Final Exam of Fundamentals of Physics (3), Electromagnetism

Open book test on May 16, 2022

Instruction to students.

a) This exam paper contains FIVE main questions and TWENTY-THREE sub-questions except the extra point problem. It comprises Six printed pages including this page.

b) This is open-book exam including any paper materials.

c) You are allowed to use a basic calculator in the exam.

d) If you carry out a computation please write down how you derived the answer. If you only write down the answer and it is wrong, no points are given for the question.

Good luck!

注意事项:

中文仅为参考翻译，一切按英文为准。

a) 本次试题共有 6 个大题，其中 5 个必答题（共 23 个小题），1 个附加题。包含本页总共 6 页。

b) 本次考试为开卷考试，可以带任何纸质资料。

c) 本次考试中可以使用基础的计算器。

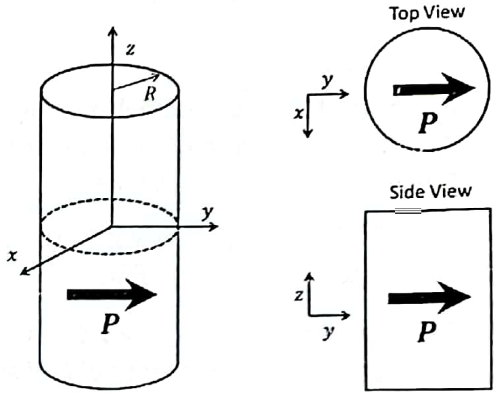
d) 计算时请写下过程，如果只写了答案并且答案是错误的，那将不得分。

1. Let's consider dielectric materials having different geometries and find the electric fields with them.

考虑不同形状的电介质材料并找出它们的电场分布。

(a-c) Let's consider a uniformly polarized ($\boldsymbol{P} = P\hat{\boldsymbol{y}}$) very long cylinder with a radius R shown in the right side.

考虑一个均匀极化($\boldsymbol{P} = P\hat{\boldsymbol{y}}$)的非常长的圆柱体，半径为 R，如右图所示。



(a) Find the bound charges of the cylinder. Find a corresponding model that can produce equivalent bound charges, similar to a uniformly polarized sphere in Example 4.3 in Griffith. Justify the equivalence of your model to the polarized cylinder.

求解该圆柱体的束缚电荷。找一个能产生等效束缚电荷的对应模型，类比于 Griffith 书的 Example 4.3。并证明其等价于该题中的圆柱体模型。

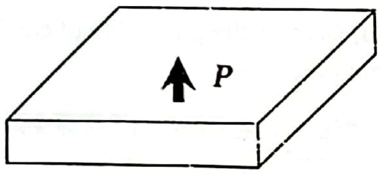
(b) Find the electric field inside the cylinder.

求解该圆柱体内部的电场。

(c) Find the potential and the electric field outside the cylinder.

求解该圆柱体外的电势和电场分布。

(d) Let's consider a uniformly polarized thin square plate ($\boldsymbol{P} = P\hat{\boldsymbol{z}}$) with thickness d. Find the bound charges of the plate and electric field inside and outside the plate.



考虑一个均匀极化($\boldsymbol{P} = P\hat{\boldsymbol{z}}$)的厚度为 d 的方形平板，如右图所示。求解该平板的束缚电荷，以及内部和外部的电场。

We assume these cylinder (e,f) and plate (g) are made of homogeneous linear dielectric material with susceptibility χ.

假设该圆柱体(e,f)和平板(g)是由均匀线性电介质做成的，其电极化率为χ。

(e) The dielectric cylinder is located in an otherwise uniform electric field $\boldsymbol{E_0} = E_0\hat{\boldsymbol{y}}$. Find the electric filed inside the cylinder (You may want to use a similar method for the problem 4.23 in Griffith.

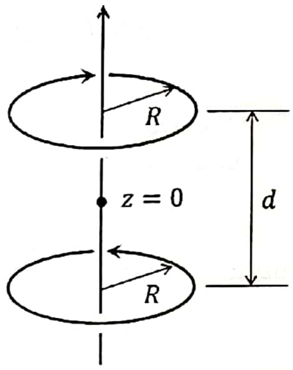
若该圆柱形电介质材料位于一个均匀电场$\boldsymbol{E_0} = E_0\hat{\boldsymbol{y}}$中，求解该圆柱体内部的电场。（你可以尝试使用 Griffith 书的 problem 4.23 的方法。）

(f) Explain the boundary conditions in cylindrical coordinates. Using the solutions of Laplace equations in cylindrical coordinates without z-dependence ($V(s,\phi) = \sum_{m=0}^{\infty}(A_m \cos m\phi + B_m \sin m\phi)(C_m s^m + D_m \frac{1}{s^m})$), find the electric field inside the cylinder and compare the result with (e).

找出该模型柱坐标下的边界条件。通过使用拉普拉斯方程（与 z 无关时）在柱坐标下的解($V(s,\phi) = \sum_{m=0}^{\infty}(A_m \cos m\phi + B_m \sin m\phi)\left(C_m s^m + D_m \frac{1}{s^m}\right)$)，求出圆柱体内部的电场，并与(e)的结果进行比较。

(g) The dielectric plate is located in an otherwise uniform electric field $\boldsymbol{E_0} = E_0\hat{\boldsymbol{z}}$. Explain the boundary conditions and find the electric field inside the dielectric plate.

若该平板电介质材料位于一个均匀电场$\boldsymbol{E_0} = E_0\hat{\boldsymbol{z}}$中，找出其边界条件，并求出该平板内部的电场。



2. (a) Find the total magnetic field produced by two circular current loops with current I everywhere in the space. Notice that directions of currents are opposite to each other.

求解由两个环形电流I所产生的全空间下的总磁场，注意，两个环形电流的方向使相反的，参数如右图所示。

(b) Find the gradients (along x, y, z directions) of the magnetic field near to the origin $(x, y, z \approx 0)$ in terms of R, d and I. Given amount of current I, find the condition to maximize the gradient of the magnetic field.

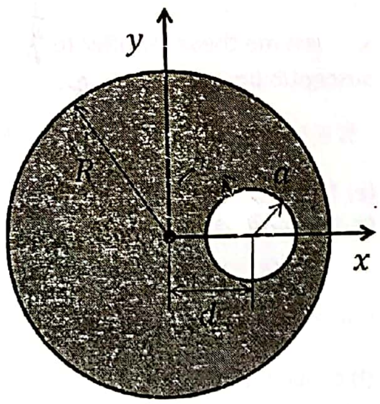
根据R, d 和 I，求解在原点位置 $(x, y, z \approx 0)$的磁场梯度(分别沿x, y, z方向)。当给定电流I时，在什么情况下磁场的梯度最大。

(c) We'd like to make the magnetic field as linear as possible, where second order magnetic field is negligible, near the origin. Determine the conditions for the most linear magnetic field.

我们想让磁场尽可能线性，原点附近的二阶磁场可以近似忽略。求解最线性情况的条件。

(d) Discuss the possibility of confining a magnetic dipole $\boldsymbol{m}$ near the origin by using the above linear magnetic field.

讨论在上述线性磁场下在原点囚禁一个磁偶极子$\boldsymbol{m}$的可能性。



3. Let's consider a cylindrical conductor of radius R that is centered about the z axis. It has a cylindrical hole of radius a bored parallel to and centered a distance d from, the z axis ($d + a < R$). The current density $\boldsymbol{J} = J\hat{\boldsymbol{z}}$ is uniform throughout the remaining metal of the cylinder and is parallel to the axis. A cross-section of the cylinder is shown in the right figure.

考虑一个半径为R，以z轴为中心的圆柱形导体。它有一个半径为a，平行并距离z轴为d的圆柱孔($d + a < R$)。该空心圆柱体具有均匀的平行于z轴的电流密度$\boldsymbol{J} = J\hat{\boldsymbol{z}}$。其横截面如右图所示。

(a) Find the magnitude and the direction of the magnetic field in the hole.

求解圆柱孔内的磁场大小和方向，

(b) Find the magnitude and the direction of the magnetic field outside the hole both inside the conductor and outside conductor.

求解圆柱孔外的磁场大小和方向(即导体内部和除圆柱孔外的导体外部）。

(c) Now imagine the situation that the magnitude of the current density varies slowly in time, that is, $\frac{dJ}{dt} \neq 0$. Find the magnitude and the direction of the induced electric field in the hole.

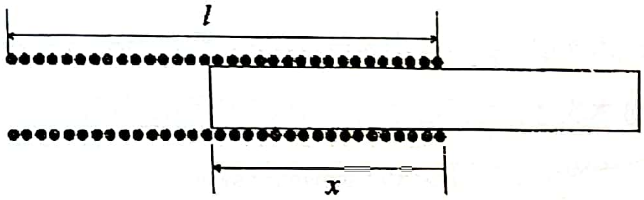
假如电流密度随着时间在缓慢变化，即$\frac{dJ}{dt} \neq 0$，求解圆柱孔内的感应电场的大小和方向。

(d) Find the magnitude and the direction of the induced electric field outside the hole both inside the conductor and outside conductor.

求解圆柱孔外的感应电场的大小和方向(即导体内部和除圆柱孔外的导体外部）。

4. Let's consider a relatively long solenoid of radius R and length l with n turns per unit length, carrying a steady current I. Inside the solenoid, a linear material with magnetic permeability μ and the same radius R is inserted up to the length of x. Let's ignore the edge effect and assume the material can be moved inside the solenoid without friction.

考虑一个半径为R，长度为l的相对较长的螺线管，单位长度内的匝数为n，带有稳定电流I。将磁导率为μ，半径为R（与螺线管相同）的线性材料插入到螺线管中，插入长度为x。忽略边缘效应以及假设该线性材料能在螺线管内部无摩擦移动。



(a) Find the magnitude and the direction of the H and B field inside the solenoid for both in and out of the linear material.

求解螺线管内磁场强度 H 和磁感应强度 B 的大小和方向（线性介质和无线性介质部分）。

(b) Find the magnetic energy of the system in the function of x.

求解这个系统磁场的能量，其结果为 x 的函数。

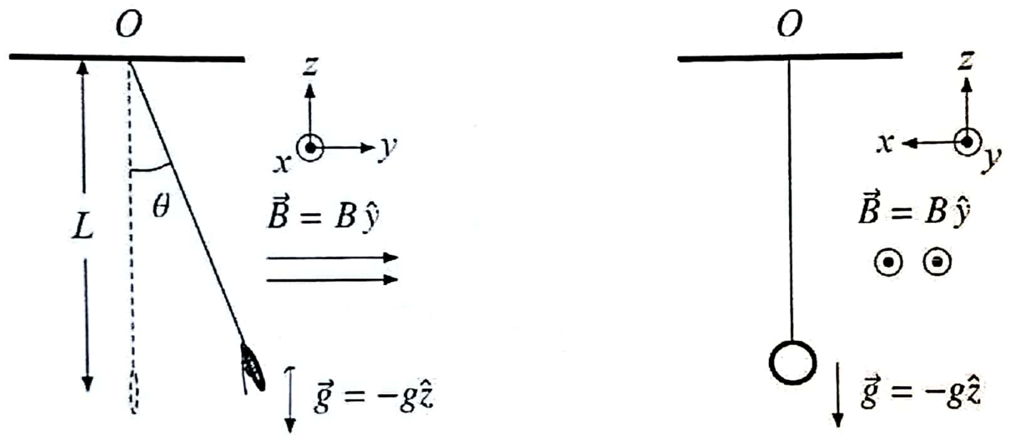
(c) Find the magnitude and the direction of the force on the linear material in the case we keep I constant.

当I为常数时，求解该线性材料受到的力的大小和方向。

(d) If the linear material is replaced by another solenoid (solenoid 2) of the same radius R with n_2 turns per unit length carrying a steady current I_2, find the magnitude and the direction of the force on the solenoid 2.

如果将该线性材料替换为另一个半径为R，单位长度内的匝数为n_2，带有稳定电流I_2的螺线管（螺线管 2），求解螺线管 2 受到的力的大小和方向。

5. Consider a circular ring made of a conducting wire. The area of the circle is A, the mass of the ring is m, and its total resistance is R. The ring is suspended at the end of a rigid, insulating rod of length L. Neglect the friction and the mass of the rod. Assume that the circle is sufficiently small ($\sqrt{A} \ll L$), so that the mass of the ring can be regarded as point-like. As the ring-rod system undergoes a pendulum motion, the rod is confined in the yz-plane, while the ring stays parallel to the x-axis (See the figure below). Finally, the pendulum is subject to a constant, uniform magnetic field $\boldsymbol{B} = B\hat{\boldsymbol{y}}$ as well as the gravitational field $g =- g\hat{\boldsymbol{z}}$.



考虑一个由导线制成的圆环，其面积为A，质量为m，总电阻为R，并悬挂在长度为 L 的刚性绝缘杆的末端，忽略该杆的摩擦和质量。假设圆环足够小($\sqrt{A} \ll L$)，其质量可以看作质点。环-杆系统在yz-平面进行摆动，且环面平行于x轴，如上图所示。最后，在该空间中，具有一个恒定的匀强磁场$\boldsymbol{B} = B\hat{\boldsymbol{y}}$以及重力场$g = - g\hat{\boldsymbol{z}}$。

(a) Suppose that the ring is in motion at an angle θ with an angular velocity $\omega = \frac{d\theta}{dt}$. Write down the induced electromotive force $\mathcal{E}$ in terms of θ and ω. Also, what is the electric current owing around the ring?

假设圆环进行运动，θ处的角速度为$\omega = \frac{d\theta}{dt}$。根据$\theta$ 和 ω，写出其感应电动势，且求出此时环内的电流。

(b) Compute the magnetic dipole moment $\boldsymbol{m}$, and the torque N due to the interaction of $\boldsymbol{m}$ with the background magnetic field $\boldsymbol{B}$.

计算该圆环的磁偶极矩$\boldsymbol{m}$，以及该磁偶极矩与背景磁场$\boldsymbol{B}$相互作用引起的力矩N。

(c) Write down the equation of motion for the pendulum, including both the magnetic force as well as the gravitational force.

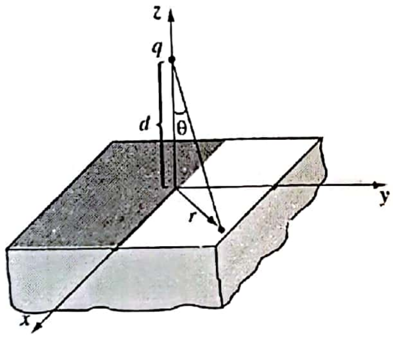
写出该圆摆的运动方程，包含磁力和重力。

(d) Using the equation of motion and taking the electric current into account, show that the energy conservation law holds.

利用其运动方程，并将电流考虑在内，证明能量守恒定律成立。

6. (Extra point problem) 附加题

Let's consider the half of the entire region below the plane $z = 0$ is filled with uniform linear dielectric material of susceptibility χ_e, and the other half (dark area) is filled with a conductor with the potential $V = 0$. Can you find which area a point charge q situated a distance d above the origin will collide? Dielectric area or conductor area? Why?



假设$z = 0$ 平面下的整个区域，一半填充满电极化率为χ_e均匀线性电介质，另外一半(深色)填充为电势为$V = 0$ 的导体。若原点上方d处有一个点电荷q，它会碰撞哪块区域，电介质还是导体？为什么？