

习题讨论课08题目：不定积分计算

★号（越）多表示题目（越）难

一、有理函数的不定积分

例 1. 计算以下不定积分：

$$\begin{aligned} (1) \int \frac{1-x^2}{1+x^2} dx & \quad (2) \int \frac{x}{3-x^2} dx & \quad (3) \int \frac{2x+1}{x^2+x+1} dx \\ (4) \int \frac{x}{x^2+x-6} dx & \quad (5) \int \frac{x^2}{1+x^6} dx & \quad (6) \int \frac{x-1}{x^2-4x+8} dx \\ (7) \int \frac{1}{(x+1)(x+2)} dx & \quad (8) \int \frac{1}{x(1+x^2)} dx & \quad (9) \int \frac{x^3+1}{x^3-5x^2+6x} dx \\ (10) \int \frac{1}{x^4-1} dx & \quad (11) \int \frac{x^4}{x^4+5x^3+4} dx & \quad (12) \int \frac{1}{1+x^3} dx \\ (13) \int \frac{x^7}{(1-x^2)^5} dx & \quad (14) \int \frac{1}{x^4(2x^2-1)} dx & \quad (15) \int \frac{1}{x(x^n+a)} dx \end{aligned}$$

二、三角函数的不定积分

例 2. 计算以下不定积分：

$$\begin{aligned} (1) \int (1-2\cot^2 x) dx & \quad (2) \int \tan x dx & \quad (3) \int \frac{\sec^2 x}{\sqrt{1+\tan x}} dx \\ (4) \int \cos^2(1-2x) dx & \quad (5) \int \cos^3 x dx & \quad (6) \int \sin \alpha x \cos \beta x dx \\ (7) \int \tan^4 x dx & \quad (8) \int \sqrt{1+\cos x} dx & \quad (9) \int \frac{\sin 2x}{1+\sin^4 x} dx \\ (10) \int \frac{\sin^4 x}{\cos^3 x} dx & \quad (11) \int \frac{1}{\sin x \cos^4 x} dx & \quad (12) \int \frac{\sin^2 x}{1+\sin^2 x} dx \\ (13) \int \frac{1+\tan x}{\sin 2x} dx & \quad (14) \int \frac{1-\tan x}{1+\tan x} dx & \quad (15) \int \frac{1}{(2+\cos x) \sin x} dx \\ (16) \int \frac{\sin x}{\sin x + \cos x} dx & \quad (17) \int \frac{1}{5+4\sin x} dx & \quad (18) \int \frac{\cos x}{\sin x + \cos x} dx \\ (19) \int \frac{\sin x \cos^3 x}{1+\cos^2 x} dx & \quad (20) \int \frac{\sqrt{1+\cos x}}{\sin x} dx, x \in (0, \pi) & \quad (21) \int \sqrt{1+\csc x} dx \end{aligned}$$

三、无理式的不定积分

例 3. 计算以下不定积分：

$$\begin{aligned} (1) \int \frac{x^2}{\sqrt{a^2+x^2}} dx & \quad (2) \int \frac{\sqrt{x^2-4}}{x} dx & \quad (3) \int \frac{1}{x\sqrt{a^2-x^2}} dx \\ (4) \int \frac{1}{x^2\sqrt{x^2-1}} dx & \quad (5) \int \frac{2x-1}{\sqrt{4x^2+4x+5}} dx & \quad (6) \int \frac{x^2}{\sqrt{3+2x-x^2}} dx \end{aligned}$$

$$\begin{aligned}
(7) \int \frac{1}{\sqrt{x}(\sqrt{x} + \sqrt[3]{x})} dx & \quad (8) \int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx & (9) \int x\sqrt{x+2} dx \\
(10) \int x^2\sqrt{1-x^2} dx & (11) \int x\sqrt{x^4+2x^2-1} dx & (12) \int x\sqrt{\frac{1+x}{1-x}} dx \\
(13) \int \sqrt{\frac{a-x}{x-b}} dx & (14) \int \frac{1-x+x^2}{\sqrt{1+x-x^2}} dx & (15) \int \frac{1}{\sqrt{(a^2-x^2)^3}} dx
\end{aligned}$$

四、换元法和分部积分

例 4. 计算以下不定积分:

$$\begin{aligned}
(1) \int \frac{1}{(1+x^2)\arctan x} dx & \quad (2) \int \frac{1}{x^2} \sinh \frac{1}{x} dx & (3) \int x \sec^2(1-x^2) dx \\
(4) \int \frac{x}{\sqrt{1+x^2}} \sin \sqrt{1+x^2} dx & (5) \int \sqrt{\frac{\arcsin x}{1-x^2}} dx & (6) \int \frac{2^x}{\sqrt{4-4^{x+1}}} dx \\
(7) \int \frac{e^x}{1+e^{2x}} dx & (8) \int \tanh x dx & (9) \int \frac{1}{x \ln x \ln \ln x} dx \\
(10) \int \frac{\sqrt{1-\ln x}}{x} dx & &
\end{aligned}$$

例 5. 计算以下不定积分:

$$\begin{aligned}
(1) \int x \cos 2x dx & \quad (2) \int x e^{-3x} dx & (3) \int x^2 \sin^2 x dx \\
(4) \int x \arctan x dx & (5) \int x \ln(x-1) dx & (6) \int \ln(x + \sqrt{1+x^2}) dx \\
(7) \int \arccos^2 x dx & (8) \int x \tan^2 x dx & (9) \int \frac{x}{\sin^2 x} dx \\
(10) \int e^x \sin^2 x dx & (11) \int \frac{\arcsin e^x}{e^x} dx & (12) \int \sin \ln x dx
\end{aligned}$$

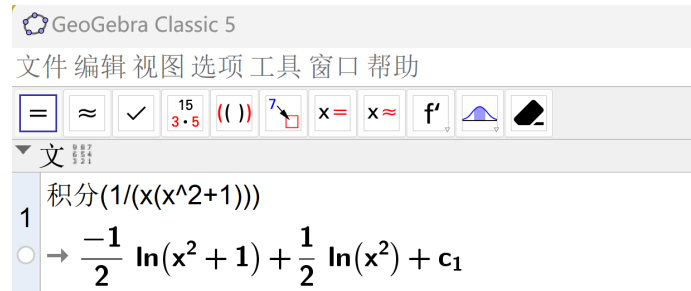
五、杂题

例 6. 以下函数是否存在原函数? 若存在, 求它的不定积分。

$$\begin{aligned}
(1) & |(x-1)(3x-2)| \\
(2) \operatorname{sgn}(x) & = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0. \end{cases} \\
(3) f(x) & = \begin{cases} -\cos \frac{1}{x} + 2x \sin \frac{1}{x}, & x \neq 0; \\ 0, & x = 0. \end{cases}
\end{aligned}$$

参考答案

请用 GeoGebra 的 CAS 自行检查计算结果



The screenshot shows the GeoGebra Classic 5 interface. At the top, there is a menu bar with "文件", "编辑", "视图", "选项", "工具", "窗口", and "帮助". Below the menu bar is a toolbar with various icons for mathematical operations. The main workspace is titled "文" and contains the following text:

1 积分($1/(x^2+1)$)
→ $-\frac{1}{2} \ln(x^2 + 1) + \frac{1}{2} \ln(x^2) + c_1$