摘自 赵凯华 《新概念物理 力学》

如图2－53所示，设傅科摆于某时刻处于位置O，过一时间Δt后它随地球自转到O'. 通过O、O'作子午线的切线，共同交地轴于N点。在O点的水平面（即地球的切面）上选直角坐标系xOy，其中Oy指北，Ox指东。将此平行移动❶到O'点，$O'x'$、$O'y'$分别与Ox、Oy平行。这时，$O'y'$与$O'N$的夹角，它等于$\angle ONO'$，就是摆面转过的角度。

$$\angle ONO' = \widehat{OO'}/\overline{ON}$$

$$= \widehat{OO'}/(\overline{OC}/\cos\theta) = \angle OCO'\cos\theta,$$

而$\angle OCO' = \omega\Delta t$（$\omega$为地球自转的角速度，$\theta$是纬度$\psi$的余角）。于是

$$\Omega = \frac{\angle ONO'}{\Delta t} = \omega\cos\theta = \omega\sin\psi, \quad (2.51)$$

上式表明，在南北极处$\psi=\pm\pi/2$，$\Omega=\pm\omega$；在赤道处$\psi=0$，$\Omega=0$.

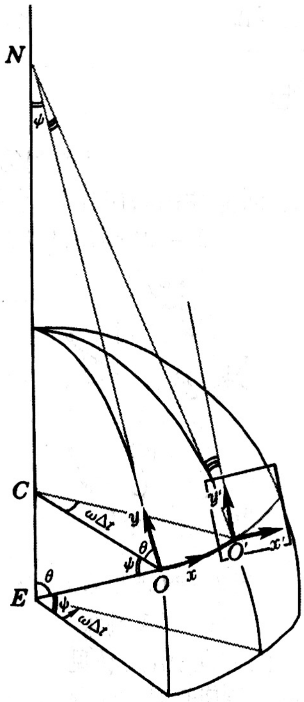


图2－53 傅科摆摆面进动的角速度

傅科摆摆面的进动非常缓慢，观察起来得有点耐心。下面介绍一个实验，可在课堂里演示，效果非常直观。如图2－54所示，两个圆盘形底座B_1和B_2互相叠放，B_1旁装有一柄H，可推着它在B_2光滑地转动（B_1和B_2间垫一层聚四氟乙烯T，以减少摩擦）。一根圆形截面的细钢丝斜插在B_1的中心，与法线成θ角，取$\theta = \arccos 3/4 = 41.41°$. 在钢丝的顶端装一质量为$m$的小球，它可沿任何横方向振动。实验开始时让小球左右振动，推H让B_1慢慢旋转一周，小球的振动方向转过$2\pi\cos\theta = 3\pi/2$，变成上下振动。读者可以自己体会一下这个实验的道理是与傅科摆一样的。

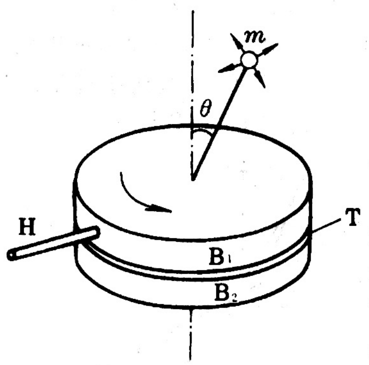


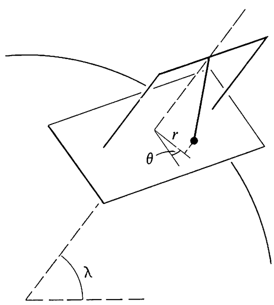
图2－54 振动面进动的演示实验

The plane of motion tends to stay fixed in inertial space while the earth rotates beneath it.

In the 1850s Foucault hung a pendulum 67 m long from the dome of the Pantheon in Paris. The bob precessed almost a centimeter on each swing, and it presented the first direct evidence that the earth is indeed rotating. The pendulum became the rage of Paris.

The next example uses our analysis of the Coriolis force to calculate the motion of the Foucault pendulum in a simple way.

Example 8.11 The Foucault Pendulum



Consider a pendulum of mass m which is swinging with frequency $\gamma = \sqrt{g/l}$, where l is the length of the pendulum. If we describe the position of the pendulum's bob in the horizontal plane by coordinates r, θ, then

$$r = r_0 \sin \gamma t,$$

where r_0 is the amplitude of the motion. In the absence of the Coriolis force, there are no tangential forces and θ is constant.

The horizontal Coriolis force $\mathbf{F}_{\mathrm{CH}}$ is

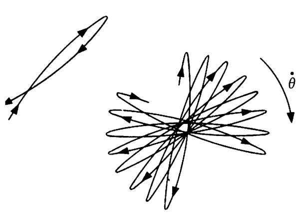
$$\mathbf{F}_{\mathrm{CH}} = -2m\Omega \sin \lambda \dot{r} \hat{\boldsymbol{\theta}}.$$

Hence, the tangential equation of motion, $ma_\theta = F_{\mathrm{CH}}$, becomes

$$m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = -2m\Omega \sin \lambda\, \dot{r}$$

or

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = -2\Omega \sin \lambda\, \dot{r}.$$



The simplest solution to this equation is found by taking $\dot{\theta}$ = constant. In this case the term $r\ddot{\theta}$ vanishes, and we have

$$\dot{\theta} = -\Omega \sin \lambda.$$

The pendulum precesses uniformly in a clockwise direction. The time for the plane of oscillation to rotate once is

$$T = \frac{2\pi}{\dot{\theta}} = \frac{2\pi}{\Omega \sin \lambda} = \frac{24\ \mathrm{h}}{\sin \lambda}.$$

Thus, at a latitude of 45°, the Foucault pendulum rotates once in 34 h.