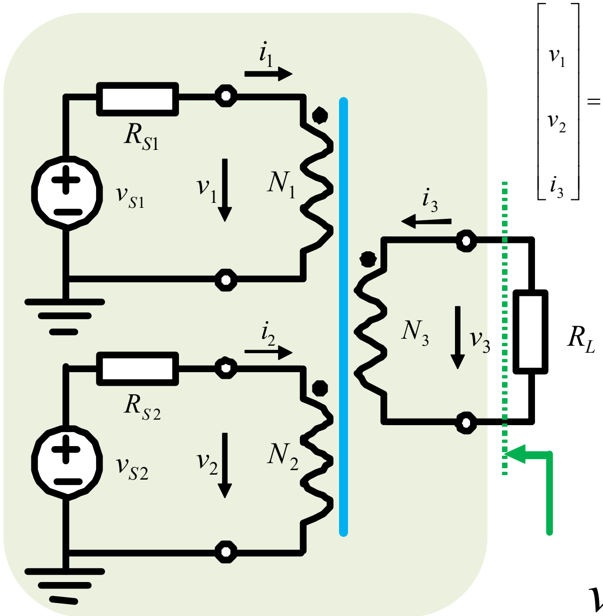




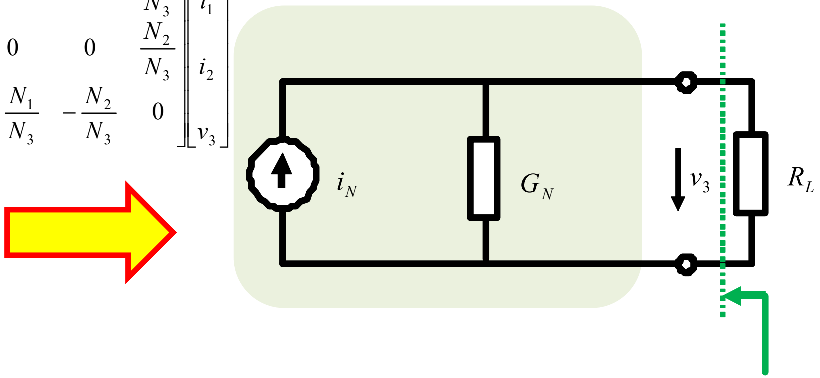
$$v_3 = \frac{\frac{N_1}{N_3}\frac{v_{S1}}{R_{S1}} + \frac{N_2}{N_3}\frac{v_{S2}}{R_{S2}}}{\frac{1}{R_L} + \left(\frac{N_1}{N_3}\right)^2 \frac{1}{R_{S1}} + \left(\frac{N_2}{N_3}\right)^2 \frac{1}{R_{S2}}} = \alpha i_{S1} + \beta i_{S2}$$

$$\alpha : \beta \sim N_1 : N_2$$

用诺顿定理求等效源电流

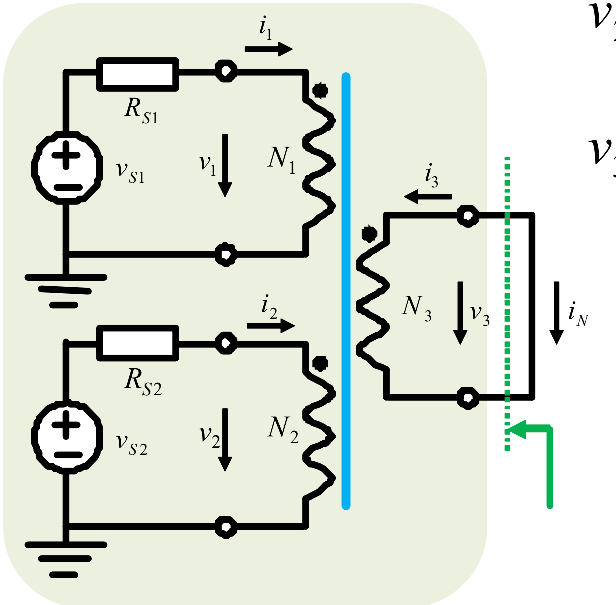


$$\begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{N_1}{N_3} \\ 0 & 0 & \frac{N_2}{N_3} \\ -\frac{N_1}{N_3} & -\frac{N_2}{N_3} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_3 \end{bmatrix}$$



电流合成：用诺顿等效相对比较适当

诺顿源电流：输出端口短路电流



$$v_1 + i_1 R_{S1} = v_{S1} \qquad v_1 = \frac{N_1}{N_3} v_3 = 0$$

$$v_2 + i_2 R_{S2} = v_{S2} \qquad v_2 = \frac{N_2}{N_3} v_3 = 0$$

$$v_3 = 0$$

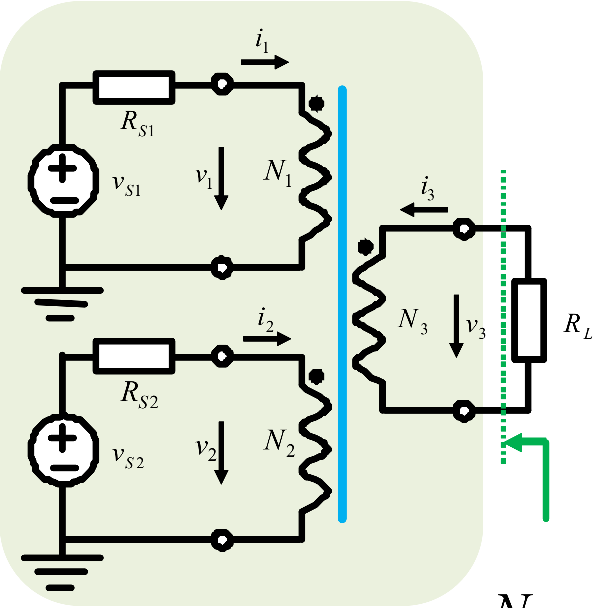
$$i_N = -i_3 = \frac{N_1}{N_3} i_1 + \frac{N_2}{N_3} i_2$$

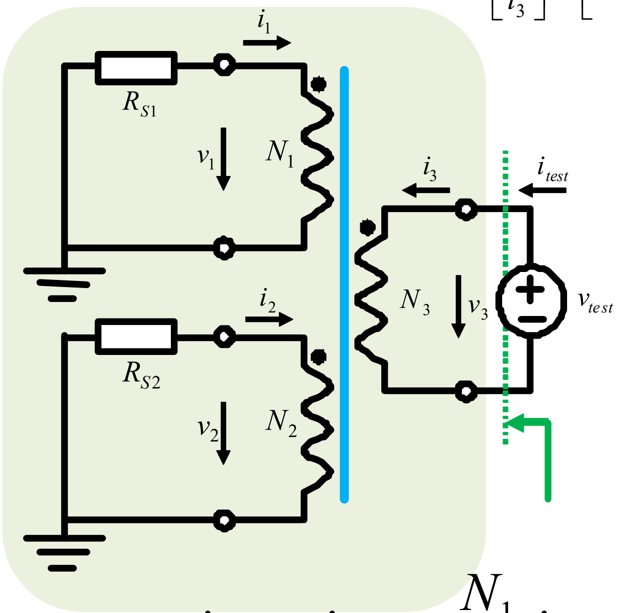
$$= \frac{N_1}{N_3} \frac{v_{S1}}{R_{S1}} + \frac{N_2}{N_3} \frac{v_{S2}}{R_{S2}}$$

$$= \frac{N_1}{N_3} i_{S1} + \frac{N_2}{N_3} i_{S2}$$

用诺顿定理求源内导

$$\begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{N_1}{N_3} \\ 0 & 0 & \frac{N_2}{N_3} \\ -\frac{N_1}{N_3} & -\frac{N_2}{N_3} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_3 \end{bmatrix}$$





$$v_1 + i_1 R_{S1} = 0$$

$$v_2 + i_2 R_{S2} = 0$$

$$v_3 = v_{test}$$

$$v_1 = \frac{N_1}{N_3} v_3 = \frac{N_1}{N_3} v_{test}$$

$$v_2 = \frac{N_2}{N_3} v_3 = \frac{N_2}{N_3} v_{test}$$

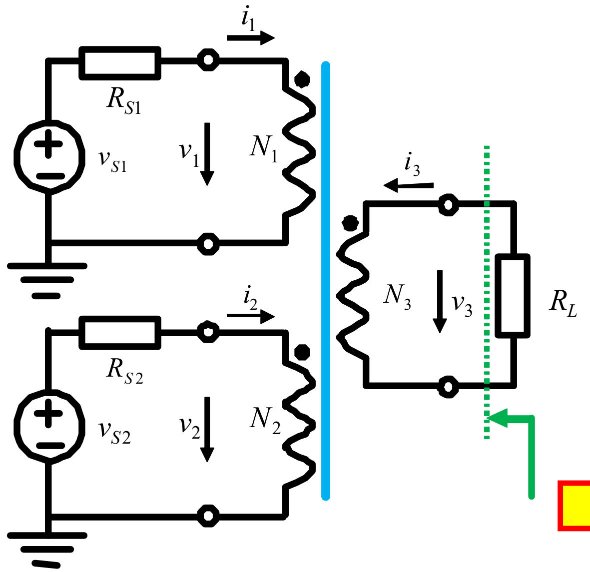
$$i_{test} = i_3 = -\frac{N_1}{N_3} i_1 - \frac{N_2}{N_3} i_2$$

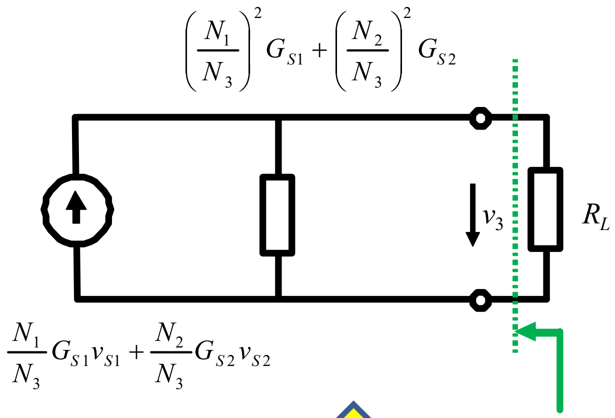
$$= \frac{N_1}{N_3} \frac{v_1}{R_{S1}} + \frac{N_2}{N_3} \frac{v_2}{R_{S2}}$$

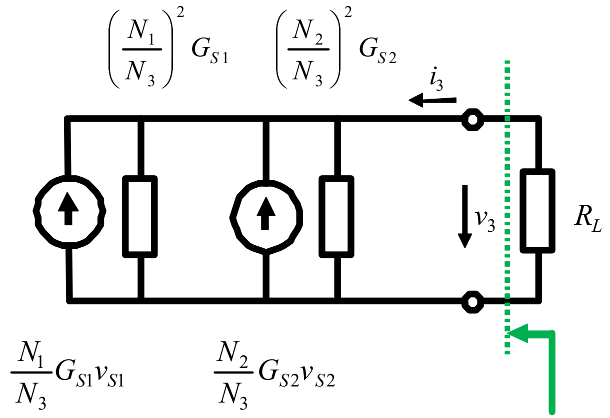
$$= \left(\frac{N_1}{N_3}\right)^2 \frac{v_{test}}{R_{S1}} + \left(\frac{N_2}{N_3}\right)^2 \frac{v_{test}}{R_{S2}}$$

$$G_N = \frac{i_{test}}{v_{test}} = \left(\frac{N_1}{N_3}\right)^2 G_{S1} + \left(\frac{N_2}{N_3}\right)^2 G_{S2}$$

等效电路理解更直观易记







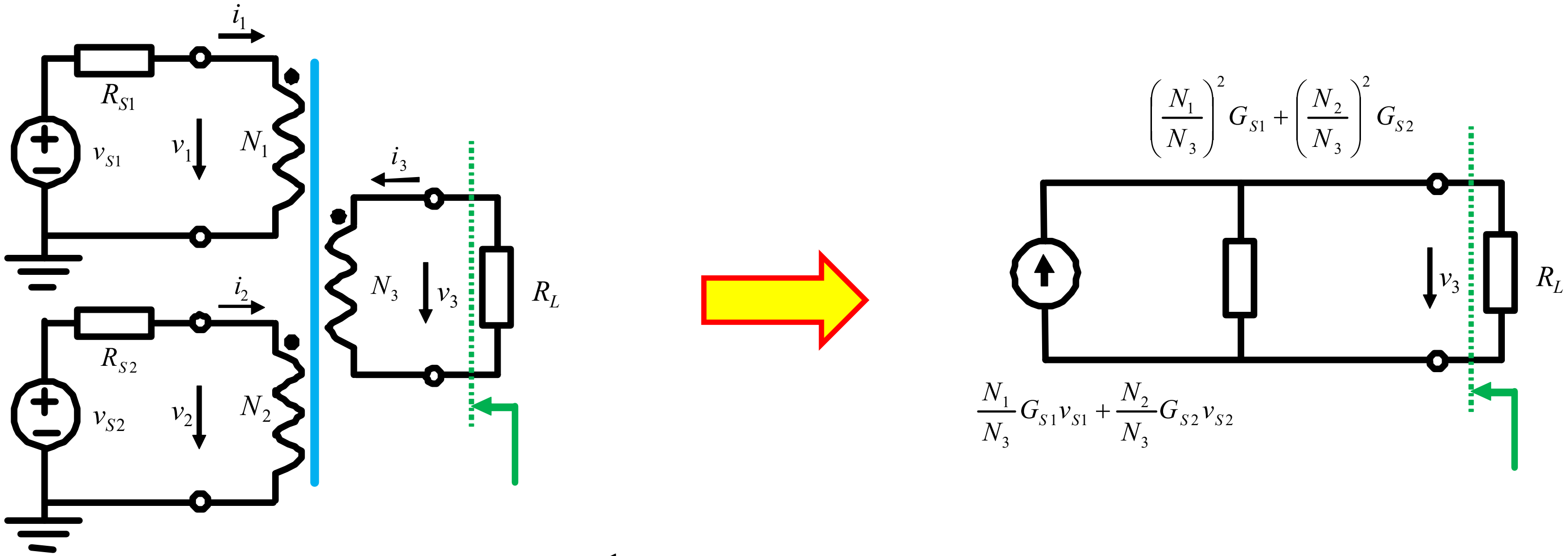
$$v_3 = \frac{\frac{N_1}{N_3} i_{S1} + \frac{N_2}{N_3} i_{S2}}{G_L + \left(\frac{N_1}{N_3}\right)^2 G_{S1} + \left(\frac{N_2}{N_3}\right)^2 G_{S2}}$$

$$= \left(\frac{N_1}{N_3} i_{S1} + \frac{N_2}{N_3} i_{S2}\right) \frac{G_L}{G_L + \frac{N_1^2}{N_3^2} G_{S1} + \frac{N_2^2}{N_3^2} G_{S2}} R_L$$

总电流　　在G_L上的分流　　流过R_L产生的电压

$$= \left(\frac{N_1}{N_3} i_{S1} + \frac{N_2}{N_3} i_{S2}\right) \lambda R_L$$

匹配？最大功率传输？

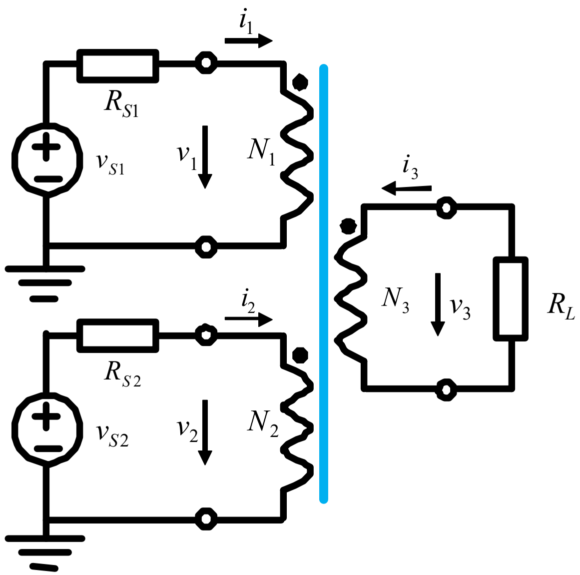


$$R_L = \frac{1}{\left(\frac{N_1}{N_3}\right)^2 G_{S1} + \left(\frac{N_2}{N_3}\right)^2 G_{S2}}$$

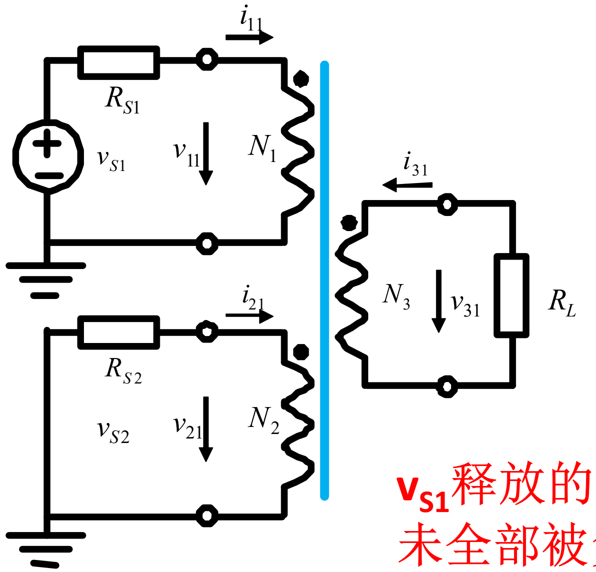
负载能够获得等效源的额定功率！！

负载吸收的功率是否就是
实际信源所释放的功率？

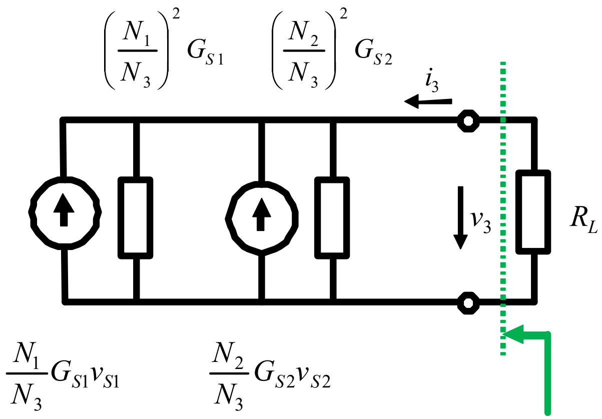
信源内阻也吸收功率？

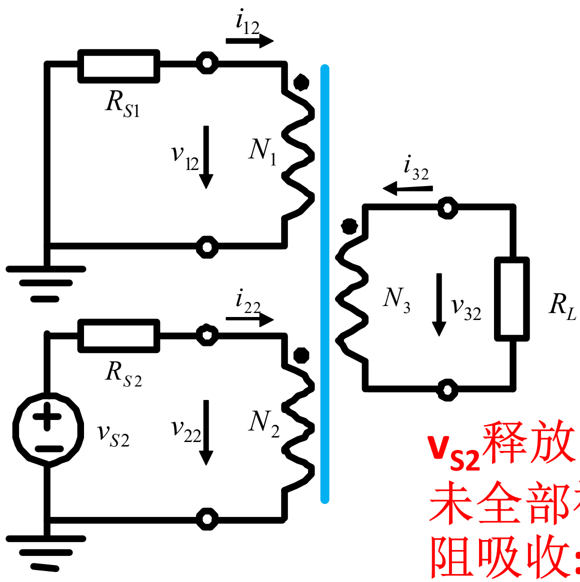


叠加定理

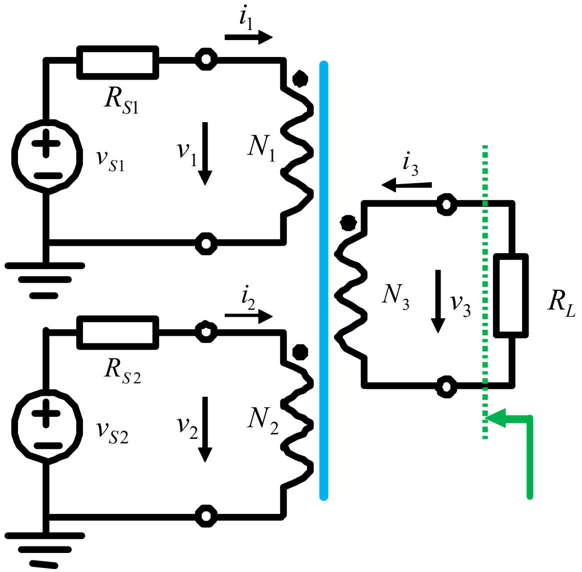


v_{S1}释放的功率并未全部被负载电阻吸收::





v_{S2}释放的功率并未全部被负载电阻吸收::



$$R_L = \frac{1}{\left(\frac{N_1}{N_3}\right)^2 G_{S1} + \left(\frac{N_2}{N_3}\right)^2 G_{S2}} \qquad N_3^2 G_L = N_1^2 G_{S1} + N_2^2 G_{S2}$$

$$v_3 = \left(\frac{N_1}{N_3} i_{S1} + \frac{N_2}{N_3} i_{S2}\right) \frac{N_3^2 G_L}{N_3^2 G_L + N_1^2 G_{S1} + N_2^2 G_{S2}} R_L$$

$$= \left(\frac{N_1}{N_3} i_{S1} + \frac{N_2}{N_3} i_{S2}\right) \frac{N_3^2}{2\left(N_1^2 G_{S1} + N_2^2 G_{S2}\right)}$$

$$P_L = \frac{v_{3,rms}^2}{R_L} = \overline{\left(\frac{N_1}{N_3} i_{S1} + \frac{N_2}{N_3} i_{S2}\right)^2} \frac{1}{4} \frac{N_3^2}{N_1^2 G_{S1} + N_2^2 G_{S2}}$$

$$= \frac{1}{4} \frac{1}{N_1^2 G_{S1} + N_2^2 G_{S2}} \left(\overline{N_1^2 i_{S1}^2} + 2\overline{N_1 N_2 i_{S1} i_{S2}} + \overline{N_2^2 i_{S2}^2}\right)$$

$$= \frac{1}{4} \frac{1}{N_1^2 G_{S1} + N_2^2 G_{S2}} \left(N_1^2 i_{S1,rms}^2 + 2 N_1 N_2 \overline{i_{S1} i_{S2}} + N_2^2 i_{S2,rms}^2\right)$$

$$P_{S1,\max} = \frac{1}{4} \frac{v_{S1,rms}^2}{R_{S1}} = \frac{1}{4} \frac{i_{S1,rms}^2}{G_{S1}}$$

$$P_{S2,\max} = \frac{1}{4} \frac{v_{S2,rms}^2}{R_{S2}} = \frac{1}{4} \frac{i_{S2,rms}^2}{G_{S2}}$$

$$P_{S,\max} = P_{S1,\max} + P_{S2,\max} = \frac{1}{4} \frac{i_{S1,rms}^2}{G_{S1}} + \frac{1}{4} \frac{i_{S2,rms}^2}{G_{S2}}$$

$$P_L = \frac{1}{4} \frac{1}{N_1^2 G_{S1} + N_2^2 G_{S2}} \left(N_1^2 i_{S1,rms}^2 + 2 N_1 N_2 \overline{i_{S1} i_{S2}} + N_2^2 i_{S2,rms}^2\right) \sim=\sim P_{S,\max} = \frac{1}{4} \frac{i_{S1,rms}^2}{G_{S1}} + \frac{1}{4} \frac{i_{S2,rms}^2}{G_{S2}}$$

$$N_1^2 i_{S1,rms}^2 + 2 N_1 N_2 \overline{i_{S1} i_{S2}} + N_2^2 i_{S2,rms}^2 \sim=\sim N_1^2 i_{S1,rms}^2 + N_2^2 \frac{G_{S2}}{G_{S1}} i_{S1,rms}^2 + N_1^2 \frac{G_{S1}}{G_{S2}} i_{S2,rms}^2 + N_2^2 i_{S2,rms}^2$$

$$2 N_1 N_2 \overline{i_{S1} i_{S2}} \sim=\sim N_2^2 \frac{G_{S2}}{G_{S1}} i_{S1,rms}^2 + N_1^2 \frac{G_{S1}}{G_{S2}} i_{S2,rms}^2 \qquad 2 N_1 N_2 \overline{i_{S1} i_{S2}} \le N_2^2 \frac{G_{S2}}{G_{S1}} i_{S1,rms}^2 + N_1^2 \frac{G_{S1}}{G_{S2}} i_{S2,rms}^2$$

最大功率传输的条件

$$P_L = \frac{1}{4}\frac{1}{N_1^2 G_{S1} + N_2^2 G_{S2}}\left(N_1^2 i_{S1,rms}^2 + 2N_1N_2\overline{i_{S1}i_{S2}} + N_2^2 i_{S2,rms}^2\right) \le P_{S,\max} = \frac{1}{4}\frac{i_{S1,rms}^2}{G_{S1}} + \frac{1}{4}\frac{i_{S2,rms}^2}{G_{S2}}$$

$$2N_1N_2\overline{i_{S1}i_{S2}} \le N_2^2\frac{G_{S2}}{G_{S1}}i_{S1,rms}^2 + N_1^2\frac{G_{S1}}{G_{S2}}i_{S2,rms}^2$$

$$2N_1N_2G_{S1}G_{S2}\overline{i_{S1}i_{S2}} \le N_2^2G_{S2}^2\overline{i_{S1}^2} + N_1^2G_{S1}^2\overline{i_{S2}^2}$$

$$N_2G_{S2}i_{S1} = N_1G_{S1}i_{S2}$$

$$\frac{i_{S1}}{N_1G_{S1}} = \frac{i_{S2}}{N_2G_{S2}}$$

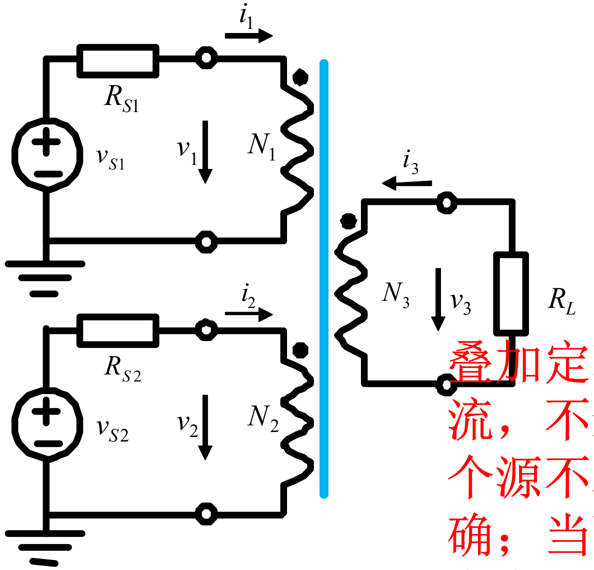
阻抗匹配条件

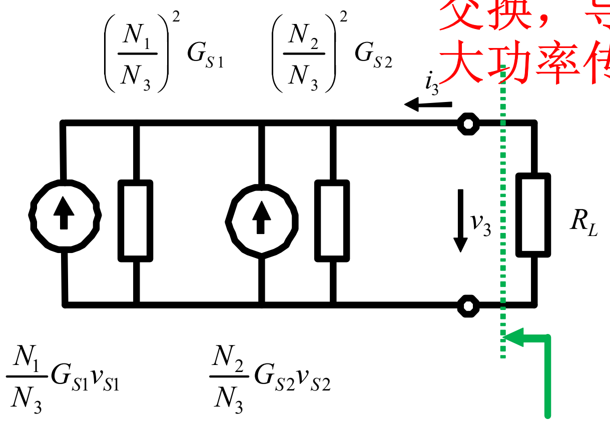
$$R_L = \frac{1}{\left(\frac{N_1}{N_3}\right)^2 G_{S1} + \left(\frac{N_2}{N_3}\right)^2 G_{S2}}$$

源相关条件 $$\frac{v_{S1}}{N_1} = \frac{v_{S2}}{N_2}$$

两个激励源不是任意的源
同步变化的源
任意时刻电压幅度比值是确定的值

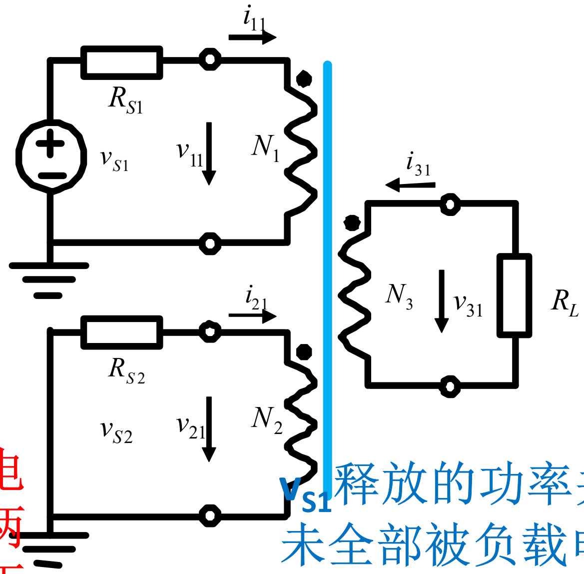
物理解释错出在哪里？



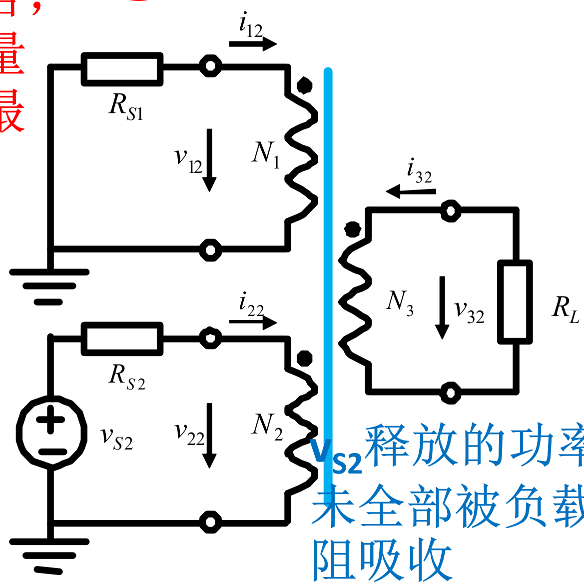


叠加定理

叠加定理仅适用电压电流，不适用功率；当两个源不相关时，分析正确；当两个源相关时，存在两个源之间的耦合，即两个源之间存在能量交换，导致最终存在最大功率传输的可能性

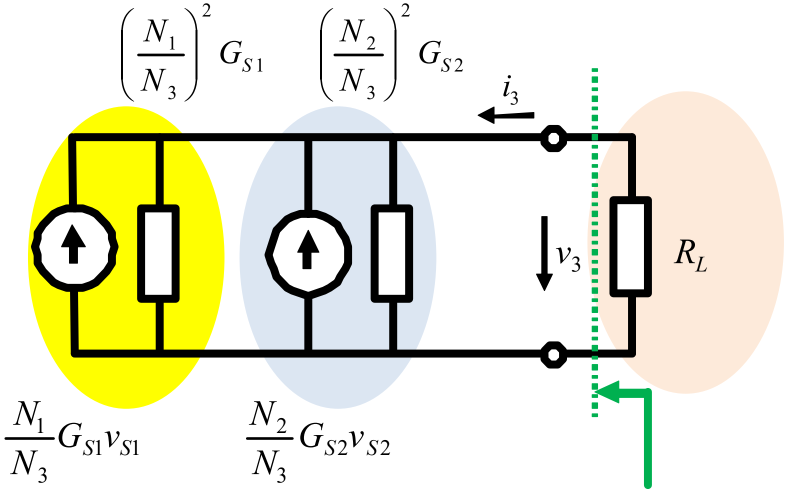


v_{S1}释放的功率并未全部被负载电阻吸收



v_{S2}释放的功率并未全部被负载电阻吸收

从等效电路看

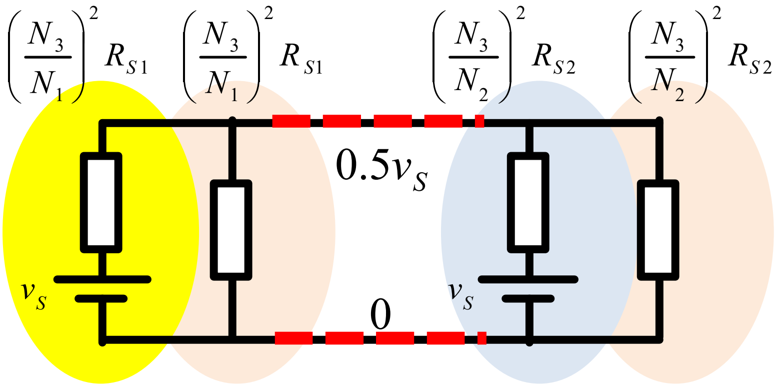


$$G_L = \left(\frac{N_1}{N_3}\right)^2 G_{S1} + \left(\frac{N_2}{N_3}\right)^2 G_{S2}$$

$$\frac{v_{S1}}{N_1} = \frac{v_{S2}}{N_2} = \frac{v_S}{N_3}$$

$$i_{N1} = \frac{N_1}{N_3} G_{S1} v_{S1} = \left(\frac{N_1}{N_3}\right)^2 G_{S1} v_S$$

$$i_{N2} = \frac{N_2}{N_3} G_{S2} v_{S2} = \left(\frac{N_2}{N_3}\right)^2 G_{S2} v_S$$



在满足阻抗匹配条件和源相关条件下：两个源阻回路之间电压相等，无电流，相当于开路：可视为两个源分别各自匹配，分别将最大功率传输给各自匹配的电阻

学会对计算结果进行物理解释

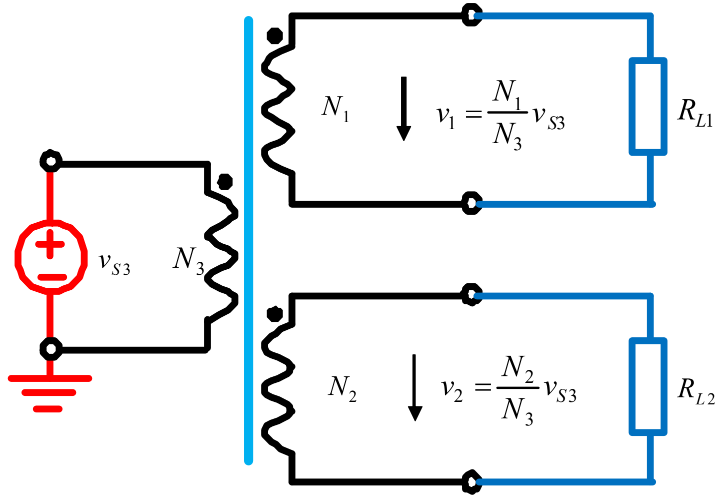
- 第一次还没有建立起物理概念时，可以通过各种方式获得解析表达式
  - 解析式可以是近似的，可以是精确的
  - 可用借助计算机辅助公式推导

- 对解析式进行物理解释，以后碰到类似问题可以直接给出结论，无需再经过列写方程、求解方程的过程
  - 建立起大量的这类直观的简单等效电路概念之后，可以帮助我们进行复杂电路的设计

- 分析过一个电路，则建立一个等效电路模型，其后直接从等效电路角度分析，从等效电路角度进行电路设计
  - 电路等效是数学方程计算过程的符号化表述，直观，易于理解
    - 给予物理解释后，直观，容易记忆，便于设计时直接利用

三端口理想变压器
端口约束方程中看到了什么？

$$\begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{N_1}{N_3} \\ 0 & 0 & \frac{N_2}{N_3} \\ -\frac{N_1}{N_3} & -\frac{N_2}{N_3} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_3 \end{bmatrix}$$



电压信号的分解

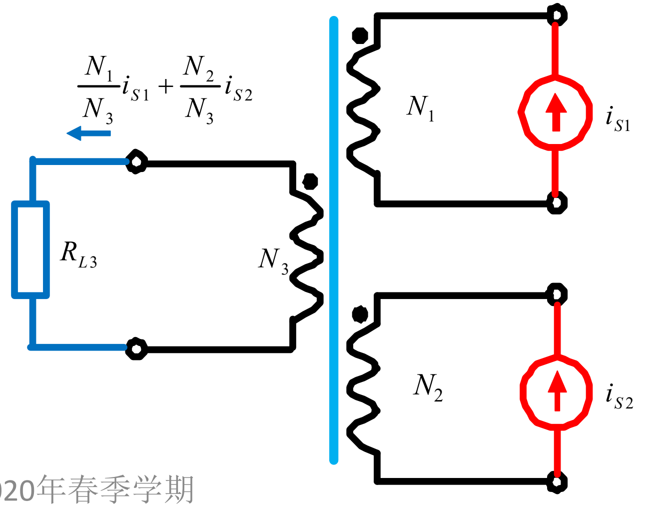
$$v_1 = \frac{N_1}{N_3} v_3$$

$$v_2 = \frac{N_2}{N_3} v_3$$

电压信号的分解

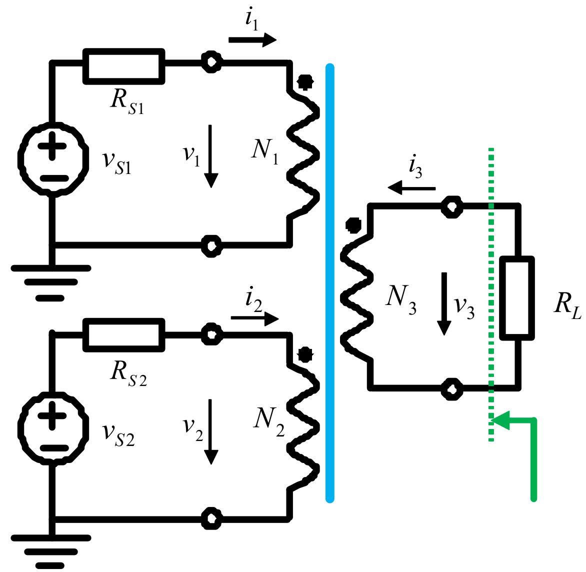
$$i_3 = -\frac{N_1}{N_3} i_1 - \frac{N_2}{N_3} i_2$$

电流信号的合成



电流信号的合成

三端口理想变压器
功率合成器



$$P_L = P_{S1,\max} + P_{S2,\max}$$

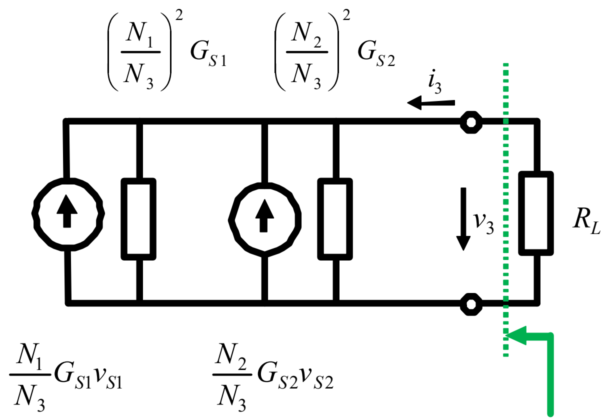
功率合成

$$G_L = \left(\frac{N_1}{N_3}\right)^2 G_{S1} + \left(\frac{N_2}{N_3}\right)^2 G_{S2}$$

阻抗匹配条件

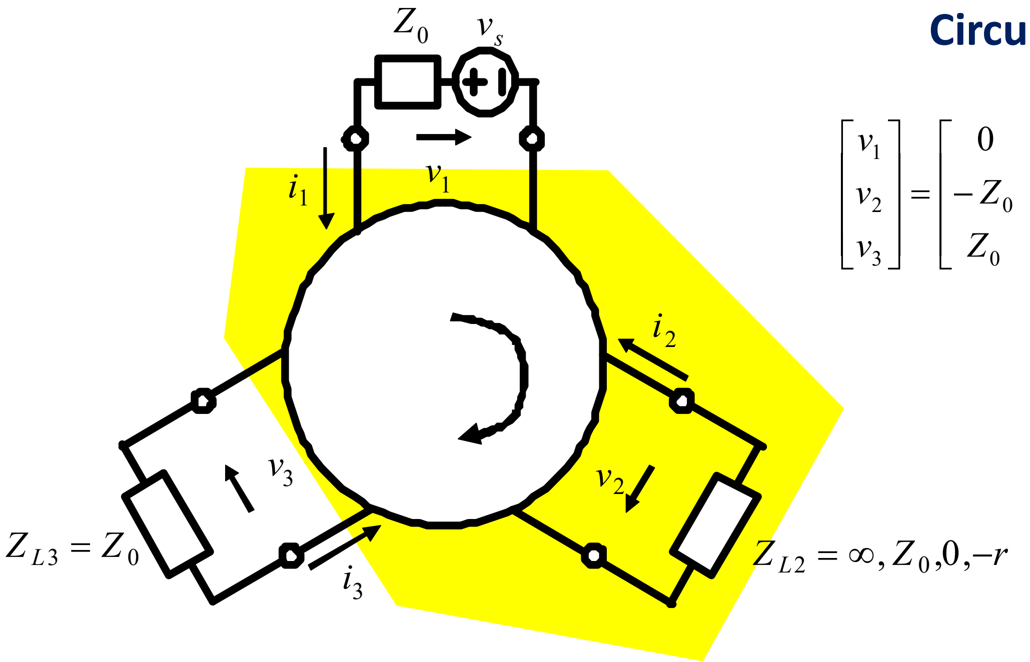
$$\frac{v_{S1}}{N_1} = \frac{v_{S2}}{N_2}$$

源相关条件



第7周作业

作业4 负阻放大器



Circulator

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 & Z_0 & -Z_0 \\ -Z_0 & 0 & Z_0 \\ Z_0 & -Z_0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

证明：（1）当端口2开路或短路时，环行器端口1吸收的功率全部从端口3送出，为端口3匹配负载吸收

（2）选作：当端口2为负阻时，环行器端口3获得功率高于端口1吸收功率，以端口2为内部端口，以端口1为输入端口，以端口3为输出端口，求该二端口网络的输入电阻、输出电阻和功率增益

端口2短路
功率全反射

$$i_1 = i_{test} \quad ❶$$

$$v_2 = 0 \quad ❷$$

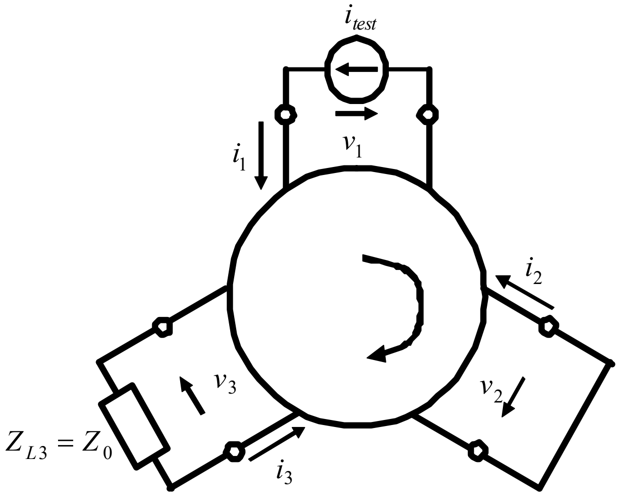
$$v_3 + i_3 Z_0 = 0 \quad ❹$$

端口对接关系，只需列写元件约束即可

$$v_1 - Z_0 i_2 + Z_0 i_3 = 0 \quad ❻$$

$$v_2 - Z_0 i_3 + Z_0 i_1 = 0 \quad ❸$$

$$v_3 - Z_0 i_1 + Z_0 i_2 = 0 \quad ❺$$



$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} Z_0 \\ 0 \\ -Z_0 \\ 1 \\ 2 \\ 1 \end{bmatrix} i_{test}$$

$$P_1 = v_1 i_1 = Z_0 i_{test} i_{test} = Z_0 i_{test}^2$$

端口1吸收功率

$$P_2 = v_2 i_2 = 0 \cdot 2 i_{test} = 0$$

端口2短路，
功率全反射

$$P_3 = v_3 i_3 = -Z_0 i_{test} \cdot i_{test} = -Z_0 i_{test}^2$$

全部从端口3释放，为匹配负载吸收

端口2开路
功率全反射

$i_1 = i_{test}$ ❶

$i_2 = 0$ ❷

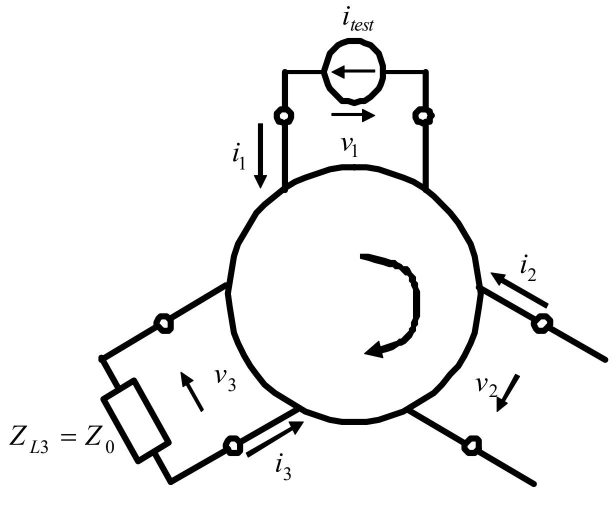
$v_3 + i_3 Z_0 = 0$ ❹

端口对接关系，只需列写元件约束即可

$v_1 - Z_0 i_2 + Z_0 i_3 = 0$ ❻

$v_2 - Z_0 i_3 + Z_0 i_1 = 0$ ❺

$v_3 - Z_0 i_1 + Z_0 i_2 = 0$ ❸



$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} Z_0 \\ -2Z_0 \\ Z_0 \\ 1 \\ 0 \\ -1 \end{bmatrix} i_{test}$$

$$P_1 = v_1 i_1 = Z_0 i_{test} i_{test} = Z_0 i_{test}^2$$

端口1吸收功率

$$P_2 = v_2 i_2 = -2Z_0 i_{test} \cdot 0 = 0$$

端口2开路，
功率全反射

$$P_3 = v_3 i_3 = Z_0 i_{test} \cdot (-i_{test}) = -Z_0 i_{test}^2$$

全部从端口3释放，为匹配负载吸收

端口2不匹配
功率反射

$$i_1 = i_{test} \quad ❶$$

$$v_2 + R_2 i_2 = 0 \quad ❹$$

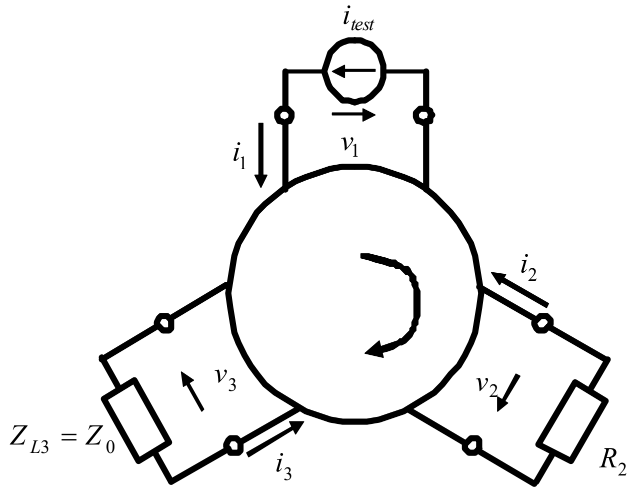
$$v_3 + i_3 Z_0 = 0 \quad ❺$$

端口对接关系，只需列写元件约束即可

$$v_1 - Z_0 i_2 + Z_0 i_3 = 0 \quad ❻$$

$$v_2 - Z_0 i_3 + Z_0 i_1 = 0 \quad ❷$$

$$v_3 - Z_0 i_1 + Z_0 i_2 = 0 \quad ❸$$

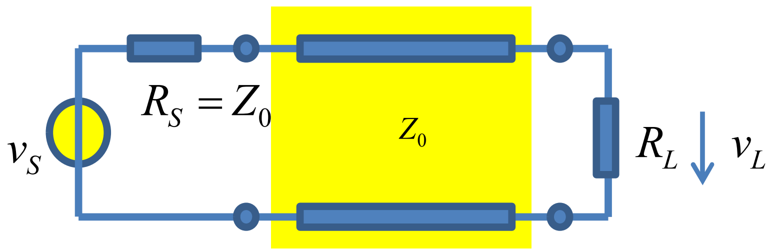


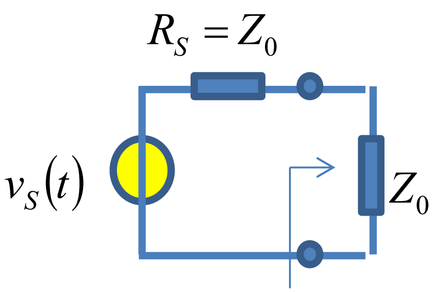
$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} Z_0 \\ -2Z_0 \dfrac{R_2}{Z_0 + R_2} \\ -Z_0 \dfrac{Z_0 - R_2}{Z_0 + R_2} \\ 1 \\ 2\dfrac{Z_0}{Z_0 + R_2} \\ \dfrac{Z_0 - R_2}{Z_0 + R_2} \end{bmatrix} i_{test}$$

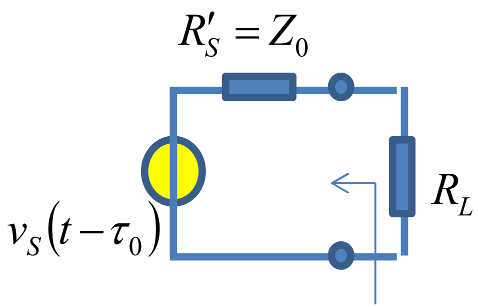
存在反射，但可能不再是全反射，而是部分吸收，部分反射

如何理解反射

要求理解反射概念







$t=0$ 信源看到的

$t=\tau_0$ 负载看到的

t=0时刻源端匹配，信源输出额定功率

$$P_{S,\max} = \frac{V_{s,rms}^2}{4R_S} = \frac{V_{s,rms}^2}{4Z_0}$$

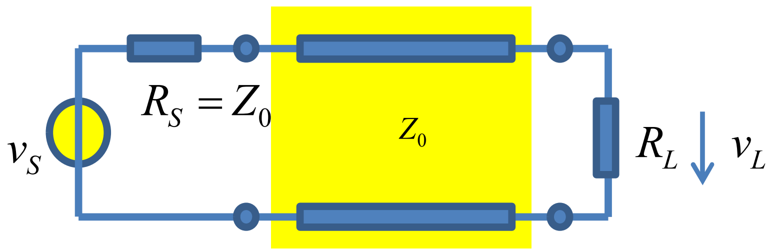
t=τ₀时刻信号到达负载端

$$i_L(t) = \frac{v_S(t-\tau_0)}{R'_S + R_L} = \frac{v_S(t-\tau_0)}{Z_0 + R_L}$$

由于负载不匹配，负载实际吸收功率小于信源输出额定功率

$$P_L = I_{L,rms}^2 R_L = \frac{R_L}{(Z_0 + R_L)^2} V_{s,rms}^2$$

功率反射



$$P_{S,\max} = \frac{V_{s,rms}^2}{4Z_0}$$

t=0时刻源端匹配，信源输出额定功率

$$P_L = \frac{R_L}{(Z_0 + R_L)^2} V_{s,rms}^2$$

t=τ_0时刻信号到达负载端
负载不匹配，负载实际吸收功率小于信源输出额定功率

多余的功率反射回去，t=2 τ_0时刻反向传输到信源，被信源匹配内阻吸收

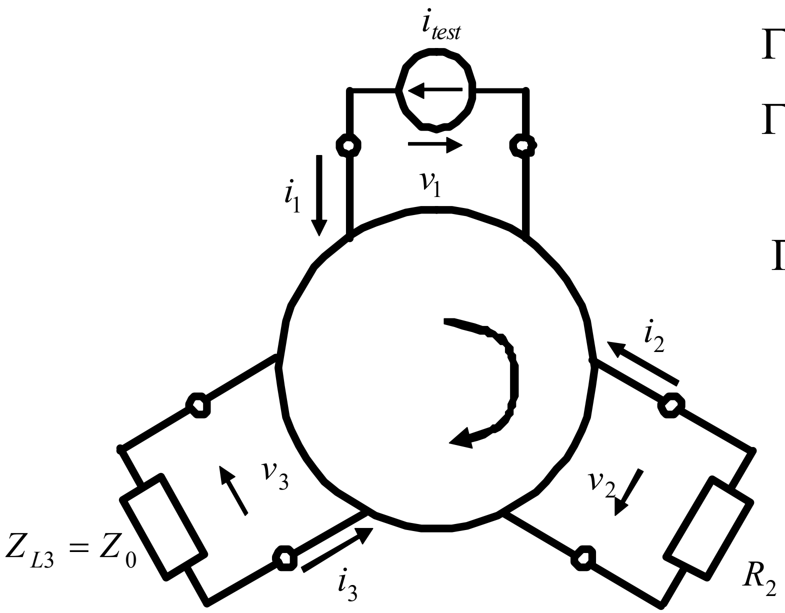
反射功率

$$P_R = P_{S,\max} - P_L = \frac{V_{s,rms}^2}{4Z_0} - \frac{R_L}{(Z_0 + R_L)^2} V_{s,rms}^2$$

$$= \frac{V_{s,rms}^2}{4Z_0}\left(1 - \frac{4Z_0 R_L}{(Z_0 + R_L)^2}\right) = P_{S,\max}\left(\frac{R_L - Z_0}{R_L + Z_0}\right)^2 = P_{S,\max}\left|\Gamma\right|^2$$

反射系数

信号功率是如何传输的？



$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} Z_0 \\ -\frac{2R_2Z_0}{Z_0+R_2} \\ -\frac{Z_0-R_2}{Z_0+R_2}Z_0 \\ 1 \\ \frac{2Z_0}{Z_0+R_2} \\ \frac{Z_0-R_2}{Z_0+R_2} \end{bmatrix} i_{test} = \begin{bmatrix} Z_0 \\ -(1+\Gamma_2)Z_0 \\ \Gamma_2 Z_0 \\ 1 \\ 1-\Gamma_2 \\ -\Gamma_2 \end{bmatrix} i_{test}$$

$$\Gamma_2\left(_{R_2=0}\right) = -1$$
$$\Gamma_2\left(_{R_2=\infty}\right) = +1$$

开路短路则全反射

$$\Gamma_2 = \frac{R_2 - Z_0}{R_2 + Z_0}$$

反射系数

$$|\Gamma_2|^2 = \frac{P_{R2}}{P_{I2}}$$

$$P_1 = v_1 i_1 = Z_0 i_{test} i_{test} = Z_0^2 i_{test} = P_S$$

端口1吸收信源释放的功率P_S

$$P_2 = -\left(1-\Gamma_2^2\right)Z_0 i_{test}^2 = -P_S + |\Gamma_2|^2 P_S$$

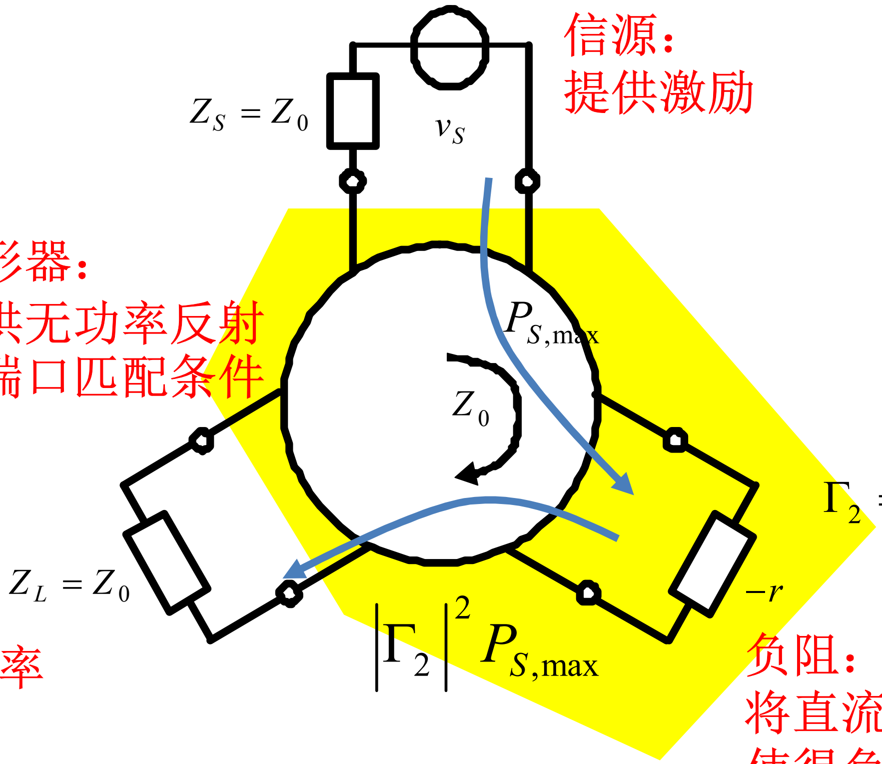
P_S在端口2全部释放，但负载不匹配，故而反射了$\Gamma_2^2 P_S$，重新被环形器端口2吸收

$$P_3 = v_3 i_3 = -|\Gamma_2|^2 Z_0 i_{test}^2 = -|\Gamma_2|^2 P_S$$

从端口2吸收的$\Gamma_2^2 P_S$功率全部从端口3释放，为端口3匹配负载吸收

反射型负阻放大器

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 & Z_0 & -Z_0 \\ -Z_0 & 0 & Z_0 \\ Z_0 & -Z_0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$



信源：
提供激励

环形器：
提供无功率反射
的端口匹配条件

负载：
吸收功率

负阻：
将直流能量转换为交流能量
使得负载获得的能量高于信源输出能量

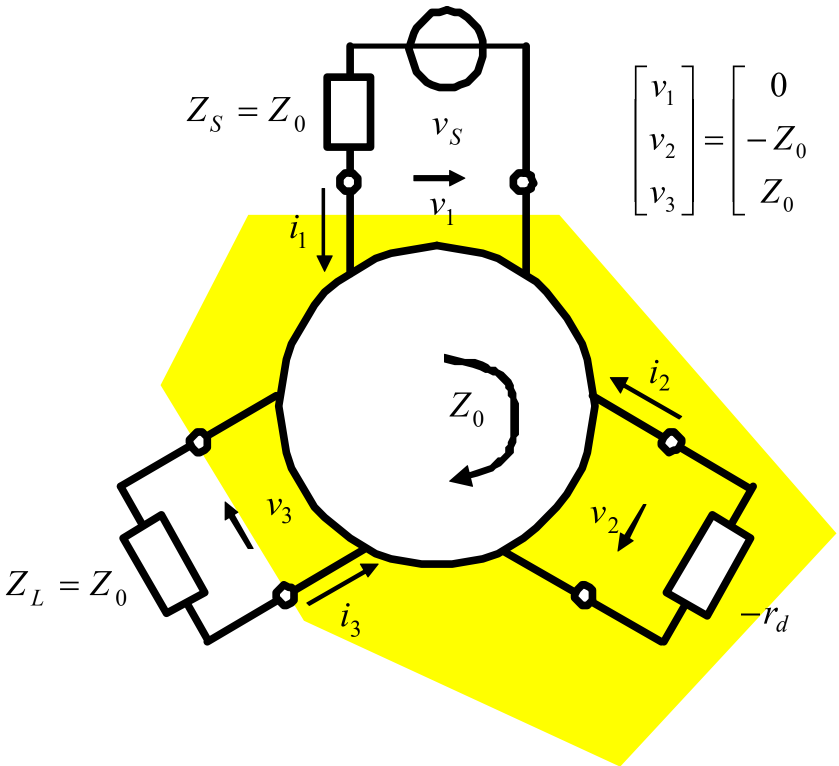
正阻吸收部分功率，
反射系数小于**1**

负阻释放功率，反
射系数大于**1** $|\Gamma| > 1$

$$\Gamma_2 = \frac{-r - Z_0}{-r + Z_0}$$

$$|\Gamma_2|^2 = \frac{P_{R2}}{P_{I2}} > 1$$

$$P_L = |\Gamma_2|^2 P_{S,\max} = \left|\frac{Z_0 + r}{Z_0 - r}\right|^2 P_{S,\max} = G_T \cdot P_{S,\max} = G_{p\max} \cdot P_{S,\max}$$



$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 & Z_0 & -Z_0 \\ -Z_0 & 0 & Z_0 \\ Z_0 & -Z_0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$$v_1 = Z_0 i_2 - Z_0 i_3$$
$$v_2 = Z_0 i_3 - Z_0 i_1$$
$$v_3 = Z_0 i_1 - Z_0 i_2$$

环行器元件约束

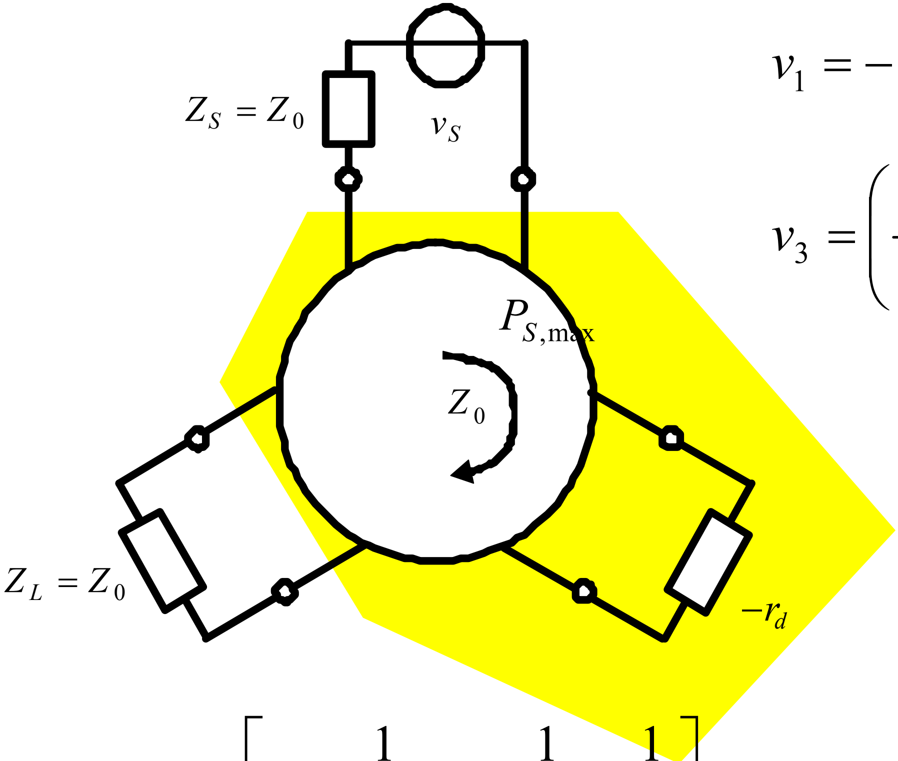
$$v_2 = r_d i_2$$

端口**2**接负阻：负阻约束

$$v_2 = Z_0 i_3 - Z_0 i_1 = r_d i_2 \qquad i_2 = \frac{Z_0}{r_d} i_3 - \frac{Z_0}{r_d} i_1$$

$$v_1 = Z_0 i_2 - Z_0 i_3 = Z_0 \left(\frac{Z_0}{r_d} i_3 - \frac{Z_0}{r_d} i_1 \right) - Z_0 i_3 = -\frac{Z_0^2}{r_d} i_1 + \left(\frac{Z_0^2}{r_d} - Z_0 \right) i_3$$

$$v_3 = Z_0 i_1 - Z_0 i_2 = Z_0 i_1 - Z_0 \left(\frac{Z_0}{r_d} i_3 - \frac{Z_0}{r_d} i_1 \right) = \left(\frac{Z_0^2}{r_d} + Z_0 \right) i_1 - \frac{Z_0^2}{r_d} i_3$$



$$v_1 = -\frac{Z_0^2}{r_d} i_1 + \left(\frac{Z_0^2}{r_d} - Z_0 \right) i_3$$

$$v_3 = \left(\frac{Z_0^2}{r_d} + Z_0 \right) i_1 - \frac{Z_0^2}{r_d} i_3$$

$$Z_0 = 250\Omega, r_d = 292\Omega$$

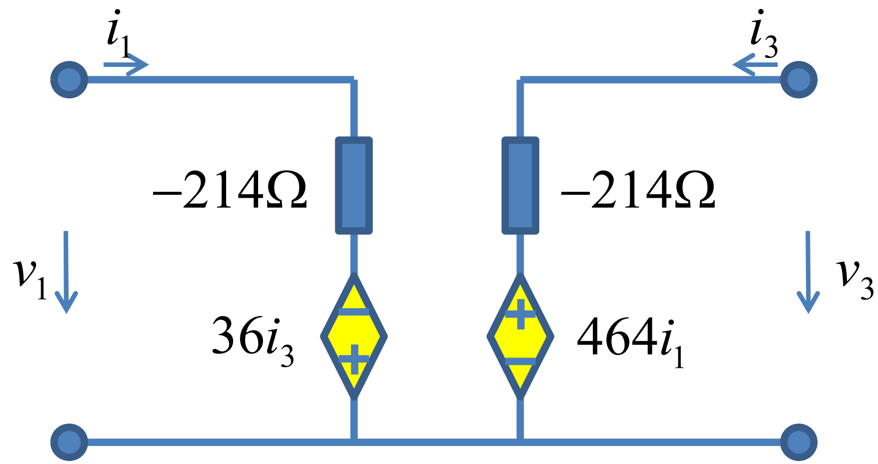
$$\mathbf{z} = \begin{bmatrix} -\frac{Z_0^2}{r_d} & \frac{Z_0^2}{r_d} - Z_0 \\ \frac{Z_0^2}{r_d} + Z_0 & -\frac{Z_0^2}{r_d} \end{bmatrix} = \begin{bmatrix} -214 & -36 \\ 464 & -214 \end{bmatrix} \Omega$$

非互易网络

$$\mathbf{y} = \mathbf{z}^{-1} = \begin{bmatrix} -\frac{1}{r_d} & \frac{1}{Z_0} - \frac{1}{r_d} \\ -\frac{1}{Z_0} - \frac{1}{r_d} & -\frac{1}{r_d} \end{bmatrix}$$

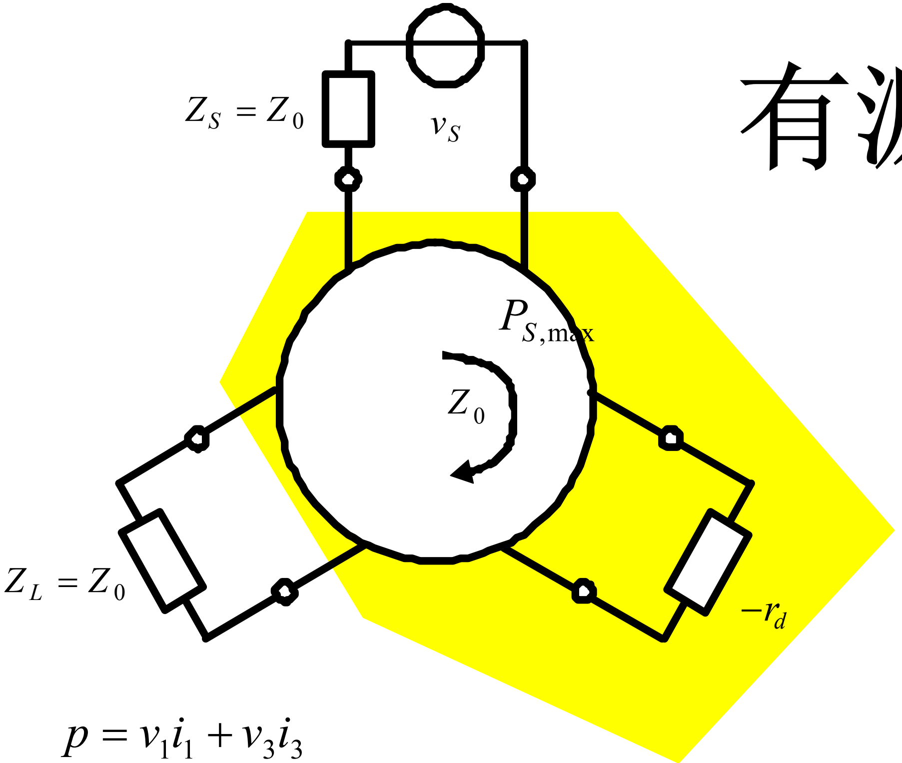
$$Z_{01} = \sqrt{\frac{z_{11}}{y_{11}}} = Z_0 = Z_{in1}(R_{L3} = Z_{03})$$

$$Z_{03} = \sqrt{\frac{z_{33}}{y_{33}}} = Z_0 = Z_{in3}(R_{L1} = Z_{01})$$



通过环行器，可以实现双端同时匹配

有源性



$$\mathbf{z} = \begin{bmatrix} -\dfrac{Z_0^2}{r_d} & \dfrac{Z_0^2}{r_d} - Z_0 \\ \dfrac{Z_0^2}{r_d} + Z_0 & -\dfrac{Z_0^2}{r_d} \end{bmatrix}$$

$$p = v_1 i_1 + v_3 i_3$$

$$= \left(-\frac{Z_0^2}{r_d} i_1 + \left(\frac{Z_0^2}{r_d} - Z_0 \right) i_3 \right) i_1 + \left(+ \left(\frac{Z_0^2}{r_d} + Z_0 \right) i_1 - \frac{Z_0^2}{r_d} i_3 \right) i_3$$

$$= -\frac{Z_0^2}{r_d} i_1^2 - \frac{Z_0^2}{r_d} i_3^2 + 2\frac{Z_0^2}{r_d} i_1 i_3$$

$$= -\frac{Z_0^2}{r_d} (i_1 - i_3)^2 < 0$$

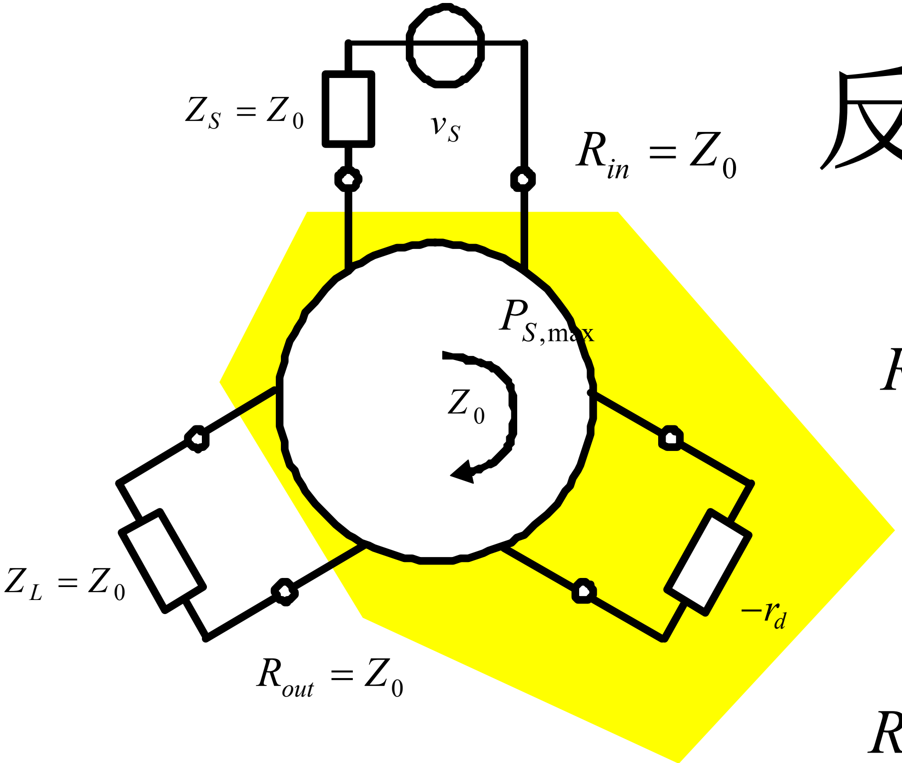
$$= -\frac{v_2^2}{r_d} < 0$$

只要端口电流之差不为**0**，只要有电压加载到负阻两端，负阻即向外输出功率，故有源

$$v_2 = Z_0 i_3 - Z_0 i_1$$

环行器+负阻：环行器为无损网络，二端口网络向外释放的纯功率全部由内部负阻提供

反射型负阻放大器



$R_{in} = Z_0$ 端口**1**吸收了信源的额定功率，而无反射，故而其输入阻抗为Z_0

输出端接匹配负载无功率反射回信源，这种情况下输入电阻为Z_0

$R_{out} = Z_0$ 端口**1**信源内阻为特征阻抗时，**3**端口进入的任何功率全部被吸收，这种情况下输出电阻为Z_0

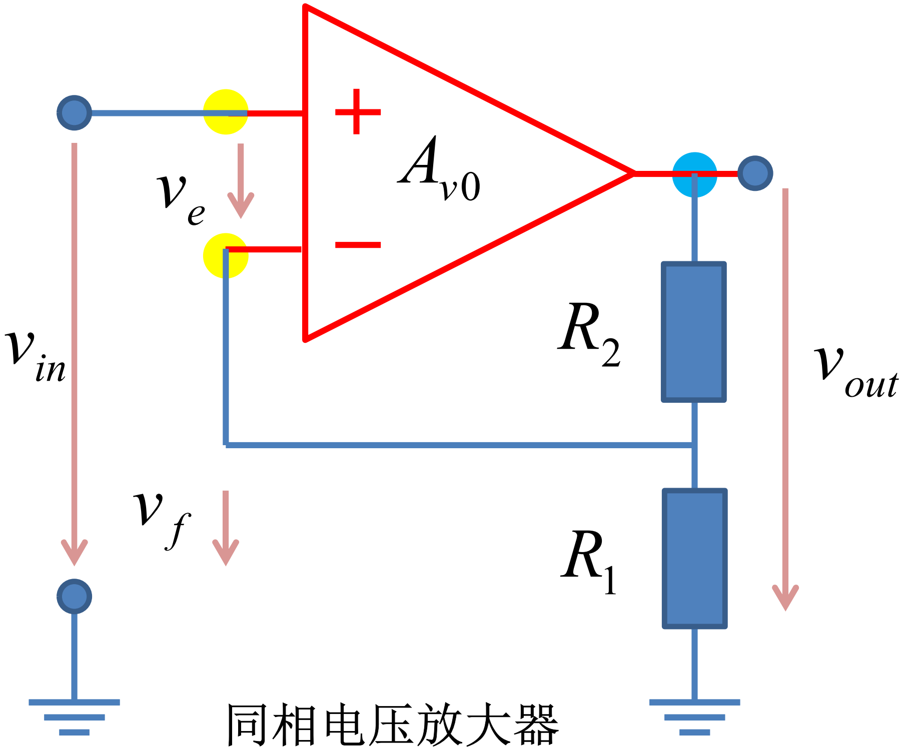
$$G_T = \frac{P_L}{P_{S,\max}} = \left|\Gamma_2\right|^2 = \left(\frac{-r_d - Z_0}{-r_d + Z_0}\right)^2 = \left(\frac{Z_0 + r_d}{Z_0 - r_d}\right)^2 > 1$$

功率增益>1，放大器

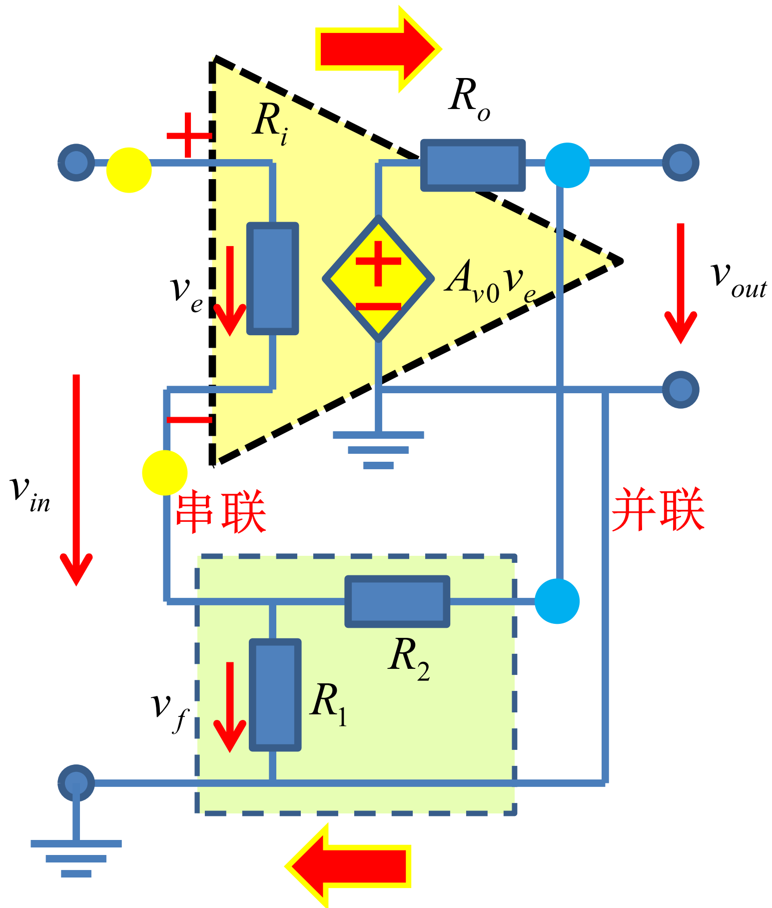
只要给出合理的电路解释，电路设计就变得简单明了
给不出电路解释，就无法给出原理性清晰的电路设计

有源则可实现功率放大

作业8 二端口网络连接关系分析



同相电压放大器



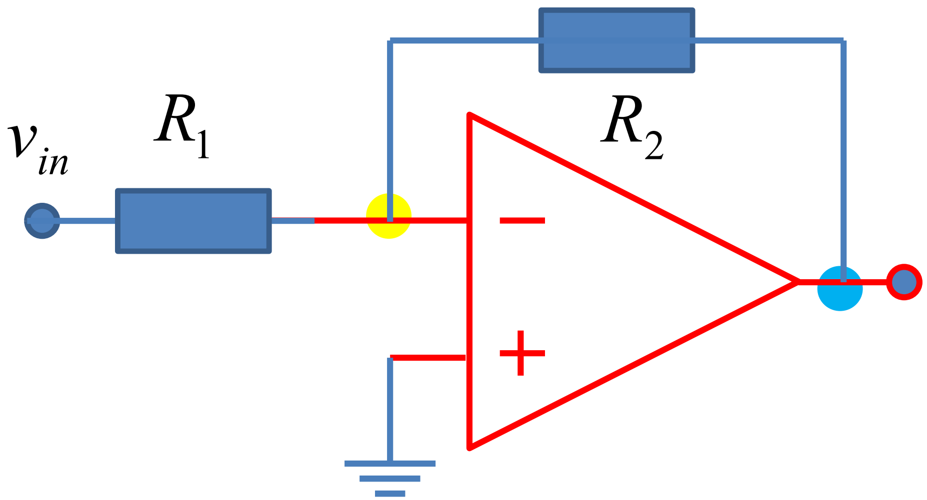
反馈网络输出点和放大网络输入点不同：串联

反馈网络输入点和放大网络输出点相同：并联

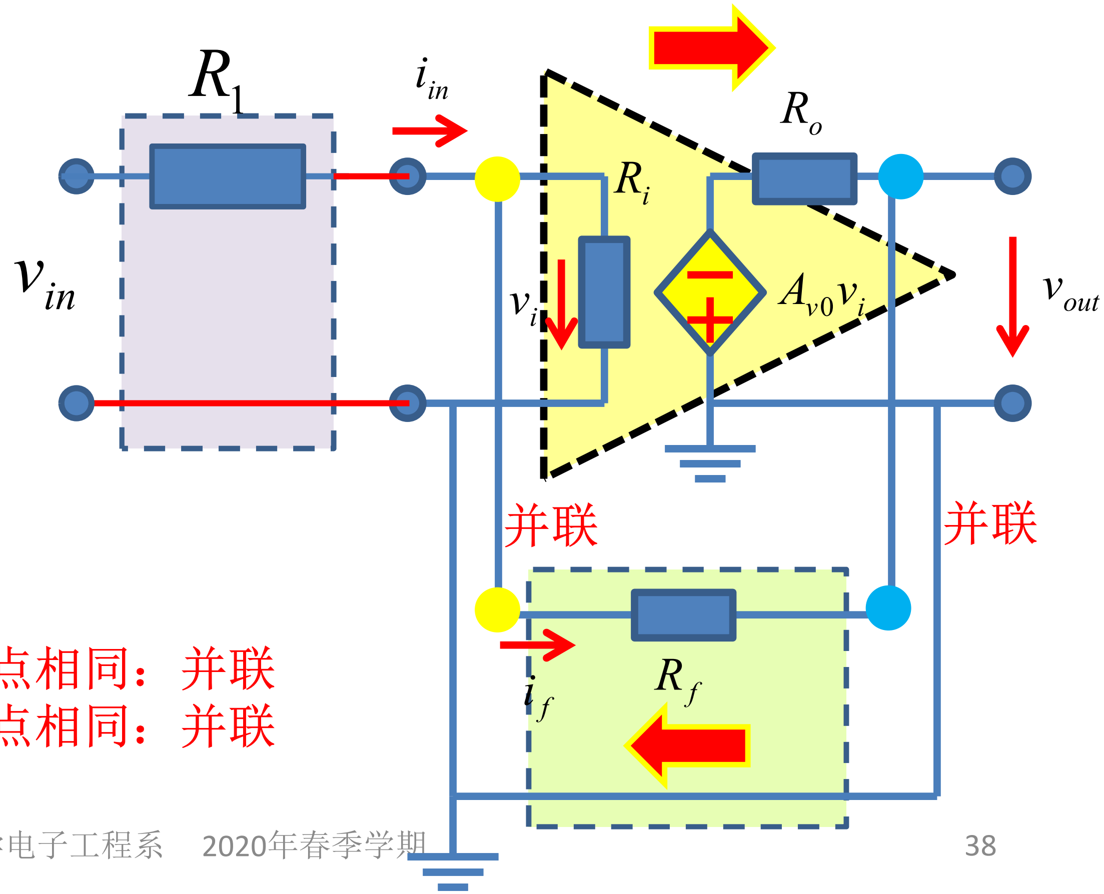
串并连接h相加

并并连接

并并连接y相加

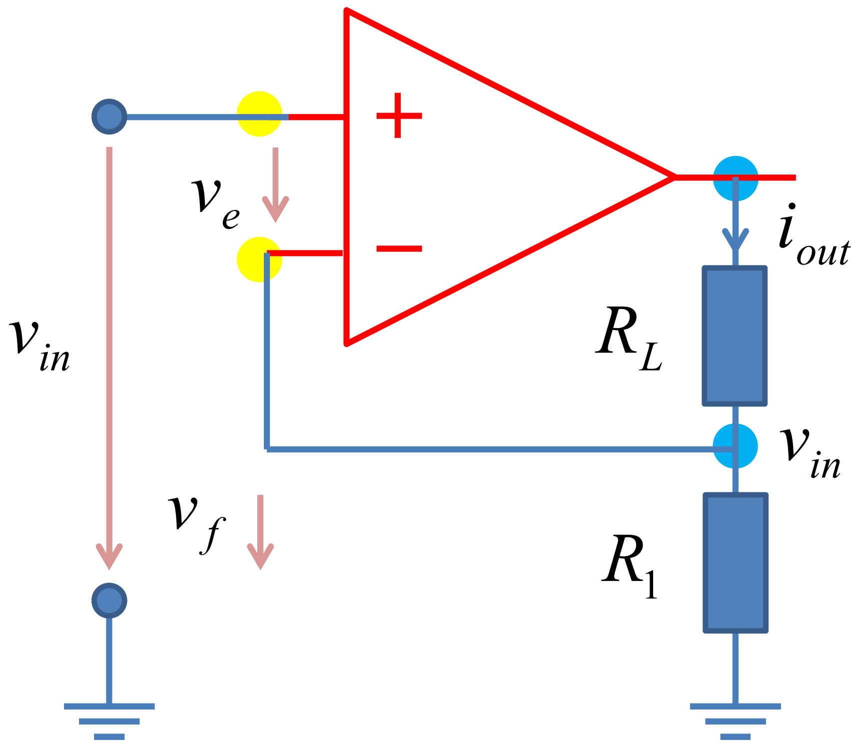


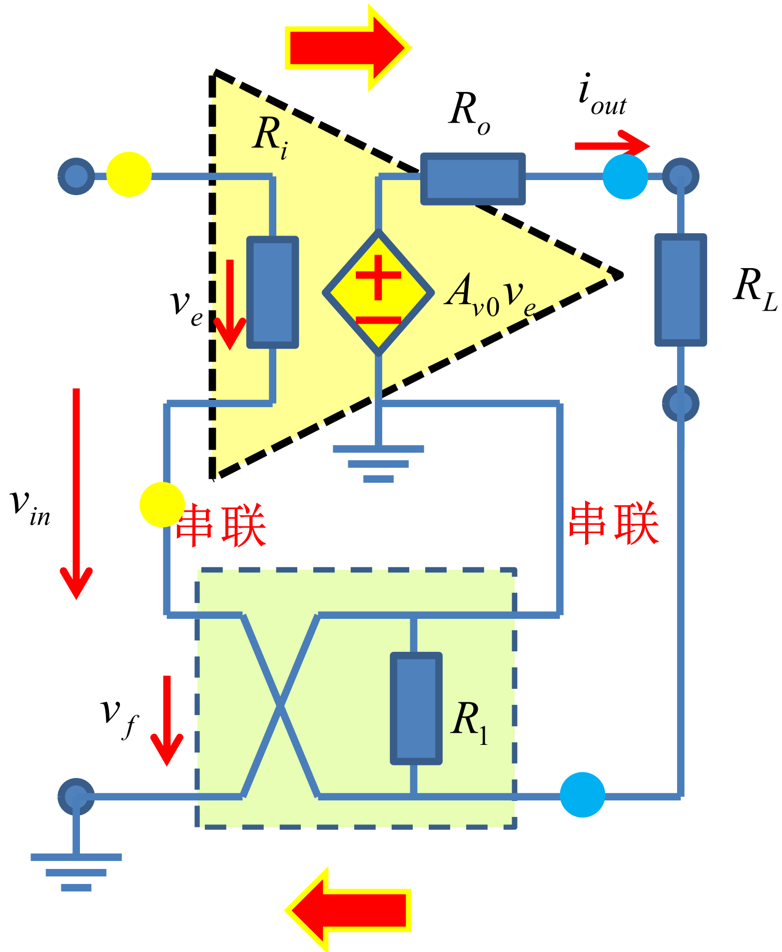
反相电压放大电路



反馈网络输出点和放大网络输入点相同：并联
反馈网络输入点和放大网络输出点相同：并联

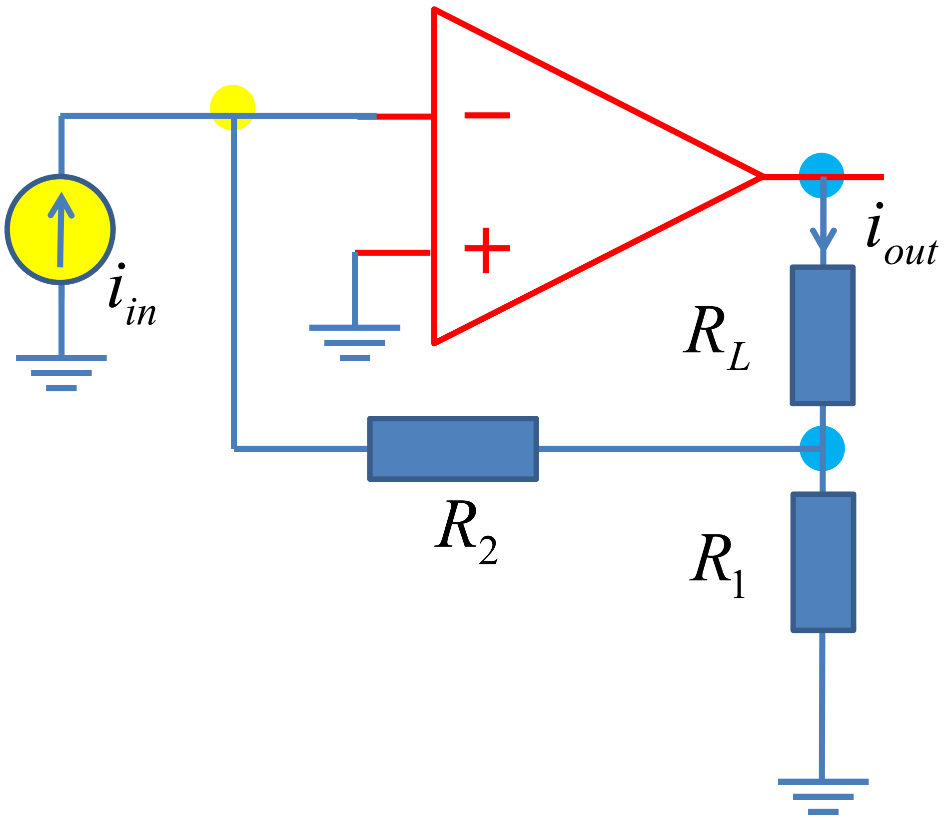
串串连接z相加

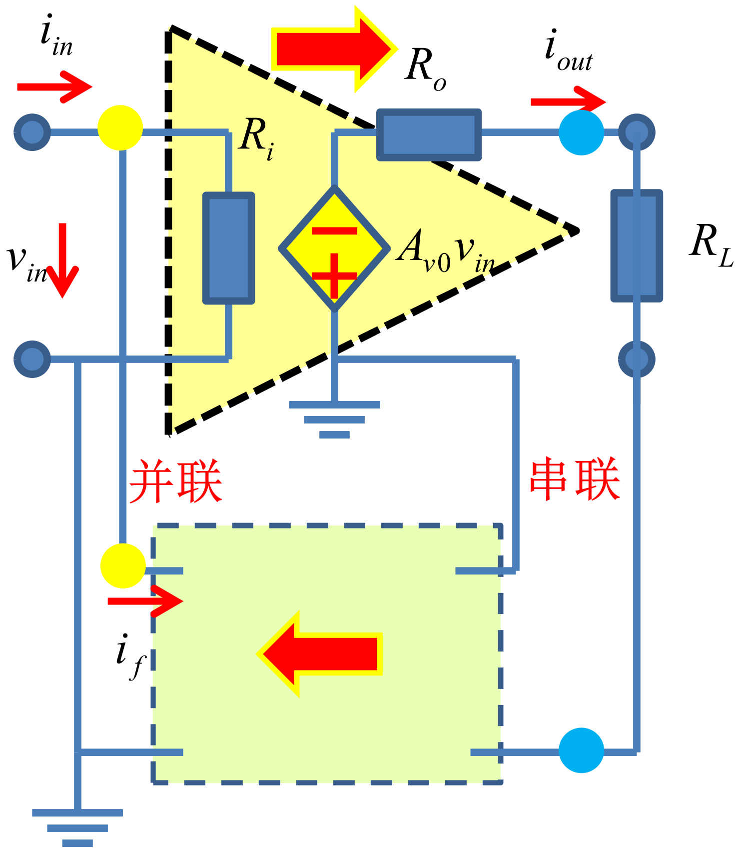




反馈网络输出点和放大网络输入点不同：串联
反馈网络输入点和放大网络输出点不同：串联

并串连接g相加



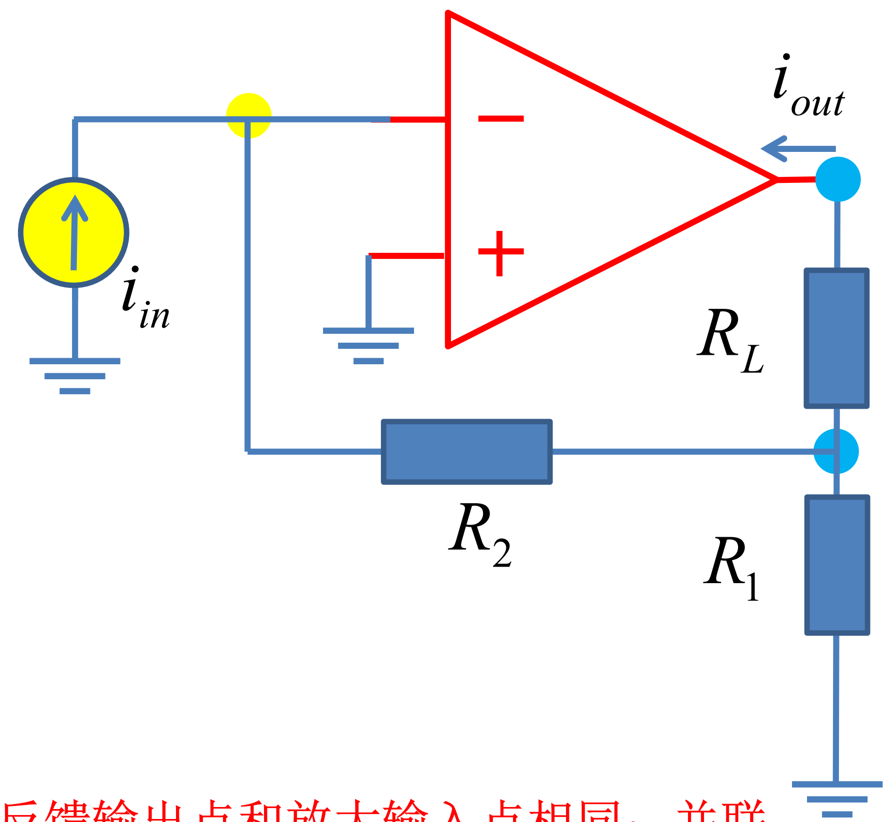


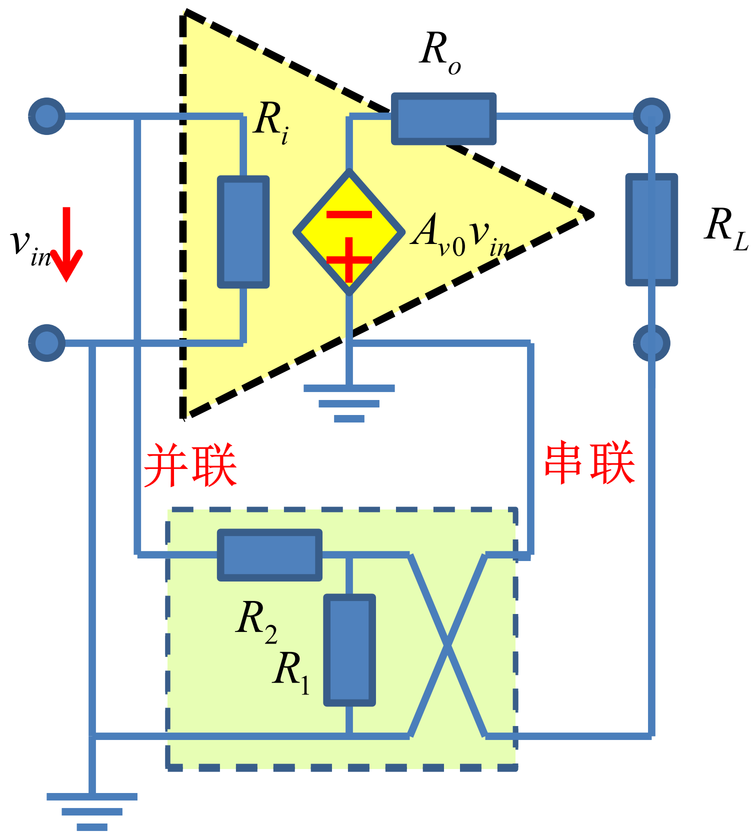
反馈网络输出点和放大网络输入点相同：并联
反馈网络输入点和放大网络输出点不同：串联

作业8 二端口网络连接（选作）

- 确认并画出两个二端口网络的连接关系
- 获得两个二端口网络的合适参量，根据网络连接关系求总网络参量
  - 并串连接**g**相加，则分别求**g**参量，再相加
- 求逆，考察$\mathbf{A_{v0}}\to\infty$时，四种连接关系接近哪种理想受控源？
  - 并串连接**g**相加，**g**求逆获得**h**，考察是否接近理想流控流源？
  - …

以并串连接为例：反馈网络



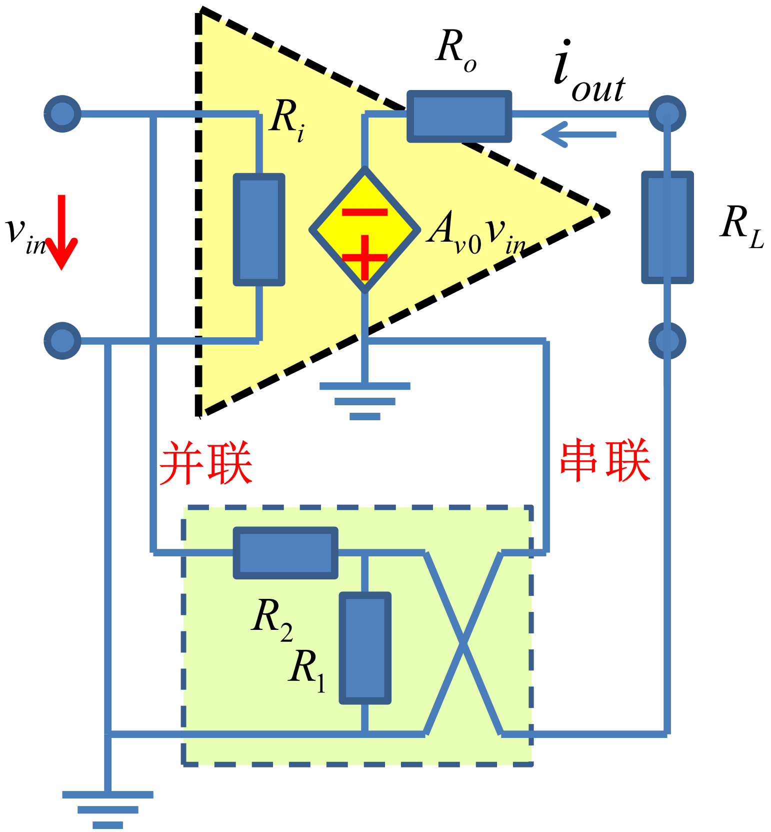


反馈输出点和放大输入点相同：并联

反馈输入点和放大输出点不同：串联

纯数学分析：并串连接g相加

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$



$$\mathbf{g}_A = \begin{bmatrix} G_{in} & 0 \\ -A_{v0} & R_{out} \end{bmatrix} \quad \mathbf{g}_F = \begin{bmatrix} \frac{1}{R_1+R_2} & \frac{R_1}{R_1+R_2} \\ -\frac{R_1}{R_1+R_2} & \frac{R_1R_2}{R_1+R_2} \end{bmatrix}$$

$$\eta = \frac{R_1}{R_1+R_2}$$

$$\mathbf{g}_{AF} = \mathbf{g}_A + \mathbf{g}_F$$

$$= \begin{bmatrix} G_{in} & 0 \\ -A_{v0} & R_{out} \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1+R_2} & \eta \\ -\eta & R_1 \parallel R_2 \end{bmatrix}$$

$$= \begin{bmatrix} G_{in} + \frac{1}{R_1+R_2} & 0 \\ -A_{v0} - \eta & R_{out} + R_1 \parallel R_2 \end{bmatrix} + \begin{bmatrix} 0 & \eta \\ 0 & 0 \end{bmatrix}$$

对数学方程的电路解释
开环放大与理想反馈

如果不给电路解释，就纯粹成了数学分析，无法给后续电路进阶---电路设计提供帮助，要想能够快速给出电路设计方案，必须充分理解对数学方程的电路解释

$$\mathbf{g}_{AF} = \mathbf{g}_A + \mathbf{g}_F = \begin{bmatrix} G_{in} & 0 \\ -A_{v0} & R_{out} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{R_1 + R_2} & \eta \\ -\eta & R_1 \parallel R_2 \end{bmatrix}$$

$$= \begin{bmatrix} G_{in} + \dfrac{1}{R_1 + R_2} & 0 \\ -A_{v0} - \eta & R_{out} + R_1 \parallel R_2 \end{bmatrix} + \begin{bmatrix} 0 & \eta \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} g_{in} & 0 \\ -A_{i0} g_{in} r_{out} & r_{out} \end{bmatrix} + \begin{bmatrix} 0 & F_i \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} r_{in} & 0 \\ A_{i0} & g_{out} \end{bmatrix}^{-1} + \begin{bmatrix} 0 & F_i \\ 0 & 0 \end{bmatrix}$$

$$r_{in} = \frac{1}{G_{in} + \dfrac{1}{R_1 + R_2}}$$

$$r_{out} = R_{out} + R_1 \parallel R_2$$

$$A_{i0} = \left(A_{v0} + \eta\right) r_{in} g_{out}$$

$$F_i = \eta$$

并串负反馈 流控流源

$$\mathbf{g}_{AF}=\begin{bmatrix} g_{in} & 0 \\ -A_{i0}g_{in}r_{out} & r_{out}\end{bmatrix}+\begin{bmatrix} 0 & F_i \\ 0 & 0\end{bmatrix}=\mathbf{g}_{Ao}+\mathbf{g}_{iF} \qquad \mathbf{h}_{Ao}=\begin{bmatrix} r_{in} & 0 \\ A_{i0} & g_{out}\end{bmatrix}$$

$$\mathbf{h}_{AF}=\mathbf{g}_{AF}^{-1}=\begin{bmatrix} g_{in} & F_i \\ -A_{i0}g_{in}r_{out} & r_{out}\end{bmatrix}^{-1}=\frac{1}{g_{in}r_{out}\left(1+A_{i0}F_i\right)}\begin{bmatrix} r_{out} & -F_i \\ A_{i0}g_{in}r_{out} & g_{in}\end{bmatrix}$$

$$=\frac{1}{1+A_{i0}F_i}\begin{bmatrix} r_{in} & -F_i\dfrac{r_{in}}{r_{out}} \\ A_{i0} & \dfrac{1}{r_{out}}\end{bmatrix}\overset{\text{单向化条件}}{\approx}\begin{bmatrix} r_{inf} & 0 \\ A_{if} & \dfrac{1}{r_{outf}}\end{bmatrix}=\frac{1}{1+A_{i0}F_i}\mathbf{h}_{Ao}$$

单向化条件：$$\left|\frac{A_{i0}F_i r_{in} g_{out}}{\left(1+A_{i0}F_i\right)^2}\right|=A_{i0}F_i r_{inf} g_{outf} \ll \left|R_S+r_{inf}\right|\cdot\left|G_L+g_{outf}\right|$$

$$R_S \gg A_{i0}F_i r_{inf}\approx r_{in}$$

$$G_L \gg A_{i0}F_i g_{outf}\approx g_{out}$$

$$R_S G_L \gg A_{i0}F_i r_{inf} g_{outf}$$

$$R_S \gg r_{in}=\frac{1}{G_{in}+\dfrac{1}{R_1+R_2}}\approx R_1+R_2$$

$$R_L \ll r_{out}=R_{out}+R_1\parallel R_2\approx R_1\parallel R_2$$

$$\frac{R_S}{R_L}\gg\frac{1}{A_{v0}\eta}=\frac{1}{A_{v0}}\left(1+\frac{R_2}{R_1}\right)$$

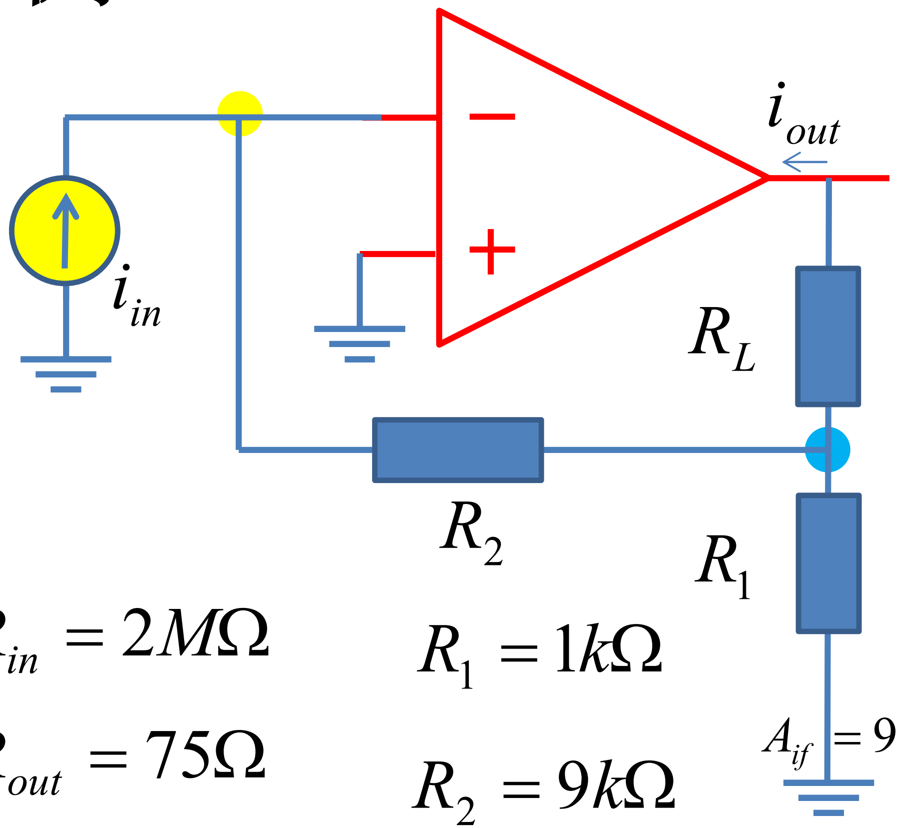
最后一条单向化条件很容易满足

数值例

$$\mathbf{g}_A=\begin{bmatrix}G_{in} & 0\\ -A_{v0} & R_{out}\end{bmatrix}=\begin{bmatrix}0.5\mu S & 0\\ -200000 & 75\Omega\end{bmatrix}$$

$$\mathbf{g}_F=\begin{bmatrix}\frac{1}{R_1+R_2} & \frac{R_1}{R_1+R_2}\\ -\frac{R_1}{R_1+R_2} & \frac{R_1R_2}{R_1+R_2}\end{bmatrix}=\begin{bmatrix}0.1mS & 0.1\\ -0.1 & 900\Omega\end{bmatrix}$$

$$\mathbf{g}_{AF}=\mathbf{g}_A+\mathbf{g}_F=\begin{bmatrix}0.1005mS & 0.1\\ -200000.1 & 975\Omega\end{bmatrix}=\begin{bmatrix}0.1005mS & 0\\ -200000.1 & 975\Omega\end{bmatrix}+\begin{bmatrix}0 & 0.1\\ 0 & 0\end{bmatrix}=\mathbf{g}_{Ao}+\mathbf{g}_{iF}$$



$$\mathbf{h}_{Ao}=\mathbf{g}_{Ao}^{-1}=\begin{bmatrix}9950\Omega & 0\\ 2041077 & 1.0256mS\end{bmatrix}\qquad F_i=0.1$$

$$A_{i0}=2041077\qquad r_{in}=9.95k\Omega$$

$$\mathbf{h}_{AF}=\mathbf{g}_{AF}^{-1}\qquad r_{out}=975\Omega$$

$$=\begin{bmatrix}0.04875\Omega & 4.999973\times10^{-6}\\ 9.999951 & 0.005025\mu S\end{bmatrix}$$

$$\approx\begin{bmatrix}0.04875\Omega & 0\\ 9.999951 & \frac{1}{199.006M\Omega}\end{bmatrix}$$

$$R_{in}=2M\Omega\qquad R_1=1k\Omega$$

$$R_{out}=75\Omega\qquad R_2=9k\Omega$$

$$A_{v0}=200000\qquad R_L=1k\Omega$$

$$A_{if}=9.999951=\frac{A_{i0}}{1+A_{i0}F_i}\qquad r_{inf}=0.04875\Omega=\frac{r_{in}}{1+A_{i0}F_i}$$

$$r_{outf}=199M\Omega=(1+A_{i0}F_i)r_{out}$$

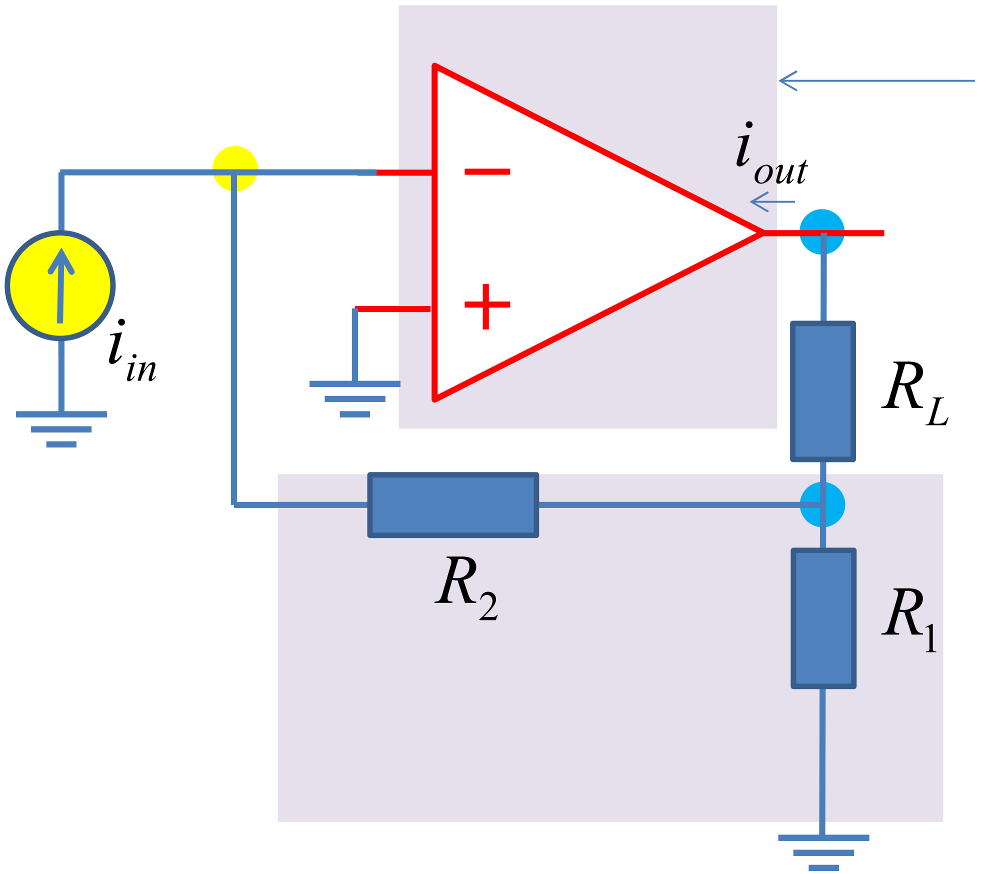
$R_S \gg r_{in} = 9.95k\Omega$

$R_L \ll r_{out} = 975\Omega$

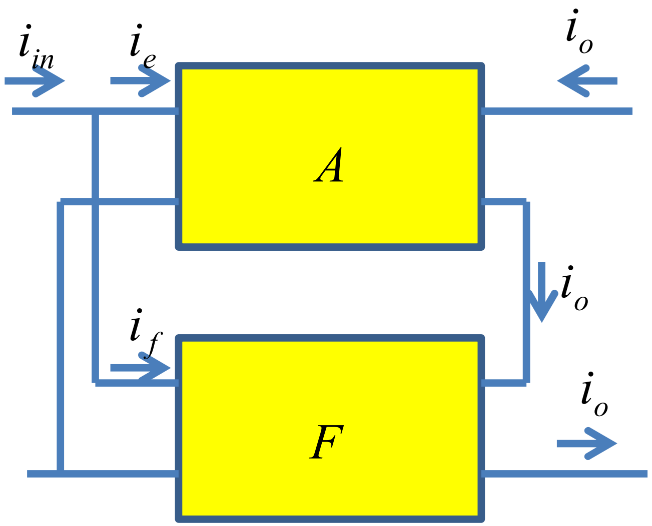
$R_L \ll A_{v0}\eta R_S = 20000R_S$

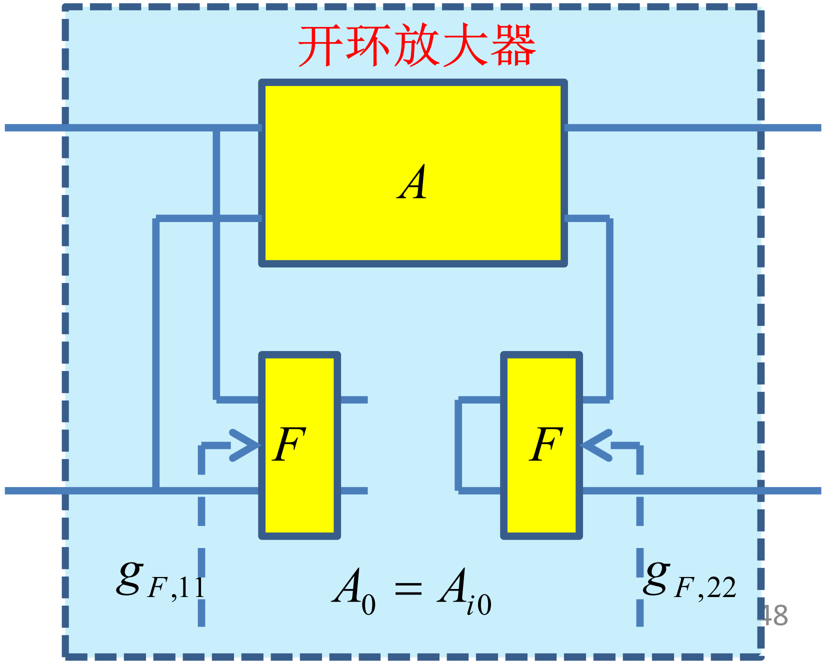
抛弃数学分析，直接进行电路操作

有些晶体管电路求网络参量显得简单问题复杂化
直接分析则显得极度简单



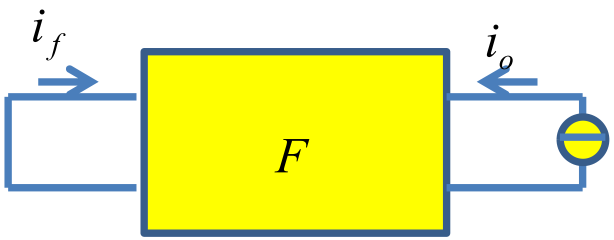
有时反馈网络和晶体管放大器网络是一体的，无法分离，单独求晶体管网络的网络参量显得困难或多此一举





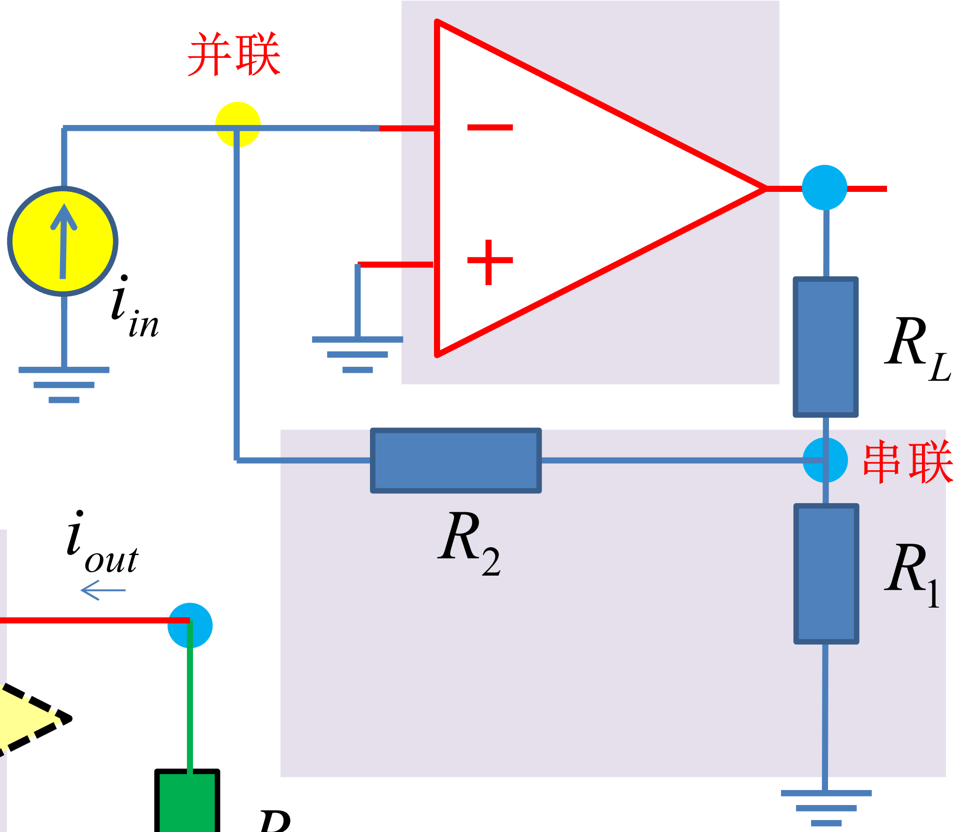
$$\mathbf{g}_{A,openloop} = \begin{bmatrix} g_{A,11} & 0 \\ g_{A,21} & g_{A,22} \end{bmatrix} + \begin{bmatrix} g_{F,11} & 0 \\ g_{F,21} & g_{F,22} \end{bmatrix}$$

$$\approx \begin{bmatrix} g_{A,11} & 0 \\ g_{A,21} & g_{A,22} \end{bmatrix} + \begin{bmatrix} g_{F,11} & 0 \\ 0 & g_{F,22} \end{bmatrix}$$

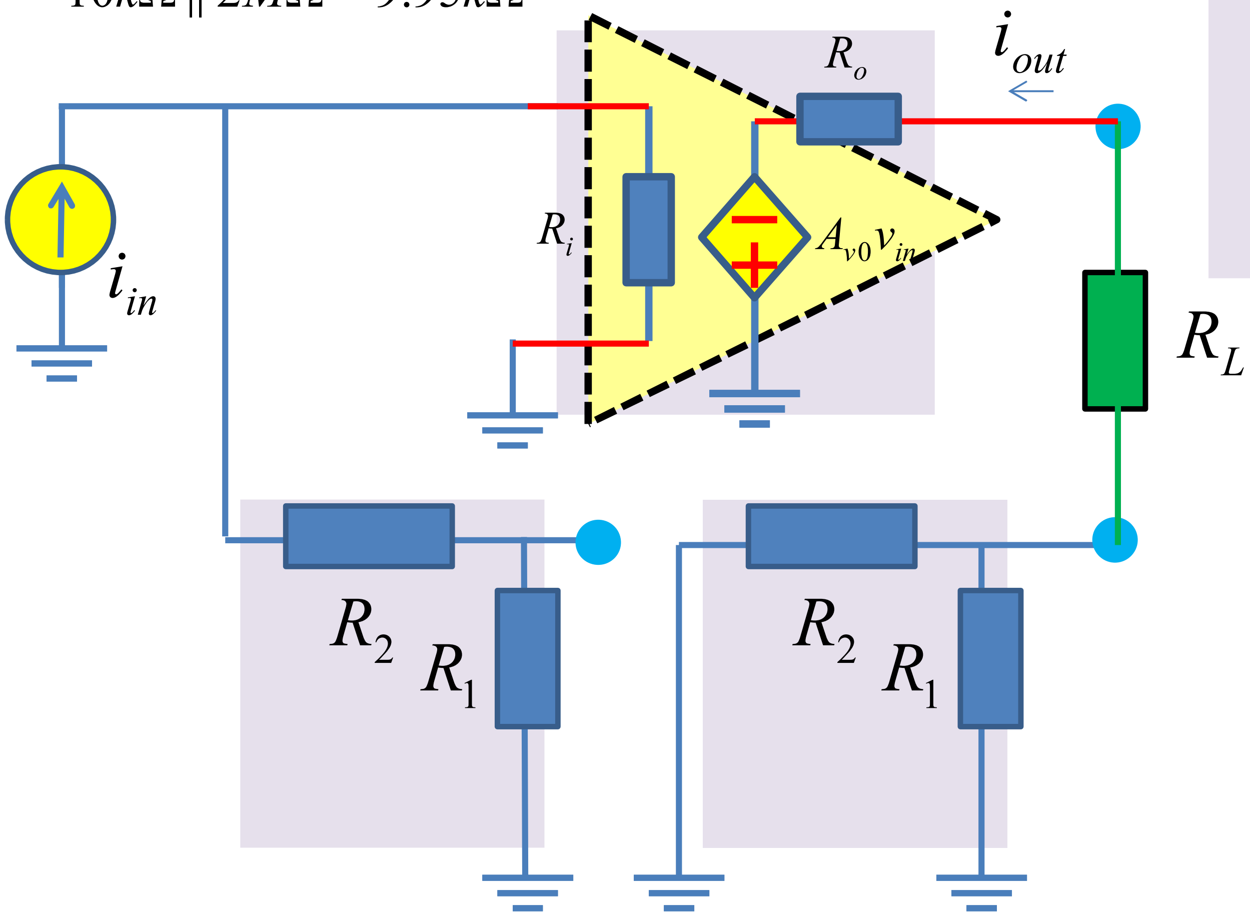


$$\mathbf{g}_{F,ideal} = \begin{bmatrix} 0 & F \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & g_{F,12} \\ 0 & 0 \end{bmatrix}$$

开环放大器



$$r_{in} = (R_2 + R_1) \| R_i$$
$$= 10k\Omega \| 2M\Omega = 9.95k\Omega$$

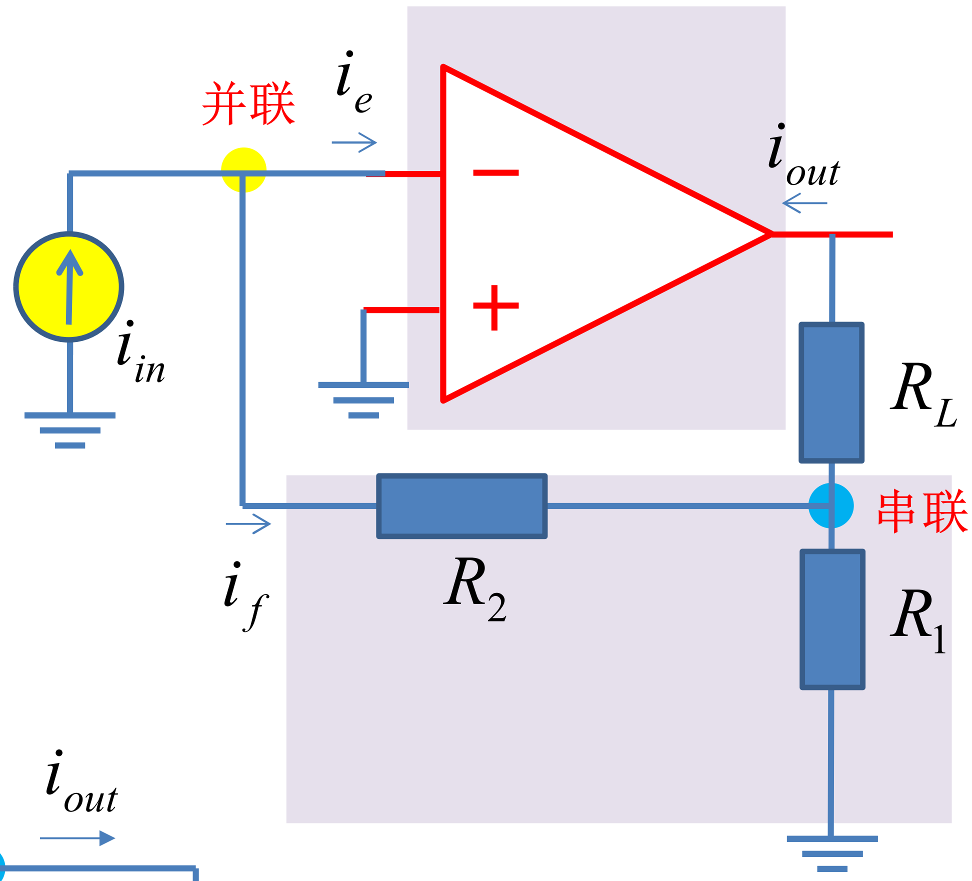


$$r_{out} = R_2 \| R_1 + R_o$$
$$= 900\Omega + 75\Omega = 975\Omega$$

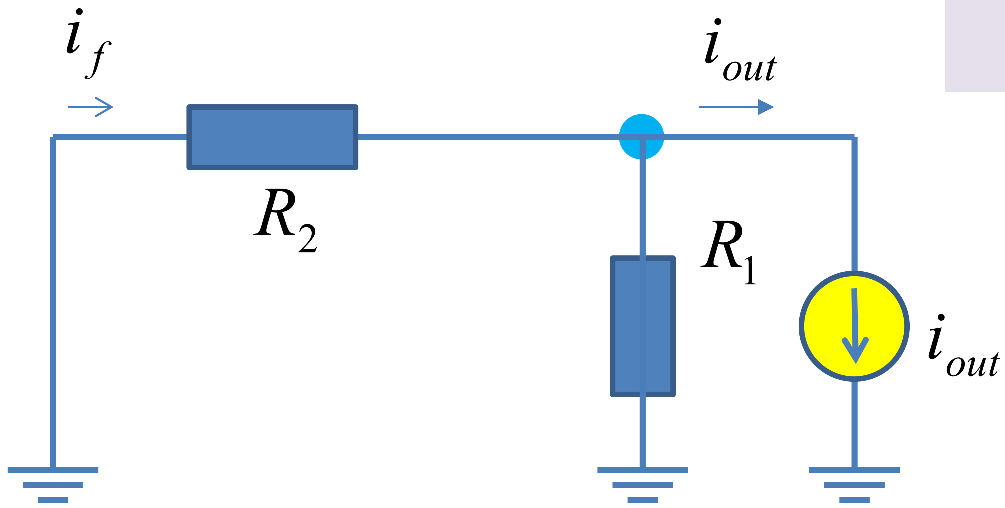
$$A_{i0} = i_N / i_{in} = \frac{v_{TH}}{r_{out}} \Bigg/ \frac{v_{in}}{r_{in}}$$
$$= A_{v0}\frac{r_{in}}{r_{out}} = 200000 \times \frac{9950}{975}$$
$$= 2041025$$

2041076/2041077

反馈系数



$$F_i = \frac{i_f}{i_{out}} = \frac{G_2}{G_1 + G_2} = \frac{R_1}{R_1 + R_2} = 0.1$$



闭环放大器

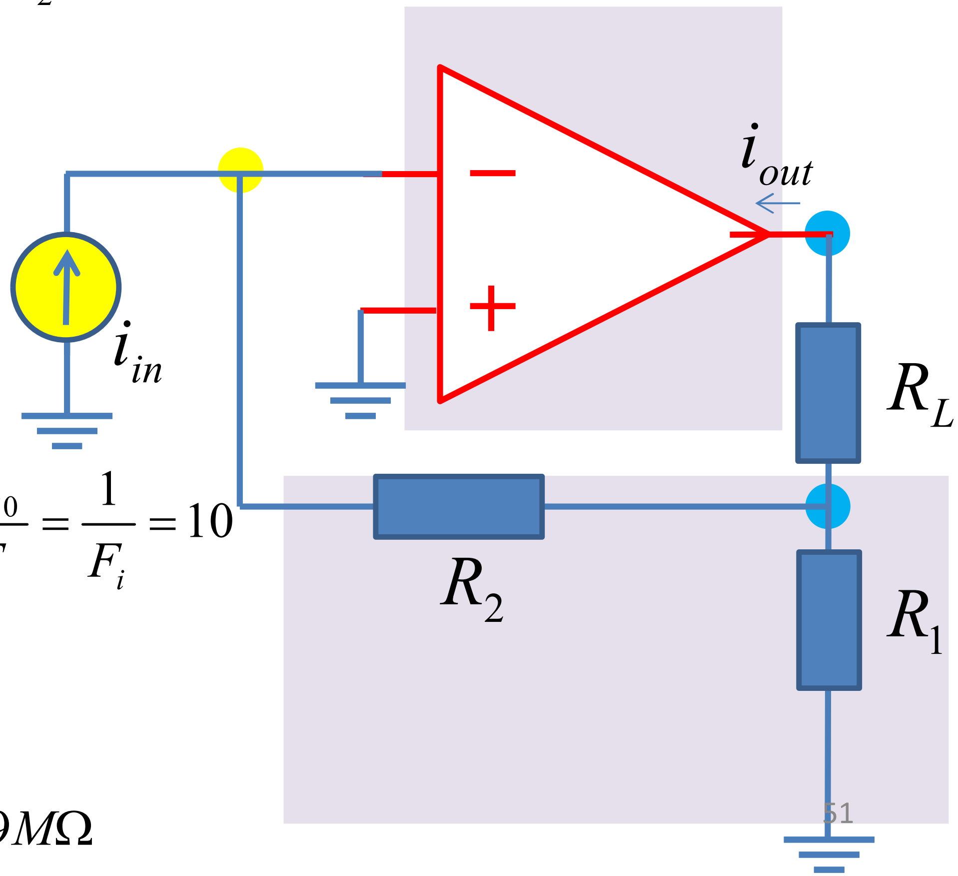
$$r_{in}=(R_2+R_1)\|R_i=10k\Omega\|2M\Omega=9.95k\Omega$$

$$r_{out}=R_2\|R_1+R_o=900\Omega+75\Omega=975\Omega$$

$$A_{i0}=A_{v0}\frac{r_{in}}{r_{out}}=200000\times\frac{9950}{975}=2041025$$

$$F_i=\frac{i_f}{i_{out}}=\frac{G_2}{G_1+G_2}=\frac{R_1}{R_1+R_2}=0.1$$

$$T=A_{i0}F_i=204103$$



$$A_{if}=\frac{A_{i0}}{1+T}=\frac{2041025}{204104}=9.999951\approx\frac{A_{i0}}{T}=\frac{1}{F_i}=10$$

$$r_{inf}=\frac{r_{in}}{1+T}=\frac{9.95k\Omega}{204104}=48.75m\Omega$$

$$r_{outf}=r_{out}(1+T)=975\Omega\times204104=199M\Omega$$

直接对负反馈放大电路进行电路操作

- 练习：对第**8**题的其他三种放大形式
  - 获得开环放大器
    - r_{in}
    - r_{out}
    - A_0
  - 获得反馈系数
    - **F**
  - 获得闭环增益：$T=A_0F$
  - 获得闭环放大参数
    - 串联阻抗放大**(1+T)**倍
    - 并联阻抗减小**(1+T)**倍
    - 闭环增益减小**(1+T)**倍，深度负反馈，近似等于**1/F**

第8讲 非线性电路分段线性化分析
作业1：直流电阻和交流电阻

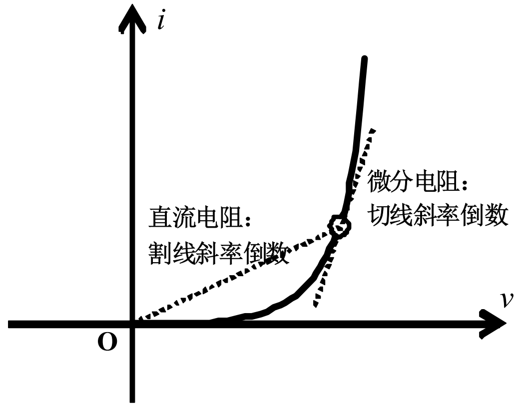
- 假设某二极管伏安特性在很大范围内都满足指数律关系
  - 该二极管的反向饱和电流I_{S0}为10fA
  - 给出直流电流为0.1mA，1mA，10mA时对应的直流电压，以及该直流工作点上的直流电阻和微分电阻
  - 分析直流电阻和微分电阻的变化规律

$$i_D = I_{S0}\left(e^{\frac{v_D}{v_T}} - 1\right)$$

i_D（mA）	v_D	R_D	r_d
0.1			
1			
10			

$$i_D = I_{S0}\left(e^{\frac{v_D}{v_T}} - 1\right) \qquad e^{\frac{v_D}{v_T}} = 1 + \frac{i_D}{I_{S0}}$$

$$v_T = \frac{kT}{q} = 26mV$$



$$v_D = v_T \ln\left(1 + \frac{i_D}{I_{S0}}\right)$$

$$I_{S0} = 10fA$$

$$i_D = 0.1mA, 1mA, 10mA$$

$$v_D = \ldots$$

非线性电阻的特征：电阻阻值和工作点有关

$$\frac{di_D}{dv_D} = \frac{I_{S0}}{v_T} e^{\frac{v_D}{v_T}}$$

$$= \frac{i_D + I_{S0}}{v_T} \approx \frac{i_D}{v_T}$$

i_D（mA）	v_D	R_D	r_d
0.1	0.5987V	5.987kΩ	260Ω
1	0.6585V	658.5Ω	26Ω
10	0.7184V	71.84Ω	2.6Ω

电流大幅变动，电压几乎不变：恒压等效的基础

$$r_d << R_D$$

$$r_d = \frac{dv_D}{di_D} = \frac{v_T}{I_D}$$

微分电阻很小，小信号的电压波动导致较大的电流波动

$$R_d = \frac{V_D}{I_D} \approx \frac{0.7V}{I_D}$$

二极管导通恒压源模型，但二极管本质是耗能的非线性电阻

$$i_D = I_{S0}\left(e^{\frac{v_D}{v_T}} - 1\right)$$

交直流分析

$$i_D = f(v_D)$$

$$v_D = V_{D0} + v_{ac}$$

$$i_D = I_{D0} + i_{ac}$$

Taylor展开

$$i_D = f(v_D) = f(V_{D0} + v_{ac}) = f(V_{D0}) + f'(V_{D0})v_{ac} + \frac{f''(V_{D0})}{2!}v_{ac}^2 + \cdots$$

$$\approx f(V_{D0}) + f'(V_{D0})v_{ac} = I_{D0} + i_{ac}$$

$|v_{ac}|$很小 高阶项影响可忽略不计

$$= f(V_{D0}) + \frac{v_{ac}}{r_d}$$

$$I_{D0} = f(V_{D0})$$

直流分析：非线性分析

只要交流信号足够小

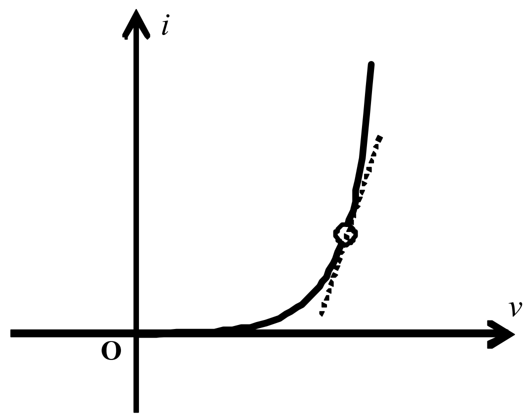
$$i_{ac} = f'(V_{D0})v_{ac} = \frac{v_{ac}}{r_d}$$

交流小信号线性分析

直流非线性分析和交流小信号线性分析可以分开分别进行

微分电阻

- **Differential Resistance**
  – 微分电阻
- **Incremental Resistance**
  – 增量电阻
- **Dynamic Resistance**
  – 动态电阻
- **Small Signal Resistance**
  – 交流小信号电阻



$$r_d = \frac{dv_D}{di_D}$$ 微分电阻

$$i = I_0 + \Delta i$$

$$v = V_0 + \Delta v = V_0 + r_d \Delta i$$

$$r_d = \frac{\Delta v}{\Delta i}$$ 增量电阻

$$i = I_0 + i_{ac}(t)$$

$$v = V_0 + v_{ac}(t) = V_0 + r_d i_{ac}(t)$$

$$r_d = \frac{v_{ac}(t)}{i_{ac}(t)}$$ 动态电阻 交流电阻

交直流功率

$$v_D = V_{D0} + v_{ac} \qquad i_D = I_{D0} + i_{ac} \approx f\left(V_{D0}\right) + \frac{v_{ac}}{r_d}$$

$$P_D = \overline{p_D} = \overline{v_D i_D} = \overline{\left(V_{D0} + v_{ac}\right)\left(I_{D0} + i_{ac}\right)}$$
$$= \overline{V_{D0}I_{D0} + v_{ac}I_{D0} + V_{D0}i_{ac} + v_{ac}i_{ac}}$$
$$= V_{D0}I_{D0} + \overline{v_{ac}}I_{D0} + V_{D0}\overline{i_{ac}} + \overline{v_{ac}i_{ac}}$$
$$= V_{D0}I_{D0} + \overline{v_{ac}i_{ac}} = P_{DC} + P_{AC}$$
$$\approx V_{D0}I_{D0} + r_d \overline{i_{ac}^2} = I_{D0}^2 R_D + I_{ac,rms}^2 r_d$$

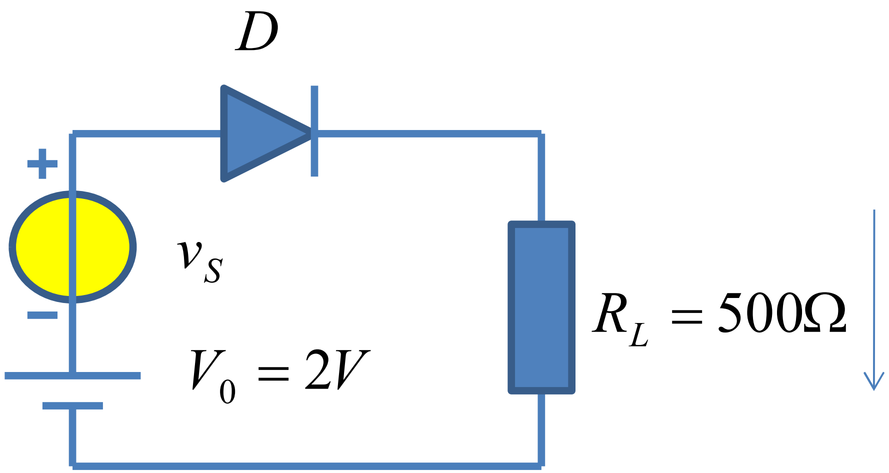
交流电阻（微分电阻）不仅可用于表述交流功率大小，同时可用于表述交流压流的线性转换关系

直流电阻可用于表述直流功率大小，在信号处理中没有什么地位

微分元件在非线性电路分析中具有重要的地位

作业2：二极管导通恒压模型的应用

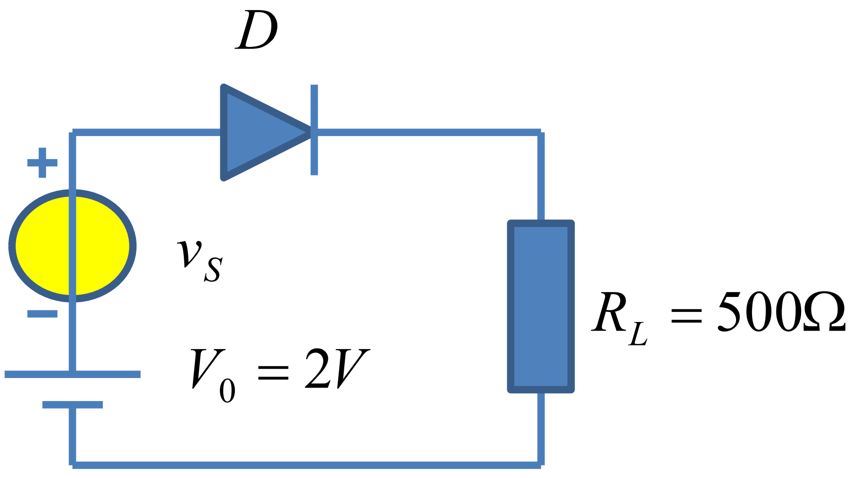
- 采用导通**0.7V**恒压源模型，分析如下电路，给出输出电阻上的电压大小



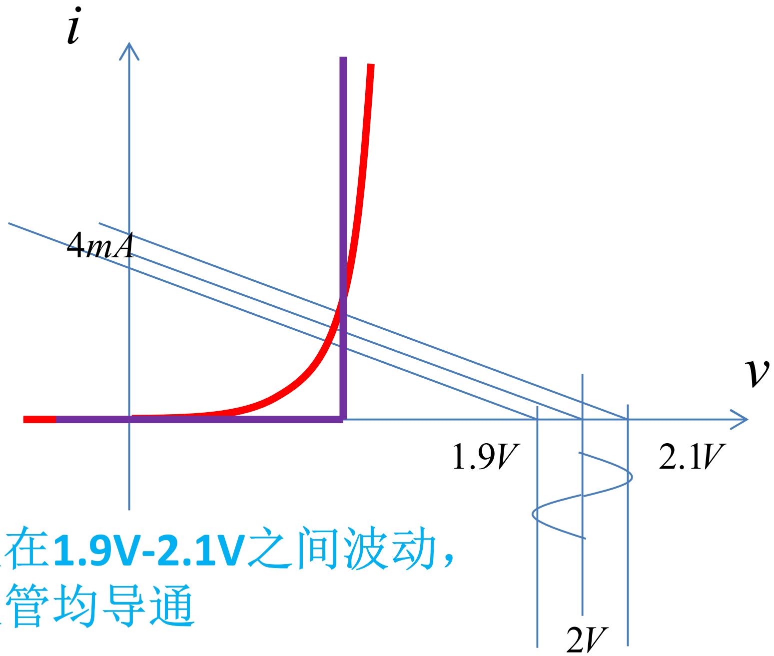
$$v_S = 100\sin\omega t\,(mV)$$

$$V_L = V_{L0} + v_L$$

直流分量　交流分量

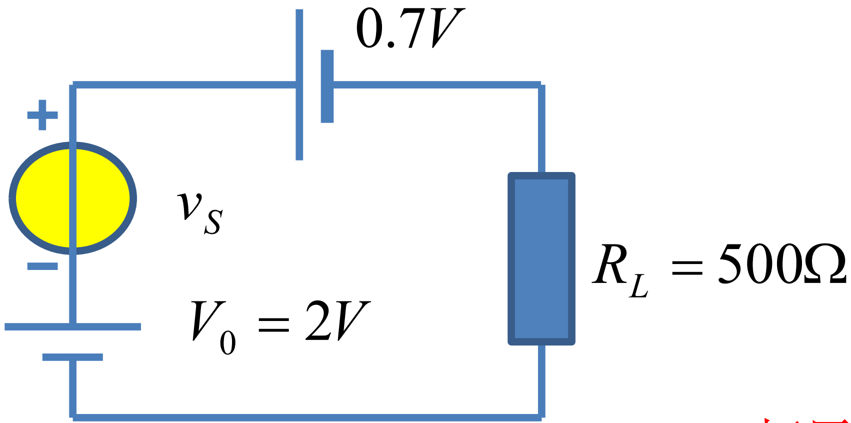


$$v_S = 100\sin\omega t\,(mV)$$



电压在**1.9V-2.1V**之间波动，
二极管均导通

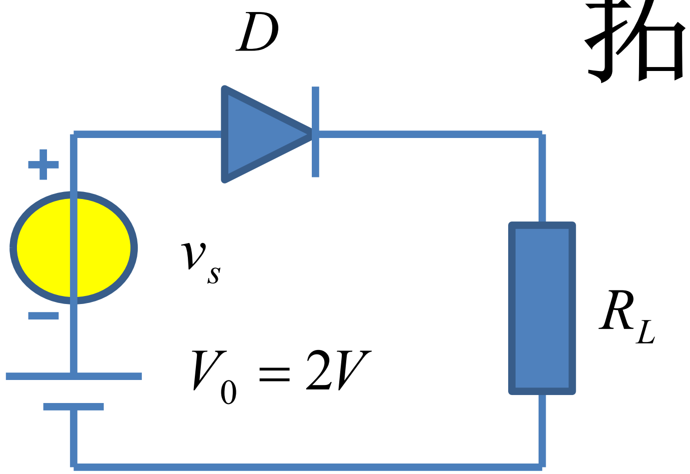
第一步，确认二极管的导通、截止状态！第二步，用折线模型替代



$$V_L = V_0 - V_{D0} + v_S$$
$$= 1300 + 100\sin\omega t\,(mV)$$
$$= V_{L0} + v_L(t)$$

0.7V恒压源模型分析结论！快速，误差有多大？

拓展分析：交直流分析



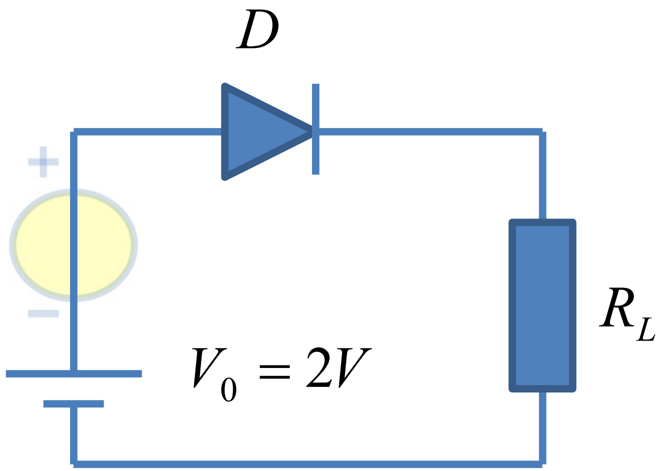
$$\begin{aligned} v_D &= V_0 + v_s(t) - (V_{L0} + v_l(t)) \\ &= (V_0 - V_{L0}) + (v_s(t) - v_l(t)) \\ &= V_{D0} + v_d(t) \end{aligned}$$

假设$v_d(t)$足够小

$$i_D = f(v_D) = f(V_{D0} + v_d(t)) \approx f(V_{D0}) + \frac{v_d(t)}{r_d} = I_0 + i_d(t)$$

$I_0 = f(V_{D0})$ 直流非线性分析

$i_d(t) = \dfrac{v_d(t)}{r_d}$ 交流小信号线性分析



$$f(v) = I_{S0}\left(e^{\frac{v}{v_T}} - 1\right)$$

直流分析

$$I_0 = f(V_{D0}) = f(V_0 - V_{L0}) = f(V_0 - I_0 R_L)$$

2V　500Ω

$$I_0 = I_{S0}\left(e^{\frac{V_0 - I_0 R_L}{v_T}} - 1\right) \Rightarrow I_0 = \frac{V_0 - v_T \ln\left(1 + \frac{I_0}{I_{S0}}\right)}{R_L}$$

10fA　26mV

简单迭代格式

可牛顿拉夫逊迭代法
数值求解

$$I_0^{(0)} = \frac{V_0 - 0.7}{R_L} = \frac{2 - 0.7}{0.5k} = 2.6mA$$

$$I_0^{(1)} = \frac{V_0 - v_T \ln\left(1 + \frac{I_0^{(0)}}{I_{S0}}\right)}{R_L} = 2.6332mA$$

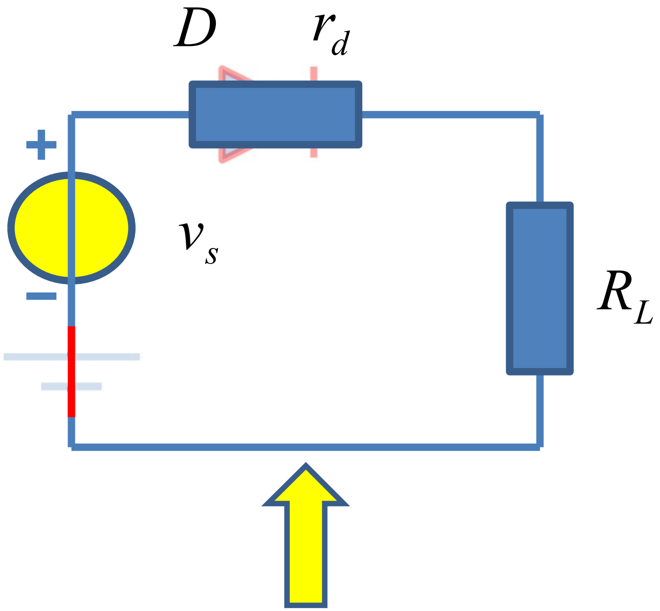
$$I_0^{(2)} = \frac{V_0 - v_T \ln\left(1 + \frac{I_0^{(1)}}{I_{S0}}\right)}{R_L} = 2.6326mA$$

$$I_0^{(3)} = \frac{V_0 - v_T \ln\left(1 + \frac{I_0^{(2)}}{I_{S0}}\right)}{R_L} = 2.6326mA$$

$$V_{D0} = v_T \ln\left(1 + \frac{I_0^{(3)}}{I_{S0}}\right) = 0.6837V$$

$$V_{L0} = V_0 - V_{D0} = 1.3163V$$

交流分析



$$
\begin{aligned}
v_D &= V_0 + v_s(t) - \left(V_{L0} + v_l(t)\right) \\
&= \left(V_0 - V_{L0}\right) + \left(v_s(t) - v_l(t)\right) \\
&= V_{D0} + v_d(t)
\end{aligned}
$$

$$i_d(t) = \frac{v_d(t)}{r_d} = \frac{v_s(t) - v_l(t)}{r_d}$$

$$r_d = \frac{v_T}{I_{D0}} = \frac{26mV}{2.6326mA} = 9.8762\Omega$$

$$v_l(t) = \frac{R_L}{R_L + r_d} v_s(t) = \frac{500}{500 + 9.8762} \times 100 \sin \omega t = 98.06 \sin \omega t (mV)$$

$$v_L(t) = V_{L0} + v_l(t) = 1316 + 98 \sin \omega t (mV)$$

v_d(t)足够小，故而交直流分析几乎精确

$$v_L(t) = 1300 + 100 \sin \omega t (mV)$$

分段折线模型误差小于**2%**，而且原理性更强，因而对于大多数二极管电路，我们更喜欢用分段折线模型

二极管小信号分析

- 当二极管电流在**mA**量级时，微分电阻**10^1**Ω量级，和**k**Ω量级负载电阻相比，一般可以忽略不计，此时二极管小信号电阻可抽象为**0**，二极管模型直接采用**0.7V**恒压源模型进行交直流分析即可

$$r_d = \frac{v_T}{I_{D0}} = \frac{26mV}{1mA} = 26\Omega$$

- 当二极管电流在**μA**量级时，微分电阻在**10k**Ω量级，和**k**Ω量级负载电阻相比，其影响不能忽略不计，此时加流小信号分析中必须将二极管微分电阻考虑在内
  - 如**BJT**的**BE**结微分电阻$\mathbf{r_{be}}$，小信号模型中一般都需要考虑在内