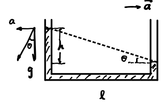
7-7



在管的非惯性系下，等势面与水平面夹角θ：$\tan\theta = \frac{a}{g}$

故由几何关系 $h = l\tan\theta = \frac{a}{g}l$

7-10 取包含A，B的流管，忽略A，B高度差 $\frac{1}{2}\rho v_A^2 + P_A = \frac{1}{2}\rho v_B^2 + P_B$ 其中 $P_B = P_0$

由连续性，流量 $Q = S_A \cdot v_A = S_B \cdot v_0$

A处吸起h水柱 $P_0 - P_A = \rho g h$

由上述各式得 $h = \frac{Q^2}{2g}\left(\frac{1}{S_A^2} - \frac{1}{S_B^2}\right)$

7-13 (1) 水柱高为h，对一水的质元m： $\frac{1}{2}mv^2 = mgH \Rightarrow$ 出口速度：$v = \sqrt{2gh}$

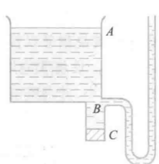
体积流量：$Q = \frac{1}{4}\pi d^2 \cdot v = \frac{1}{4}\pi d^2\sqrt{2gH}$

(2) 取喷嘴流管 $\frac{1}{2}\rho v_0^2 + P = \frac{1}{2}\rho v^2 + \rho g h + P_0$

连续性 $Q = \frac{1}{4}\pi D^2 \cdot v_0 \Rightarrow v_0 = \frac{d^2}{D^2}\sqrt{2gH}$

由上述各式得 $P = P_0 + \rho g\left[h + H\left(1 - \frac{d^4}{D^4}\right)\right]$

7-14



题 7-14 图

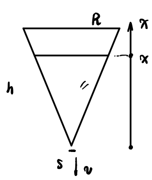
(1) 连通器：$h_1 = h_A$

(2) 取包含A，B，C的流管 $\rho g h_A + P_0 = \rho g h_B + \frac{1}{2}\rho v^2 + P_B = \rho g h_C + \frac{1}{2}\rho v^2 + P_0$

B处示数h_2 $P_B = P_0 + \rho g(h_2 - h_B)$

由上述各式得 $h_2 = h_C$

7-17



取从水面到孔流管（S为小孔，$\frac{h}{2} \le x \le h$，故忽略水面移动速度）

$$\rho g x + P_0 = \frac{1}{2}\rho v^2 + P_0 \Rightarrow v = \sqrt{2gx}$$

流量 $Q = S \cdot v = S\sqrt{2gx}$

$$Q = \frac{dV}{dt} = -\frac{\pi R^2}{h^2}x^2\frac{dx}{dt}$$

故 $t = -\frac{\pi R^2}{Sh^2}\frac{1}{\sqrt{2g}}\int_h^{\frac{h}{2}} x^{\frac{3}{2}}dx = \frac{\pi R^2}{20S}\sqrt{\frac{h}{g}}\,(4\sqrt{2} - 1)$

7-19 (1) 取小段，由泊肃叶公式 $Q = \frac{\pi R^4 dP}{8\eta\, dl}$ 其中 $dP = \rho g\, dl$

故 $\eta = \frac{\pi\rho g R^4}{8Q}$

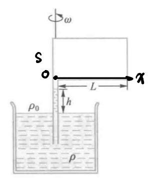
(2) 管轴处: $v_0 = \frac{R^2 dP}{4\eta dl} = \frac{2Q}{\pi R^2}$

7-20 斯托克斯公式 $f = 6\pi\eta r v_r$

稳态 $f = \rho g \frac{4}{3}\pi r^3$

故 $v_r = \frac{2\rho g r^2}{9\eta} = 1.425\times10^{-2}\ m/s$

7-23



题 7-23 图

考虑等温状态下压强的玻尔兹曼分布 $\frac{P}{\rho} = Const = \frac{p_0}{\rho_0}$

对一小段空气 $S\,dP = \rho S x\omega^2 dx$

$$\Rightarrow \rho_0\omega^2 x\,dx = p_0\frac{dP}{P}$$

积分 $P = p_1 e^{\frac{\rho_0\omega^2x^2}{2p_0}}$ 其中 p_1 为 $x=0$ 处的压强

$\rho = \rho_1 e^{\frac{\rho_0\omega^2x^2}{2p_0}}$ ρ_1 为 $x=0$ 处的密度

归一化, 总质量 $m = \rho_0 S L = \int_0^L \rho S\,dx = \rho_1 S\int_0^L e^{\frac{\rho_0\omega^2x^2}{2p_0}}dx$

$$\Rightarrow \rho_1 = \rho_0\cdot\frac{L}{\int_0^L e^{\frac{\rho_0\omega^2x^2}{2p_0}}dx}, \quad p_1 = p_0\cdot\frac{L}{\int_0^L e^{\frac{\rho_0\omega^2x^2}{2p_0}}dx}$$

ω很小, $\frac{\rho_0\omega^2x^2}{2p_0}\ll1$, 做近似 $\int_0^L e^{\frac{\rho_0\omega^2x^2}{2p_0}}dx \doteq \int_0^L\left(1+\frac{\rho_0\omega^2x^2}{2p_0}\right)dx = L\left(1+\frac{\rho_0\omega^2L^2}{6p_0}\right)$

$$p_1 \doteq p_0\cdot\frac{1}{1+\frac{\rho_0\omega^2L^2}{6p_0}} \doteq p_0 - \frac{1}{6}\rho_0\omega^2L^2$$

吸起 h 水柱 $\rho g h = p_0 - p_1 = \frac{1}{6}\rho_0\omega^2L^2$

得 $h = \frac{\rho_0\omega^2L^2}{6\rho g}$