2021 Final exam for GPII

(2021-6-17)

1. (10 points, 2 for each) Calculate the values for the following quiz (no complicated computation involved)

1) For electron with energy of 1eV, encounter a potential barrier with height of 10eV, what is the threshold of barrier width a_0 for e to tunnel, $a_0 = ?\,\text{Å}$

2) For the $|E_2>$, the 2nd eigenstate of energy in an infinite potential well with width of L, calculate $<E_2|\boldsymbol{P^2}|E_2> = ?$ $\boldsymbol{P^2}$ is momentum operator squared.

3) For the $|E_n>$ the (n+1)th eigenstate of energy for harmonic oscillator with $V=\frac{1}{2}kx^2$, calculate the expectation for X: $<E_n|\mathbf{X}|E_n> = ?$

4) For the $|E_2>$, the 3rd eigenstate of energy for harmonic oscillator, $\omega = \sqrt{k/m}$; calculate the average kinetics energy: $<E_2\left|\frac{\boldsymbol{P^2}}{2m}\right|E_2> = ?$

5) For the electron in H atom, calculate the value: $<2p_x|\widehat{L_z}|3p_x> = ?$ (L_z is the z-component of orbital angular momentum; p_x is a linear combination of Y_l^m: $p_x = \frac{1}{\sqrt{2}}(Y_1^{-1} - Y_1^1)$)

2. (16 points) The state of the single electron in Li^{2+} ion (Li, atomic number Z=3, the 2+ ion then only has one electron, it is a **hydrogen-like ion** with Z=3) is expressed by a normalized state function:

$\psi(r,\theta,\varphi) = (\frac{1}{3})^{1/2}R_{42}(r)Y_2^{-1}(\theta,\varphi) + (\frac{2}{3})iR_{32}(r)Y_2^1(\theta,\varphi) - (\frac{2}{9})^{1/2}R_{10}(r)Y_0^0(\theta,\varphi)$ The spin state can be equally either up or down. R, Y are hydrogen-like orbitals with nlm quantum number listed as usual.

(1) (7 points) In an ensemble of ions prepared in such state, if the energy is measured, what values will be found? (express it in atomic unit: **Hartree**, where the n=1 Hydrogen ground state energy is -1/2 Hartree); If different E values were found, what is the probability of finding each energy value, and what is the average energy?

(2) (5 points) If the **magnitude** of electron orbit angular momentum is measured, what values will be found? (express it in unit of $\hbar$); If multi-values were found, what is the probability for each result and what is average value?

(3) (2 points) If the orbital angular momentum along z direction is measured (by Zeeman), then what values can be found? (in unit of $\hbar$)

(4) (2 points) If the spin of electron along z direction are measured (by SG), then what values can be found and what are their probability?

3. (10 points) We had studied spin-1/2 electrons (spin quantum number $j = s = \frac{1}{2}$; $s_z = m = \pm\frac{1}{2}$). In this problem we are going to investigate matrix form of $\boldsymbol{J_z}, \boldsymbol{J_x}, \boldsymbol{J_y}$ **operator**

($\boldsymbol{J_z}$ **represents angular momentum component along z direction ….**) for spin-1 particle: Spin-1 particle has spin quantum number $j = s = 1$; $s_z = m = 1, 0, -1$; the eigenvectors of $\boldsymbol{J_z}$ will be used as base vectors:

$$|1>=|j=1,m=1>=\begin{pmatrix}1\\0\\0\end{pmatrix};|2>=|j=1,m=0>=\begin{pmatrix}0\\1\\0\end{pmatrix};|3>=|j=1,m=-1>=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$\boldsymbol{J_z}|j,m>=m\hbar|j,m>$.

1) (2 points) Write out the $\boldsymbol{J_z}$ **matrix** in the |1>,|2>,|3> base, in forms of $\boldsymbol{J_z} = \hbar(\boldsymbol{matrix})$

To find out $\boldsymbol{J_x}, \boldsymbol{J_y}$, we define two operators:

$\boldsymbol{J_+} = \boldsymbol{J_x} + \boldsymbol{iJ_y}$; $\boldsymbol{J_-} = \boldsymbol{J_x} - \boldsymbol{iJ_y}$

We can find out matrix of $\boldsymbol{J_+}, \boldsymbol{J_-}$ by the following formula (take for granted here)

$$<j,m'|\boldsymbol{J_+}|j,m> = \hbar\sqrt{j(j+1)-m(m+1)}\delta_{m',m+1}$$

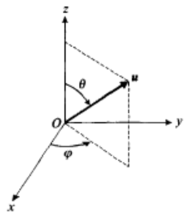
$$<j,m'|\boldsymbol{J_-}|j,m> = \hbar\sqrt{j(j+1)-m(m-1)}\delta_{m',m-1}$$

2) (4 points) Write out the $\boldsymbol{J_+}$ **and** $\boldsymbol{J_-}$ **matrix** in the |1>,|2>,|3> base

3) Write out the $\boldsymbol{J_x}, \boldsymbol{J_y}$ **matrix** in the $|1>,|2>,|3>$ base in forms of $\boldsymbol{J_{x,y}} = \frac{\hbar}{\sqrt{2}}(\boldsymbol{matrix})$

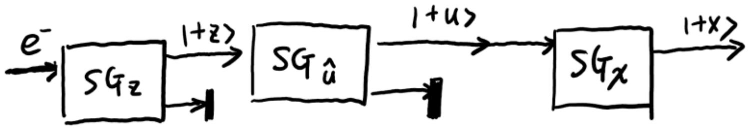
4. (14 points) In the following Stern-Gerlach experiment, we adopt |+z> to be the eigenstate for projection of spin along z to be $\frac{1}{2}\hbar$; and |-z> be eigenstate with projection of -$\frac{1}{2}\hbar$. |+x>,|-x>; |+y>,|-y> has similar meaning, corresponding to eigenstates with spin projection along x,y.

When the SG magnetic field is along $\hat{u}$ direction, the eigenstate is:



$$|+u>=cos(\frac{\theta}{2})|+Z>+sin(\frac{\theta}{2})e^{i\varphi}|-Z>$$

$$|-u>=-sin(\frac{\theta}{2})|+Z>+cos(\frac{\theta}{2})e^{i\varphi}|-Z>$$



Consider the above setup (neglect any B field from earth in this problem), we know that after passing first SGz, there are N electrons been prepared to |+z> state

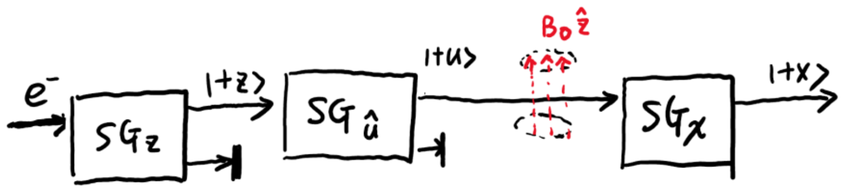
1) (2 points) By passing the $SG_{\hat{u}}$, $\hat{u}$ direction is shown in the previous graph in terms of θ, φ; the spin S projection along $\hat{u}$ will be measured, what are possible values? and

what is the probability of getting each?

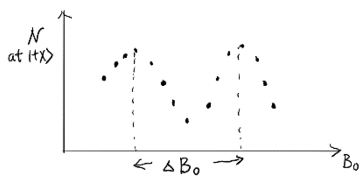
2) (4 points) Only allow |+u> state passing the last SGx setup, we will measure the spin S projection along x. What are the possible values? What is the probability for each value?

How many electrons will be in final |+x> state (in terms of N, θ, φ) ?

3) (8 points) We insert a Helmholtz ring (it can generate uniform B field)as shown in the figure below:



The B field is along $+\hat{z}$, and we can adjust current to get various uniform B_0. The time electron spent in the B_0 field will depend on diameter of the ring and electron momentum, call this time T.



We change the value of B_0 and record the number of electrons at |+x> state as shown:
Please calculate the ΔB_0, in terms of $g_s = -\frac{e}{m_e}, T, \theta, \varphi$ (g_s is gyro constant for electron spin)

5. (14 points) Consider a particle with mass=m in a potential given by:
$U(x,y,z) = A(x^2 + y^2 + xy) + B(z^2 + 2z)$
and A,B are positive constants
Please find the energy forms of the particle in this potential expressed by the given m, A,B, physical constant and any quantum numbers with their possible values.
Hint: for the x-y part, think of rotating the x-y base by angle θ and thus in the x'-y', the potential is simpler. If you adopt this approach, please show the $cos^2\theta$ value for partial credit.

6. (16 points) In this problem you shall find the bond length (distance between atomic nuclei in molecule) of carbon monoxide CO. We shall use an approximation that the bond length is fixed (we know it can oscillate, but here we treat it as a constant for simplicity; basically we measured the averaged bond length). This is the model of so-called rigid rotor in physics.
   1) (1 point) Consider two mass point m_1 and m_2; distance between them is fixed at d (that is why it is called rigid rotor); the potential energy between them is taken as zero. (since d is fixed the interaction between the two points is a constant and we take that as 0). Write out the spatial representation of the total Hamiltonian H_{tot} for this rigid rotor, in terms of x_1, y_1, z_1 the position of m_1 and x_2, y_2, z_2 the position of m_2.

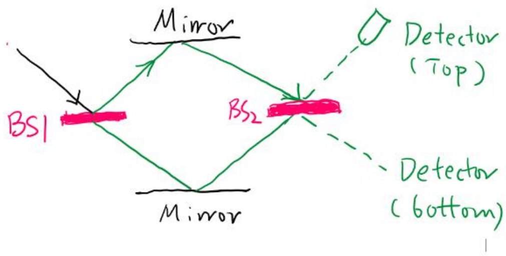
2) (2 points) The total H_{tot} can be decoupled into: $H_{tot} = H_{CM} + H_{relative}$; where H_{CM} only depends on the Center of Mass position: X_{CM}, Y_{CM} *and* Z_{CM}; $H_{relative}$ only depends on the relative position between the two mass points x, y, z ($x = x_2 - x_1, etc$); write out the spatial representation for H_{CM} *in terms of* X_{CM}, Y_{CM} *and* Z_{CM}; and $H_{relative}$ *in terms of* x, y, z.

3) (2 points)The relative motion between the mass points (rigid rotor) is governed by $H_{relative}$; its expression in x,y,z as in 2) is inconvenient, Please express the $H_{relative}$ of the rigid rotor in spherical coordinate. ($\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)$ in spherical coordinate)

4) (6 points) What is the eigenvalues (energies) for the $H_{relative}$? A) Express its general form in terms of *the reduced mass* μ *of* m_1 *and* m_2, distance d between them; any physical constant and necessary quantum number; B) Also please specify the possible values of the quantum number; C) List out the 3 lowest energy values; D)And which special function is the eigenfunction for $H_{relative}$? (no need to give its analytical expression, only tell me the name and symbol of it)

5) (5 points) When the CO molecule subject to microwave light, transition between the above calculated energy levels will be observed (this is called rotational spectroscopy). The lowest observed microwave absorption frequency of $^{12}C^{16}O$ is: $\upsilon = 115.27GHz$ ($1GHz = 10^9 Hz$); please compute the bond length d in $^{12}C^{16}O$.

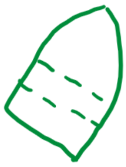
7. (20 points) Flitzur-Vaidman bomb detection:



For the Mach-Zehnder interference setup to detect the F-V bomb, we know that BS1, BS2 are identical R=50%-T=50% beam splitters. In terms of reflection and transmission coefficients: $R = |r^2|; T = |t^2| = 1/2$, and the phase difference between the reflected light and transmitted light at **each** beam splitter is: $\frac{r}{t} = e^{i\frac{\pi}{2}}$.

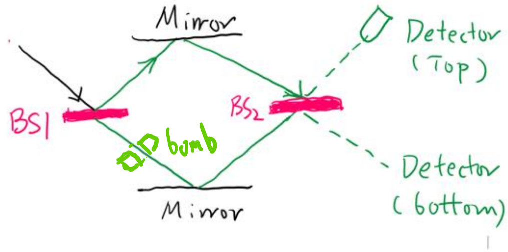
1) (2 points) if we set up an interference for **single photon** using MZ interferometer as shown in the figure above: The incoming photon comes from top, the optical path difference between top and bottom path is adjusted to be zero $\Delta l = 0$, and the reflection mirrors introduce no change in phase. What is the probability for the top detector and bottom detector records photon in each experiment, i.e. P(top) and P(bot)=?

We are going to use this to detect F-V bomb:



The part of the bomb between the dash lines is transparent embedded with photo-sensitive material which can absorb single photon of certain wavelength with 100% efficiency. Once the photon absorbed, the bomb will be detonated and explode. If this photo sensitive material is defective (bad egg), then all light can pass (just like vacuum) and the bomb cannot be detonated. Suppose the only way to know whether the photo-sensitive material works or not is by shining light of that wavelength, but this immediately raise a dilemma: If you shine the light directly, the good bomb will explode; only bad bombs left. The chance of successfully test, i.e. making sure the bomb works and not detonate it, is zero. Now with quantum and interference, we have certain probability to successfully test the bomb. Here is how:

In the following questions, the setup let incoming single photon along the top path before BS1 as shown in the figure, and the bomb will always be placed at the bottom path:



2) (2 points) We adjust the path length difference $\Delta l = 0$ as in 1), if the bomb is a bad egg, what is P(top) and P(bot)?

3) (2 points) If the bomb is a good one, you do the single photon test once, what is the probability you determine it is a good egg without detonating it?

4) (4 points)Now suppose we have many ZV bombs, there are N good ones. You can repeat the above test until you definitively pick out the good ones (or explosion), how many good eggs you successfully pick out with such R=T=50% BS?

5) Now we will try to **increase** the probability of successful detection: Using two identical Beam Splitters with R,T of reflection and transmission: $(R = |r|^2)$, $(T = |t|^2)$; R+T=1; R>T

5A) (2 points) How you adjust the path length difference $\Delta l = ?$ In your planned test?

5B) (1 point) If the bomb is a bad egg, what is P(top) and P(bot)?

5C) (2 points) If the bomb is a good one, you do the single photon test once, what is the probability you determine it is a good egg without detonating it? Express it in terms of R

5D) (3 points) Now suppose we have many ZV bombs, there are N good ones. You can repeat the above test until you definitively pick out the good ones (or explosion), how many good eggs you will successfully pick out? In terms of R, N

5E) (2 points) At what R you may have the largest probability, and what is its value; i.e. R=? and P(max)=?

中文试卷

1. (10 分， 每小问 2 分)计算下列小问题的数值（不该用到复杂计算）

1） 一个能量为 1eV 的电子，遇到一个势垒，高度为 10eV; 电子有隧道效应时，势垒的阈值宽度a_0=? Å

2） 对于$|E_2>$，在一维无限深势阱中的第二个能量本征态，势阱宽度为 L，计算动量算符平方$\boldsymbol{P^2}$的期望值：$<E_2|\boldsymbol{P^2}|E_2>$=?

3） $V=\frac{1}{2}kx^2$的简谐振子，$|E_n>$是它的第（n+1）能量本征态，计算位置 X 的期望值：$<E_n|\mathbf{X}|E_n>$=?

4） 对于$|E_2>$, 简谐振子($\omega=\sqrt{k/m}$)的第 3 个能量本征态, 计算平均动能值: $<E_2\left|\frac{\boldsymbol{P^2}}{2m}\right|E_2>$ =?

5） 对氢原子中的电子，$<2p_x|\widehat{L_z}|3p_x>$=? （$L_z$表示轨道角动量沿 z 方向分量，$p_x=\frac{1}{\sqrt{2}}(Y_1^{-1}-Y_1^1)$， 是$Y_l^m$的一个线性组合。）

2. (16 分) 考虑类 H 原子的锂的 2 价正离子 Li^{2+} （锂原子序数 Z=3， 因而 2 价正离子中仅有一个电子)，该电子的波函数为：

$$\psi(r,\theta,\varphi)=(\tfrac{1}{3})^{\frac{1}{2}}R_{42}(r)Y_2^{-1}(\theta,\varphi)+(\tfrac{2}{3})iR_{32}(r)Y_2^1(\theta,\varphi)-(\tfrac{2}{9})^{1/2}R_{10}(r)Y_0^0(\theta,\varphi)$$

自旋状态可能同等地朝上或朝下， R， Y 是类氢原子轨道，量子数 nlm 的含义也相同。

（1）（7 分） 若离子们被制备到这样的状态，进行能量测量，可能得到的值是多少？（用原子单位 Hartree 表示，氢原子基态 n=1 的能量值为 -1/2Hartree）. 若测量到多个数值，每一个值的概率为多少？ 平均能量为多大？

（2）（5 分） 若测量电子轨道角动量的大小，可能得到的值是多少？（用$\hbar$表示）；若测量到多个数值，每一个值的概率为多少？ 平均轨道角动量为多大？

（3）（2 分） 若测量电子轨道角动量沿 Z 方向分量的大小（比如用 Zeeman），可能得到的值是多少？（用$\hbar$表示）

（4）（2 分）若测量电子自旋角动量沿 Z 方向分量的大小，可能得到的值为多少？各自概率为多大？

3. （10 分）我们学习过了自旋-1/2 电子(spin quantum number $\mathrm{j}=\mathrm{s}=\frac{1}{2}$; $s_z=m=\pm\frac{1}{2}$)。

在本题中，我们研究$\boldsymbol{J_z},\boldsymbol{J_x},\boldsymbol{J_y}$算符的矩阵表达，对象是 自旋-1 的粒子： $\mathrm{j}=\mathrm{s}=1$; $s_z=m=1,0,-1$。 $\boldsymbol{J_z}$的本征态用作基矢量： $\boldsymbol{J_z}|\mathrm{j},\mathrm{m}>=\mathrm{m}\hbar|\mathrm{j},\mathrm{m}>$

$$|1>=|\mathrm{j}=1,\mathrm{m}=1>=\begin{pmatrix}1\\0\\0\end{pmatrix};|2>=|\mathrm{j}=1,\mathrm{m}=0>=\begin{pmatrix}0\\1\\0\end{pmatrix};|3>=|\mathrm{j}=1,\mathrm{m}=-1>=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

1） （2 分）写出$\boldsymbol{J_z}$ 矩阵在$|1>,|2>,|3>$基下的表达，表示为： $\boldsymbol{J_z}=\hbar(\boldsymbol{matrix})$

为求出$\boldsymbol{J_x},\boldsymbol{J_y}$，我们定义下面算符：

$\boldsymbol{J_+}=\boldsymbol{J_x}+\boldsymbol{iJ_y}$; $\boldsymbol{J_-}=\boldsymbol{J_x}-\boldsymbol{iJ_y}$

$\boldsymbol{J_+}$ and $\boldsymbol{J_-}$ matrix 可由下列关系求出（下列关系当作给定）

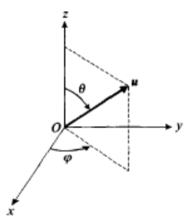
$< \mathrm{j,m'}|\boldsymbol{J}_+|\mathrm{j,m}> = \hbar\sqrt{j(j+1)-m(m+1)}\delta_{m',m+1}$

$< \mathrm{j,m'}|\boldsymbol{J}_-|\mathrm{j,m}> = \hbar\sqrt{j(j+1)-m(m-1)}\delta_{m',m-1}$

2） （4 分）写出$\boldsymbol{J}_+$ **and** $\boldsymbol{J}_-$ **matrix** 在$|1>, |2>, |3>$基下的表达

3） （4 分）写出$\boldsymbol{J}_x$ **and** $\boldsymbol{J}_y$ **matrix** 在$|1>, |2>, |3>$基下的表达： $\boldsymbol{J}_{x,y} = \frac{\hbar}{\sqrt{2}}(\boldsymbol{matrix})$

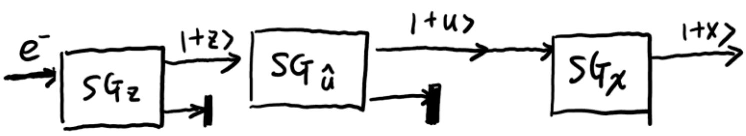
4. （14 分）在下面的 Stern-Gerlach 实验中， 我们采用通常的表达 |+z> 代表电子自旋沿 z 方向投影为$\frac{1}{2}\hbar$的本征态； |-z>代表电子自旋沿 z 方向投影为-$\frac{1}{2}\hbar$的本征态。 |+x>,|-x>; |+y>,|-y>含义相似，但对应 x, y 方向投影。



SG 磁场沿$\hat{u}$方向时：对应本征态的表达为

$$|+u>= cos(\frac{\theta}{2})|+Z>+sin(\frac{\theta}{2})e^{i\varphi}|-Z>$$

$$|-u>= -sin(\frac{\theta}{2})|+Z>+cos(\frac{\theta}{2})e^{i\varphi}|-Z>$$

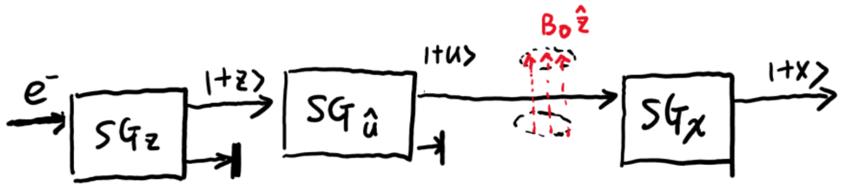


考虑上述的装置 (本题中忽略地球磁场，它被屏蔽或平衡)，已知经过第一个 SGz 制备 N 个电子到|+z>态

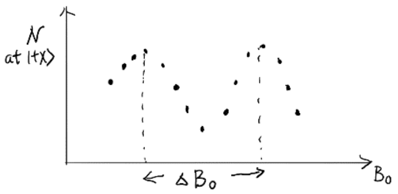
1） (2 分)经过一个$SG_{\hat{u}}$，$\hat{u}$ 方向如图所示，由θ，φ表示。 测量自旋 S 沿$\hat{u}$ 方向分量，可能得到的值有哪些？ 各自的概率是多少？

2） (6 分) 挡住部分$SG_{\hat{u}}$的输出，只让|+u>态经过后一个 SGx 装置， 测量得到自旋 x 分量数值可能有哪些，各自的概率多大？ 有多少个电子在|+x>态？ （用 N，θ，φ等表示）

3） (8 分) 如图，现在将一个 Helmholtz 线圈放入|+u>经过的空间，如图所示。



磁场方向沿$+\hat{z}$；可调节电流获得均匀磁场B_0；电子经过磁场的时间可由它的动量与线圈直径确定， 记作T。



现在调节磁场B_0， 可以观察到处于|+x>态电子数随B_0变化：

请计算ΔB_0，用$g_s = -\frac{e}{m_e}, T, \theta, \varphi$等表示。($g_s$ is gyro constant for electron spin)

5. （14 分）质量为 m 的粒子处在如下的三维势阱中：

$$U(x,y,z) = A(x^2 + y^2 + xy) + B(z^2 + 2z)$$

A， B 是正的常数。

求出该粒子的能量表达式，用给定参数 m, A, B，物理常数和量子数（及量子数的可能取值）

表达。

提示： 对于 x-y 部分， 旋转 x-y 坐标一个角度θ， 使得在新的 x'-y'系中，势能简化。 若你采用这个思路， 写出$cos^2\theta$的数值以获得部分分数。

6. （16 分）本题中你将会计算一氧化碳 CO 分子中键的长度（分子中原子核之间的距离）。我们假定键的长度是固定的（我们知道键长会有振动伸缩，本题中用固定长度简化处理；也可说是键的平均长度）， 这个模型在物理中被称为刚体转子。

1) （1 分） 考虑两个质点m_1 and m_2； 它们之间的距离固定为 d (这就是被称为刚体的原因)。 它们之间的势能被定为 0（由于 d 固定，相互之间的作用能量为常数，因此我们可以把相互作用能量定为 0)。写出刚体转子总的 Hamiltonian 算符 H_{tot}在空间中的表达形式，用m_1的位置x_1, y_1, z_1， 和m_2的位置x_2, y_2, z_2来表示。

2) (2 分) H_{tot}可以分解为$H_{tot} = H_{CM} + H_{relative}$；其中$H_{CM}$只与质心的位置$X_{CM}, Y_{CM}$ and Z_{CM}有关；$H_{relative}$只与质点间相对位置 x, y, z ($x = x_2 - x_1, etc$)有关； 写出H_{CM}的表达式，用X_{CM}, Y_{CM} and Z_{CM}来表示；写出$H_{relative}$的表达式，用 x, y, z 表示。

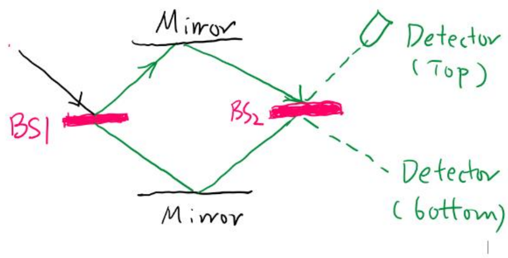
3) （2 分）$H_{relative}$描绘了刚体转子质点之间的相对运动，但在 2）中的 x, y, z 表示式并不是最方便的形式， 请在球坐标下给出$H_{relative}$的表达形式。 ($\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)$ in spherical coordinate)

4)（6 分） 关于$H_{relative}$的本征值（本征能量） A)写出它的一般表达式， 用$the\ reduced\ mass\ \mu\ of\ m_1\ and\ m_2$， 它们的间距 d，物理常数和必要的量子数来表达。 B)

明确指出量子数的可能取值 C）列出能量最低的 3 个能量值； D）哪种特殊函数是$H_{relative}$的本征函数？（不必给出解析式只需它的名称与符号）

5) （5 分）当一氧化碳 CO 分子被微波频率范围的光照射时，可以观测到上述能级之间的跃迁（这被称为转动光谱）。 在微波波段观测到的${}^{12}C^{16}O$的最小的吸收频率为：$\upsilon = 115.27GHz$（$1GHz = 10^9 Hz$）；请算出${}^{12}C^{16}O$中键长度 d.

7 （20 分）单光子 MZ 实验与 Elitzur-Vaidman 炸弹的检测：

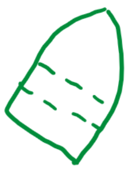


用来检测 F-V 炸弹的 Mach-Zehnder 干涉装置， 有两个相同的 50%-50%分束器，即反射与透射的百分比$R = T = 50\%$，（$R = |r|^2$，r *为振幅反射比*；$T = |t|^2$=1/2)，经每个分束器反射光与透射光之间存在位相差：

$\frac{r}{t} = e^{i\frac{\pi}{2}}$

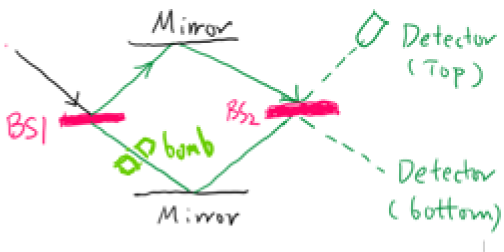
1) （2 分）在单光子实验中，如果我们将上下两路光程差调成一样$\Delta l = 0$， 单光子入射如图所示（从上路），M 反射镜不引入额外位相。 那么每次实验两个探测器 top 和 bottom 探测到光子的概率各为多少？ i.e. P(top) and P(bot)=?

Elitzur-Vaidman 炸弹. 示意图如下：



这个炸弹虚线之间是透明的，有感光物质，对特定波长的光（即使单光子照射），一定吸收，那就可以触发炸弹；但这一物质可能失效，那么透明部分就变成完全透光（类似空气），炸弹也无法触发。 要想确定炸弹是否工作（即感光材料是否有效），假定只能用特定波长的光照射它。 但常规用此波长光照立刻引起个矛盾：如果炸弹工作（好蛋），光一照就爆炸废掉了；如果炸弹不工作（废蛋），当然怎么照都不会炸。这样只能检测出废弹。 成功检测，即确定是好蛋，且它不爆炸的概率为 0. 现在我们用量子与干涉能够有一定概率实现成功检测：

下述都是入射单光子如下图所示入射，炸弹都放在下路：



2) (2 分) 光路调成 1) 问中$\Delta l = 0$; 如果放入下路的炸弹是坏蛋，那么 P(top) and P(bot)=?

3) （2 分） 如果放入的是好蛋， 一次测量，你有多大的概率确定它是好蛋而不让它爆炸？

4) (4 分) 现在有一堆 ZV 式炸弹，其中有 N 个好蛋；你可以不断测量直到成功挑出好蛋（或

爆炸)，那么用这种 50%-50%分束器，最终有多少好蛋成功检测?

5)现在我们尝试提高成功检测的概率,采用两个相同的分束器,每个的反射透射率为 R（$R = |r|^2$），T ($T = |t|^2$)；R+T=1；R>T.

5A) （2 分）你如何调节光路的光程差？即$\Delta l =$?

5B) （1 分）在你采用的光路中，放入的是坏蛋，P(top) and P(bot)=?

5C) （2 分）放入的是好蛋，一次检测，成功的概率是多少？用 R 表示；

5D)（3 分）反复检测，从 N 个好蛋中可以成功挑出多少个好蛋？用 R,N 表示

5E) （2 分）R 为多少时，成功挑出好蛋最多，可挑出好蛋的百分比为多少？