电子电路与系统基础I

习题课第九讲

第六周作业讲解（部分）
第七周作业讲解（部分）

李国林
清华大学电子工程系

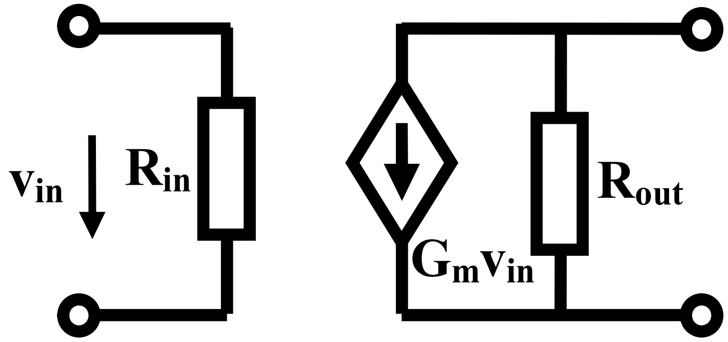
第5周作业

作业7：放大器的有源性条件

- 请推导（方法不限）：
  - （1）跨导放大器满足什么条件时，它才是有源的（能够向外输出功率）？
  - （2）满足上述有源性条件前提下，又满足什么条件时，基本放大器可向外输出最大功率？最高功率增益为多少？

电压放大器的有源性条件 $\left|A_v\right| > 2\sqrt{\dfrac{R_o}{R_i}}$

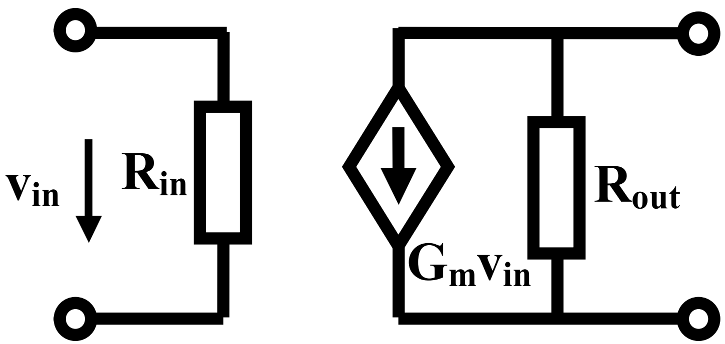
跨导放大器



$$\mathbf{y} = \begin{bmatrix} G_{in} & 0 \\ G_m & G_{out} \end{bmatrix}$$

$$p_{\Sigma} = v_1 i_1 + v_2 i_2 < 0$$

如果存在这种可能性，则有源
如果没有这种可能性，则无源



$$\mathbf{y}=\begin{bmatrix} G_{in} & 0 \\ G_m & G_{out} \end{bmatrix}$$

$$p_{\Sigma}=v_1 i_1+v_2 i_2=v_1\left(G_{in}v_1\right)+v_2\left(G_m v_1+G_{out}v_2\right)=G_{in}v_1^2+G_m v_1 v_2+G_{out}v_2^2<0$$

$G_{in}<0$

只需令端口2短路v_2=0，端口1加压，则有功率输出：有源

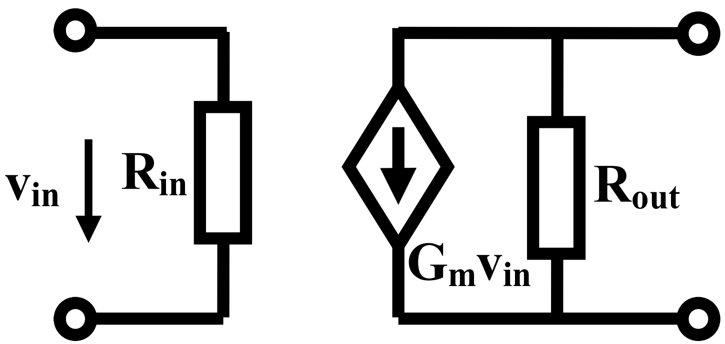
$$p_{\Sigma}=G_{in}v_1^2<0$$

负阻是有源的

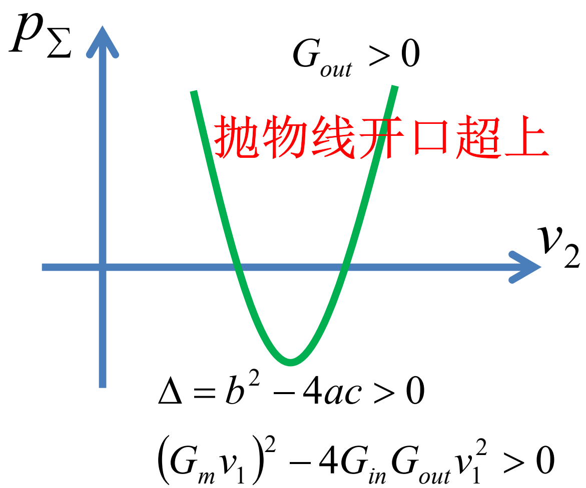
$G_{out}<0$

只需令端口1短路v_1=0，端口2加压，有功率输出：有源

$$p_{\Sigma}=G_{out}v_2^2<0$$



$$\mathbf{y}=\begin{bmatrix} G_{in} & 0 \\ G_m & G_{out} \end{bmatrix}$$



$$p_\Sigma = G_{in}v_1^2 + G_m v_1 v_2 + G_{out}v_2^2$$

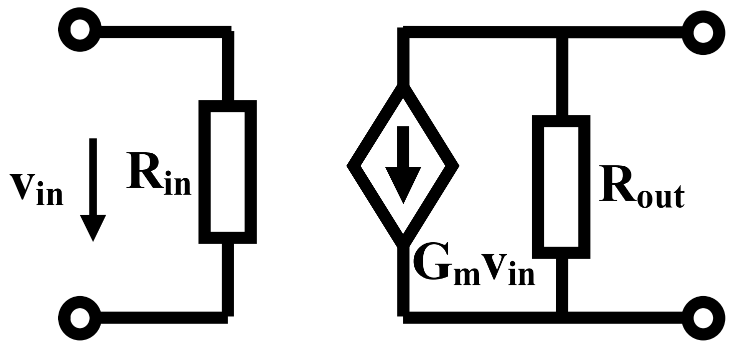
$$= G_{out}\left(v_2+\frac{G_m v_1}{2G_{out}}\right)^2 + v_1^2\left(G_{in}-\frac{1}{4}\frac{G_m^2}{G_{out}}\right) < 0$$

$G_{in}>0$ $\qquad$ $G_{out}>0$

只需令 $R_L=R_{out}$ ，如果 $G_m^2>4G_{in}G_{out}$

端口**1**加压，则端口**2**负载获得功率高于端口**1**吸收功率：总体是向外输出功率的

有源性条件



$$\mathbf{y} = \begin{bmatrix} G_{in} & 0 \\ G_m & G_{out} \end{bmatrix}$$

$G_{in} < 0$　　输入电阻为负阻，可向外提供能量

$G_{out} < 0$　　输出电阻为负阻，可向外提供能量

$$G_m^2 > 4G_{in}G_{out} \qquad (G_{in}, G_{out} > 0)$$

跨导增益足够大，其提供能量不仅补偿内阻消耗能量，还有额外的能量向外输出

三个条件满足其一，则有源

有源则可作为放大器使用，也可形成振荡器

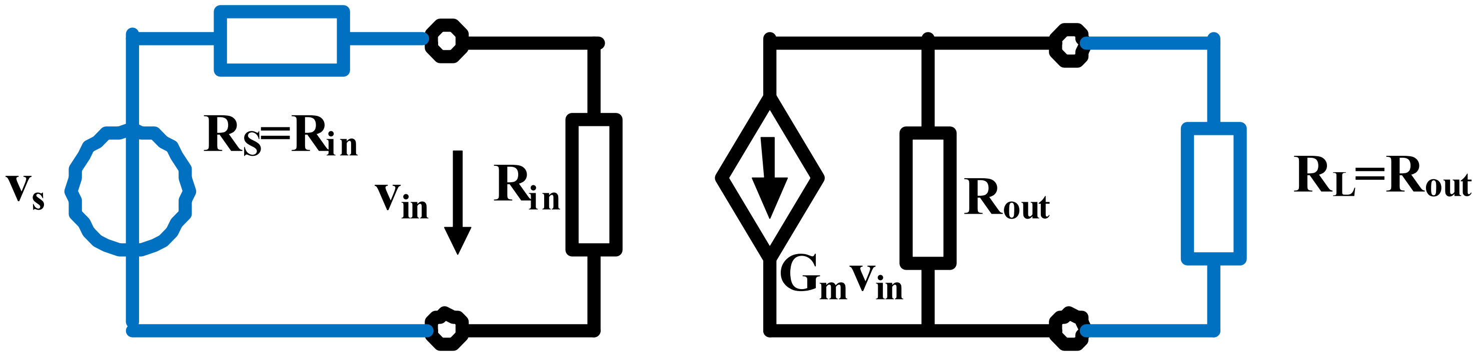
对放大器而言

另外两个有源性条件：负阻条件

$G_{in}<0$或$G_{out}<0$

通过无损器件环行器作用，形成反射型的负阻放大器，放大器功率大于1

- 有源性条件　等价于　功率增益大于1



$$G_{T,\max}=\frac{P_{L,\max}}{P_{S,\max}}=\frac{\dfrac{\left(-G_m V_{in,rms}0.5R_{out}\right)^2}{R_{out}}}{\dfrac{V_{s,rms}^2}{4R_{in}}}$$

$$=\frac{G_m^2V_{in,rms}^2R_{out}R_{in}}{V_{s,rms}^2}=\frac{G_m^2R_{out}R_{in}}{4}>1$$

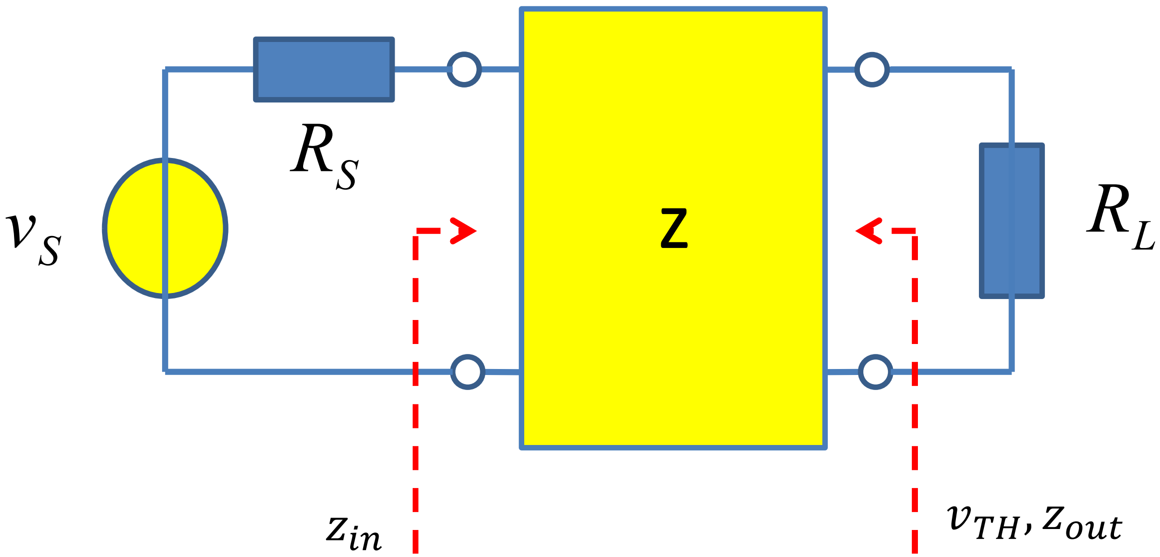
⟺

$$G_m^2>4G_{in}G_{out}$$

功率增益大于1，意味着输出端口输出功率大于输入端口吸收功率，和有源性定义要求一致

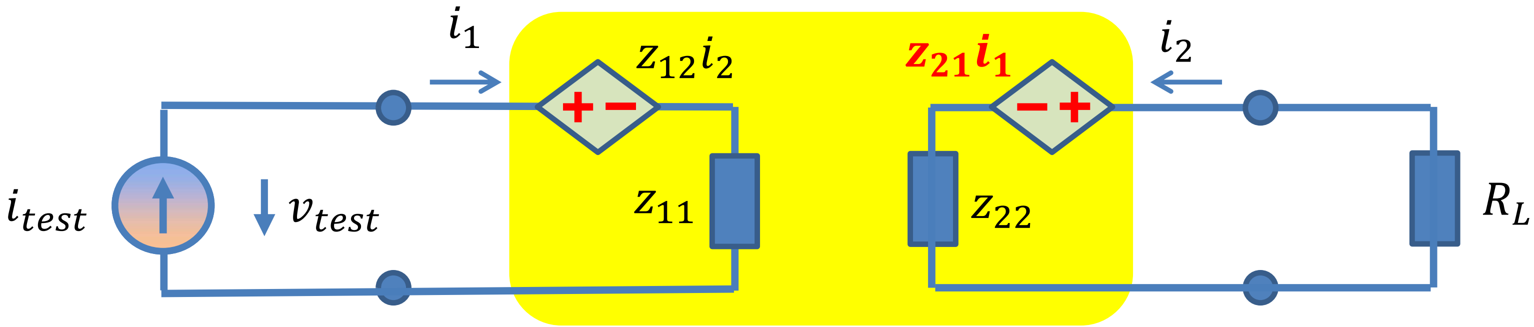
第6周作业

作业1



- 已知二端口网络的**z**参量，**1**端口接信源（v_s,R_s），**2**端口接负载R_L
  - 求输入阻抗Z_{in}
  - 求输出端戴维南等效v_{TH}，Z_{out}
    - 要求有详细的推导步骤：要求用电路语言分析
  - 在此基础上，考察单向网络的表达式与等效电路之间的关系
    - **z**参量单向网络：将$z_{12}=0$，$z_{21}=R_m$代入表达式即可
    - 通过等效电路图分析，比对解表达式，理解对电路中的分压、分流关系

加流求压获得输入电阻



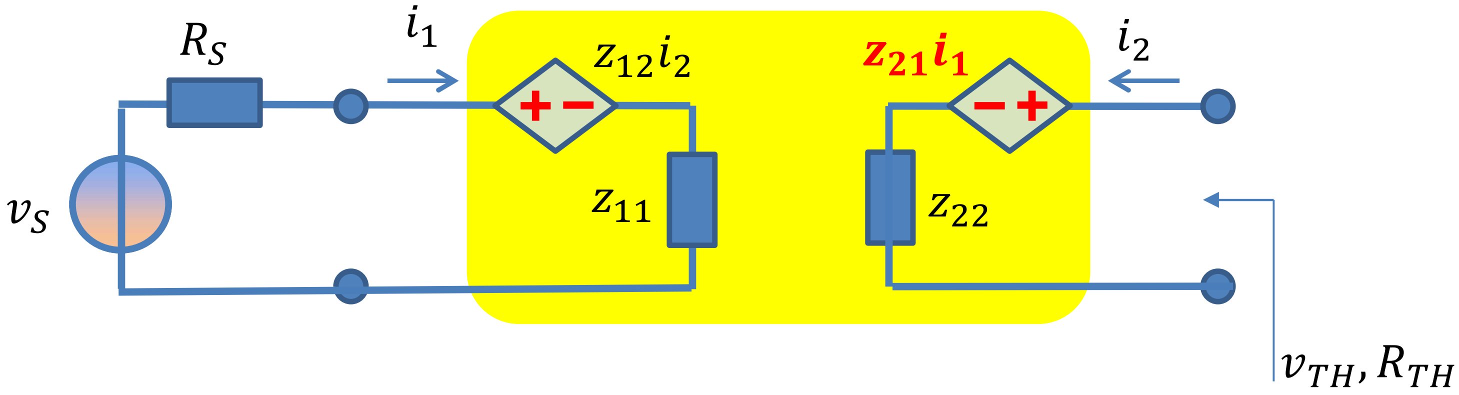
$$i_2 = \frac{-z_{21}i_{test}}{z_{22} + R_L}$$

$$v_{test} = z_{12}i_2 + z_{11}i_{test}$$

$$z_{in} = \frac{v_{test}}{i_{test}} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + R_L}$$

$$z_{out} = z_{22} - \frac{z_{21}z_{12}}{z_{11} + R_S}$$

等效戴维南源电压



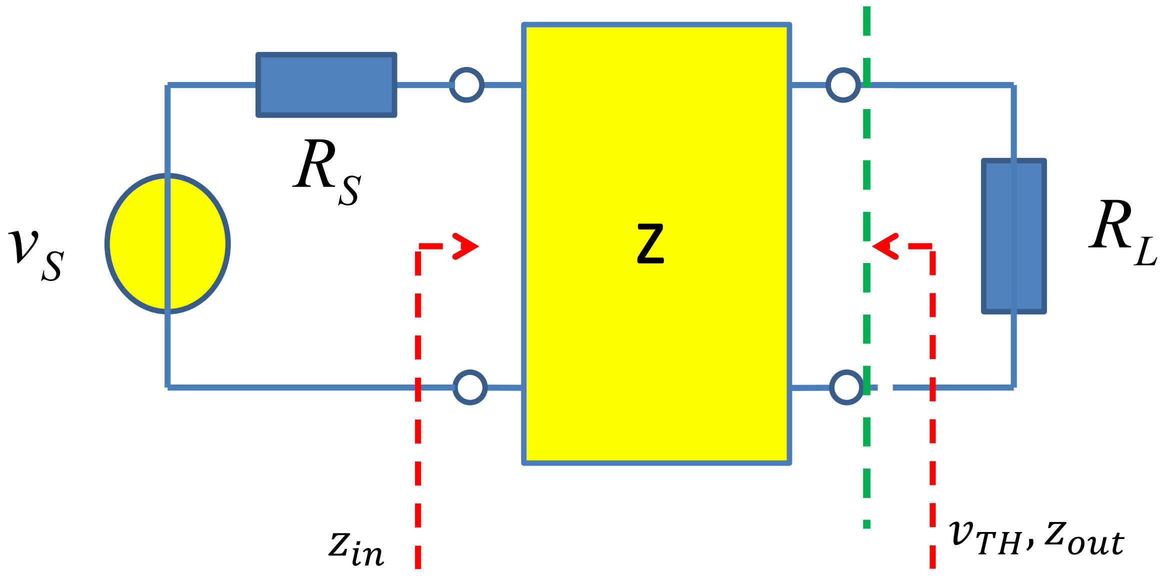
$$v_{TH} = z_{21}\frac{1}{R_S + z_{11}}v_S$$

被本征跨阻增益转换为输出回路开路电压

输出开路，输入回路流控压源短路，激励电压被输入回路总电阻转化为输入回路电流

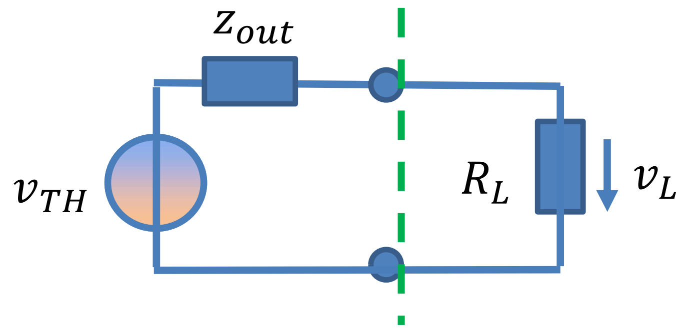
戴维南源电压可以直接给出，无需复杂的求解过程

求输出电压表达式



$$v_{TH}=z_{21}\frac{1}{R_S+z_{11}}v_S$$

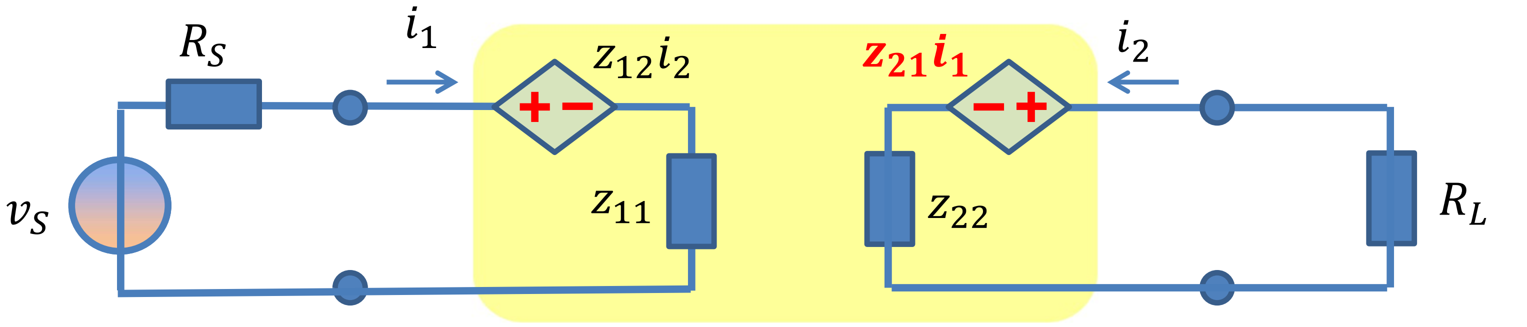
$$z_{out}=z_{22}-\frac{z_{21}z_{12}}{z_{11}+R_S}$$



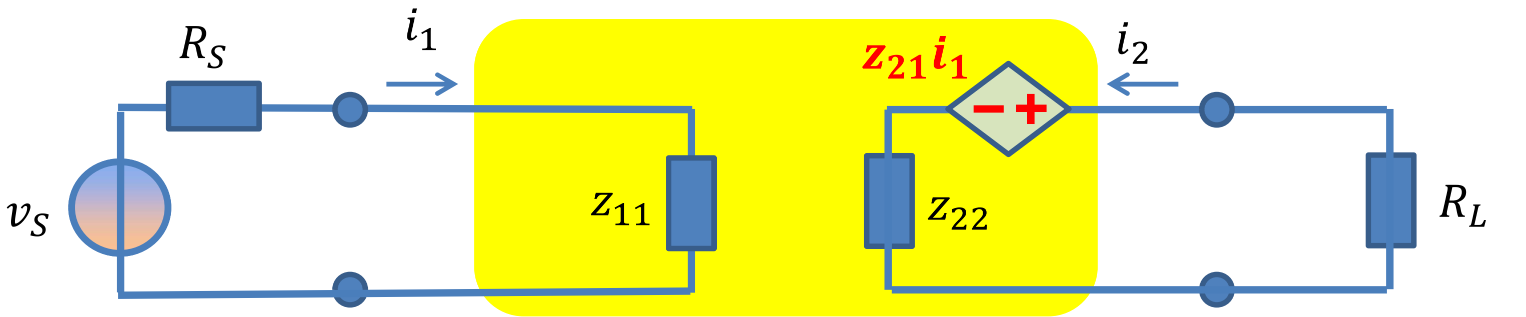
$$v_L=\frac{R_L}{z_{out}+R_L}v_{TH}=\frac{R_L}{z_{22}-\dfrac{z_{21}z_{12}}{z_{11}+R_S}+R_L}z_{21}\frac{1}{R_S+z_{11}}v_S$$

$$=\frac{z_{21}R_L}{(z_{22}+R_L)(z_{11}+R_S)-z_{21}z_{12}}v_S$$

单向网络表达式



$$v_L = \frac{z_{21}R_L}{(z_{22}+R_L)(z_{11}+R_S)-z_{21}z_{12}}v_S \overset{z_{12}=0}{\triangleq} \frac{z_{21}R_L}{(R_L+z_{22})(R_S+z_{11})}v_S$$



$$v_L = \frac{R_L}{R_L+z_{22}} z_{21} \frac{1}{R_S+z_{11}} v_S$$

输出回路电压在负载电阻上的分压为负载电压

被本征跨阻增益转换为输出回路电压

输入电压被输入回路总电阻转化为输入回路电流

单向网络传递函数=分网络传递函数之积 $H = H_1 H_2 H_3$

物理意义明确：单向传输

拓展讨论：阻抗变换功能

$$z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + R_L}$$

$$z_{out} = z_{22} - \frac{z_{21}z_{12}}{z_{11} + R_S}$$

可以理解为R_L被变换为z_{in}，R_S被变换为z_{out}

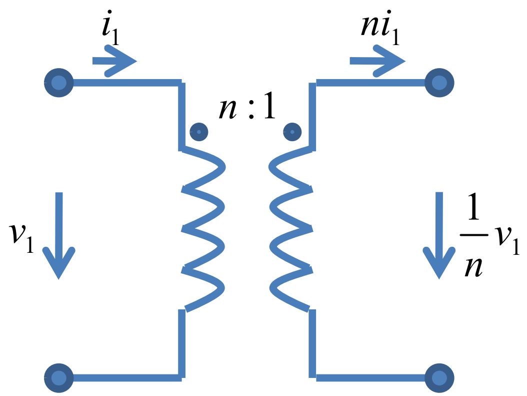
$z_{12}z_{21} \neq 0$　　双向二端口网络具有阻抗变换功能

$z_{12}z_{21} = 0$　　单向二端口网络输入阻抗（导纳）和输出阻抗（导纳）完全由二端口网络自身决定，和端口所接负载无关

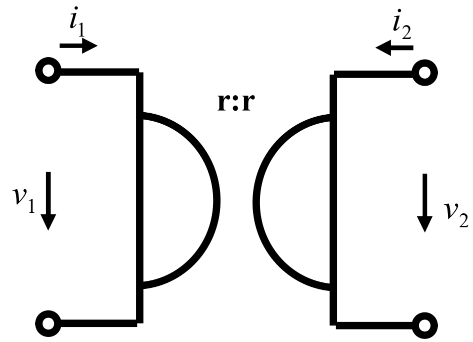
基本放大器：均属单向网络，输入（输出）阻抗和负载（信源内阻）无关；实际放大器：存在反向作用，双向网络，输入阻抗和负载有关；反馈由人为设计或寄生；我们期望实际放大器接近单向：$|z_{12}| << |z_{21}|$，反向作用小到使得$|z_{12}z_{21}| << |z_{11}z_{22}|$，输入电阻和输出电阻则基本可确定为 $z_{in} \approx z_{11}$，$z_{out} \approx z_{22}$

阻抗变换网络一定是双向网络

$z_{21}z_{12} \neq 0$ $y_{21}y_{12} \neq 0$ $h_{21}h_{12} \neq 0$ $g_{21}g_{12} \neq 0$ $AD - BC \neq 0$



存在**hg**（互易无损），无**zy**表述



存在**zy**（非互易无损），无**hg**表述

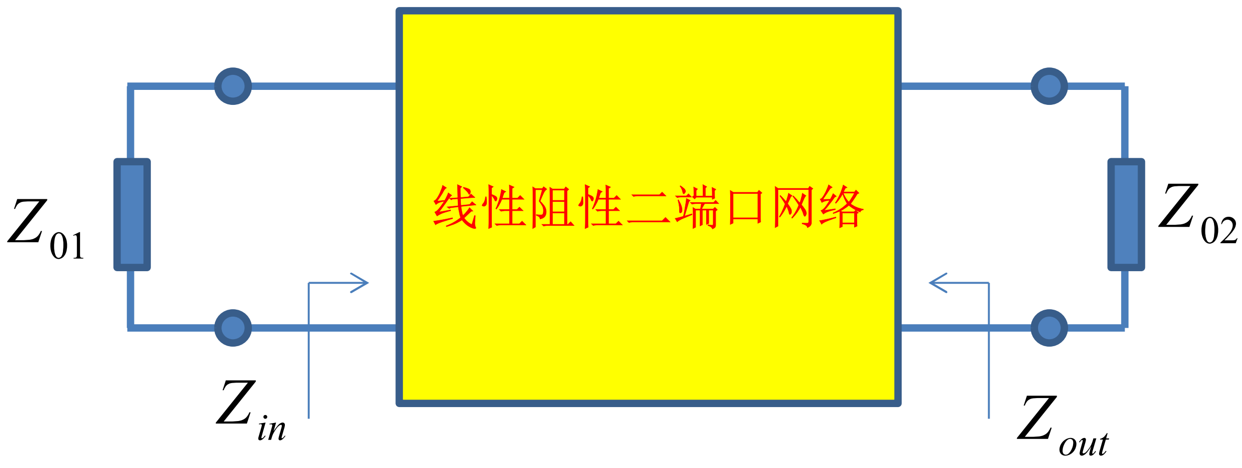
$$\mathbf{h} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \qquad \mathbf{ABCD} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

$$\mathbf{ABCD} = \begin{bmatrix} 0 & r \\ \frac{1}{r} & 0 \end{bmatrix} \qquad \mathbf{z} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}$$

理想变压器和回旋器是两个最典型的阻抗变换网络，它们的特点是戴维南-诺顿等效网络参量（**zyhg**矩阵）的**11**参量和**22**参量为**0**，只有**12**参量和**21**参量，这意味着它们的特征阻抗为任意值：具有任意的阻抗变换匹配功能

$$Z_{01} = \sqrt{\frac{z_{11}}{y_{11}}} = \sqrt{\frac{h_{11}}{g_{11}}} \qquad Z_{02} = \sqrt{\frac{z_{22}}{y_{22}}} = \sqrt{\frac{g_{22}}{h_{22}}}$$

特征阻抗



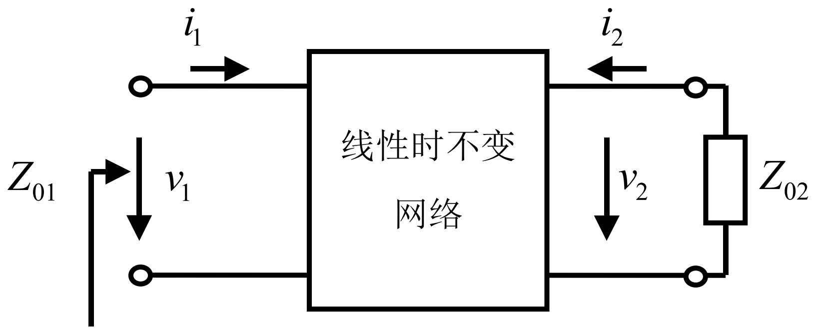
$$Z_{in} = Z_{01} \qquad\qquad Z_{out} = Z_{02}$$

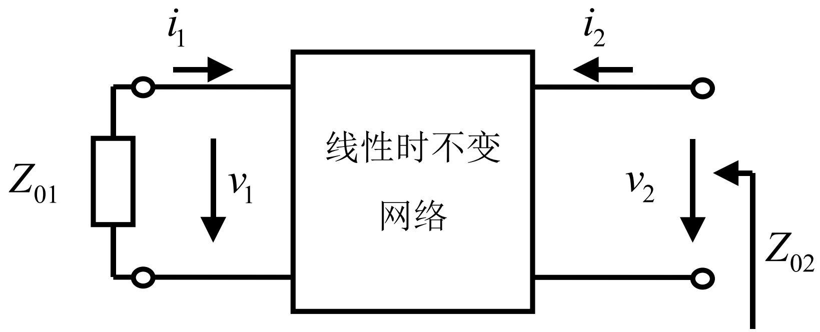
如果端接特征阻抗，则两个端口都匹配

$$Z_{01} = \sqrt{\frac{z_{11}}{y_{11}}} = \sqrt{\frac{h_{11}}{g_{11}}} = \sqrt{Z_{in,short} \cdot Z_{in,open}} \qquad Z_{02} = \sqrt{\frac{z_{22}}{y_{22}}} = \sqrt{\frac{g_{22}}{h_{22}}} = \sqrt{Z_{out,short} \cdot Z_{out,open}}$$

便于简单电路计算，但两个特征阻抗之间的内在关联看不清楚

用ABCD参量表述特征阻抗





$$Z_{01}=\frac{AZ_{02}+B}{CZ_{02}+D}$$

$$Z_{02}=\frac{DZ_{01}+B}{CZ_{01}+A}$$

$$Z_{01}=\sqrt{\frac{A}{D}}\cdot\sqrt{\frac{B}{C}}=\sqrt{\frac{A_{i0}}{A_{v0}}}\cdot\sqrt{\frac{R_{m0}}{G_{m0}}}$$

$$=nZ_0$$

$$Z_{02}=\sqrt{\frac{D}{A}}\cdot\sqrt{\frac{B}{C}}=\sqrt{\frac{A_{v0}}{A_{i0}}}\cdot\sqrt{\frac{R_{m0}}{G_{m0}}}$$

$$=\frac{1}{n}Z_0$$

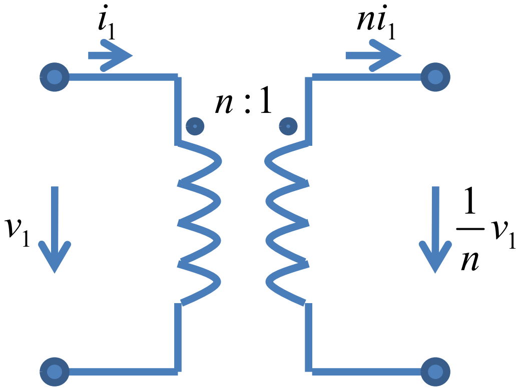
对称网络：$z_{11}=z_{22}, z_{12}=z_{21}$

：A=D，AD-BC=1

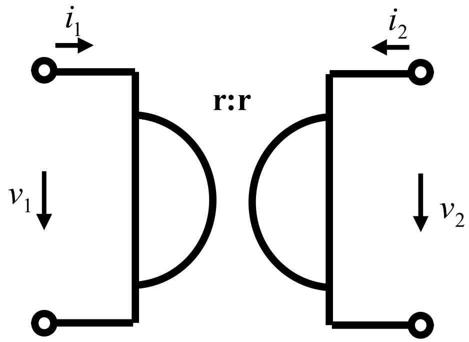
n^2：阻抗变换系数，$Z_{01}=n^2Z_{02}$：网络不对称性的体现

对称网络：A=D，n=1，$Z_{01}=Z_{02}=Z_0$

理想变压器和理想回旋器



$$\mathbf{ABCD}=\begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$



$$\mathbf{ABCD}=\begin{bmatrix} 0 & r \\ \frac{1}{r} & 0 \end{bmatrix}$$

$$Z_{01}=\sqrt{\frac{A}{D}}\cdot\sqrt{\frac{B}{C}}=nZ_0$$

$$Z_{02}=\sqrt{\frac{D}{A}}\cdot\sqrt{\frac{B}{C}}=\frac{1}{n}Z_0$$

阻抗变换比**n²**确定
Z_0任意取值

$$Z_{01}=n^2Z_{02}$$

$$Y_{01}=n^{-2}Y_{02}$$

任意阻抗可变换**n²**倍
阻抗属性不变：
电阻仍然是电阻，…

$$Z_{01}=\sqrt{\frac{A}{D}}\cdot\sqrt{\frac{B}{C}}=nr$$

$$Z_{02}=\sqrt{\frac{D}{A}}\cdot\sqrt{\frac{B}{C}}=\frac{1}{n}r$$

Z_0=r确定
阻抗变换比**n²**
任意取值

下周习题课
作业讨论

$$Z_{01}=\frac{r^2}{Z_{02}}=r^2Y_{02}$$

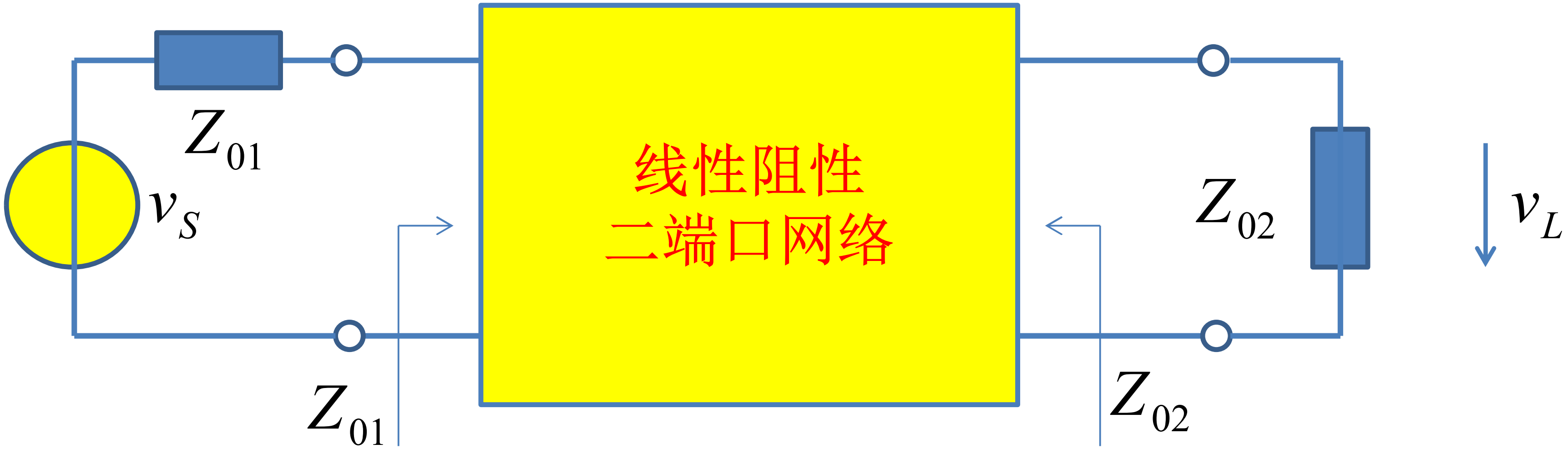
$$Y_{01}=r^{-2}Z_{02}$$

实现了对偶变换
阻抗属性对偶变换
电导变电阻，…

$$Z_{01}=\sqrt{\frac{A}{D}}\cdot\sqrt{\frac{B}{C}}=nZ_0$$

最大功率传输

$$Z_{02}=\sqrt{\frac{D}{A}}\cdot\sqrt{\frac{B}{C}}=\frac{1}{n}Z_0$$



两端同时最大功率传输匹配，故而必将获得最大功率增益

$$H=2\sqrt{\frac{R_S}{R_L}}\frac{v_L}{v_S}=2\sqrt{\frac{Z_{01}}{Z_{02}}}\frac{v_L}{v_S}=\frac{2}{A\sqrt{\frac{Z_{02}}{Z_{01}}}+B\frac{1}{\sqrt{Z_{02}Z_{01}}}+C\sqrt{Z_{02}Z_{01}}+D\sqrt{\frac{Z_{01}}{Z_{02}}}}=\frac{1}{\sqrt{AD}+\sqrt{BC}}$$

$$G_{p,\max}=|H|^2=\frac{1}{\left|\sqrt{AD}+\sqrt{BC}\right|^2}$$

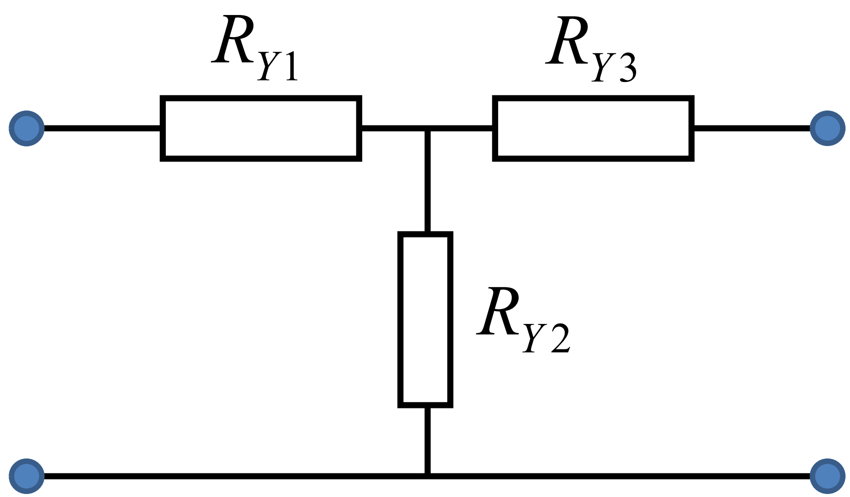
这个公式仅对阻性线性二端口网络成立

动态线性二端口网络公式相对复杂

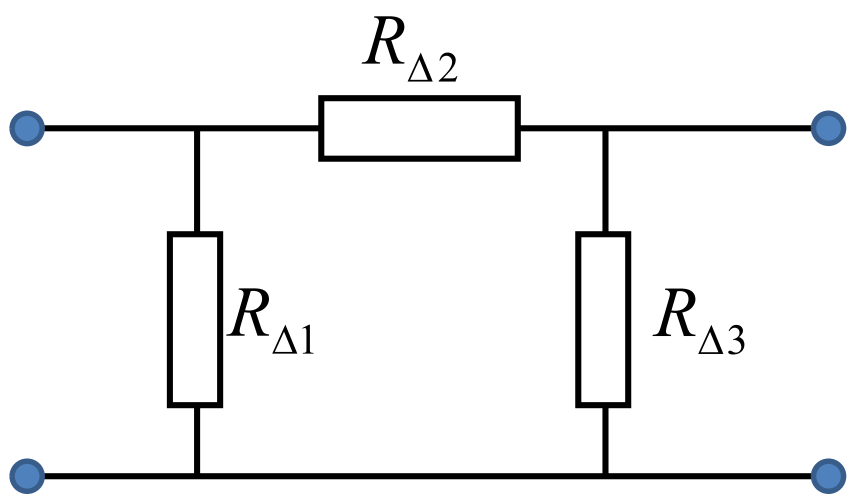
作业2：Y-Δ转换关系的推导

- 如果两个二端口网络具有相同的网络参量矩阵，这两个二端口网络则可认为是等效的
  - 如果图示**Y**形网络和Δ形网络等价，它们的电阻必须满足某种关系
    - 求**Y**形网络的**z**矩阵，求逆获得其**y**矩阵
    - 求Δ形网络的**y**矩阵
    - 两者相等，求出**Y-Δ**转换关系：$\mathbf{R}_\Delta$如何用$\mathbf{R}_\mathbf{Y}$表示？
    - 反之，$\mathbf{R}_\mathbf{Y}$如何用$\mathbf{R}_\Delta$表示？

ABCD参量是否更简单？



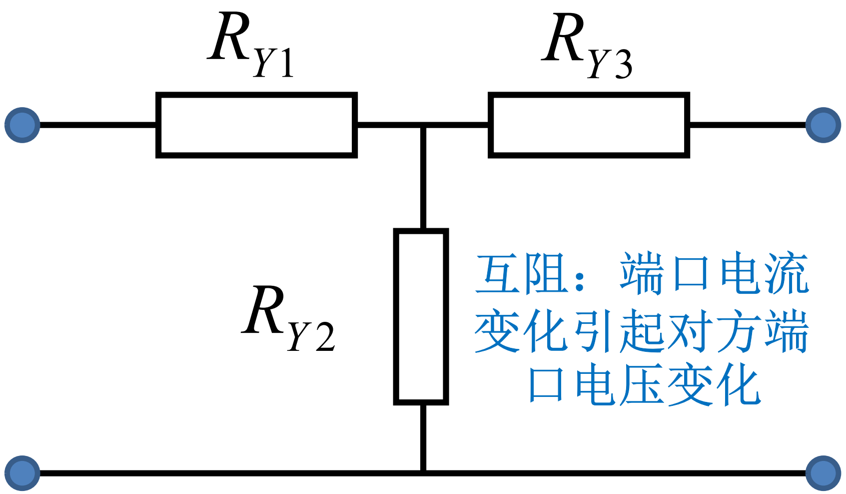
T形网络，**Y**形网络

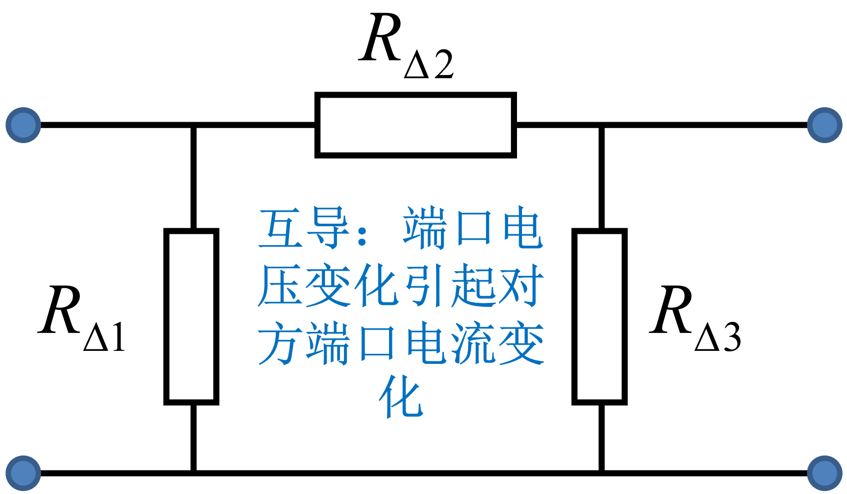


π形网络，Δ形网络

网络参量相同，两网络则为等效电路

网络参量就是等效电路模型，等效电路模型一致，网络则等价
从外端口看是等价的





$$\mathbf{z}=\begin{bmatrix} R_{Y1}+R_{Y2} & R_{Y2} \\ R_{Y2} & R_{Y3}+R_{Y2} \end{bmatrix}$$

互阻　　自阻

二端口电阻

$$\mathbf{y}=\begin{bmatrix} G_{\Delta 1}+G_{\Delta 2} & -G_{\Delta 2} \\ -G_{\Delta 2} & G_{\Delta 3}+G_{\Delta 2} \end{bmatrix}$$

互导　　自导

二端口电导

等价要求网络参量一致

$$\mathbf{z}=\begin{bmatrix} R_{Y1}+R_{Y2} & R_{Y2} \\ R_{Y2} & R_{Y3}+R_{Y2} \end{bmatrix}$$

$$\mathbf{y}=\begin{bmatrix} G_{\Delta1}+G_{\Delta2} & -G_{\Delta2} \\ -G_{\Delta2} & G_{\Delta3}+G_{\Delta2} \end{bmatrix}$$

$$\mathbf{y}=\mathbf{z}^{-1}=\frac{\begin{bmatrix} R_{Y3}+R_{Y2} & -R_{Y2} \\ -R_{Y2} & R_{Y1}+R_{Y2} \end{bmatrix}}{R_{Y1}R_{Y3}+R_{Y3}R_{Y2}+R_{Y2}R_{Y1}}$$

$$\mathbf{z}=\mathbf{y}^{-1}=\frac{\begin{bmatrix} G_{\Delta3}+G_{\Delta2} & G_{\Delta2} \\ G_{\Delta2} & G_{\Delta1}+G_{\Delta2} \end{bmatrix}}{G_{\Delta1}G_{\Delta3}+G_{\Delta3}G_{\Delta2}+G_{\Delta2}G_{\Delta1}}$$

$$G_{\Delta2}=\frac{R_{Y2}}{R_{Y1}R_{Y3}+R_{Y3}R_{Y2}+R_{Y2}R_{Y1}}$$

$$G_{\Delta1}=\frac{R_{Y3}}{R_{Y1}R_{Y3}+R_{Y3}R_{Y2}+R_{Y2}R_{Y1}}$$

$$G_{\Delta3}=\frac{R_{Y1}}{R_{Y1}R_{Y3}+R_{Y3}R_{Y2}+R_{Y2}R_{Y1}}$$

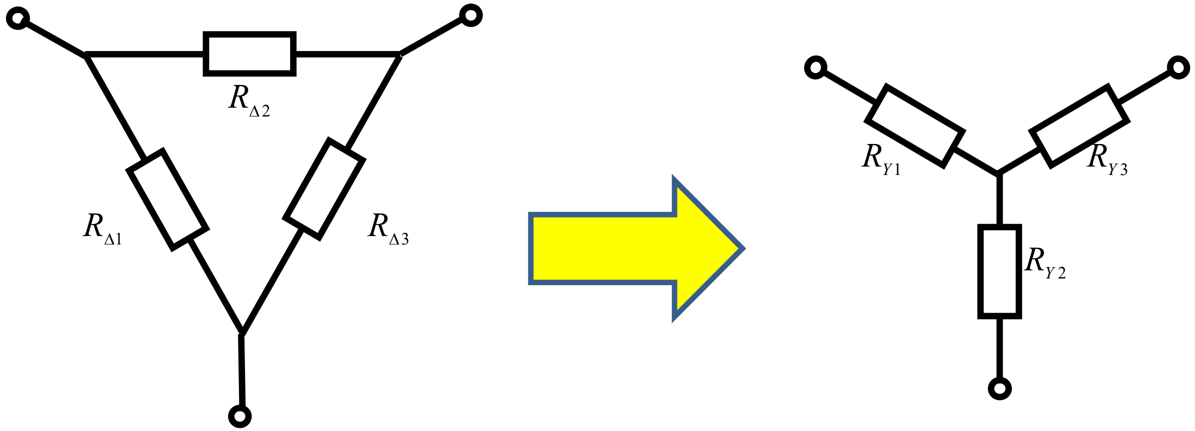
对偶电路，对偶表述

$$R_{Y2}=\frac{G_{\Delta2}}{G_{\Delta1}G_{\Delta3}+G_{\Delta3}G_{\Delta2}+G_{\Delta2}G_{\Delta1}}$$

$$R_{Y1}=\frac{G_{\Delta3}}{G_{\Delta1}G_{\Delta3}+G_{\Delta3}G_{\Delta2}+G_{\Delta2}G_{\Delta1}}$$

$$R_{Y3}=\frac{G_{\Delta1}}{G_{\Delta1}G_{\Delta3}+G_{\Delta3}G_{\Delta2}+G_{\Delta2}G_{\Delta1}}$$

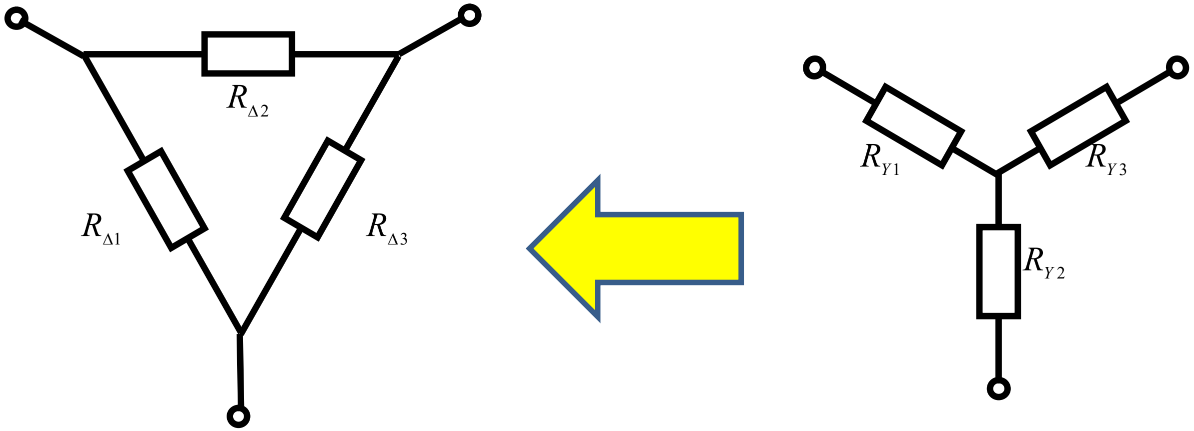
Δ Y 转换



$$R_{Y2} = \frac{G_{\Delta 2}}{G_{\Delta 1}G_{\Delta 3} + G_{\Delta 3}G_{\Delta 2} + G_{\Delta 2}G_{\Delta 1}} = \frac{R_{\Delta 1}R_{\Delta 3}}{R_{\Delta 1} + R_{\Delta 2} + R_{\Delta 3}}$$

$$R_{Y1} = \frac{G_{\Delta 3}}{G_{\Delta 1}G_{\Delta 3} + G_{\Delta 3}G_{\Delta 2} + G_{\Delta 2}G_{\Delta 1}} = \frac{R_{\Delta 1}R_{\Delta 2}}{R_{\Delta 1} + R_{\Delta 2} + R_{\Delta 3}}$$

$$R_{Y3} = \frac{G_{\Delta 1}}{G_{\Delta 1}G_{\Delta 3} + G_{\Delta 3}G_{\Delta 2} + G_{\Delta 2}G_{\Delta 1}} = \frac{R_{\Delta 3}R_{\Delta 2}}{R_{\Delta 1} + R_{\Delta 2} + R_{\Delta 3}}$$



Y Δ 转换

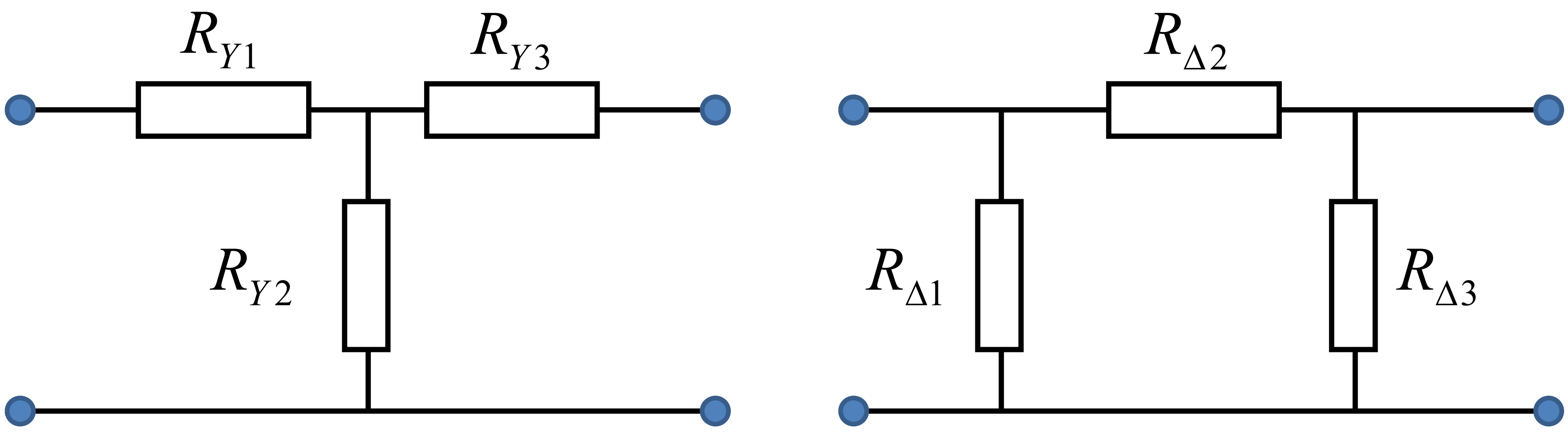
$$R_{\Delta 2} = G_{\Delta 2}^{-1} = \frac{R_{Y1}R_{Y3} + R_{Y3}R_{Y2} + R_{Y2}R_{Y1}}{R_{Y2}} = R_{Y3} + R_{Y1} + \frac{R_{Y1}R_{Y3}}{R_{Y2}}$$

$$R_{\Delta 1} = G_{\Delta 1}^{-1} = \frac{R_{Y1}R_{Y3} + R_{Y3}R_{Y2} + R_{Y2}R_{Y1}}{R_{Y3}} = R_{Y1} + R_{Y2} + \frac{R_{Y1}R_{Y2}}{R_{Y3}}$$

$$R_{\Delta 3} = G_{\Delta 3}^{-1} = \frac{R_{Y1}R_{Y3} + R_{Y3}R_{Y2} + R_{Y2}R_{Y1}}{R_{Y1}} = R_{Y2} + R_{Y3} + \frac{R_{Y2}R_{Y3}}{R_{Y1}}$$

公式无需特别记忆，等价原理清楚后，随时随手可以推导出来

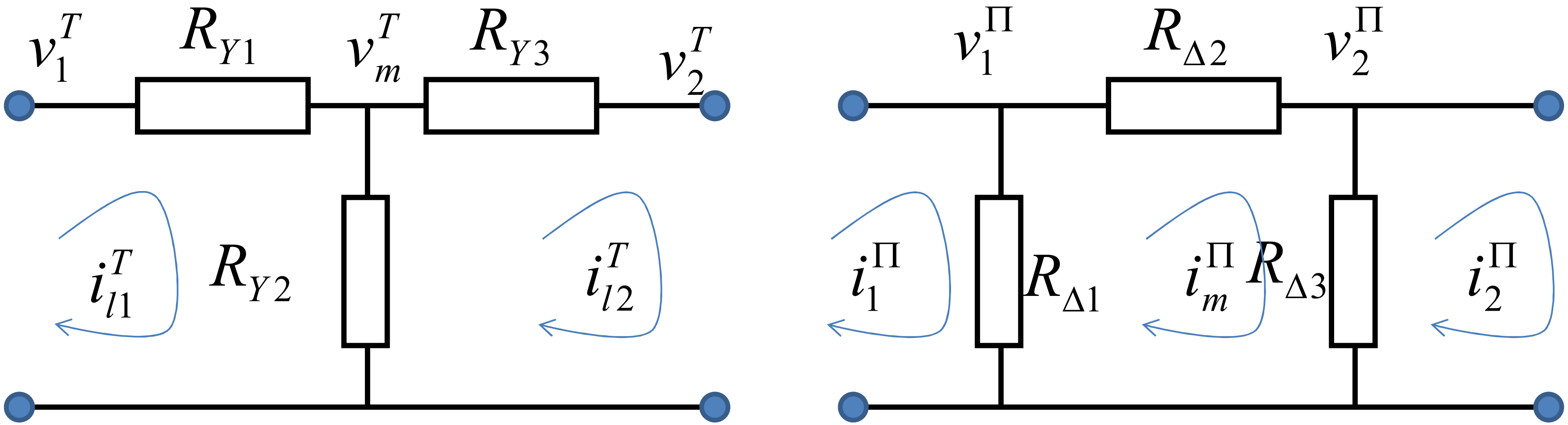
zy为何不对偶？



$$\mathbf{z} = \begin{bmatrix} R_{Y1} + R_{Y2} & R_{Y2} \\ R_{Y2} & R_{Y3} + R_{Y2} \end{bmatrix}$$

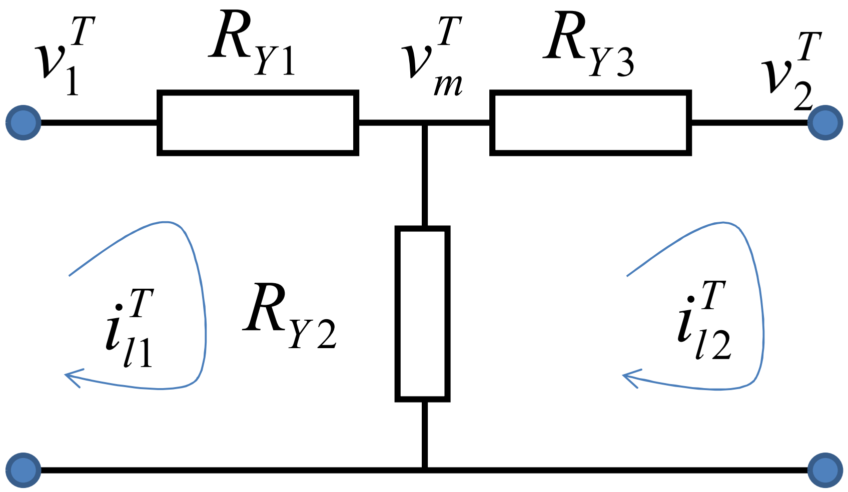
$$\mathbf{y} = \begin{bmatrix} G_{\Delta 1} + G_{\Delta 2} & -G_{\Delta 2} \\ -G_{\Delta 2} & G_{\Delta 3} + G_{\Delta 2} \end{bmatrix}$$

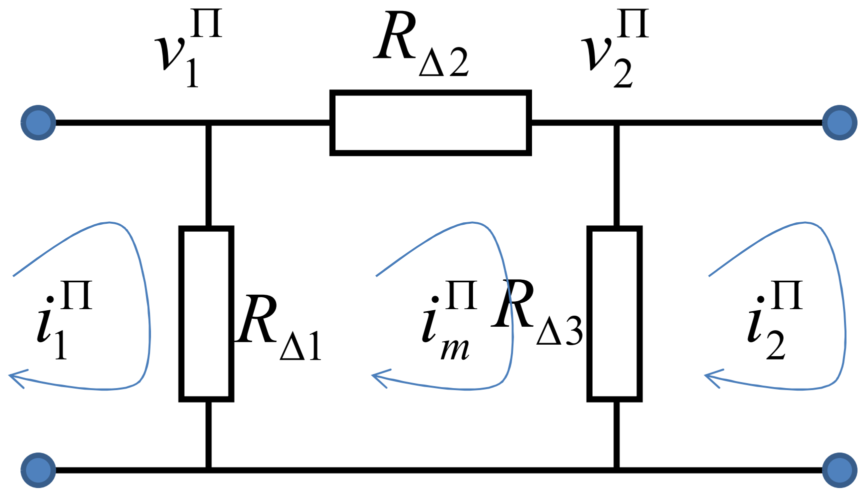
对偶关系



v_1^T v_1 v_m^T v_2^T v_2 i_{l1}^T i_1 i_{l2}^T $-i_2$

i_1^Π i_1 i_m^Π i_2^Π $-i_2$ v_1^Π v_1 v_2^Π v_2





$$\mathbf{z} = \begin{bmatrix} R_{Y1}+R_{Y2} & R_{Y2} \\ R_{Y2} & R_{Y3}+R_{Y2} \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} G_{\Delta1}+G_{\Delta2} & -G_{\Delta2} \\ -G_{\Delta2} & G_{\Delta3}+G_{\Delta2} \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} R_{Y1}+R_{Y2} & R_{Y2} \\ R_{Y2} & R_{Y3}+R_{Y2} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

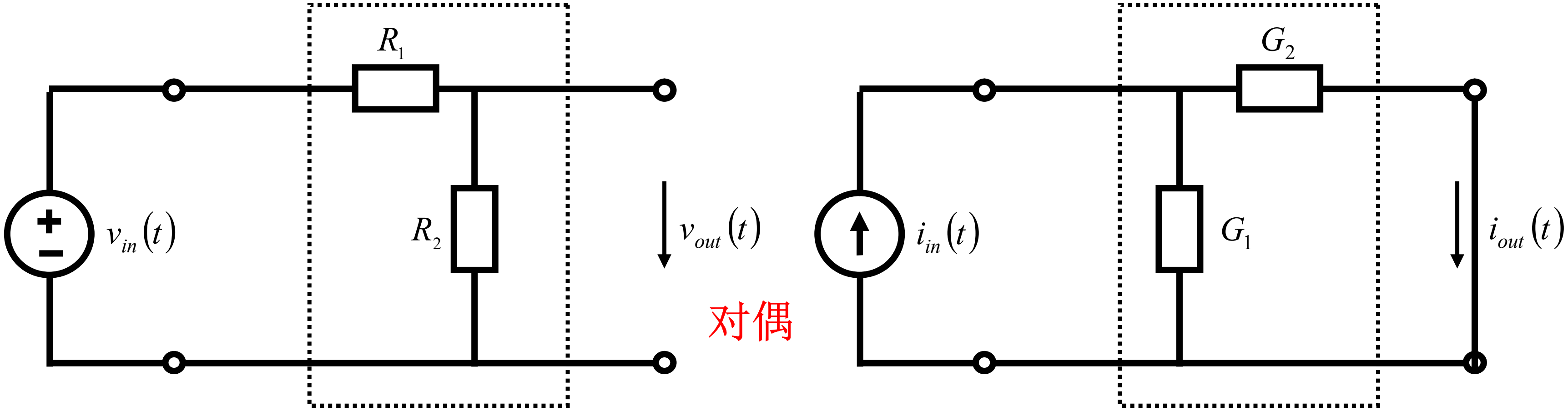
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} G_{\Delta1}+G_{\Delta2} & -G_{\Delta2} \\ -G_{\Delta2} & G_{\Delta3}+G_{\Delta2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} R_{Y1}+R_{Y2} & -R_{Y2} \\ R_{Y2} & -R_{Y3}-R_{Y2} \end{bmatrix} \begin{bmatrix} i_1 \\ -i_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ -i_2 \end{bmatrix} = \begin{bmatrix} G_{\Delta1}+G_{\Delta2} & -G_{\Delta2} \\ G_{\Delta2} & -G_{\Delta3}-G_{\Delta2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

对偶量置换，才能确保对偶表达式

分压 对偶 分流



$$v_{out} = \frac{R_2}{R_1 + R_2} v_{in}$$

$$i_{out} = \frac{G_2}{G_1 + G_2} i_{in}$$

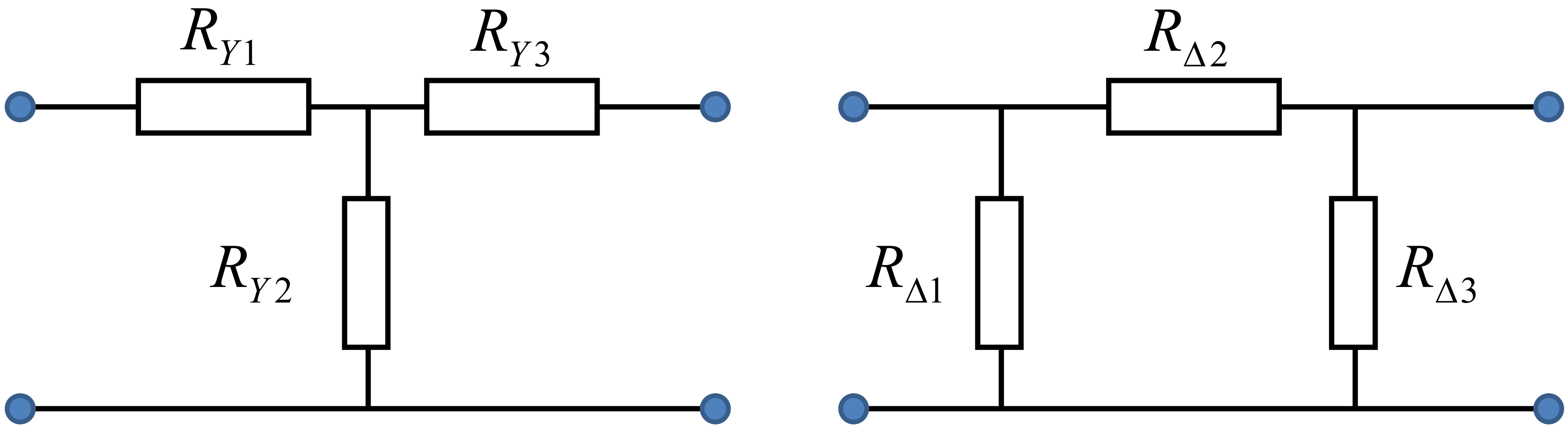
i_{out} 对偶 v_{out}

$-i_2$ v_2

i_{in} 对偶 v_{in}

i_1 v_1

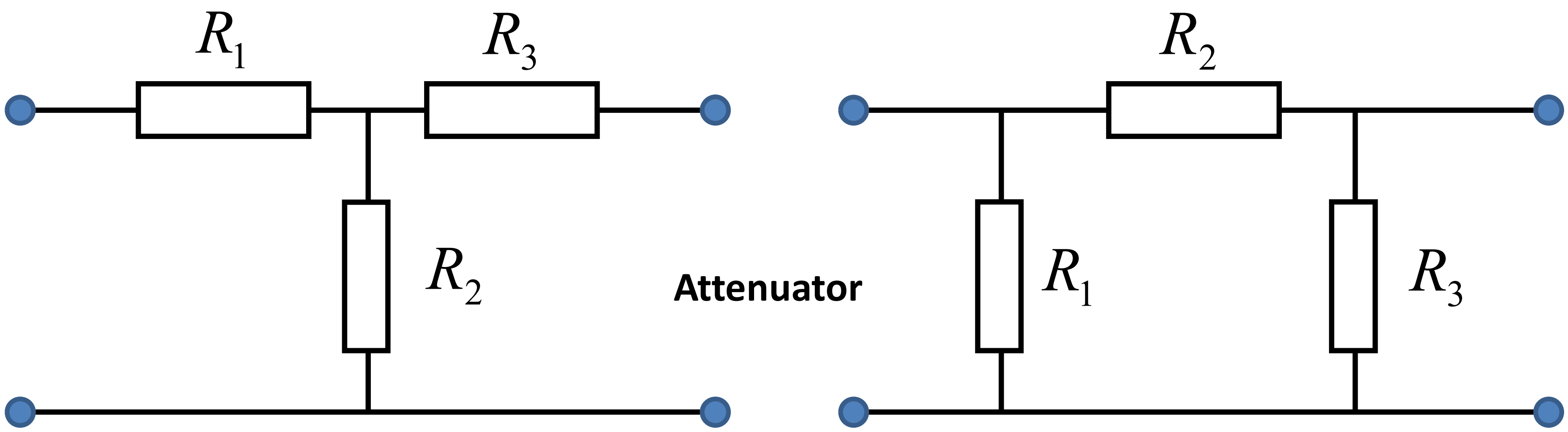
ABCD是否更合适？



$$ABCD_T = \begin{bmatrix} 1 & R_{Y1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{1}{R_{Y2}} & 1 \end{bmatrix} \begin{bmatrix} 1 & R_{Y3} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{R_{Y1}}{R_{Y2}} & R_{Y1} + R_{Y3} + \dfrac{R_{Y1}R_{Y3}}{R_{Y2}} \\ \dfrac{1}{R_{Y2}} & 1 + \dfrac{R_{Y3}}{R_{Y2}} \end{bmatrix}$$

$$ABCD_{\Pi} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{R_{\Delta 1}} & 1 \end{bmatrix} \begin{bmatrix} 1 & R_{\Delta 2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{1}{R_{\Delta 3}} & 1 \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{R_{\Delta 2}}{R_{\Delta 3}} & R_{\Delta 2} \\ \dfrac{1}{R_{\Delta 1}} + \dfrac{1}{R_{\Delta 3}} + \dfrac{R_{\Delta 2}}{R_{\Delta 1}R_{\Delta 3}} & 1 + \dfrac{R_{\Delta 2}}{R_{\Delta 1}} \end{bmatrix}$$

第7次作业　作业1 匹配衰减器



根据对偶性给出T性电阻衰减器的设计公式并根据公式设计一个50Ω系统到75Ω系统转换的20dB匹配衰减器，并给出该T型电阻衰减器的z参量和s参量矩阵

对偶：
串联/并联、回路/结点
电阻/电导
特征阻抗/特征导纳
T/π

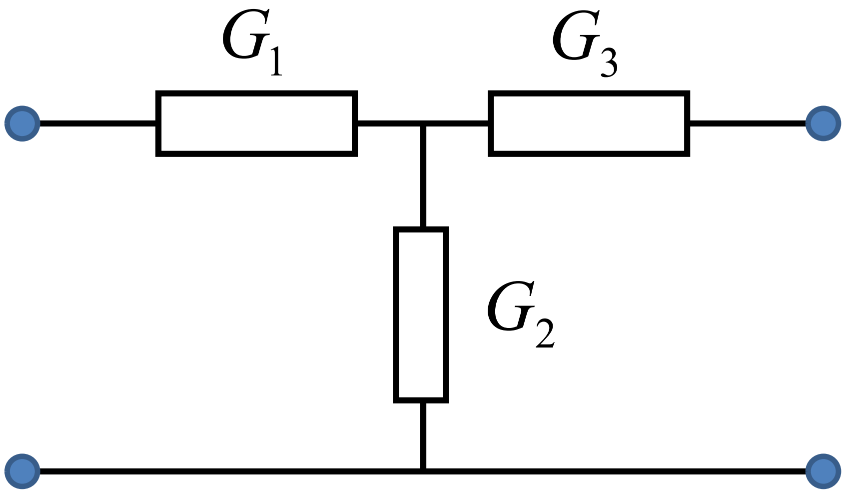
$$R_2 = 0.5\left(\beta - \beta^{-1}\right)\sqrt{Z_{01}Z_{02}}$$

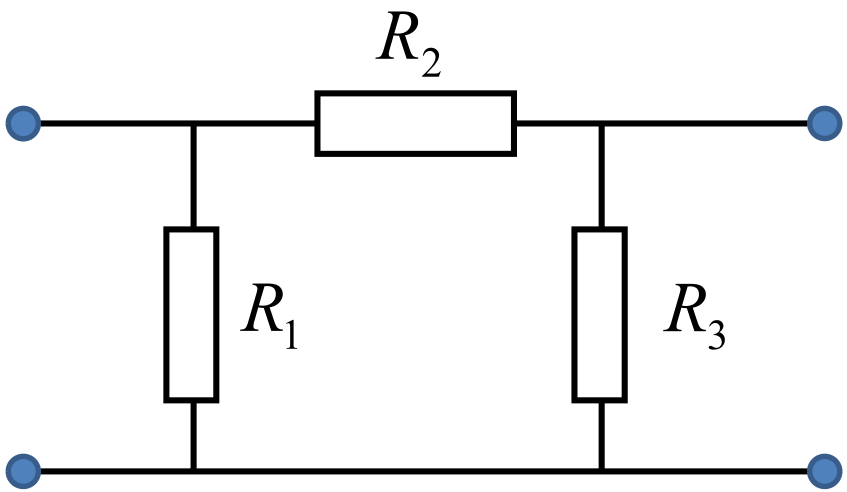
$$R_1 = \frac{1}{\frac{1}{Z_{01}}\frac{\beta+\beta^{-1}}{\beta-\beta^{-1}} - \frac{1}{R_2}}$$

$$R_3 = \frac{1}{\frac{1}{Z_{02}}\frac{\beta+\beta^{-1}}{\beta-\beta^{-1}} - \frac{1}{R_2}}$$

$$\beta = 10^{\frac{L}{20}}$$

根据对偶性列写设计公式





$$G_2 = 0.5\left(\beta - \beta^{-1}\right)\sqrt{Y_{01}Y_{02}}$$

$$R_2 = 0.5\left(\beta - \beta^{-1}\right)\sqrt{Z_{01}Z_{02}}$$

$$G_1 = \frac{1}{\frac{1}{Y_{01}}\frac{\beta+\beta^{-1}}{\beta-\beta^{-1}} - \frac{1}{G_2}}$$

$$R_1 = \frac{1}{\frac{1}{Z_{01}}\frac{\beta+\beta^{-1}}{\beta-\beta^{-1}} - \frac{1}{R_2}}$$

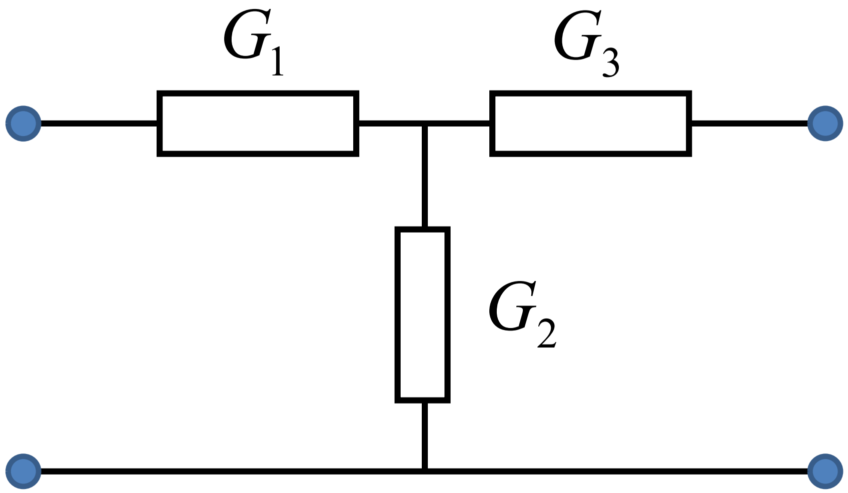
$$G_3 = \frac{1}{\frac{1}{Y_{02}}\frac{\beta+\beta^{-1}}{\beta-\beta^{-1}} - \frac{1}{G_2}}$$

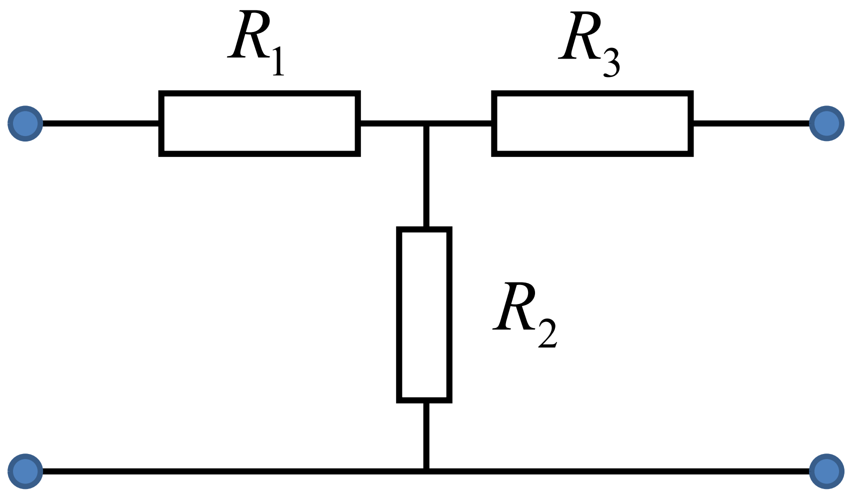
$$\beta = 10^{\frac{L}{20}}$$

$$R_3 = \frac{1}{\frac{1}{Z_{02}}\frac{\beta+\beta^{-1}}{\beta-\beta^{-1}} - \frac{1}{R_2}}$$

$$\beta = 10^{\frac{L}{20}}$$

转换为阻抗表述形式





$$G_2 = 0.5\left(\beta - \beta^{-1}\right)\sqrt{Y_{01}Y_{02}}$$

$$R_2 = \frac{1}{0.5\left(\beta - \beta^{-1}\right)\sqrt{Y_{01}Y_{02}}} = \frac{2\sqrt{Z_{01}Z_{02}}}{\beta - \beta^{-1}}$$

$$G_1 = \frac{1}{\frac{1}{Y_{01}}\frac{\beta + \beta^{-1}}{\beta - \beta^{-1}} - \frac{1}{G_2}}$$

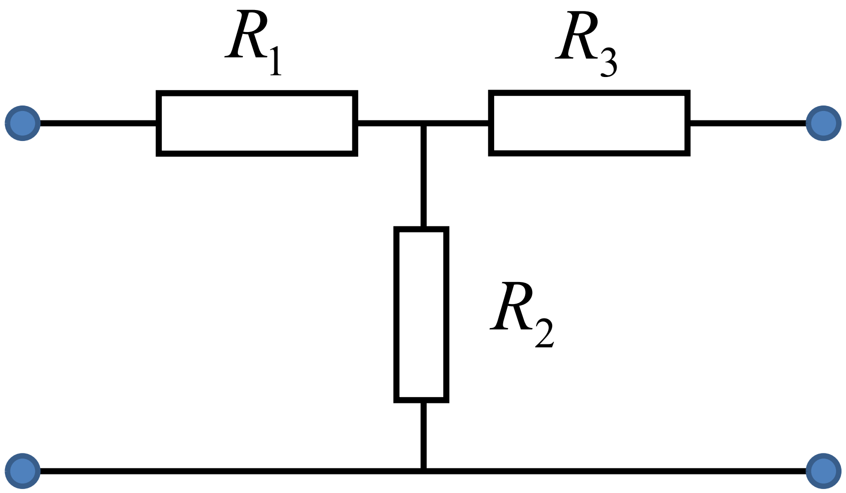
$$R_1 = Z_{01}\frac{\beta + \beta^{-1}}{\beta - \beta^{-1}} - R_2$$

$$G_3 = \frac{1}{\frac{1}{Y_{02}}\frac{\beta + \beta^{-1}}{\beta - \beta^{-1}} - \frac{1}{G_2}}$$

$$\beta = 10^{\frac{L}{20}}$$

$$R_3 = Z_{02}\frac{\beta + \beta^{-1}}{\beta - \beta^{-1}} - R_2$$

具体数值设计结果



$$R_2 = \frac{2\sqrt{Z_{01}Z_{02}}}{\beta-\beta^{-1}}$$

$$R_1 = Z_{01}\frac{\beta+\beta^{-1}}{\beta-\beta^{-1}} - R_2$$

$$\beta = 10^{\frac{L}{20}}$$

$$R_3 = Z_{02}\frac{\beta+\beta^{-1}}{\beta-\beta^{-1}} - R_2$$

$$L = 20dB$$

$$Z_{01} = 50\Omega \qquad Z_{02} = 75\Omega$$

$$\beta = 10^{\frac{L}{20}} = 10^1 = 10$$

$$R_2 = \frac{2\sqrt{Z_{01}Z_{02}}}{\beta-\beta^{-1}} = \frac{2\times\sqrt{50\times 75}}{10-0.1} = 12.4\Omega$$

$$R_1 = Z_{01}\frac{\beta+\beta^{-1}}{\beta-\beta^{-1}} - R_2$$

$$= 50\times\frac{10+0.1}{10-0.1} - 12.4 = 38.6\Omega$$

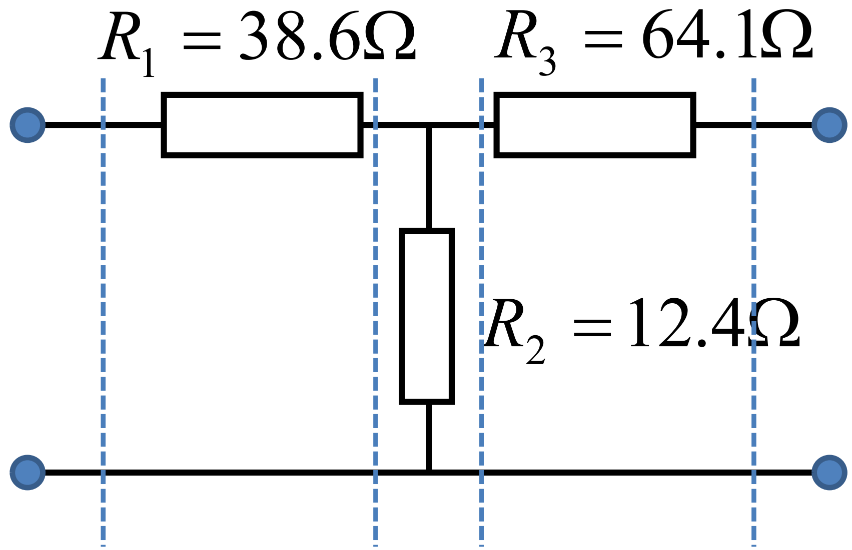
$$R_3 = Z_{02}\frac{\beta+\beta^{-1}}{\beta-\beta^{-1}} - R_2$$

$$= 75\times\frac{10+0.1}{10-0.1} - 12.4 = 64.1\Omega$$

$$\mathbf{ABCD}=\begin{bmatrix}1 & R_1\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 0\\G_2 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & R_3\\0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 38.6\Omega\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 0\\80.8mS & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 64.1\Omega\\0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}4.12 & 302.6\Omega\\80.8mS & 6.18\end{bmatrix}$$



验证方法1
ABCD参量

$$Z_{01}=\sqrt{\frac{A}{D}\cdot\frac{B}{C}}=\sqrt{\frac{4.12}{6.18}\frac{302.6}{0.0808}}=49.96\approx 50\Omega$$

$$Z_{02}=\sqrt{\frac{D}{A}\cdot\frac{B}{C}}=\sqrt{\frac{6.18}{4.12}\frac{302.6}{0.0808}}=74.96\approx 75\Omega$$

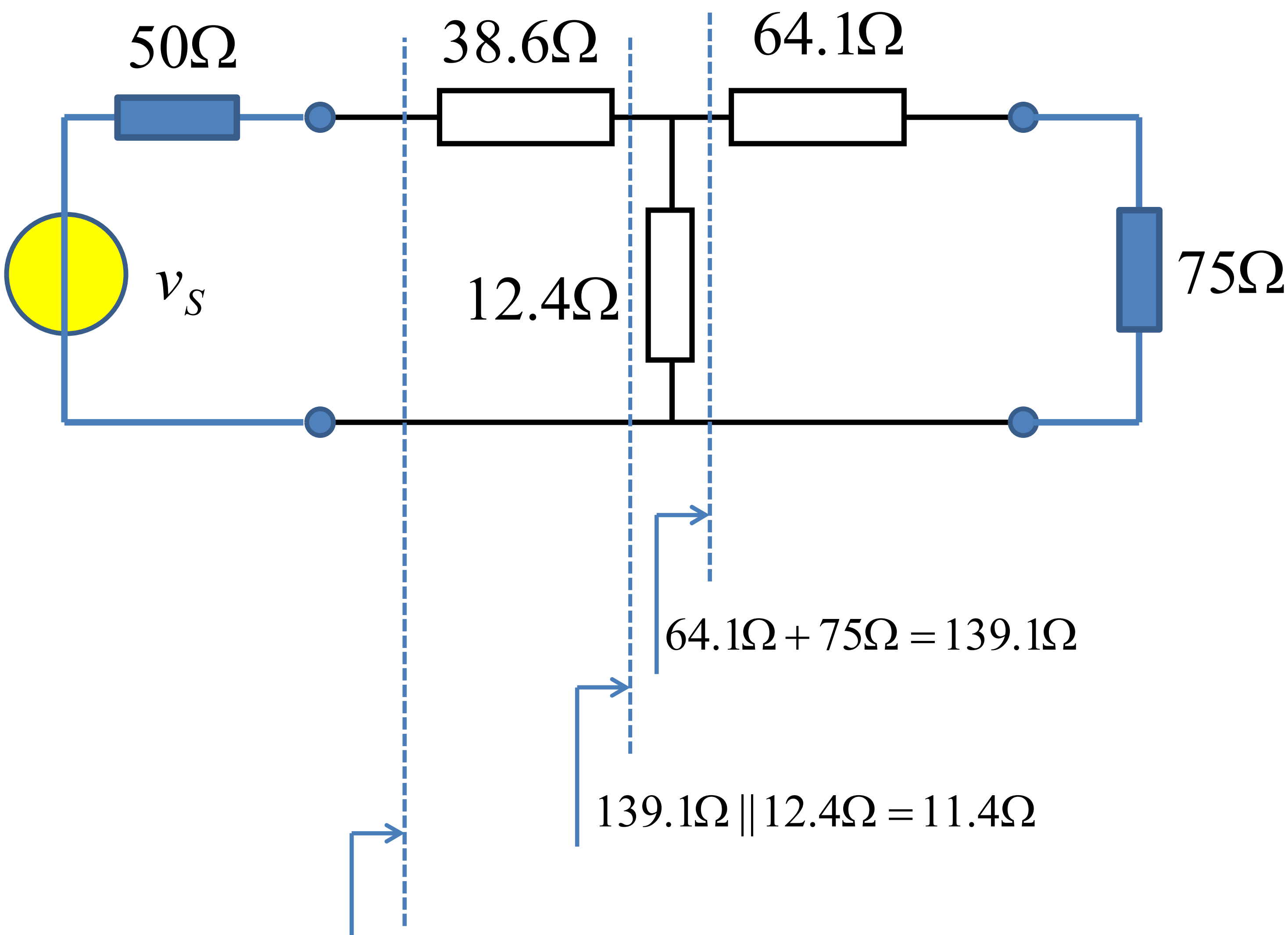
$$H=\frac{2}{A\sqrt{\frac{R_L}{R_S}}+B\frac{1}{\sqrt{R_SR_L}}+C\sqrt{R_SR_L}+D\sqrt{\frac{R_S}{R_L}}}=\frac{2}{A\sqrt{\frac{Z_{02}}{Z_{01}}}+B\frac{1}{\sqrt{Z_{01}Z_{02}}}+C\sqrt{Z_{01}Z_{02}}+D\sqrt{\frac{Z_{01}}{Z_{02}}}}$$

$$=\frac{2}{4.12\cdot\sqrt{\frac{75}{50}}+64.1\cdot\frac{1}{\sqrt{50\cdot 75}}+0.0808\cdot\sqrt{50\cdot 75}+6.18\cdot\sqrt{\frac{50}{75}}}=0.1001=-20dB$$

$$H=\frac{1}{\sqrt{AD}+\sqrt{BC}}=\frac{1}{\sqrt{4.12\times 6.18}+\sqrt{302.6\times 0.0808}}=\frac{1}{5.05+4.94}=\frac{1}{9.99}=0.1001=-20dB$$

验证方法2

简单串并联



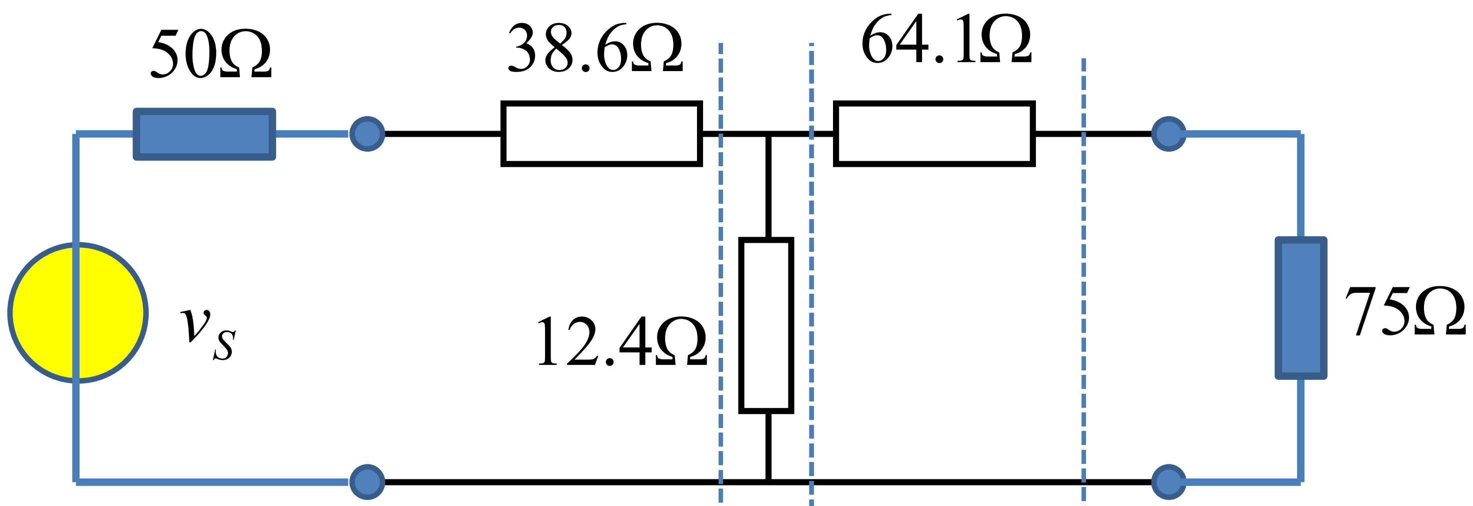
$$64.1\Omega + 75\Omega = 139.1\Omega$$

$$139.1\Omega \,||\, 12.4\Omega = 11.4\Omega$$

$$38.6\Omega + 11.4\Omega = 50\Omega$$

验证方法2

戴维南等效



$$P_{S,\max} = \frac{V_{S,rms}^2}{4R_S} = \frac{V_{S,rms}^2}{4\times 50} = 0.005V_{S,rms}^2$$

$$R_{TH} = 50 + 38.6 = 88.6\Omega$$

$$v_{TH} = v_S$$

$$R_{TH} = 88.6 \,||\, 12.4 = 10.9\Omega$$

$$v_{TH} = \frac{12.4}{88.6+12.4} v_S = 0.123 v_S$$

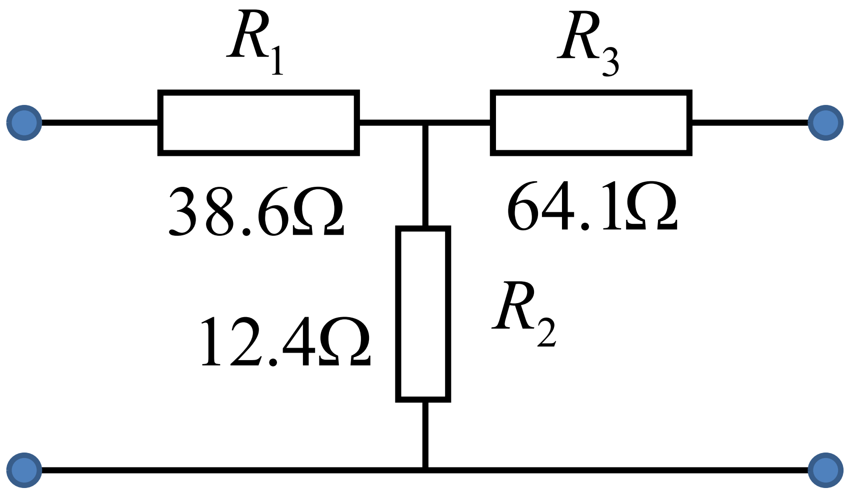
$$R_{TH} = 10.9 + 64.1 = 75\Omega$$

$$v_{TH} = 0.123 v_S$$

$$v_L = 0.5 \times 0.123 v_S = 0.061 v_S$$

$$P_L = \frac{V_{L,rms}^2}{R_L} = \frac{\left(0.061 V_{S,rms}\right)^2}{75} = 0.00005 V_{S,rms}^2 = 0.01 P_{S,\max}$$

$$L = 20dB$$



z参量
s参量

$$\boldsymbol{z}=\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}=\begin{bmatrix} R_1+R_2 & R_2 \\ R_2 & R_3+R_2 \end{bmatrix}=\begin{bmatrix} 51\Omega & 12.4\Omega \\ 12.4\Omega & 76.5\Omega \end{bmatrix}$$

$$\boldsymbol{s}_{\boldsymbol{R_S=Z_{01}=50\Omega,R_L=Z_{02}=75\Omega}}=\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}=\begin{bmatrix} \frac{z_{in}-R_S}{z_{in}+R_S} & 2\sqrt{\frac{R_L}{R_S}}\frac{v_{L1}}{v_{S2}} \\ 2\sqrt{\frac{R_S}{R_L}}\frac{v_{L2}}{v_{S1}} & \frac{z_{out}-R_L}{z_{out}+R_L} \end{bmatrix}_{\boldsymbol{R_S=Z_{01},R_L=Z_{02}}}=$$

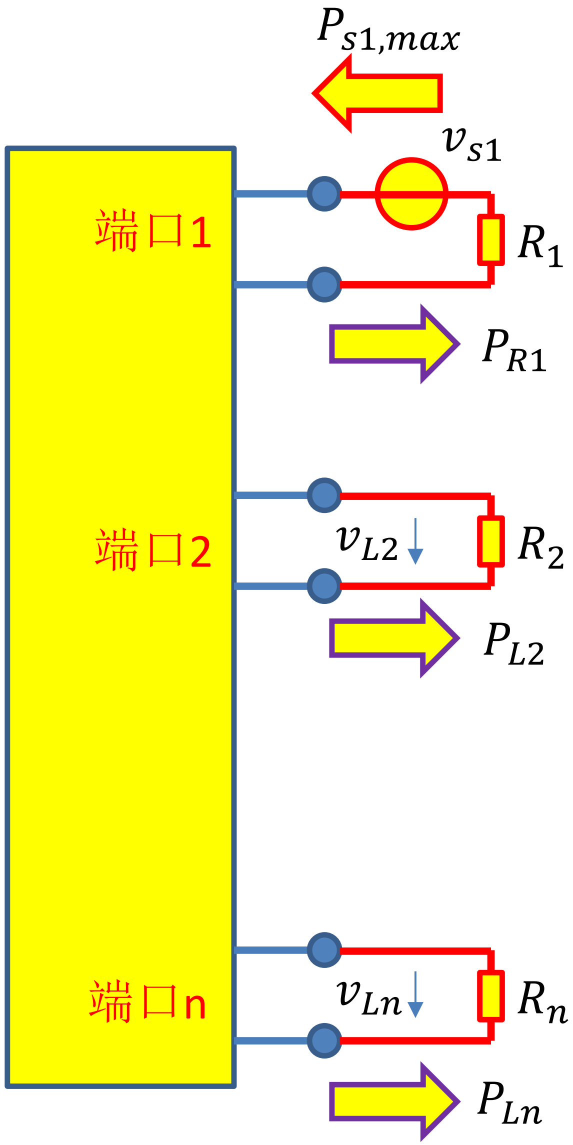
$$\begin{bmatrix} 0 & s_{12,max} \\ s_{21,max} & 0 \end{bmatrix}_{\boldsymbol{R_S=Z_{01},R_L=Z_{02}}}=\begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}_{\boldsymbol{R_S=Z_{01},R_L=Z_{02}}}$$

S参量测量

各个端口外接负载电阻不改变
只改变激励源的位置

$$\boldsymbol{s}=\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{bmatrix}$$

端口1加源测试 | 端口2加源测试 | 端口n加源测试



$$s_{11}=\frac{z_{in1}-R_1}{z_{in1}+R_1}$$

反射系数

$$|s_{11}|^2=\frac{P_{R1}}{P_{s1,max}}$$

$$s_{21}=2\sqrt{\frac{R_1}{R_2}}\frac{v_{L2}}{v_{s1}}$$

传输系数

$$|s_{21}|^2=\frac{P_{L2}}{P_{s1,max}}$$

$$P_{s1,max}=\frac{v_{s1,rms}^2}{4R_1}$$

$$P_{L2}=\frac{v_{L2,rms}^2}{R_2}$$

$$P_{Ln}=\frac{v_{Ln,rms}^2}{R_n}$$

$$s_{n1}=2\sqrt{\frac{R_1}{R_n}}\frac{v_{Ln}}{v_{s1}}$$

传输系数

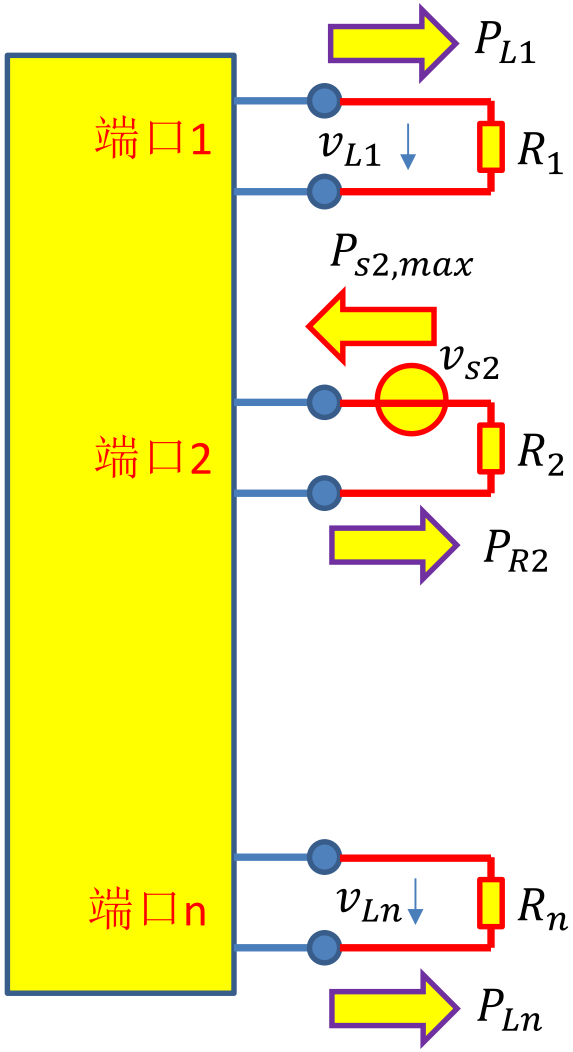
$$|s_{n1}|^2=\frac{P_{Ln}}{P_{s1,max}}$$

S参量测量

对角元为反射系数
非对角元为传输系数

$$\mathbf{s} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{bmatrix}$$

端口1加源测试　端口2加源测试　端口n加源测试



$$s_{12} = 2\sqrt{\frac{R_2}{R_1}}\frac{v_{L1}}{v_{s2}}$$

$$|s_{12}|^2 = \frac{P_{L1}}{P_{s2,max}}$$

$$s_{22} = \frac{z_{in2} - R_2}{z_{in} + R_2}$$

$$|s_{22}|^2 = \frac{P_{R2}}{P_{s2,max}}$$

$$P_{s2,max} = \frac{v_{s2,rms}^2}{4R_2}$$

$$P_{L1} = \frac{v_{L1,rms}^2}{R_1}$$

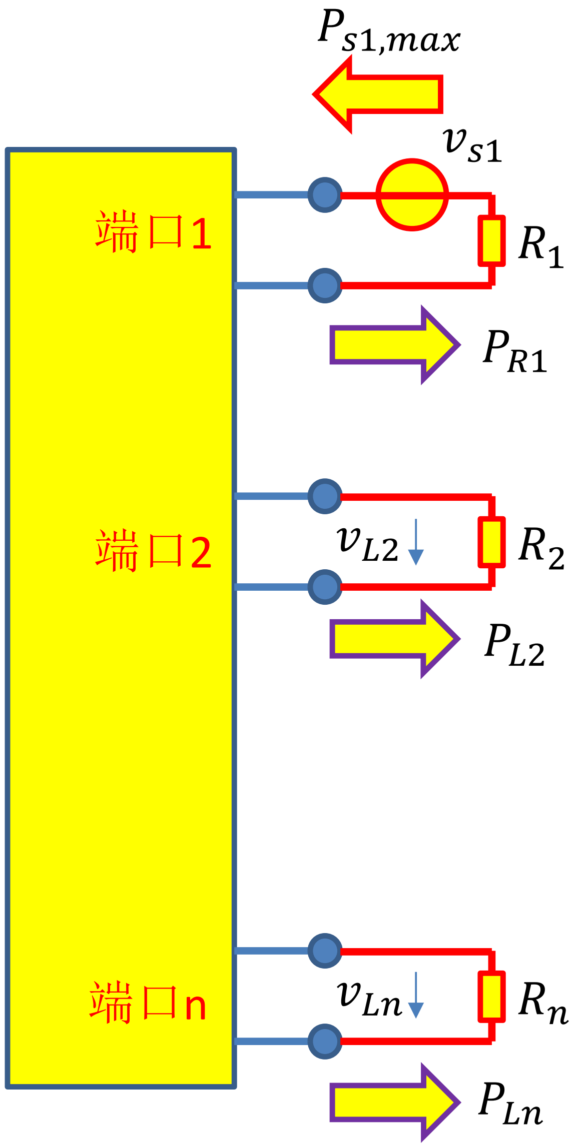
$$P_{Ln} = \frac{v_{Ln,rms}^2}{R_n}$$

$$s_{n2} = 2\sqrt{\frac{R_2}{R_n}}\frac{v_{Ln}}{v_{s2}}$$

$$|s_{n2}|^2 = \frac{P_{Ln}}{P_{s2,max}}$$

无损网络

$$\boldsymbol{s}=\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{bmatrix}$$



$$|s_{11}|^2=\frac{P_{R1}}{P_{s1,max}}$$

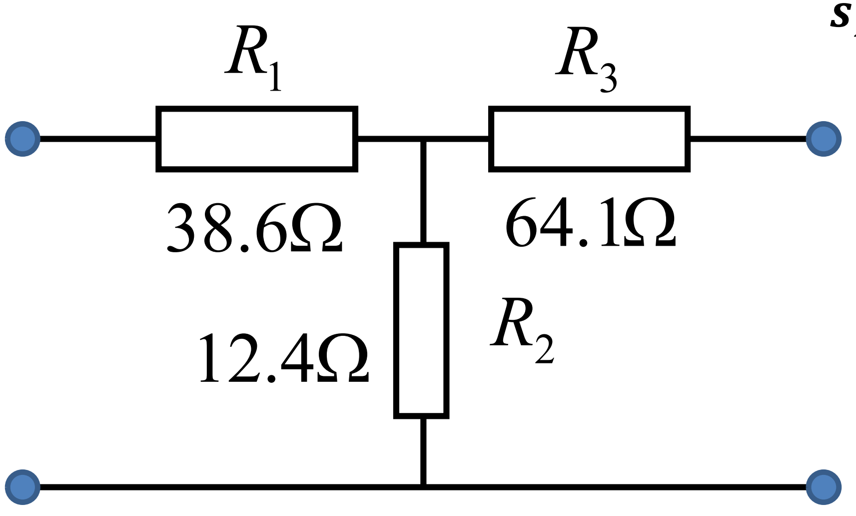
$$P_{s1,max}=P_{R1}+P_{L2}+\cdots+P_{Ln}$$

$$|s_{21}|^2=\frac{P_{L2}}{P_{s1,max}}$$

$$1=\frac{P_{R1}}{P_{s1,max}}+\frac{P_{L2}}{P_{s1,max}}+\cdots+\frac{P_{Ln}}{P_{s1,max}}$$

$$|s_{n1}|^2=\frac{P_{Ln}}{P_{s1,max}}$$

$$|s_{11}|^2+|s_{21}|^2+\cdots+|s_{n1}|^2=1$$



$$\mathbf{s}_{R_S=Z_{01}=50\Omega,R_L=Z_{02}=75\Omega} = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}_{R_S=Z_{01},R_L=Z_{02}}$$

S参量计算

$$s_{11} = \frac{z_{in1} - R_1}{z_{in1} + R_1} \qquad s_{12} = 2\sqrt{\frac{R_2}{R_1}}\frac{v_{L1}}{v_{s2}}$$

$$s_{21} = 2\sqrt{\frac{R_1}{R_2}}\frac{v_{L2}}{v_{s1}} \qquad s_{22} = \frac{z_{in2} - R_2}{z_{in2} + R_2}$$

$$\mathbf{s}_{R_S=50\Omega,R_L=50\Omega} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} = \begin{bmatrix} -0.002 & 0.098 \\ 0.098 & 0.2 \end{bmatrix}$$

$$z_{in1}(R_L = 50\Omega) = (50 + 64.1)||12.4 + 38.6 = 49.8\Omega$$

$$s_{11} = \frac{49.8 - 50}{49.8 + 50} = -0.002$$

$$\frac{v_{L2}}{v_{s1}} = \frac{49.8}{99.8}\frac{11.2}{49.8}\frac{50}{114.1} = 0.0492$$

$$s_{21} = 2 \times \sqrt{\frac{50}{50}} \times 0.0492 = 0.098$$

不匹配
互易
有损

$$z_{in}\ (R_S = 50\Omega) = (50 + 38.6)||12.4 + 64.1 = 75.0\Omega$$

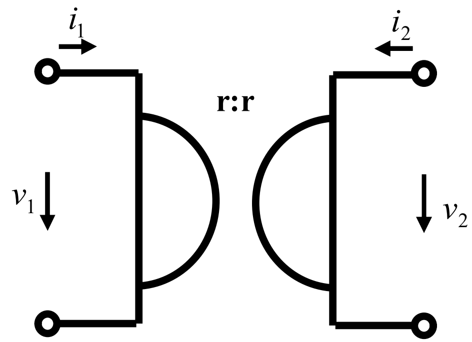
$$s_{22} = \frac{75 - 50}{75 + 50} = 0.2$$

$$\frac{v_{L1}}{v_{s2}} = \frac{75}{125}\frac{10.9}{75}\frac{50}{88.6} = 0.0492$$

$$s_{12} = 2 \times \sqrt{\frac{50}{50}} \times 0.0492 = 0.098$$

作业5 理想回旋器

Gyrator



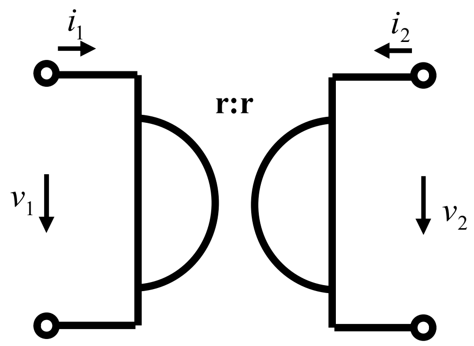
回旋器电路符号

理想回旋器是一种二端口网络，其端口描述方程为

$$v_1 = -ri_2 \qquad v_2 = ri_1$$

（**1**）假设我们可以实现理想受控源，如何实现回旋器
（**2**）给出回旋器的**6**个网络参量及等效电路（如果存在）
（**2**）证明：回旋器可实现对偶变换**---**它可以将电容**C**转换为电感**L**，将电感**L**转换为电容**C**，将并联**RLC**转换为串联**GCL**，将恒压源转换为恒流源，将开路转换为短路，**…**
（**4**）回旋器是有源的还是无源的？是无损还是非无损？

回旋器等效电路及其实现方法1

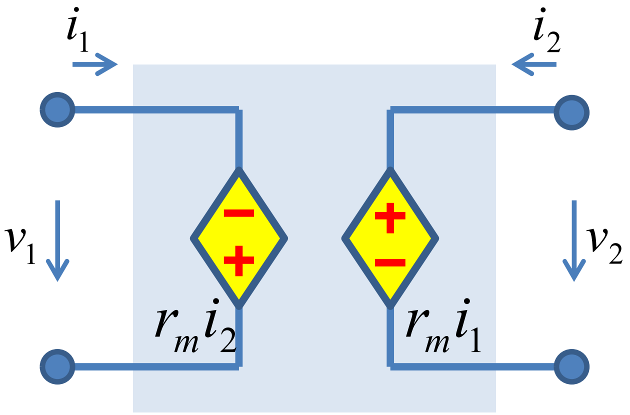


$$v_1 = -r_m i_2$$

$$v_2 = r_m i_1$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & -r_m \\ r_m & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

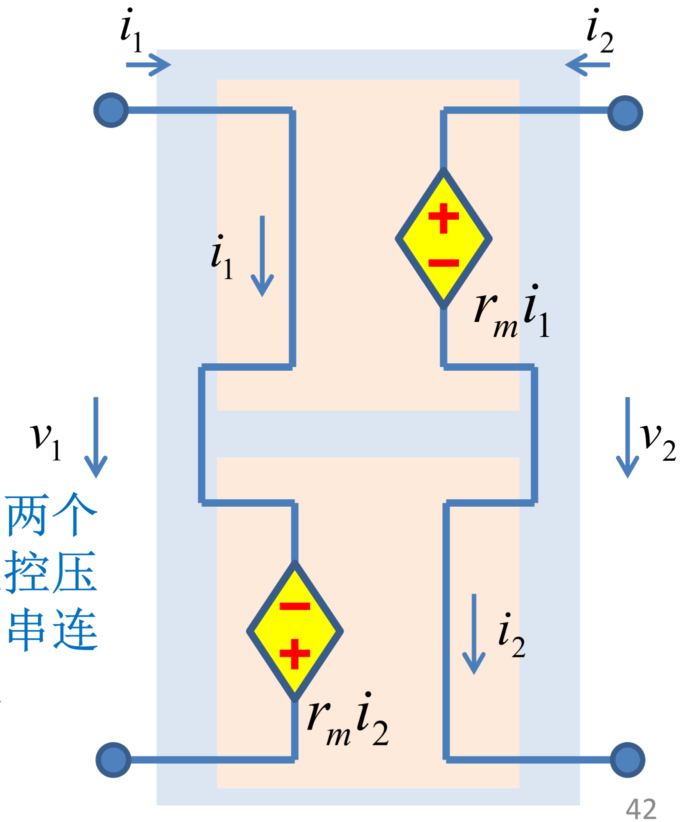
$$\mathbf{z} = \begin{bmatrix} 0 & -r_m \\ r_m & 0 \end{bmatrix}$$



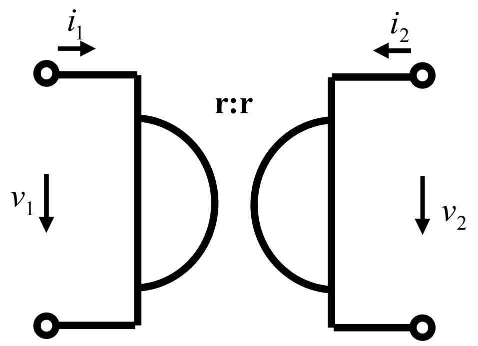
等效电路

$$\mathbf{z} = \begin{bmatrix} 0 & -r_m \\ r_m & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ r_m & 0 \end{bmatrix} + \begin{bmatrix} 0 & -r_m \\ 0 & 0 \end{bmatrix}$$

可以用两个理想流控压源的串串连接实现



回旋器等效电路及其实现方法2



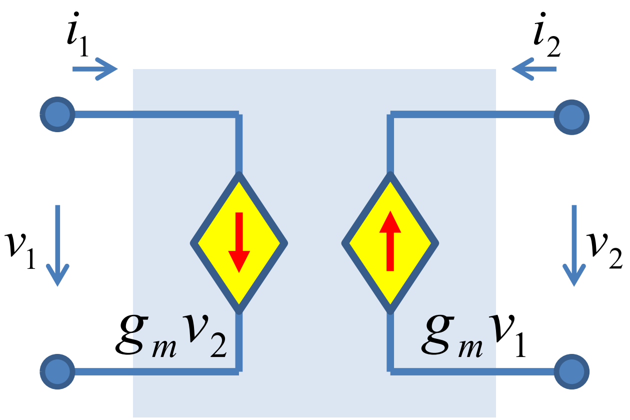
$$i_1 = g_m v_2$$

$$i_2 = -g_m v_1$$

$$v_1 = -r_m i_2 \qquad v_2 = r_m i_1$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g_m \\ -g_m & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

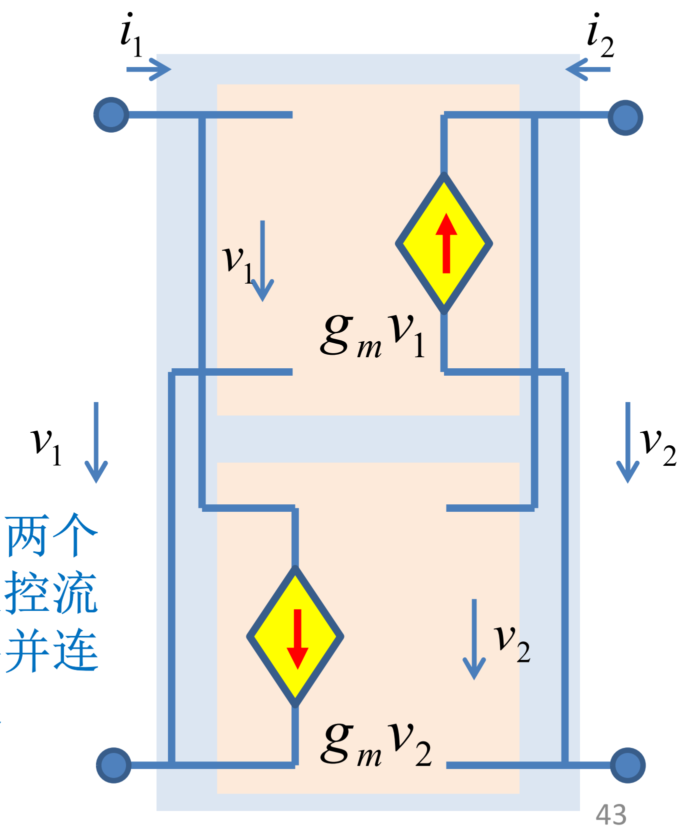
$$\mathbf{y} = \begin{bmatrix} 0 & g_m \\ -g_m & 0 \end{bmatrix}$$



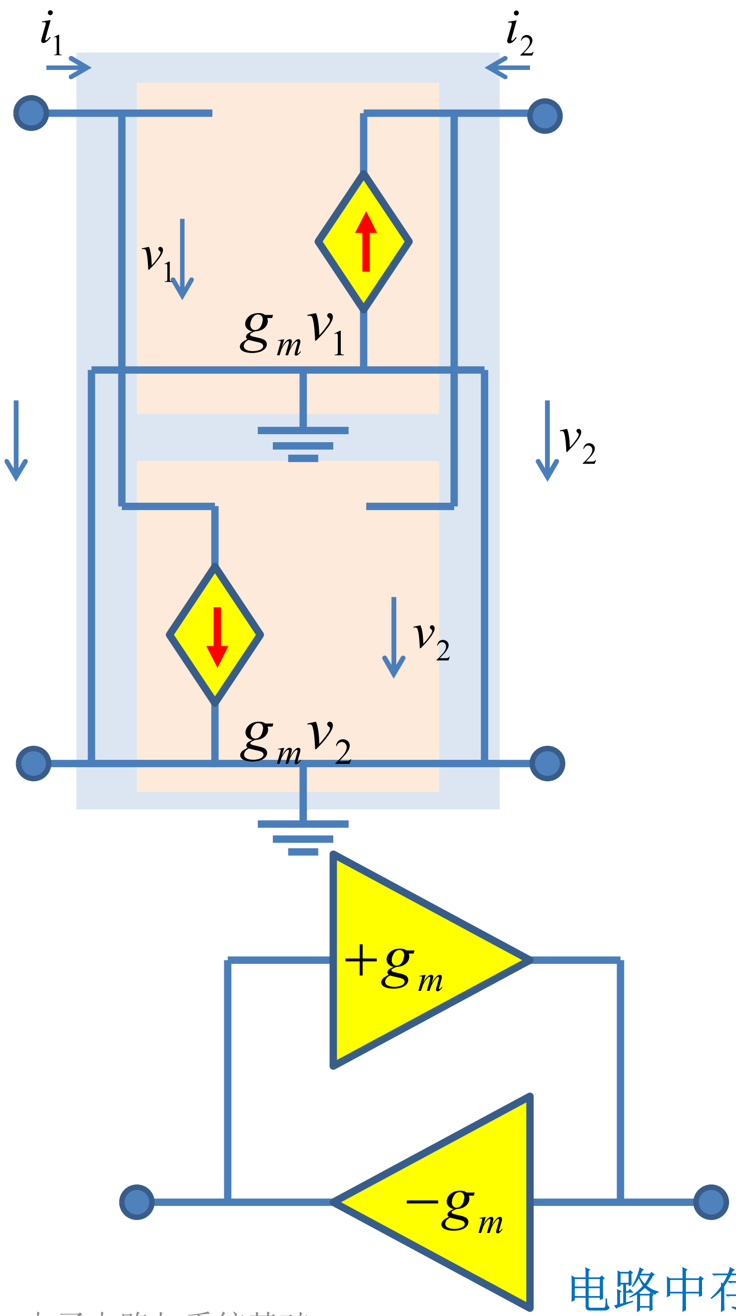
等效电路

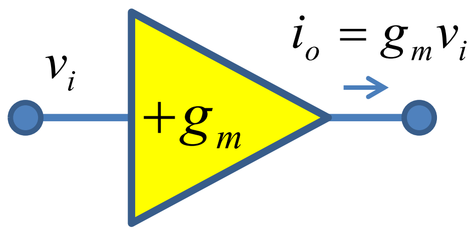
$$\mathbf{y} = \begin{bmatrix} 0 & g_m \\ -g_m & 0 \end{bmatrix} = \begin{bmatrix} 0 & g_m \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -g_m & 0 \end{bmatrix}$$

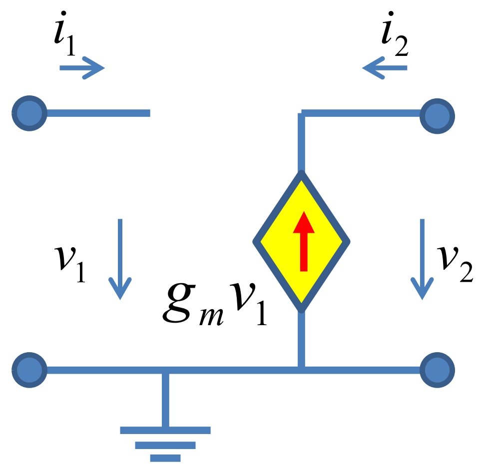
可以用两个理想压控流源的并并连接实现



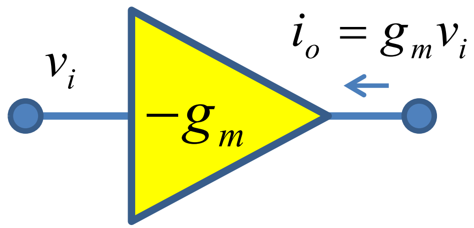
跨导器实现方案较常见

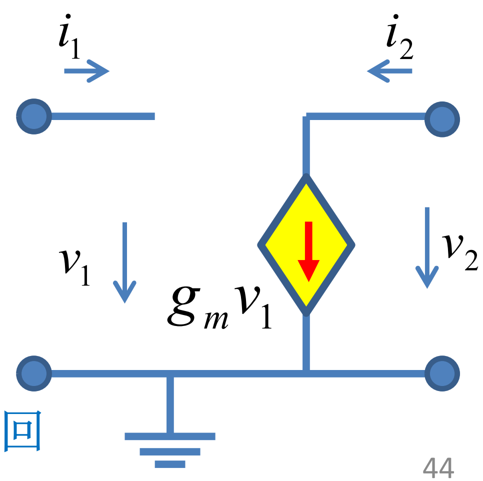






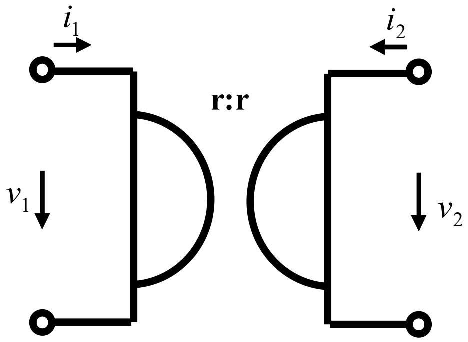
有默认地端点作为端口的一个端点

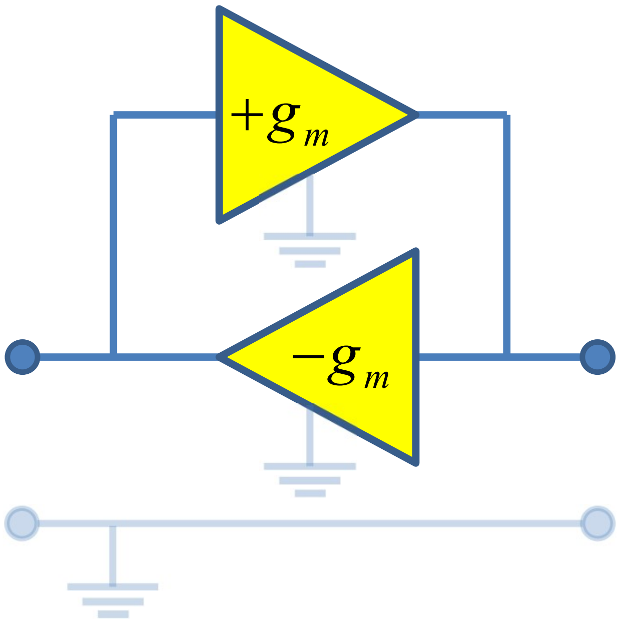




电路中存在这种结构，则为回旋器：集成滤波器常见结构

回旋器的6个网络参量





非互易

$$\mathbf{z}=\begin{bmatrix}0 & -r_m\\ r_m & 0\end{bmatrix} \qquad \mathbf{y}=\begin{bmatrix}0 & g_m\\ -g_m & 0\end{bmatrix}$$

$$v_1=-r_m i_2 \qquad v_2=r_m i_1$$

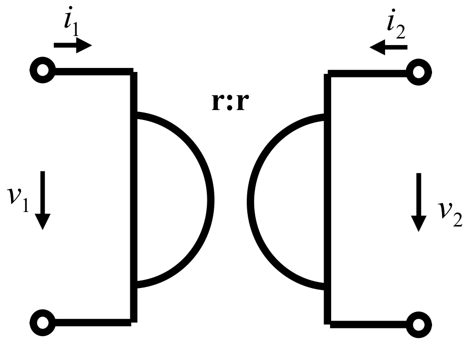
h、g参量不存在

$$\mathbf{ABCD}=\begin{bmatrix}0 & r_m\\ g_m & 0\end{bmatrix}$$

非互易

$$\mathbf{abcd}=\begin{bmatrix}0 & -r_m\\ -g_m & 0\end{bmatrix}$$

网络性质



$$v_1 = -r_m i_2$$

$$v_2 = r_m i_1$$

$$\mathbf{z} = \begin{bmatrix} 0 & -r_m \\ r_m & 0 \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} 0 & g_m \\ -g_m & 0 \end{bmatrix}$$

$$\mathbf{ABCD} = \begin{bmatrix} 0 & r_m \\ g_m & 0 \end{bmatrix} \qquad \mathbf{abcd} = \begin{bmatrix} 0 & -r_m \\ -g_m & 0 \end{bmatrix}$$

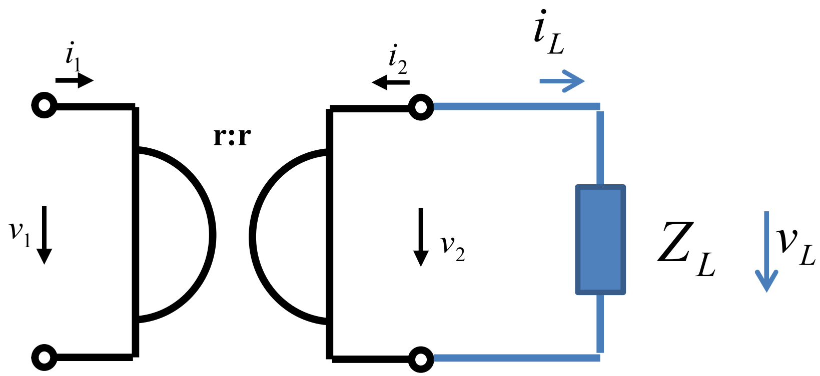
$z_{12} \neq z_{21}$ 非互易网络 $\Delta_T = -1 \neq +1$

$$p_\Sigma = v_1 i_1 + v_2 i_2 = -r_m i_2 i_1 + r_m i_1 i_2 = 0$$

无损网络：一个端口吸收的功率另外一个端口全部释放出去

无源网络：实际实现则需要用两个有源网络实现

阻抗变换



$$v_1 = -r_m i_2 \qquad v_2 = r_m i_1$$

$$f(v_L, i_L) = 0$$

$$f(v_2, -i_2) = 0$$

$$f(r_m i_1, g_m v_1) = 0$$

$f(v_L, i_L) = 0$　　端口**2**负载元件约束关系

↕ ↕　对偶变换：电压电流互换位置

$f(r_m i_1, g_m v_1) = 0$　　端口**1**等效负载元件约束关系

端口2负载元件约束关系

$$f(v_L, i_L) = 0$$

对偶变换：元件约束电压电流互换位置

端口1等效负载元件约束关系

$$f(r_m i_1, g_m v_1) = 0$$

开路 $i_L = 0$

$$f(v_L, i_L) = 0 \cdot v_L + 1 \cdot i_L = 0$$

$v_1 = 0$ 短路

$$f(r_m i_1, g_m v_1) = 0 \cdot r_m i_1 + 1 \cdot g_m v_1 = 0$$

恒压源 $v_L = V_{S0}$

$$f(v_L, i_L) = 1 \cdot v_L + 0 \cdot i_L - V_{S0} = 0$$

$i_1 = g_m V_{S0} = I_{S0}$ 恒流源

$$f(r_m i_1, g_m v_1) = 1 \cdot r_m i_1 + 0 \cdot g_m v_1 - V_{S0} = 0$$

电感

$$v_L = L \frac{di_L}{dt}$$

$$f(v_L, i_L) = v_L - L \frac{di_L}{dt} = 0$$

$$i_1 = g_m^2 L \frac{dv_1}{dt} = C \frac{dv_1}{dt}$$

电容

$$f(r_m i_1, g_m v_1) = r_m i_1 - L \frac{dg_m v_1}{dt} = 0$$

RLC串联

$$v_L = Ri_L + L \frac{di_L}{dt} + \frac{1}{C} \int i_L dt$$

$$f(v_L, i_L) = v_L - \left(Ri_L + L \frac{di_L}{dt} + \frac{1}{C} \int i_L dt \right) = 0$$

GCL并联

$$i_1 = Rg_m^2 v_1 + g_m^2 L \frac{dv_1}{dt} + \frac{1}{r_m^2 C} \int v_1 dt$$

$$= G_1 v_1 + C_1 \frac{dv_1}{dt} + \frac{1}{L_1} \int v_1 dt$$

$$f(r_m i_1, g_m v_1)$$

$$= r_m i_1 - \left(Rg_m v_1 + L \frac{dg_m v_1}{dt} + \frac{1}{C} \int g_m v_1 dt \right) = 0$$

对偶变换

- 回旋器可实现对偶变换
  - 短路变开路，开路变短路
  - 恒压源变恒流源，恒流源变恒压源
  - 电容变电感，电感变电容
    - 集成滤波器典型设计方案
  - 并联变串联，串联变并联
    - 结点变回路，回路变结点
    - 串联**RLC**变并联**GCL**（并联**RLC**）
  - **N**型负导变**S**型负阻，**S**型负阻变**N**型负导
    - **N**型负导**---N**型负阻
  - **...**

对偶变换：元件约束电压电流互换位置，方程形式不变：对偶元件

作业7 无损网络

- 某阻性线性二端口网络是无损网络，证明无损性意味着其网络参量具有如下特性
  - 证明其一即可

$R_{11}=0 \quad R_{22}=0 \quad R_{12}=-R_{21}$ z参量

$g_{11}=0 \quad g_{22}=0 \quad g_{12}=-g_{21}$ g参量

$AC=0 \quad BD=0 \quad AD+BC=1$ $ABCD$参量

无损网络

- **Lossless Network and lossy Network**

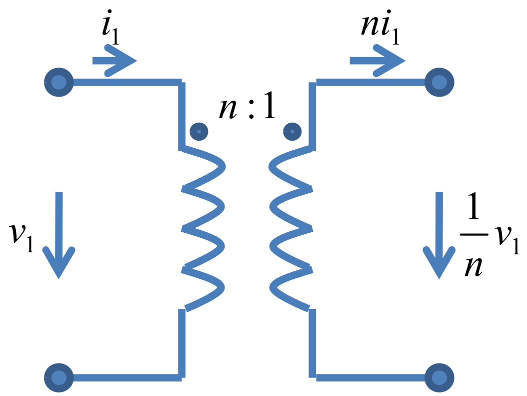
- 对于不存在电容、电感的无源阻性网络
  - 如果其端口总吸收功率恒等于**0**，则为无损网络

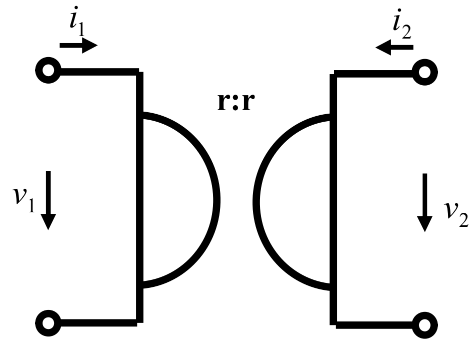
$$P=\sum_{k=1}^{n}p_k=\sum_{k=1}^{n}v_k i_k=\mathbf{v}^T\mathbf{i}=0 \qquad \left(\forall \mathbf{v},\mathbf{i},\mathbf{f}(\mathbf{v},\mathbf{i})=0\right)$$

  - 否则有损

无损二端口网络

$$P=\sum_{k=1}^{2}p_k=v_1i_1+v_2i_2\equiv 0$$





$$\mathbf{h}=\begin{bmatrix}0 & n\\ -n & 0\end{bmatrix}$$

$$\mathbf{z}=\begin{bmatrix}0 & -r\\ r & 0\end{bmatrix}$$

$$\begin{aligned}P&=v_1i_1+v_2i_2\\ &=(h_{11}i_1+h_{12}v_2)i_1+v_2(h_{21}i_1+h_{22}v_2)\\ &=h_{11}i_1^2+(h_{12}+h_{21})v_2i_1+h_{22}v_2^2\\ &\equiv 0\end{aligned}$$

$$\begin{aligned}P&=v_1i_1+v_2i_2\\ &=(z_{11}i_1+z_{12}i_2)i_1+(z_{21}i_1+z_{22}i_2)i_2\\ &=z_{11}i_1^2+(z_{12}+z_{21})i_2i_1+z_{22}i_2^2\\ &\equiv 0\end{aligned}$$

存在端口间相互作用的无损二端口网络只有两个

$$P_h = v_1 i_1 + v_2 i_2$$
$$= (h_{11} i_1 + h_{12} v_2) i_1 + v_2 (h_{21} i_1 + h_{22} v_2)$$
$$= h_{11} i_1^2 + (h_{12} + h_{21}) v_2 i_1 + h_{22} v_2^2 \equiv 0$$

$$h_{11} = 0, h_{22} = 0, h_{12} = -h_{21}$$

理想变压器：互易无损网络

$$g_{11} = 0, g_{22} = 0, g_{12} = -g_{21}$$

$$P_z = v_1 i_1 + v_2 i_2$$
$$= (z_{11} i_1 + z_{12} i_2) i_1 + (z_{21} i_1 + z_{22} i_2) i_2$$
$$= z_{11} i_1^2 + (z_{12} + z_{21}) i_2 i_1 + z_{22} i_2^2 \equiv 0$$

无损条件

$$z_{11} = 0, z_{22} = 0, z_{12} = -z_{21}$$

理想回旋器：非互易无损网络

$$y_{11} = 0, y_{22} = 0, y_{12} = -y_{21}$$

用ABCD参量表述

ABCD存在，意味着端口1对端口2存在作用关系

$$P_{ABCD}=v_1i_1+v_2i_2$$
$$=\left(Av_2-Bi_2\right)\left(Cv_2-Di_2\right)+v_2i_2$$
$$=ACv_2^2+BDi_2^2+\left(1-AD-BC\right)v_2i_2\equiv 0$$

$AC=0 \qquad BD=0 \qquad AD+BC=1$ 无损条件

$$\begin{cases}A=0 \\ C=0\end{cases} \qquad \begin{matrix}BC=1 \\ AD=1\end{matrix} \qquad \begin{matrix}D=0 \\ B=0\end{matrix}$$

除了这两种情况外，别无它解？

$$\mathbf{ABCD}_{ideal\ \ Transformer}=\begin{bmatrix}n & 0 \\ 0 & \dfrac{1}{n}\end{bmatrix} \qquad \mathbf{ABCD}_{ideal\ \ gyrator}=\begin{bmatrix}0 & r \\ \dfrac{1}{r} & 0\end{bmatrix}$$

作业6 网络单向化及其有源性

- 已知某双向阻性网络的z参量矩阵为

$$\mathbf{z}=\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \qquad R_{21}R_{12}\neq 0 \qquad R_{11}>0 \qquad R_{22}>0$$

双向网络　　端口开路输入电阻为正阻

- （1）已知该网络有源，请给出该网络的有源性条件
- （2）请设法将该双向有源网络转化为单向有源网络（提示：和无损二端口网络连接）
- （3）选作：证明变换后的单向网络（基本放大器）的‘最大功率增益大于1’等价于‘双向网络的有源性条件’

有源性

$$\mathbf{z} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \qquad R_{21}R_{12} \neq 0 \qquad R_{11} > 0 \qquad R_{22} > 0$$

$$\begin{aligned} p &= v_1 i_1 + v_2 i_2 \\ &= (R_{11}i_1 + R_{12}i_2)i_1 + (R_{21}i_1 + R_{22}i_2)i_2 \\ &= R_{11}i_1^2 + (R_{12} + R_{21})i_1 i_2 + R_{22}i_2^2 < 0 \end{aligned}$$

$$R_{11} > 0 \qquad R_{22} > 0$$

$$\Delta = b^2 - 4ac = (R_{12} + R_{21})^2 - 4R_{11}R_{22} > 0$$

$$(R_{12} + R_{21})^2 > 4R_{11}R_{22}$$ 有源性条件：**zyhg**，完全相同的形式

$$\mathbf{z}=\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$$

$R_{21}R_{12} \neq 0$ $R_{11} > 0$ $R_{22} > 0$

$(R_{12}+R_{21})^2 > 4R_{11}R_{22}$

单向化

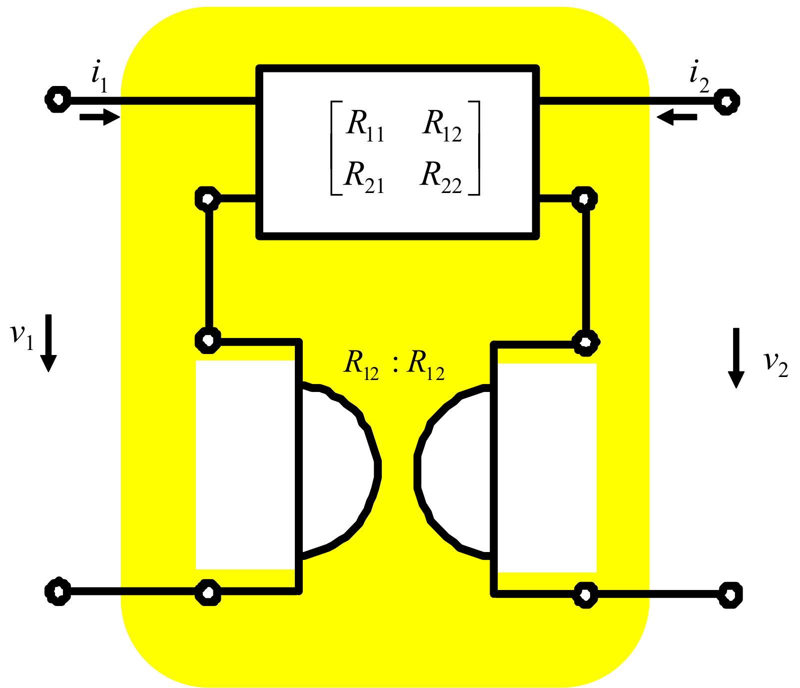
$$\mathbf{z}_T = \mathbf{z} + z_{gyrator} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} + \begin{bmatrix} 0 & -r_m \\ r_m & 0 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12}-r_m \\ R_{21}+r_m & R_{22} \end{bmatrix}$$

$$\overset{r_m=R_{12}}{=} \begin{bmatrix} R_{11} & 0 \\ R_{21}+R_{12} & R_{22} \end{bmatrix}$$

串串连接z相加

理想回旋器是无损网络
新网络有源性不会改变

理想回旋器是非互易网络
新网络互易性可能改变



功率增益

$$p = v_1 i_1 + v_2 i_2$$
$$= (R_{11} i_1) i_1 + ((R_{21} + R_{12}) i_1 + R_{22} i_2) i_2$$
$$= R_{11} i_1^2 + (R_{12} + R_{21}) i_1 i_2 + R_{22} i_2^2 < 0 \qquad (R_{12} + R_{21})^2 > 4 R_{11} R_{22}$$

有源性条件没有改变

$$R_S = R_{11}, \qquad R_L = R_{22}$$

两端同时最大功率传输匹配

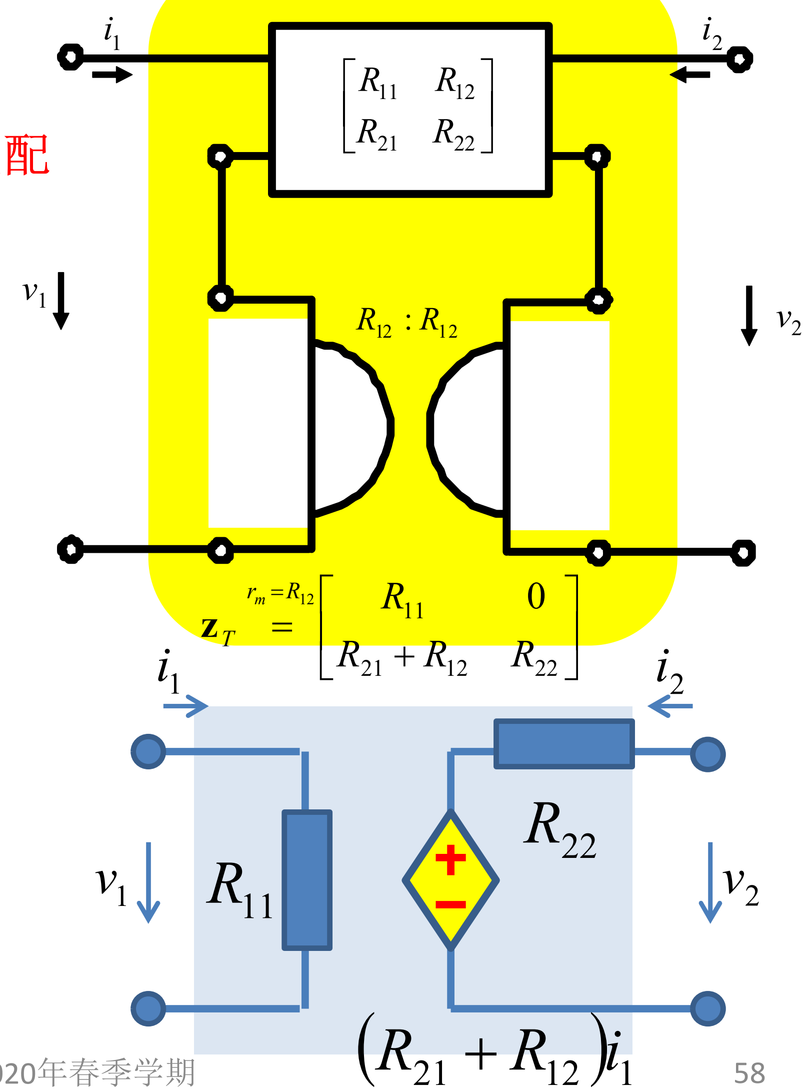
$$P_L = \frac{1}{4} \frac{((R_{21} + R_{12}) i_{1,rms})^2}{R_{22}} = \frac{1}{4} \frac{(R_{21} + R_{12})^2 (i_{1,rms})^2}{R_{22}}$$

$$= \frac{1}{4} \frac{(R_{21} + R_{12})^2 \left(\frac{1}{2} \frac{v_{s,rms}}{R_{11}} \right)^2}{R_{22}} = \frac{1}{16} \frac{(R_{21} + R_{12})^2 v_{s,rms}^2}{R_{22} R_{11}^2}$$

$$= \frac{1}{4} \frac{(R_{21} + R_{12})^2}{R_{22} R_{11}} \frac{1}{4} \frac{v_{s,rms}^2}{R_{11}} = G_{p,\max} P_{S,\max}$$

$$G_{p,\max} = \frac{1}{4} \frac{(R_{21} + R_{12})^2}{R_{22} R_{11}} > 1$$

有源则功率增益大于1



连接无损二端口网络不改变原网络有源性

- 满足无损条件的二端口网络
  - 理想变压器
    - 从互感变压器（无损二端口电感）抽象而来
  - 理想回旋器
    - 没有真实对应的无源网络，需要用两个有源网络的串串连接或并并连接实现

- 原理上，用理想回旋器理解有源性和功率增益大于**1**的等价性最简单

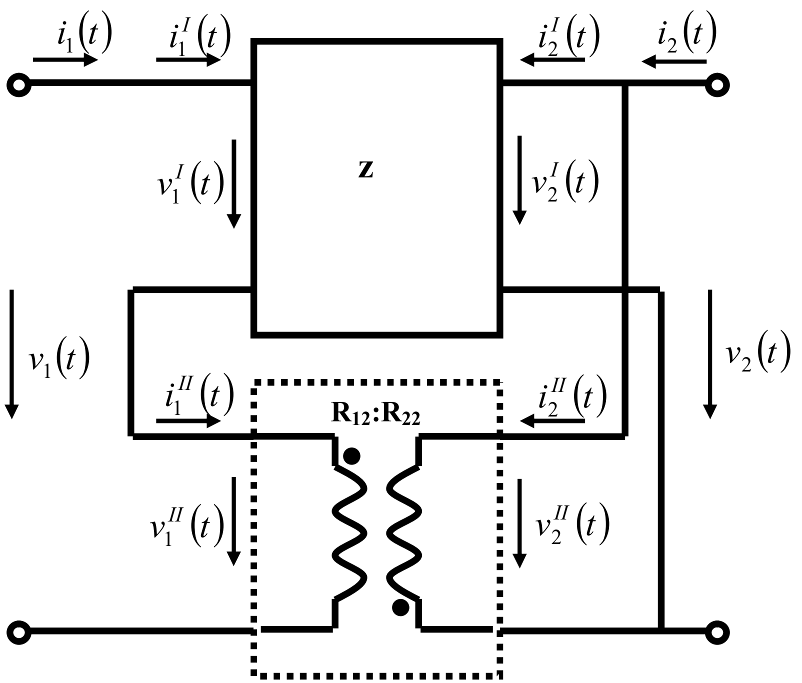
- 课下自学，充分理解
  - 用理想变压器实现单向网络，并证明变换后的单向网络（基本放大器）的‘最大功率增益大于**1**’等价于‘双向网络的有源性条件’
    - 如果单向网络的输入阻抗或输出阻抗为负值，可实现负阻放大器，使得其功率增益大于**1**，功率增益大于**1**与双向网络的有源性条件等同
    - 理想变压器是互易网络，不改变原网络的互易性

提示

$$\mathbf{h}^{I}=\frac{\begin{bmatrix}\Delta_z & z_{12}\\ -z_{21} & 1\end{bmatrix}}{z_{22}}=\begin{bmatrix}\dfrac{R_{11}R_{22}-R_{12}R_{21}}{R_{22}} & \dfrac{R_{12}}{R_{22}}\\ -\dfrac{R_{21}}{R_{22}} & \dfrac{1}{R_{22}}\end{bmatrix}$$

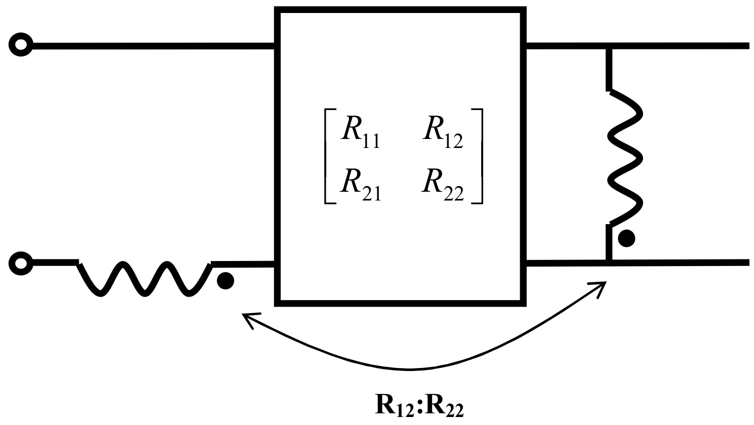
$$\mathbf{h}^{II}=\begin{bmatrix}0 & -n\\ n & 0\end{bmatrix}$$

$$n=\frac{R_{12}}{R_{22}}$$



$n:1=-R_{12}:R_{22}$

串并连接h相加，h变成单向网络



转化为单向网络后，考虑互易非互易，
输入电阻为正为负等各种情况