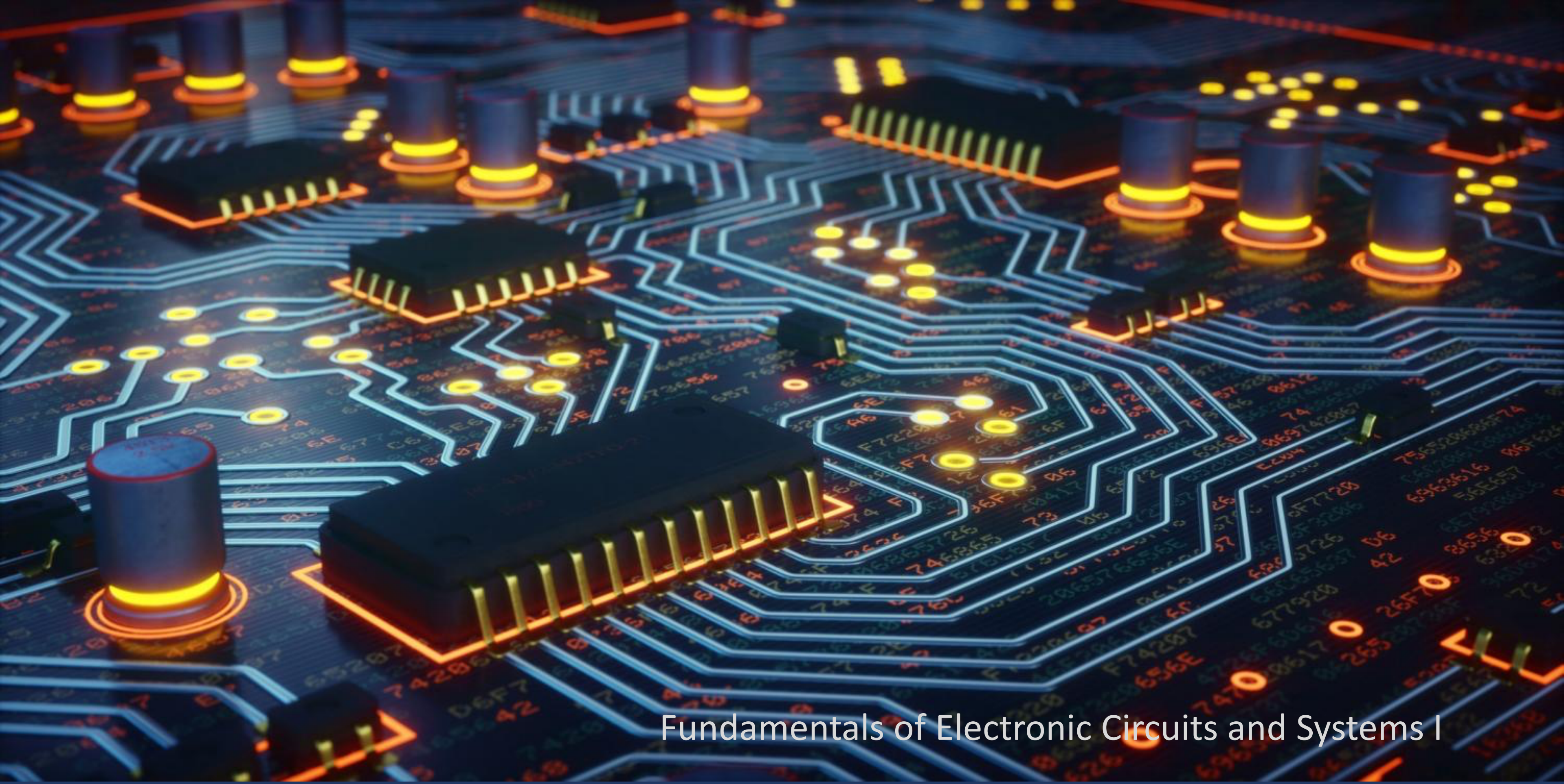


Fundamentals of Electronic Circuits and Systems I

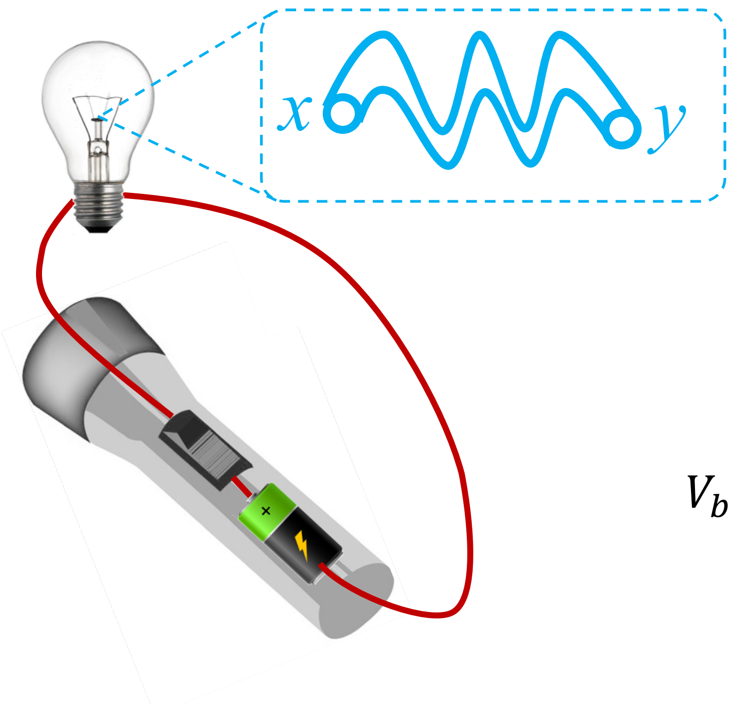
Resistive Circuits

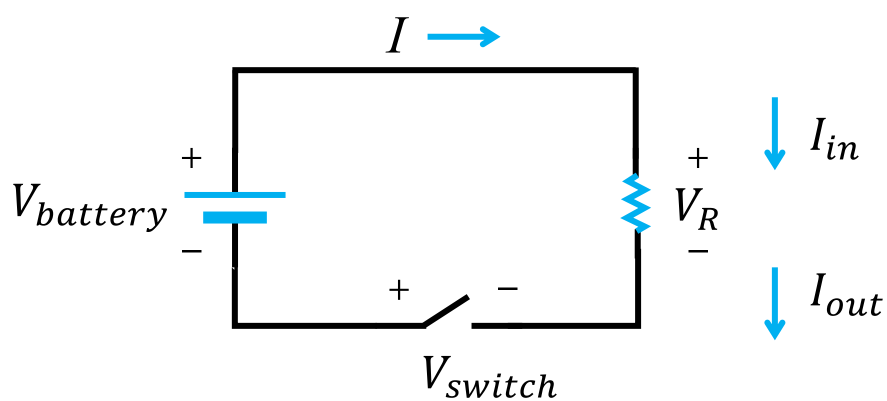
Milin Zhang
Dept of EE, Tsinghua University

Outlines

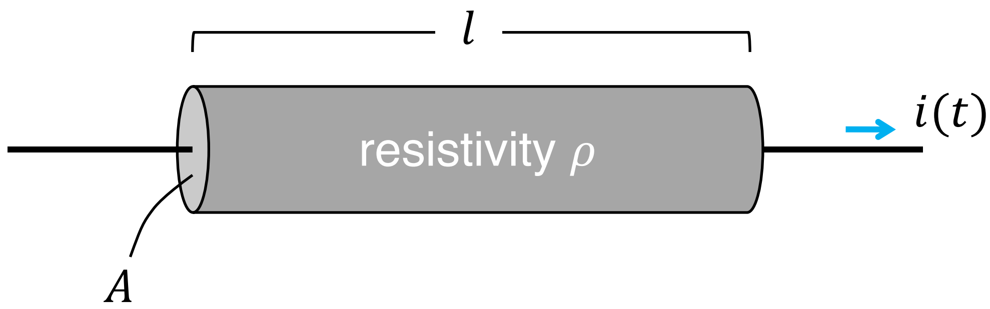
- Linear resistor & Ohm's law
- Power & energy
- KVL/KCL
- Equivalent circuit

Review: a simple circuit





Resistance & Conductance



$$R = \rho \frac{l}{A} \quad (\Omega = V/A)$$

Resistance, R, represents the ability to resist flow of electric current

Conductance, $G = \frac{1}{R}$ $\quad (S = A/V)$

* Resistivity of some common materials

Graphene	$1 \times 10^{-8} \Omega \cdot m$	Aluminum	$2.82 \times 10^{-8} \Omega \cdot m$
Silver	$1.59 \times 10^{-8} \Omega \cdot m$	Iron	$9.71 \times 10^{-8} \Omega \cdot m$
Copper	$1.68 \times 10^{-8} \Omega \cdot m$	Platinum	$1.06 \times 10^{-7} \Omega \cdot m$
Gold	$2.44 \times 10^{-8} \Omega \cdot m$	Titanium	$4.20 \times 10^{-7} \Omega \cdot m$

What is a resistor?

Resistors are passive electrical components that reduce the flow of electrical current in a circuit.

Axial resistor

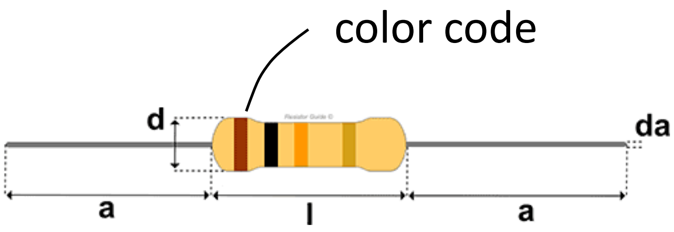
Surface Mount Devices (SMD) resistor

Metal Electrode Leadless Face (MELF) resistor

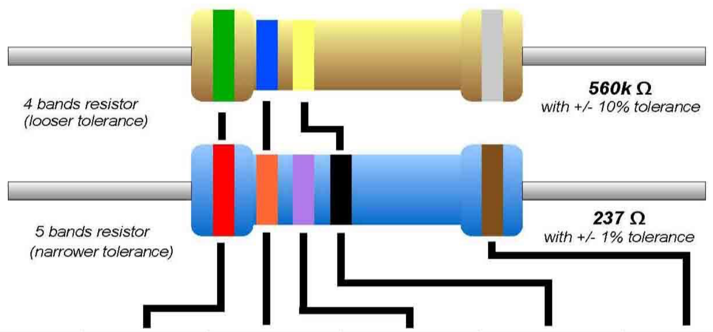
Resistor properties

- **Resistance**
  - represents the ability to resist flow of electric current
- **Resistor noise**
  - Thermal noise: main contributor to resistor noise
  - Current noise: declines when frequency is increased
- **Temperature coefficient of resistance (TCR)**
  - Relationship between resistance and temperature
  - Measured in $ppm/°C$
- **Power rating**
  - Maximum energy a resistor can dissipate without damaging or altering the properties

Color code for axial resistors

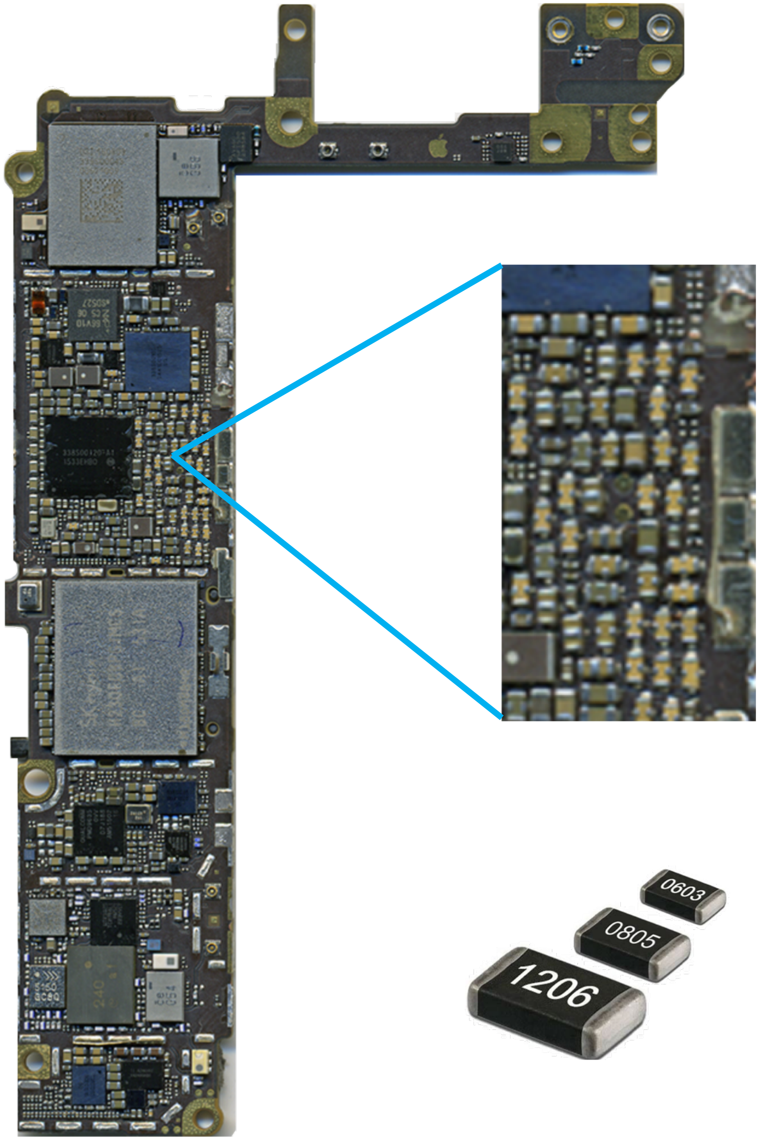


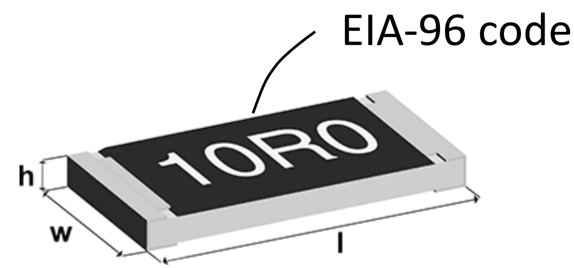
d	l	power
1.8 ± 0.3	3.0 ± 0.3	1/8
2.5 ± 0.3	6.5 ± 0.5	1/4
3.2 ± 0.3	8.5 ± 0.5	1/2
5 ± 0.3	11 ± 1	1



Color	1st Band	2nd Band	3rd Band	Multiplier	Tolerance
Black	0	0	0	x 1 Ω	
Brown	1	1	1	x 10 Ω	+/- 1%
Red	2	2	2	x 100 Ω	+/- 2%
Orange	3	3	3	x 1K Ω	
Yellow	4	4	4	x 10K Ω	
Green	5	5	5	x 100K Ω	+/- .5%
Blue	6	6	6	x 1M Ω	+/- .25%
Violet	7	7	7	x 10M Ω	+/- .1%
Grey	8	8	8		+/- .05%
White	9	9	9		
Gold				x .1 Ω	+/- 5%
Silver				x .01 Ω	+/- 10%

SMD resistors



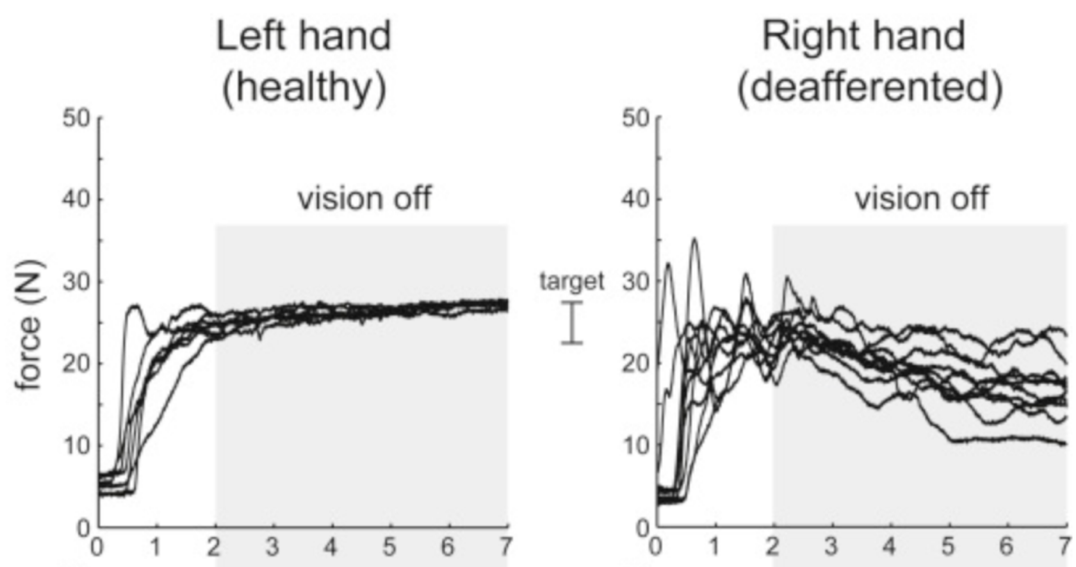


Size code	Size / mils	power
0201	20 x 10	1/20
0402	40 x 20	1/16
0603	60 x 30	1/10
0805	80 x 50	1/8
1206	120 x 60	1/4
2010	200 x 100	3/4
2512	250 x 120	1

SMD = Surface mounted device

“Resistors” are useful

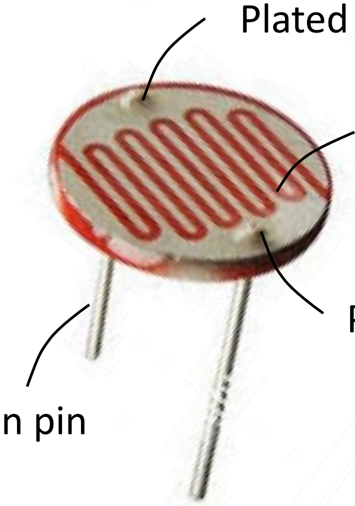
Force sensitive resistor

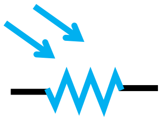


[Cortex 2016]

“Resistors” are useful

Photo resistor



Symbol 

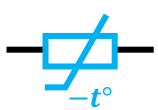
- Light dependent resistor (LDR)
- Sensitivity varies with the light wavelength

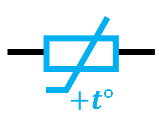
“Resistors” are useful

Thermistor

- Thermistor is temperature sensitive resistor
- Positive temperature coefficient (PTC) thermistor : resistance increases as the temperature increases
- Negative temperature coefficient (NTC) thermistor : resistance decreases as the temperature increases

Symbol

NTC 

PTC 

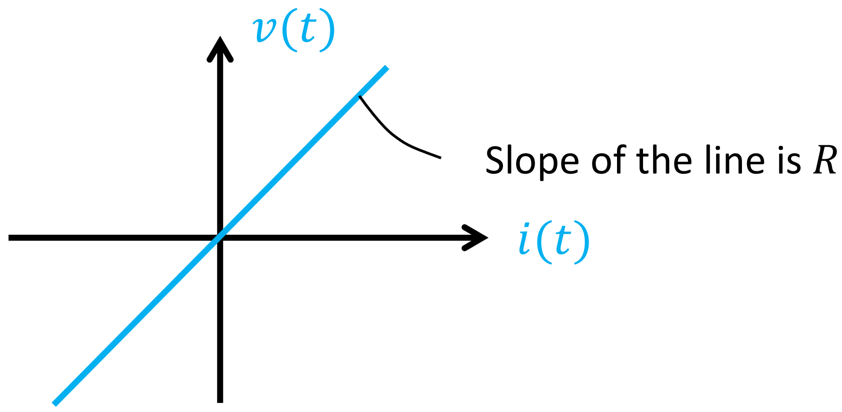
Outlines

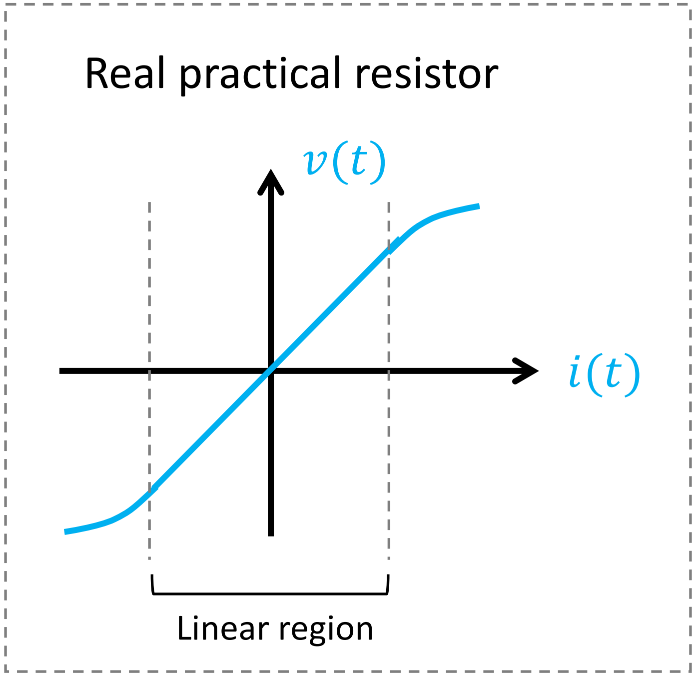
- Linear resistor & Ohm's law
  - Resistance & conductance
  - **Ohm's law**

Ohm's Law

Ohm's law states that the voltage across a resistance is directly proportional to the current flowing through it.

Mathematical relationship of Ohm's law: $v(t) = Ri(t)$

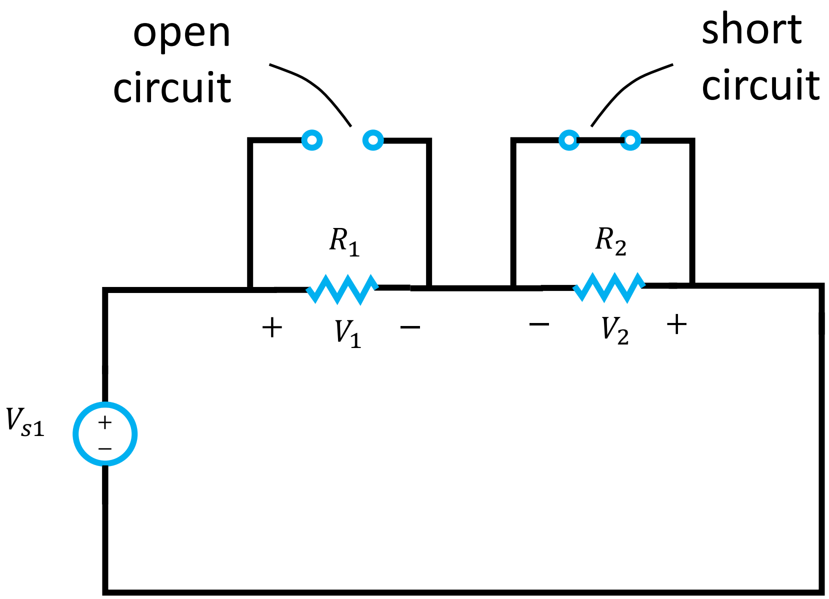




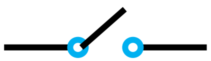
- The resistance measured in ohm (Ω)
- The resistance is the constant of proportionality between the voltage and the current

Open Circuit & Short Circuit

- **Open circuit:** no path for current flow → $R = \infty$
- **Short circuit:** no voltage drop → $R = 0$



open switch = open circuit

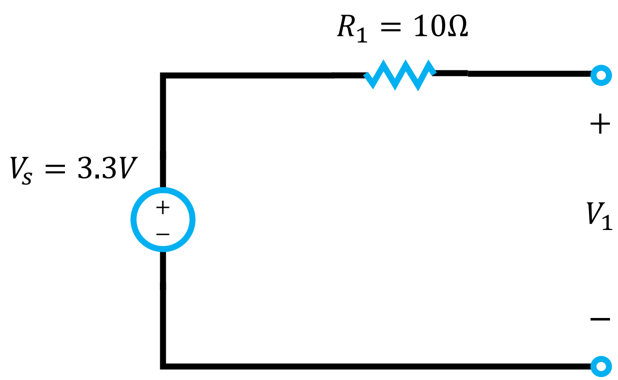


closed switch = short circuit

*Note:
- open/short circuit is the basis for modeling a switch behavior.
- No perfect switches

Example 1

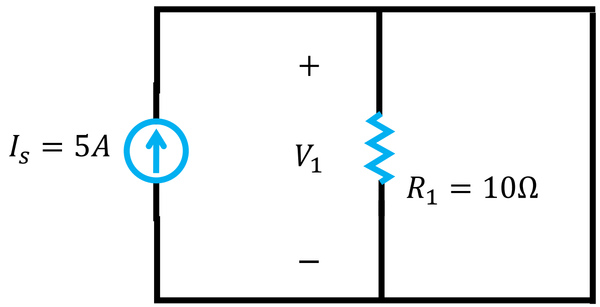
QUESTION: Find the voltage at $\boldsymbol{V_1}$ in the circuit below:



ANSWER: $V_1 = 3.3V$

? How much current flows through R_1

QUESTION: Find the voltage across the resistor $\boldsymbol{V_1}$ in the circuit below:



ANSWER: $V_1 = 0V$

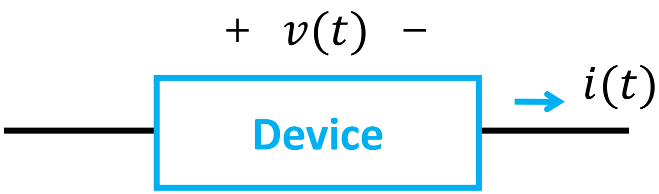
? What is the voltage difference across I_S

Outlines

- Linear resistor & Ohm's law
  - Resistance & conductance
  - Ohm's law
  - Open circuit & short circuit
- **Power & energy**

Power

The **POWER** delivered to or supplied by a device is the product of the voltage and the current

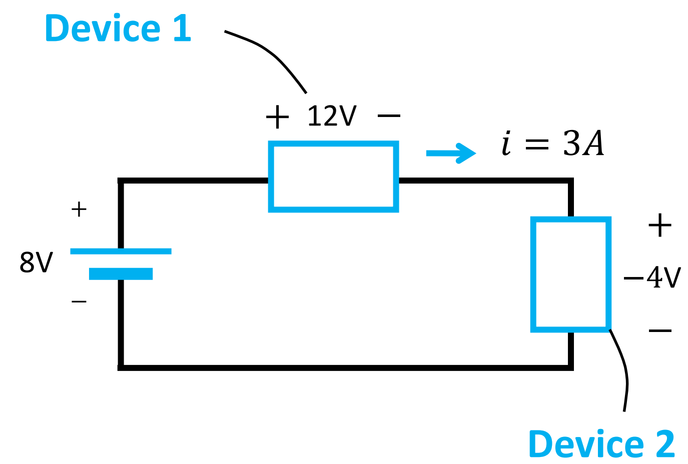


- Instantaneous power

$$p(t) = v(t)i(t)$$

- Average power

$$\bar{P} = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt$$



$p > 0$ Power delivered to device

$p < 0$ Power supplied by device

*Note: The average value of any periodic waveform can be computed by integrating the function over a complete period and dividing this result by the period

Power in dB

Decibel (dB) is a unitless measurement for expressing ratios

- For power

$$10log_{10}\frac{P}{1W}$$ ➔dB

$$10log_{10}\frac{P}{1mW}$$ ➔dBm

- For voltage

$$20log_{10}\frac{v}{1V}$$ ➔dBV

$$20log_{10}\frac{v}{1mV}$$ ➔dBmV

Example 2

QUESTION: Assume the load of a transceiver is $R_L = 50\Omega$. The voltage measured at the output point is $V_o dBV$. Find the output power in dBm

The output power $P_{out} = \frac{V_{out}^2}{R_L}$

$$= 10log_{10}\frac{P_{out}}{1mW}dBm$$

$$= 10log_{10}\frac{V_{out}^2}{R_L}\frac{1}{1mW}dBm$$

$$= \left(10log_{10}\frac{1}{R_L \times 1mW} + 20log_{10}V_{out}\right)dBm$$

$$= \mathbf{(13 + V_o dBV)dBm}$$

Energy & Effective I/V

If a constant amount of power P is delivered over an interval T the **ENERGY** w supplied is

$$w = \int_0^T p(t)dt$$

EFFECTIVE VALUE is found by first determining the square of the current/voltage, then computing the average or mean value

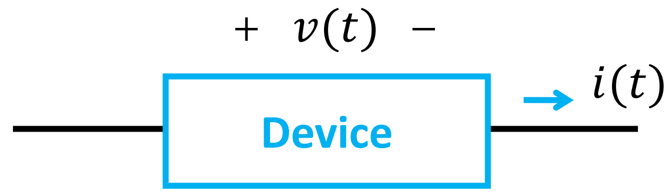
$$v_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)dt}$$

$$i_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt}$$

Example 3

Given the voltage $v(t) = V_P \cos(\omega t + \varphi_V)$

Given the current $i(t) = I_P \cos(\omega t + \varphi_I)$



- Instantaneous power $p(t) = v(t)i(t) = V_P \cos(\omega t + \varphi_V)\, I_P \cos(\omega t + \varphi_I)$

$$= \frac{1}{2} V_P I_P \cos(\varphi_V - \varphi_I) + \frac{1}{2} V_P I_P \cos(2\omega t + \varphi_V + \varphi_I)$$

- Average power $\bar{P} = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt$

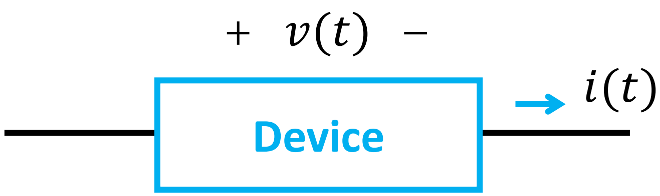
$$= \frac{1}{T}\int_{t_0}^{t_0+T} \left(\frac{1}{2} V_P I_P \cos(\varphi_V - \varphi_I) + \frac{1}{2} V_P I_P \cos(2\omega t + \varphi_V + \varphi_I)\right) dt$$

$$= \frac{1}{2} V_P I_P \cos(\varphi_V - \varphi_I) \quad = \frac{V_P}{\sqrt{2}} \frac{I_P}{\sqrt{2}} \cos(\varphi_V - \varphi_I)$$

Example 3

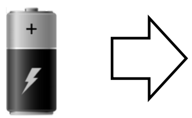
Given the voltage $v(t) = V_P\cos(\omega t + \varphi_V)$

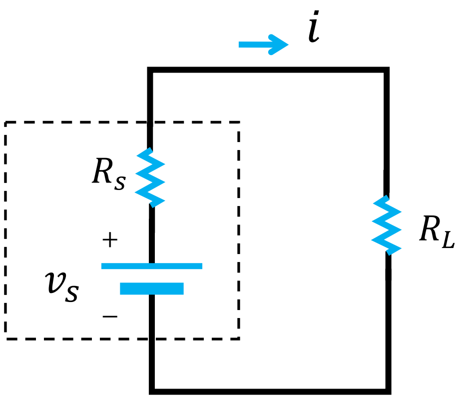
Given the current $i(t) = I_P\cos(\omega t + \varphi_I)$



- Instantaneous power $$p(t) = \frac{1}{2}V_P I_P\cos(\varphi_V - \varphi_I) + \frac{1}{2}V_P I_P\cos(2\omega t + \varphi_V + \varphi_I)$$
- Average power $$\bar{P} = \frac{V_P}{\sqrt{2}}\frac{I_P}{\sqrt{2}}\cos(\varphi_V - \varphi_I) = v_{rms} i_{rms}\cos(\varphi_V - \varphi_I)$$
- Effective voltage $$v_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)dt} = \frac{V_P}{\sqrt{2}}$$
- Effective current $$i_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt} = \frac{I_P}{\sqrt{2}}$$

Maximum Power Transfer





Practical voltage source

- Voltage of R_L

$$v_{R_L} = v_s - iR_s$$

- Power at the load R_L

$$P_L = v_{R_L} i = (v_s - iR_s)i \;\; = -R_s\left(i^2 - \frac{v_s}{R_s}i\right) = -R_s\left(i - \frac{1}{2}\frac{v_s}{R_s}\right)^2 + \frac{1}{4}\frac{v_s^2}{R_s}$$

$$\leq \frac{1}{4}\frac{v_{s,rms}^2}{R_s}$$

The maximum power being absorbed by the load

- When $R_s = R_L$ $\quad P_L = P_{L,max} = \frac{1}{4}\frac{v_{s,rms}^2}{R_s}$

Outlines

- Linear resistor & Ohm's law
  - Resistance & conductance
  - Ohm's law
  - Open circuit & short circuit
- Power & energy
  - Definition of power
  - Power in dB
  - Definition of energy
  - Effective current/voltage
  - Maximum power transfer
- **KVL/KCL**

Review: Lumped Matter Discipline (LMD)

- The rate of change of magnetic flux linked with any portion of the circuit must be zero for all time.

$$\oint E \cdot dl = -\frac{\partial \Phi_B}{\partial t} = 0$$

- The rate of change of the charge at any node in the circuit must be zero for all time. A node is any point in the circuit at which two or more element terminals are connected using wires.

$$\int_{S_x} J \cdot dS - \int_{S_y} J \cdot dS = \frac{\partial q}{\partial t} = 0$$

- The signal timescales must be much larger than the propagation delay of electromagnetic waves through the circuit.

LMD:

The fundamental of circuit and system theory

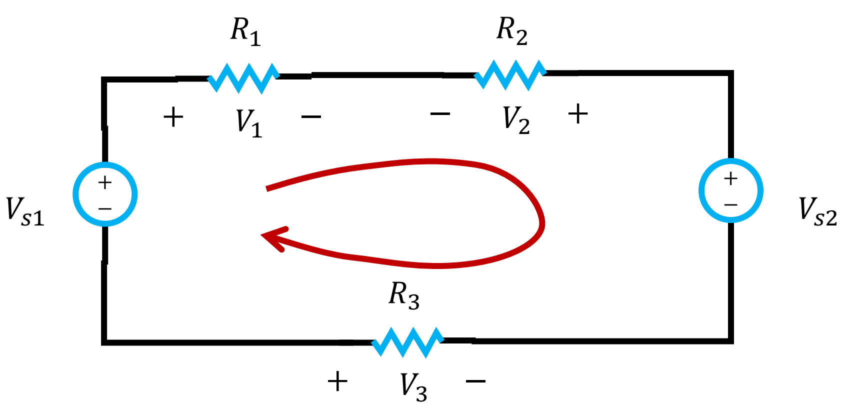
Kirchhoff's Voltage Law (KVL)

- The rate of change of magnetic flux linked with any portion of the circuit must be zero for all time.

$$\oint E \cdot dl = -\frac{\partial \Phi_B}{\partial t} = 0 \quad \Rightarrow \quad \sum_{n-1}^{N} v_n = 0$$

KVL: Sum of voltages around a closed path is zero

Sum of voltage drops = Sum of voltage rises



$$V_{s1} - V_1 + V_2 - V_{s2} + V_3 = 0$$

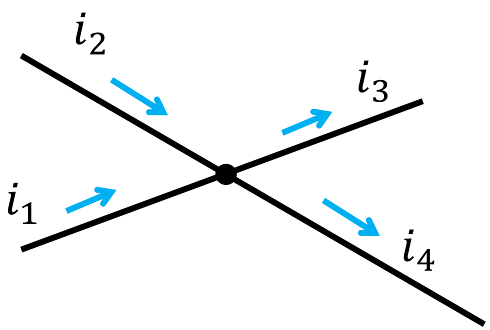
* Note
- Add up the voltages in a systematic clockwise movement around the loop
- Assign a positive sign to the voltage across an element if the (+) side of that voltage is encountered first, and assign a negative sign if the (-) side is encountered first

Kirchhoff's Current Law (KCL)

- The rate of change of the charge at any node in the circuit must be zero for all time. A node is any point in the circuit at which two or more element terminals are connected using wires.

$$\int_{S_x} J \cdot dS - \int_{S_y} J \cdot dS = \frac{\partial q}{\partial t} = 0 \quad \Rightarrow \quad \boldsymbol{\sum_{n-1}^{N} i_n = 0}$$

KCL: Sum of currents entering a node is zero



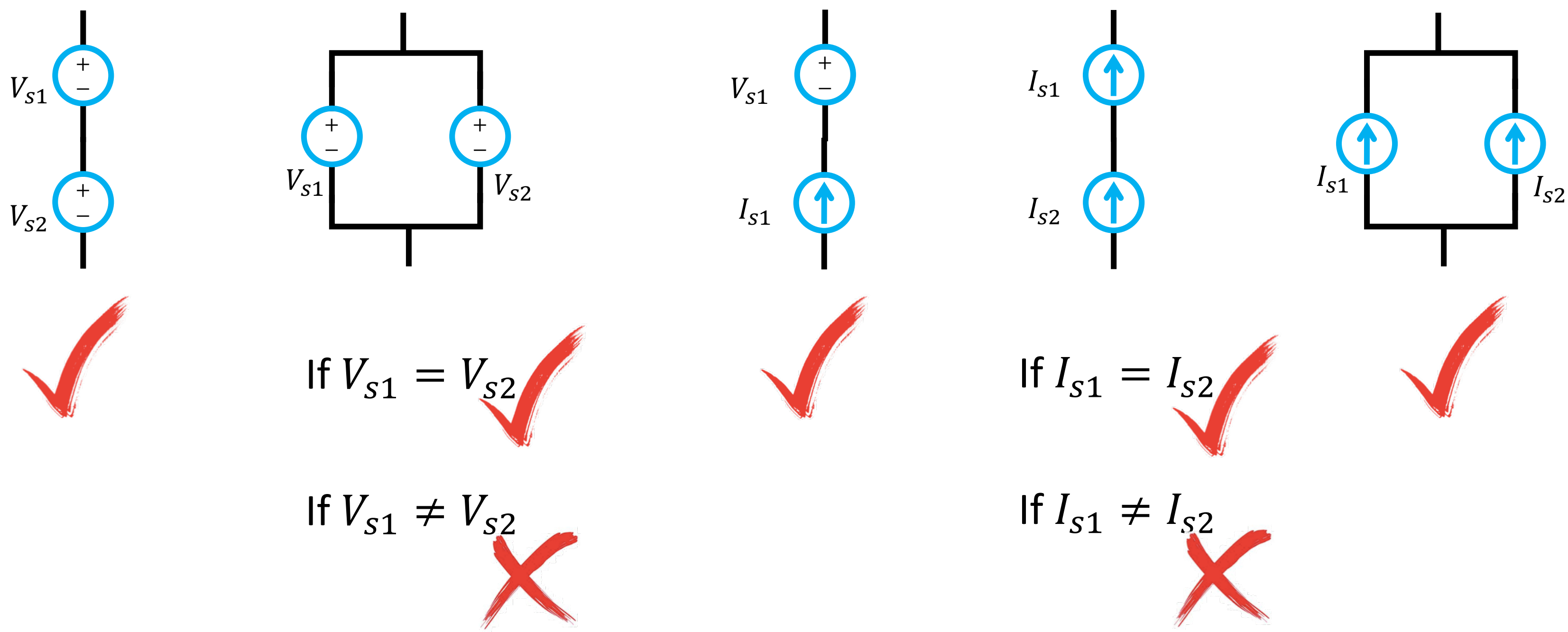
$$i_1 + i_2 - i_3 - i_4 = 0$$

* Note: sign convention in Engineering

- Assign a positive sign to current entering a node
- Assign a negative sign to current entering a node

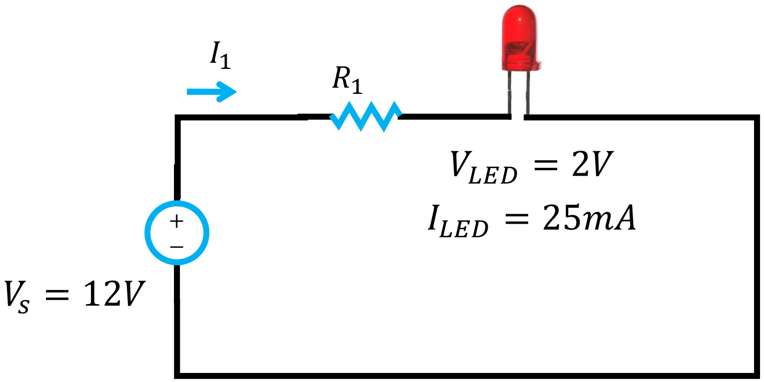
Example 4: power source connection

QUESTION: Find if the connection of the circuit is right or wrong



Example 5: Ballast resistor

QUESTION: Find if the resistance of R_1



- According to KVL

$$V_s - V_{R_1} - V_{LED} = 0$$

$$\rightarrow V_{R_1} = 10V$$

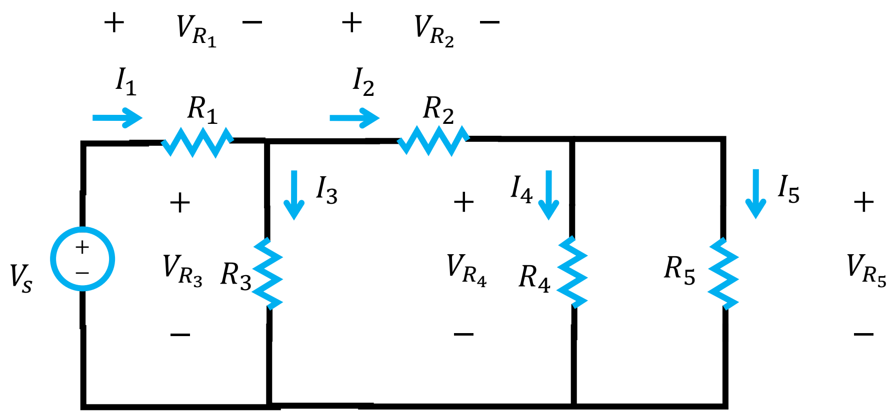
- According to Ohm's law

$$R_1 = \frac{V_{R_1}}{I_{LED}} = 40\Omega$$

A ballast resistor is used to limit the current through a device, and to prevent that it burns

Example 6

QUESTION: Find the output current of the voltage source



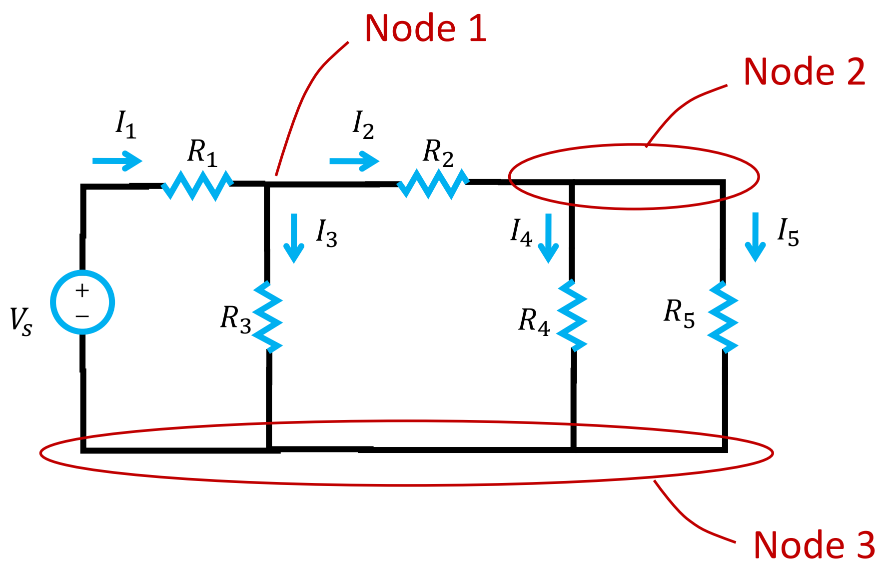
Step1 define i and v

Step 2 According to Ohm's law

$$\begin{cases} V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \end{cases}$$

Example 6

QUESTION: Find the output current of the voltage source



- According to KCL

$$\begin{cases} I_1 - I_2 - I_3 = 0 \\ I_2 - I_4 - I_5 = 0 \\ I_1 - I_3 - I_4 - I_5 = 0 \end{cases}$$

Step1 define i and v

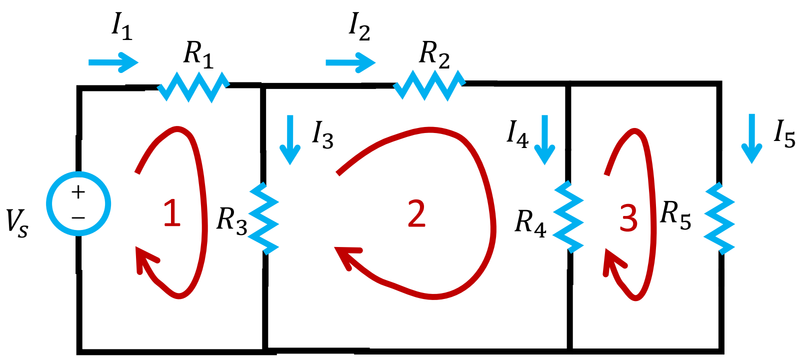
Step 2 According to Ohm's law

$$\begin{cases} V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \end{cases}$$

Step 3 Check the nodes

Example 6

QUESTION: Find the output current of the voltage source



Step1 define i and v

Step 2 According to Ohm's law

$$\begin{cases} V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \\ V_{R_1} = I_1 R_1 \end{cases}$$

- According to KCL

$$\begin{cases} I_1 - I_2 - I_3 = 0 \\ I_2 - I_4 - I_5 = 0 \\ I_1 - I_3 - I_4 - I_5 = 0 \end{cases}$$

- According to KVL

$$\begin{cases} V_s - V_{R_1} - V_{R_3} = 0 \\ V_{R_3} - V_{R_2} - V_{R_4} = 0 \\ V_{R_4} - V_{R_5} = 0 \end{cases}$$

Step 3 Check the nodes

Step 4 Check the loops

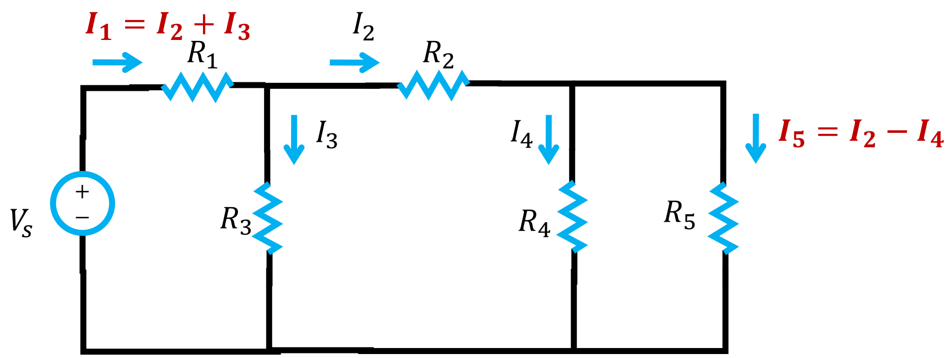
Example 6

$$
\begin{bmatrix}
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
R_s & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & R_2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & R_3 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & R_4 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & R_L & 0 & 0 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_L \\ V_{R_s} \\ V_{R_2} \\ V_{R_3} \\ V_{R_4} \\ V_{R_L}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix} V_s
$$

Very friendly to numerical calculation !

Example 6

QUESTION: Find the output current of the voltage source



- According to KVL

$$\begin{cases} V_s - V_{R_1} - V_{R_3} = 0 \\ V_{R_3} - V_{R_2} - V_{R_4} = 0 \\ V_{R_4} - V_{R_5} = 0 \end{cases}$$

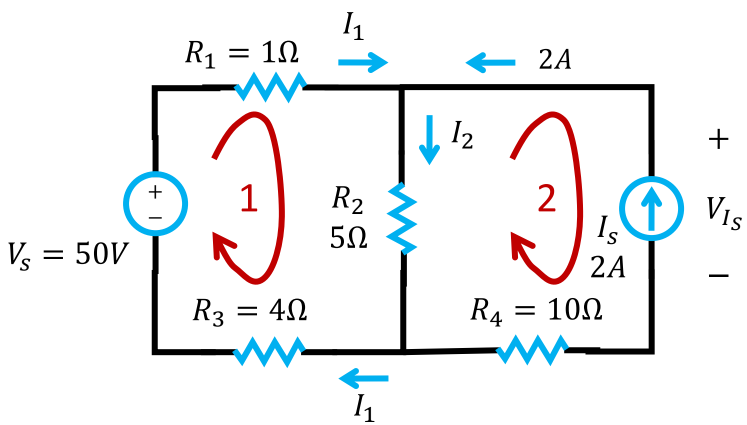
$$\begin{cases} V_s - (I_2 + I_3)R_1 - I_3 R_3 = 0 \\ I_3 R_3 - I_2 R_2 - I_4 R_4 = 0 \\ I_4 R_4 - (I_2 - I_4) R_5 = 0 \end{cases}$$

$$\begin{bmatrix} R_1 & R_1 & R_3 \\ -R_2 & R_3 & -R_4 \\ -R_5 & 0 & R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix}$$

3 equations in 3 unknowns

Example 7

QUESTION: Find the output current of the voltage source



- According to KVL

$$\begin{cases} V_s - V_{R_1} - V_{R_2} - V_{R_3} = 0 \\ -V_{I_S} + V_{R_2} + V_{R_4} = 0 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} 50 - I_1 - 5I_2 - 4I_1 = 0 \\ -V_{I_S} + 5I_2 + 10\times 2 = 0 \end{cases}$$

- According to KCL

$$I_1 + 2 = I_2$$

$$\begin{cases} 50 - I_1 - 5I_2 - 4I_1 = 0 \\ -V_{I_S} + 5I_2 + 10\times 2 = 0 \\ I_1 + 2 = I_2 \end{cases}$$

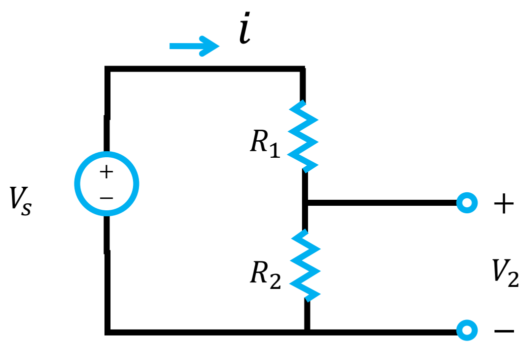
3 equations in 3 unknowns

Outlines

- Linear resistor & Ohm's law
  - Resistance & conductance
  - Ohm's law
  - Open circuit & short circuit
- Power & energy
  - Definition of power
  - Power in dB
  - Definition of energy
  - Effective current/voltage
  - Maximum power transfer
- KVL/KCL
- **Equivalent circuit**

Resistors in series

VOLTAGE DIVIDER



According to KVL $V_s = iR_1 + iR_2$

➔ $i = \frac{V_s}{R_1+R_2}$

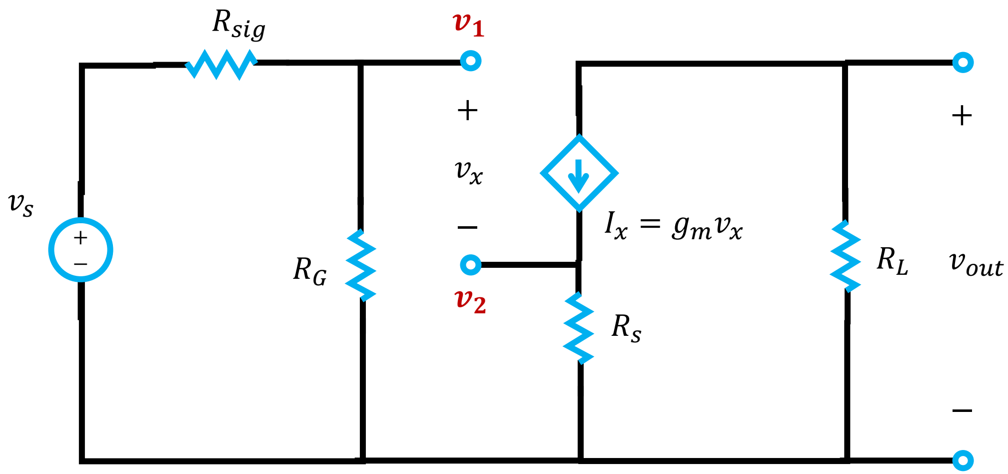
$$V_2 = iR_2 = \frac{R_2}{R_1 + R_2} V_s$$

Voltage divided over resistors

- **Equivalent resistance, R_{eq}, (series) is sum of resistances** $$R_{eq} = \sum_{i=1}^{N} R_i$$
- **Voltage difference across a single resistance of resistors in series** $$V_i = \frac{R_i}{R_{eq}} V_s$$

Example 8

QUESTION: Find the output voltage on R_L

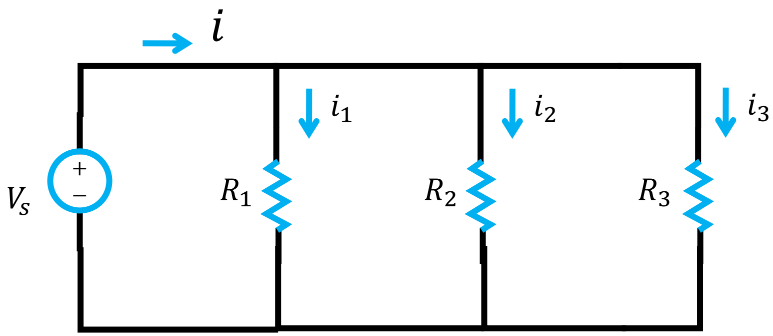


- According to the rule of voltage divided over resistors $v_1 = \frac{R_G}{R_{sig} + R_G} v_s$
- According to Ohm's law $v_2 = R_s I_x$

$$= R_s g_m v_x = R_s g_m (v_1 - v_2) \Rightarrow v_2 = \frac{1}{1 + g_m R_s} v_1$$

- According to Ohm's law, the voltage on R_L $\boldsymbol{v_{out} = -g_m v_x R_L}$

Resistors in parallel



According to KCL

$$i = i_1 + i_2 + i_3 = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3}$$

$$= \frac{V_s}{R_{eq}}$$

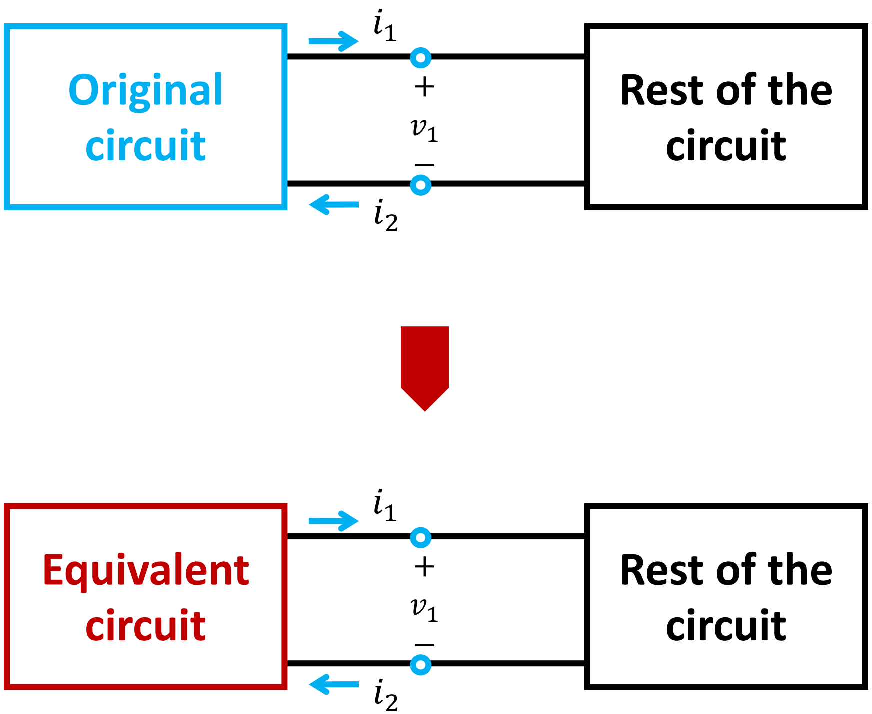
- **Equivalent resistance, R_{eq}, (series) is sum of resistances**

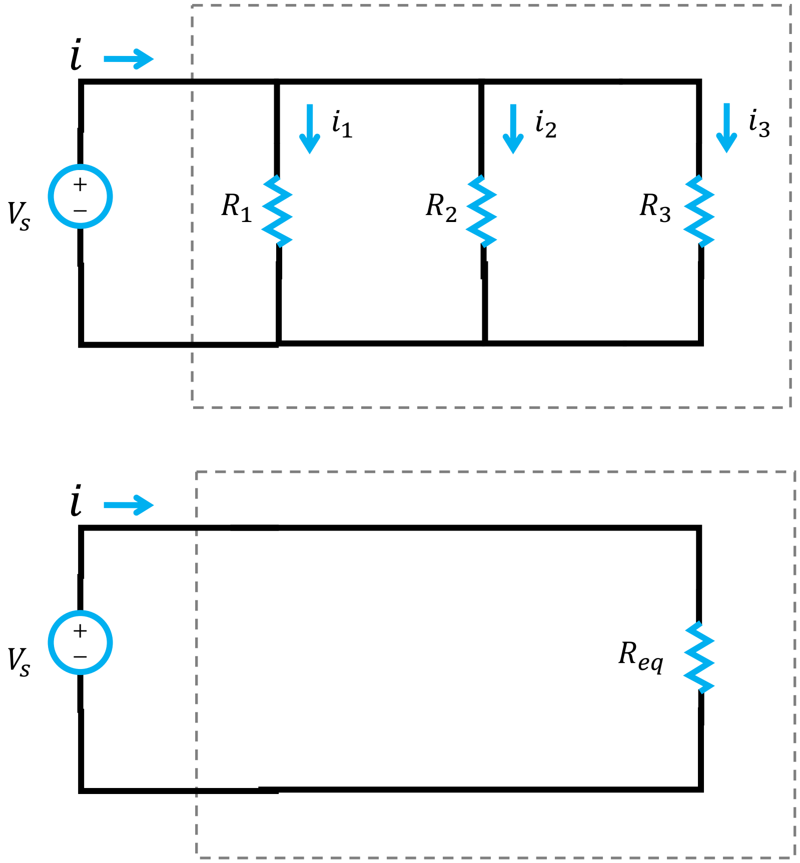
$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

- **Current flowing through a single resistance of resistors in parallel**

$$i_i = \frac{R_{eq}}{R_i} i_s$$

Equivalent circuits



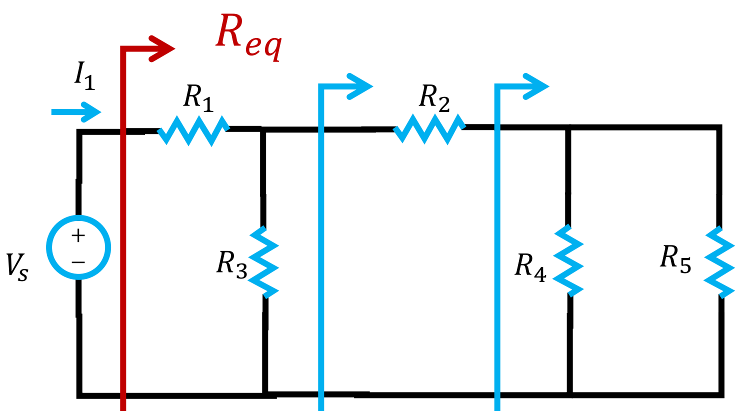


- If the current & voltage at nodes are identical, the circuits are considered "equivalent"
- Identifying equivalent circuits simplifies analysis

Example 9

QUESTION: Find the value of the equivalent resistance, R_{eq}

Find the output current of the voltage source



$$R' = \frac{R_4 R_5}{R_4 + R_5}$$

$$R'' = \frac{R_4 R_5}{R_4 + R_5} + R_2$$

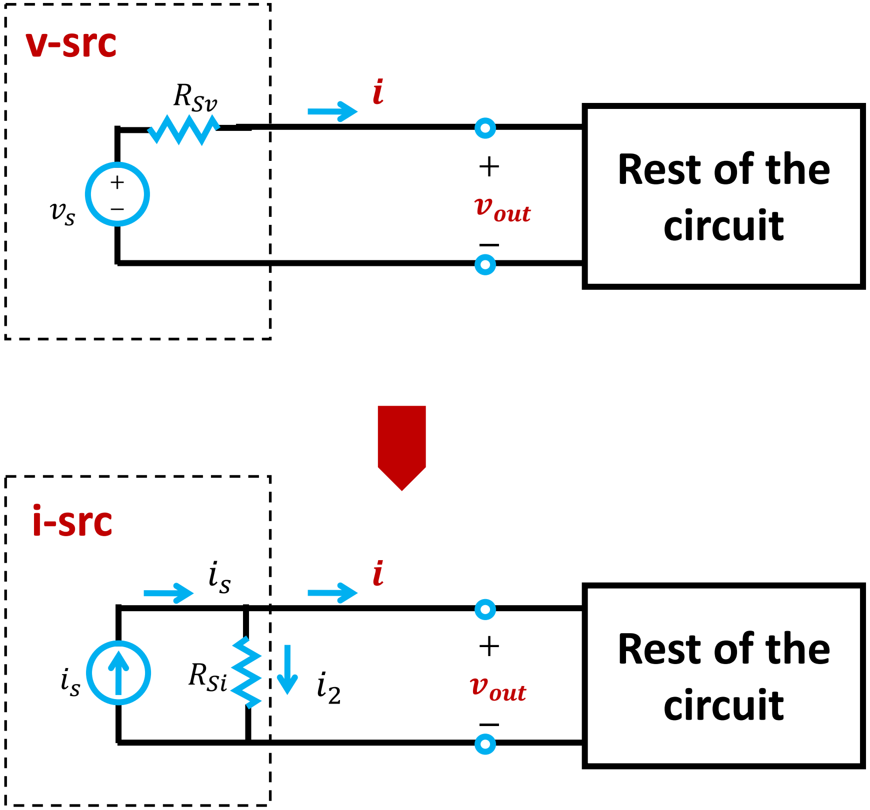
ANSWER:

$$R_{eq} = \frac{R_3 R''}{R_3 + R''} + R_1$$

$$I_1 = \frac{V_S}{R_{eq}}$$

Source Transformation

The goal of source transformation is to turn a voltage source into current source, and vice-versa



- For the v-src, according to KVL

$$-v_s + iR_{Sv} + v_{out} = 0$$

- For the i-src, according to KCL

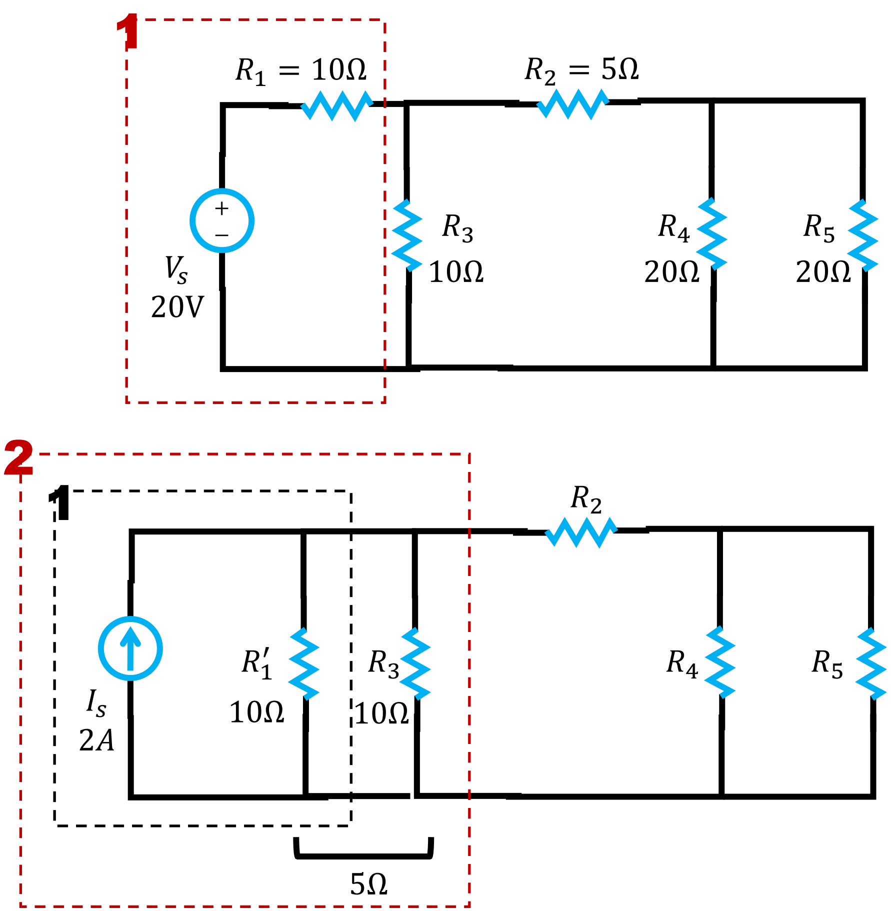
$$\begin{cases} i_s = i + i_2 \\ i_2 = \dfrac{v_{out}}{R_{Si}} \end{cases}$$

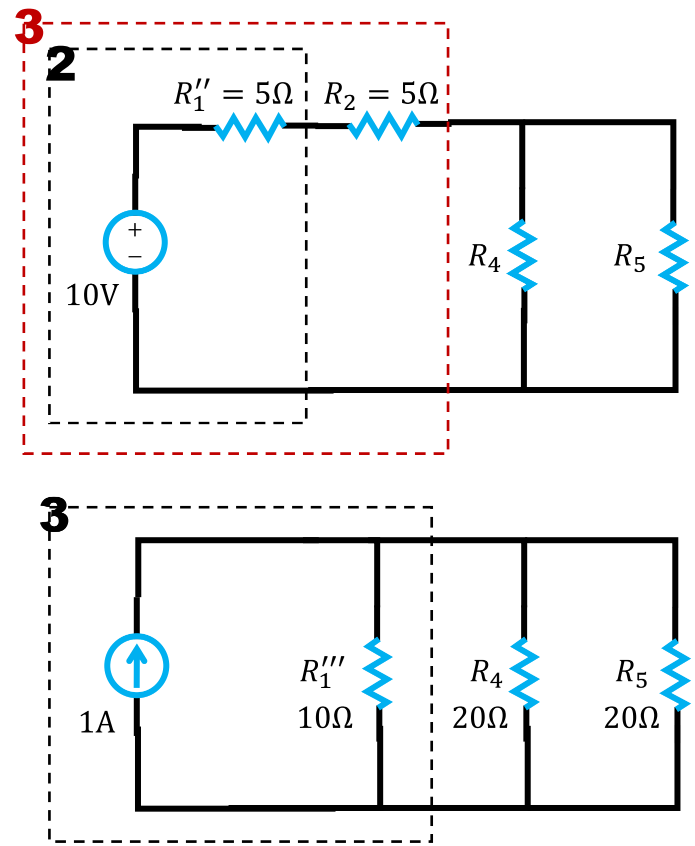
- If $R_{Sv} = R_{Si} = R_S$

$$\boldsymbol{v_s = i_s R_S}$$

Example 10

QUESTION: Find the current of R_5





ANSWER: $I_{R_5} = 0.25A$

? Shall we calculate the output current of V_s using this method

Outlines

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- KVL/KCL
- Equivalent circuit
  - Resistors in series/parallel
  - Source transformation

Reading tasks & learning goals

- Reading tasks
  - Basic Engineering Circuit Analysis (10th edition)
    - Chapter 2.1 – 2.6, 2.8, 2.10
    - Chapter 9.1, 9.2, 9.4, 9.5
- Learning goals
  - Be able to use **Ohm's law** for circuit analysis
  - Be able to calculate **power/energy/effective V/I** of a circuit
  - Be able to use **KVL/KCL** for circuit analysis
  - Know how to combine **resistors in series/parallel**
  - Know how to do **source transformation**
  - Be able to analysis circuit containing **dependent sources**