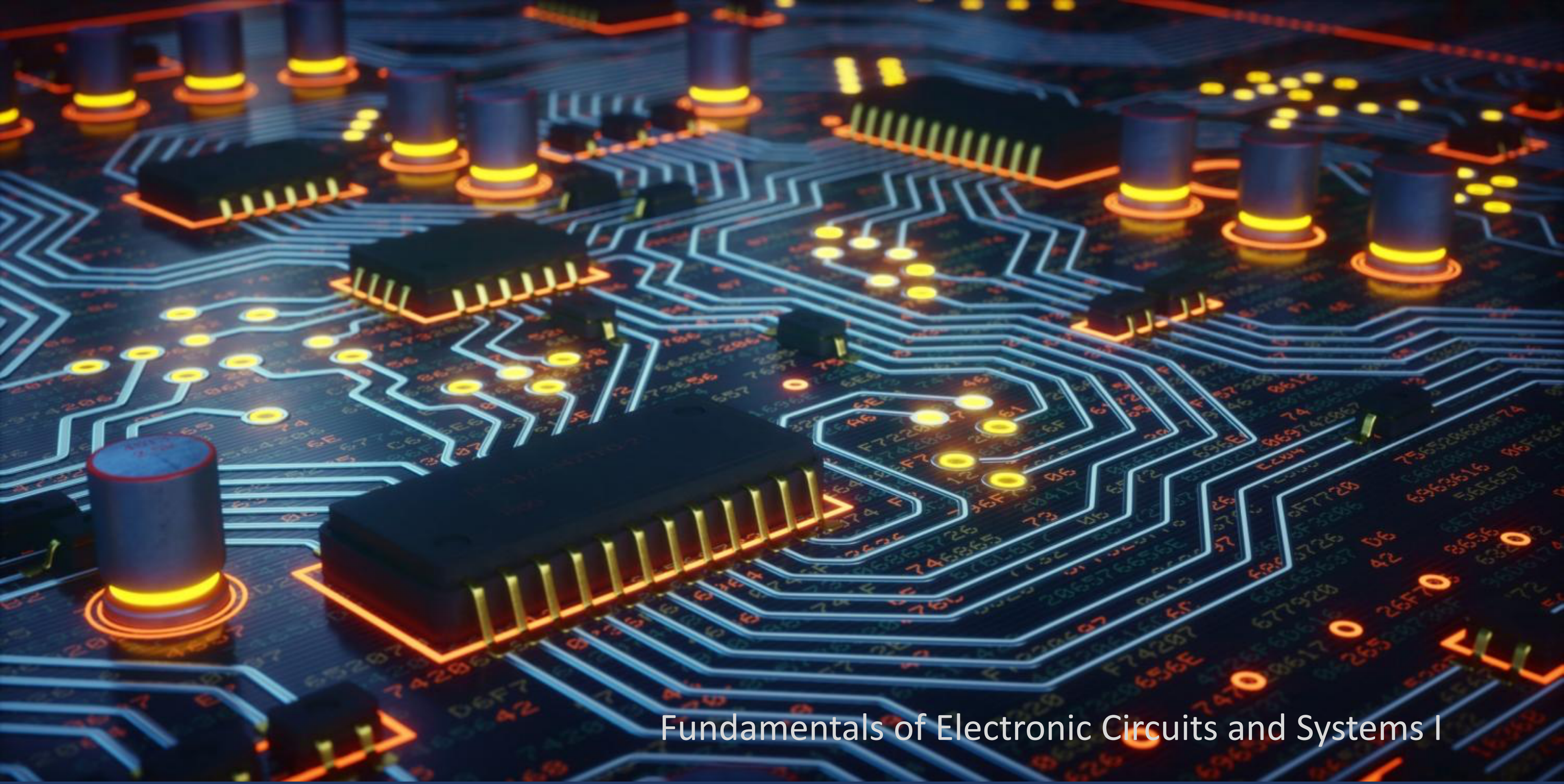


Fundamentals of Electronic Circuits and Systems I

Circuit Analysis Techniques

Milin Zhang

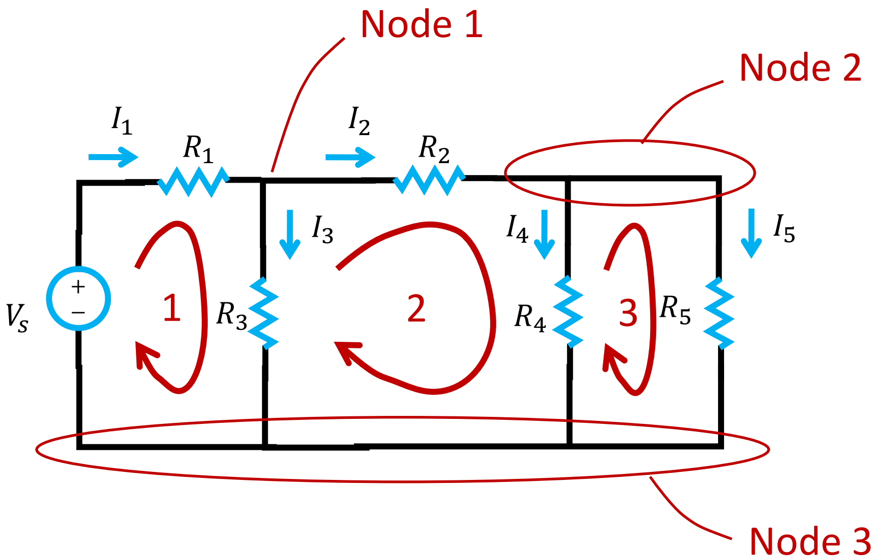
Dept of EE, Tsinghua University

Outlines

- Node-voltage method
- Mesh-current method
- Linearity & Superposition
- Thévenin and Norton Equivalent Circuits

Review: an example from last lecture

QUESTION: Find the output current of the voltage source

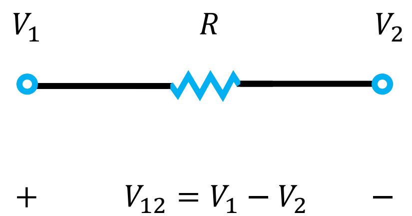


- According to KVL/KCL/Ohm's law

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ R_s & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & R_L & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_L \\ V_{R_s} \\ V_{R_2} \\ V_{R_3} \\ V_{R_4} \\ V_{R_L} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_s$$

A lot of equations! A lot of work!

Basis of node voltage



Node voltages
Voltage at the node

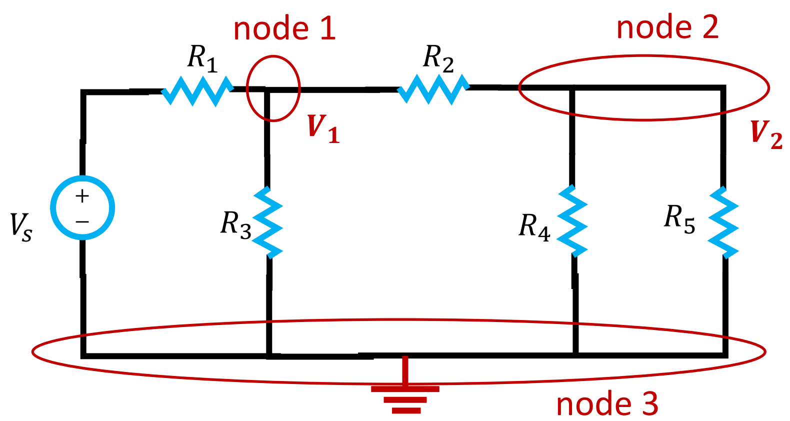
Current in terms of node voltages

$$I = \frac{V_1 - V_2}{R}$$

Elemental voltage
The voltage across the element

Node-Voltage Method

QUESTION: if all the resistor values and V_s are known. Find all voltage & current in circuit



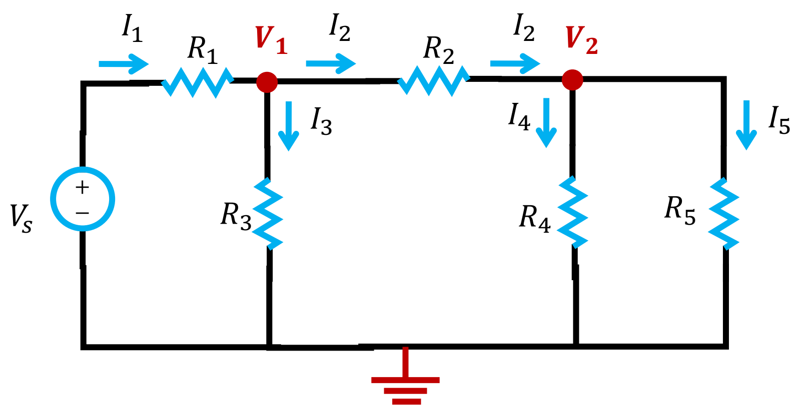
- Step 1a: identify all extraordinary nodes
- Step 1b: Assign one extraordinary node as **ground**

  node 3 = **ground** = **0V**

- Step 1c: assign node voltages to rest of the extraordinary nodes

Node-Voltage Method

QUESTION: if all the resistor values and V_s are known. Find all voltage & current in circuit



- Step 1a: identify all extraordinary nodes
- Step 1b: Assign one extraordinary node as **ground**. node 3 = **ground** = **0V**
- Step 1c: assign node voltages to rest of the extraordinary nodes

- Step 2a: perform **KCL** at each node except the ground node

@node 1

$$I_1 - I_2 - I_3 = 0$$

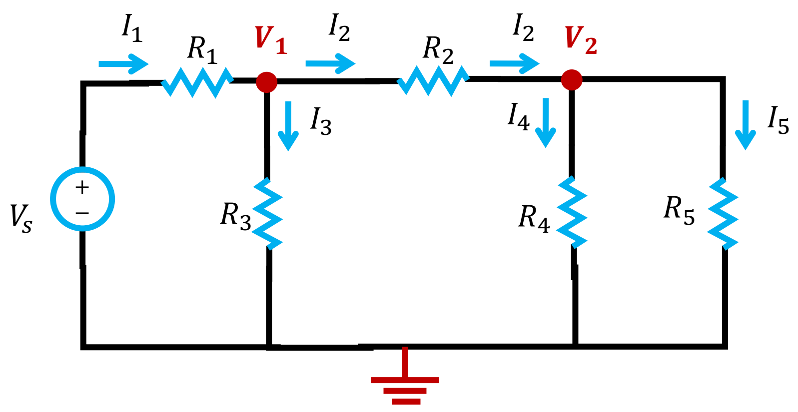
@node 2

$$I_2 - I_4 - I_5 = 0$$

[* Note: Assign a positive sign to current entering a node]

Node-Voltage Method

QUESTION: if all the resistor values and V_s are known. Find all voltage & current in circuit



- Step 1a: identify all extraordinary nodes
- Step 1b: Assign one extraordinary node as **ground**. node 3 = **ground** = **0V**
- Step 1c: assign node voltages to rest of the extraordinary nodes

- Step 2a: perform **KCL** at each node except the ground node
- Step 2b: apply ohm's law for each current in terms of node voltages

@node 1

$$I_1 - I_2 - I_3 = 0$$

$$\frac{V_s - V_1}{R_1} - \frac{V_1 - V_2}{R_2} - \frac{V_1}{R_3} = 0$$

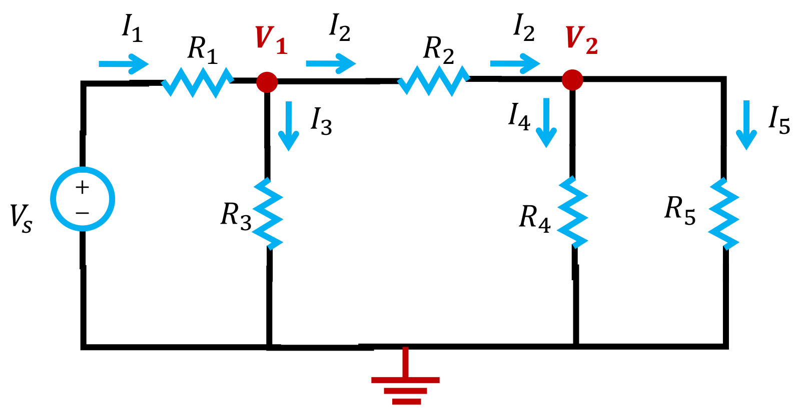
@node 2

$$I_2 - I_4 - I_5 = 0$$

$$\frac{V_1 - V_2}{R_2} - \frac{V_2}{R_4} - \frac{V_2}{R_5} = 0$$

Node-Voltage Method

QUESTION: if all the resistor values and V_s are known. Find all voltage & current in circuit



$$\begin{cases} \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)V_1-\frac{1}{R_2}V_2=\frac{1}{R_1}V_s \\ \frac{1}{R_2}V_1-\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right)V_2=0 \end{cases}$$

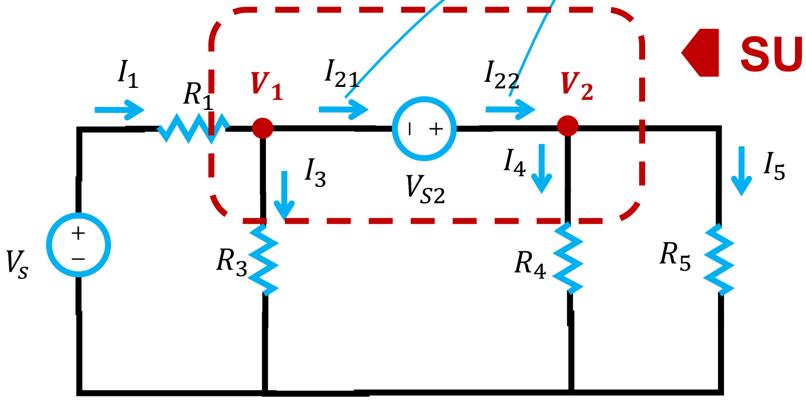
2 equations in 2 unknowns

- Step 1a: identify all extraordinary nodes
- Step 1b: Assign **ground** node
- Step 1c: assign node voltages to the rest
- Step 2a: perform **KCL** at each node
- Step 2b: apply ohm's law for each current
- Step 3: solve the system of equations

[* Note:
the # of equations to solve =
# of extraordinary nodes – 1
Since we don't write an equation
for GND node. It's redundant]

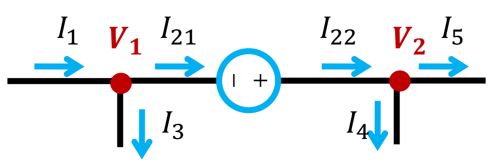
Supernode

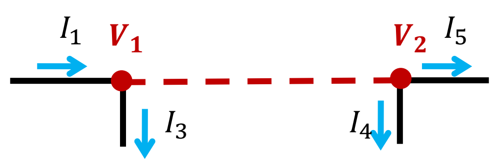
No way to determine this current!!



SUPERNODE

a SUPERNODE is formed
when a voltage source connects
two extraordinary nodes





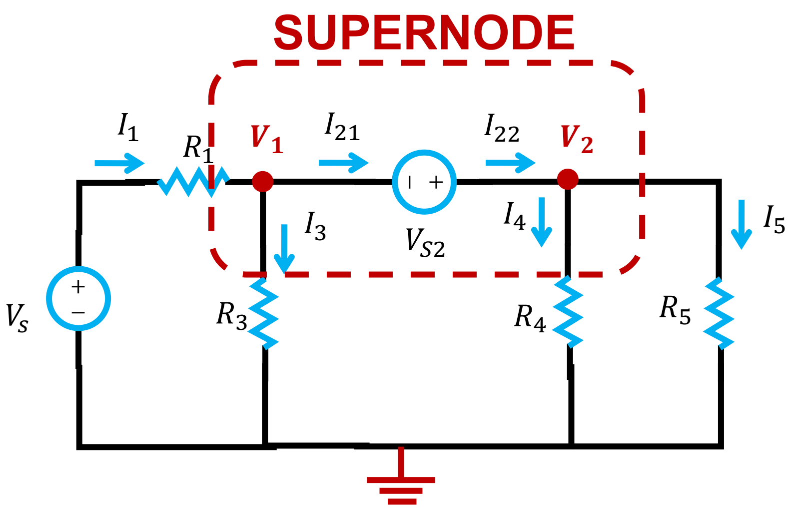
- According to KCL

$I_1 - I_3 = I_{21} \quad I_{22} = I_4 + I_5$

$I_1 - I_3 - I_4 - I_5 = 0$

The "internal" current (I_{21}, I_{22}) cancels

Supernode w/ node-voltage method



@ the supernode

$$I_1 - I_3 - I_4 - I_5 = 0$$

$$\frac{V_s - V_1}{R_1} - \frac{V_1}{R_3} - \frac{V_2}{R_4} - \frac{V_2}{R_5} = 0$$

1 equations in 2 unknowns

- Step 2c: generate an auxiliary equation at the supernode

$$V_2 - V_1 = V_{s2}$$

2 equations in 2 unknowns

- Step 1a: identify all extraordinary nodes
- Step 1b: Assign **ground** node
- Step 1c: assign node voltages to the rest
- Step 2a: perform **KCL** at each node
- Step 2b: apply ohm's law for each current

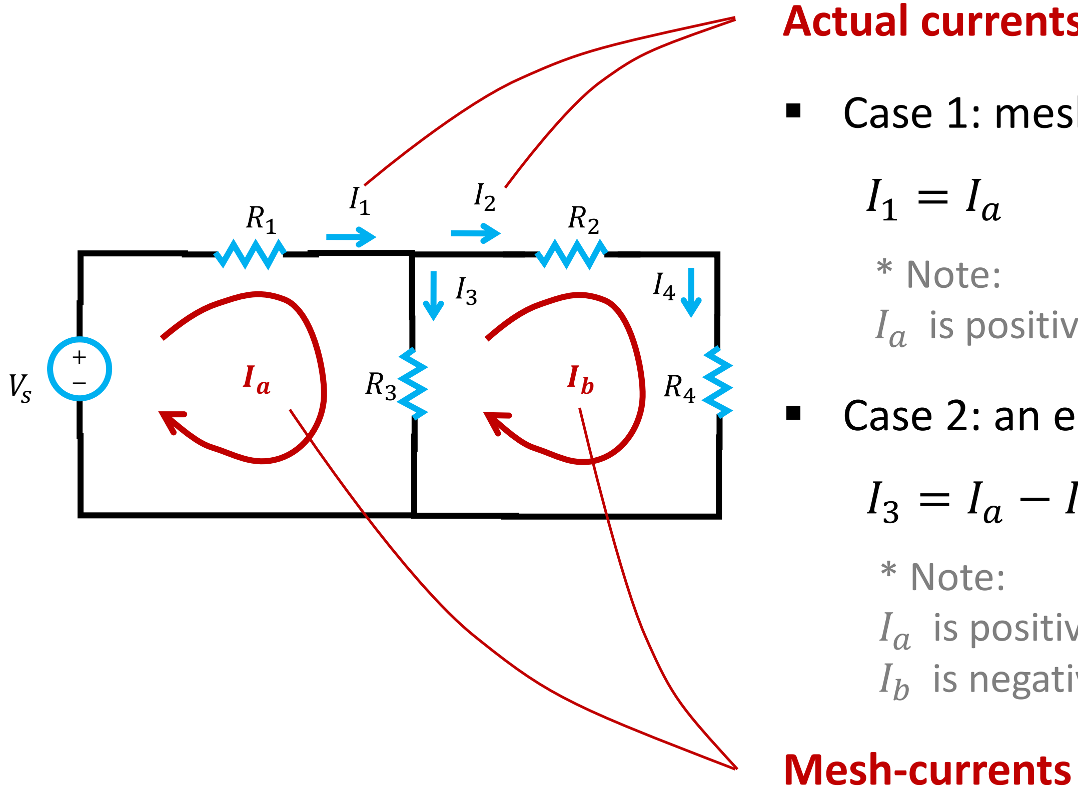
Outlines

- Node-voltage method
  - Basis of node voltage
  - Node-voltage method for circuit analysis
  - Supernode w/ node-voltage method

Outlines

- Node-voltage method
  - Basis of node voltage
  - Node-voltage method for circuit analysis
  - Supernode w/ node-voltage method
- Mesh-current method

Basis of mesh-current



- Case 1: mesh-current equals actual current

$$I_1 = I_a$$

* Note:
I_a is positive as it agrees with I_1's direction

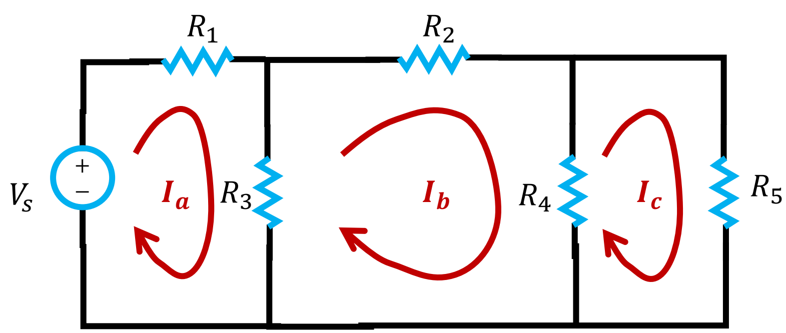
- Case 2: an element is shared by 2 meshes

$$I_3 = I_a - I_b$$

* Note:
I_a is positive as it agrees with I_3's direction
I_b is negative as it flows in opposite to I_3's direction

Mesh-current method

QUESTION: Find the output current of the voltage source



- Step 1a: identify all mesh
- Step 1b: assign each mesh an unknown current, usually in clockwise direction

- Step 2a: apply KVL to each mesh

In mesh a

$$-V_s + I_a R_1 + (I_a - I_b) R_3 = 0$$

In mesh b

$$(I_b - I_a) R_3 + I_b R_2 + (I_b - I_c) R_4 = 0$$

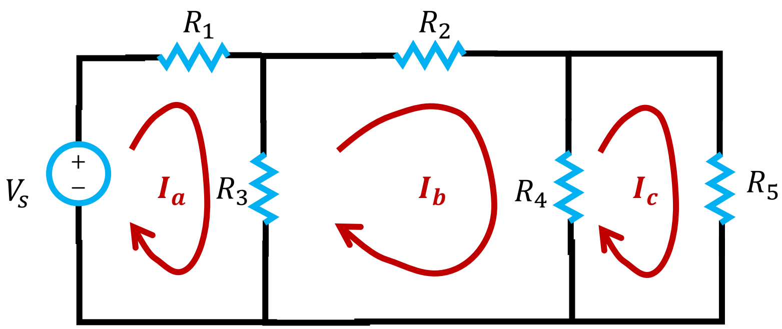
In mesh c

$$(I_c - I_b) R_4 + I_c R_5 = 0$$

* Note:
- Put all voltages in terms of mesh-current
- Each term is just Ohm's law
- Sign of the current

Mesh-current method

QUESTION: Find the output current of the voltage source



- Step 1a: identify all mesh
- Step 1b: assign each mesh an unknown current, usually in clockwise direction
- Step 2a: apply KVL to each mesh
- Step 2b: group terms by mesh-current

In mesh a

$$-V_s + I_a R_1 + (I_a - I_b) R_3 = 0$$

$\Rightarrow$ $I_a(R_1 + R_3) - I_b R_3 = V_s$

In mesh b

$$(I_b - I_a)R_3 + I_b R_2 + (I_b - I_c)R_4 = 0$$

$\Rightarrow$ $-I_a R_3 + I_b(R_3 + R_2 + R_4) - I_c R_4 = 0$

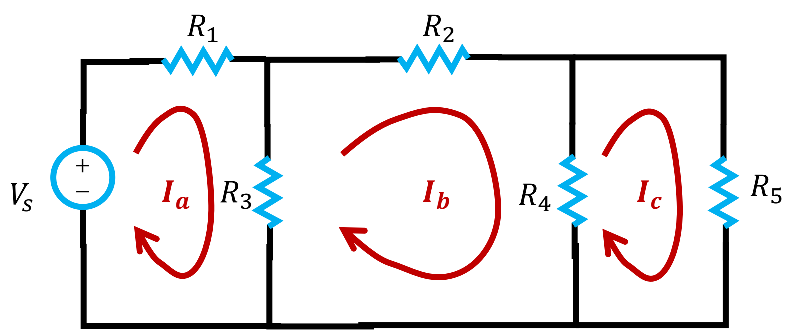
In mesh c

$$(I_c - I_b)R_4 + I_c R_5 = 0$$

$\Rightarrow$ $-I_b R_4 + I_c(R_4 + R_5) = 0$

Mesh-current method

QUESTION: Find the output current of the voltage source



$$\begin{cases} I_a(R_1+R_3) - I_bR_3 = V_s \\ -I_aR_3 + I_b(R_3+R_2+R_4) - I_cR_4 = 0 \\ -I_bR_4 + I_c(R_4+R_5) = 0 \end{cases}$$

- Step 1a: identify all mesh
- Step 1b: assign each mesh an unknown current, usually in clockwise direction
- Step 2a: apply KVL to each mesh
- Step 2b: group terms by mesh-current
- Step 3: solve the system of equations

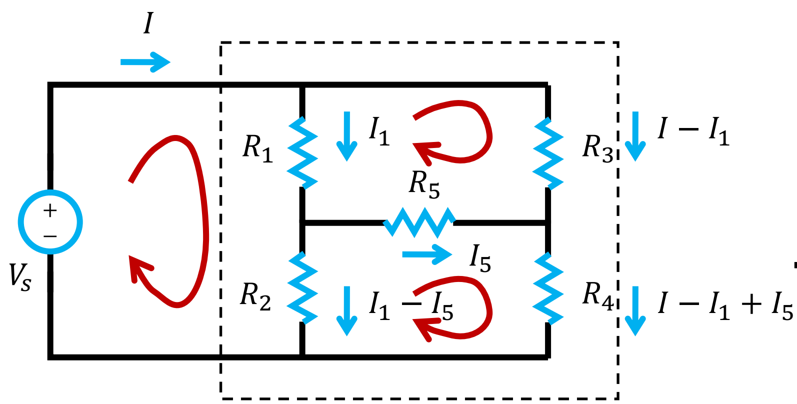
3 equations in 3 unknowns

[* Note:
All the actual currents can be calculated from the mesh-current, such as

$$I_{R_3} = I_a - I_b$$]

Example 1 Wheatstone Bridge

QUESTION: Find the equivalent resistance of the circuit in the block



According to mesh-current method

$$\begin{cases} -V_s + R_1 I_1 + (I_1 - I_5)R_2 = 0 \\ R_1 I_1 + R_5 I_5 - (I - I_1)R_3 = 0 \\ -(I_1 - I_5)R_2 + R_5 I_5 + (I - I_1 + I_5)R_4 = 0 \end{cases}$$

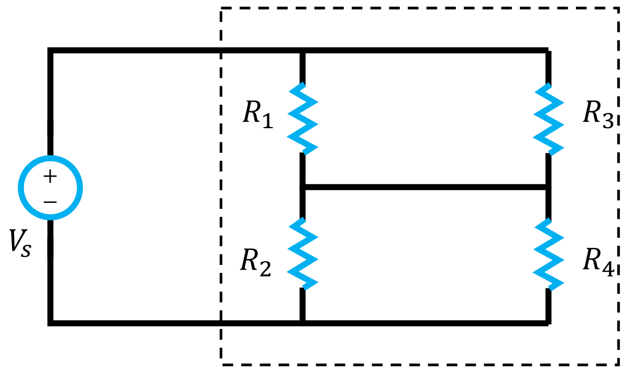
3 equations in 3 unknowns

$$\begin{bmatrix} 0 & R_1 + R_2 & -R_2 \\ -R_3 & R_1 + R_3 & R_5 \\ -R_4 & R_2 + R_4 & -R_2 - R_4 - R_5 \end{bmatrix} \begin{bmatrix} I \\ I_1 \\ I_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} V_s$$

$$R_{eq} = \frac{V_s}{I} = \frac{R_5(R_1 + R_2)(R_3 + R_4) + R_2R_4(R_1 + R_3) + R_1R_3(R_2 + R_4)}{R_5(R_1 + R_2 + R_3 + R_4) + (R_2 + R_4)(R_1 + R_3)}$$

Example 1 Wheatstone Bridge

$$R_{eq} = \frac{V_s}{I} = \frac{R_5(R_1+R_2)(R_3+R_4) + R_2R_4(R_1+R_3) + R_1R_3(R_2+R_4)}{R_5(R_1+R_2+R_3+R_4) + (R_2+R_4)(R_1+R_3)}$$

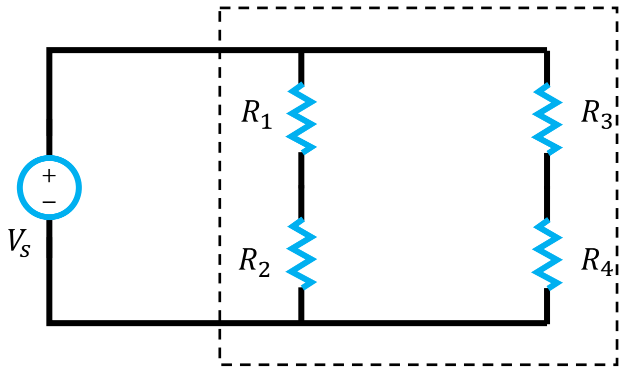


- If $R_5 = 0$ ➧ short circuit

$$R_{eq} = \frac{R_2R_4(R_1+R_3) + R_1R_3(R_2+R_4)}{(R_2+R_4)(R_1+R_3)}$$

$$= \frac{R_2R_4}{R_2+R_4} + \frac{R_1R_3}{R_1+R_3}$$

$$= R_2||R_4 + R_1||R_3$$

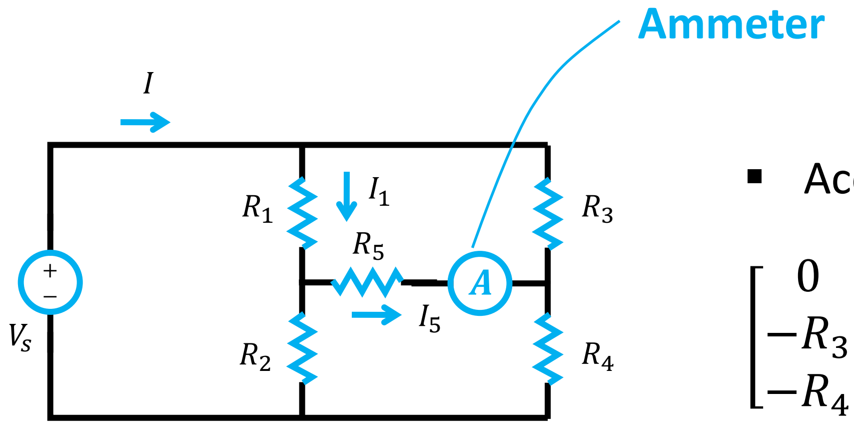


- If $R_5 = \infty$ ➧ open circuit

$$R_{eq} = \frac{(R_1+R_2)(R_3+R_4)}{R_1+R_2+R_3+R_4}$$

$$= (R_1+R_2)||(R_3+R_4)$$

Example 1 Wheatstone Bridge



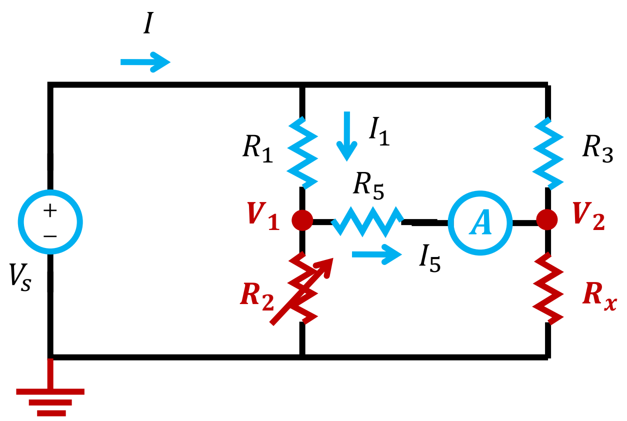
- According to system of equations

$$\begin{bmatrix} 0 & R_1 + R_2 & -R_2 \\ -R_3 & R_1 + R_3 & R_5 \\ -R_4 & R_2 + R_4 & -R_2 - R_4 - R_5 \end{bmatrix} \begin{bmatrix} I \\ I_1 \\ I_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} V_s$$

$$I_5 = \frac{R_2 R_3 - R_1 R_4}{R_5(R_1 + R_2)(R_3 + R_4) + R_2 R_4 (R_1 + R_3) + R_1 R_3 (R_2 + R_4)} V_s$$

- Balanced condition: $I_5 = 0$ → $\boldsymbol{R_2 R_3 = R_1 R_4}$

Example 1 Wheatstone Bridge



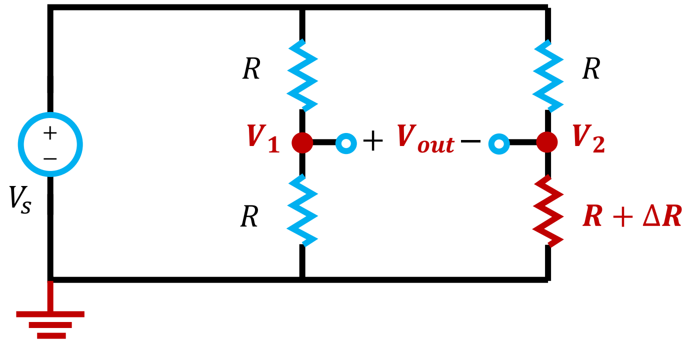
- Balanced condition: $I_5 = 0$ ➡ $\boldsymbol{V_1 = V_2}$
- According to voltage division

$$\begin{cases} V_1 = \dfrac{R_2}{R_1 + R_2} V_s \\ V_2 = \dfrac{R_x}{R_3 + R_x} V_s \end{cases} \quad \Rightarrow \quad \boldsymbol{R_x = \dfrac{R_2 R_3}{R_1}}$$

Wheatstone bridge can be used to determine unknown resistance based on "balanced" condition

Example 1 Wheatstone Bridge

QUESTION: Find the voltage V_{out}



- According to voltage division

$$\begin{cases} V_1 = \dfrac{R}{R+R} V_s = \dfrac{1}{2} V_s \\ V_2 = \dfrac{(R+\Delta R)}{R+(R+\Delta R)} V_s = \dfrac{R+\Delta R}{2R+\Delta R} V_s \end{cases}$$

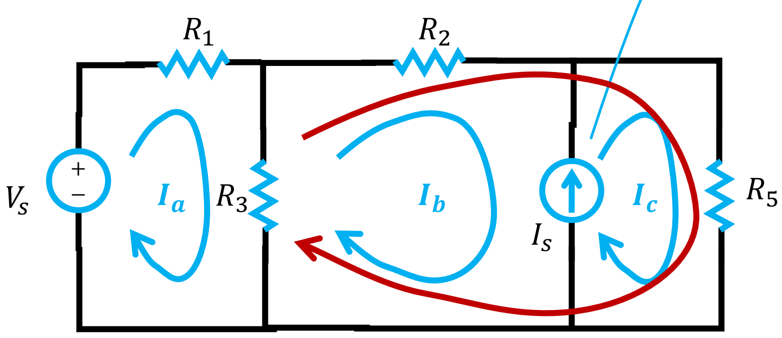
$$V_{out} = V_2 - V_1 = \frac{R+\Delta R}{2R+\Delta R} V_s - \frac{1}{2} V_s = \frac{\Delta R}{4R+2\Delta R} V_s = \frac{\Delta R}{4R\left(1+\frac{\Delta R}{2R}\right)} V_s \approx \frac{\Delta R}{4R} V_s$$

Since $\frac{\Delta R}{R} \ll 1$

In Wheatstone bridge, by measuring v_{out}, we can find ΔR

Supermesh

No way to determine this voltage!!



In mesh a

$$-V_s + I_a R_1 + (I_a - I_b) R_3 = 0$$

$$I_a (R_1 + R_3) - I_b R_3 = V_s$$

In the supermesh

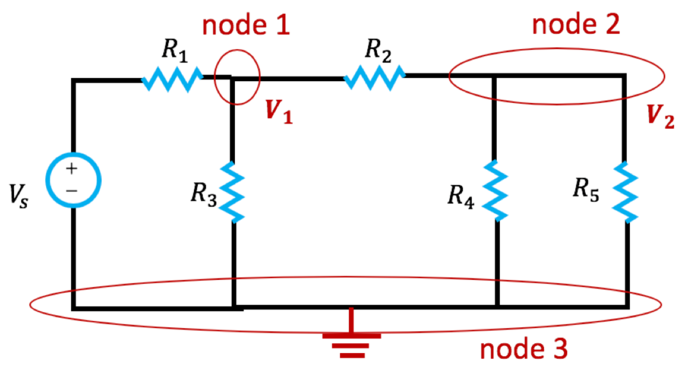
$$(I_b - I_a) R_3 + I_b R_2 + I_c R_5 = 0$$

Auxiliary equation for the supermesh

$$I_c - I_b = I_s$$

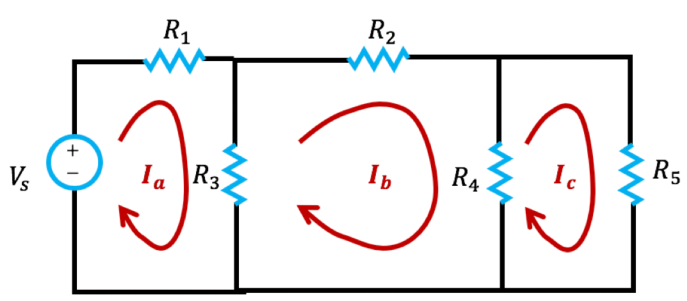
- Step 1b: assign each mesh an unknown current, usually in clockwise direction
- Step 2a: apply KVL to each mesh

Nodal versus Mesh



Node-voltage method

- Node voltages are UNKNOWN
- # of equations =
  # of extraordinary node - 1



Mesh-current method

- Currents flowing in meshes are UNKNOWN
- # of equations = # of meshes

Outlines

- Node-voltage method
  - Basis of node voltage
  - Node-voltage method for circuit analysis
  - Supernode w/ node-voltage method
- Mesh-current method
  - Basis of mesh-current
  - Mesh-current method for circuit analysis
  - Supermesh w/ mesh-current method
  - Node-voltage v.s. mesh-current method

Outlines

- Node-voltage method
  - Basis of node voltage
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- Linearity & Superposition

Review: linear function

A function is linear if it satisfies 2 properties:

- **Additivity** (superposition) $f(x+y) = f(x) + f(y)$
- **Homogeneity** (scaling) $f(ax) = a\,f(x)$

A linear function therefore must exhibit both these 2 properties simultaneously:

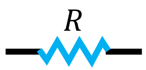
$$f(ax+by) = af(x) + bf(y)$$

Linearity describes a relationship between cause & effect

Linear device

A DEVICE is linear if it satisfies 2 properties:

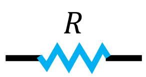
- **Additivity**: response to SUM OF INPUTS is the SUM OF THE RESPONSES to each input applied separately



  - According to Ohm's Law, if $v_1 = i_1 R$ and $v_2 = i_2 R$
  - If a current $\boldsymbol{i = i_1 + i_2}$ is applied to R

$$\boldsymbol{v} = (i_1 + i_2)R = i_1 R + i_2 R = \boldsymbol{v_1 + v_2}$$

- **Homogeneity**: if input (excitation) is multiplied by a constant, then output (response) is multiplied by the SAME constant



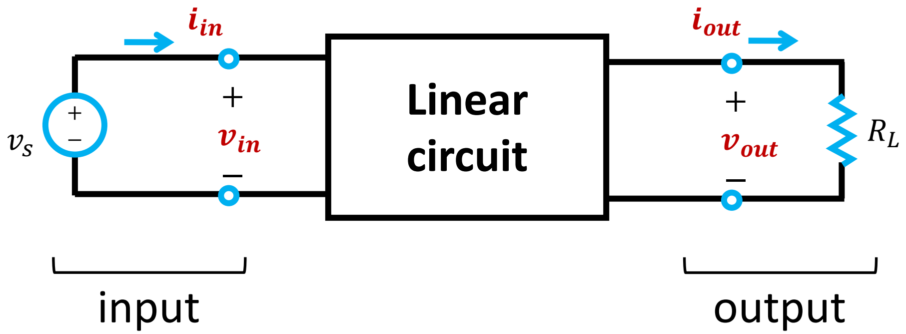
  - According to Ohm's Law, if $v_1 = i_1 R$
  - If the current is increased by k, $\boldsymbol{i_2 = k i_1}$

$$\boldsymbol{v_2} = i_2 R = k i_1 R = \boldsymbol{k v_1}$$

A resistor is a linear device since its voltage-current relationship satisfies both the additivity and homogeneity properties

Linear circuit

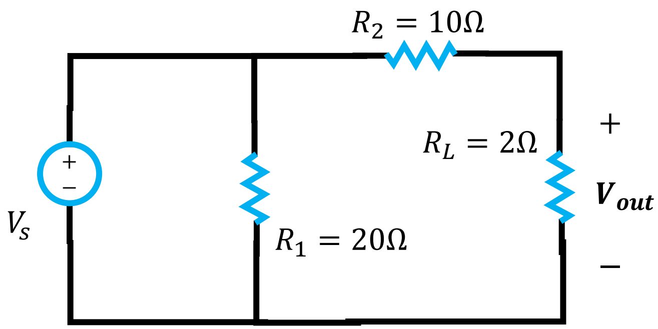
A CIRCUIT is linear if it possesses the properties of additive and homogeneity



- A circuit is linear when its output is directly proportional to its input
- A linear circuit's output must scale with its input
- In general: a linear circuit consists of only
  - linear devices (resistors, capacitors, inductors)
  - linear dependent sources
  - independent sources

Example 2

QUESTION: if $V_s = 24V$, Find the output voltage of the load R_L



- Assume $V_{out-test} = 1\text{V}$
- According to Ohm's law

$$I_{R_L} = \frac{V_{out-test}}{R_L} = 0.5A$$

$$V_{s-test} = V_{R_2} + V_{out-test}$$

$$= I_{R_L}R_2 + V_{out-test} = 6\text{V}$$

- This is a linear circuit

$$V_s = 4V_{s-test}$$

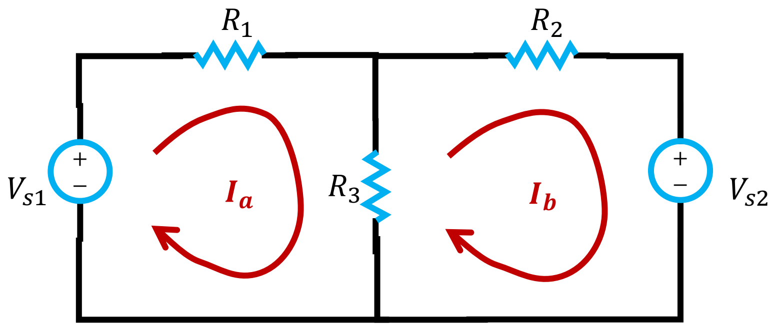
$\boldsymbol{V_{out} = 4V}$

Outlines

- Node-voltage method
  - Basis of node voltage
  - Node-voltage method for circuit analysis
  - Supernode w/ node-voltage method
- Mesh-current method
  - Basis of mesh-current
  - Mesh-current method for circuit analysis
  - Supermesh w/ mesh-current method
  - Node-voltage v.s. mesh-current method
- Linearity & Superposition
  - What is a linear circuit
  - Additivity & homogeneity of linear circuit
  - Homogeneity in circuit analysis
  - **Superposition in circuit analysis**

Example 3

QUESTION: Find the voltage on R_3



- According to mesh-current method

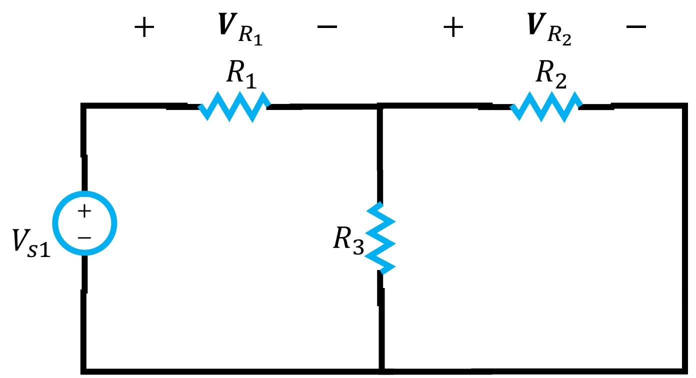
$$\begin{cases} -V_{s1} + R_1 I_a + R_3(I_a - I_b) = 0 \\ R_3(I_b - I_a) + R_2 I_b - V_{s2} = 0 \end{cases}$$

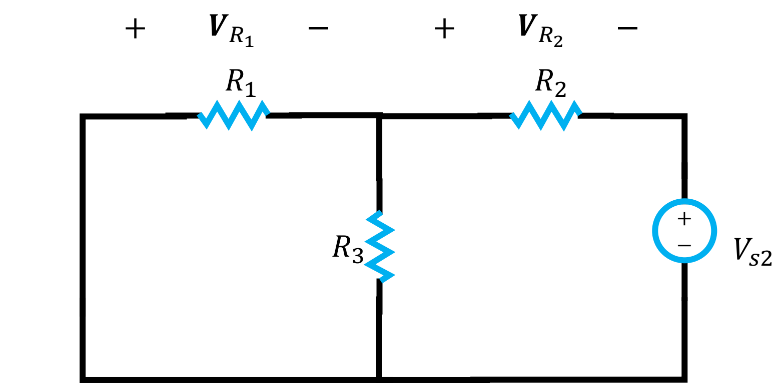
$$\Rightarrow \begin{cases} I_a = \dfrac{R_3 V_{s1} + (R_1 + R_3)V_{s2}}{R_1R_2 + R_1R_3 + R_2R_3} \\ I_b = \dfrac{(R_2 + R_3)V_{s1} + R_3 V_{s2}}{R_1R_2 + R_1R_3 + R_2R_3} \end{cases}$$

$$\Rightarrow V_{R_3} = R_3(I_a - I_b)$$

$$= \frac{R_2R_3V_{s1} - R_1R_3V_{s2}}{R_1R_2 + R_1R_3 + R_2R_3}$$

Example 3





- According to voltage division

$$V'_{R_3} = \frac{R_2||R_3}{R_1 + R_2||R_3} V_{s1}$$

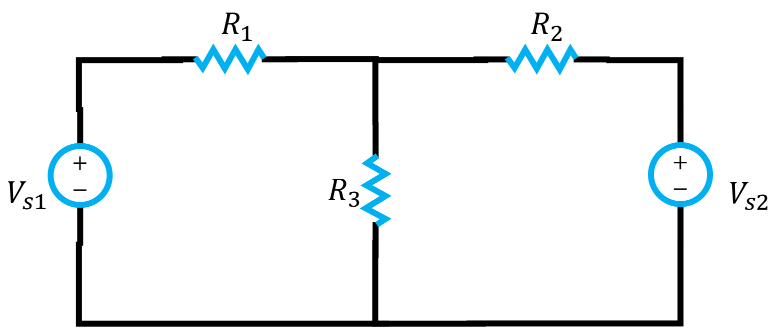
$$= \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} V_{s1}$$

$$V''_{R_3} = \frac{R_1||R_3}{R_2 + R_1||R_3} V_{s2}$$

$$= -\frac{R_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} V_{s2}$$

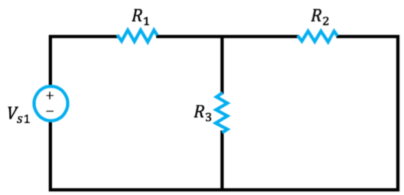
Example 3

QUESTION: Find the voltage on R_3



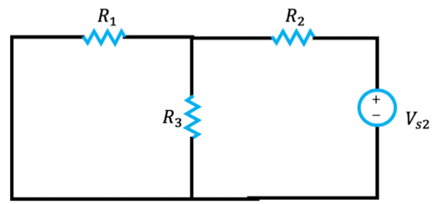
$$V_{R_3} = \frac{R_2 R_3 V_{s1} - R_1 R_3 V_{s2}}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

=



$$V'_{R_3} = \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} V_{s1}$$

+



$$V''_{R_3} = -\frac{R_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} V_{s2}$$

The superposition property of a linear circuit can help analyze circuits with more than 1 source

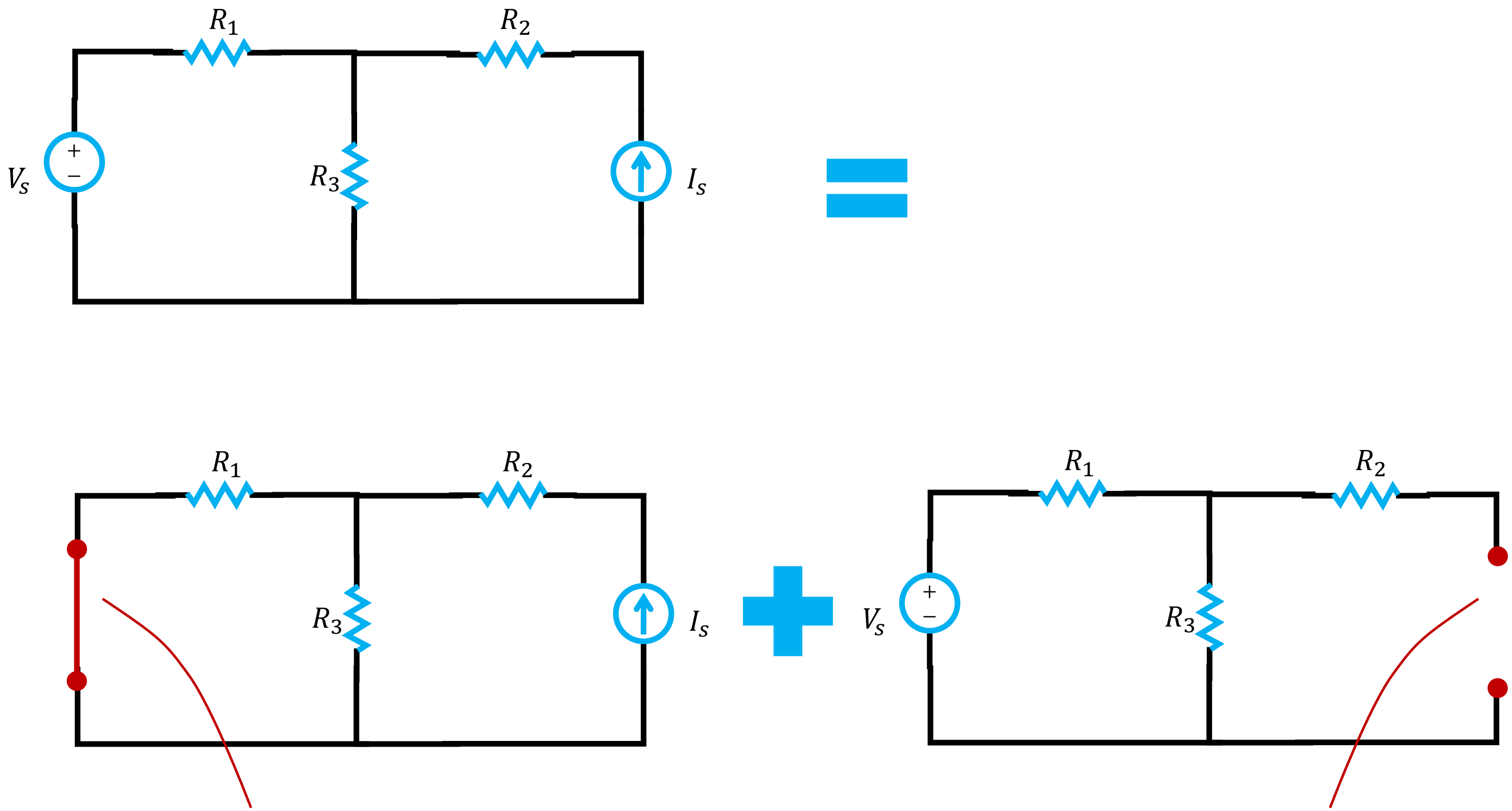
Superposition in Action

Superposition trades off the examination of several simpler circuits in place of one complex circuit

How to apply superposition to circuit analysis

- Consider one **INDEPENDENT** source at a time, by "turning off" all other **INDEPENDENT** sources
- **DEPENDENT** sources are left intact because they are controlled by circuit variables

How to “turn off” an independent src



Short circuit –
Turn off voltage source

Open circuit –
Turn off current source

Outlines

- Node-voltage method
  - Basis of node voltage
  - Node-voltage method for circuit analysis
  - Supernode w/ node-voltage method
- Mesh-current method
  - Basis of mesh-current
  - Mesh-current method for circuit analysis
  - Supermesh w/ mesh-current method
  - Node-voltage v.s. mesh-current method
- Linearity & Superposition
  - What is a linear circuit
  - Additivity & homogeneity of linear circuit
  - Homogeneity in circuit analysis
  - Superposition in circuit analysis

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- Node-voltage method
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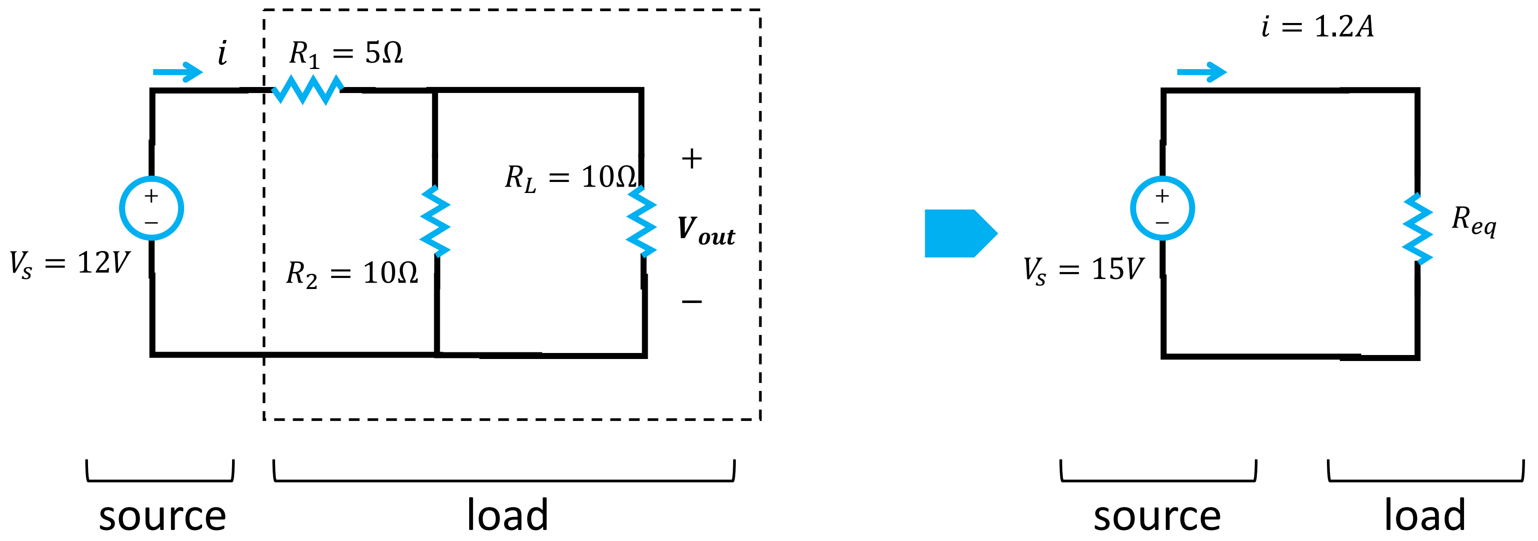
- **Thévenin and Norton Equivalent Circuits**

Review: equivalent resistance

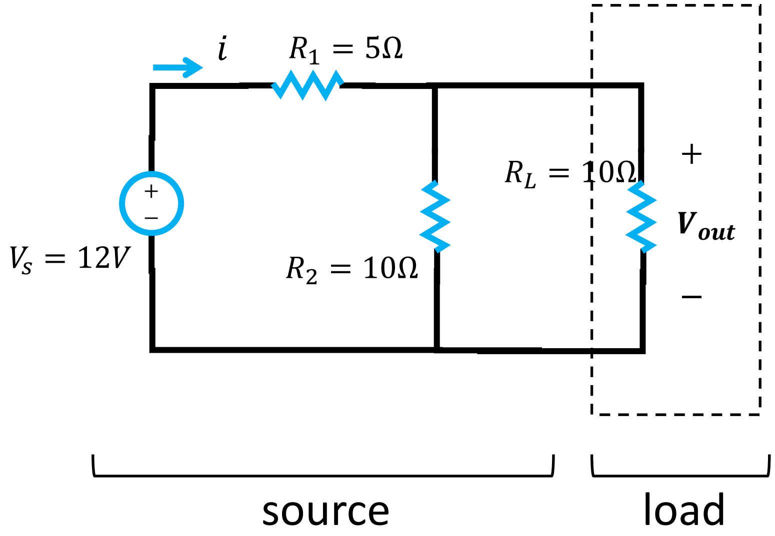


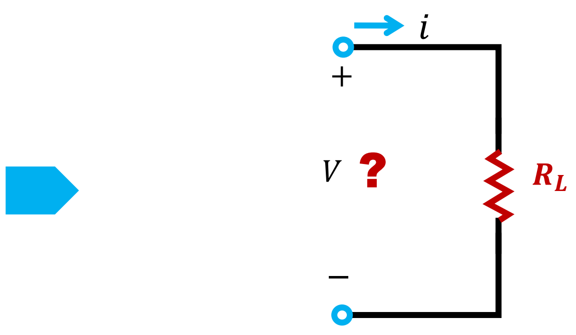
$$R_{eq} = R_1 + (R_2||R_L) = 10\Omega$$

$$i = \frac{V_s}{R_{eq}} = 1.2A$$

- **Source sees the load as just "resistance"**
- **Source supplies the amount of current demanded by load**

What does R_L see?

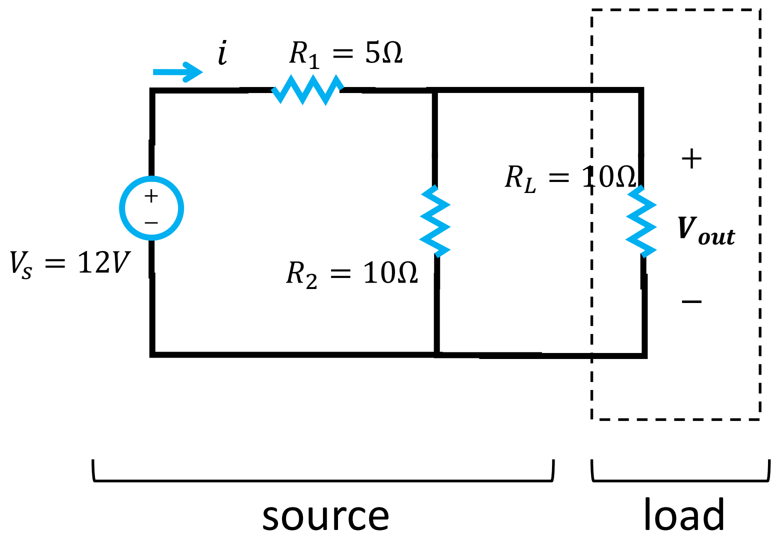


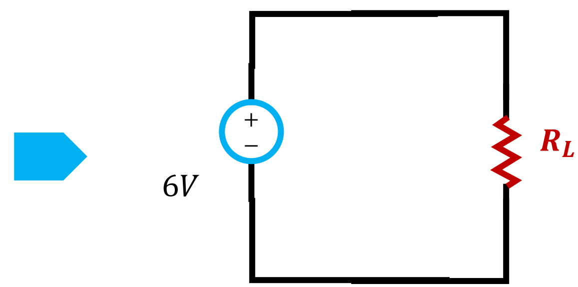


$$V_{R_L} = V_s - iR_1 = 6V$$

$$i_{R_L} = \frac{V_{R_L}}{R_L} = 0.6A$$

What does R_L see?





$$V_{R_L} = V_s - iR_1 = 6V$$

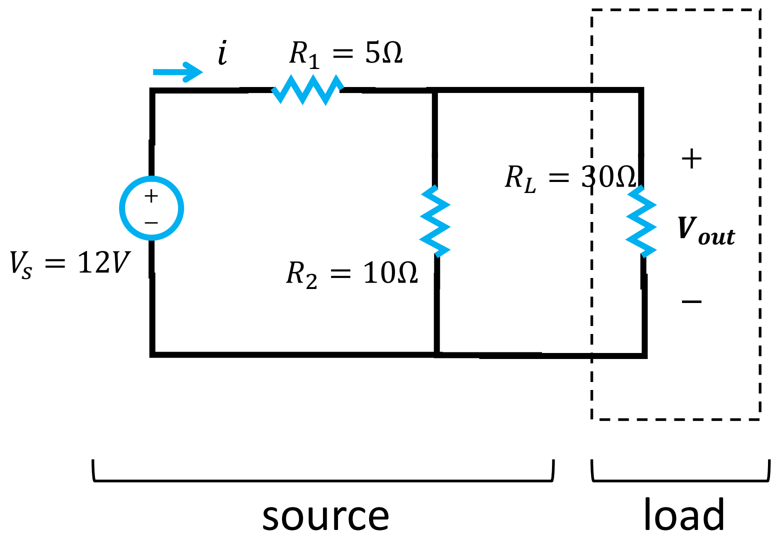
$$i_{R_L} = \frac{V_{R_L}}{R_L} = 0.6A$$

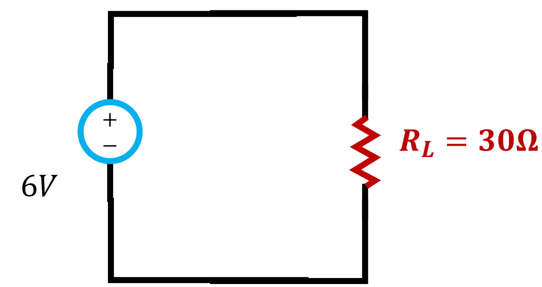
Are these two circuit equivalent?

NO!!!

What does R_L see?

What if R_L increases to 30Ω?





These two circuit are NOT equivalent!!!

$$V_{R_L} = V_s - iR_1 = 7.2V$$

$$i_{R_L} = \frac{V_{R_L}}{R_L} = 0.24A$$

$$\neq$$

$$i_{R_L} = \frac{V_s}{R_L} = 0.2A$$

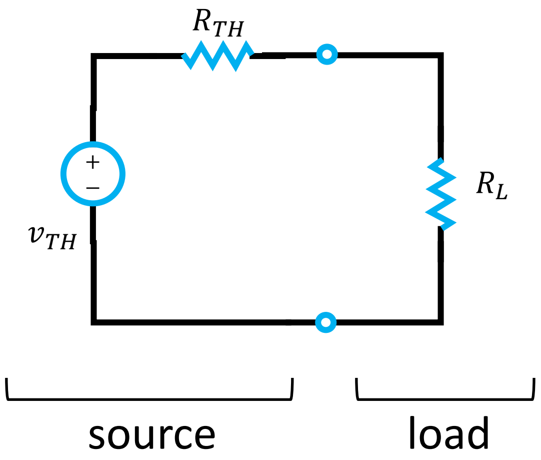
$$V_{R_L} = V_s = 6V$$

Equivalent circuits supply the same voltage & current to ANY load

Circuit equivalent

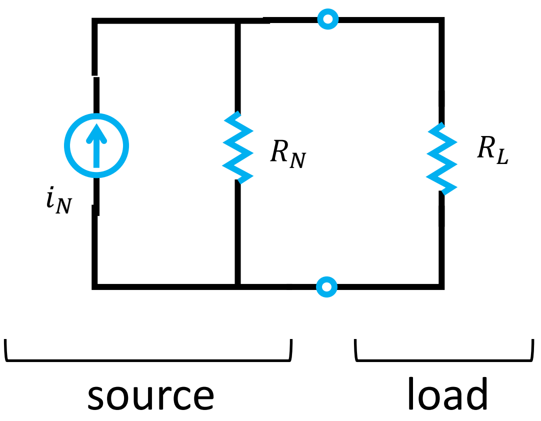
Thévenin's theorem

LINEAR two-terminal circuit can be replaced by an equivalent circuit composed of a **voltage source** and a **series resistor**



Norton's theorem

LINEAR two-terminal circuit can be replaced by an equivalent circuit composed of a **current source** and a **parallel resistor**

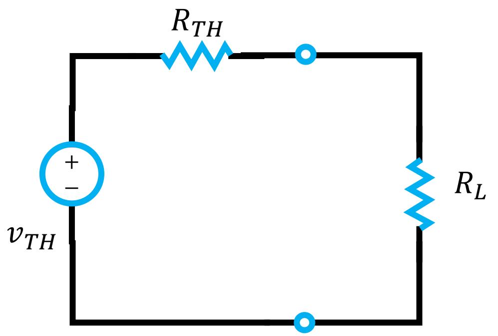


Thévenin & Norton equivalency

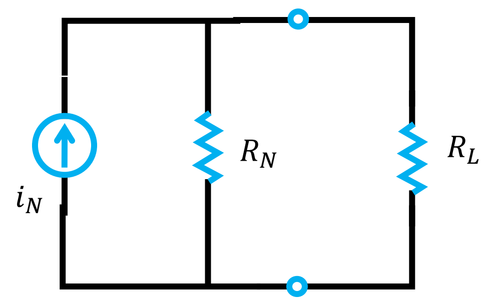
- According to source transformation

$$\begin{cases} i_N = \dfrac{v_{TH}}{R_{TH}} \\ R_{TH} = R_N \end{cases}$$

- Norton's Theorem is just a "source transformation" of Thévenin's theorem
- <u>Superposition</u> and <u>Additivity</u> make these theorems possible



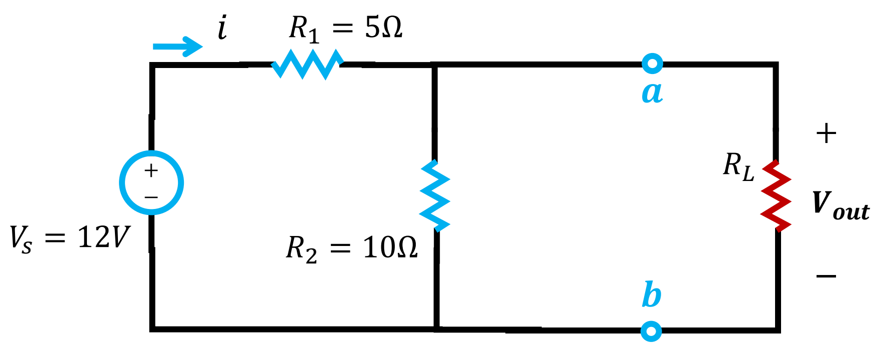
Thévenin's theorem

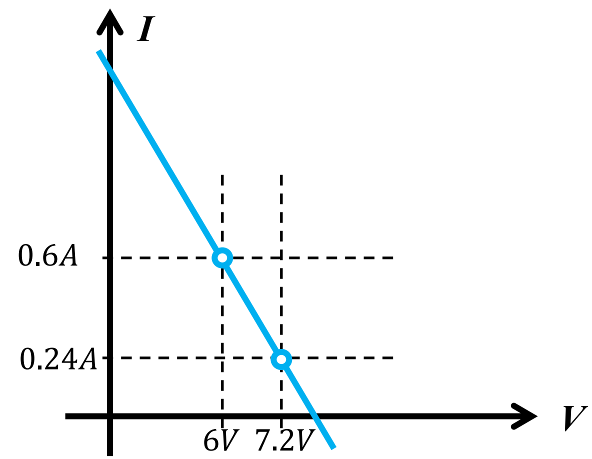


Norton's theorem

Example 4

QUESTION: Find the Thévenin equivalent circuit of the network at the terminals a & b



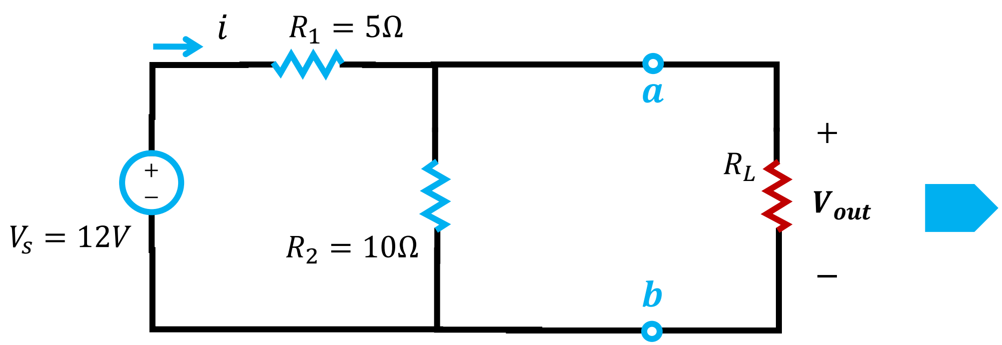


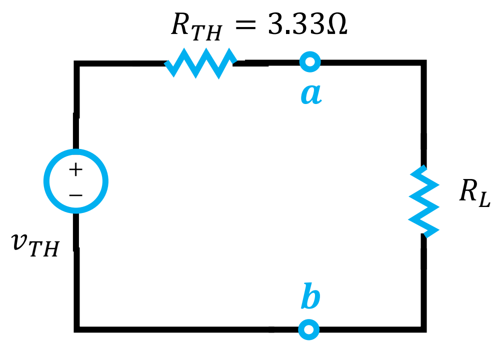
- When $R_L = 10\Omega$ $\begin{cases} i_{R_L} = 0.6A \\ V_{R_L} = 6V \end{cases}$
- When $R_L = 30\Omega$ $\begin{cases} i_{R_L} = 0.24A \\ V_{R_L} = 7.2V \end{cases}$

$$R_{TH} = \frac{\Delta V}{\Delta I} = \frac{7.2V - 6V}{0.6A - 0.24A} = 3.33\Omega$$

Example 4

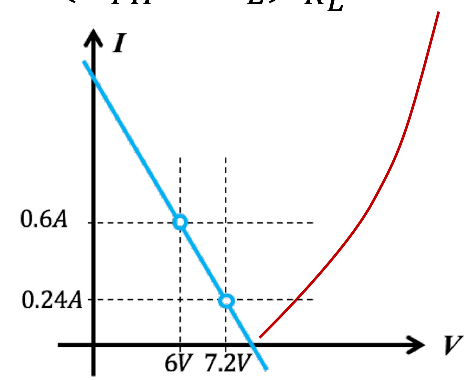
QUESTION: Find the Thévenin equivalent circuit of the network at the terminals a & b





- When $R_L = 10\Omega$ $\begin{cases} i_{R_L} = 0.6A \\ V_{R_L} = 6V \end{cases}$
- When $R_L = 30\Omega$ $\begin{cases} i_{R_L} = 0.24A \\ V_{R_L} = 7.2V \end{cases}$

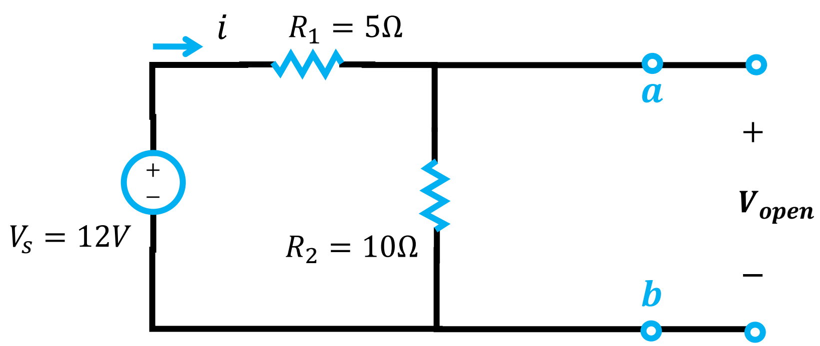
$$v_{TH} = (R_{TH} + R_L)i_{R_L} = 8\text{V}$$



v_{TH} is the maximum voltage @open circuit ($i = 0$)

Example 4

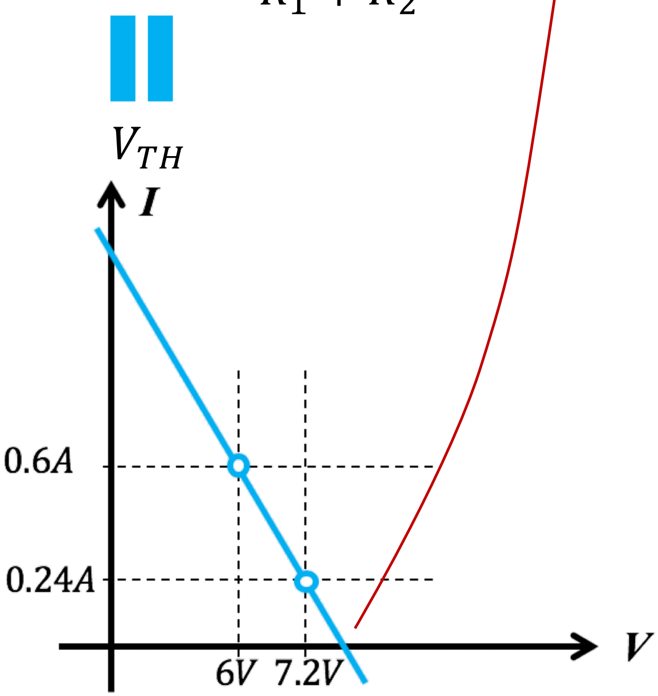
QUESTION: Find the Thévenin equivalent circuit of the network at the terminals a & b



- Step 1: remove the load
- Step 2: find V_{open}

- According to voltage division

$$V_{open} = \frac{R_2}{R_1 + R_2} V_s = 8V$$



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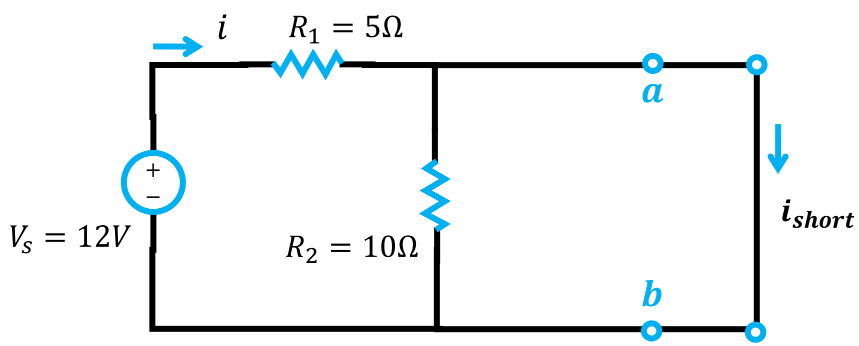


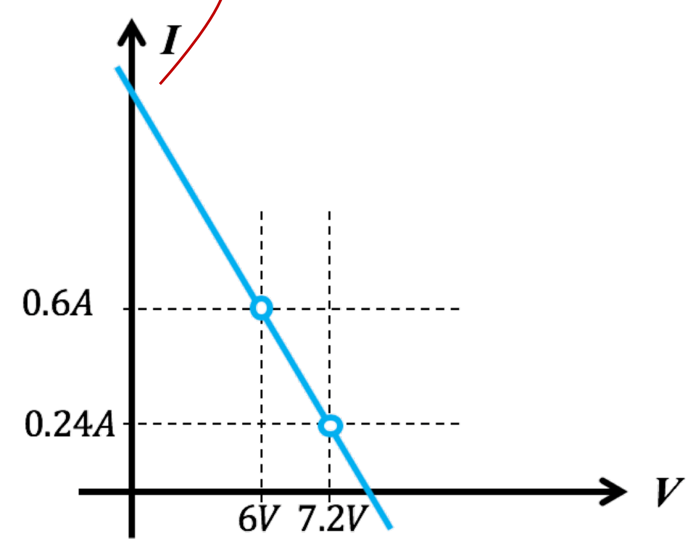
- Step 1: remove the load
- Step 2: find V_{open}
- Step 3: find i_{short}

- According to Ohm's law

$$i_{open} = \frac{V_s}{R_1} = 2.4A$$

$\parallel$

i_N



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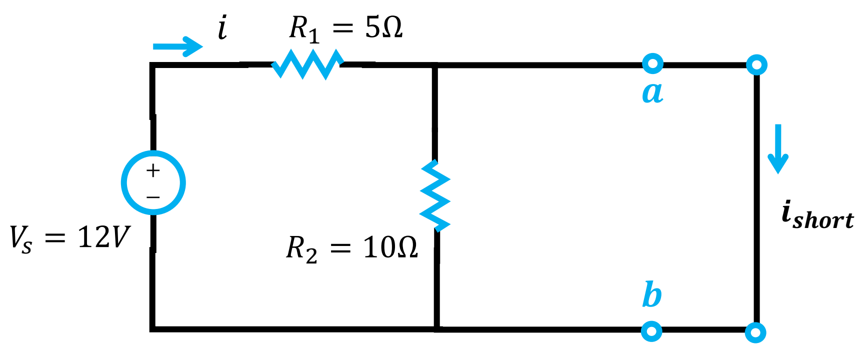
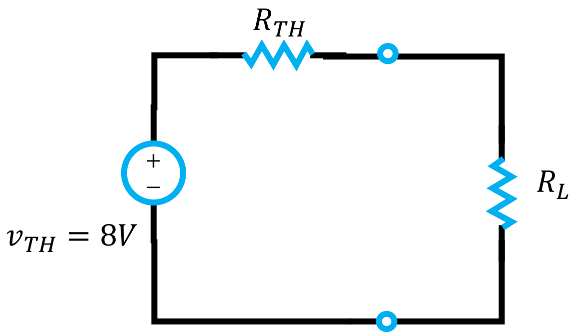
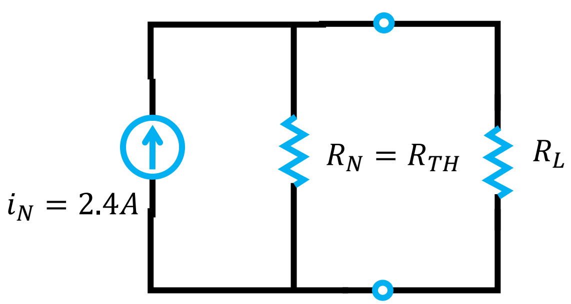
QUESTION: Find the Thévenin equivalent circuit of the network at the terminals a & b





- Step 1: remove the load
- Step 2: find $V_{open} = v_{TH}$
- Step 3: find $i_{short} = i_N$
- Step 4: find R_{TH}

$$R_{TH} = \frac{v_{TH}}{i_N} = 3.33\Omega$$



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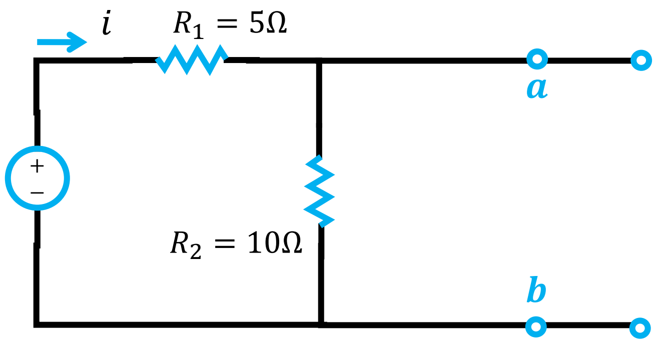


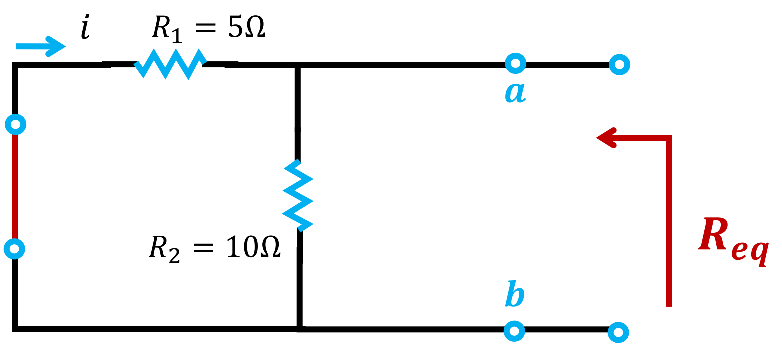
If a circuit contains ONLY independent sources

Method to find R_{TH}

- Step 1: remove the load

Example 4

QUESTION: Find the Thévenin equivalent circuit of the network at the terminals a & b



$$R_{TH} = R_{eq} = R_1||R_2 = 3.33\Omega$$

If a circuit contains ONLY independent sources

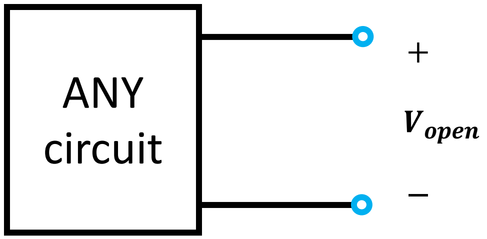
Method to find R_{TH}

- Step 1: remove the load
- Step 2: turn off ALL independent sources
  - SHORT all voltage sources
  - OPEN all current sources
- Step 3: find R_{eq} from the perspective of the load

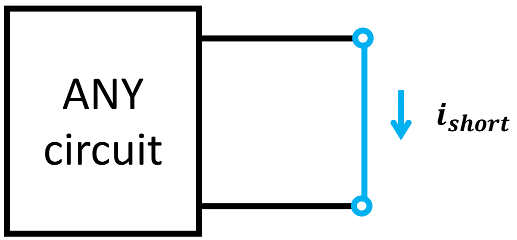
Thévenin & Norton equivalency

How to find Thévenin / Norton equivalent circuit

Method applied to ANY circuit



- Step 1: remove the load
- Step 2: $v_{TH} = V_{open}$
- Step 3: $i_N = i_{short}$
- Step 4: $R_{TH} = \frac{v_{TH}}{i_N}$



Method applied to circuit ONLY contains independent sources

- Step 1: remove the load
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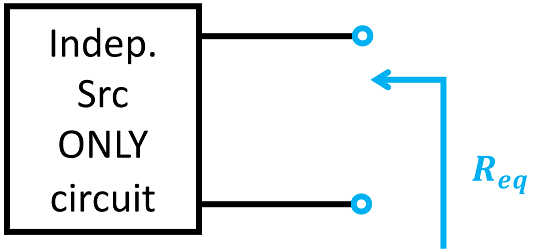
- Step 3: find R_{eq} from the perspective of the load



Outlines

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  - Basis of node voltage
  - Node-voltage method for circuit analysis
  - Supernode w/ node-voltage method
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  - Additivity & homogeneity of linear circuit
  - Homogeneity in circuit analysis
  - Superposition in circuit analysis
- Thévenin and Norton Equivalent Circuits
  - What is Thévenin and Norton Equivalency
  - How to calculate Thévenin and Norton Equivalent circuit

Reading tasks & learning goals

- Reading tasks
  - Basic Engineering Circuit Analysis, 10th edition
    - Chapter 3, 5.1-5.3
- Learning goals
  - Be able to calculate all V/I in circuits contain multi-nodes/loops
  - Be able to use **node-voltage** method for circuit analysis
  - Be able to use **mesh-current** method for circuit analysis
  - Be able to calculate a **Thévenin equivalent** circuit for a linear circuit
  - Be able to calculate a **Norton equivalent** circuit for a linear circuit