Final Exam of Fundamentals of Physics (3), Thermal Physics

Open book test on June 18, 2022

Instruction to students.

a) This exam paper contains FOUR main questions and SEVENTEEN sub-questions except the extra point problem.

b) This is open-book exam including any paper materials.

c) You are allowed to use a basic calculator in the exam.

d) If you carry out a computation please write down how you derived the answer. If you only write down the answer and it is wrong, no points are given for the question.

Good luck!

注意事项:

中文仅为参考翻译，一切按英文为准。

a) 本次试题共有 4 大题，17 个小题（除附加题外）。

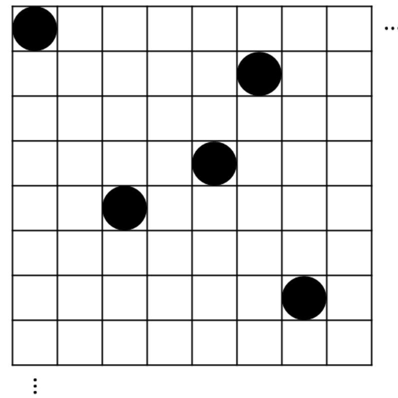
b) 本次考试为开卷考试，可以带任何纸质资料。

c) 本次考试中可以使用基础的计算器。

d) 计算时请写下过程，如果只写了答案并且答案是错误的，那将不得分。

祝好!

1. Let's consider an ideal lattice gas model. In the model, the position of a monoatomic gas particle is restricted to a volume v_0 in the 3D space. The figure at right side shows the projection of the lattice gas model to the 2D space. The total volume is V, number of lattice sites is $M = V/v_0$, and the number of particles is N, where $M > N$. Here we consider classical indistinguishable particles and ignore the momentum of the particle.



考虑一个理想晶格气体模型。在这个模型中，单原子气体粒子的位置被限制在一个体积为v_0的三维空间。右图为该晶格气体模型在二维空间的投影。总体积为V，晶格的格点数为$M = V/v_0$，粒子数为N，且$M > N$。这里我们考虑经典的不可区分粒子，并且忽略粒子的动量。

(a) Let's find Ω, the probability distribution of the model and the entropy given as $S = k_B \ln \Omega$.

求出该模型的概率分布Ω，以及熵($S = k_B \ln \Omega$)。

(b) If $M \gg 1$, $N \gg 1$ and $M - N \gg 1$, what will the entropy per lattice, $s \equiv S/M$? Let's define p as the probability of filling a lattice and write s as a function of p.

如果$M \gg 1$, $N \gg 1$且$M - N \gg 1$，每个格点的熵($s \equiv S/M$)是多少？定义p为填充到格点的概率，将s写成p的函数。

(c) Given the density $\bar{\rho} = \frac{p}{v_0}$, the average number of particles per unit volume, and find the standard deviation of $\bar{\rho}$.

给定密度$\bar{\rho} = \frac{p}{v_0}$(单位体积的平均粒子数)，求$\bar{\rho}$的标准差。

(d) In the limit of a dilute gas $p \ll 1$, derive the ideal gas law from the calculated entropy assuming T, the temperature of the system, is known. (Hint: use the equation (8.25) in the textbook, you can also find the equation in the end of this paper.)

在稀释气体($p \ll 1$)的极限下，通过已经求得的熵，推导理想气体方程，假设系统的温度T是已知的。(提示：使用课本中的方程(8.25)，也可在试卷末尾找到。)

(e) Assuming the energy per particle is ϵ, find the temperature of the system in terms of M, N, and ϵ.

假设每个粒子的能量为ϵ，根据M, N, 和 ϵ，求出系统的温度。

(extra point) In this system, does the negative temperature exist? What is the meaning of it?

(附加题) 在这个系统中，负温度存在吗？它的意义是什么？

2. Let's consider a classical ideal gas model in a volume V that consists of N distinguishable particles moving with speed of light c, where the Hamiltonian is given by $H(p,q)=\sum_{i=1}^{N} c\sqrt{p_{x,i}^2+p_{y,i}^2+p_{z,i}^2}$, where $q_i=\left(q_{x,i},q_{y,i},q_{z,i}\right)$ is the spatial coordinate and $p_i=\left(p_{x,i},p_{y,i},p_{z,i}\right)$ is the momentum of i-th particle.

考虑一个体积为V，包含N个可区分粒子的经典理想气体模型，其以光速c在运动，哈密顿量可以表示为$H(p,q)=\sum_{i=1}^{N} c\sqrt{p_{x,i}^2+p_{y,i}^2+p_{z,i}^2}$，其中$q_i=\left(q_{x,i},q_{y,i},q_{z,i}\right)$是第$i$个粒子的空间坐标，$p_i=\left(p_{x,i},p_{y,i},p_{z,i}\right)$是第$i$个粒子的动量坐标。

(a) Find the entropy of the total system. (Hint: use the equation (7.45) in the textbook)

求出整个系统的熵。(提示：使用课本中的(7.45)等式，也可在试卷末尾找到。)。

(b) Find the temperature T, pressure P, and chemical potential μ from the entropy.

根据熵，求出温度T，压强P，和化学势μ。

(c) Find the energy E $(or\ U)$ and Helmholtz free energy F from the entropy.

根据熵，求出能量E(或U)，以及亥姆霍兹自由能F。

(d) Find the specific heat at constant volume c_V and at constant pressure c_P.

求出定体热容c_V和定压热容c_P。

3. Let's consider a thermally isolated box with two compartments which are filled with ideal gases and separated by a partition (line). The initial pressure, volume, and temperature of each section after gas filling are given as in the figure below.

考虑一个绝热的盒子，它由两个充满理想气体的隔间，并用一个隔板隔开。装好气体后两部分的初始压强，体积和温度如图所示。

(a) If the partition becomes diathermal, or thermally conducting, find the final equilibrium temperature.

如果隔板是可以进行热传导的，求出最后平衡时的温度。

(b) If we release the partition to move freely along the horizontal direction, find the final equilibrium temperature and pressure of the system.

如果释放隔板，让它水平方向自由移动，求出系统最后平衡时的温度和压强。

(c) Calculate the increase of the total internal energy and the entropy of the system after the relaxation process of (b).

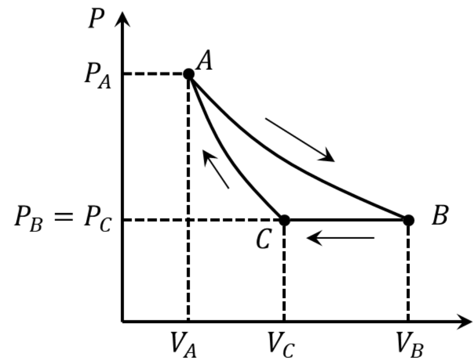
计算在(b)过程以后，系统的总内能和熵的增量。

(d) If we suddenly remove the partition, calculate the change of the total entropy of the system after long enough time to reach to an equilibrium.

如果我们突然移走隔板，系统经过足够长时间后到达平衡，求出系统总熵的变化。

4. Let's consider the quasi-static processes of monatomic classical ideal gas of N particles shown in right figure.

考虑右图中 N 个粒子的单原子经典理想气体的准静态过程。



The process from A to B is isothermal, that is, constant temperature process. The process from B to C is Isobaric, that is, constant pressure process. The process C to A is adiabatic, where no heat exchanges. Here $V_B = 32\, V_A$.

从 A 到 B 的过程是等温的，即温度是恒定的。从 B 到 C 的过程是等压的，即压强是恒定的。从 C 到 A 的过程是绝热的，即没有热量交换。其中$V_B = 32\, V_A$。

(a) In the process from A to B, find the amount of work done by the gas ΔW, heat absorbed to the system ΔQ, change of internal energy ΔU and entropy ΔS.

在 A 到 B 的过程的，求出气体所做的功ΔW，吸收的热量ΔQ，内能的改变量ΔU和熵的改变量ΔS。

(b) What is V_C? Write it in terms of V_A.

根据V_A求出V_C。

(c) When the system starts from A and returns to A through the whole processes, find $\Delta W, \Delta Q, \Delta U$, and ΔS.

当系统从 A 出发，又回到 A 点，在这整个过程中，求出$\Delta W, \Delta Q, \Delta U$, 和 ΔS。

(d) At A, the process is changed to adiabatic free expansion to the volume V_C, find $\Delta W, \Delta Q, \Delta U$, and ΔS.

在 A 点时，如果系统经历一个自由膨胀过程到体积V_C，求出该过程的$\Delta W, \Delta Q, \Delta U$, 和 ΔS。

$$S_\alpha(E_\alpha, V_\alpha, N_\alpha) = k \ln \Omega_\alpha(E_\alpha, V_\alpha, N_\alpha) \tag{7.45}$$
$$= k \ln \left[\frac{1}{h^{3N_\alpha} N_\alpha!} \int dq_\alpha \int dp_\alpha \, \delta(E_\alpha - H_\alpha) \right].$$

$$\left(\frac{\partial S}{\partial V} \right)_{E,N} = \frac{P}{T}. \tag{8.25}$$