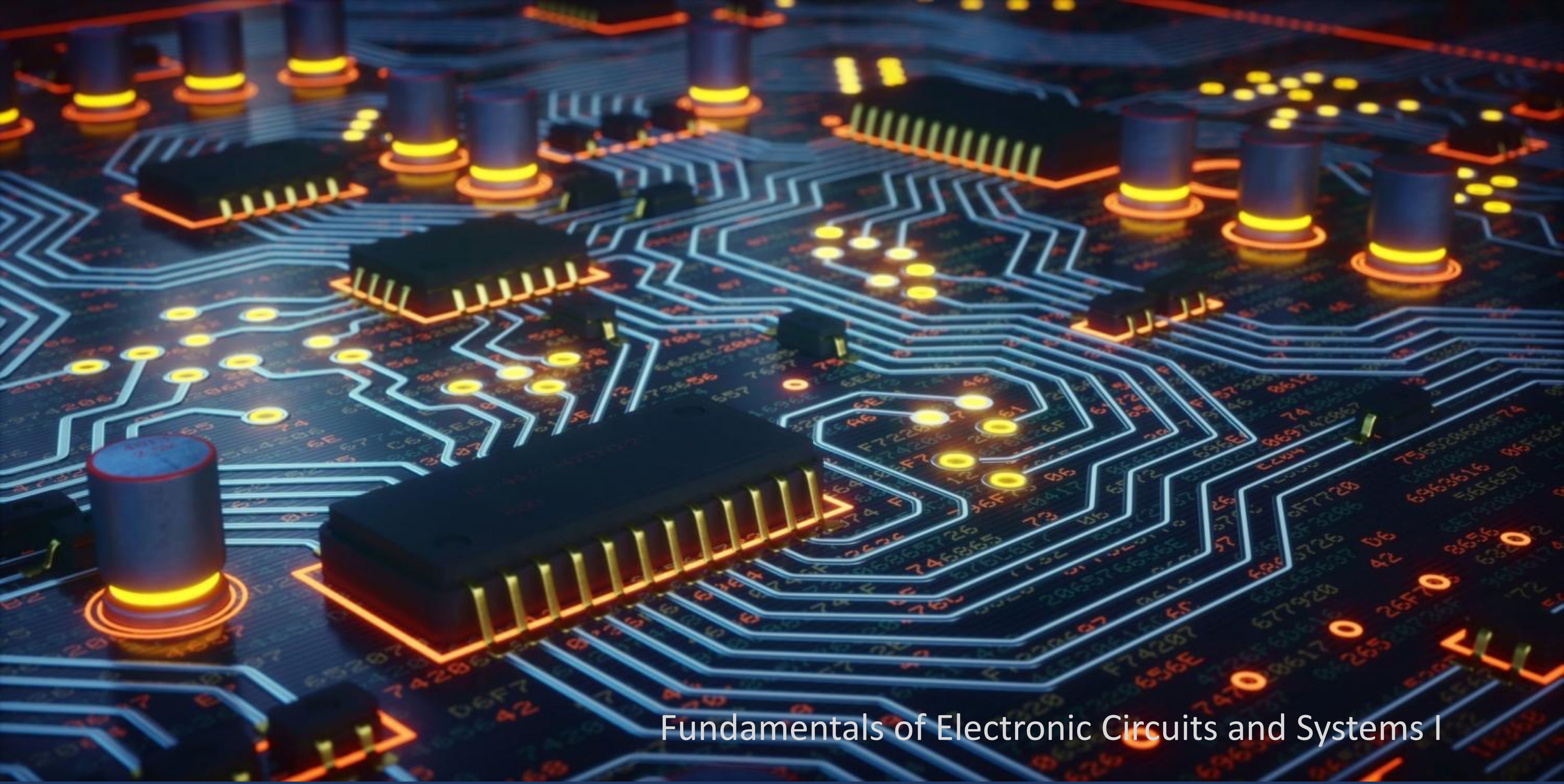

Fundamentals of Electronic Circuits and Systems I

Two-Port Networks

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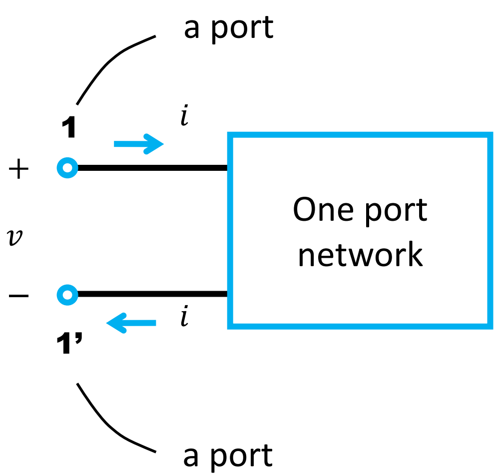
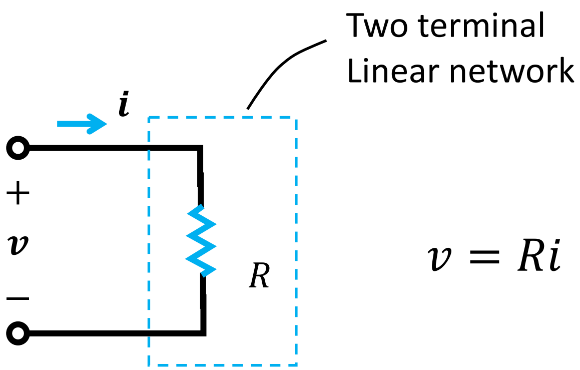
Outlines

- One port network
- Two port network & Parameters

One-port network

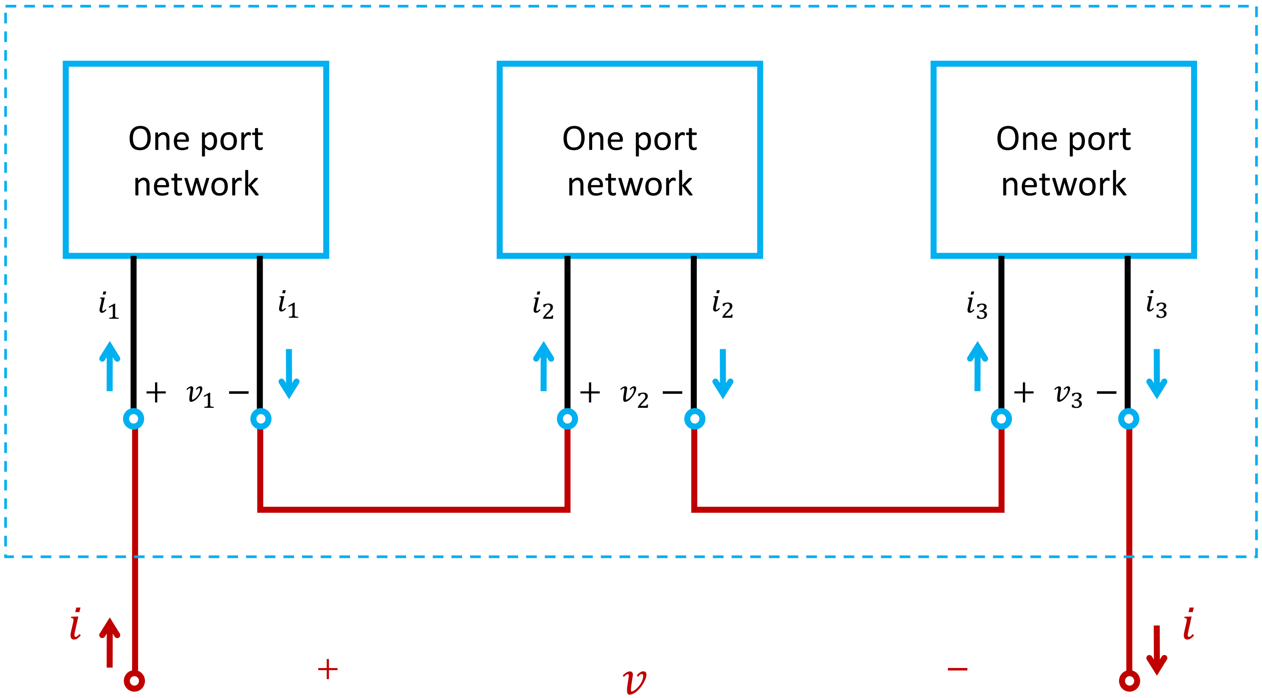
ONE PORT NETWORK is a two terminal electrical network in which, current enters through one terminal and leaves through another terminal.





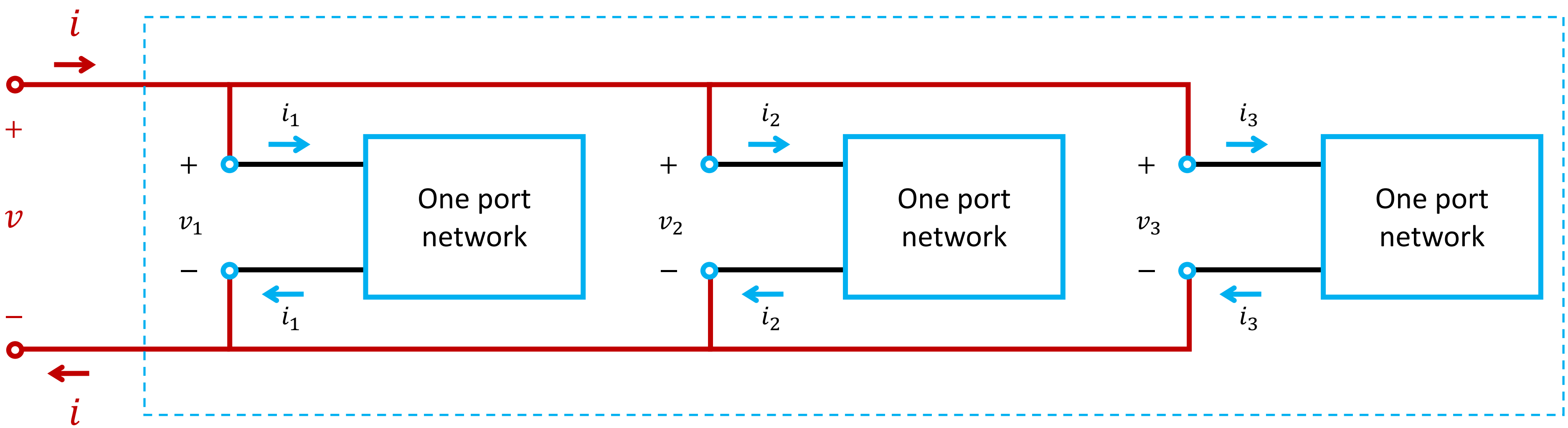
$$v = Ri$$

One-port network in series



- According to KVL $\boldsymbol{v = v_1 + v_2 + v_3}$
- According to KCL $\boldsymbol{i = i_1 = i_2 = i_3}$

One-port network in parallel

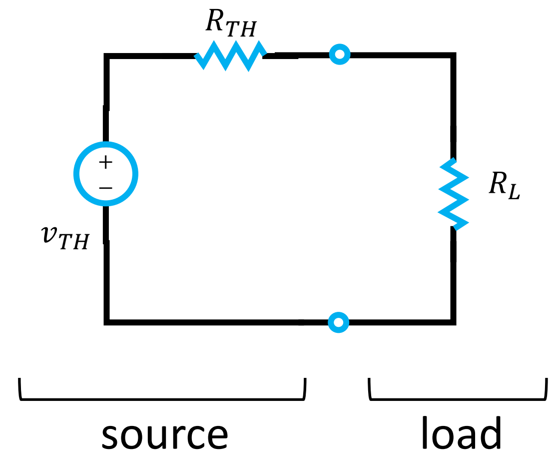


- According to KVL $v = v_1 = v_2 = v_3$
- According to KCL $i = i_1 + i_2 + i_3$

Recall: Circuit equivalent

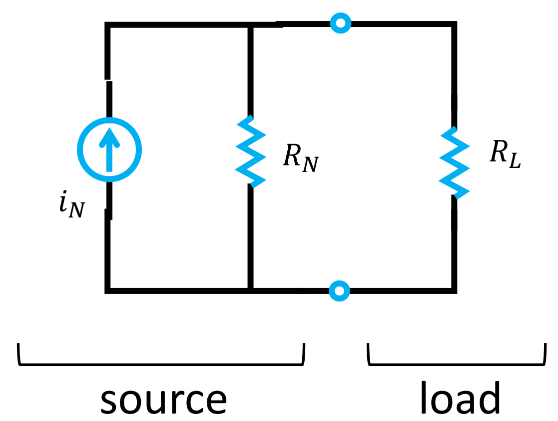
Thévenin's theorem

LINEAR two-terminal circuit can be replaced by an equivalent circuit composed of a **voltage source** and a **series resistor**

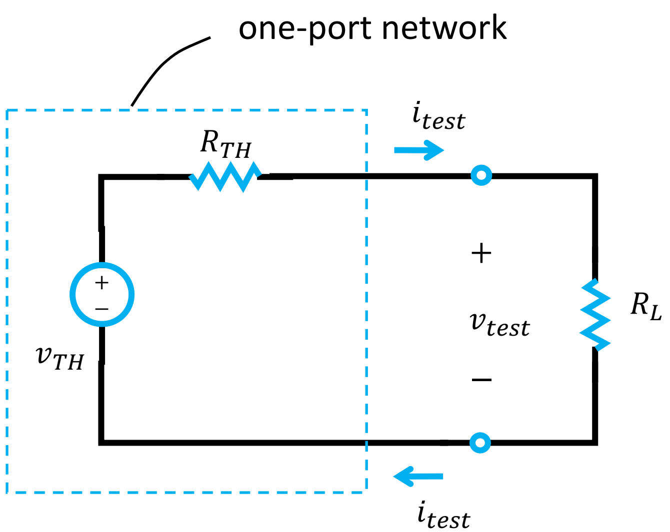


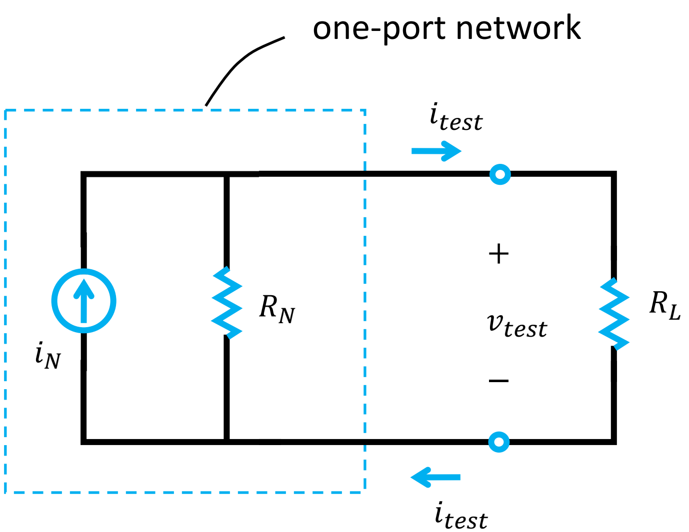
Norton's theorem

LINEAR two-terminal circuit can be replaced by an equivalent circuit composed of a **current source** and a **parallel resistor**



Circuit equivalent as one-port network





$$v_{test} = i_{test}R_{TH} + v_{TH}$$

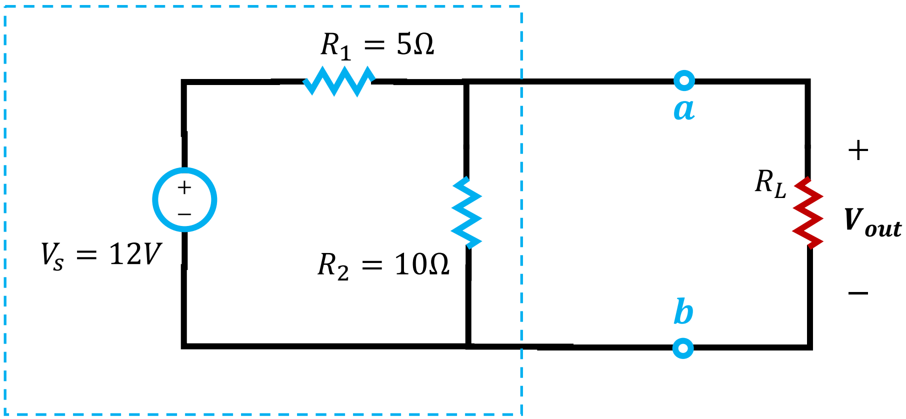
$$i_{test} = \frac{v_{test}}{R_N} + i_N$$

Another way to find the circuit equivalent

- Step 1: apply a voltage v_{test} to the terminal of a one-port network
- Step 2: find the relationship between the port current i_{test} and v_{test}

Example 1

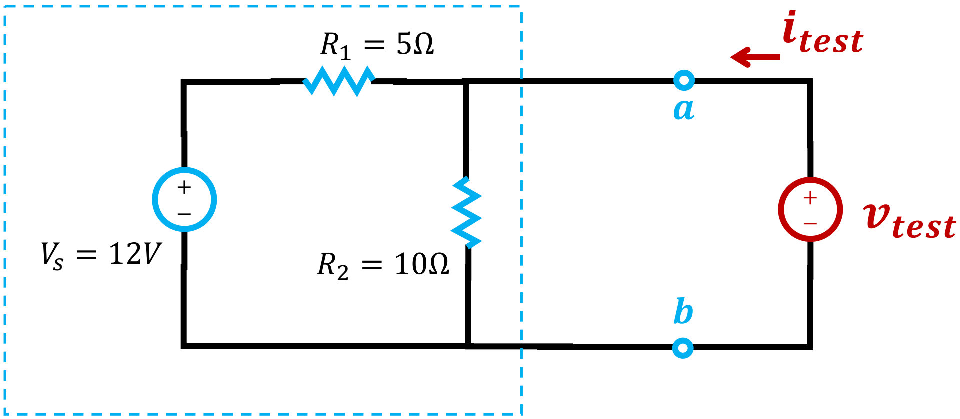
QUESTION: Find the Thévenin equivalent circuit of the network at the terminals a & b



- Step 1a: remove the load R_L

Example 1

QUESTION: Find the Thévenin equivalent circuit of the network at the terminals a & b



- Step 1a: remove the load R_L
- Step 1b: apply v_{test} to a and b
- Step 2: find the relationship between i_{test} and v_{test}

This is a practical way to find out circuit equivalent when the topology is unknown

- According to KCL

$$i_{test} = i_{R_1} + i_{R_2}$$

$$= \frac{v_{test} - V_S}{R_1} + \frac{v_{test}}{R_2}$$

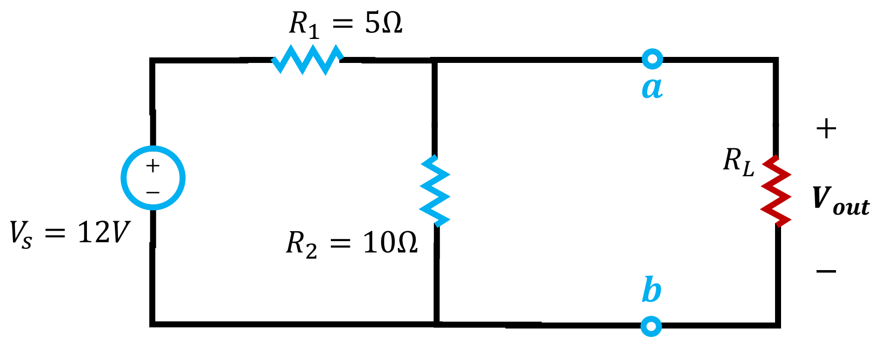
$$= \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_{test} - \frac{V_S}{R_1}$$

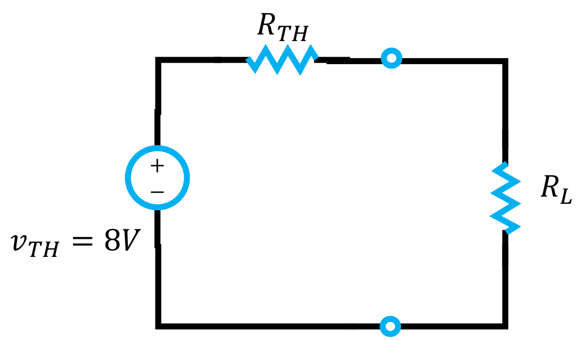
- Relationship between i_{test} & v_{test}

$$v_{test} = \underbrace{3.3\Omega}_{R_{TH}} \cdot i_{test} + \underbrace{8V}_{V_{TH}}$$

Recall: example 4 in L3

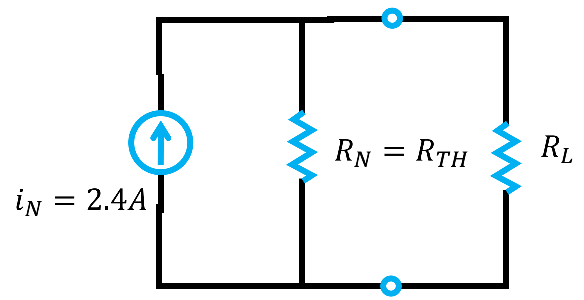
QUESTION: Find the Thévenin equivalent circuit of the network at the terminals a & b





- Step 1: remove the load
- Step 2: find $V_{open} = v_{TH}$
- Step 3: find $i_{short} = i_N$
- Step 4: find R_{TH}

$$R_{TH} = \frac{v_{TH}}{i_N} = 3.33\Omega$$

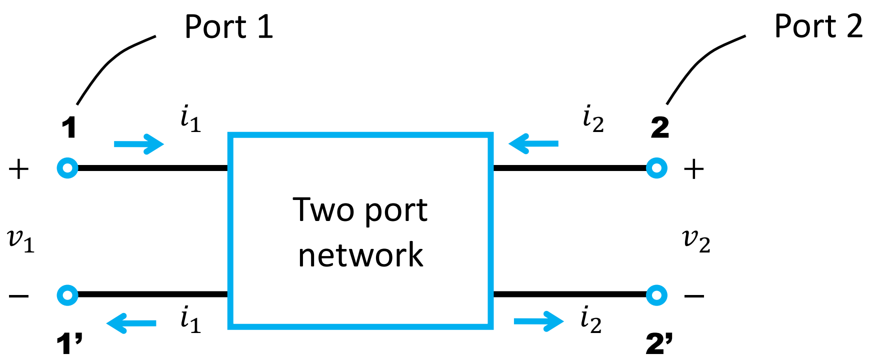


Outlines

- One-port network
  - Definition of one-port network
  - One-port network in series / parallel
  - Circuit equivalent as one-port network
- Two-port network & Parameters

Two-port network

TWO PORT NETWORK is a pair of two terminal electrical network in which, current enters through one terminal and leaves through another terminal.

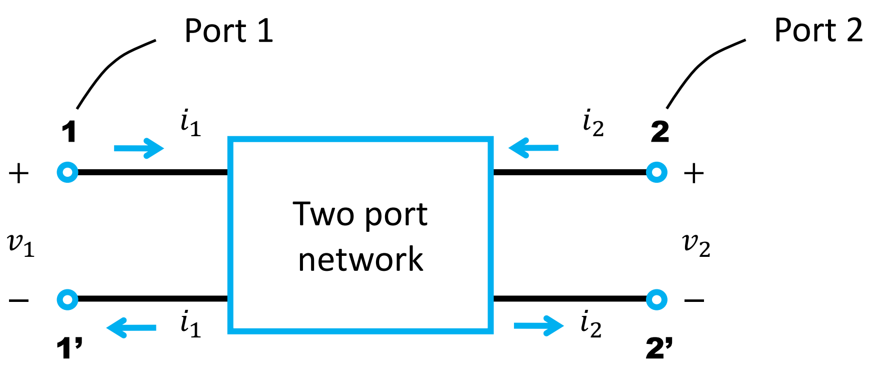


Four variables
v_1, v_2, i_1, i_2

If two of the four variables are independent and another two variables as dependent, the coefficients of the independent variables are called as PARAMETERS

Z parameters

If two of the four variables are independent and another two variables as dependent, the coefficients of the independent variables are called as PARAMETERS



Four variables: v_1, v_2, i_1, i_2

- v_1, v_2 are dependent
- i_1, i_2 are independent

→ Z parameters

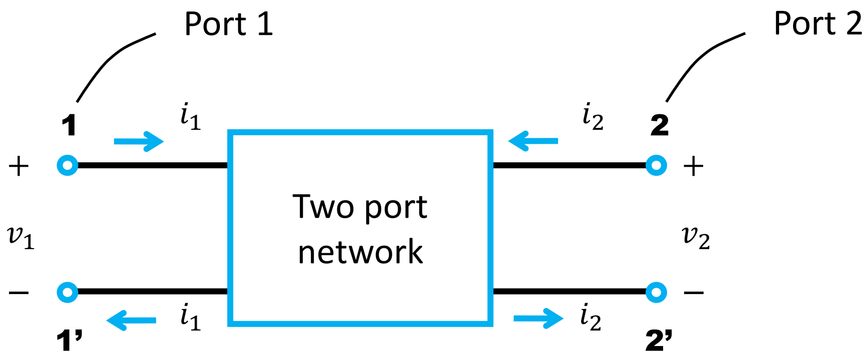
$$\begin{cases} v_1 = Z_{11} i_1 + Z_{12} i_2 \\ v_2 = Z_{21} i_1 + Z_{22} i_2 \end{cases}$$

or

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

unit: Ω

Z parameters



→ Z parameters

$$\begin{cases} v_1 = Z_{11} i_1 + Z_{12} i_2 \\ v_2 = Z_{21} i_1 + Z_{22} i_2 \end{cases}$$

or

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

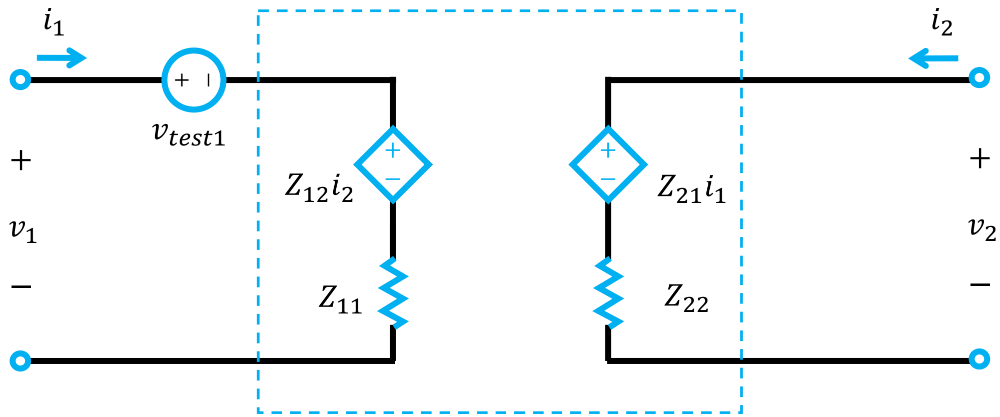
$$Z_{11} = \frac{v_1}{i_1} \quad \text{when } i_2 = 0$$

$$Z_{12} = \frac{v_1}{i_2} \quad \text{when } i_1 = 0$$

$$Z_{21} = \frac{v_2}{i_1} \quad \text{when } i_2 = 0$$

$$Z_{22} = \frac{v_2}{i_2} \quad \text{when } i_1 = 0$$

Z parameters



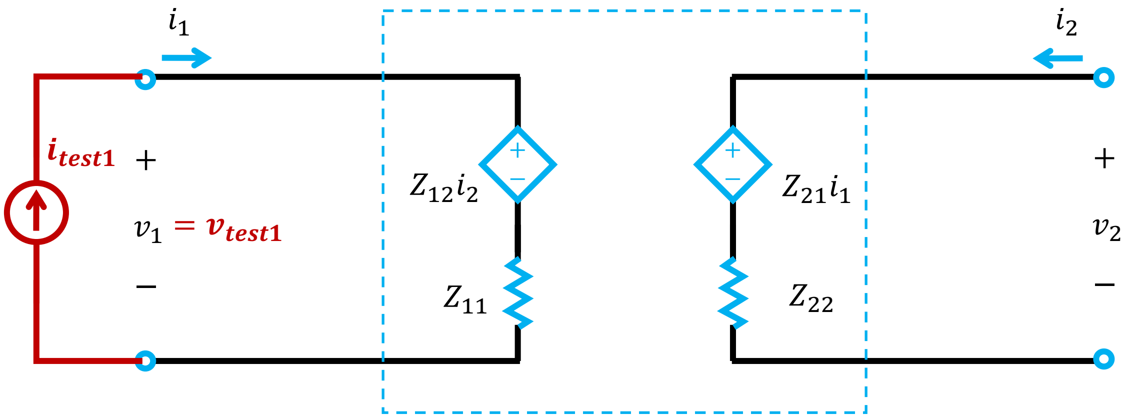
$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

What does v_1 mean?

- Step 1: leave port 1&2 open, thus $i_1 = i_2 = 0$
- Step 2: apply a voltage v_{test1} to port 1
- Step 3: we can find v_1

$$v_1 = v_{test1}\Big|_{i_1=0,\, i_2=0}$$

Z parameters



- According to KVL

$$\begin{cases} v_1 = Z_{11}i_1 + Z_{12}i_2 \\ v_2 = Z_{21}i_1 + Z_{22}i_2 \end{cases}$$

Z Parameters are also called as open-circuit impedance parameters

What does Z_{11} mean?

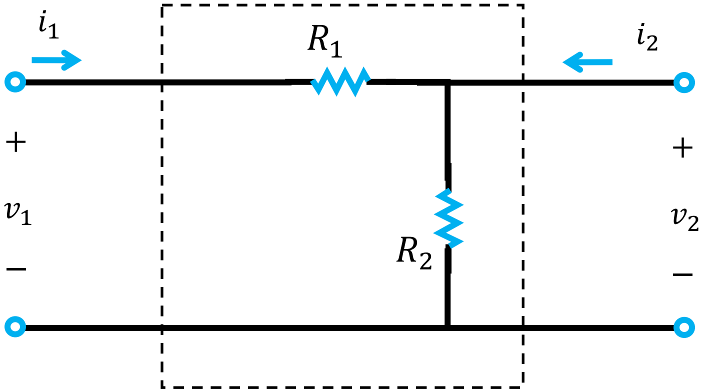
- Step 1: leave port 2 open, thus $i_2 = 0$
- Step 2: apply a voltage i_{test1} to port 1
- Step 3: we can find Z_{11} according to the current i_{test1} measured in port 1

$$Z_{11} = \left.\frac{v_{test1}}{i_{test1}}\right|_{i_2=0}$$

INPUT IMPEDANCE

Example 2

QUESTION: find the Z parameters of the network labeled in the dash line



Z parameters

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$Z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} = R_1 + R_2$$

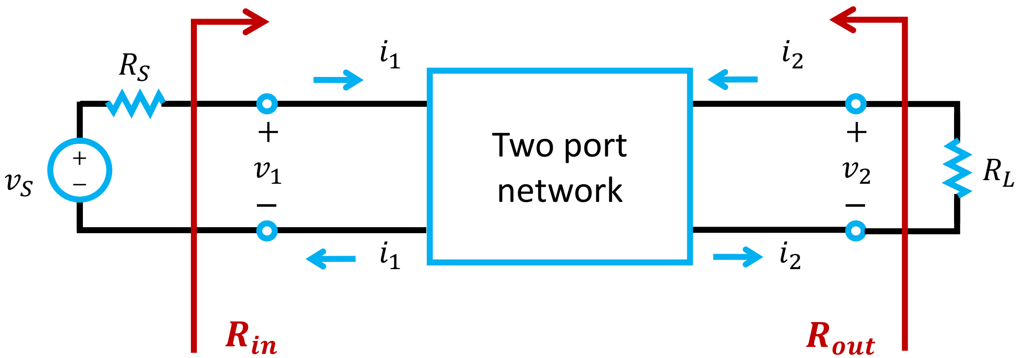
$$Z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} = R_2$$

$$Z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0} = R_2$$

$$Z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = R_2$$

Example 3

QUESTION: find the transfer function $H = \frac{v_L}{v_s}$, the input impedance R_{in} & output impedance R_{out} of the circuit below. The Z parameters of the network is known.



Z parameters

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

- According to Ohm's law

$$v_L = -R_L i_2$$

- the transfer function

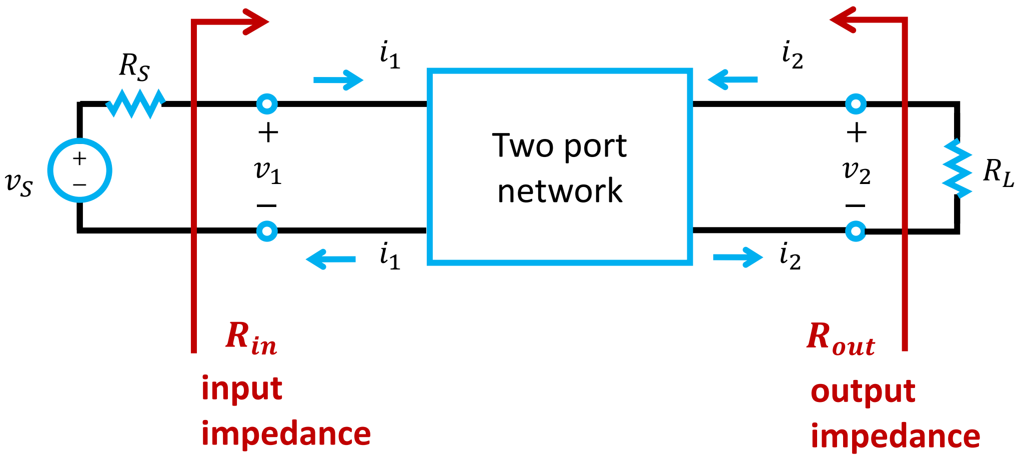
$$H = \frac{v_L}{v_s} = \frac{-R_L i_2}{\left(R_S + Z_{11} - \frac{Z_{12}Z_{21}}{R_L + Z_{22}}\right) i_1}$$

$$= \frac{-R_L}{\left(R_S + Z_{11} - \frac{Z_{12}Z_{21}}{R_L + Z_{22}}\right)} \cdot \frac{-Z_{21}}{R_L + Z_{22}}$$

$$= \frac{R_L Z_{21}}{(R_S + Z_{11})(R_L + Z_{22}) - Z_{12}Z_{21}}$$

Example 3

QUESTION: find the transfer function $H = \frac{v_L}{v_s}$, the input impedance R_{in} & output impedance R_{out} of the circuit below. The Z parameters of the network is known.



Z parameters

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

- According to KVL

$$\begin{cases} v_S = R_S i_1 + v_1 & (1) \\ v_2 + R_L i_2 = 0 & (2) \end{cases}$$

- Take (2) to $v_2 = Z_{21} i_1 + Z_{22} i_2$

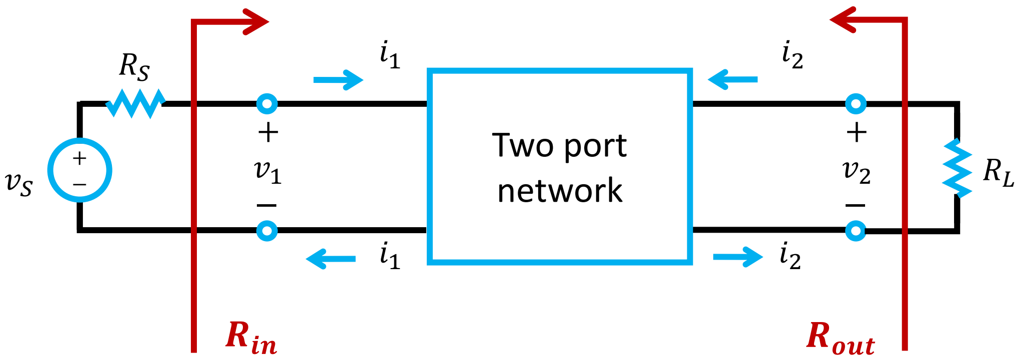
$$\rightarrow i_2 = -\frac{Z_{21}}{R_L + Z_{22}} i_1 \qquad (3)$$

- Take (1) & (2) to $v_1 = Z_{11} i_1 + Z_{12} i_2$

$$\rightarrow v_s = \left(R_S + Z_{11} - \frac{Z_{12} Z_{21}}{R_L + Z_{22}}\right) i_1$$

Example 3

QUESTION: find the transfer function $H = \frac{v_L}{v_s}$, the input impedance R_{in} & output impedance R_{out} of the circuit below. The Z parameters of the network is known.



Z parameters

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

- Let's find R_{in}, recall

$$v_s = \left(R_S + Z_{11} - \frac{Z_{12} Z_{21}}{R_L + Z_{22}} \right) i_1$$

- According to KVL

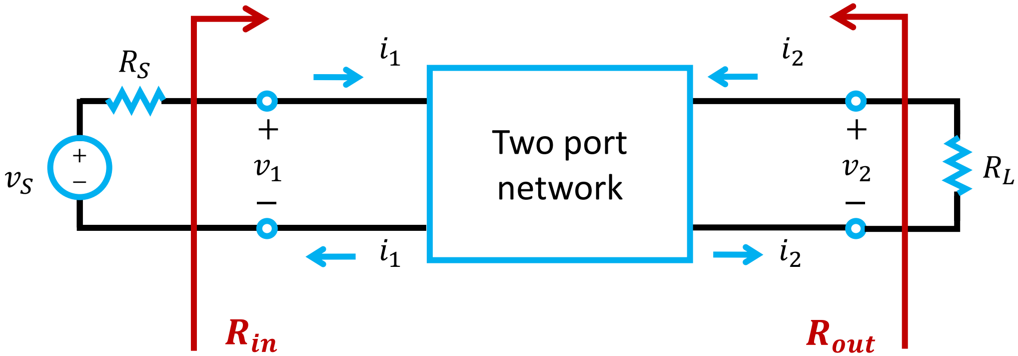
$$v_s = R_S i_1 + R_{in} i_1$$

- Thus, the input impedance

$$R_{in} = Z_{11} - \frac{Z_{12} Z_{21}}{R_L + Z_{22}}$$

Example 3

QUESTION: find the transfer function $H = \frac{v_L}{v_s}$, the input impedance R_{in} & output impedance R_{out} of the circuit below. The Z parameters of the network is known.



$$\left[\begin{aligned} R_{in} &= Z_{11} - \frac{Z_{12}Z_{21}}{R_L + Z_{22}} \\ i_2 &= -\frac{Z_{21}}{R_L + Z_{22}} i_1 \end{aligned}\right]$$

- When R_L is very high

$$i_2 \xrightarrow{R_L \to \infty} 0$$

$$R_{in} \xrightarrow{R_L \to \infty} Z_{11}$$

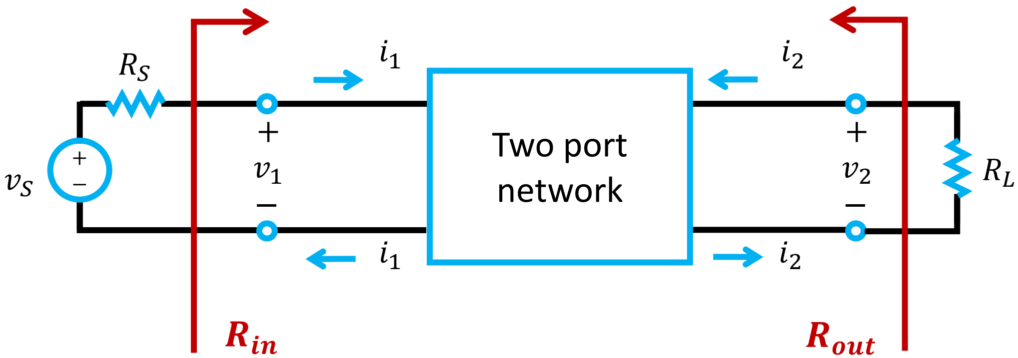
- When R_L is very low

$$i_2 \xrightarrow{R_L \to 0} -\frac{Z_{21}}{Z_{22}} i_1$$

$$R_{in} \xrightarrow{R_L \to 0} Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}}$$

Example 3

QUESTION: find the transfer function $H = \frac{v_L}{v_S}$, the input impedance R_{in} & output impedance R_{out} of the circuit below. The Z parameters of the network is known.



$$\begin{cases} i_2 = -\frac{Z_{21}}{R_L + Z_{22}} i_1 & (3) \\ v_S = \left(R_S + Z_{11} - \frac{Z_{12}Z_{21}}{R_L + Z_{22}}\right) i_1 & (4) \end{cases}$$

- Let's find R_{out}, Take (4) to (3)

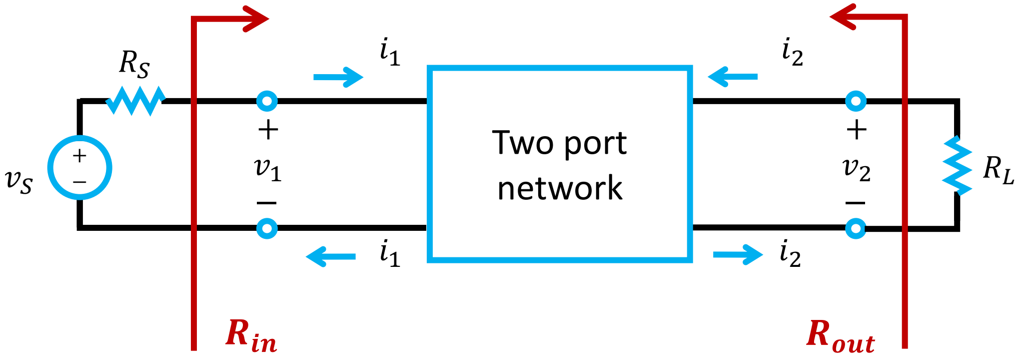
$$i_2 = -\frac{Z_{21}}{R_L + Z_{22}} \cdot \frac{v_S}{R_S + Z_{11} - \frac{Z_{12}Z_{21}}{R_L + Z_{22}}}$$

- Find the relationship

$$\frac{Z_{21}}{R_S + Z_{11}} v_S = \underbrace{-i_2 R_L}_{v_2} - \underbrace{\left(Z_{22} - \frac{Z_{12}Z_{21}}{R_S + Z_{11}}\right)}_{R_{out}} i_2$$

Example 3

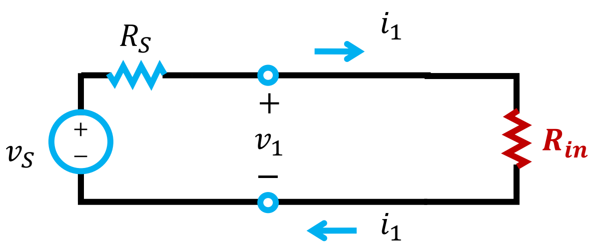
QUESTION: find the transfer function $H = \frac{v_L}{v_S}$, the input impedance R_{in} & output impedance R_{out} of the circuit below. The Z parameters of the network is known.

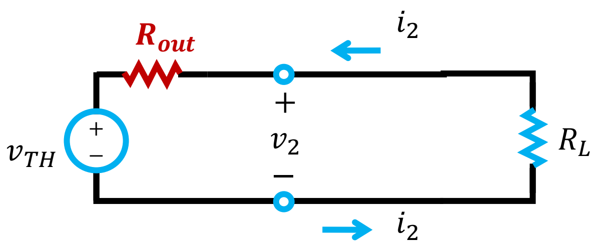


What does R_{in} / R_{out} mean?

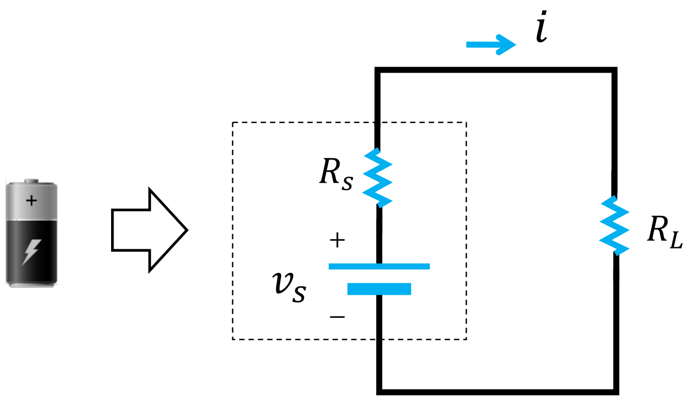
input impedance $R_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{R_L+Z_{22}}$

output impedance $R_{out} = Z_{22} - \frac{Z_{12}Z_{21}}{R_S+Z_{11}}$





Recall: Max. Power Transfer



MAXIMUM POWER TRANSFER occurs in the load when the load resistance, R_L, is equal in value to the source resistance, R_S

- Power at the load R_L

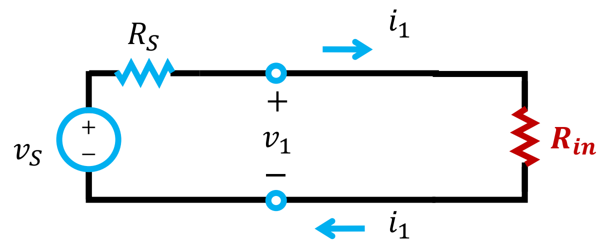
$$P_L = v_{R_L} i = (v_s - iR_s)i \;\; = -R_s\left(i^2 - \frac{v_s}{R_s} i\right) = -R_s\left(i - \frac{1}{2}\frac{v_s}{R_s}\right)^2 + \frac{1}{4}\frac{v_s^2}{R_s}$$

$$\leq \frac{1}{4}\frac{v_{s,rms}^2}{R_s}$$

The maximum power being absorbed by the load

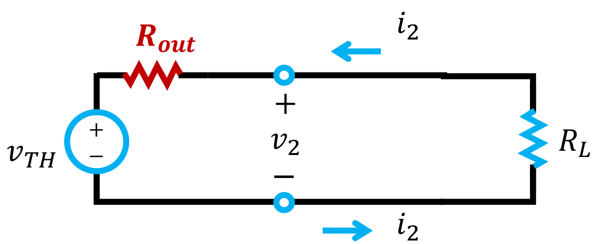
- When $R_s = R_L$ $\quad P_L = P_{L,max} = \dfrac{1}{4}\dfrac{v_{s,rms}^2}{R_s}$

Impedance matching



Maximum power transfer is expected when

$$R_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{R_L + Z_{22}} = R_S$$



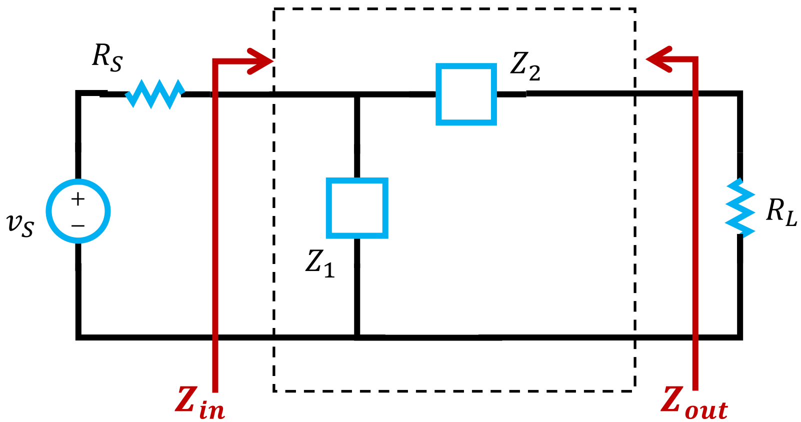
Maximum power transfer is expected when

$$R_{out} = Z_{22} - \frac{Z_{12}Z_{21}}{R_S + Z_{11}} = R_L$$

$$\begin{cases} R_S = \sqrt{Z_{11}\dfrac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}}} = \sqrt{R_{in}\big|_{R_L=0} \cdot R_{in}\big|_{R_L=\infty}} \\ R_L = \sqrt{Z_{22}\dfrac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{11}}} = \sqrt{R_{out}\big|_{R_S=0} \cdot R_{out}\big|_{R_S=\infty}} \end{cases}$$

Example 4

QUESTION: find the value of Z_1 and Z_2 to maximize the output power on R_L



- A maximum power transfer is expected when

$$Z_{in} = \sqrt{Z_{in}\Big|_{R_L=0} \cdot Z_{in}\Big|_{R_L=\infty}} = R_S$$

$$Z_{out} = \sqrt{Z_{out}\Big|_{R_S=0} \cdot Z_{out}\Big|_{R_S=\infty}} = R_L$$

$$Z_{in}\Big|_{R_L=0} = Z_1||Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

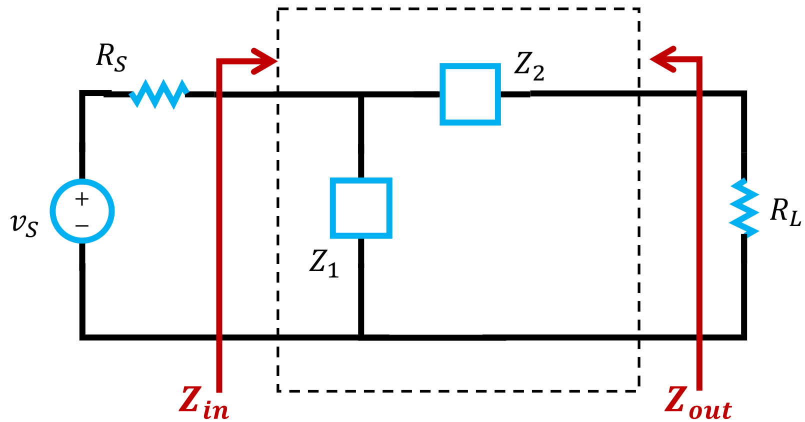
$$Z_{out}\Big|_{R_S=0} = Z_2$$

$$Z_{in}\Big|_{R_L=\infty} = Z_1$$

$$Z_{out}\Big|_{R_S=\infty} = Z_1 + Z_2$$

Example 4

QUESTION: find the value of Z_1 and Z_2 to maximize the output power on R_L



- A maximum power transfer is expected when

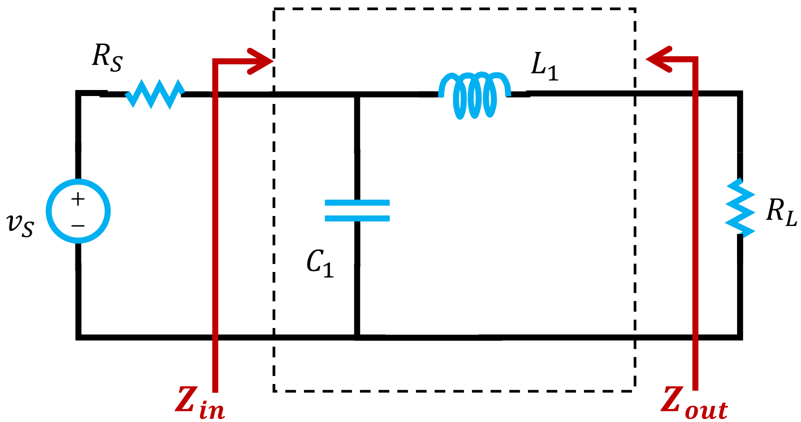
$$Z_{in} = \sqrt{Z_{in}\big|_{R_L=0} \cdot Z_{in}\big|_{R_L=\infty}} = R_S$$

$$Z_{out} = \sqrt{Z_{out}\big|_{R_S=0} \cdot Z_{out}\big|_{R_S=\infty}} = R_L$$

$$\begin{cases} R_S = \sqrt{\dfrac{Z_1 Z_2}{Z_1 + Z_2} \cdot Z_1} \\ R_L = \sqrt{Z_2(Z_1 + Z_2)} \end{cases} \Rightarrow \begin{cases} Z_1 = \pm j R_S \sqrt{\dfrac{R_L}{R_S - R_L}} \\ Z_2 = \mp j R_L \sqrt{\dfrac{R_S - R_L}{R_L}} \end{cases}$$

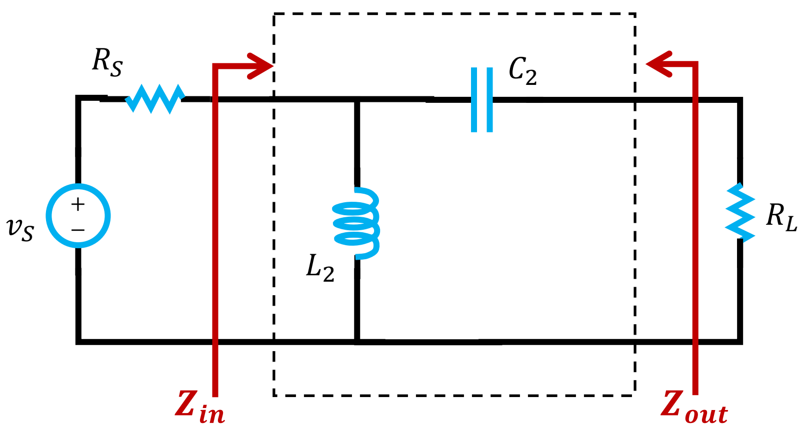
Example 4

QUESTION: find the value of Z_1 and Z_2 to maximize the output power on R_L



- Solution 1

$$\begin{cases} Z_1 = -jR_S\sqrt{\dfrac{R_L}{R_S - R_L}} \\ Z_2 = +jR_L\sqrt{\dfrac{R_S - R_L}{R_L}} \end{cases} \Rightarrow \begin{cases} C_1 = \dfrac{1}{R_S}\sqrt{\dfrac{R_S - R_L}{R_L}} \\ L_1 = R_L\sqrt{\dfrac{R_S - R_L}{R_L}} \end{cases}$$



- Solution 2

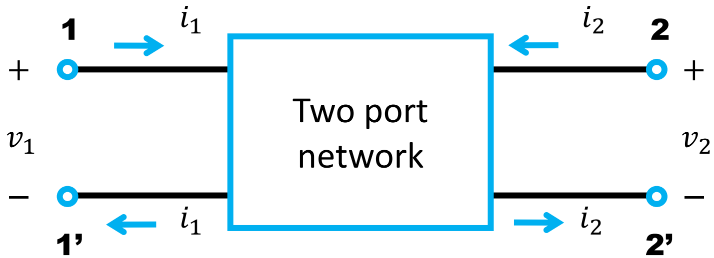
$$\begin{cases} Z_1 = +jR_S\sqrt{\dfrac{R_L}{R_S - R_L}} \\ Z_2 = -jR_L\sqrt{\dfrac{R_S - R_L}{R_L}} \end{cases} \Rightarrow \begin{cases} C_2 = \dfrac{1}{R_L}\sqrt{\dfrac{R_L}{R_S - R_L}} \\ L_2 = R_S\sqrt{\dfrac{R_L}{R_S - R_L}} \end{cases}$$

Outlines

- One-port network
  - Definition of one-port network
  - One-port network in series / parallel
  - Circuit equivalent as one-port network
- Two-port network & Parameters
  - Definition of two–port network
  - Z parameters / **$\boldsymbol{Y}$ parameters**

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Y parameters



- i_1, i_2 are dependent
- v_1, v_2 are independent

$$Y_{11} = \frac{i_1}{v_1} \quad \text{when } v_2 = 0$$

$$Y_{12} = \frac{i_1}{v_2} \quad \text{when } v_1 = 0$$

$$Y_{21} = \frac{i_2}{v_1} \quad \text{when } v_2 = 0$$

$$Y_{22} = \frac{i_2}{v_2} \quad \text{when } v_1 = 0$$

→ *Y* parameters

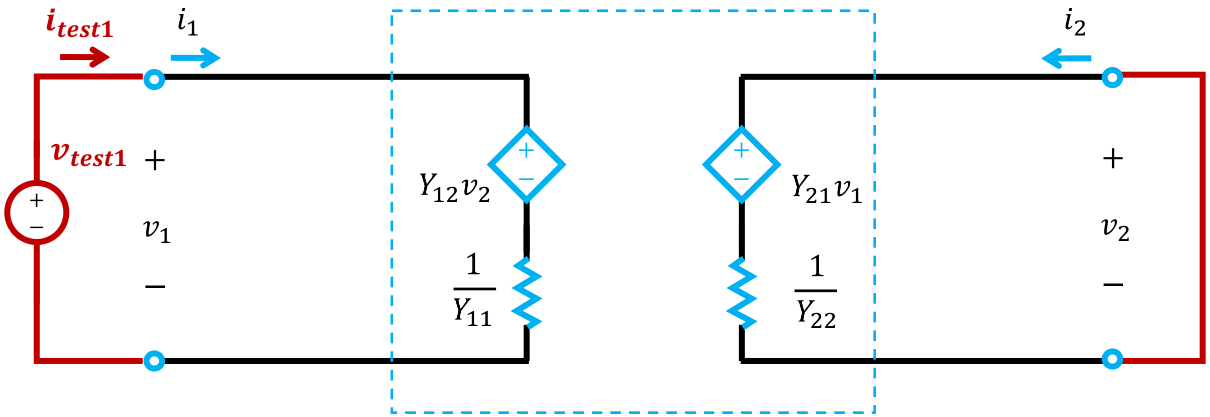
$$\begin{cases} i_1 = Y_{11}v_1 + Y_{12}v_2 \\ i_2 = Y_{21}v_1 + Y_{22}v_2 \end{cases}$$

or

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

unit: *S*

Y parameters



- According to KVL

$$\begin{cases} i_1 = Y_{11}v_1 + Y_{12}v_2 \\ i_2 = Y_{21}v_1 + Y_{22}v_2 \end{cases}$$

Y Parameters are also called as short-circuit admittance parameters

What does Y_{11} mean?

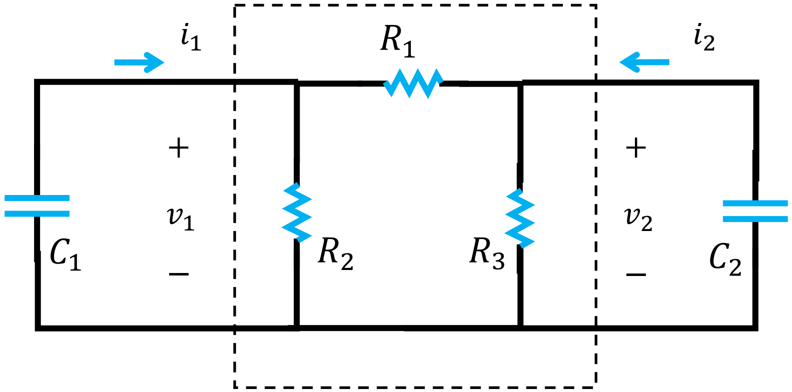
- Step 1: short port 2 open, thus $v_2 = 0$
- Step 2: apply a current v_{test1} to port 1
- Step 3: we can find Y_{11} according to the voltage v_{test1} measured in port 1

$$Y_{11} = \left.\frac{i_{test1}}{v_{test1}}\right|_{v_2=0}$$

INPUT ADMITTANCE

Example 5

QUESTION: find the Y parameters of the network labeled in the dash line



Y parameters

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$Y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} = \frac{1}{R_1} + \frac{1}{R_2}$$

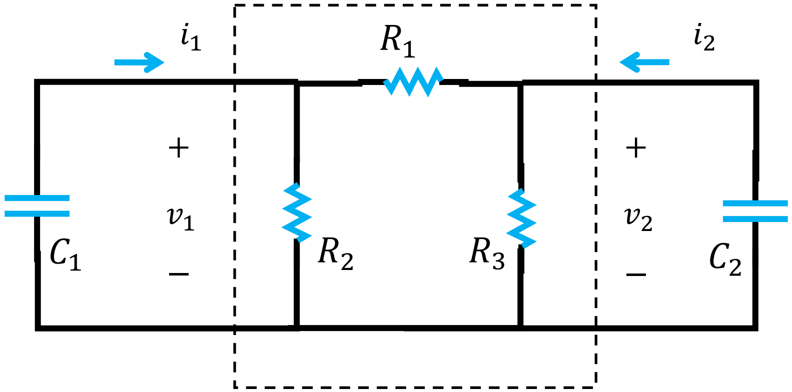
$$Y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} = -\frac{1}{R_1}$$

$$Y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} = -\frac{1}{R_1}$$

$$Y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = \frac{1}{R_1} + \frac{1}{R_3}$$

Example 5

QUESTION: find the Y parameters of the network labeled in the dash line



The Y parameters of the network

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

- According to the $i - v$ relationship of the capacitors

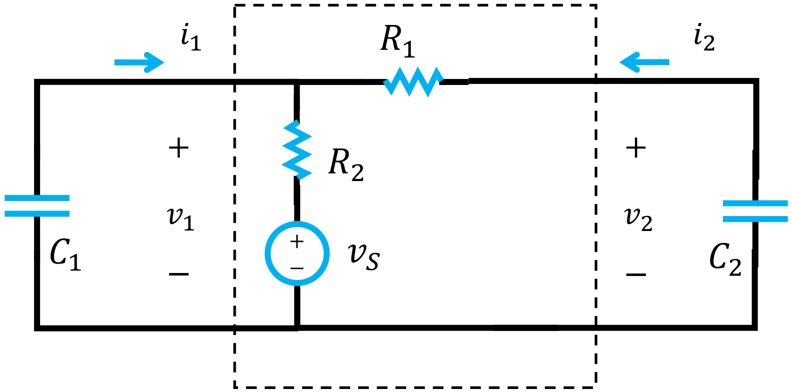
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} C_1 \frac{dv_1}{dt} \\ C_2 \frac{dv_2}{dt} \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} C_1 v_1 \\ C_2 v_2 \end{bmatrix}$$

- The transient function is as

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} C_1 v_1 \\ C_2 v_2 \end{bmatrix}$$

Example 6

QUESTION: find the relationship of the voltages/currents for the network labeled in the dash line



- According to KCL

$$i_1 + i_2 = \frac{v_1 - v_S}{R_2}$$

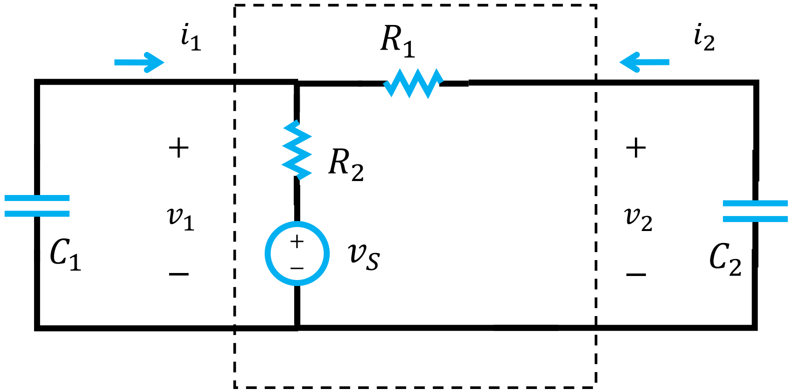
$$i_2 = \frac{v_2 - v_1}{R_1}$$

- The Y parameters

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{R_2} v_s \\ 0 \end{bmatrix}$$

Example 6

QUESTION: find the Y parameters of the network labeled in the dash line



The Y parameters of the network

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{R_2} v_s \\ 0 \end{bmatrix}$$

- According to the $i - v$ relationship of the capacitors $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} C_1 \frac{dv_1}{dt} \\ C_2 \frac{dv_2}{dt} \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} C_1 v_1 \\ C_2 v_2 \end{bmatrix}$

$$\frac{d}{dt} \begin{bmatrix} C_1 v_1 \\ C_2 v_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) & -\frac{1}{C_1 R_1} \\ -\frac{1}{C_2 R_1} & \frac{1}{C_2 R_1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{C_1 R_2} v_s \\ 0 \end{bmatrix}$$

constants — forcing func.

Outlines

- One-port network
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  - Z parameters / Y parameters
  - **T parameters / T' parameters**

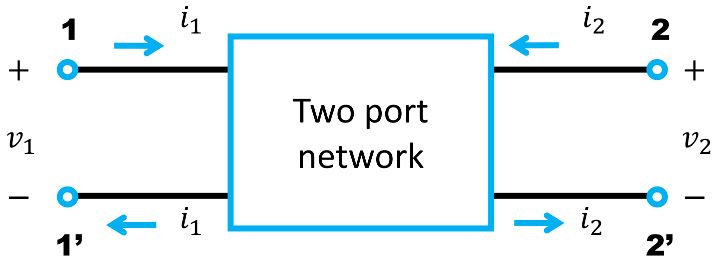
Z parameters

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Y parameters

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

T parameters



- v_1, i_1 are dependent
- v_2, i_2 are independent

→ T **parameters**

$$\begin{cases} v_1 = Av_2 - Bi_2 \\ i_1 = Cv_2 - Di_2 \end{cases}$$

or

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

$A = \frac{v_1}{v_2}$ when $i_2 = 0$

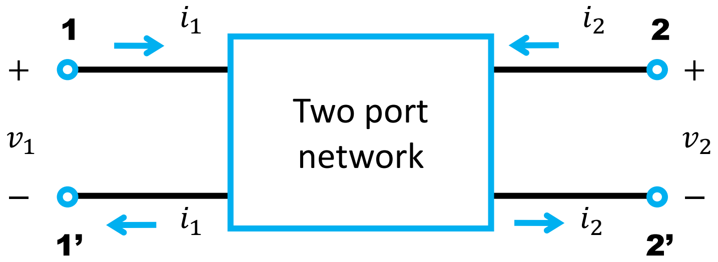
$B = -\frac{v_1}{i_2}$ when $v_2 = 0$

$C = \frac{i_1}{v_2}$ when $i_2 = 0$

$D = -\frac{i_1}{i_2}$ when $v_2 = 0$

T Parameters are also called as $ABCD$ parameters

T’ parameters



- v_1, i_1 are dependent
- v_2, i_2 are independent

→ T' **parameters**

$$\begin{cases} v_2 = av_1 - bi_1 \\ i_2 = cv_1 - di_1 \end{cases}$$

or

$$\begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} v_1 \\ -i_1 \end{bmatrix}$$

$$a = \frac{v_2}{v_1} \qquad \text{when } i_1 = 0$$

$$b = -\frac{v_2}{i_1} \qquad \text{when } v_1 = 0$$

$$c = \frac{i_2}{v_1} \qquad \text{when } i_1 = 0$$

$$d = -\frac{i_2}{i_1} \qquad \text{when } v_1 = 0$$

T' Parameters are also called as $abcd$ parameters

Outlines

- One-port network
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  - Definition of two-port network
  - Z parameters / Y parameters
  - T parameters / T' parameters
  - **h parameters / g parameters**

Z parameters

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Y parameters

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

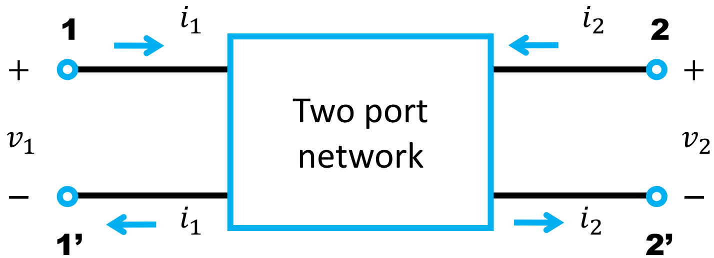
T parameters

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

T' parameters

$$\begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} v_1 \\ -i_1 \end{bmatrix}$$

h parameters



- v_1, i_2 are dependent
- v_2, i_1 are independent

→ ***h* parameters**

$$\begin{cases} v_1 = h_{11} i_1 + h_{12} v_2 \\ i_2 = h_{21} i_1 + h_{22} v_2 \end{cases}$$

or

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

h_{11}: Ω; h_{12}: no unit; h_{21}: no unit; h_{22}: S

$$h_{11} = \frac{v_1}{i_1} \quad \text{when } v_2 = 0$$

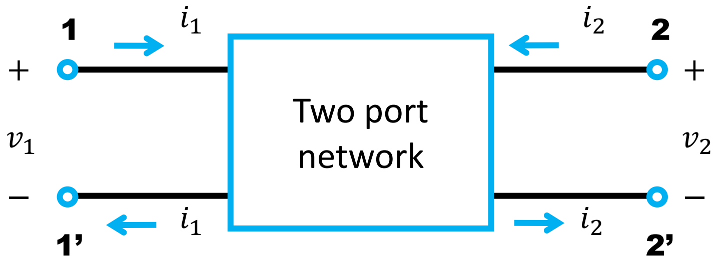
$$h_{12} = \frac{v_1}{v_2} \quad \text{when } i_1 = 0$$

$$h_{21} = \frac{i_2}{i_1} \quad \text{when } v_2 = 0$$

$$h_{22} = \frac{i_2}{v_2} \quad \text{when } i_1 = 0$$

***h* Parameters are also called as hybrid parameters**

g parameters



- i_1, v_2 are dependent
- i_2, v_1 are independent

→ g **parameters**

$$\begin{cases} i_1 = g_{11}v_1 + g_{12}i_2 \\ v_2 = g_{21}v_1 + g_{22}i_2 \end{cases}$$

or

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

g_{11}: S; g_{12}: no unit; g_{21}: no unit; g_{22}: Ω

$g_{11} = \frac{i_1}{v_1}$ when $i_2 = 0$

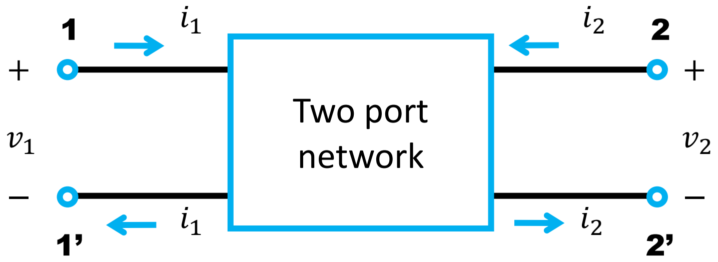
$g_{12} = \frac{i_1}{i_2}$ when $v_1 = 0$

$g_{21} = \frac{v_2}{v_1}$ when $i_2 = 0$

$g_{22} = \frac{v_2}{i_2}$ when $v_1 = 0$

g Parameters are also called as inverse hybrid parameters

Network parameters



***Z* parameters**

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

***Y* parameters**

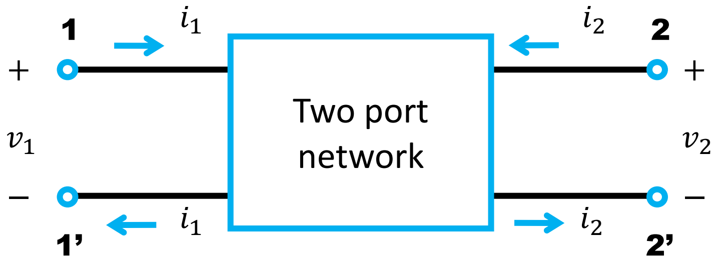
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

➡ $\boldsymbol{Z = Y^{-1}}$

Network parameters



***Z* parameters**

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

***Y* parameters**

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\boldsymbol{Z} = \boldsymbol{Y}^{-1}$$

***T* parameters**

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

***T'* parameters**

$$\begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} v_1 \\ -i_1 \end{bmatrix}$$

$$\boldsymbol{T} = \boldsymbol{T}'^{-1}$$

***h* parameters**

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

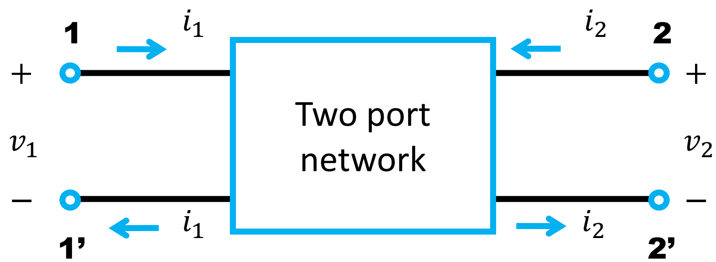
***g* parameters**

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

$$\boldsymbol{h} = \boldsymbol{g}^{-1}$$

Example 5

QUESTION: Find the Z parameters according to the T parameters



T parameters

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

Z parameters

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

- According to the eq. of i_1

$$i_1 = Cv_2 - Di_2 \quad \rightarrow v_2 = \frac{1}{C} i_1 + \frac{D}{C} i_2$$

- Take the eq. of v_2 to the eq. of v_1

$$v_1 = Av_2 - Bi_2 = A\left(\frac{1}{C} i_1 + \frac{D}{C} i_2\right) - Bi_2$$

$$= \frac{A}{C} i_1 + \left(\frac{AD}{C} - B\right) i_2$$

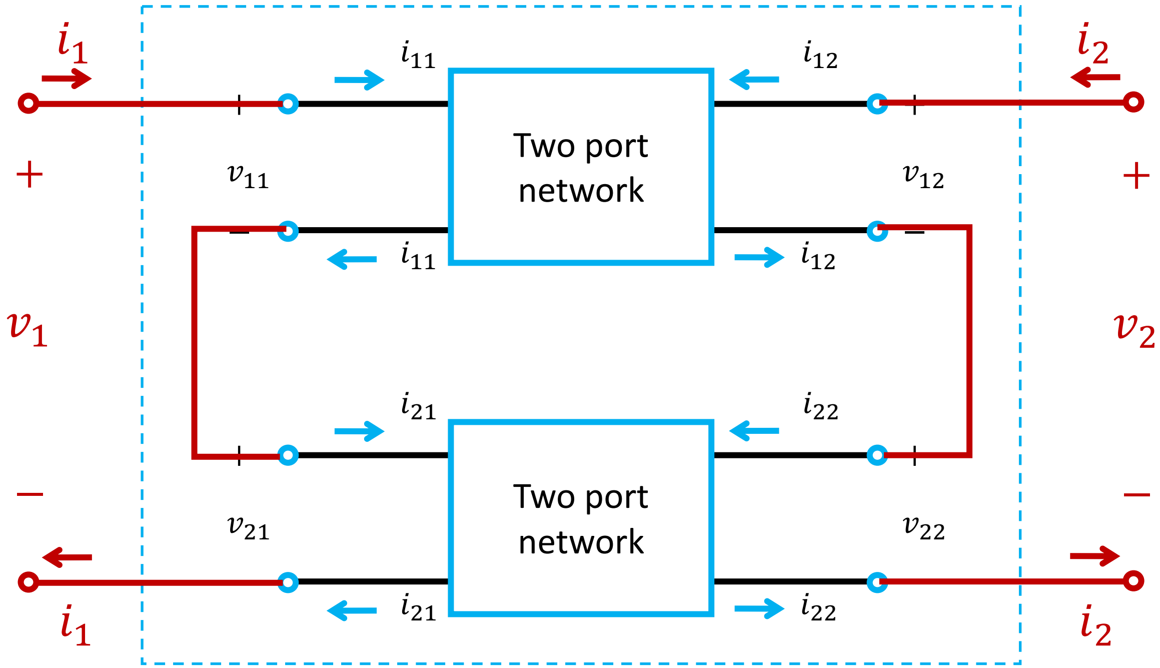
- The Z parameters are

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{C} \begin{bmatrix} A & AD - BC \\ 1 & D \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Outlines

- One-port network
  - Definition of one-port network
  - One-port network in series / parallel
  - Circuit equivalent as one-port network
- Two-port network & Parameters
  - Definition of two-port network
  - Z / Y / T / T' / h / g parameters
  - **Series / parallel / cascading connections**

Two-port network in series



- The relationship of the currents & voltages

$$\begin{cases} v_1 = v_{11} + v_{21} \\ v_2 = v_{12} + v_{22} \end{cases}$$

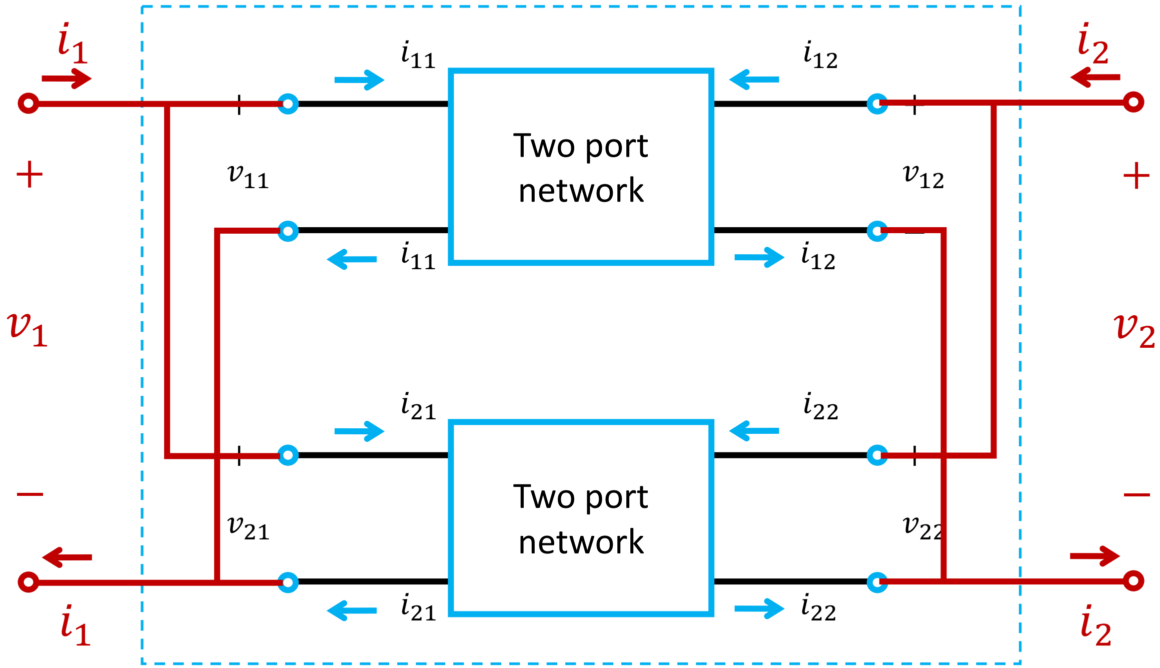
$$\begin{cases} i_1 = i_{11} = i_{12} \\ i_2 = i_{21} = i_{22} \end{cases}$$

- The parameters of the series connected network

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} + \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_1 \begin{bmatrix} i_{11} \\ i_{12} \end{bmatrix} + \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_2 \begin{bmatrix} i_{21} \\ i_{22} \end{bmatrix}$$

$$= \left(\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_1 + \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_2 \right) \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Two-port network in parallel



- The relationship of the currents & voltages

$$\begin{cases} v_1 = v_{11} = v_{21} \\ v_2 = v_{12} = v_{22} \end{cases}$$

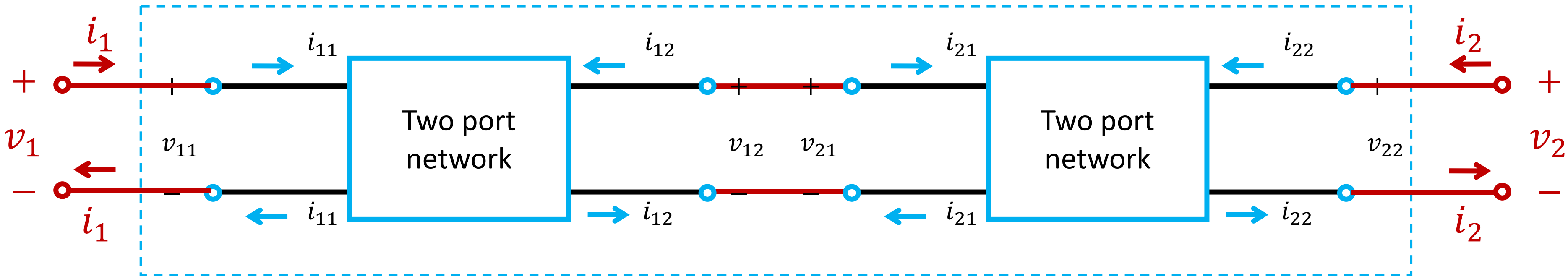
$$\begin{cases} i_1 = i_{11} + i_{21} \\ i_2 = i_{12} + i_{22} \end{cases}$$

- The parameters of the parallel connected network

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_{11} \\ i_{12} \end{bmatrix} + \begin{bmatrix} i_{21} \\ i_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_1 \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} + \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_2 \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix}$$

$$= \left(\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_1 + \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_2 \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Cascading two-port network



- The relationship of the currents & voltages

$$\begin{cases} v_1 = v_{11} \\ v_{12} = v_{21} \\ v_2 = v_{22} \end{cases} \qquad \begin{cases} i_1 = i_{11} \\ i_{12} = i_{21} \\ i_2 = i_{22} \end{cases}$$

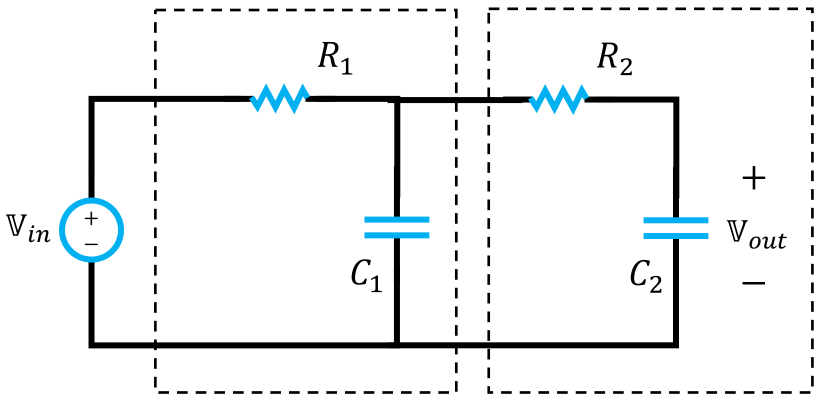
- The parameters of the cascading network

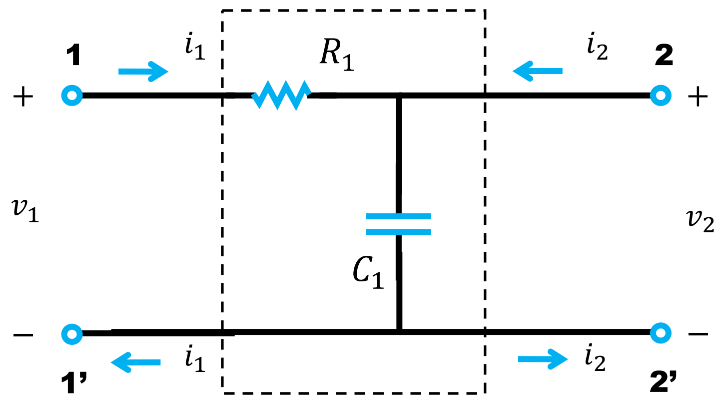
$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} v_{11} \\ i_{11} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} v_{12} \\ -i_{12} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} v_{21} \\ i_{21} \end{bmatrix}$$

$$= \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} v_{22} \\ -i_{22} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

Example 6

QUESTION: find the T parameters of the circuit below





- The goal is to find the T parameters of a RC network

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

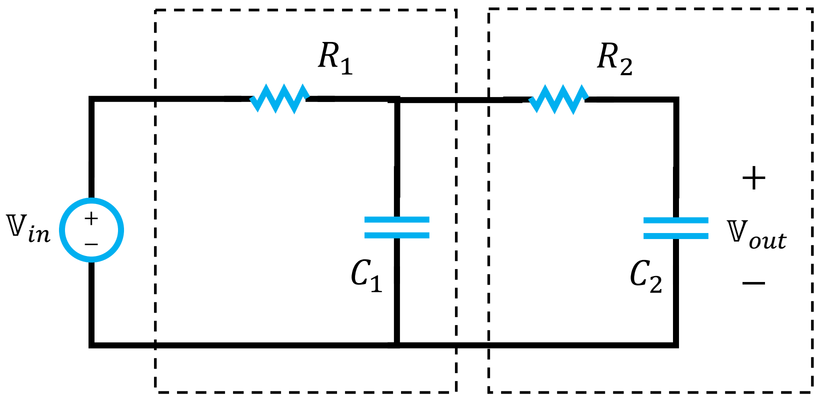
- Keep port 2 open circuit

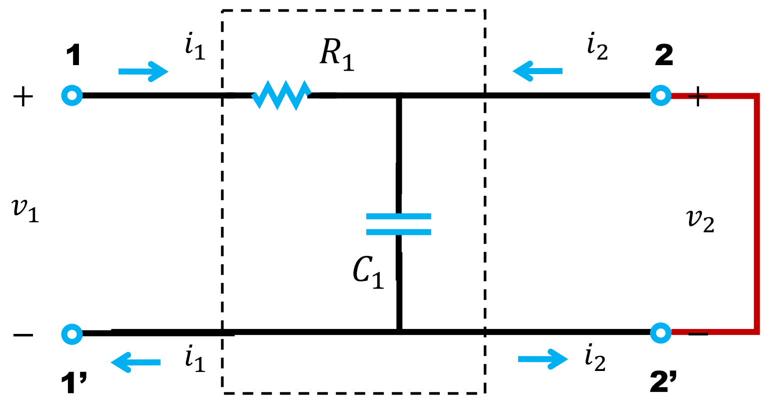
$$A = \left.\frac{v_1}{v_2}\right|_{i_2=0} = \frac{R_1 + \frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1}} = 1 + j\omega R_1 C_1$$

$$C = \left.\frac{i_1}{v_2}\right|_{i_2=0} = j\omega C_1$$

Example 6

QUESTION: find the T parameters of the circuit below





- Keep port 2 short circuit

$$B = -\left.\frac{v_1}{i_2}\right|_{v_2=0} = R_1$$

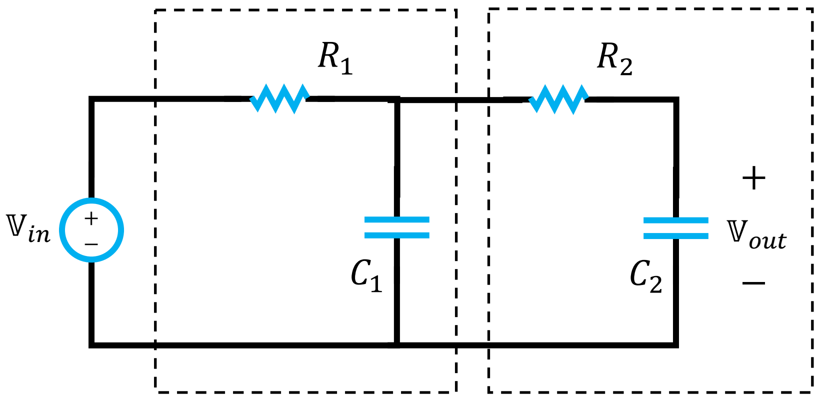
$$D = -\left.\frac{i_1}{i_2}\right|_{v_2=0} = 1$$

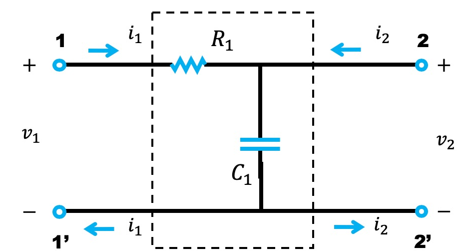
- The T parameters of the RC network

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} 1 + j\omega R_1 C_1 & R_1 \\ j\omega C_1 & 1 \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

Example 6

QUESTION: find the T parameters of the circuit below





$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} 1 + j\omega R_1 C_1 & R_1 \\ j\omega C_1 & 1 \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

$$\begin{bmatrix} v_{in} \\ i_{in} \end{bmatrix} = \begin{bmatrix} 1 + j\omega R_1 C_1 & R_1 \\ j\omega C_1 & 1 \end{bmatrix} \begin{bmatrix} 1 + j\omega R_2 C_2 & R_2 \\ j\omega C_2 & 1 \end{bmatrix} \begin{bmatrix} v_{out} \\ -i_{out} \end{bmatrix}$$

$$= \begin{bmatrix} (1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_1 C_2 & j\omega R_1 R_2 C_2 + R_1 + R_2 \\ j\omega (C_1 + C_2) - \omega^2 R_2 C_1 C_2 & 1 + j\omega R_2 C_2 \end{bmatrix} \begin{bmatrix} v_{out} \\ -i_{out} \end{bmatrix}$$

Outlines

- One-port network
  - Definition of one-port network
  - One-port network in series / parallel
  - Circuit equivalent as one-port network
- Two-port network & Parameters
  - Definition of two-port network
  - Z / Y / T / T' / h / g parameters
  - Series / parallel / cascading connections

Reading tasks & learning goals

- Learning goals
  - Know how to find the **circuit equivalent** by applying a voltage/current to the port.
  - Understand the concept of **input impedance** and **equivalent impedance** from a port
  - Know how to calculate the **network parameters** of a two-port network