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6.30 (1) ~~总体~~样本均值是总体均值的无偏估计

即 $\mu = E(\bar{X}) = 1200$ (hours)

(2) 样本方差 $s^2 = 10000$ (hours^2)

$$\hat{s}^2 = \frac{n}{n-1} s^2 = \frac{10}{10-1} \times 10000 = \frac{100000}{9} \quad (\text{hours}^2)$$

$\hat{s}^2$是~~样~~总体方差的无偏估计

$$\sigma = \sqrt{E(\hat{s}^2)} = \sqrt{\frac{100000}{9}} = 105.4 \text{ (hours)}$$

6.33 (1) $T = \dfrac{\bar{X} - \mu}{\hat{s}/\sqrt{n}}$, $\hat{s} = \sqrt{\dfrac{n}{n-1} s^2} = \sqrt{\dfrac{250}{249} \times 5.8^2 \times 10^{-8}}$, $\bar{X} = 0.72642$

T服从 $v = n-1 = 249$的 t分布. v很大可认为 ~~T接近正态分布~~ $v \to \infty$

$P(-t \le T \le t) = 0.99 \Rightarrow P(T \le t) = 0.995$

$t_{0.995} = 2.58$

$$P\left(-t \le \frac{\bar{X} - \mu}{\hat{s}/\sqrt{n}} \le t\right) = P\left(\bar{X} - \frac{\hat{s}}{\sqrt{n}} t \le \mu \le \bar{X} + \frac{\hat{s}}{\sqrt{n}} t\right)$$

$$\bar{X} \pm \frac{\hat{s}}{\sqrt{n}} t = 0.72642 \pm \sqrt{\frac{1}{249} \times 5.8^2 \times 10^{-8}} \times \overset{2.58}{\cancel{0.995}} = 0.72642 \pm 0.000095 \text{ (inch)}$$

(2) $P(-t \le T \le t) = 0.98 \Rightarrow P(T \le t) = 0.99$

$t_{0.99} = 2.33$

$$\bar{X} \pm \frac{\hat{s}}{\sqrt{n}} t = 0.72642 \pm \sqrt{\frac{1}{249} \times 5.8^2 \times 10^{-8}} \times 2.33 = 0.72642 \pm 0.000085 \text{ (inch)}$$

(3) $P(-t \le T \le t) = 0.95 \Rightarrow P(T \le t) = 0.975$

$t_{0.975} = 1.96$

$$\bar{X} \pm \frac{\hat{s}}{\sqrt{n}} t = 0.72642 \pm \sqrt{\frac{1}{249} \times 5.8^2 \times 10^{-8}} \times 1.96 = 0.72642 \pm 0.000072 \text{ (inch)}$$

(4) $P(-t \le T \le t) = 0.90 \Rightarrow P(T \le t) = 0.95$

$t_{0.95} = 1.645$

$$\bar{X} \pm \frac{\hat{s}}{\sqrt{n}} t = 0.72642 \pm \sqrt{\frac{1}{249} \times 5.8^2 \times 10^{-8}} \times 1.645 = 0.72642 \pm 0.000060 \text{ (inch)}$$

6.35

(1) ~~$S=100$，设 $T=\frac{\bar{X}-\mu}{S/\sqrt{n}}$，$\frac{S}{\sqrt{n}}=\frac{S}{\sqrt{n-1}}$~~

~~T服从自由度 $\nu=n-1$ 的 t-分布~~

~~$P(|\bar{X}-\mu|<20)=0.95$~~

~~$P(|T|<\frac{20}{S/\sqrt{n-1}})=P(-\frac{\sqrt{n-1}}{5}<T<\frac{\sqrt{n-1}}{5})=0.95$~~

$\sigma=100$，设 $Z=\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

则 $Z\sim N(0,1)$

$P(|\bar{X}-\mu|\le 20)=P(|Z|\le\frac{\sqrt{n}}{5})=0.95$

即 $P(Z\le\frac{\sqrt{n}}{5})=0.975$

解得 $\frac{\sqrt{n}}{5}=1.96$，$n=96.04$，至少需要97个样本才能满足条件

(2) $P(|Z|\le\frac{\sqrt{n}}{5})=0.90 \Rightarrow P(Z\le\frac{\sqrt{n}}{5})=0.95$

~~解得 $\frac{\sqrt{n}}{5}=1.65$，$n=(5\times1.65)^2=68.1$，至少~~

解得 $\frac{\sqrt{n}}{5}=1.645$，$n=25\times1.645^2=67.7$，

至少要 68个样本才能满足条件

(3) $P(|Z|\le\frac{\sqrt{n}}{5})=0.99 \Rightarrow P(Z\le\frac{\sqrt{n}}{5})=0.995$

解得 $\frac{\sqrt{n}}{5}=2.58$，$n=\cancel{166.8}\ 166.4$

至少要 167个样本

(4) $P(|Z|\le\frac{\sqrt{n}}{5})=0.9973$，$P(-\frac{\sqrt{n}}{5}\le Z\le\frac{\sqrt{n}}{5})=P(-3\sigma\le Z\le 3\sigma)$

~~故 $\frac{\sqrt{n}}{5}=3\sigma=300$，$n=1500^2=$~~

故 $\frac{\sqrt{n}}{5}=3$，$n=225$

至少要225个样本

6.39 (1) $\bar{X}=\frac{1}{5}(0.28+0.30+0.27+0.33+0.31)=0.298$

$\frac{\hat{S}}{\sqrt{n}}=\sqrt{\frac{(0.28-0.298)^2+\cdots+(0.31-0.298)^2}{5\times4}}=\cancel{0.10}\ 0.010677$

$T=\frac{\bar{X}-\mu}{\hat{S}/\sqrt{n}}$ 满足 $\nu=4$ 的 t-分布

$P(-t\le T\le t)=0.95 \Rightarrow P(T\le t)=0.975$，

$t_{0.975}=2.78$

故 $\bar{X}\pm\frac{\hat{S}}{\sqrt{n}}t=0.298\pm0.010677\times2.78=0.298\pm0.030$

(2) $P(-t\le T\le t)=0.99 \Rightarrow \cancel{t=0.99}\ P(T\le t)=0.995$

$t_{0.995}=4.60$

故 $\bar{X}\pm\frac{\hat{S}}{\sqrt{n}}t=0.298\pm0.010677\times4.60=0.298\pm0.049$

6.41 (1) 设样本容量 n，$X_i=1$ 表示第 i 个球是红色

$S_n=\sum X_i$，$S_n \sim B(n,p)$，$p=0.7$

记 $S_n^*=\dfrac{S_n-np}{\sqrt{np(1-p)}}$，$S_n^* \sim N(0,1)$

$P\left(\left|\dfrac{S_n}{n}-p\right|\le 0.05\right)=P\left(|S_n^*|\le \dfrac{1}{20}\sqrt{\dfrac{n}{p(1-p)}}\right)=0.95$

故 $P\left(S_n^*\le \dfrac{1}{20}\sqrt{\dfrac{n}{p(1-p)}}\right)=0.975$

$\dfrac{1}{20}\sqrt{\dfrac{n}{p(1-p)}}\ge t_{0.975}=1.96$

$n\ge 1.96^2\times 400\times p(1-p)\approx 322.7$

n 至少是 323

(2) $P\left(|S_n^*|\le \dfrac{1}{20}\sqrt{\dfrac{n}{p(1-p)}}\right)=0.99$，$P\left(S_n^*\le \dfrac{1}{20}\sqrt{\dfrac{n}{p(1-p)}}\right)=0.995$

$\dfrac{1}{20}\cdot\sqrt{\dfrac{n}{p(1-p)}}\ge t_{0.995}=2.58$，$n\ge 2.58^2\times 400\times 0.21=559.1$

故 n 至少是 560

(3) $P\left(|S_n^*|\le \dfrac{1}{20}\sqrt{\dfrac{n}{p(1-p)}}\right)=0.9973$ 则 $\dfrac{1}{20}\sqrt{\dfrac{n}{p(1-p)}}\ge 3$

$n\ge 9\times 400\times 0.21=756$

n 至少是 756

6.46 (1) 根据例 6.19 可知，

置信区间是 $S\pm z_c\cdot\dfrac{\sigma}{\sqrt{2n}}$

$p=0.95$ 时，$z_c=t_{0.975}=1.96$

$S\pm z_c\cdot\dfrac{\sigma}{\sqrt{2n}}=1800\pm\dfrac{1.96\times 1800}{\sqrt{200}}=1800\pm 249$

(2) $p=0.99$ 时，$z_c=2.58=t_{0.995}$

$S\pm z_c\cdot\dfrac{\sigma}{\sqrt{2n}}=1800\pm\dfrac{2.58\times 1800}{\sqrt{200}}=1800\pm 328$

(3) $p=0.9973$ 时，$z_c=3$

$S\pm z_c\cdot\dfrac{\sigma}{\sqrt{2n}}=1800+\dfrac{3\times 1800}{\sqrt{200}}=1800\pm 382$

6.57 记 $L = f_1(x_1, k) f_2(x_2, k) \cdot \cdots \cdot f_n(x_n, k)$

$$= \begin{cases} (k+1)^n (x_1 x_2 \cdots x_n)^k, & 0 \le x \le 1 \\ 0, & \cancel{else}\ otherwise \end{cases}$$

$$\ln L = n \ln(k+1) + \cancel{k \ln}\ k \sum_{i=1}^{n} \ln x_i$$

两边同时对 k 求导

$$\frac{1}{L} \cdot \frac{\partial L}{\partial k} = \frac{n}{k+1} + \sum_{i=1}^{n} \ln x_i$$

令 $\frac{\partial L}{\partial k} = 0$, $\cancel{k+1 = n}$

$$\therefore k = -1 - \frac{n}{\sum_{i=1}^{n} \ln x_i}$$

k 的极大似然估计是 $-1 - \frac{n}{\sum_{i=1}^{n} \ln x_i}$

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