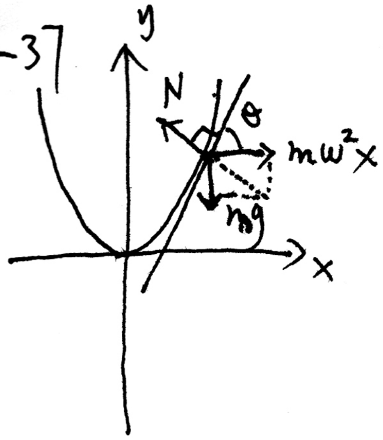
2-37



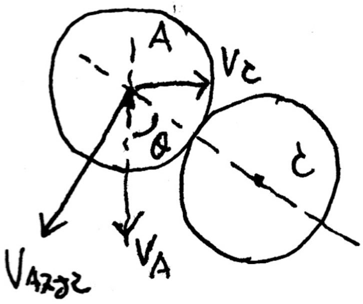
钢丝系:

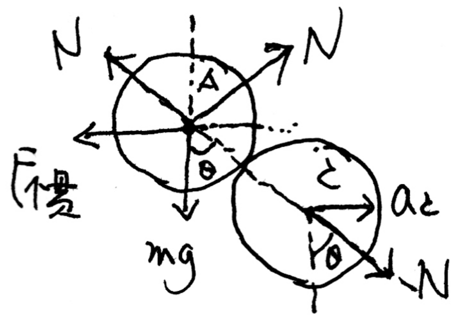
$$\frac{m\omega^2 x}{mg} = tg\theta = \frac{dy}{dx}$$

$$\Rightarrow dy = \frac{\omega^2}{g} x\,dx$$

$$\Rightarrow y = \frac{\omega^2}{2g} x^2$$

4-22





E守恒: $mg2R(\cos 30^\circ - \cos\theta) = \frac{1}{2} m V_A^2 + m V_c^2$

相对运动: $V_A = V_{A对c}\sin\theta$

$V_c = V_{A对c}\cos\theta$

c系中A的圆心运动:

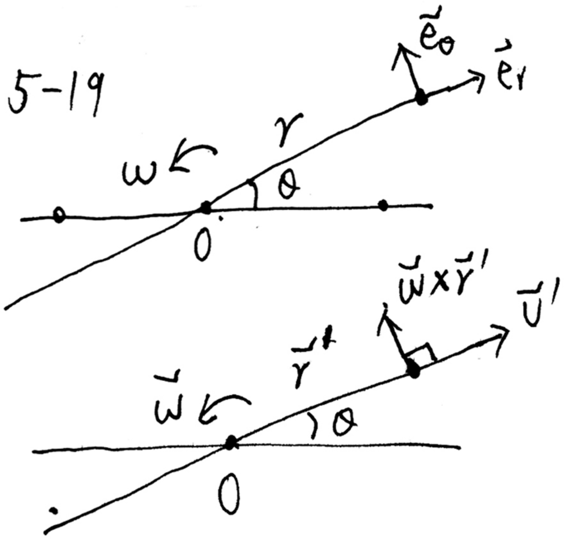
$$mg\cos\theta - F_{惯}\sin\theta - N + N\cos(\pi - 2\theta) = m\frac{V_{A对c}}{2R}$$

$$F_{惯} = m a_c$$

$$m a_c = N\sin\theta$$

$$\Rightarrow N = \frac{mg\cos\theta}{1+\sin^2\theta+\cos 2\theta} - \frac{2mg(\cos 30^\circ - \cos\theta)}{(1+\cos^2\theta)(1+\sin^2\theta+\cos 2\theta)}$$

5-19



运动学：

① 极坐标系：$V_\theta = r\dot{\theta}$ $V_r = \dot{r}$

或用② 相对运动关系（杆这转动系）

$$\vec{v} = \vec{v}' + \vec{\omega}\times\vec{r}' \qquad \vec{r}' = \vec{r}$$

$$v' = \dot{r} = V_r$$

$$|\vec{\omega}\times\vec{r}'| = \omega r = \dot{\theta} r = V_\theta$$

对O点L守恒：

$$2rmr\dot{\theta} = 2ama\omega_0 \qquad ①$$

Ek守恒：

$$2\frac{1}{2}m(r^2\dot{\theta}^2+\dot{r}^2) = 2\frac{1}{2}m(a\omega_0)^2 \qquad ②$$

方程①②消去$\dot{\theta}$得：

$$\dot{r} = \frac{a\omega_0}{r}\sqrt{r^2-a^2} = \frac{dr}{dt}$$

分离变量，两边积分：

$$\int_a^r \frac{r\,dr}{\sqrt{r^2-a^2}} = \int_0^t a\omega_0\,dt$$

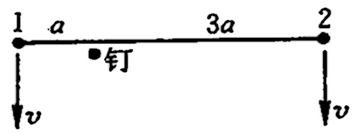
$$\Rightarrow \quad r = \sqrt{a^2 + a^2\omega_0^2 t^2}$$

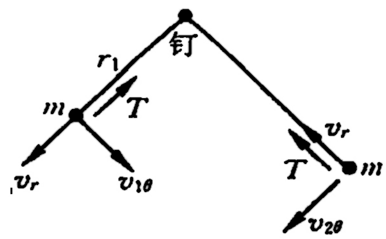
代回①式得：

$$\omega = \dot{\theta} = \frac{\omega_0}{1+\omega_0^2 t^2}$$

$$\alpha = \dot{\omega} = \frac{-2\omega_0^3 t}{(1+\omega_0^2 t^2)^2}$$

4-24





运动学关系 $v_{1r}=v_{2r}=v_r$

各自对钉的角动量守恒

$$mv_{1\theta}r_1 = mva \qquad ①$$

$$mv_{2\theta}(4a - r_1) = mv3a \qquad ②$$

动能守恒 $\frac{1}{2}m(v_{1\theta}^2 + v_r^2) + \frac{1}{2}m(v_{2\theta}^2 + v_r^2) = 2 \times \frac{1}{2}mv^2 \qquad ③$

(1) 当球 1 与钉的距离 r_1 达最大时,必有 $v_r=0$,由 ③ 得

$$\frac{1}{2}mv_{1\theta}^2 + \frac{1}{2}mv_{2\theta}^2 = 2 \times \frac{1}{2}mv^2 \qquad ④$$

由 ① ② ④ 消去 $v_{1\theta}$,$v_{2\theta}$,可得 r_1 的最大值方程

$$\left(\frac{a}{r_1}\right)^2 + \left(\frac{3a}{4a - r_1}\right)^2 = 2$$

(2) 对球 1 有

$$T = - ma_r$$

$$a_r = \frac{\mathrm{d}^2 r_1}{\mathrm{d}t^2} - r_1\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 = \frac{\mathrm{d}v_r}{\mathrm{d}t} - \frac{v_{1\theta}^2}{r_1} = \frac{\mathrm{d}v_r}{\mathrm{d}t} - \frac{v^2a^2}{r_1^3}$$

①②③联立可得

$$\left(\frac{va}{r_1}\right)^2 + \left(\frac{3va}{4a - r_1}\right)^2 + 2v_r^2 = 2v^2$$

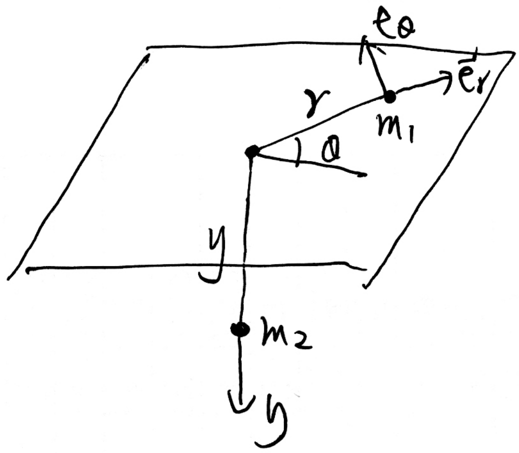
两边对 t 求导得 $\frac{\mathrm{d}v_r}{\mathrm{d}t} = \frac{v^2a^2}{2r_1^3} - \frac{9v^2a^2}{2(4a - r_1)^3}$

代入 T 表达式得

$$T = \frac{mv^2}{2a}\left[\frac{a^3}{r_1^3} + \frac{9a^3}{(4a - r_1)^3}\right]$$

T 极值 $T_{\mathrm{Min}} = \frac{(\sqrt{3} + 1)^4}{128}m\frac{v^2}{a}$

4.25



(1) $m_2 g = T = m_1 \omega_0^2 r_0$ ---- ①

$$\omega_0 = \sqrt{\frac{m_2 g}{m_1 r_0}}$$

(2) 几何约束关系:

$$y + r = c$$

$$\Rightarrow \quad a_2 = -\ddot{r} \quad \text{----} ②$$

牛顿方程:

$$m_2 g - T = m_2 a_2 \quad \text{----} ③$$

$$-T = m_1 \ddot{r} - m_1 r \dot{\theta}^2 \quad \text{----} ④$$

对O点 m_1的L守恒:

$$m_1 r^2 \dot{\theta} = m_1 r_0^2 \omega_0 \quad \text{----} ⑤$$

由①②③④⑤得:

$$(m_1+m_2)\ddot{r} + m_1 \omega_0^2 r_0 - \frac{m_1 r_0^4 \omega_0^2}{r^3} = 0$$

$$r = r_0 + \delta$$

$$\Rightarrow \quad \ddot{r} = \ddot{\delta}$$

$$(m_1+m_2)\ddot{\delta} + m_1 \omega_0^2 r_0 - \frac{m_1 r_0^4 \omega_0^2}{(r_0+\delta)^3} = 0$$

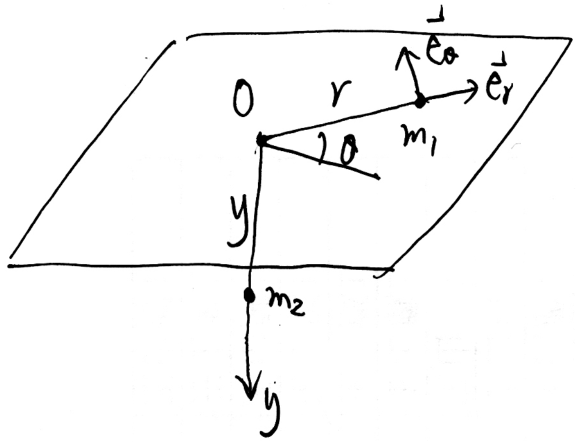
$$(m_1+m_2)\ddot{\delta} + m_1 \omega_0^2 r_0 \left[1 - \frac{1}{(1+\frac{\delta}{r_0})^3}\right] = 0$$

$$(m_1+m_2)\ddot{\delta} + m_1 \omega_0^2 r_0 \left[1 - (1 - 3\frac{\delta}{r_0})\right] = 0$$

$$\ddot{\delta} + \frac{3m_1}{m_1+m_2}\omega_0^2 \delta = 0$$

另法:

$m_2 g = m_1 \omega_0^2 r_0$ ①



几何约束:

$y + r = C_1$

$v_2 = -v_r = -\dot{r}$ ②

m_1对O点L守恒:

$m_1 r^2 \dot{\theta} = m_1 r_0^2 \omega_0$

$\dot{\theta} = \frac{r_0^2 \omega_0}{r^2}$ ③

m_1+m_2的E守恒(以圆周运动时为势能0点)

$\frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_1 (v_r^2 + v_\theta^2) + m_2 g (r - r_0) = C_2$ ④

①②③代入④:

$\frac{1}{2}(m_1+m_2)\dot{r}^2 + \frac{1}{2} m_1 r^2 \dot{\theta}^2 + m_1 \omega_0^2 r_0 (r - r_0) = C_2$

$\frac{1}{2}(m_1+m_2)\dot{r}^2 + \frac{1}{2} m_1 \frac{r_0^4 \omega_0^2}{r^2} + m_1 \omega_0^2 r_0 (r - r_0) = C_2$ ⑤

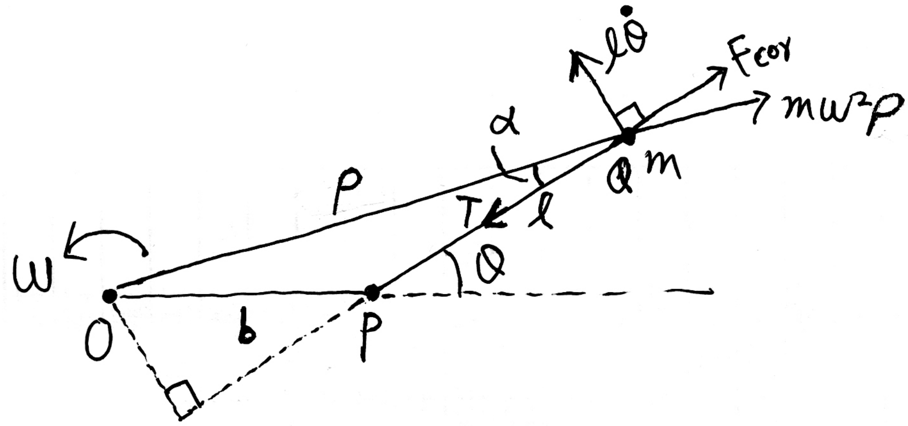
⑤式两边对t求导:

$(m_1+m_2)\dot{r}\ddot{r} - m_1 \frac{r_0^4 \omega_0^2}{r^3}\dot{r} + m_1 \omega_0^2 r_0 \dot{r} = 0$

$(m_1+m_2)\ddot{r} + m_1 \omega_0^2 r_0 - \frac{m_1 r_0^4 \omega_0^2}{r^3} = 0$

后面同前.

4.9



以台面为参考系，以P为参考点，张力T、科氏力F_{cor}力矩为0.

惯性离心力力矩（设逆时针为正）

$$M = -l\sin\alpha\, m\omega^2\rho = -m\omega^2 l b\sin\theta$$

对P点角动量：

$$L = l m l\dot{\theta}$$

由角动量定理 $\frac{dL}{dt} = M$ 得：

$$ml^2\ddot{\theta} = -m\omega^2 l b\sin\theta$$

即 $\ddot{\theta} = -\frac{\omega^2 b}{l}\sin\theta$

另法：列极坐标系下横向运动方程：

$$m(2\dot{r}\dot{\theta} + r\ddot{\theta}) = -m\omega^2\rho\sin\alpha = -m\omega^2 b\sin\theta$$

$$r = l \quad \dot{r} = 0$$

$$\Rightarrow \quad \ddot{\theta} = -\frac{\omega^2 b}{l}\sin\theta$$