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11.105

$f(x|\theta)=\binom{n}{x}\theta^{x}(1-\theta)^{n-x}$, $x=0,1,2,\cdots,n$

(X,θ)的联合密度函数 $h(x,\theta)=\pi(\theta)f(x|\theta)=4\binom{n}{x}\theta^{x+3}(1-\theta)^{n-x}$, $x=0,1,2,\cdots,n$, $\theta\in(0,1)$

X的边际分布 $f(x)=\int_0^1 4\binom{n}{x}\theta^{x+3}(1-\theta)^{n-x}dx=4\binom{n}{x}B(x+4,n-x+1)$, $x=0,1,2,\cdots n$

故后验分布 $\pi(\theta|x)=\dfrac{h(\theta,x)}{f(x)}=\dfrac{4\binom{n}{x}\theta^{x+3}(1-\theta)^{n-x}}{4\binom{n}{x}B(x+4,n-x+1)}=\dfrac{\theta^{x+3}(1-\theta)^{n-x}}{B(x+4,n-x+1)}$, $\theta\in(0,1)$

11.106

设10个中次品的个数有X个，θ在$(0,1)$上一致分布，$\pi(\theta)=1$，$\theta\in(0,1)$

$P(X=x|\theta)=\binom{n}{x}\theta^{x}(1-\theta)^{n-x}$, $x=0,1,\cdots,n$, $\theta\in(0,1)$ 是 $f(x|\theta)$

(θ,X)的联合分分布 $h(\theta,x)=\binom{n}{x}\theta^{x}(1-\theta)^{n-x}$

X的边际分布 $f(x)=\int_0^1\binom{n}{x}\theta^{x}(1-\theta)^{n-x}dx=\binom{n}{x}B(x+1,n-x+1)$

故θ的后验分布 $\pi(\theta|x)=\dfrac{h(\theta,x)}{f(x)}=\dfrac{\theta^{x}(1-\theta)^{n-x}}{B(x+1,n-x+1)}$ $\theta\in(0,1)$, $x=0,1,2,\cdots,n$

F θ服从 $\alpha=x+1$, $\beta=n-x+1$ 的Beta分布

$E(\theta)=\dfrac{\alpha}{\alpha+\beta}=\dfrac{x+1}{n+2}$, $\cancel{x+1=2,\ x=1}$

$x=2$. $n=10$, 故 $\alpha=3$, $\beta=9$

故 θ后验分布服从 $\alpha=3$, $\beta=9$的Beta分布

11.109

(1) 设六个月内发生事故X次

λ的先验分布 $\pi(\lambda)=\cancel{\dfrac{\lambda^{\alpha-1}e^{-\frac{\lambda}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)}}\ \dfrac{\lambda^{\alpha-1}e^{-\frac{\lambda}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)}=\dfrac{1}{50}\lambda e^{-\frac{\lambda}{5}}$, $\lambda>0$

~~X的似然函数 f(x|λ)=~~ X服从参数为λ的泊松分布

$P(X=x|\lambda)=\dfrac{\lambda^{x}}{x!}e^{-\lambda}$, $x=0,1,2,\cdots$, $\lambda>0$

(λ,x)的联合密度函数 $h(\lambda,x)=\dfrac{\lambda^{x+1}}{50x!}e^{-\frac{6}{5}\lambda}$, $\lambda>0$, $x=0,1,2,\cdots$

X的边际分布 $P(X=x)=\int_0^{+\infty}h(\lambda,x)d\lambda=\dfrac{1}{50x!}\int_0^{+\infty}\lambda^{x+1}e^{-\frac{6}{5}\lambda}d\lambda$

$=\dfrac{1}{50x!}\cdot\left(\dfrac{5}{6}\right)^{x+2}\cdot\Gamma(x+2)$

故λ的后验分布 $\pi(\lambda|x)=\dfrac{h(\lambda,x)}{P(X=x)}=\dfrac{\lambda^{x+1}e^{-\frac{6}{5}\lambda}}{\left(\frac{5}{6}\right)^{x+2}\Gamma(x+2)}$

$x=14$, 故 $\pi(\lambda|x)=\dfrac{\lambda^{15}e^{-\frac{6}{5}\lambda}}{\left(\frac{5}{6}\right)^{16}\Gamma(16)}$. λ服从 $\alpha=x+2=16$, $\beta=\frac{5}{6}$的Gamma分布

(2) 后验分布的期望 $E(\lambda)=\alpha\beta=16\times\frac{5}{6}=\frac{40}{3}$

(3) 方差 $\sigma^2=\alpha\beta^2=16\times(\frac{5}{6})^2=\frac{100}{9}$

11.112

设个体反应时间为X，$X\sim N(\theta,0.3^2)$

θ先验分布 $\theta\sim N(1.5,0.1)$

$\pi(\theta)=\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(\theta-\mu_1)^2}{2\sigma_1^2}}$ ，$\sigma_1^2=0.1$，$\mu_1=1.5$，$\theta\in R$

X的密度函数 $f(x|\theta)=(\frac{1}{\sqrt{2\pi}\sigma_0})^n e^{-\frac{1}{2\sigma_0^2}\sum_i(x_i-\theta)^2}$ ，$\sigma_0^2=0.09$

(X,θ)的联合密度函数 $h(x,\theta)=\frac{1}{(2\pi)^{\frac{n+1}{2}}\sigma_0^n\sigma_1}\cdot exp[-\frac{1}{2\sigma_0^2}\sum_i(x_i-\theta)^2-\frac{1}{2\sigma_1^2}(\theta-\mu_1)^2]$

$$=k_1\ exp[-\frac{(\theta-BA^{-1})^2}{2A^{-1}}-\frac{C-B^2A^{-1}}{2}]$$

其中 $k_1=\frac{1}{(2\pi)^{\frac{n+1}{2}}\sigma_0^n\sigma_1}$ ，$\bar{x}=\frac{1}{n}(x_1+\cdots+x_n)$，$A=\frac{n}{\sigma_0^2}+\frac{1}{\sigma_1^2}$，$B=\frac{n\bar{x}}{\sigma_0^2}+\frac{\mu_1}{\sigma_1^2}$，$C=\frac{\sum x_i^2}{\sigma_0^2}+\frac{\mu_1^2}{\sigma_1^2}$

X的边际密度 $f(x)=\int_{-\infty}^{+\infty}h(x,\theta)d\theta=\sqrt{\frac{2\pi}{A}}k_1\ exp(-\frac{C-B^2A^{-1}}{2})$

$\pi(\theta|x)=\frac{h(x,\theta)}{f(x)}=\sqrt{\frac{A}{2\pi}}\ exp[-\frac{(\theta-BA^{-1})^2}{2A^{-1}}]$

服从 $N(BA^{-1},A^{-1})$，$BA^{-1}=\frac{\frac{n\bar{x}}{\sigma_0^2}+\frac{\mu_1}{\sigma_1^2}}{\frac{n}{\sigma_0^2}+\frac{1}{\sigma_1^2}}=\frac{\frac{20\times2}{0.09}+\frac{1.5}{0.1}}{\frac{20}{0.09}+\frac{1}{0.1}}=1.98$

$A^{-1}=\frac{1}{\frac{n}{\sigma_0^2}+\frac{1}{\sigma_1^2}}=\frac{1}{\frac{20}{0.09}+\frac{1}{0.1}}=0.0043$

故后验分布服从 $N(1.98,0.0043)$