电子电路与系统基础I

习题课第六讲

1、第四周作业讲解

2、网络参量应用

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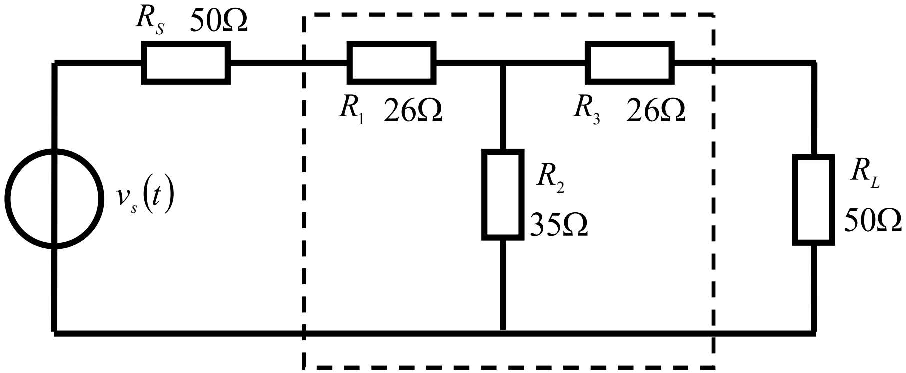
习题课第六讲　　大纲

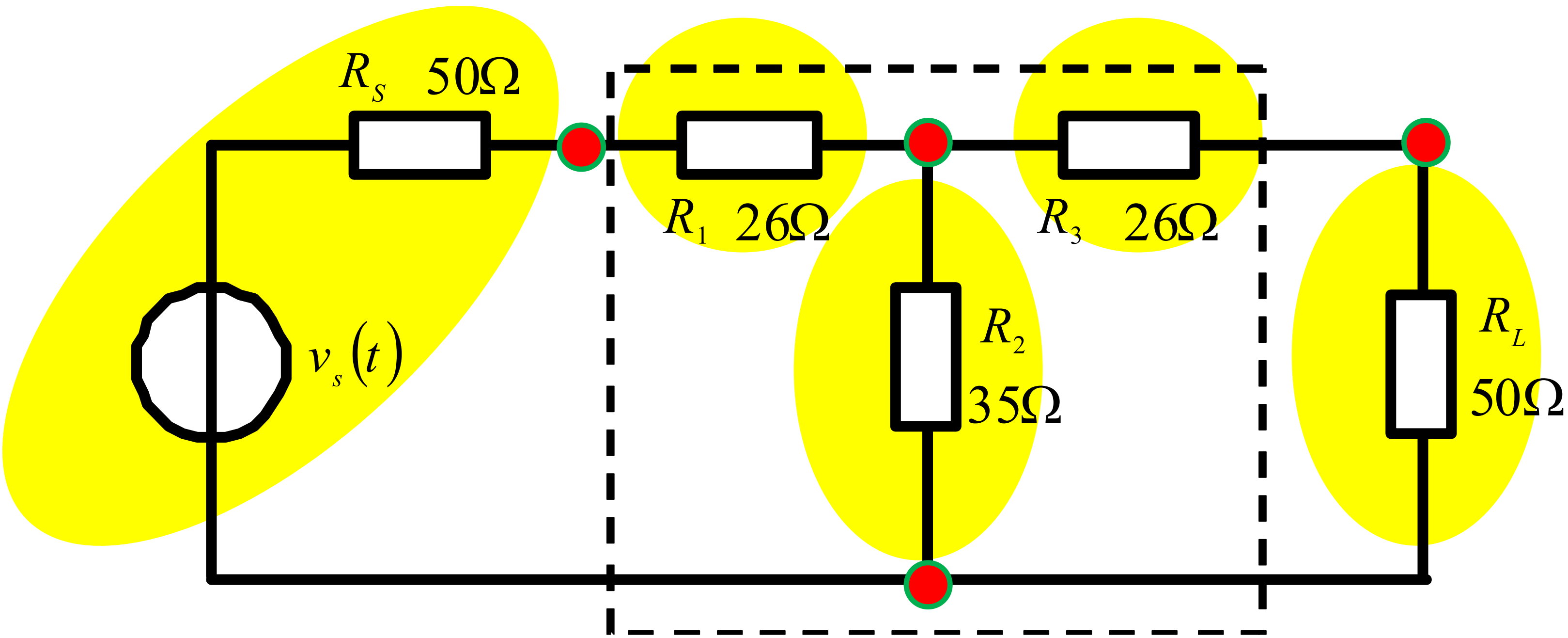
- 第四周作业讲解
- 二端口网络网络参量及传递函数应用
- 二端口网络噪声分析（例3.7.4）
- 一个三端口网络例（例3.7.2）

作业1：T型电阻衰减网络



- 用支路电压电流法、支路电流法、回路电流法、结点电压法列写上述电路的电路方程
  - 选取其中的一种方法，矩阵求逆求解，获得负载电压与源电压之间的比值关系，说明衰减系数为多大

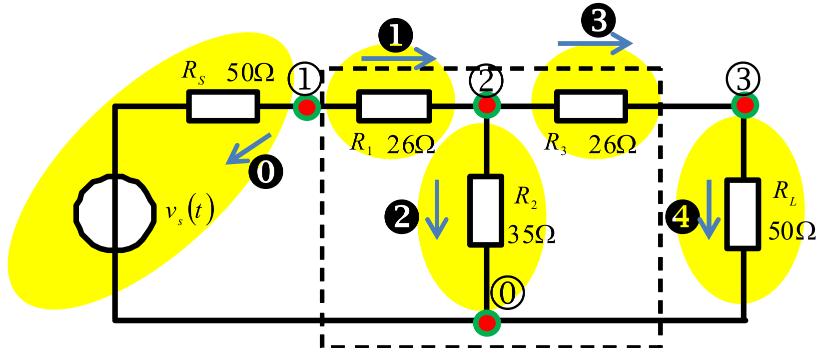
电路结构分析



b=5　　5条支路

n=4　　4个结点

支路电压电流法



b=5个支路元件约束方程

$$v_0 - i_0 R_s = v_s$$

$$v_1 - i_1 R_1 = 0$$

$$v_2 - i_2 R_2 = 0$$

$$v_3 - i_3 R_3 = 0$$

$$v_4 - i_4 R_4 = 0$$

n-1=3个KCL方程

$$-i_0 - i_1 = 0$$

$$i_1 - i_2 - i_3 = 0$$

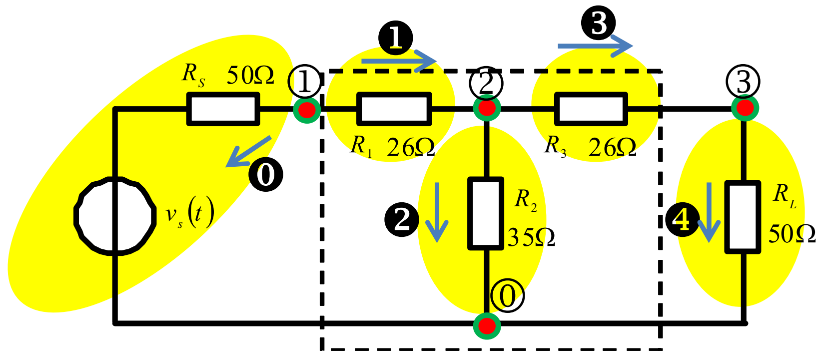
$$i_3 - i_4 = 0$$

b-n+1=2个KVL方程

$$-v_0 + v_1 + v_2 = 0$$

$$-v_2 + v_3 + v_4 = 0$$

支路电压电流法
矩阵方程



$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -26 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -35 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -26 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -50 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
-1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}
=
\begin{bmatrix} v_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

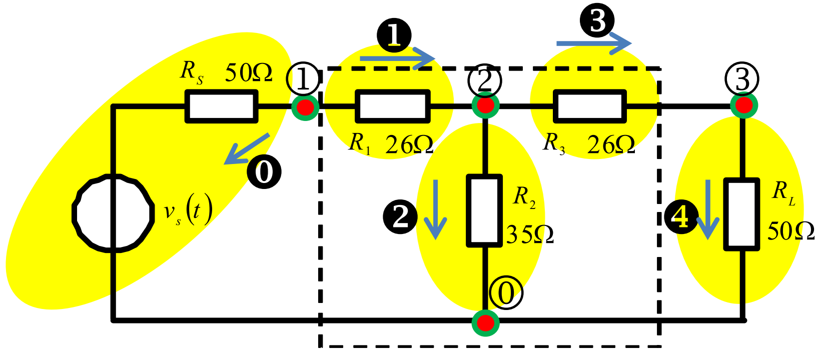
Matlab辅助计算

```
>>  A=[1 0 0 0 0 -50 0 0 0 0;0 1 0 0 0 0 -26 0 0 0;0 0 1 0 0 0 0 -35 0 0;0 0 0 1 0 0 0 0 -26 0; 0 0 0 0 1 0 0 0 0 -50; 0

A =

     1     0     0     0     0   -50     0     0     0     0
     0     1     0     0     0     0   -26     0     0     0
     0     0     1     0     0     0     0   -35     0     0
     0     0     0     1     0     0     0     0   -26     0
     0     0     0     0     1     0     0     0     0   -50
     0     0     0     0     0    -1    -1     0     0     0
     0     0     0     0     0     0     1    -1    -1     0
     0     0     0     0     0     0     0     0     1    -1
    -1     1     1     0     0     0     0     0     0     0
     0     0    -1     1     1     0     0     0     0     0

>> inv(A)

ans =

    0.4998    0.5002    0.3425    0.1577    0.1577  -24.9910  -11.9863   -7.8857   -0.5002   -0.1577
    0.2601    0.7399   -0.1781   -0.0820   -0.0820  -13.0047    6.2329    4.1006    0.2601    0.0820
    0.2397   -0.2397    0.5205    0.2397    0.2397  -11.9863  -18.2192  -11.9863    0.2397   -0.2397
    0.0820   -0.0820    0.1781    0.7399   -0.2601   -4.1006   -6.2329   13.0047    0.0820    0.2601
    0.1577   -0.1577    0.3425   -0.5002    0.4998   -7.8857  -11.9863  -24.9910    0.1577    0.5002
   -0.0100    0.0100    0.0068    0.0032    0.0032   -0.4998   -0.2397   -0.1577   -0.0100   -0.0032
    0.0100   -0.0100   -0.0068   -0.0032   -0.0032   -0.5002    0.2397    0.1577    0.0100    0.0032
    0.0068   -0.0068   -0.0137    0.0068    0.0068   -0.3425   -0.5205   -0.3425    0.0068   -0.0068
    0.0032   -0.0032    0.0068   -0.0100   -0.0100   -0.1577   -0.2397    0.5002    0.0032    0.0100
    0.0032   -0.0032    0.0068   -0.0100   -0.0100   -0.1577   -0.2397   -0.4998    0.0032    0.0100

>> |
```

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -26 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -35 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -26 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -50 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
-1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}
=
\begin{bmatrix} v_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
\begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}
=
\begin{bmatrix} 0.4998 \\ 0.2601 \\ 0.2397 \\ 0.0820 \\ 0.1577 \\ -0.0100 \\ 0.0100 \\ 0.0068 \\ 0.0032 \\ 0.0032 \end{bmatrix} v_s
$$

结果分析



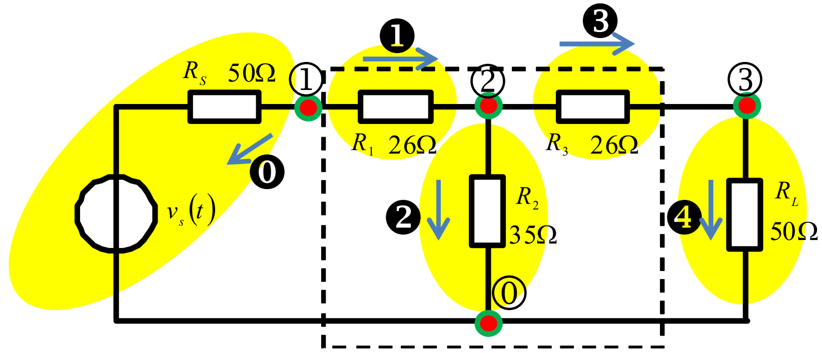
$$\begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0.4998 \\ 0.2601 \\ 0.2397 \\ 0.0820 \\ 0.1577 \\ -0.0100 \\ 0.0100 \\ 0.0068 \\ 0.0032 \\ 0.0032 \end{bmatrix} v_s$$

$$v_L = v_4 = 0.1577 v_s$$

$$L = \frac{P_{S,\max}}{P_L} = \frac{\dfrac{V_{S,rms}^2}{4R_S}}{\dfrac{V_{L,rms}^2}{R_L}} = \frac{R_L}{4R_S}\left(\frac{V_{S,rms}}{V_{L,rms}}\right)^2 = \frac{1}{4}\left(\frac{1}{0.1577}\right)^2 = 10.05 = 10dB$$

10dB的衰减器：负载电阻只获得了信源额定功率的10%

支路电流法



$$\begin{bmatrix} -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -50 & 26 & 35 & 0 & 0 \\ 0 & 0 & -35 & 26 & 50 \end{bmatrix} \begin{bmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ v_s \\ 0 \end{bmatrix}$$

n-1=3个KCL方程

$$-i_0 - i_1 = 0$$

$$i_1 - i_2 - i_3 = 0$$

$$i_3 - i_4 = 0$$

b-n+1=2个KVL方程

$$-i_0 R_s + i_1 R_1 + i_2 R_2 = v_s$$

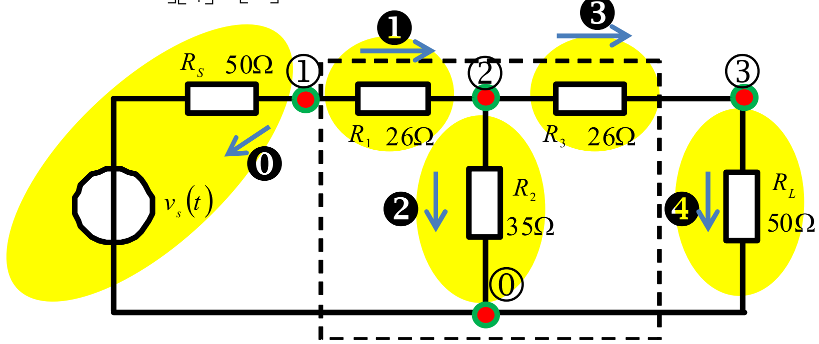
$$-i_2 R_2 + i_3 R_3 + i_4 R_4 = 0$$

$$\begin{bmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -0.0100 \\ 0.0100 \\ 0.0068 \\ 0.0032 \\ 0.0032 \end{bmatrix} v_S$$

Matlab辅助计算

```
>> A=[-1 -1 0 0 0;0 1 -1 -1 0;0 0 0 1 -1;-50 26 35 0 0;0 0 -35 26 50]

A =

    -1    -1     0     0     0
     0     1    -1    -1     0
     0     0     0     1    -1
   -50    26    35     0     0
     0     0   -35    26    50

>> inv(A)

ans =

   -0.4998   -0.2397   -0.1577   -0.0100   -0.0032
   -0.5002    0.2397    0.1577    0.0100    0.0032
   -0.3425   -0.5205   -0.3425    0.0068   -0.0068
   -0.1577   -0.2397    0.5002    0.0032    0.0100
   -0.1577   -0.2397   -0.4998    0.0032    0.0100

>> vL=ans(5,4)*50

vL =

    0.1577

>> 20*log10(2*vL)

ans =

  -10.0220
```

$$\begin{bmatrix} -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -50 & 26 & 35 & 0 & 0 \\ 0 & 0 & -35 & 26 & 50 \end{bmatrix} \begin{bmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ v_s \\ 0 \end{bmatrix}$$

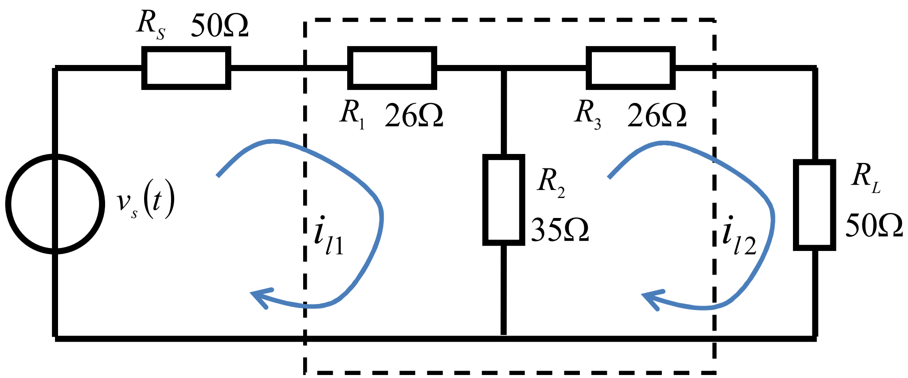


$$\begin{bmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -0.0100 \\ 0.0100 \\ 0.0068 \\ 0.0032 \\ 0.0032 \end{bmatrix} v_S$$

$$v_L = i_4 R_L = 0.1577 v_S$$

$$G_T = 10\log\frac{P_L}{P_{S,\max}} = 10\log\frac{\frac{1}{2}\frac{v_L^2}{R_L}}{\frac{1}{8}\frac{v_S^2}{R_S}} = 10\log\left(4\frac{R_S}{R_L}\left(\frac{v_L}{v_S}\right)^2\right) = 20\log\left(2\frac{v_L}{v_S}\right)$$

回路电流法



$$\begin{bmatrix} R_S+R_1+R_2 & -R_2 \\ -R_2 & R_2+R_3+R_4 \end{bmatrix}\begin{bmatrix} i_{l1} \\ i_{l2} \end{bmatrix}=\begin{bmatrix} v_s \\ 0 \end{bmatrix}$$

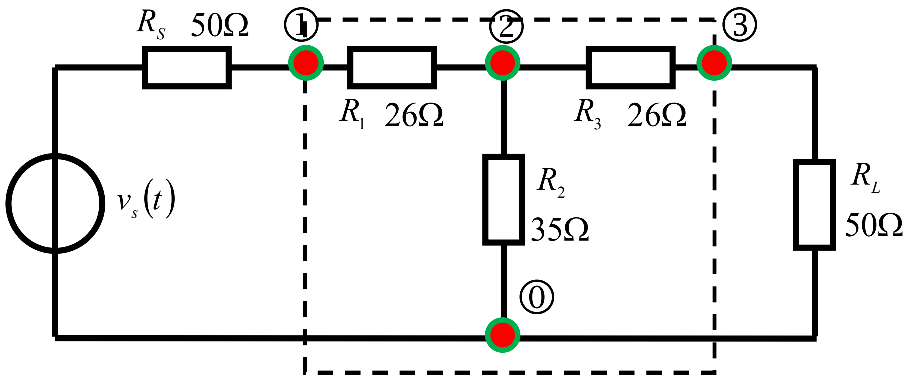
$$\begin{bmatrix} 50+26+35 & -35 \\ -35 & 35+26+50 \end{bmatrix}\begin{bmatrix} i_{l1} \\ i_{l2} \end{bmatrix}=\begin{bmatrix} v_s \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 111 & -35 \\ -35 & 111 \end{bmatrix}\begin{bmatrix} i_{l1} \\ i_{l2} \end{bmatrix}=\begin{bmatrix} v_s \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i_{l1} \\ i_{l2} \end{bmatrix}=\begin{bmatrix} 111 & -35 \\ -35 & 111 \end{bmatrix}^{-1}\begin{bmatrix} v_s \\ 0 \end{bmatrix}=\begin{bmatrix} 0.0100 & 0.0032 \\ 0.0032 & 0.0100 \end{bmatrix}\begin{bmatrix} v_s \\ 0 \end{bmatrix}=\begin{bmatrix} 0.0100 \\ 0.0032 \end{bmatrix}v_s$$

$$v_L=i_{l2}R_L=0.1577v_S$$

结点电压法



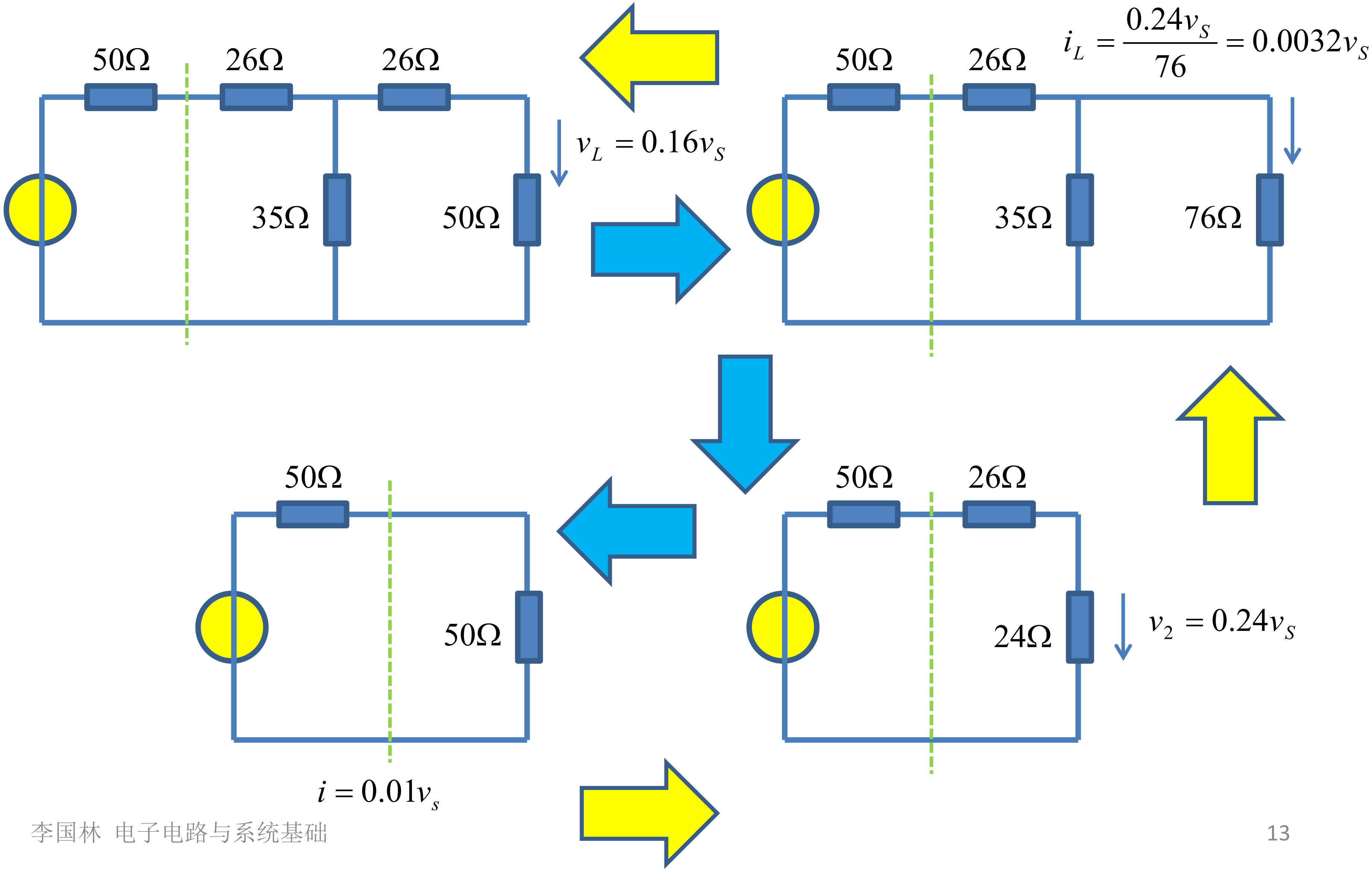
$$\begin{bmatrix} G_S+G_1 & -G_1 & 0 \\ -G_1 & G_1+G_2+G_3 & -G_3 \\ 0 & -G_3 & G_3+G_L \end{bmatrix}\begin{bmatrix} v_{\langle 1\rangle} \\ v_{\langle 2\rangle} \\ v_{\langle 3\rangle} \end{bmatrix}=\begin{bmatrix} G_S v_S \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 58.46 & -38.46 & 0 \\ -38.46 & 105.49 & -38.46 \\ 0 & -38.46 & 58.46 \end{bmatrix}\begin{bmatrix} v_{\langle 1\rangle} \\ v_{\langle 2\rangle} \\ v_{\langle 3\rangle} \end{bmatrix}=\begin{bmatrix} 20v_S \\ 0 \\ 0 \end{bmatrix}$$

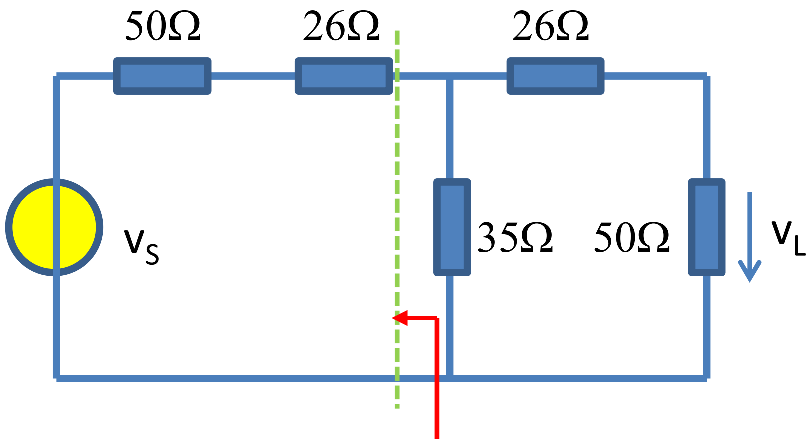
$$\begin{bmatrix} v_{\langle 1\rangle} \\ v_{\langle 2\rangle} \\ v_{\langle 3\rangle} \end{bmatrix}=\begin{bmatrix} 58.46 & -38.46 & 0 \\ -38.46 & 105.49 & -38.46 \\ 0 & -38.46 & 58.46 \end{bmatrix}^{-1}\begin{bmatrix} 20v_S \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} 0.0250 & 0.0120 & 0.0079 \\ 0.0120 & 0.0182 & 0.0120 \\ 0.0079 & 0.0120 & 0.025 \end{bmatrix}\begin{bmatrix} 20v_S \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} 0.4998 \\ 0.2397 \\ 0.1577 \end{bmatrix}v_S$$

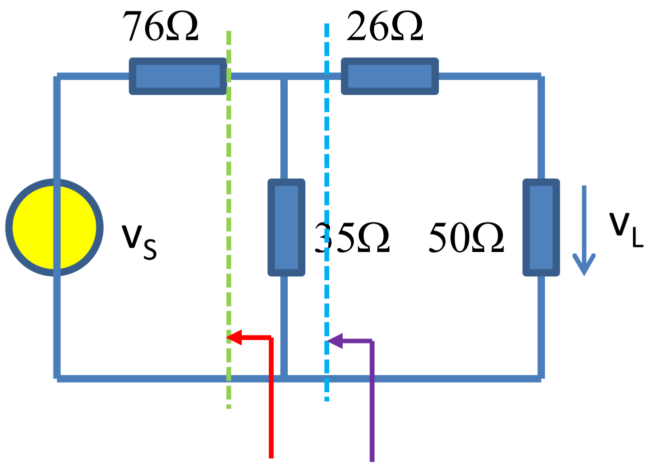
$$v_L=v_{\langle 3\rangle}=0.1577v_S$$

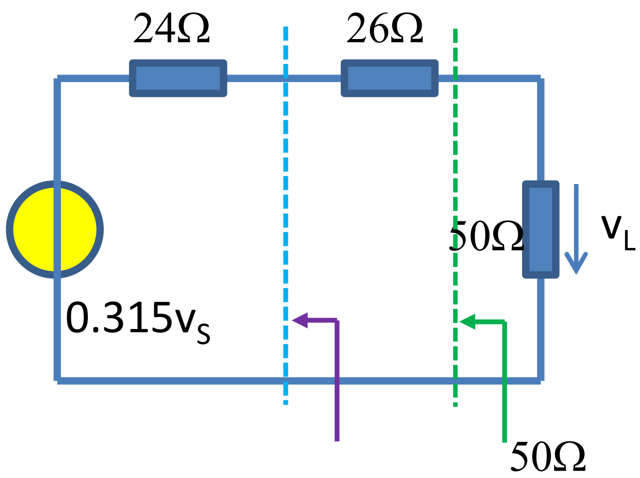
简单串并联等效电路法（1）



简单串并联等效电路法（2）

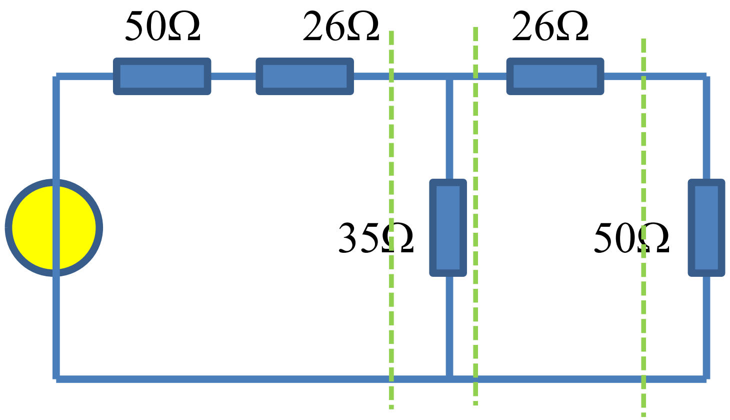






$$v_L = 0.5 * 0.315 v_S = 0.158 v_S$$

网络参量法



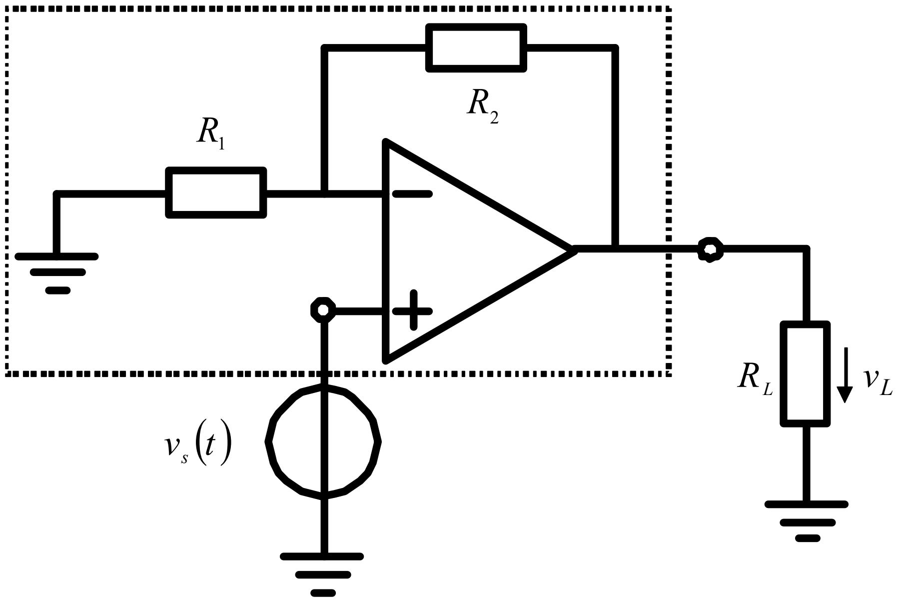
$$G_p = 4\frac{R_S}{R_L}\left(\frac{v_L}{v_S}\right)^2 = \frac{4}{A^2} = \frac{4}{6.34^2}$$

$$= 0.0995 = -10dB$$

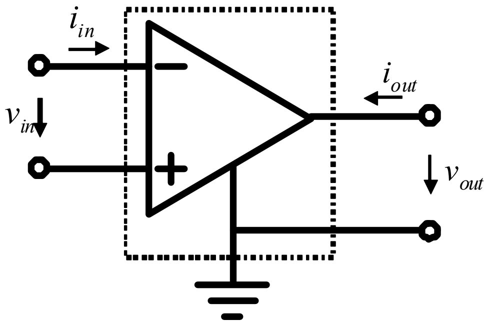
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 76 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{35} & 1 \end{bmatrix} \begin{bmatrix} 1 & 26 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{50} & 1 \end{bmatrix}$$

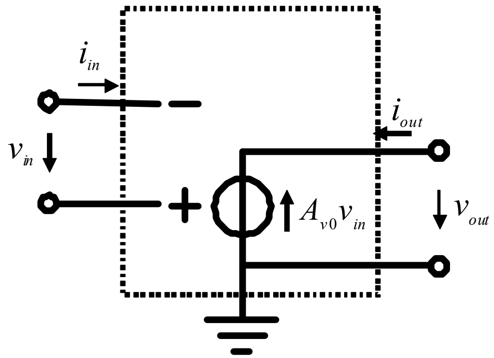
$$= \begin{bmatrix} 1+\frac{76}{35} & 76 \\ \frac{1}{35} & 1 \end{bmatrix} \begin{bmatrix} 1+\frac{26}{50} & 26 \\ \frac{1}{50} & 1 \end{bmatrix} = \begin{bmatrix} \left(1+\frac{76}{35}\right)\left(1+\frac{26}{50}\right)+\frac{76}{50} & \dots \\ \dots & \dots \end{bmatrix}$$

作业2：同相电压放大器



- 用任意方法列写电路方程，求解，分析电路功能
  - 先假设运放存在有限增益，再延拓到无限大增益情况

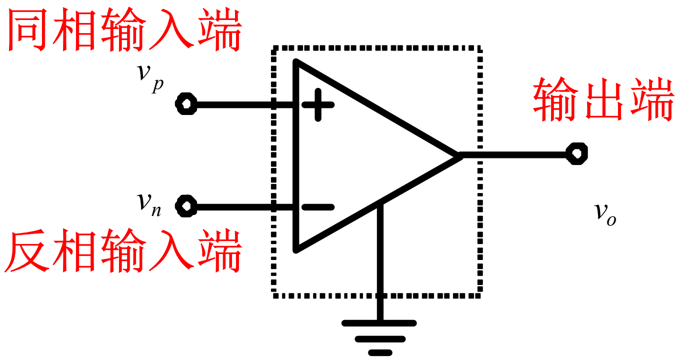




运算放大器参量：

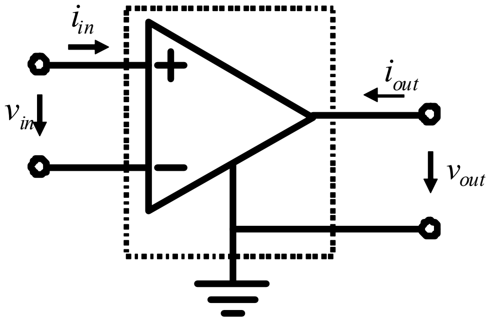
输入电阻无穷大
输出电阻为**0**
电压增益极大

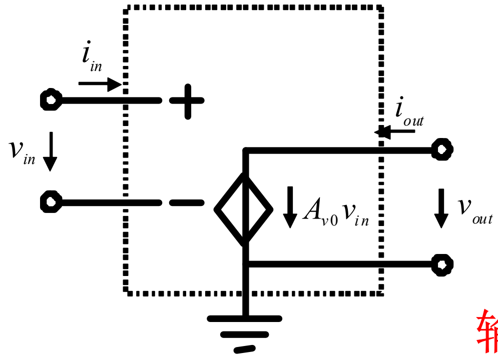
运算放大器



$$v_o = A_{v0}\left(v_p - v_n\right)$$

输出开路电压



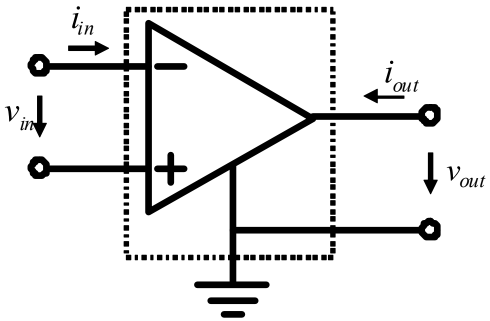


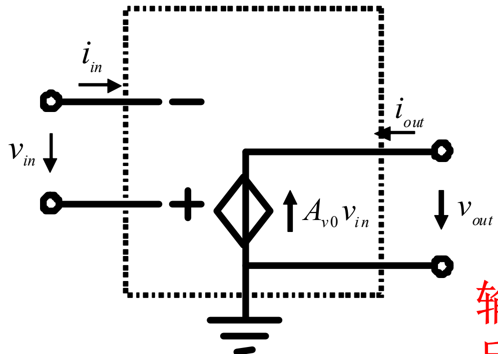
$$v_{in} = v_p - v_n$$

$$v_{out} = A_{v0} v_{in}$$

输出和输入同相位

同相电压放大器





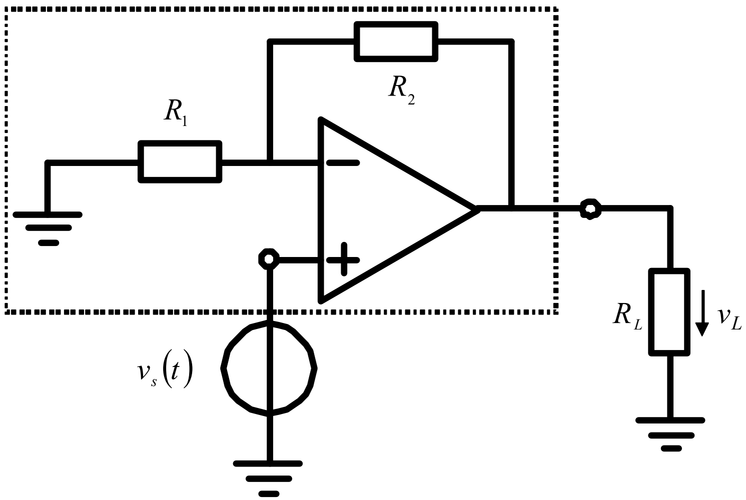
$$v_{in} = v_n - v_p$$

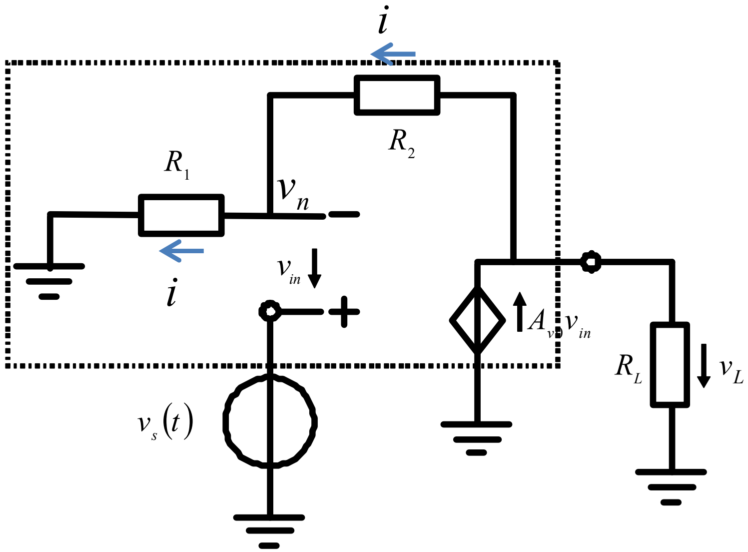
$$v_{out} = -A_{v0} v_{in}$$

输出和输入反相位

反相电压放大器

任意方法：手工计算以简单为首要原则





$$\frac{-A_v(v_n - v_s) - v_n}{R_2} = i = \frac{v_n}{R_1}$$

$$v_n = \frac{v_s}{1 + \frac{1}{A_v}\left(1 + \frac{R_2}{R_1}\right)} \xrightarrow{A_v \to \infty} v_s$$

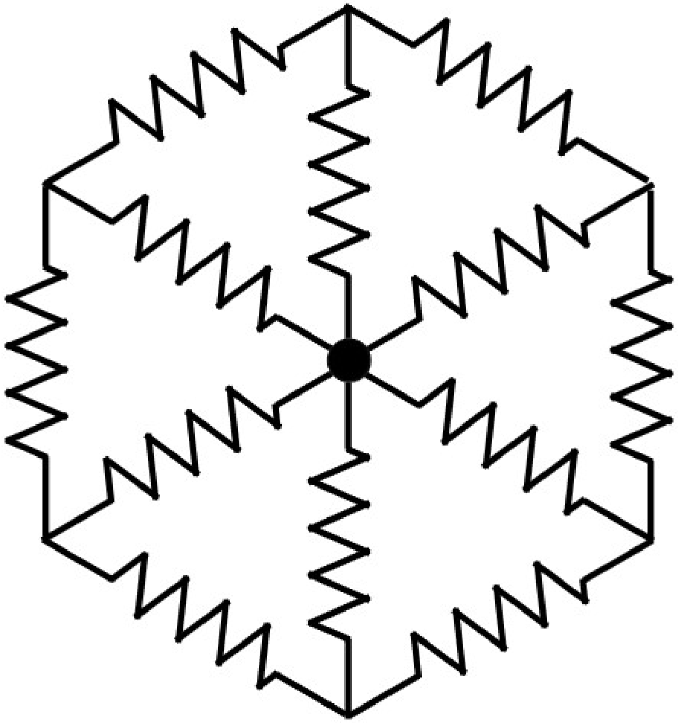
输出和输入同相位
同相电压放大器

$$v_L = -A_v(v_n - v_s) = \frac{1}{1 + \frac{1}{A_v}\left(1 + \frac{R_2}{R_1}\right)}\left(1 + \frac{R_2}{R_1}\right)v_s \xrightarrow{A_v \to \infty} \left(1 + \frac{R_2}{R_1}\right)v_s$$

放大倍数由外部器件决定

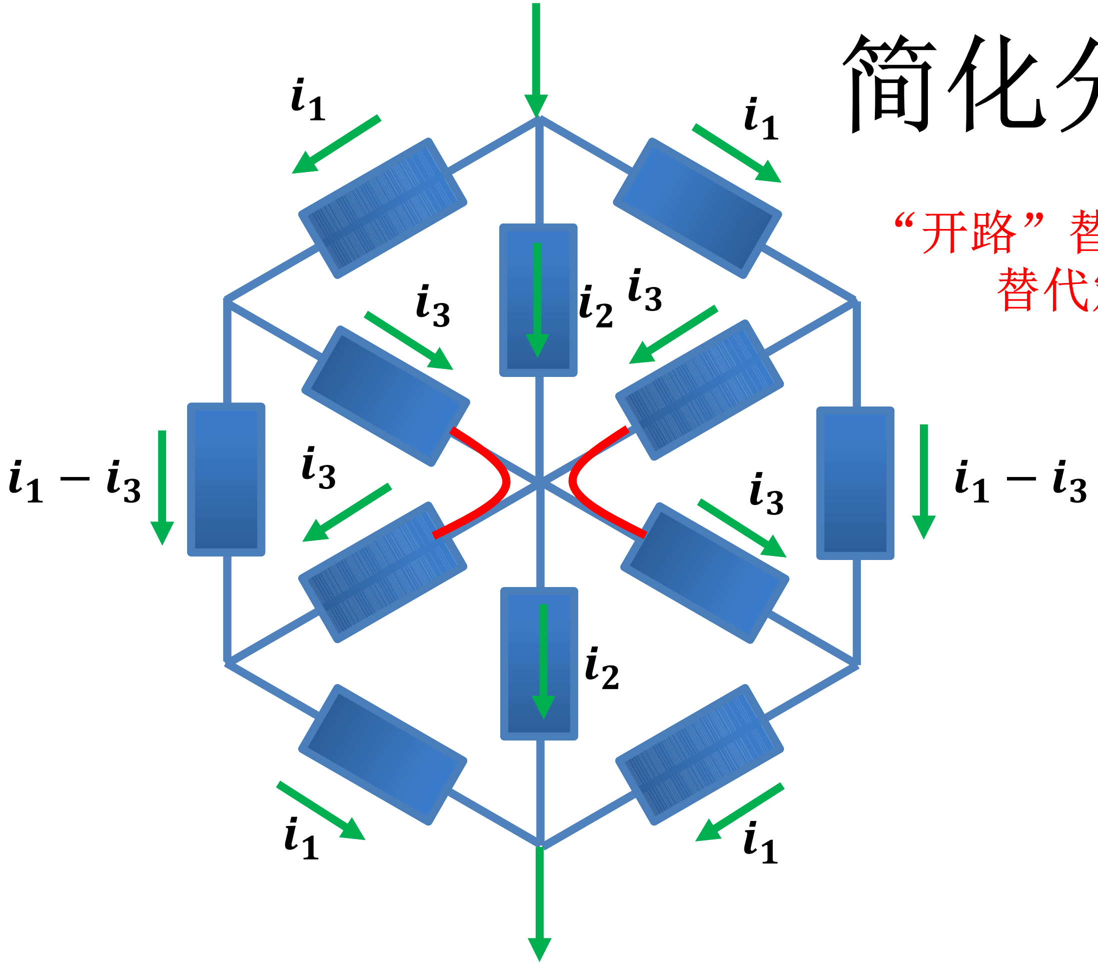
设计运放时，应尽量使得其
电压放大倍数足够高

作业3 求电阻

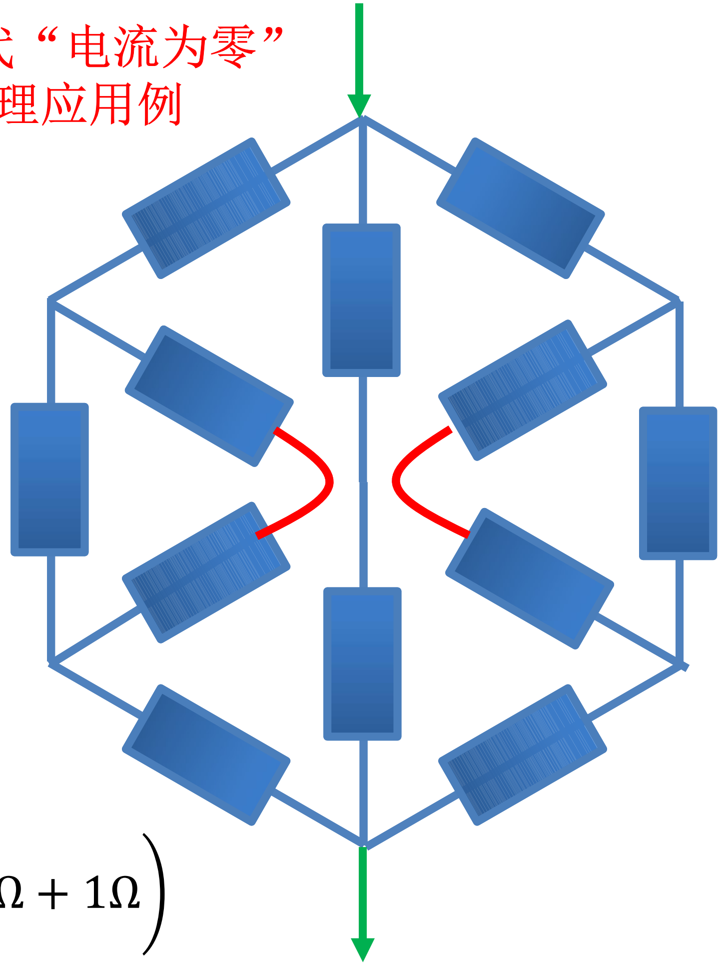


求如图所示电路A、B节点间的电阻。所有电阻均为1Ω。

简化分析要点：对称性



"开路"替代"电流为零"
替代定理应用例



$$R=\left(1\Omega+\frac{1\cdot 2}{1+2}\Omega+1\Omega\right)||(1\Omega+1\Omega)||\left(1\Omega+\frac{1\cdot 2}{1+2}\Omega+1\Omega\right)$$

$$=\frac{8}{3}\Omega||2\Omega||\frac{8}{3}\Omega=\frac{1}{\frac{3}{8}+\frac{4}{8}+\frac{3}{8}}\Omega=\frac{8}{10}\Omega=0.8\Omega$$

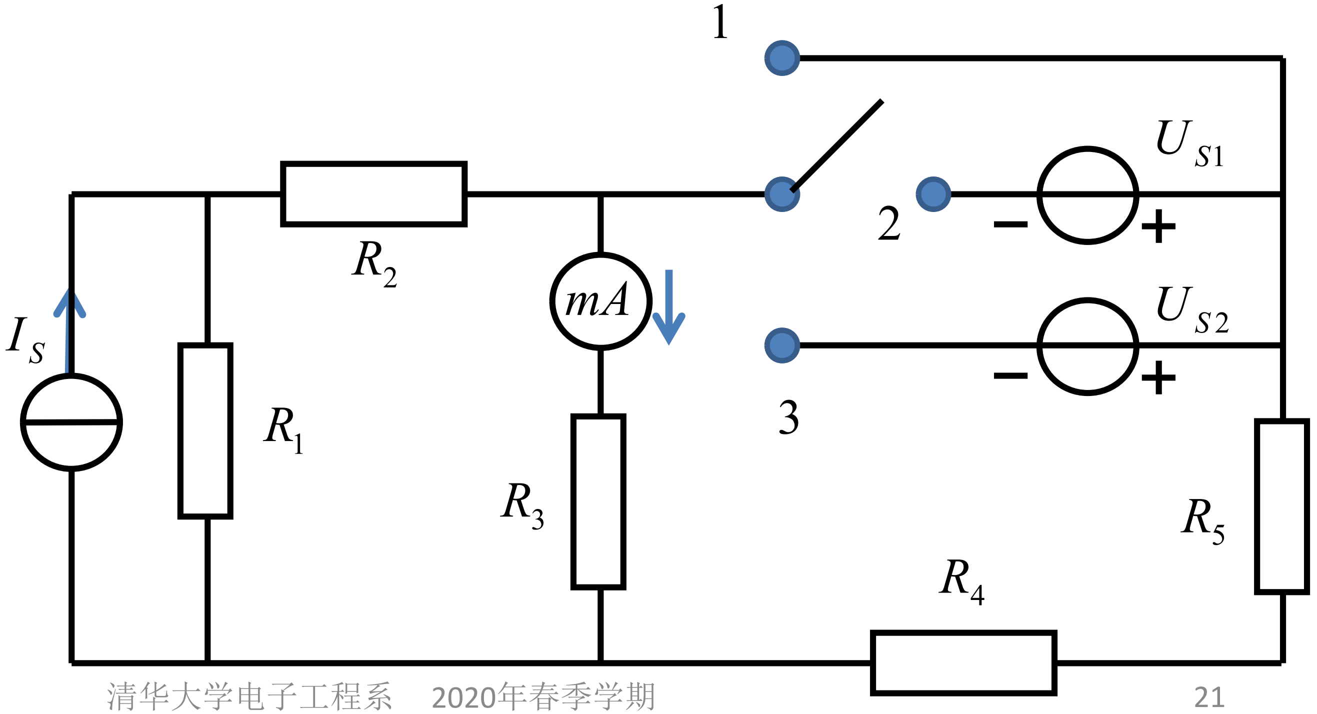
作业4

- 当开关置于位置1时，毫安表读数为40mA，当开关置于位置2时，毫安表读数为-60mA，问：当开关置于位置3时，毫安表读数是多少？
  - 毫安表可视为短路线，毫安表检测该短路线电流并显示出来

$U_{S1}=10V$

$U_{S2}=15V$

- 分析：
- 1、这是线性电路
- 2、有多个源
- 叠加定理可以采用



假设仅I_S作用时，毫安表读数为$I(I_S,0)$
假设仅U_{S1}作用时，毫安表读数为$I(0,U_{S1})$
假设仅U_{S2}作用时，毫安表读数为$I(0,U_{S2})$

$$U_{S1}=10V$$

$$U_{S2}=15V$$

开关位于位置1时，仅I_S起作用，毫安表读数为40mA

$$I(I_S,0)=40mA$$

线性电路满足叠加性：开关位于位置2时，I_S和U_{S1}同时起作用，毫安表读数为-60mA

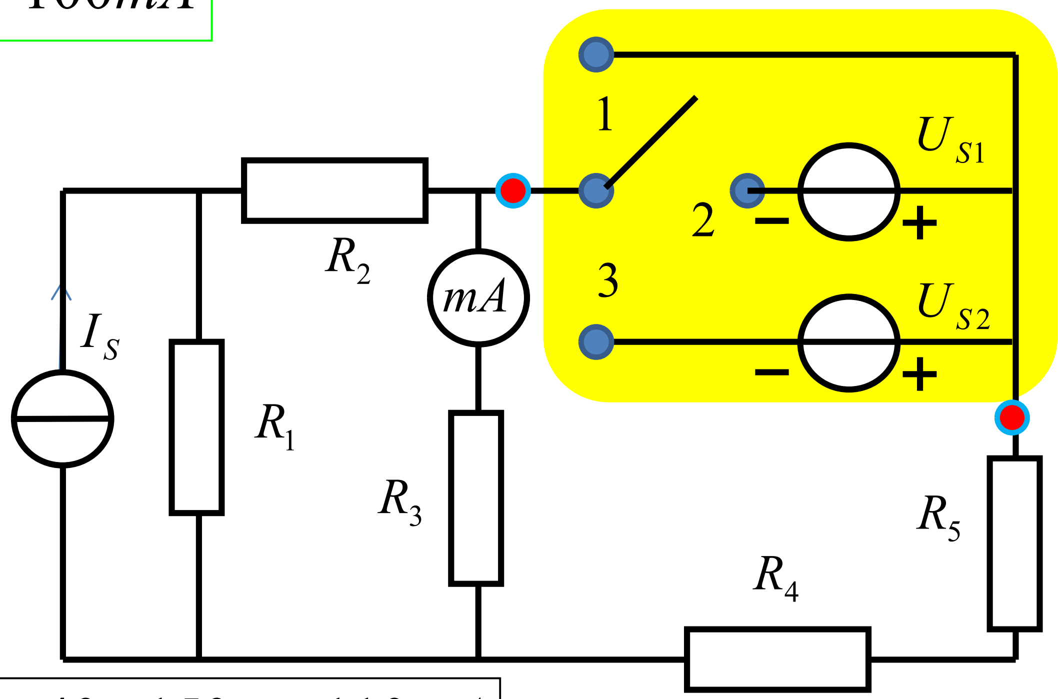
$$I(I_S,U_{S1})=I(I_S,0)+I(0,U_{S1})=-60mA$$

$$I(0,U_{S1})=-100mA$$

线性电路满足齐次性

$$I(0,U_{S2})=U_{S2}I(0,1)$$

$$=\frac{U_{S2}}{U_{S1}}I(0,U_{S1})=-150mA$$



线性电路满足叠加性：开关位于位置3时，I_S和U_{S2}同时起作用，毫安表读数为

$$I(I_S,U_{S2})=I(I_S,0)+I(0,U_{S2})=40-150=-110mA$$

习题课第六讲　　大纲

- 第四周作业讲解
- 简单二端口网络的网络参量简单列写方法（熟练度要求）
  - 附带讨论：奇异矩阵，病态问题
- 二端口网络噪声分析（例3.7.4）
- 一个三端口网络例（例3.7.2）

二端口网络参量求取方法

- 一般均可根据端口加压、加流测量定义求

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_{TH1} \\ v_{TH2} \end{bmatrix}$$

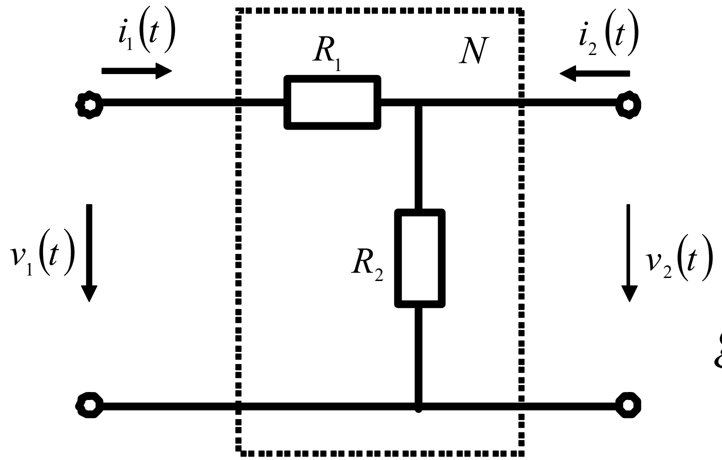
$$v_{TH1} = v_1 \Big|_{i_1=0, i_2=0}$$

$$v_{TH2} = v_2 \Big|_{i_1=0, i_2=0}$$

$$R_{11} = \frac{v_1}{i_1} \Big|_{i_2=0, v_{TH1}=0} \qquad R_{12} = \frac{v_1}{i_2} \Big|_{i_1=0, v_{TH1}=0}$$

$$R_{21} = \frac{v_2}{i_1} \Big|_{i_2=0, v_{TH2}=0} \qquad R_{22} = \frac{v_2}{i_2} \Big|_{i_1=0, v_{TH2}=0}$$

例1：理论课例题



$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\mathbf{v} = \mathbf{z}\mathbf{i}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\mathbf{i} = \mathbf{y}\mathbf{v}$$

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

$$R_{11} = \frac{v_1}{i_1}\bigg|_{i_2=0} = R_1 + R_2$$

$$R_{21} = \frac{v_2}{i_1}\bigg|_{i_2=0} = R_2$$

$$R_{12} = \frac{v_1}{i_2}\bigg|_{i_1=0} = R_2$$

$$R_{22} = \frac{v_2}{i_2}\bigg|_{i_1=0} = R_2$$

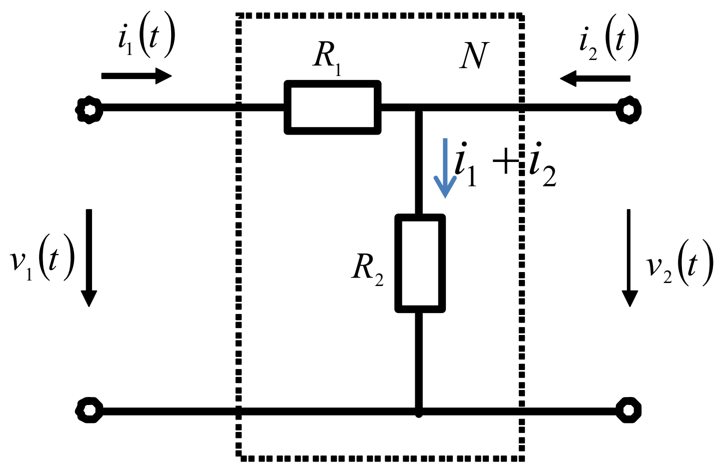
$$g_{11} = \frac{i_1}{v_1}\bigg|_{i_2=0} = \frac{1}{R_1 + R_2}$$

$$g_{21} = \frac{v_2}{v_1}\bigg|_{i_2=0} = \frac{R_2}{R_1 + R_2}$$

$$g_{12} = \frac{i_1}{i_2}\bigg|_{v_1=0} = -\frac{G_1}{G_1 + G_2} = -\frac{R_2}{R_1 + R_2}$$

$$g_{22} = \frac{v_2}{i_2}\bigg|_{v_1=0} = \frac{R_1 R_2}{R_1 + R_2}$$

简单电路



复杂电路、封装黑匣子严格按定义测量获得网络参量

可见其内部结构的简单电路，可直接观察，用**KVL/KCL/OL**直接获得端口描述方程

$$v_1 = i_1 R_1 + (i_1 + i_2) R_2 = i_1 (R_1 + R_2) + i_2 R_2$$

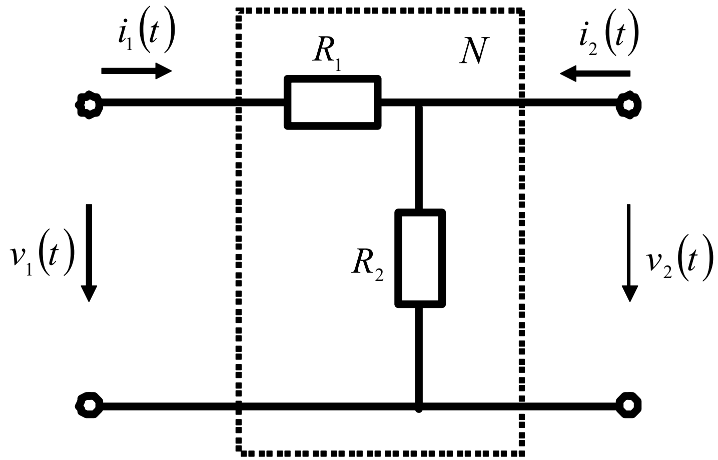
$$v_2 = (i_1 + i_2) R_2 = i_1 R_2 + i_2 R_2$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

根据观察，直接获得**z**参量矩阵
相当于两个端口同时加流测试

矩阵求逆，获得**y**参量矩阵

$$\mathbf{y} = \mathbf{z}^{-1} = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 \end{bmatrix}^{-1} = \frac{1}{R_1 R_2} \begin{bmatrix} R_2 & -R_2 \\ -R_2 & R_1 + R_2 \end{bmatrix} = \begin{bmatrix} G_1 & -G_1 \\ -G_1 & G_1 + G_2 \end{bmatrix}$$



直接列写端口描述方程

$$v_1 = i_1(R_1 + R_2) + i_2 R_2$$

$$v_2 = i_1 R_2 + i_2 R_2$$

$$i_2 R_2 = v_2 - i_1 R_2$$

$$i_2 = -i_1 + \frac{1}{R_2} v_2$$

$$v_1 = i_1(R_1 + R_2) + \left(-i_1 + \frac{1}{R_2} v_2\right) R_2$$

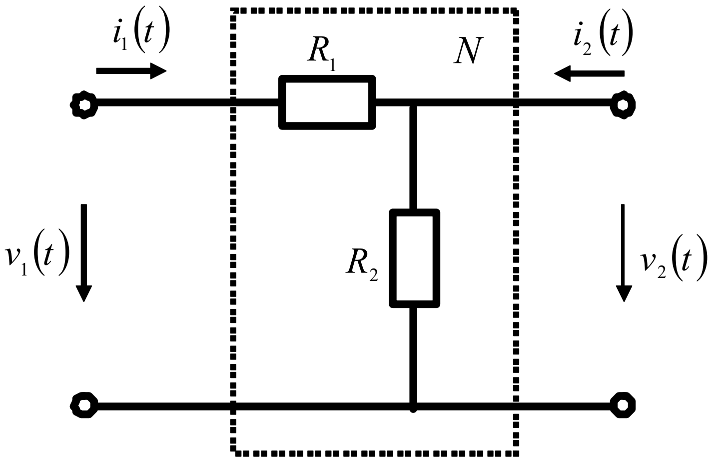
$$= i_1 R_1 + v_2$$

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} R_1 & 1 \\ -1 & G_2 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \mathbf{h} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

$$\mathbf{g} = \mathbf{h}^{-1} = \frac{1}{1 + \frac{R_1}{R_2}} \begin{bmatrix} \frac{1}{R_2} & -1 \\ 1 & R_1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{R_2 + R_1} & -\frac{R_2}{R_2 + R_1} \\ \frac{R_2}{R_2 + R_1} & \frac{R_1 R_2}{R_2 + R_1} \end{bmatrix}$$

传输矩阵



$$v_1 = i_1(R_1 + R_2) + i_2 R_2$$

$$v_2 = i_1 R_2 + i_2 R_2$$

$$i_1 R_2 = v_2 - i_2 R_2$$

$$i_1 = \frac{1}{R_2} v_2 - i_2$$

$$v_1 = i_1(R_1 + R_2) + i_2 R_2$$

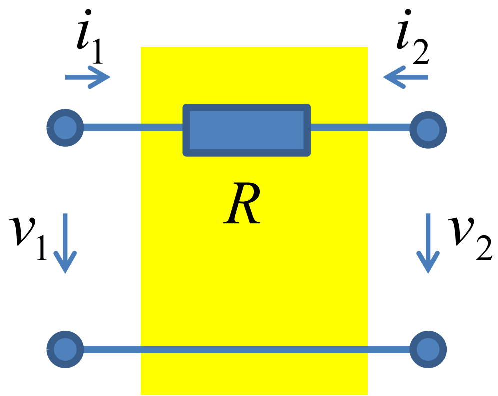
$$= \frac{R_1 + R_2}{R_2} v_2 - R_1 i_2$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} \frac{R_1 + R_2}{R_2} & R_1 \\ \frac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

更简单的方法是**ABCD**相乘获取

$$\mathbf{abcd} = \begin{bmatrix} \frac{R_1 + R_2}{R_2} & -R_1 \\ -\frac{1}{R_2} & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & R_1 \\ \frac{1}{R_2} & \frac{R_1 + R_2}{R_2} \end{bmatrix}$$

例2：串臂电阻例：奇异矩阵



$$i_1 = -i_2$$

$$v_1 = i_1 R + v_2 = v_2 - i_2 R$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} \qquad \mathbf{ABCD} = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$$

$$i_2 = \frac{1}{R} v_2 - \frac{1}{R} v_1 = G v_2 - G v_1$$

$$i_1 = -i_2 = G v_1 - G v_2$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} G & -G \\ -G & G \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} G & -G \\ -G & G \end{bmatrix}$$

$$\mathbf{z} = \mathbf{y}^{-1} = ?$$

病态矩阵 ill-conditioned Matrix

- 奇异矩阵
  - **Singular Matrix**：行列式为0的矩阵
    - 奇异矩阵可导致无解或无穷多解

- 病态矩阵

病态问题：无解、无穷多解、数值求解误差极大

  - **Ill-conditioned Matrix**：求逆计算误差极大的矩阵
    - 病态矩阵一般都是一些接近奇异矩阵的矩阵，极小的数值偏差导致极大的计算误差

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

数值计算结果和真实解误差极大

为何奇异（可能病态）

存在强制性约束条件

- 极致化抽象
  - 短路：**R=0**　　**G=∞**　　$v=0$　和**i**无关
  - 开路：**G=0**　　**R=∞**　　$i=0$　和**v**无关
  - 恒压：**R=0**　　$v=v_S$　和**i**无关
  - 恒流：**G=0**　　$i=i_S$　和**v**无关

- 本来是单端口网络，强行拓展为二端口网络

$$\mathbf{y}=\begin{bmatrix} G & -G \\ -G & G \end{bmatrix}$$

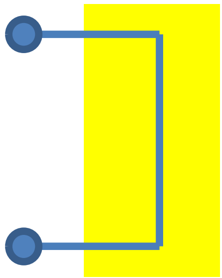
$$\mathbf{z}=\mathbf{y}^{-1}=?$$

$i_2+i_1=0$　和**v**无关

i满足该强制条件，两个**i**不独立，无法做自变量

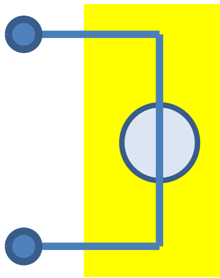
病态网络

存在强制约束



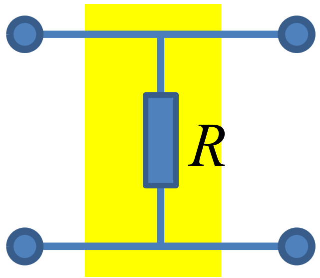
$v = 0$

$R = 0$



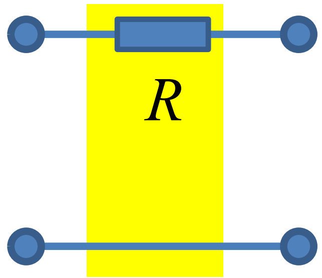
$v = v_S$

$R = 0$



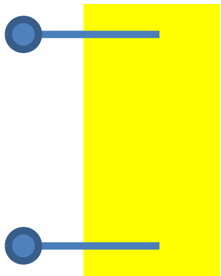
$v_1 = v_2$

$$\mathbf{z} = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$$



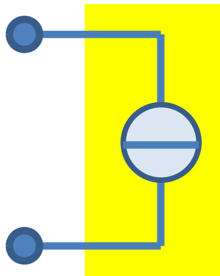
$i_1 = -i_2$

$$\mathbf{y} = \begin{bmatrix} G & -G \\ -G & G \end{bmatrix}$$



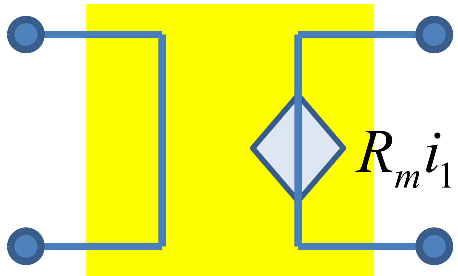
$i = 0$

$G = 0$



$i = i_S$

$G = 0$



$v_1 = 0$

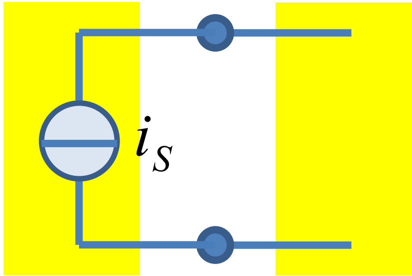
$v_2 = R_m i_1$

$$\mathbf{z} = \begin{bmatrix} 0 & 0 \\ R_m & 0 \end{bmatrix}$$

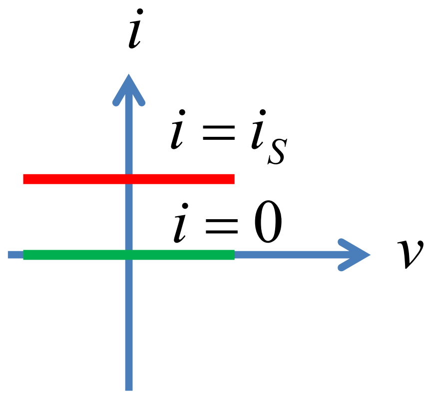
这些病态网络，要么是极致化抽象走极端，要么是多端口过度拓展，使得其参量矩阵奇异

病态二端口线性网络的**z**、**y**、**g**、**h**、**ABCD**（或**abcd**）参量矩阵至少其一是不存在的

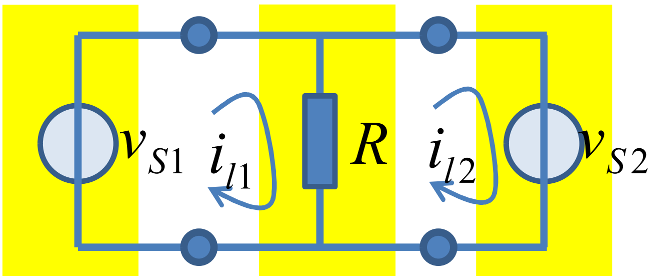
小心病态网络的连接



恒流源和开路对接



无解：伏安特性曲线无交点：违反**KCL**



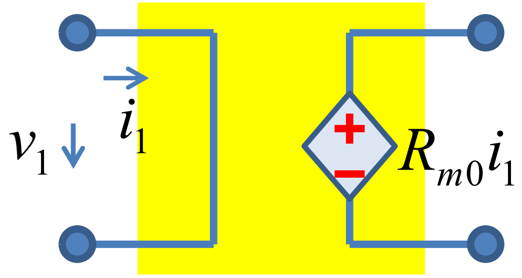
恒压源和恒压源对接

$$\begin{bmatrix} R & -R \\ -R & R \end{bmatrix}\begin{bmatrix} i_{l1} \\ i_{l2} \end{bmatrix} = \begin{bmatrix} v_{S1} \\ -v_{S2} \end{bmatrix}$$

$v_{S1} \neq v_{S2}$　无解：违反**KVL**

$v_{S1} = v_{S2}$　无穷多解

记忆：理想受控源

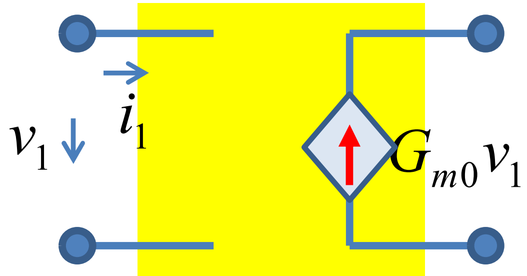


$$v_1 = 0$$
$$v_2 = R_{m0}i_1$$

$$\mathbf{z} = \begin{bmatrix} 0 & 0 \\ R_{m0} & 0 \end{bmatrix}$$

跨阻增益

$$\mathbf{ABCD} = \begin{bmatrix} 0 & 0 \\ \dfrac{1}{R_{m0}} & 0 \end{bmatrix}$$

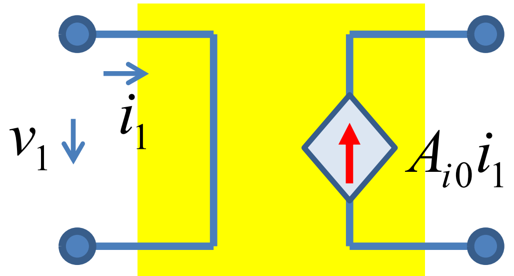


$$i_1 = 0$$
$$i_2 = -G_{m0}v_1$$

$$\mathbf{y} = \begin{bmatrix} 0 & 0 \\ -G_{m0} & 0 \end{bmatrix}$$

跨导增益

$$\mathbf{ABCD} = \begin{bmatrix} 0 & \dfrac{1}{G_{m0}} \\ 0 & 0 \end{bmatrix}$$

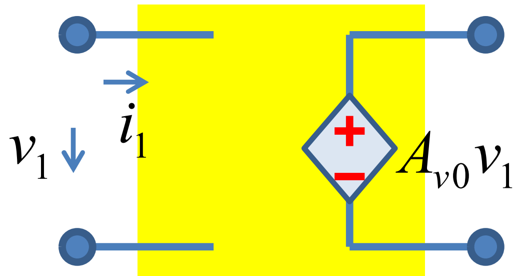


$$v_1 = 0$$
$$i_2 = -A_{i0}i_1$$

$$\mathbf{h} = \begin{bmatrix} 0 & 0 \\ -A_{i0} & 0 \end{bmatrix}$$

电流增益

$$\mathbf{ABCD} = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{1}{A_{i0}} \end{bmatrix}$$



$$i_1 = 0$$
$$v_2 = A_{v0}v_1$$

$$\mathbf{g} = \begin{bmatrix} 0 & 0 \\ A_{v0} & 0 \end{bmatrix}$$

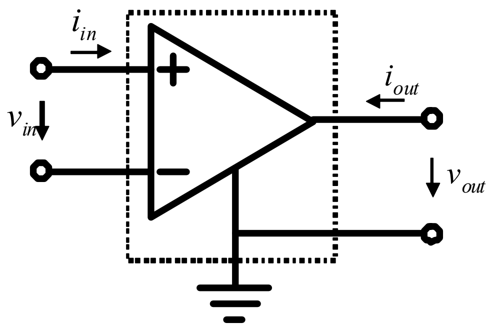
电压增益

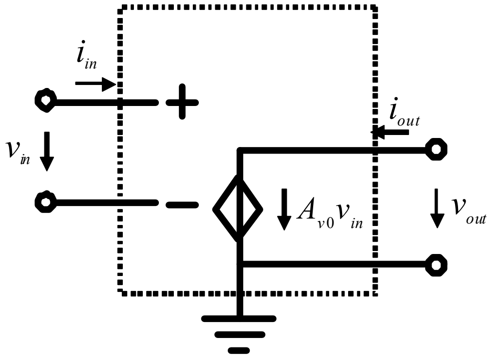
$$\mathbf{ABCD} = \begin{bmatrix} \dfrac{1}{A_{v0}} & 0 \\ 0 & 0 \end{bmatrix}$$

理想受控源有其唯一的加压加流测量参量矩阵
还有传输参量ABCD也存在外，其他参量不存在

例3：理想运算放大器

- 极致化抽象的电压放大器
  - 输入电阻无穷大，输出电阻为0，电压增益无穷大





$$\mathbf{g}=\begin{bmatrix}0 & 0\\ A_{v0} & 0\end{bmatrix}$$

$$\mathbf{ABCD}=\begin{bmatrix}\frac{1}{A_{v0}} & 0\\ 0 & 0\end{bmatrix}$$

$$i_1=0$$
$$v_2=A_{v0}v_1$$

$\longrightarrow$

$$i_1=0$$
$$v_1=\frac{1}{A_{v0}}v_2 \overset{A_v\to\infty}{=} 0$$

$\longrightarrow$

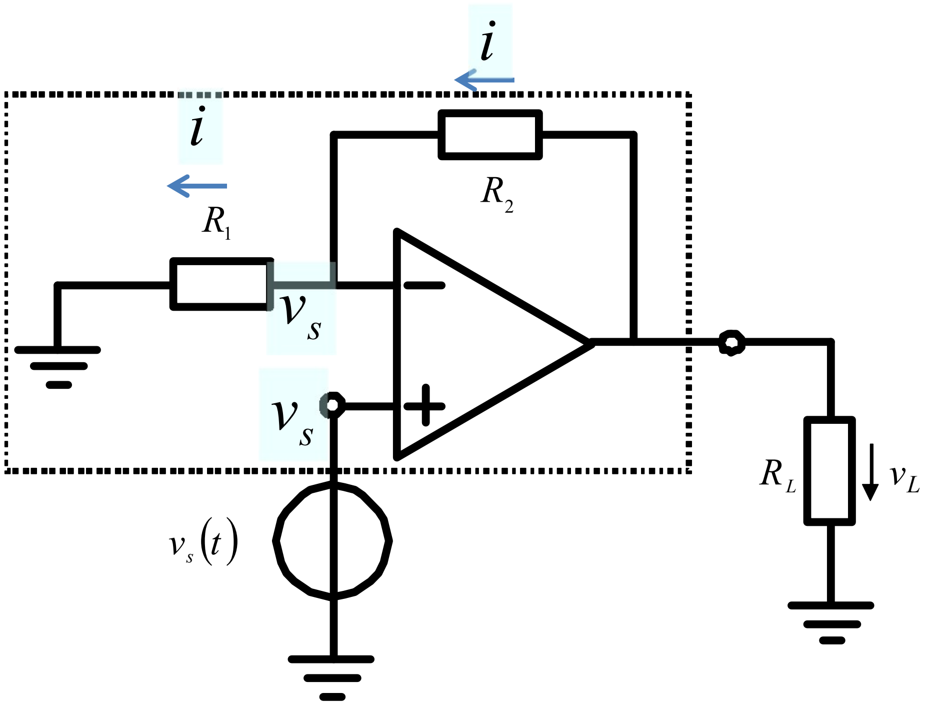
$$\mathbf{ABCD}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

理想运放只能用0传输矩阵表述

理想运放黄金法则

$i_1 = 0$ 虚断

$v_1 = 0$ 虚短



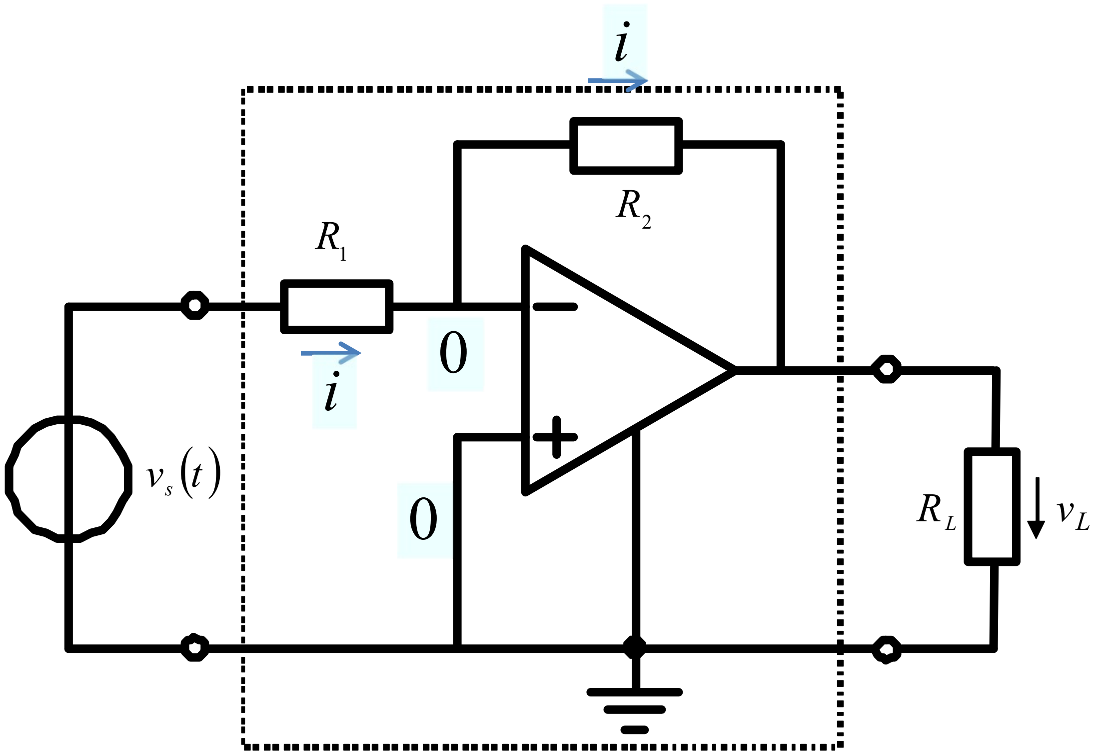
碰到运放电路，分析将变得极度简化：

直接采用理想运放模型获得原理性结果

$$\frac{v_L - v_S}{R_2} = i = \frac{v_S}{R_1} \Rightarrow v_L - v_S = R_2 \frac{v_S}{R_1} \Rightarrow v_L = \left(1 + \frac{R_2}{R_1}\right) v_S$$

同相电压放大器

理想运放黄金法则：虚断和虚短



741运放
输入电阻**2MΩ**
输出电阻**75Ω**
电压增益**200000**

极致化抽象

理想运放
输入电阻无穷大
输出电阻**0**
电压增益无穷大

$$\frac{v_S - 0}{R_1} = i = \frac{0 - v_L}{R_2} \quad \Rightarrow \quad v_L = -\frac{R_2}{R_1} v_S$$

反相电压放大器

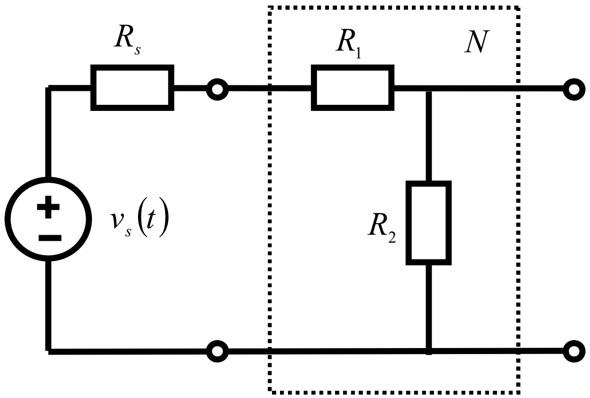
大纲

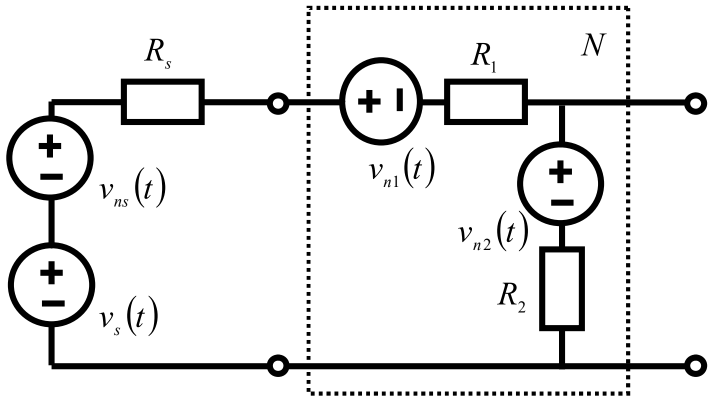
习题课讲解的噪声分析不做要求，只需了解有一个噪声、噪声系数的基本概念即可：知道分析微弱信号需要考虑噪声问题，知道目前的知识已经可以进行简单的噪声分析即可

- 第四周作业讲解
- 简单二端口网络的网络参量简单列写方法（熟练度要求）
- 二端口网络噪声分析（例**3.7.4**）
- 一个三端口网络例（例**3.7.2**）

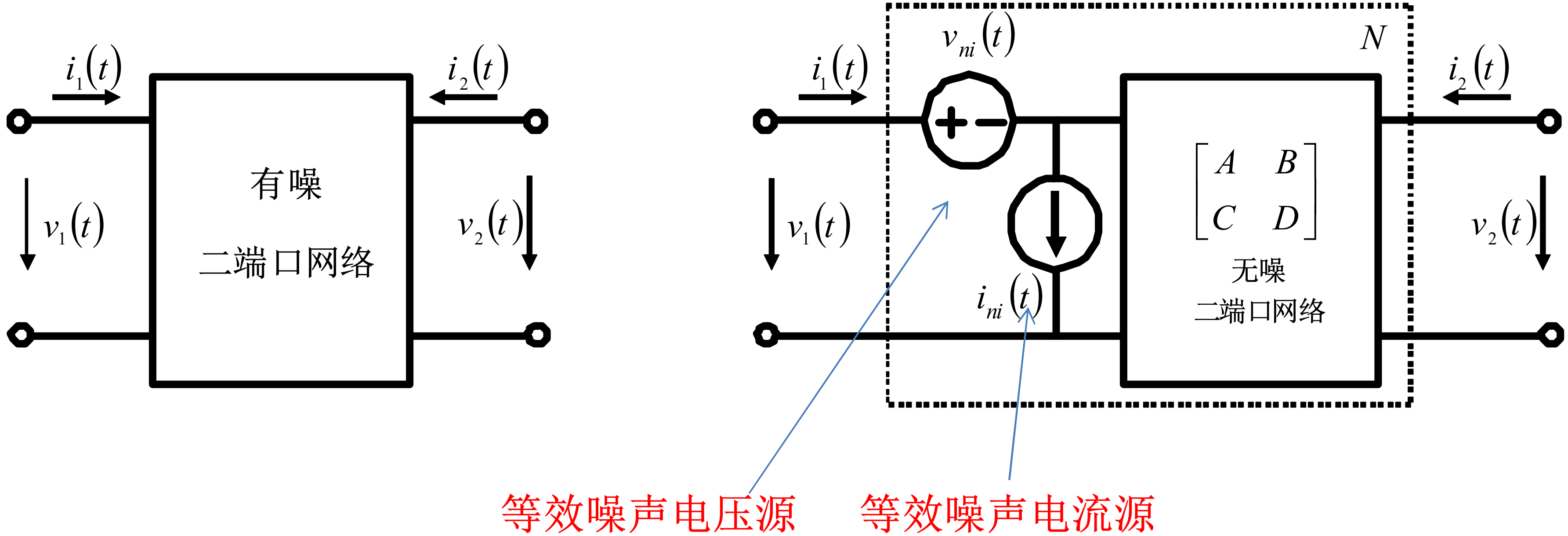
例4 二端口网络噪声性能分析 教材例3.7.4

- 对于如图所示二端口电阻网络，假设它处理的信号比较微弱，因而需要考虑网络内部噪声的影响。假设输入信号源的信噪比为$\mathbf{SNR_i}$，经过该网络作用后，输出信噪比$\mathbf{SNR_o}$比输入信噪比$\mathbf{SNR_i}$下降了多少？





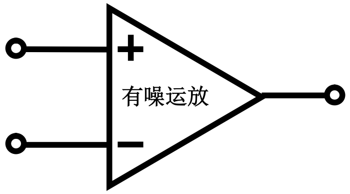
噪声分析

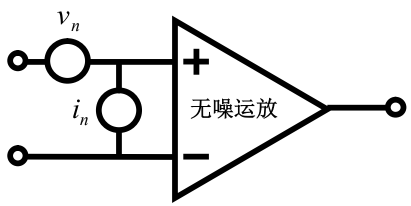


将二端口线性网络内部噪声折合到输入端考察是研究二端口线性网络噪声性能的通常做法：折合到输入端的噪声是可直接比较的，等效噪声源电压、源电流越大，噪声性能则越差

输出端比较噪声大小不适当：需要考虑网络本身的放大衰减等因素

运算放大器例





intersil：HA2600

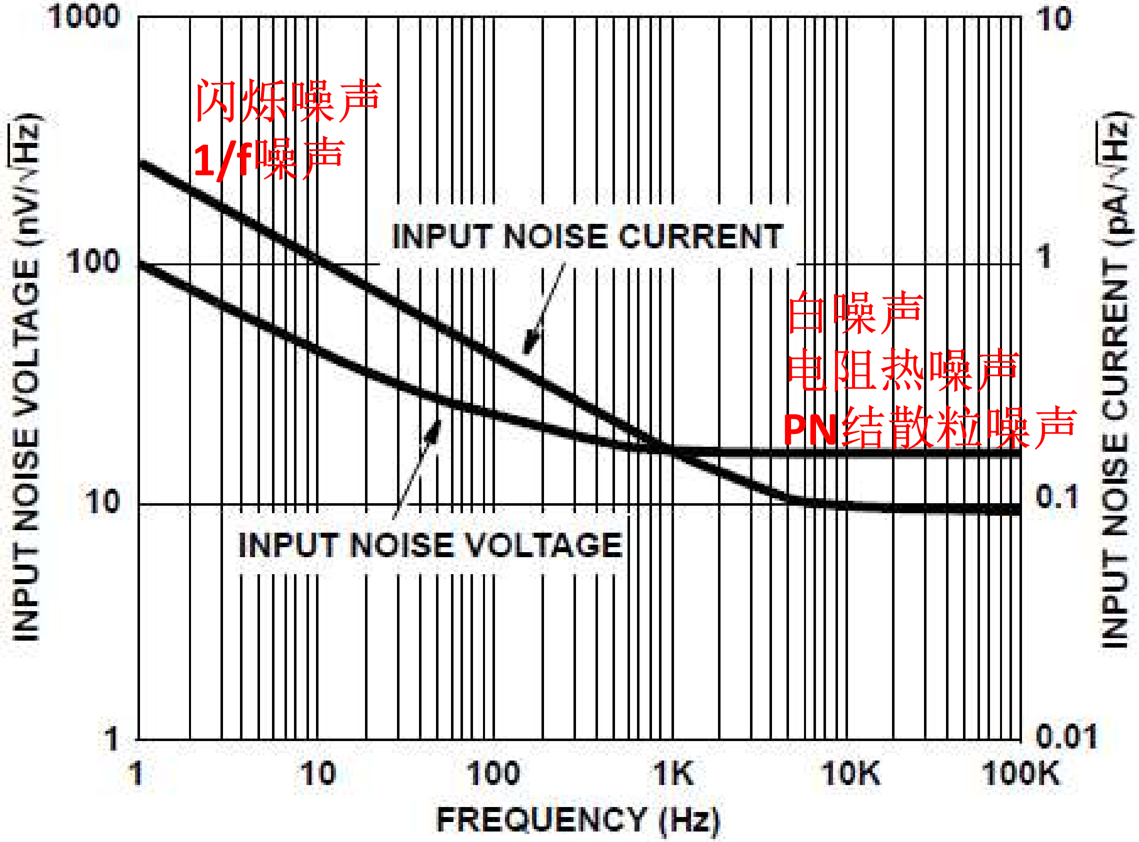


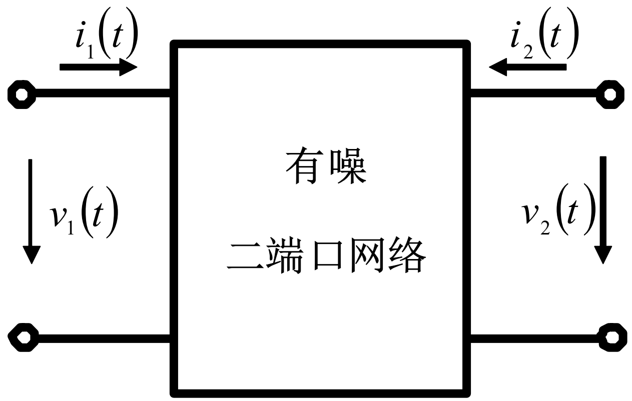
FIGURE 18. NOISE DENSITY vs FREQUENCY

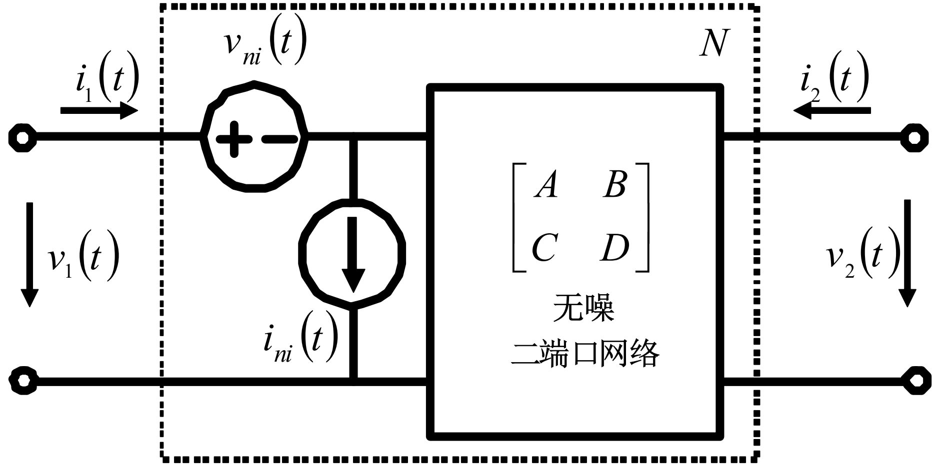
$$\sqrt{\frac{d\overline{v_{n,in}^2}}{df}} : nV/\sqrt{Hz}$$

$$\sqrt{\frac{d\overline{i_{n,in}^2}}{df}} : pA/\sqrt{Hz}$$

如果二端口网络处理的是弱信号，噪声问题则不可回避

等效噪声源

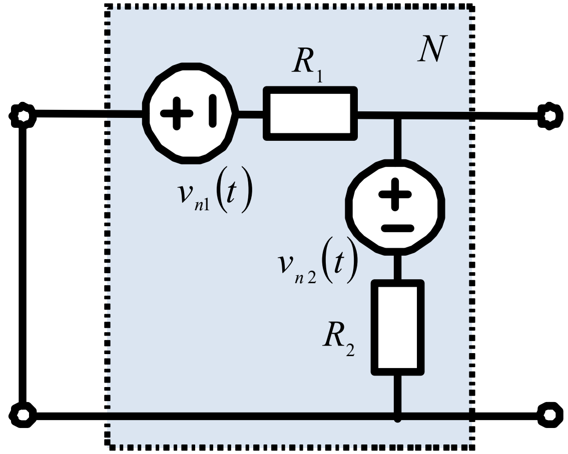


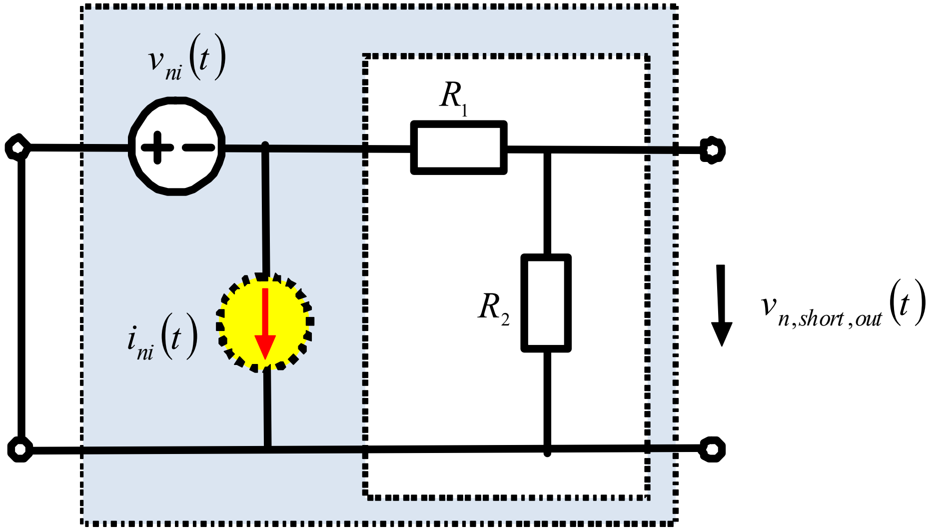


$$v_{n,short,out} = A_{v0} \cdot (-v_{ni}) \qquad v_{ni} = -\frac{v_{n,short,out}}{A_{v0}} = -Av_2\Big|_{v_1=0,i_2=0} = -\frac{v_2\big|_{v_1=0,i_2=0}}{g_{21}}$$

$$v_{n,open,out} = R_{m0} \cdot (-i_{ni}) \qquad i_{ni} = -\frac{v_{n,open,out}}{R_{m0}} = -Cv_2\Big|_{i_1=0,i_2=0} = -\frac{v_2\big|_{i_1=0,i_2=0}}{z_{21}}$$

输入短路，输出开路电压



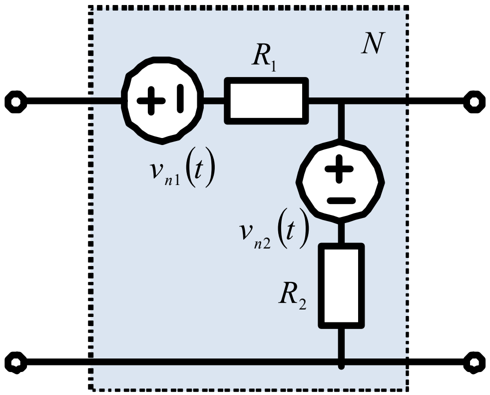


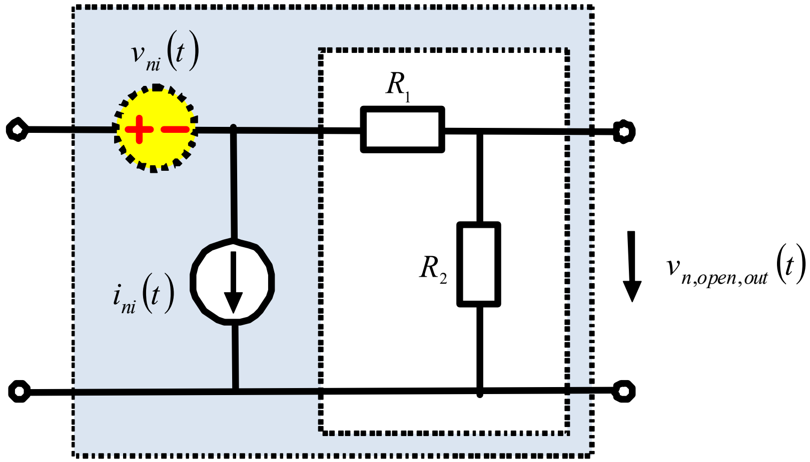
$$v_{n,short,out}=\left(-\frac{v_{n1}}{R_1}+\frac{v_{n2}}{R_2}\right)\frac{R_1R_2}{R_1+R_2}=\frac{R_1}{R_1+R_2}v_{n2}-\frac{R_2}{R_1+R_2}v_{n1}$$

$$v_{n,short,out}=-v_{ni}\left(\frac{R_2}{R_1+R_2}\right)\qquad v_{ni}=-\frac{R_1+R_2}{R_2}v_{n,short,out}=v_{n1}-\frac{R_1}{R_2}v_{n2}$$

$g_{21}=A_{v0}$

输入开路，输出开路电压





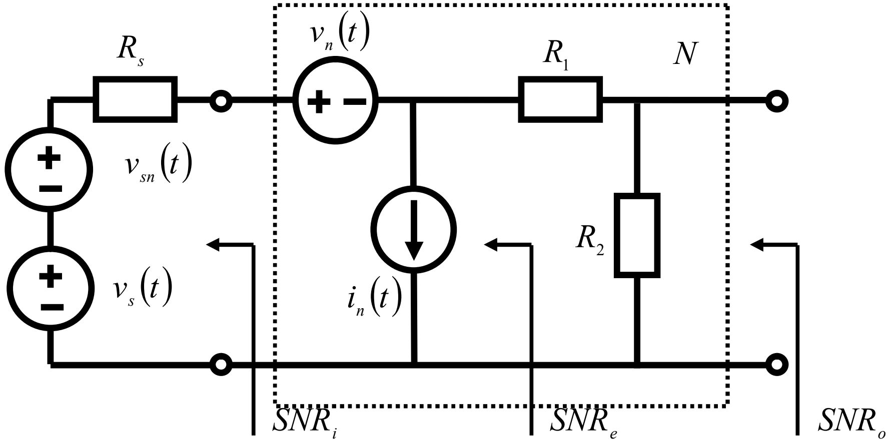
$$v_{n,open,out} = v_{n2}$$

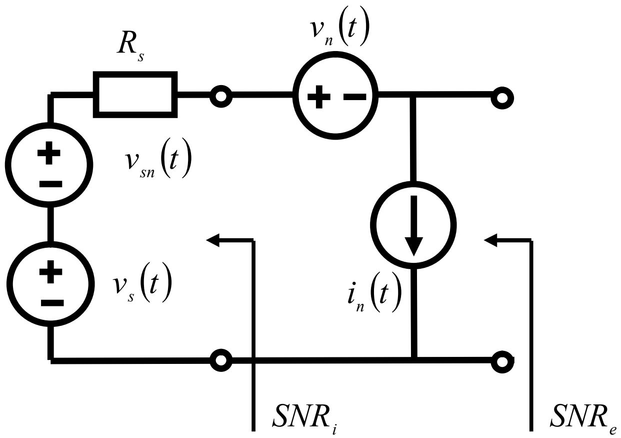
$$v_{n,open,out} = -i_{ni}R_2$$

$$i_{ni} = -\frac{v_{n2}}{R_2}$$

$z_{21}=R_{m0}$

噪声系数





二端口网络附加了额外噪声，使得输出端信号质量下降

$$v_n(t) = v_{n1} - \frac{R_1}{R_2} v_{n2}$$

$$i_n(t) = -\frac{v_{n2}}{R_2}$$

$$F_n = \frac{SNR_i}{SNR_o} \qquad NF = 10\log_{10} F_n$$

信噪比：信号质量的描述

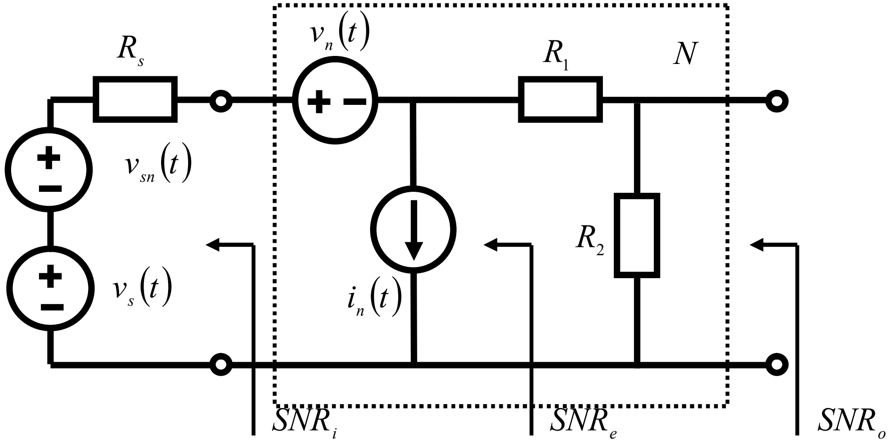
噪声系数：二端口网络噪声性能的描述

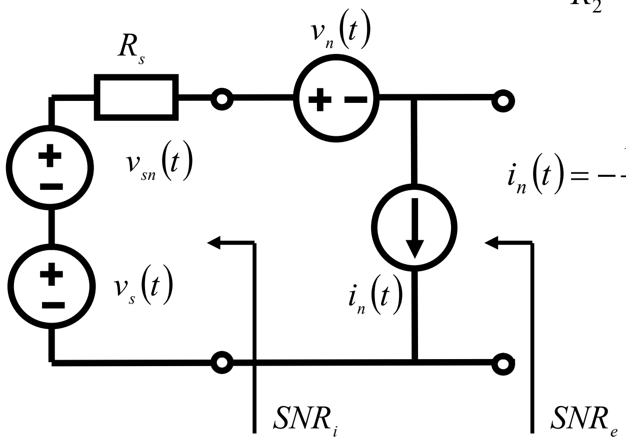
$$F_n = \frac{SNR_i}{SNR_o} = \frac{SNR_i}{SNR_e}$$

噪声折合到输入端才有可比性

信号与噪声

$$v_n(t) = v_{n1} - \frac{R_1}{R_2} v_{n2}$$





$$i_n(t) = -\frac{v_{n2}}{R_2}$$

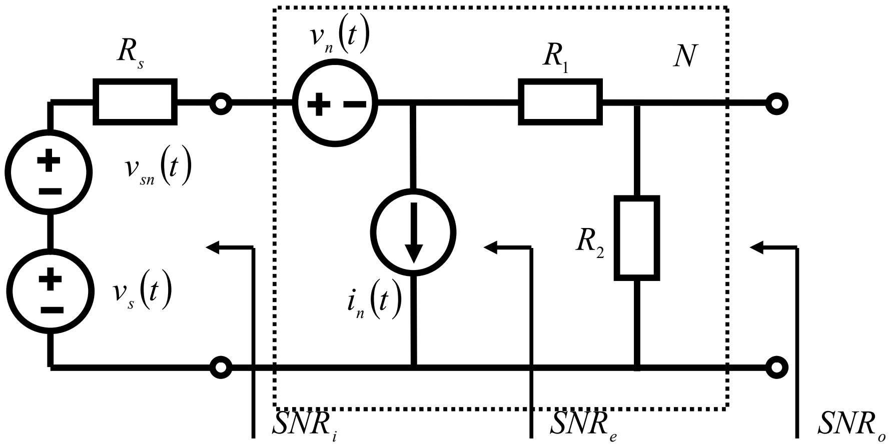
$$\overline{\left(v_s(t)+v_{ns}(t)-v_n(t)-i_n(t)R_s\right)^2} = \overline{v_s^2(t)} + \overline{v_{ns}^2(t)} + \overline{\left(v_n(t)+i_n(t)R_s\right)^2} = \overline{v_s^2(t)} + \overline{v_{ns}^2(t)} + \overline{\left(v_{n1}(t) - v_{n2}(t)\frac{R_s+R_1}{R_2}\right)^2}$$

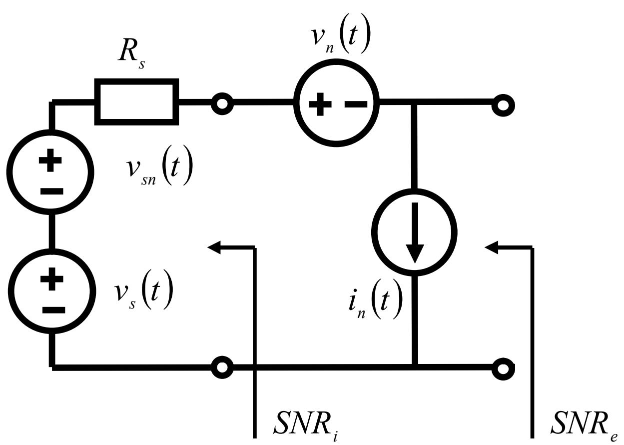
$$= \overline{v_s^2(t)} + \overline{v_{ns}^2(t)} + \overline{v_{n1}^2(t)} + \overline{v_{n2}^2(t)}\left(\frac{R_s+R_1}{R_2}\right)^2 = V_{s,rms}^2 + 4kTR_s\Delta f + 4kTR_1\Delta f + 4kTR_2\Delta f\left(\frac{R_s+R_1}{R_2}\right)^2$$

$$= V_{s,rms}^2 + 4kT\left(R_s + R_1 + \frac{(R_s+R_1)^2}{R_2}\right)\Delta f$$

$$SNR_i = \frac{P_{sim}}{P_{nim}} = \frac{\overline{v_s^2(t)}}{\overline{v_{ns}^2(t)}} = \frac{V_{s,rms}^2}{4kTR_s\Delta f} \qquad SNR_e = \frac{P_{sem}}{P_{nem}} = \frac{\overline{v_s^2(t)}}{\overline{v_{ne}^2(t)}} = \frac{V_{s,rms}^2}{4kT\left(R_s + R_1 + \frac{(R_s+R_1)^2}{R_2}\right)\Delta f}$$

噪声系数





$$SNR_i = \frac{P_{sim}}{P_{nim}} = \frac{\overline{v_s^2(t)}}{\overline{v_{ns}^2(t)}} = \frac{V_{s,rms}^2}{4kTR_s\Delta f} \qquad SNR_e = \frac{P_{sem}}{P_{nem}} = \frac{\overline{v_s^2(t)}}{\overline{v_{ne}^2(t)}} = \frac{V_{s,rms}^2}{4kT\left(R_s + R_1 + \frac{(R_s+R_1)^2}{R_2}\right)\Delta f}$$

$$F_n = \frac{SNR_i}{SNR_o} = \frac{SNR_i}{SNR_e} = \frac{R_s + R_1 + \frac{(R_s+R_1)^2}{R_2}}{R_s} = 1 + \frac{R_1}{R_s} + \frac{(R_s+R_1)^2}{R_sR_2} > 1$$

信号质量恶化

关于噪声系数的结论

$$F_n = 1 + \frac{R_1}{R_s} + \frac{(R_s + R_1)^2}{R_s R_2}$$

- 噪声系数大于**1**
  - 通带内信号经过一级处理后，信号质量变差
- 噪声系数和信源内阻有关，存在最佳信源内阻，使得噪声系数最小
  - $R_S=R_{Sopt}$,最小噪声系数匹配

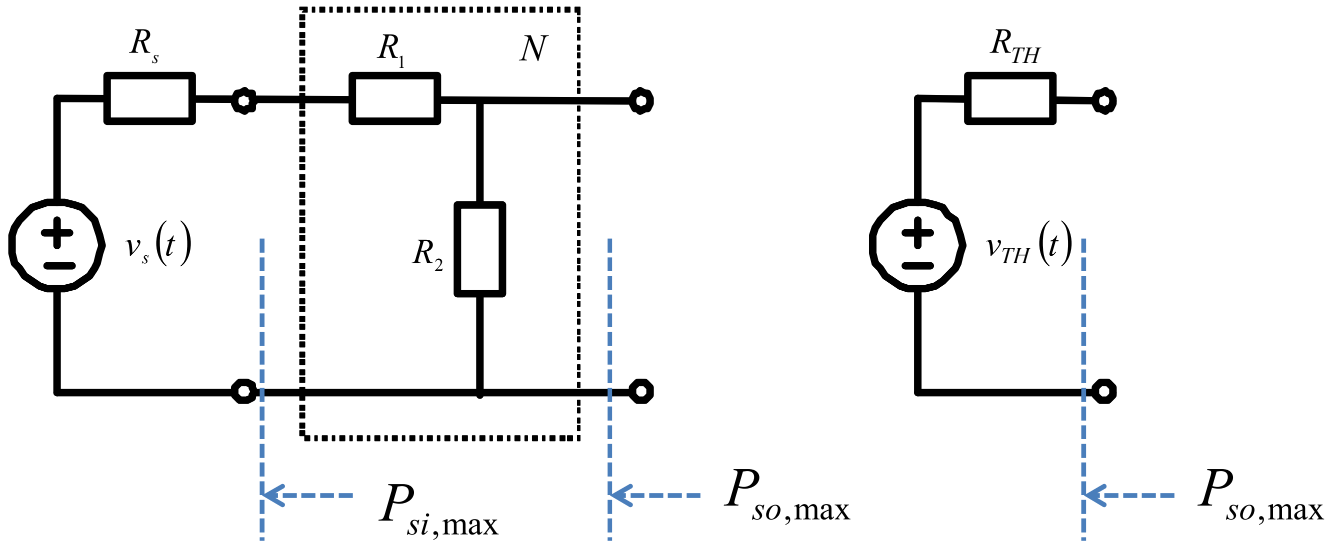
练习：自行分析上例的最佳信源内阻

- 对于只有电阻热噪声的无源网络，噪声系数等于衰减系数

$$F_n = \frac{SNR_i}{SNR_o} = \frac{P_{sim}/P_{nim}}{P_{som}/P_{nom}} = \frac{P_{sim}}{P_{som}} = \frac{1}{G_A} = L$$

$$P_{nim} = P_{nom} = kT\Delta f$$

方法2：无源网络衰减系数

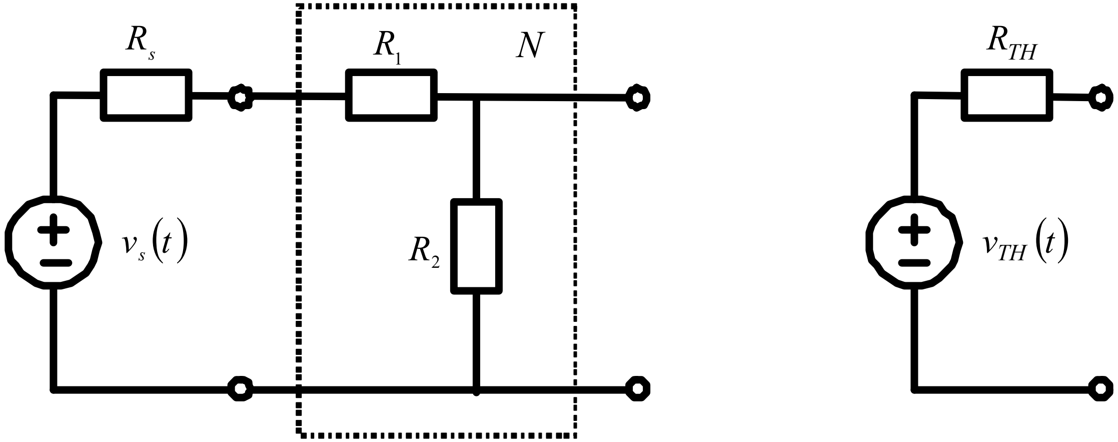


$$P_{si,\max} = \frac{V_{s,rms}^2}{4R_s}$$

$$v_{TH}(t) = \frac{R_2}{R_s + R_1 + R_2} v_s(t)$$

$$P_{so,\max} = \frac{V_{TH,rms}^2}{4R_{TH}}$$

$$R_{TH} = \frac{(R_1 + R_s)R_2}{R_s + R_1 + R_2}$$

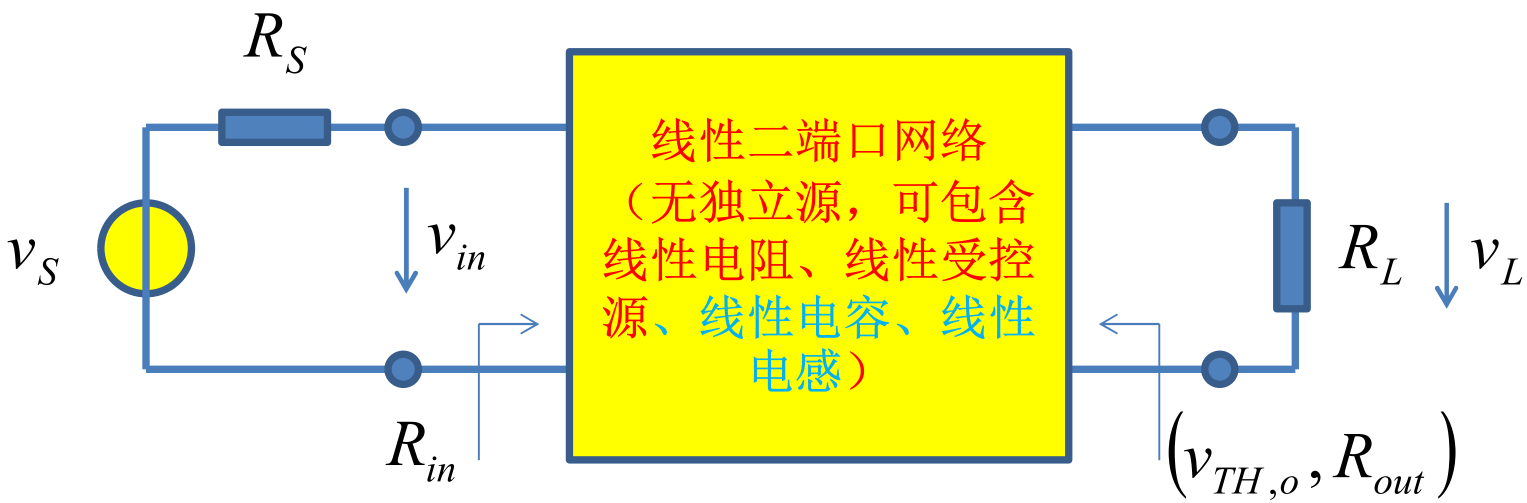


$$P_{so,\max}=\frac{V_{TH,rms}^2}{4R_{TH}}=\frac{\left(\frac{R_2}{R_s+R_1+R_2}\right)^2 V_{s,rms}^2}{4\frac{(R_1+R_s)R_2}{R_s+R_1+R_2}}=\frac{\left(\frac{R_2}{R_s+R_1+R_2}\right)^2 R_s}{\frac{(R_1+R_s)R_2}{R_s+R_1+R_2}}\frac{V_{s,rms}^2}{4R_s}$$

$$=\frac{R_sR_2}{(R_1+R_s)(R_s+R_1+R_2)}P_{si,\max}$$

$$F_n=L=\frac{1}{G_A}=\frac{P_{si,\max}}{P_{so,\max}}=\frac{(R_1+R_s)(R_s+R_1+R_2)}{R_sR_2}=1+\frac{R_1}{R_s}+\frac{(R_s+R_1)^2}{R_sR_2}$$

功率增益的三种定义



$$G_T = \frac{P_L}{P_{sim}}$$

转换功率增益**Transducer power gain**

通常定义：如滤波器，射频放大器、…

$$G_A = \frac{P_{som}}{P_{sim}}$$

资用功率增益**Available power gain**

考察线性二端口网络噪声性能时采用

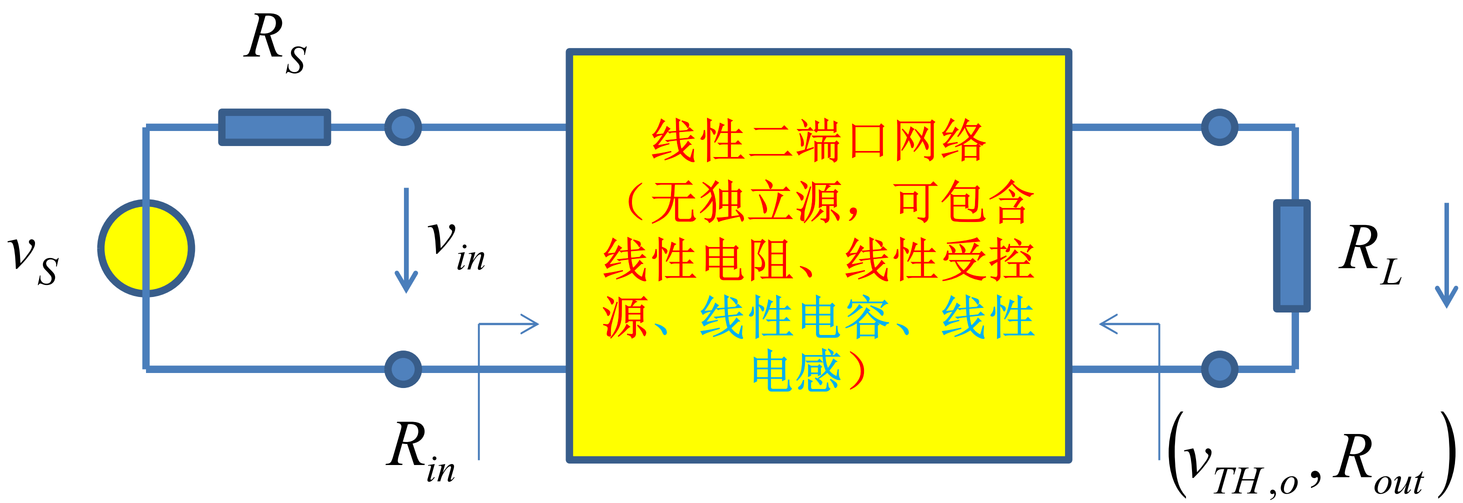
$$G_p = \frac{P_L}{P_{in}}$$

工作功率增益**Operating power gain**

低频线性电路常用，能量转换电路能量转换效率定义

$$P_L = \frac{V_{L,rms}^2}{R_L} \qquad P_{sim} = \frac{V_{S,rms}^2}{4R_S} \qquad P_{som} = \frac{V_{TH,o,rms}^2}{4R_{out}} \qquad P_{in} = \frac{V_{in,rms}^2}{R_{in}}$$

最大功率增益



$$G_T = \frac{P_L}{P_{sim}}$$

$$G_A = \frac{P_{som}}{P_{sim}}$$

$$G_p = \frac{P_L}{P_{in}}$$

$$G_T \le G_A$$

$$G_T \le G_p$$

两个端口同时最大功率传输匹配时，三个功率增益等同

$$R_{in} = R_S \qquad R_{out} = R_L$$

$$G_T = G_A = G_p = G_{p,\max}$$

特征阻抗与最大功率增益

多端口网络的特征阻抗定义：Z_{01}，Z_{02}，…，Z_{0n}为n端口网络n个端口的特征阻抗，只需满足如下要求：当其他n-1个端口端接各自的特征阻抗时，从第i个端口看入的输入阻抗为Z_{0i}。

$$Z_{in} = Z_{01} \qquad\qquad Z_{out} = Z_{02}$$

特征阻抗是网络自身属性，可以证明：对于线性阻性二端口网络，如果取$R_S=Z_{01}$，$R_L=Z_{02}$，则可获得最大功率增益

二端口网络的特征阻抗

$Z_{in} = Z_{01}$ $\quad$ $Z_{out} = Z_{02}$

假设z参量已知

$$Z_{01} = Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + R_L} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_{02}}$$

$$Z_{02} = Z_{out} = z_{22} - \frac{z_{21}z_{12}}{z_{11} + R_S} = z_{22} - \frac{z_{21}z_{12}}{z_{11} + Z_{01}}$$

二端口网络特征阻抗

$$Z_{01}=z_{11}-\frac{z_{12}z_{21}}{z_{22}+Z_{02}}\qquad Z_{02}=z_{22}-\frac{z_{21}z_{12}}{z_{11}+Z_{01}}$$

$$Z_{01}=z_{11}-\frac{z_{12}z_{21}}{z_{22}+Z_{02}}=z_{11}-\frac{z_{12}z_{21}}{z_{22}+z_{22}-\frac{z_{21}z_{12}}{z_{11}+Z_{01}}}=z_{11}-\frac{z_{12}z_{21}z_{11}+z_{12}z_{21}Z_{01}}{2z_{22}z_{11}+2z_{22}Z_{01}-z_{21}z_{12}}$$

$$2z_{22}z_{11}Z_{01}+2z_{22}Z_{01}^2-z_{21}z_{12}Z_{01}=2z_{22}z_{11}^2+2z_{11}z_{22}Z_{01}-z_{11}z_{21}z_{12}-z_{12}z_{21}z_{11}-z_{12}z_{21}Z_{01}$$

$$2z_{22}Z_{01}^2=2z_{22}z_{11}^2-2z_{11}z_{21}z_{12}$$

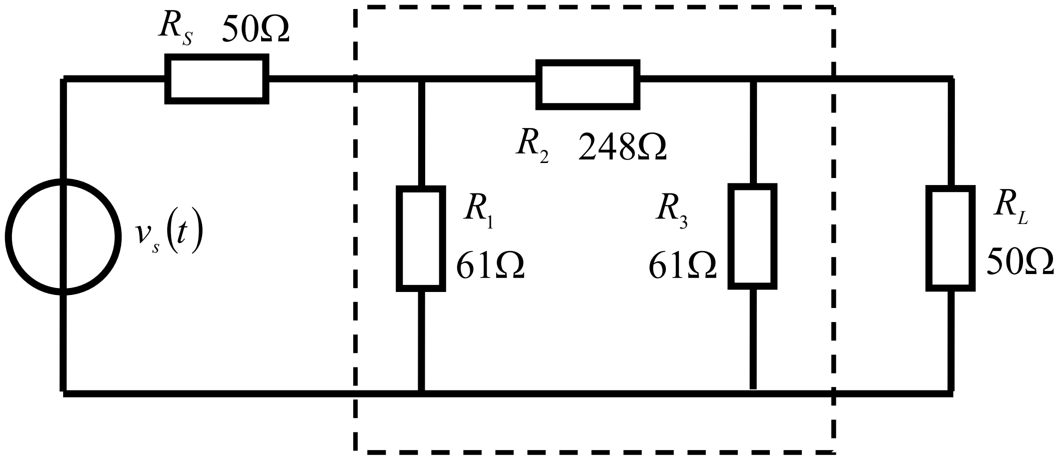
$$Z_{01}^2=z_{11}^2-z_{11}\frac{z_{21}z_{12}}{z_{22}}$$

$$Z_{01}=\sqrt{z_{11}\left(z_{11}-\frac{z_{21}z_{12}}{z_{22}}\right)}\qquad Z_{02}=\sqrt{z_{22}\left(z_{22}-\frac{z_{12}z_{21}}{z_{11}}\right)}$$

二端口网络的特征阻抗

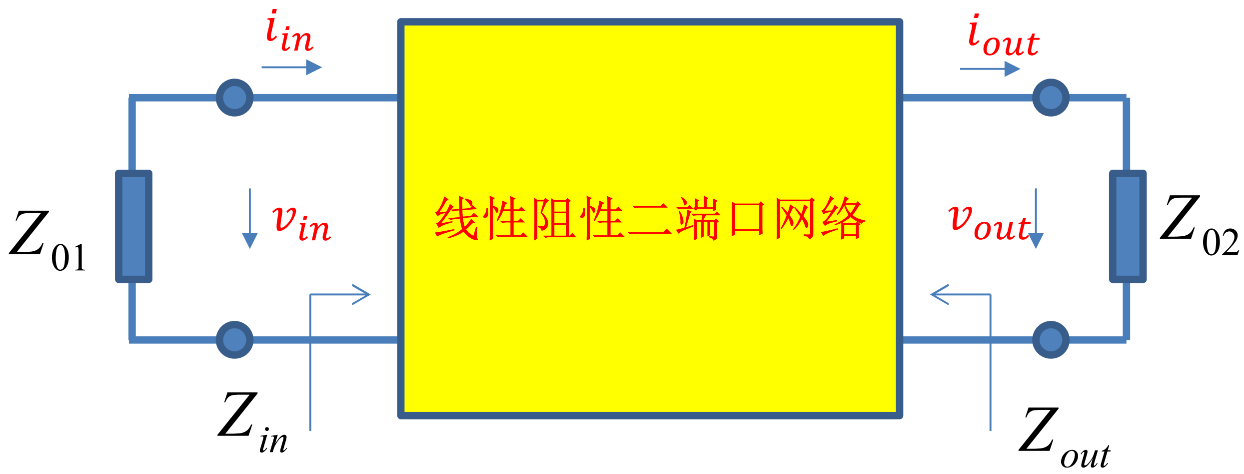
$$Z_{01}=\sqrt{z_{11}\left(z_{11}-\frac{z_{21}z_{12}}{z_{22}}\right)}=\sqrt{z_{11}\left(\frac{\Delta_z}{z_{22}}\right)}=\sqrt{\frac{z_{11}}{y_{11}}}=\sqrt{Z_{in,2open}Z_{in,2short}}$$

$$Z_{02}=\sqrt{z_{22}\left(z_{22}-\frac{z_{12}z_{21}}{z_{11}}\right)}=\sqrt{z_{22}\left(\frac{\Delta_z}{z_{11}}\right)}=\sqrt{\frac{z_{22}}{y_{22}}}=\sqrt{Z_{out,1open}Z_{out,1sh}}$$



$$Z_{01}=\sqrt{Z_{in,2open}Z_{in,2short}}=\sqrt{\big(61||(248+61)\big)\times(61||248)}$$
$$=\sqrt{50.94\times 48.96}=49.94\approx 50\Omega$$

ABCD参量表述的特征阻抗



$$Z_{in} = Z_{01}$$

$$Z_{out} = Z_{02}$$

假设ABCD参量已知

$$Z_{01} = \frac{v_{in}}{i_{in}} = \frac{Av_{out} + Bi_{out}}{Cv_{out} + Di_{out}} = \frac{AZ_{02} + B}{CZ_{02} + D}$$

$$Z_{02} = \frac{v'_{in}}{i'_{in}} = \frac{av'_{out} + bi'_{out}}{cv'_{out} + di'_{out}} = \frac{aZ_{01} + b}{cZ_{01} + d} = \frac{DZ_{01} + B}{CZ_{01} + A}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} A & -B \\ -C & D \end{bmatrix}^{-1} = \frac{1}{AD - BC}\begin{bmatrix} D & B \\ C & A \end{bmatrix}$$

二端口网络特征阻抗

$$Z_{01}=\frac{AZ_{02}+B}{CZ_{02}+D}\qquad\qquad Z_{02}=\frac{DZ_{01}+B}{CZ_{01}+A}$$

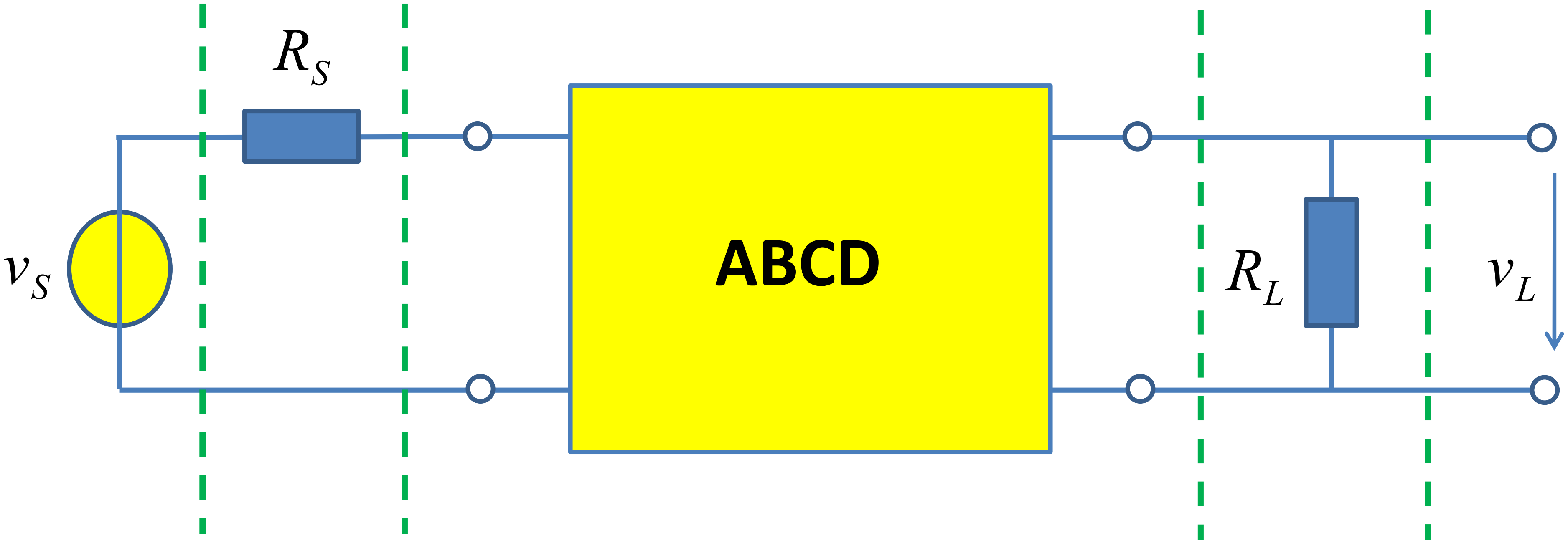
$$Z_{01}=\frac{AZ_{02}+B}{CZ_{02}+D}=\frac{A\dfrac{DZ_{01}+B}{CZ_{01}+A}+B}{C\dfrac{DZ_{01}+B}{CZ_{01}+A}+D}=\frac{ADZ_{01}+AB+BCZ_{01}+BA}{CDZ_{01}+CB+DCZ_{01}+DA}$$

$$2CDZ_{01}^2+{\color{red}ADZ_{01}}+{\color{green}BCZ_{01}}={\color{red}ADZ_{01}}+{\color{green}BCZ_{01}}+2AB$$

$$CDZ_{01}^2=AB$$

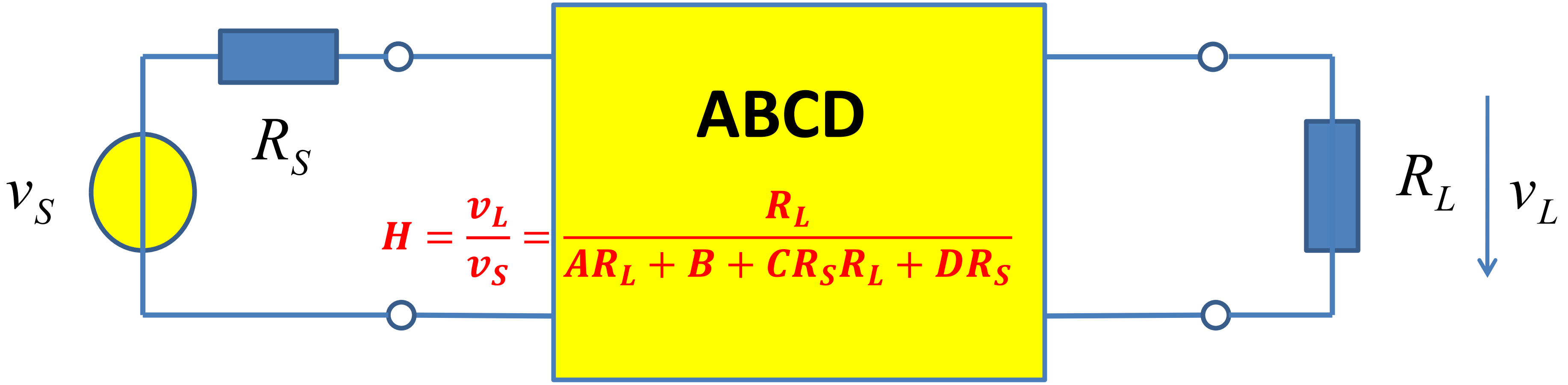
$$Z_{01}=\sqrt{\frac{A}{D}}\sqrt{\frac{B}{C}}=\sqrt{\frac{A_{i0}}{A_{v0}}}\sqrt{\frac{R_{m0}}{G_{m0}}}\qquad\qquad Z_{02}=\sqrt{\frac{D}{A}}\sqrt{\frac{B}{C}}=\sqrt{\frac{A_{v0}}{A_{i0}}}\sqrt{\frac{R_{m0}}{G_{m0}}}$$

为何端接特征阻抗
则可获得最大功率增益？



$$\begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} = \begin{bmatrix} A_S & B_S \\ C_S & D_S \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} A_L & B_L \\ C_L & D_L \end{bmatrix} = \begin{bmatrix} 1 & R_S \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_L} & 1 \end{bmatrix} = \begin{bmatrix} A + CR_S + \frac{B + DR_S}{R_L} & \cdots \\ \cdots & \cdots \end{bmatrix}$$

$$H = \frac{v_L}{v_S} = \frac{1}{A_c} = \frac{1}{A + CR_S + \frac{B + DR_S}{R_L}} = \frac{R_L}{AR_L + B + CR_SR_L + DR_S}$$



$$H=\frac{v_L}{v_S}=\frac{R_L}{AR_L+B+CR_SR_L+DR_S}$$

$$G_T=\frac{P_L}{P_{S,max}}=\frac{\frac{V_{Lrms}^2}{R_L}}{\frac{V_{Srms}^2}{4R_S}}=4\frac{R_S}{R_L}\frac{V_{Lrms}^2}{V_{Srms}^2}=4\frac{R_S}{R_L}\left(\frac{R_L}{AR_L+B+CR_SR_L+DR_S}\right)^2$$

$$=\frac{4R_SR_L}{(AR_L+B+CR_SR_L+DR_S)^2}$$

$$\frac{\partial G_T}{\partial R_S}=0 \qquad \frac{\partial G_T}{\partial R_L}=0$$ 求极值

$$\le\frac{4R_SR_L}{\left(2\sqrt{AR_LDR_S}+2\sqrt{BCR_SR_L}\right)^2}=\frac{1}{\left(\sqrt{AD}+\sqrt{BC}\right)^2}=G_{p,max}$$ 中学数学求极值

获得极大值等号成立的条件：

$$AR_L=DR_S \qquad B=CR_SR_L \quad\Rightarrow\quad R_S=Z_{01}=\sqrt{\frac{A}{D}}\sqrt{\frac{B}{C}} \qquad R_L=Z_{02}=\sqrt{\frac{D}{A}}\sqrt{\frac{B}{C}}$$

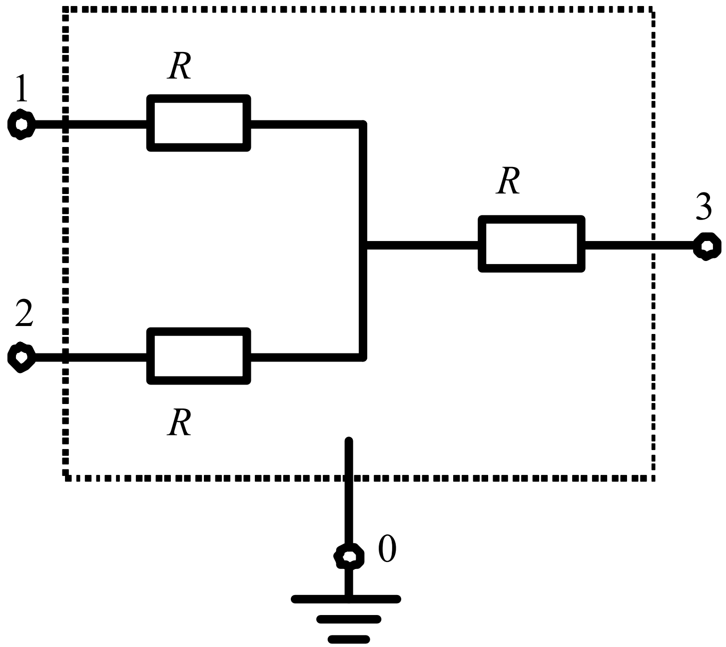
两端同时最大功率传输匹配，获得最大功率增益

大纲

- 第四周作业讲解
- 简单二端口网络的网络参量简单列写方法（熟练度要求）
- 二端口网络噪声分析（例**3.7.4**）
- 一个三端口网络例（例**3.7.2**）

例5　阻性信号合成网络

讲义例3.7.2

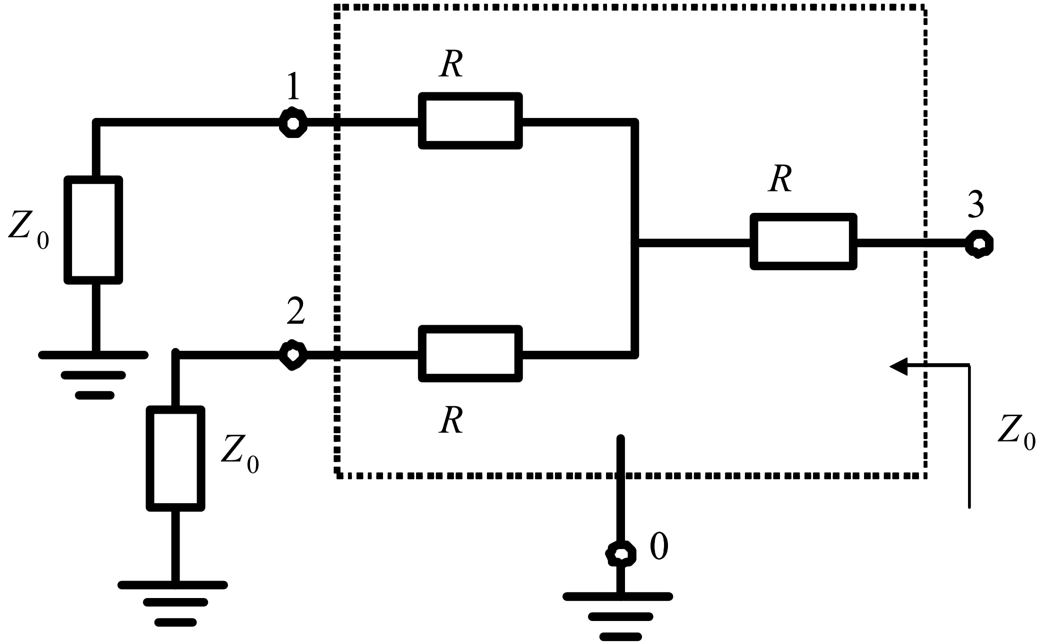


1、求该三端口网络的特征阻抗
2、证明端接匹配时，端口3输出为端口1、端口2输入信号的合成
3、求三端口网络参量

特征阻抗

完全对称结构，故而三个端口特征阻抗相等

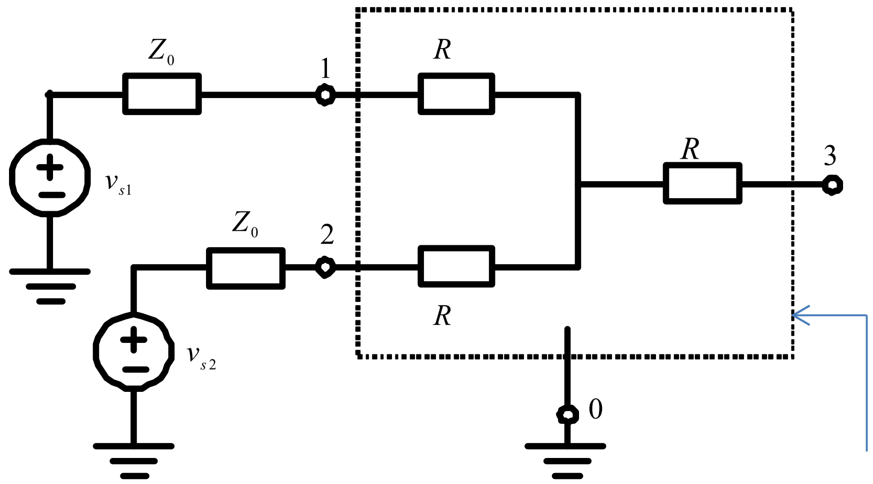
$$Z_{01} = Z_{02} = Z_{03} = Z_0 = ?$$



$$Z_0 = Z_{03} = (Z_{01} + R) \| (Z_{02} + R) + R$$
$$= (Z_0 + R) \| (Z_0 + R) + R$$
$$= \frac{Z_0 + R}{2} + R = 0.5Z_0 + 1.5R$$

$$Z_0 = 3R$$

信号合成



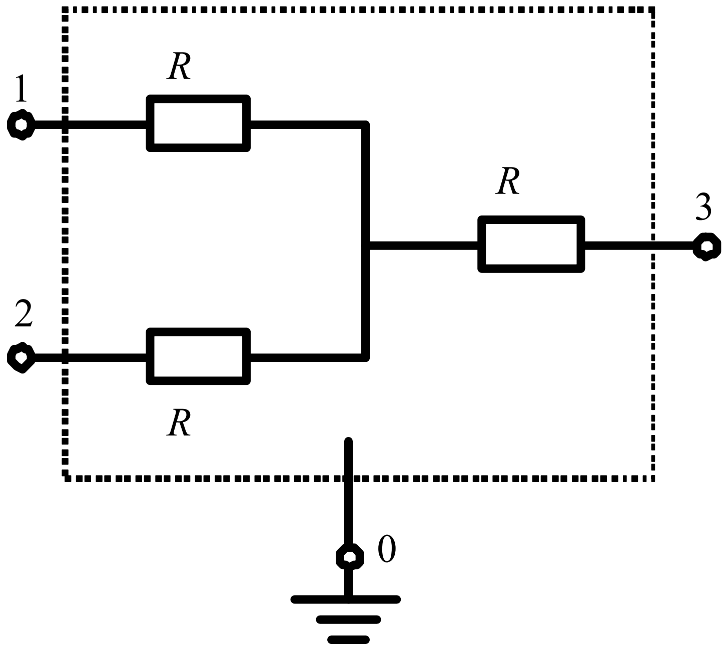
v_{TH}， $R_{TH}=Z_0=3R$

$$v_{TH} = v_{TH1} + v_{TH2}$$
$$= \frac{R+Z_0}{2(R+Z_0)} v_{S1} + \frac{R+Z_0}{2(R+Z_0)} v_{S2}$$
$$= 0.5(v_{S1} + v_{S2})$$

信号合成

y参量矩阵

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$



$$i_1 = G_{11} v_1 + G_{12} v_2 + G_{13} v_3$$

$$G_{11} = \frac{i_1}{v_1}\bigg|_{v_2=0, v_3=0} = \frac{1}{R \parallel R + R} = \frac{1}{1.5R} = \frac{2}{3R} = \frac{2}{Z_0} = 2Y_0$$

$$G_{12} = \frac{i_1}{v_2}\bigg|_{v_1=0, v_3=0} = -\frac{\left(\frac{v_2}{1.5R}\right) \cdot \frac{1}{2}}{v_2} = -\frac{1}{3R} = -\frac{1}{Z_0} = -Y_0$$

$$G_{13} = \frac{i_1}{v_3}\bigg|_{v_1=0, v_2=0} = -Y_0$$

$$\mathbf{y} = \begin{bmatrix} 2Y_0 & -Y_0 & -Y_0 \\ -Y_0 & 2Y_0 & -Y_0 \\ -Y_0 & -Y_0 & 2Y_0 \end{bmatrix}$$

$$\mathbf{z} = \mathbf{y}^{-1} = ?$$

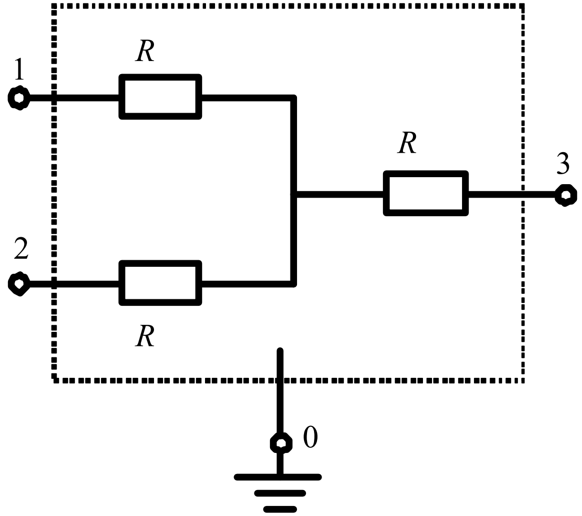
不存在z参量矩阵

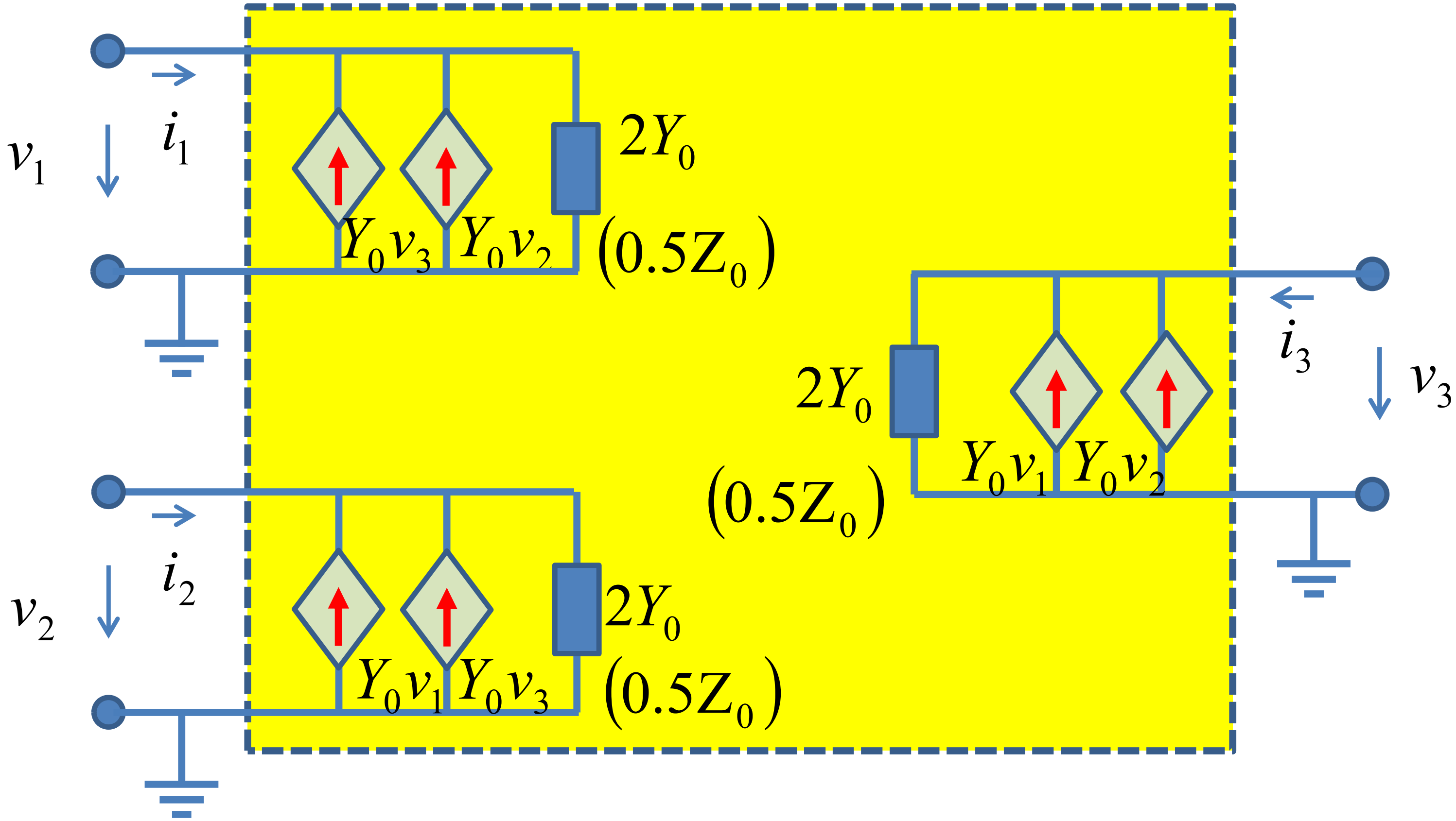
病态问题：二端口拓展为三端口，有强制性约束条件存在

$i_1 + i_2 + i_3 = 0$ 三个电流不独立，无法做自变量

Y参量等效电路

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2Y_0 & -Y_0 & -Y_0 \\ -Y_0 & 2Y_0 & -Y_0 \\ -Y_0 & -Y_0 & 2Y_0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$





混合参量表述

$$i_1 = +2Y_0 v_1 - Y_0 v_2 - Y_0 v_3$$
$$i_2 = -Y_0 v_1 + 2Y_0 v_2 - Y_0 v_3$$
$$i_3 = -Y_0 v_1 - Y_0 v_2 + 2Y_0 v_3$$

$$v_3 = 0.5v_1 + 0.5v_2 + 0.5Z_0 i_3$$

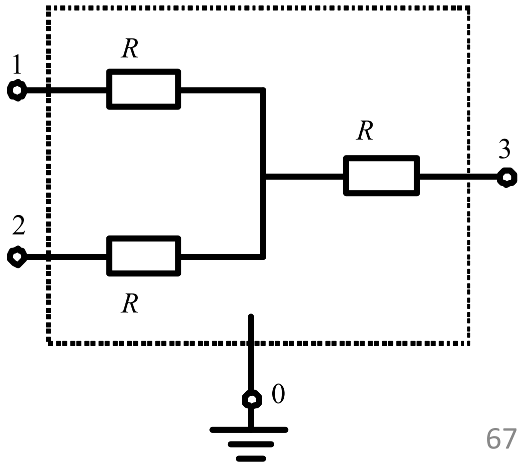
$$i_1 = +2Y_0 v_1 - Y_0 v_2 - Y_0 v_3 = +2Y_0 v_1 - Y_0 v_2 - \left(0.5Y_0 v_1 + 0.5Y_0 v_2 + 0.5i_3\right)$$
$$= 1.5Y_0 v_1 - 1.5Y_0 v_2 - 0.5i_3$$
$$i_2 = -Y_0 v_1 + 2Y_0 v_2 - Y_0 v_3 = -Y_0 v_1 + 2Y_0 v_2 - \left(0.5Y_0 v_1 + 0.5Y_0 v_2 + 0.5i_3\right)$$
$$= -1.5Y_0 v_1 + 1.5Y_0 v_2 - 0.5i_3$$

电流信号分解

$$\begin{bmatrix} i_1 \\ i_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1.5Y_0 & -1.5Y_0 & -0.5 \\ -1.5Y_0 & 1.5Y_0 & -0.5 \\ 0.5 & 0.5 & 0.5Z_0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix}$$

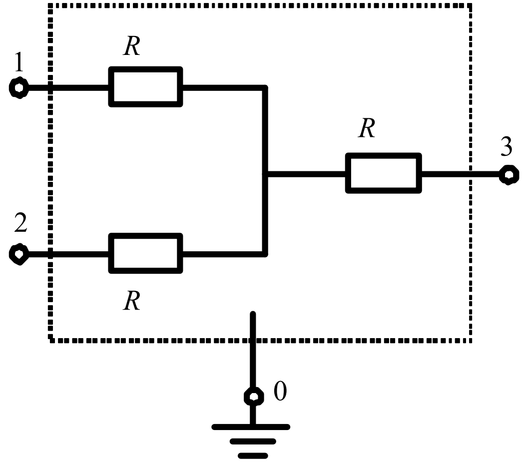
电压信号合成

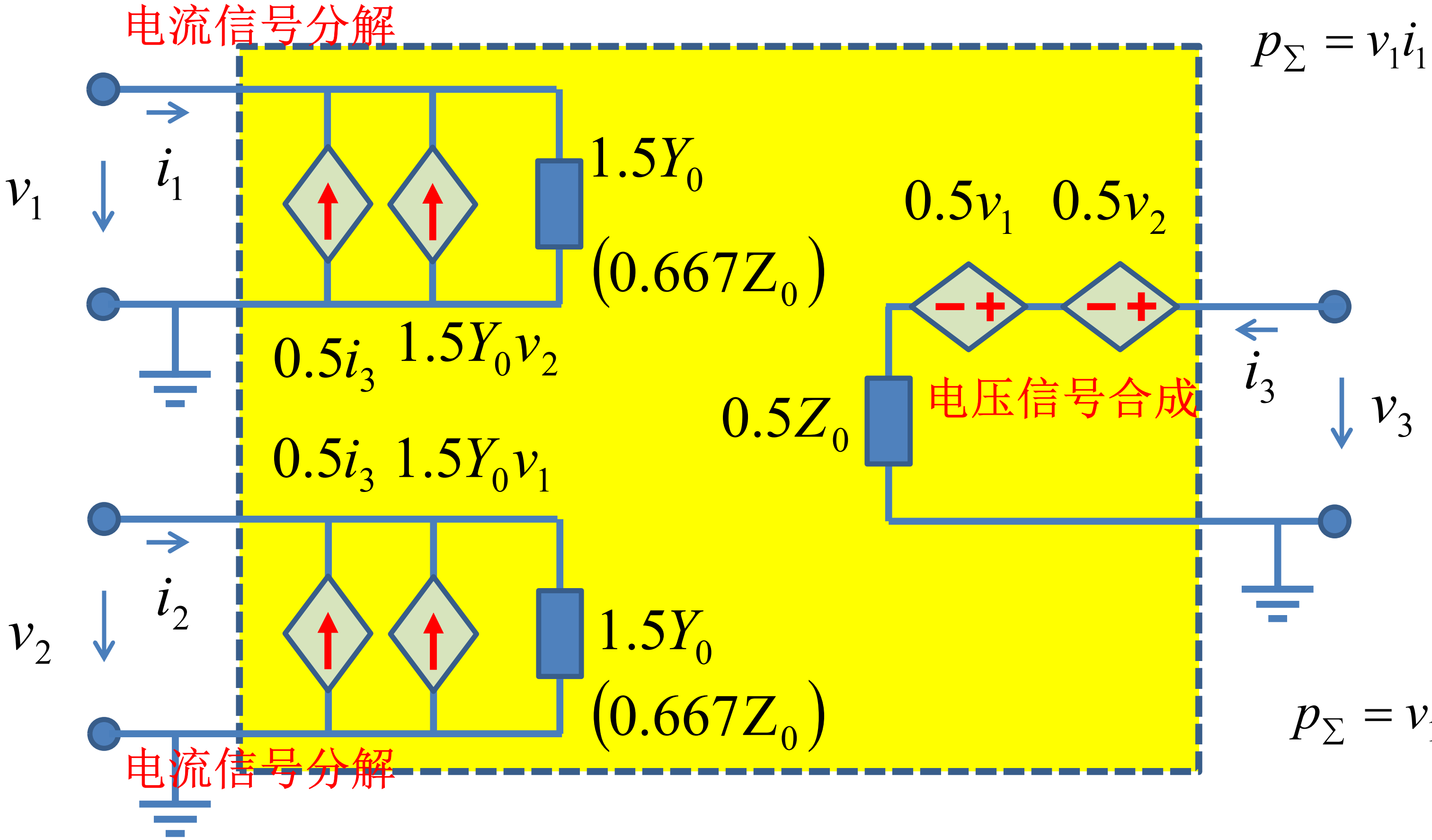
从混合参量看信号合成与分解



混合参量等效电路

$$\begin{bmatrix} i_1 \\ i_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1.5Y_0 & -1.5Y_0 & -0.5 \\ -1.5Y_0 & 1.5Y_0 & -0.5 \\ 0.5 & 0.5 & 0.5Z_0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix}$$





$$p_\Sigma = v_1 i_1 + v_2 i_2 + v_3 i_3 \geq 0$$

本例为有损信号分解与合成例，无损信号分解与合成可以采用无损的电感、电容、传输线、变压器等方案实现：下节理论课---理想变压器信号分解与合成例

$$p_\Sigma = v_1 i_1 + v_2 i_2 + v_3 i_3 \equiv 0$$