6-8

解：(1) 对于3m与m圆盘整体：以转轴为轴．角动量守恒．

故 $J_{3m}\omega_0 = (J_{3m}+J_m)\omega_末$ ∴ $\omega_末 = \frac{3}{4}\omega_0$．

设摩擦力矩为：M

则 $dM = r\times df = r\times \mu dN = r\mu\cdot\rho\cdot\pi[(r+dr)^2-r^2] = 2\pi\mu\rho r^2 dr$

∴ $M = \int_0^R 2\pi\mu\rho\cdot r^2 dr = 2\pi\mu\rho\cdot\frac{R^3}{3}$ 又∵ ρ 为单位面积质量．$\rho = \frac{3mg}{\pi R^2}$

∴ $M = 2\cancel{\pi}\mu\frac{R^{\cancel{3}}}{\cancel{3}}\cdot\frac{\cancel{3}mg}{\cancel{\pi R^2}} = 2R\mu mg$

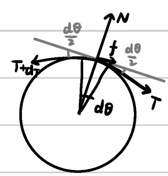
由 $M = J_{3m}\alpha$ 可得：$\alpha = \frac{2\cancel{R}\mu \cancel{m}g}{\frac{1}{2}\cdot 3\cancel{m}R^{\cancel{2}}} = \frac{4\mu g}{3R}$

故 $t = \frac{\omega_0-\frac{3}{4}\omega_0}{\alpha} = \frac{\frac{1}{4}\omega_0}{\frac{4\mu g}{3R}} = \frac{3R\omega_0}{16\mu g}$

(2) 由(1)分析知：$\omega_末 = \frac{3}{4}\omega_0$

6-9.

解：(1)
$$\begin{cases} m_1 g - F_{T1} = m_1 a & ① \\ F_{T2} - m_2 g = m_2 a. & ② \end{cases}$$



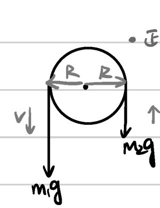
$$\begin{cases} N = (T+dT)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2} & ③ \\ (T+dT)\cos\frac{d\theta}{2} = T\cos\frac{d\theta}{2} + \mu N & ④ \end{cases}$$

∵ $\sin\frac{d\theta}{2} = \frac{d\theta}{2}$ $\cos\frac{d\theta}{2} = 1$ 则由③④解得 $\frac{dT}{T} = \mu d\theta$

两边积分：$\int_{F_{T2}}^{F_{T1}}\frac{dT}{T} = \int_0^{\pi}\mu d\theta$ ∴ $\frac{F_{T1}}{F_{T2}} = e^{\mu\pi}$ ⑤

联立①②⑤ 可得：$F_{T2} = \frac{2m_1m_2 g}{m_1+m_2e^{\mu\pi}}$ $a = \frac{m_1-m_2e^{\mu\pi}}{m_1+m_2e^{\mu\pi}}g$

(2) 对 $m_1.m_2$ 物体及滑轮整体：$M_外 = Rm_1g - Rm_2g$



角动量定理：$\int M_外 dt = Rm_1v + Rm_2v + \frac{1}{2}mR^2\cdot\omega$

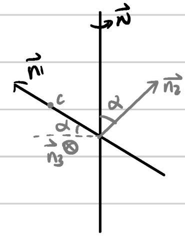
两边对t求导：$M_外 = R(m_1+m_2)a + \frac{1}{2}mR^2\cdot\alpha$

∴ $\alpha = \frac{4m_1m_2g(e^{\pi\mu}-1)}{mR(m_1+m_2e^{\pi\mu})}$

6-13

解：质心运动定理：$ma_c = F_{外}$

∴ $F_a + F_b = m\cdot w^2\cdot\frac{R}{2}\cdot\cos\alpha = \frac{1}{2}mw^2R\cos\alpha$ ①



角动量：$\vec{w} = w_1\hat{n}_1 + w_2\hat{n}_2$

$= w\sin\alpha\,\hat{n}_1 + w\cos\alpha\,\hat{n}_2$

转动角动量$\vec{L}'$ $= J_1 w_1\hat{n}_1 + J_2 w_2\hat{n}_2$

$= \frac{1}{4}mR^2 w\sin\alpha\,\hat{n}_1 + \frac{1}{2}mR^2 w\cos\alpha\,\hat{n}_2$

质心的角动量：$\vec{L}_c = \vec{r}_c\times m\vec{v}_c = mw\frac{R^2}{4}\cos\alpha\cdot\hat{n}_2$

∴ $\vec{L} = \vec{L}' + \vec{L}_c = \frac{1}{4}mR^2 w\sin\alpha\,\hat{n}_1 + \frac{3}{4}mwR^2\cos\alpha\,\hat{n}_2$

∴ $\frac{d\vec{L}}{dt} = \vec{w}\times\vec{L} = \frac{1}{2}mR^2w^2\sin\alpha\cos\alpha\,\hat{n}_3 = \vec{M}$ ②

力矩：$F_a l\,\hat{n}_3 - F_b\cdot l\cdot\hat{n}_3 - mg\cdot\frac{R}{2}\cdot\cos\alpha\,\hat{n}_3 = \vec{M}$ ③

联立②③得：$(F_a-F_b)l - \frac{1}{2}mgR\cos\alpha = \frac{1}{2}mR^2w^2\sin\alpha\cos\alpha$

联立①④得：
$$\begin{cases} F_a - F_b = \frac{1}{2l}mR\cos\alpha(g+Rw^2\sin\alpha) & ④ \\ F_a + F_b = \frac{1}{2}mw^2R\cos\alpha \end{cases}$$

$F_a = \frac{1}{4l}mR\cos\alpha(g+Rw^2\sin\alpha+w^2l)$

$F_b = \frac{1}{4l}mR\cos\alpha(w^2l-Rw^2\sin\alpha-g)$

6-16

解：(1) 对圆锥体及小滑块整体分析：角动量守恒

故：$J_0W_0 = J_0W_{末} + RmW_{末}R$　　解得：$W_{末} = \frac{J_0W_0}{J_0+mR^2}$

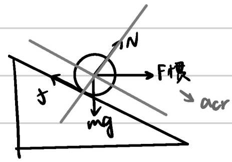
(2) 对整体：能量守恒：故有：$\frac{1}{2}J_0w_0^2 + mgh = \frac{1}{2}J_0W_{末}^2 + \frac{1}{2}mv^2$

∴ $J_0w_0^2\left[1-\frac{J_0^2}{(J_0+mR^2)^2}\right] + 2mgh = mv^2$

∴ $v = \sqrt{\frac{(2J_0+mR^2)J_0w_0^2R^2}{(J_0+mR^2)^2} + 2gh}$

6-19

解：以斜面为参考系：$\begin{cases} m a_{cr} = mg\sin\theta + m a_0 \cos\theta - f \\ N + m a_0 \sin\theta = mg\cos\theta \end{cases}$



无滑动滚动：　$a_{cr} = \alpha R$　　$M = J\alpha$　　$J = \frac{1}{2}mR^2$

又∵ $M = f \cdot R$　　∴ $3f = mg\sin\theta + m a_0 \cos\theta$ ①

$N = mg\cos\theta - m a_0 \sin\theta$ ②

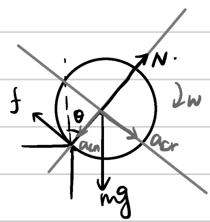
对斜面受力分析：　$m a_0 = N\sin\theta - f\cos\theta$ ③

将①②代入③：　$a_0 = \dfrac{2g\sin\theta\cos\theta}{6\sin^2\theta + 4\cos^2\theta} = \dfrac{g}{3\tan\theta + 2\cot\theta}$

6-20

解：滑动条件：$\mu = \frac{f}{N}$

根据质心运动定理：$\begin{cases} m a_{cr} = mg\sin\theta - f & ① \\ m w^2 \cdot R = mg\cos\theta - N & ② \end{cases}$



∵以与桌面接触轴为定轴：　$J = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$

又∵ $\vec{M} = J\alpha$　$\vec{M} = R \cdot \sin\theta \cdot mg$　∴ $\alpha = \dfrac{2g\sin\theta}{3R}$ ③

又∵无滑滚动故有 $a_{cr} = \alpha \cdot R$ ④

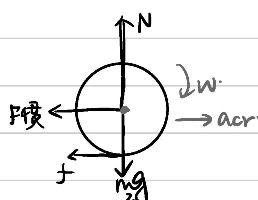
将③④代入① 得：$f = \frac{1}{3}mg\sin\theta$ ⑤

∵机械能守恒：$mgR - mgR\cos\theta = \frac{1}{2}Jw^2$　　∴ $w^2 = \dfrac{4g(1-\cos\theta)}{3R}$ ⑥

将⑥代入② 得：$N = mg\cos\theta - \frac{4}{3}mg(1-\cos\theta)$ ⑦

联立 $\mu = \frac{f}{N}$ 及⑤⑦式：得 $7\cos\theta - 4\sin\theta = 4$.　故 $\cos\theta = \frac{56}{65}$

6-21

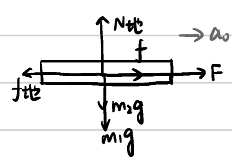


解：(1) 以m_1为参考系：$m a_{cr} = -m_2 a_0 - f$ ①

∵纯滚动：　$a_{cr} = \alpha \cdot R$ ②

又∵ $\vec{M} = J\vec{\alpha}$，$J = \frac{1}{2}mR^2$　$\vec{M} = f \cdot R = J\alpha$　∴ $\alpha = \dfrac{2f}{mR}$ ③

联立①②③可得：　$f = \dfrac{-m_2 a_0}{3}$



对木板m_1受力分析：$m_1 a_0 = f + F - \mu(m_1 + m_2)g$

∴ $a_0 = \dfrac{F - \mu(m_1+m_2)g}{m_1 + \frac{1}{3}m_2}$

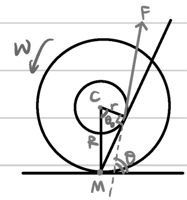
(2) 由(1)问知：$N = m_2 g$

$$f = -\frac{m_2}{3}a_0 = -\frac{m_2}{3}\cdot\frac{F-\mu(m_1+m_2)g}{m_1+\frac{m_2}{3}}$$

不产生滑动的条件：$|f| \le \mu N$

即 $\frac{m_2}{3}\cdot\frac{F-\mu(m_1+m_2)g}{m_1+\frac{1}{3}m_2} \le \mu m_2 g \quad \therefore F \le (4m_1+2m_2)\mu g$

6-27



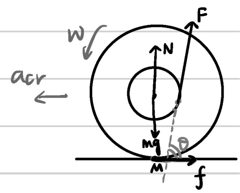
解：(1) (i) 作纯滚动，故以M轴为定轴分析：

要使其向后滚动故应使 $\theta > \theta_0 \quad \because \cos\theta_0 = \frac{r}{R}$

$\therefore \theta > \arccos\frac{r}{R}$

(ii) $\vec{M} = rF + Rf \quad \vec{M} = J\alpha \quad \therefore \alpha = \frac{rF+Rf}{J}$

又∵作纯滚动 $\therefore a_{cr} = \alpha\cdot R = \frac{Rf+rF}{J}\cdot R$



又∵ $\begin{cases} ma_{cr} = -f - F\cdot\cos\theta \\ N + F\cdot\sin\theta = mg \end{cases}$

又∵ $f \le \mu N \quad \therefore \mu \ge \frac{|f|}{N} = \frac{F(mrR+J\cos\theta)}{(mg-F\sin\theta)(mR^2+J)}$

(iii) 由(ii)知：$f = \frac{-F(mrR+J\cos\theta)}{mR^2+J}$

又∵ $a_{cr} = \frac{1}{m}(-f-F\cos\theta) = \frac{1}{m}\frac{F(mrR+J\cos\theta)-F\cos\theta(mR^2+J)}{mR^2+J}$

$$= \frac{FR(r-R\cos\theta)}{mR^2+J}$$

(2) (i) $\theta < \arccos\theta\frac{r}{R}$

(ii) 由(1)(ii)知：求法完全一致. $\therefore \mu \ge \frac{F(mrR+J\cos\theta)}{(mg-F\sin\theta)(mR^2+J)}$

(iii) 由(1)(iii)知：$a_{cr}' = -a_{cr} = -\frac{FR(r-R\cos\theta)}{mR^2+J}$

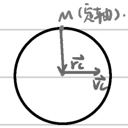
6-30

解: 情况1: 对子弹和圆盘系统: 角动量守恒:

$$Rmv_0 = J\omega_1 + m\omega_1 R^2 \qquad \therefore \omega_1 = \frac{Rmv_0}{\frac{1}{2}m_0R^2 + mR^2} = \frac{mv_0}{\frac{1}{2}m_0R + mR}$$

$$\therefore E_{k1} = \frac{1}{2}m(\omega_1 R)^2 + \frac{1}{2}J\omega_1^2$$

$$= (\frac{1}{2}m + \frac{1}{4}m_0)(\omega_1 R)^2 = (\frac{1}{2}m + \frac{1}{4}m_0)\frac{m^2v_0^2}{(\frac{1}{2}m_0 + m)^2} = \frac{\frac{1}{2}m^2v_0^2}{\frac{1}{2}m_0 + m}$$



情况2: 对子弹+圆盘: 角动量守恒: $0 = J_c \cdot \omega_2 - R m_0 v_c$

$$\therefore v_c = \frac{1}{2}R\omega_2$$

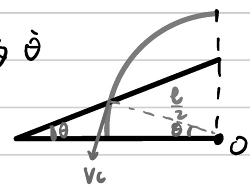
∵ 自由运动: 动量守恒: $mv_0 = m(v_c + \omega_2 R) + m_0 v_c$ $\quad \therefore \omega_2 = \frac{mv_0}{R(\frac{3}{2}m + \frac{1}{2}m_0)}$

$$\therefore E_{k2} = \frac{1}{2}m(v_c + \omega_2 R)^2 + \frac{1}{2}J\omega_2^2 + \frac{1}{2}m_0 v_c^2$$

$$= \frac{1}{2}m(\frac{3}{2}\omega_2 R)^2 + \frac{1}{4}m_0(R\omega_2)^2 + \frac{1}{2}m_0\frac{1}{4}(R\omega_2)^2 = \frac{m^2v_0^2}{\cancel{\frac{1}{4}}(3m+m_0)^{\cancel{2}}}\cdot\frac{3}{\cancel{8}\,2}\cdot\cancel{(3m+m_0)}$$

$$\therefore \frac{E_{k1}}{E_{k2}} = \frac{\cancel{\frac{1}{2}}\cancel{m^2v_0^2}}{\frac{1}{2}m_0 + m} \times \frac{\cancel{2}(3m+m_0)}{3\cancel{m^2v_0^2}} = \frac{2(3m+m_0)}{3(m_0+2m)}$$

6-31



解: ∵ 质心的运动为圆周运动, 且相对于O点的角速度为 $\dot{\theta}$

记 $\dot{\theta} = \omega$

则杆的运动可看作: 质心的平动 + 绕质心转动,

绕质心转动的角速度为 $\dot{\theta} = \omega$

又∵ 能量守恒: $mg\cdot\frac{l}{2}\sin\theta = \frac{1}{2}m(\omega\cdot\frac{l}{2})^2 + \frac{1}{2}J\omega^2$

$$\therefore \omega = \sqrt{\frac{3g\sin\theta}{l}}$$

又∵ $v_c = \omega\frac{l}{2} + \omega\frac{l}{2} = \omega l = \sqrt{3gl\sin\theta}$

6-32

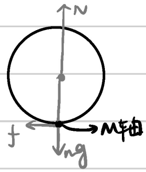
解:(1) ∵ 碰撞: 动量守恒: $mv_0 = mv_1 + mv_2$

∵ 摩擦可忽略 ∴ 碰撞后瞬间角速度不变

又∵ 弹性碰撞: $\frac{1}{2}mv_0^2 + \frac{1}{2}J\omega^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}J\omega^2 + \frac{1}{2}mv_2^2$

$$\therefore v_1 = 0 \quad v_2 = v_0$$

分别滚动时　对M轴：角动量守恒：



$$Jw_1 + RmV_1 = Jw_1' + RmV_1'$$

故 $Jw = Jw_1' + RmV_1'$

∵末状态纯滚动：$V_1' = w_1'R$　　∴解得 $V_1 = \frac{2}{7}V_0$

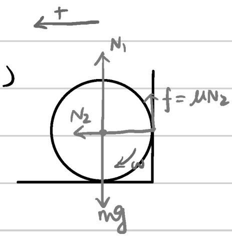
$$\begin{cases} RmV_0 = Jw_2' + RmV_2' \\ w_2'R = V_2' \end{cases} \qquad V_2 = \frac{5}{7}V_0$$

(2) $\Delta E = \frac{1}{2}mV_0^2 + \frac{1}{2}Jw^2 - (\frac{1}{2}mV_1'^2 + \frac{1}{2}Jw_1'^2 + \frac{1}{2}mV_2'^2 + \frac{1}{2}Jw_2'^2)$

$= \frac{7}{10}m \times \frac{20}{49}V_0^2 = \frac{2}{7}mV_0^2$

6-33

解：(1)



根据动量定理：$Ndt = 2mV_0$

角动量定理：$-f\cdot Rdt = Jw_1 - Jw_0$

∴ $-\mu R(Ndt) = Jw_1 - Jw_0$

∴ $w_1 - w_0 = \frac{-\cancel{2}\mu \cancel{R}\cancel{m}V_0}{\frac{2}{5}\cancel{m}R^{\cancel{2}}} = \frac{-5\mu V_0}{R}$

又∵做纯滚动 ∴ $w_0 = \frac{V_0}{R}$　∴ $w_1 = \frac{V_0}{R}(1-5\mu)$

又∵角动量守恒：$RmV_0 - Jw_1 = RmV_末 + Jw_末$　　$V_末 = w_末 R$

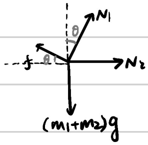
∴ $V_末 = -(\frac{3}{7} + \frac{10}{7}\mu)V_0$ (向左)

(2) $\begin{cases} f \le mg \\ f = \mu N \\ N\Delta t = -2mV_0 \end{cases}$　　∴ $\mu \le \frac{g\Delta t}{2V_0}$

6-40

解：刚体静平衡：$\Sigma F_i = 0$　$\Sigma M_i = 0$

受力分析：



$$\begin{cases} N_2 + N_1\sin\theta = f\cos\theta & ① \\ (m_1+m_2)g = N_1\cos\theta + f\sin\theta & ② \end{cases}$$

力矩：$(\frac{a}{\cos\theta} - l\frac{m_2}{m_1+m_2})\cos\theta(m_1+m_2)g = N_2 a\tan\theta$ ③

又∵ $\frac{a}{\cos\theta} - l\frac{m_2}{m_1+m_2} \geq 0$ ∴ $1+\frac{m_1}{m_2} \geq \frac{l\cos\theta}{a}$

将①② 联立: 解得 $N_2 = \frac{(m_1+m_2)g\cos\theta}{\sin\theta} - N_1\frac{\cos^2\theta}{\sin\theta} - N_1\sin\theta$ ④

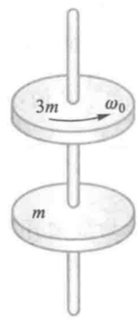
将④代入③ 解得: $N_1 = \frac{m_2 l g\cos^2\theta}{a}$ ⑤

将⑤代入② 并由 $f \leq \mu N$ 解得: $1+\frac{m_1}{m_2} \leq \frac{l}{a}\cos^2\theta(\cos\theta+\mu\sin\theta)$.

又∵ 由①得 $N_1\sin\theta + N_2 \leq \mu N_1\cos\theta$ ∴ $\mu \geq \tan\theta + \frac{N_2}{N_1\cos\theta} \geq \tan\theta$.

故: $\frac{l}{a}\cos\theta < 1+\frac{m_1}{m_2} < \frac{l}{a}\cos^2\theta(\cos\theta+\mu\sin\theta)$

6-8 两个半径均为 R、质量分别为 $3m$ 和 m 的圆盘装在同一轴上，均可绕轴无摩擦地旋转，如题 6-8 图所示. 质量为 $3m$ 的圆盘的初角速度为 ω_0，而另一个圆盘开始时静止. 现将上圆盘放下，使两者相互接触.



题 6-8 图

(1) 若两者之间的摩擦因数为 μ，则需经多少时间两圆盘将以相同角速度旋转?

(2) 试求出此角速度.

(1) $J_1\omega_0=(J_1+J_2)\omega$

$\frac{3}{2}mR^2\omega_0=(\frac{3}{2}m+\frac{m}{2})R^2\omega$

$\omega=\frac{3}{4}\omega_0$

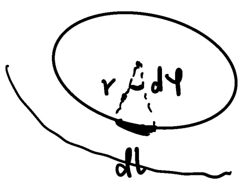
对 3m 的圆盘，规定向上为正

$J\Delta\omega=\int_0^t J\alpha dt=\int_0^t M_f dt$

$M_f=-\int_0^R \frac{\mu 3mg\cdot 2\pi r}{\pi R^2} r dr=-\frac{6\mu mg}{R^2}\frac{r^3}{3}\Big|_0^R=-2\mu mgR$

$J(-\frac{\omega_0}{4})=-2\mu mgRt$ $\quad 2\mu mgRt=\frac{3m\omega_0R^2}{8}$

$t=\frac{3\omega_0R}{16\mu g}$

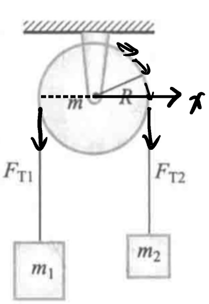


(2) $\omega=\frac{3}{4}\omega_0$

6-9 在如题 6-9 图所示的阿特伍德机中，两物体的质量分别为 m_1 和 m_2，滑轮的半径为 R、质量为 m. 若物体运动时，绳与滑轮之间有相对滑动，两者之间的摩擦因数为 μ. 设绳不可伸长，忽略滑轮轴承处的摩擦. 试求：

(1) m_1 与 m_2 的加速度 a；

(2) 滑轮的角加速度 α.



题 6-9 图

(1) 若 $m_1=m_2$，绳与滑轮均静止，无相对滑动

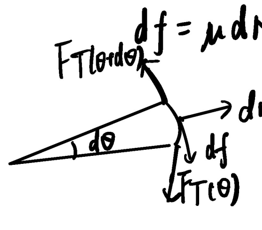
不妨设 $m_1>m_2$

$m_1g-F_{T_1}=m_1a$ ①

$F_{T_2}-m_2g=m_2a$ ②

$F_{T_1}=F_{T_2}+\int df$

$df=\mu dN$



$dN=2F_T(\theta)\sin\frac{d\theta}{2}$
$=F_T(\theta)d\theta$

$\begin{cases}F_T(0)=F_{T_2}\\F_T(\pi)=F_{T_1}\end{cases}$

$$df = \mu F_T d\theta$$

$$F_T(\theta+d\theta) = F_T(\theta) + df$$

$$dF_T = \mu F_T d\theta \qquad \int \frac{dF_T}{F_T} = \int \mu d\theta \Rightarrow \ln F_T = \mu\theta + C_1$$

$$F_T = Ce^{\mu\theta}$$

$$F_T(0) = C = F_{T_2}$$

$$F_T(\theta) = F_{T_2}e^{\mu\theta} \qquad F_T(\pi) = F_{T_2}e^{\mu\pi} = F_{T_1}$$

将 $F_{T_1} = e^{\mu\pi}F_{T_2}$ 代入①②得 $a = \dfrac{m_1 - m_2 e^{\mu\pi}}{m_1 + m_2 e^{\mu\pi}}g$

$$F_{T_2}e^{\mu\theta}R\Big|_0^\pi$$

(2) 由(1)得 $F_{T_2} = \dfrac{2m_1m_2e^{\mu\pi}}{m_1+m_2e^{\mu\pi}}g$

对滑轮，摩擦力的力矩为 $M_f = \int_0^\pi df R = \int_0^\pi \mu F_{T_2}e^{\mu\theta}R\,d\theta$

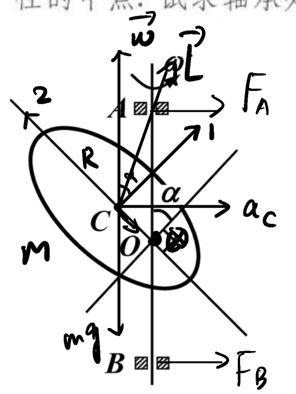
$$= F_{T_2}Re^{\mu\pi} - F_{T_2}R$$

$$= \frac{2m_1m_2e^{\mu\pi}(e^{\mu\pi}-1)}{m_1+m_2e^{\mu\pi}}gR$$

由 $M_f = J\alpha$ 其中 $J = \frac{1}{2}mR^2$

得 $\alpha = \dfrac{4m_1m_2g(e^{\mu\pi}-1)}{mR(m_1+m_2e^{\mu\pi})}$

6-13 在§6-2的例5题中，若竖直轴并非通过盘心，而是通过某一半径的中点．试求轴承处所受水平作用力，设 $OA=OB=l$.



规定逆时针为正

$$a_c = \omega^2 \frac{R}{2}\cos\alpha$$

$$F_A + F_B = ma_c = m\omega^2\frac{R}{2}\cos\alpha$$

圆盘的角动量 $L = \vec{L}_c + \vec{L}'$

$$\vec{L}_c = m\frac{R}{2}\omega\frac{R}{2}\cos\alpha\,\hat{n}_1 = m\frac{R^2}{4}\omega\cos\alpha\,\hat{n}_1$$

$$\vec{\omega} = \vec{\omega}_1\hat{n}_1 + \vec{\omega}_2\hat{n}_2$$

$$= \omega\cos\alpha\,\hat{n}_1 + \omega\sin\alpha\,\hat{n}_2$$

$$\vec{L}' = J_1\omega_1\hat{n}_1 + J_2\omega_2\hat{n}_2$$

$$= \frac{1}{2}mR^2\omega\cos\alpha\,\hat{n}_1 + \frac{1}{4}mR^2\omega\sin\alpha\,\hat{n}_2$$

$$\vec{L} = L_c + L' = \frac{3}{4}mR^2\omega\cos\alpha\,\hat{n}_1 + \frac{1}{4}mR^2\omega\sin\alpha\,\hat{n}_2$$

$$\vec{M} = \frac{d\vec{L}}{dt} = \vec{\omega}\times\vec{L} = (\omega\cos\alpha, \omega\sin\alpha)\times\left(\frac{3}{4}mR^2\omega\cos\alpha, \frac{1}{4}mR^2\omega\sin\alpha\right)$$

$$= -\frac{1}{2}mR^2\omega^2\sin\alpha\cos\alpha$$

解答：$\vec{M}$点力矩垂直纸面向里

$$\vec{M} = -F_A l + F_B l + mg\frac{R}{2}\cos\alpha$$

$$F_A - F_B = \frac{1}{2}m\frac{R^2}{l}\omega^2\sin\alpha\cos\alpha + mg\frac{R}{2l}\cos\alpha$$

$$F_A + F_B = m\omega^2\frac{R}{2}\cos\alpha$$

$mR^2\omega^2\sin\alpha\cos\alpha + mgR\cos\alpha + m\omega^2 lR\cos\alpha$

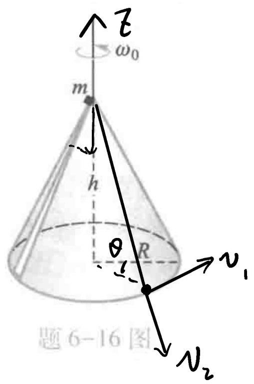
$$F_A = \frac{1}{4}m\frac{R^2}{l}\omega^2\sin\alpha\cos\alpha + \frac{mg}{4l}R\cos\alpha + \frac{1}{4}m\omega^2R\cos\alpha = \frac{mR\cos\alpha(\omega^2R\sin\alpha + \omega^2 l + g)}{4l}$$

$$F_B = \frac{1}{4}m\omega^2R\cos\alpha - \frac{1}{4}m\frac{R^2}{l}\omega^2\sin\alpha\cos\alpha - \frac{mg}{4l}R\cos\alpha = \frac{mR\cos\alpha(\omega^2 l - \omega^2R\sin\alpha - g)}{4l}$$

6-16 一个高为 h、底半径为 R 的圆锥体，可以绕其固定的竖直轴自由旋转. 在其表面沿母线刻有一条光滑的斜槽，如题 6-16 图所示. 开始时，锥体以角速度 ω_0 旋转. 此时，将质量为 m 的小滑块从槽顶无初速地释放. 设在滑块沿槽滑下的过程中始终不脱离斜槽，而锥体绕竖直轴的转动惯量为 J_0. 试问：

(1) 当滑体到达底边时，圆锥体的角速度为多大？

(2) 当滑块到达底边时，滑块相对地面的速度为多大？



题 6-16 图

(1) 对 m 和圆锥体组成的系统.

由于 m 受到的重力平行于 z 轴. $M_z = 0$

系统角动量守恒

小球绕 z 轴转动的转动惯量 $J_1 = mr^2$

当小球落至底端时 $J_1 = mR^2$

$$J_0\omega_0 = (J_0 + mR^2)\omega$$

$$\omega = \frac{J_0\omega_0}{J_0 + mR^2}$$

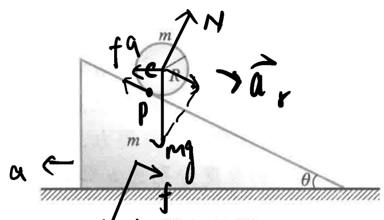
(2) 由机械能守恒

$$mgh + \frac{1}{2}J_0\omega_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}J_0\omega^2$$

$$v^2 = 2gh + \frac{J_0}{m}(\omega_0^2 - \omega^2)$$

$$\Rightarrow v = \sqrt{\frac{J_0(2J_0 + mR^2)R^2}{(J_0 + mR^2)^2}\omega_0^2 + 2gh}$$

6-19 如题 6-19 图所示，一质量为 m、倾角为 θ 的斜面体放置在光滑水平面上，另一质量也为 m、半径为 R 的圆柱体沿斜面无滑动地滚下. 求斜面体的运动加速度.



题 6-19 图

设圆柱体质心为C

$\vec{a}_c = \vec{a}_r + \vec{a}$

其中 $\vec{a}$ 为斜面体的加速度，水平向左．$\vec{a}_r$ 为圆柱体相对于斜面体的加速度

由于圆柱体纯滚动．$\vec{a}_r = R\alpha$

$$fR = J\alpha = \frac{1}{2}mR^2\alpha \qquad \vec{a}_r = \frac{2f}{m}$$

对斜面体 $N\sin\theta - f\cos\theta = ma$

对圆柱体 $mg\sin\theta - f = m(a_r - a\cos\theta)$

$$mg\cos\theta - N = ma\sin\theta$$

$$\Rightarrow \quad a = \frac{\sin2\theta}{4+2\sin^2\theta}g = \frac{2\sin\theta\cos\theta}{6\sin^2\theta+4\cos^2\theta}g = \frac{1}{2\cot\theta+3\tan\theta}g$$

6-20 如题 6-20 图所示，一圆柱体从桌边角处由静止开始滚下，圆柱体与桌边角处的摩擦因数 $\mu=0.25$. 滚下过程中所转过的角度用 θ 表示，试求圆柱体开始相对桌边角滑动时的 θ 值.

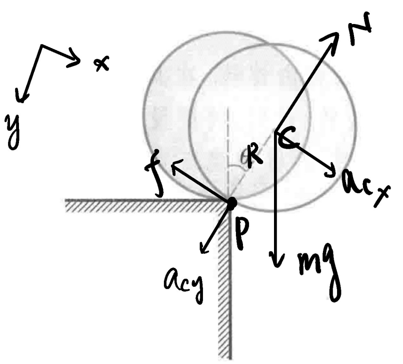
由运动定律，对P点

$$mgR\sin\theta = J\alpha$$

由平行轴定理 $J = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$

$$\alpha = \frac{2g\sin\theta}{3R}$$

即将相对滑动的瞬间，$\begin{cases} a_{cx} = R\alpha = \frac{2}{3}g\sin\theta \\ f = \mu N \end{cases}$



题 6-20 图

对圆柱体

$$\begin{cases} mg\cos\theta - N = ma_{cy} & ① \\ mg\sin\theta - f = ma_{cx} & ② \end{cases}$$

$$a_{cy} = \omega_c^2 R$$

由动能定理，对P点，

$$\int_0^\theta mgR\sin\varphi \, d\varphi = \frac{1}{2}J\omega_c^2 - 0$$

$$\omega_c^2 = \frac{4g}{3R}(1-\cos\theta)$$

联立得 $\Rightarrow \quad 7\cos\theta = 4(\sin\theta+1)$

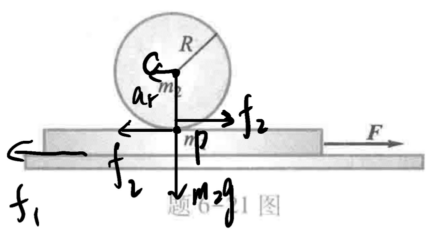
$\Rightarrow \sin\theta = \frac{33}{65}$ 或 $\sin\theta = -1$ (舍)

$\Rightarrow \theta = \arcsin\frac{33}{65}$

6-21 如题 6-21 图所示，质量为 m_1 的板受水平力 F 的作用沿水平面运动，板与水平面之间的摩擦因数为 μ. 板上放着一质量为 m_2、半径为 R 的圆柱体.

（1）若圆柱体在板上的运动为纯滚动，求板的加速度；

（2）为使圆柱体在板上作纯滚动，求 F 的最大值. 设圆柱体与板之间的摩擦因数也是 μ.



题 6-21 图

(1) 设 m_2 的质心为 C. 由受力分析有

$$\begin{cases} f_2 = m_2 a_c \\ F - f_1 - f_2 = m_1 a_1 \end{cases}$$

其中 $f_1 = \mu(m_1+m_2)g$

对 m_2 的质心 C

$$f_2 R = J_2\alpha = \frac{1}{2}m_2R^2\alpha$$

$a_c = a_1 - a_r$ ，a_r 为相对加速度

由于圆柱体在板上的运动为纯滚动，$a_r = R\alpha$

代入得 $f_2 = \frac{m_2}{3}a_1$

$\Rightarrow$ 木板加速度 $a_1 = \dfrac{F-\mu(m_1+m_2)g}{m_1+\frac{m_2}{3}} = \dfrac{3[F-\mu(m_1+m_2)g]}{3m_1+m_2}$

(2) 为使圆柱体在板上做纯滚动，则 f_2 为静摩擦力

$$f_2 = \frac{m_2}{3}a_1 \le \mu m_2 g$$

$$F \le \mu(4m_1+2m_2)g$$

F 的最大值为 $\mu(4m_1+2m_2)g$

6-27 质量为 m 的线轴上绕有细线，如题 6-27 图所示，其外半径为 R，绕线部分的半径为 r. 设其绕轴线的转动惯量为 J. 今将绕线的一端以一恒力 F 拉它，F 与水平面成 θ 角. 设 $F\sin\theta<mg$.

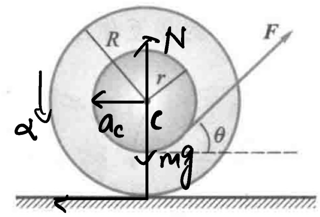
(1) 为使线轴向后（即与 F 的水平分量方向相反）作纯滚动，

(i) 求 θ 的范围；

(ii) 若 θ 满足 (i) 中条件，则摩擦因数 μ 至少应为多大？

(iii) 求出线轴质心运动的加速度.

(2) 为使线轴向前作纯滚动，同样求出 (1) 中的三个问题.



题 6-27 图

(1) (i) 对线轴受力分析，要使线轴向后做纯滚动

$$f > F\cos\theta, \quad f - F\cos\theta = ma_c$$

$$a_c = R\alpha$$

对 c 使用转动定律，规定垂直纸面向外为正，

$$Fr - fR = J\alpha$$

$$\Rightarrow f = \frac{mRr + J\cos\theta}{J + mR^2} F$$

$$f - F\cos\theta > 0 \Rightarrow \cos\theta < \frac{r}{R} \quad \theta > \arccos\frac{r}{R}$$

(ii) 纯滚动意味着 f 为静摩擦力, $f \le \mu N$

$$N + F\sin\theta = mg \quad N = mg - F\sin\theta$$

$$\mu \ge \frac{f}{N} = \frac{mRr + J\cos\theta}{(mg - F\sin\theta)(J + mR^2)} F$$

(iii) $$a_c = \frac{f}{m} - \frac{F}{m}\cos\theta = F\left(\frac{mRr + J\cos\theta}{m(J + mR^2)} - \frac{J\cos\theta + mR^2\cos\theta}{m(J + mR^2)}\right) = \frac{FR(r - R\cos\theta)}{J + mR^2}$$

(2) 线轴向前作纯滚动, $F\cos\theta < f$ a_c 向右. $a_c = R\alpha$

$$F\cos\theta - f = ma_c$$

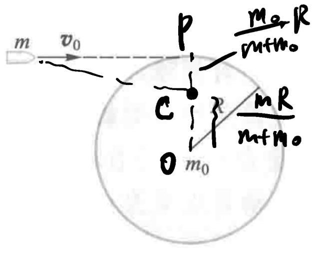
$$Fr - fR = J\alpha$$

解得 (i) $\theta < \arccos\frac{r}{R}$

(ii) $$\mu \ge \frac{F(J\cos\theta + mRr)}{(mg - F\sin\theta)(J + mR^2)}$$

(iii) $$a_c = \frac{FR(R\cos\theta - r)}{J + mR^2}$$

6-30 质量为 m 的子弹，以速度 v_0 水平射入放在光滑水平面上质量为 m_0、半径为 R 的圆盘的边缘，并留在该处，v_0 的方向与射入处的半径垂直，如题 6-30 图所示. 试就以下两种情况：(1) 盘心有一竖直的光滑固定轴；(2) 圆盘是自由的，求子弹射入后圆盘系统总动能之比 E_{k1}/E_{k2}.



题 6-30 图

(1) 由角动量守恒. $mRv_0 = m\omega R^2 + J_0\omega_1$

$J_0 = \frac{1}{2}m_0R^2 \quad mv_0R = m\omega_1R^2 + \frac{1}{2}m_0\omega R^2$

$$\omega = \frac{2mv_0}{(2m+m_0)R}$$

$$E_{k1} = \frac{1}{2}m(\omega_1R)^2 + \frac{1}{2}J_0\omega_1^2 = \frac{m^2v_0^2}{m_0+2m}$$

(2) 若圆盘是自由的

设圆盘和子弹组成的系统为C点. 则 $\overline{CP} = \frac{m_0}{m+m_0}R$ $\overline{CO} = \frac{m}{m+m_0}R$

对C点，由角动量守恒，规定垂直纸面向里为正

$$m v_0 \overline{CP} = J\omega$$

$$J = \frac{1}{2}m_0R^2 + m_0\overline{CO}^2 + m\overline{CP}^2 = \frac{m_0(3m+m_0)}{2(m+m_0)}R^2$$

$$\Rightarrow \omega = \frac{2mv_0}{(3m+m_0)R}$$

由动量守恒，$mv_0 = (m+m_0)v_c$

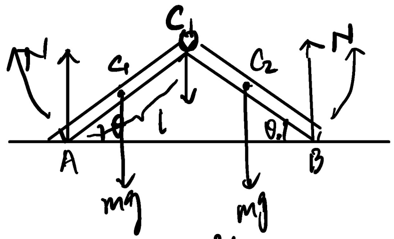
$$\Rightarrow v_c = \frac{m}{m+m_0}v_0$$

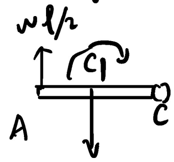
$$E_{k2} = \frac{1}{2}(m+m_0)v_c^2 + \frac{1}{2}J\omega^2$$

$$= \frac{1}{2}\frac{m^2v_0^2}{m+m_0} + \frac{m^2m_0}{(m+m_0)(3m+m_0)}v_0^2$$

$$\frac{E_{k1}}{E_{k2}} = \frac{\frac{m^2 \cancel{v_0^2}}{m_0+2m}}{\frac{1}{2}\frac{m^2}{m+m_0}\cancel{v_0^2} + \frac{m^2m_0}{(m+m_0)(3m+m_0)}\cancel{v_0^2}} = \frac{\frac{1}{m_0+2m}}{\frac{1}{2(m+m_0)} + \frac{m_0}{(m+m_0)(3m+m_0)}} = \frac{2(m_0+3m)}{3(m_0+2m)}$$

6-31 两根质量均为 m、长度均为 l 的相同均质细杆 AC 与 CB，两杆的 C 端用一光滑的铰链相连. 将两杆分开一定角度，让 A、B 端与光滑地面接触，并使两杆均在竖直平面内. 开始时，两杆与地面间的夹角均为 θ. 现无初速地释放两杆，问两杆着地时 C 点的速度.





对细杆AC或CB 有 $v_{c_1}=v_{c_2}$. $J_{c_1}=J_{c_2}$

由机械能守恒

$$2mg\frac{l}{2}\sin\theta=\frac{1}{2}2mv_{c_1}^2+2\frac{1}{2}J_{c_1}\omega^2 \quad ①$$

$$J_{c_1}=\frac{1}{12}ml^2$$

$$v_A=v_{c_1}-\frac{1}{2}l\omega=0 \qquad v_{c_1}=\frac{1}{2}l\omega$$

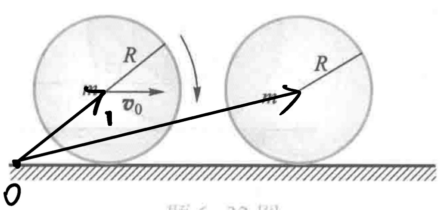
代入①得 $(\omega l)^2=3gl\sin\theta$

$$v_C=v_{c_1}+\frac{1}{2}\omega l=\omega l=\sqrt{3gl\sin\theta}$$

6-32 一质量为 m、半径为 R 的均质球 1 在水平面上作纯滚动，球心速度为 v_0，与另一完全相同的静止球 2 发生对心碰撞，如题 6-32 图所示. 设碰撞时各接触面间的摩擦均可以忽略，碰撞是弹性的.

（1）碰撞后，各自经过一段时间，两球开始作纯滚动，求出此时各球心的速度；

（2）求此过程中系统机械能的损失.



题 6-32 图

(1) 碰撞时,两小球受到的作用力均经过圆心

两小球的角动量各自守恒.

碰撞后 $\omega_{左}=v_0/R \qquad \omega_{右}=0$

由动量守恒. $mv_0=mv_{左}+mv_{右}$

由动能守恒

$$\frac{1}{2}mv_0^2+\frac{1}{2}J\left(\frac{v_0}{R}\right)^2=\frac{1}{2}mv_{左}^2+\frac{1}{2}J\left(\frac{v_0}{R}\right)^2+\frac{1}{2}mv_{右}^2$$

$$\Rightarrow \begin{cases} v_{左}=0 \\ v_{右}=v_0 \end{cases}$$

选定水平面上一定点O. 对两小球. 由角动量守恒.

以碰撞后到达到纯滚动

$$\omega'_{左}=v'_{左}/R \qquad \omega'_{右}=v'_{右}/R \qquad J=\frac{2}{5}mR^2$$

$$\begin{cases} mv_{左}R+J\frac{v_0}{R}=mv'_{左}R+J\left(\frac{v'_{左}}{R}\right) \\ mv_{右}R=mv'_{右}R+J\left(\frac{v'_{右}}{R}\right) \end{cases}$$

$$v'_{左}=\frac{2}{7}v_0 \qquad v'_{右}=\frac{5}{7}v_0$$

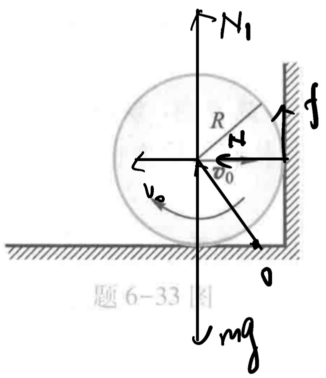
$$\Delta E_k=\frac{1}{2}mv_0^2+\frac{1}{2}J\left(\frac{v_0}{R}\right)^2-\frac{1}{2}m(v'_{左})^2-\frac{1}{2}J\left(\frac{v'_{左}}{R}\right)^2-\frac{1}{2}m(v'_{右})^2-\frac{1}{2}J\left(\frac{v'_{右}}{R}\right)^2$$

$$=\frac{2}{7}mv_0^2$$

6-33 如题 6-33 图所示，质量为 m、半径为 R 的弹性球在水平面上作纯滚动，球心速度为 v_0，与一粗糙的墙面发生碰撞后，以相同的球心速度反弹. 设球与墙面间的摩擦因数为 μ，在碰撞时球与水平面间的摩擦可以忽略.

（1）碰撞后，球经过一段时间开始作纯滚动，求出此时的球心速度；

（2）若球与墙面间的碰撞时间为 Δt，为使碰撞时球不会跳起，则摩擦因数应满足什么关系？设碰撞中的相互作用力为恒力.



题 6-33 图

(1) 规定向右为正，逆时针为正

碰撞时

由动量定理 $N\Delta t = mv_0 - m(-v_0)$

由冲量矩定理 $fR\Delta t = J\omega - J(-\frac{v_0}{R})$

$f = \mu N$

$J = \frac{2}{5}mR^2$

$\Rightarrow \omega = (5\mu+1)\frac{v_0}{R}$

从碰撞后到达到纯滚动

选取地面上一定点为轴，f 经过轴，角动量守恒

$mRv_0 + J\omega = mv'R + J\frac{v'}{R}$

$v' = \frac{10\mu+3}{7}v_0$

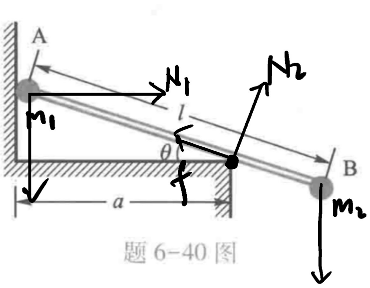
(2) $N_1 + f = mg$

$f = \mu N = \mu\frac{2mv_0}{\Delta t}$

$N_1 = mg - f \geq 0$

$\mu \leq \frac{g\Delta t}{2v_0}$

6-40 质量分别为 m_1 和 m_2 的两小球 A 和 B 用长为 l 的轻杆相连，静置于题 6-40 图所示位置；A 球与光滑墙面接触，杆斜搁在桌角上，已知杆与桌角之间的摩擦因数为 μ，墙与桌角之间的水平距离为 a，试问：为了使此系统能保持平衡，则已知量 m_1、m_2、μ、θ、a、l 之间应满足什么条件？



题 6-40 图

由 $\sum F_x = 0$ 得

$N_1 + N_2\sin\theta = f\cos\theta$

由 $\sum F_y = 0$

$N_2\cos\theta + f\sin\theta = m_1 g + m_2 g$

选墙角点为支点，垂直纸面向外为正

$$M = m_1 g a - N_1 a\tan\theta - m_2 g(l\cos\theta - a) = 0$$

$$N_2 = m_2 g \frac{l}{a}\cos^2\theta$$

$$0 < f = \frac{m_1 + m_2}{\sin\theta} g - m_2 g \frac{l}{a}\frac{\cos^3\theta}{\sin\theta} \leq \mu N_2$$

$$m_2 l\cos^2\theta(\mu\sin\theta + \cos\theta) \geq (m_1 + m_2)a$$

$$N_1 = \frac{m_1 g a - m_2 g(l\cos\theta - a)}{a\tan\theta} > 0$$

$$\Rightarrow (m_1 + m_2)a > m_2 l\cos\theta$$

综上 $$m_2 l\cos\theta < (m_1 + m_2)a < m_2 l\cos^2\theta(\mu\sin\theta + \cos\theta)$$

6-8 角动量守恒： $I_0\omega_0=(I+I_0)\omega$ 其中 $I_0=\frac{1}{2}\cdot 3mR^2$， $I=\frac{1}{2}\cdot mR^2$

得最后 $\omega=\frac{3}{4}\omega_0$

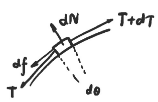
摩擦力力矩： $M=\int_0^R \mu\sigma 2\pi r g\cdot r\,dr$ 其中 $\sigma=\frac{3m}{\pi R^2}$

得 $M=2\mu mgR$

对 m， $M\cdot t=I(\omega-0)$

故 (1) $t=\frac{3R\omega_0}{16\mu g}$ (2) $\omega=\frac{3}{4}\omega_0$

6-9 系统对轴角动量 $L=m_1vR+m_2vR+I\omega$ 其中 $I=\frac{1}{2}mR^2$



对轮上一个质元： $\begin{cases} dN=Td\theta \\ dT=df=\mu\cdot dN \end{cases}$ $\Rightarrow$ $\frac{dT}{T}=\mu d\theta$ 积分得 $F_{T_1}=F_{T_2}\cdot e^{\mu\pi}$

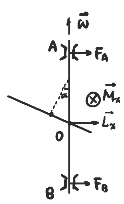
对 m_1, m_2 $\begin{cases} m_1g-F_{T_1}=m_1a \\ F_{T_2}-m_2g=m_2a \end{cases}$

由三式得 $a=\frac{m_1-m_2e^{\mu\pi}}{m_1+m_2e^{\mu\pi}}g$

对轮： $F_{T_1}R-F_{T_2}R=I\alpha$ 其中 $I=\frac{1}{2}mR^2$

得： $\alpha=\frac{4m_1m_2(e^{\mu\pi}-1)g}{m(m_1+m_2e^{\mu\pi})R}$

6-13



质心运动定律，水平方向 $F_A+F_B=m\frac{R}{2}\cos\alpha\cdot\omega^2$

O点主轴惯量张量 $I=\begin{bmatrix}\frac{3}{4}mR^2 & & \\ & \frac{1}{4}mR^2 & \\ & & \frac{1}{2}mR^2\end{bmatrix}$， $\vec{\omega}=\begin{bmatrix}\cos\alpha \\ \sin\alpha \\ 0\end{bmatrix}\omega$

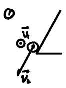
垂直轴角动量分量 $L_x=\frac{3}{4}mR^2\cdot\omega\cos\alpha\cdot\sin\alpha-\frac{1}{4}mR^2\omega\sin\alpha\cdot\cos\alpha=\frac{1}{2}mR^2\omega\sin\alpha\cos\alpha$

$M_x=F_Al-F_Bl-mg\frac{R}{2}\cos\alpha=L_x\omega$

由上述各式得 $F_A=\frac{1}{4}mgR\cos\alpha\left[\frac{\omega^2}{g}\left(1+\frac{R}{l}\sin\alpha\right)+\frac{1}{l}\right]$

$F_B=\frac{1}{4}mgR\cos\alpha\left[\frac{\omega^2}{g}\left(1-\frac{R}{l}\sin\alpha\right)-\frac{1}{l}\right]$

6-16 机械能守恒 $\frac{1}{2}J_0\omega_0^2 + mgh = \frac{1}{2}J_0\omega^2 + \frac{1}{2}m(v_1^2 + v_2^2)$ ①



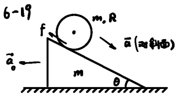
其中 $v_1 = \omega R$ ②

角动量守恒 $J_0\omega_0 = J\omega + mRv_1$ ③

(1) 由②③得 $\omega = \frac{J_0}{J_0 + mR^2}\omega_0$

(2) 结合①知 $|\vec{v}| = \sqrt{v_1^2 + v_2^2} = \sqrt{2gh + \frac{J_0\omega_0^2}{m}\left[1 - \left(\frac{J_0}{J_0 + mR^2}\right)^2\right]}$

6-19



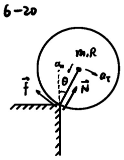
在斜面系下, $mg\sin\theta + ma_0\cos\theta - f = ma$

$fR = I\cdot\beta$ 其中 $I = \frac{1}{2}mR^2$, $\beta = \frac{a}{R}$

系统水平方向合外力为0: $m(a\cos\theta - a_0) - ma_0 = 0$

由上述各式得 $a_0 = g\cdot\frac{\sin\theta\cos\theta}{3 - 2\cos^2\theta}$

6-20



动能定理 $mgR(1-\cos\theta) = \frac{1}{2}I\dot{\theta}^2$

其中 $I = mR^2 + \frac{1}{2}mR^2 = \frac{3}{2}mR^2$

得 $\dot{\theta}^2 = \frac{4g}{3R}(1-\cos\theta)$ $mg\left(\cancel{\cos\theta} - \frac{4}{3} + \frac{4}{3}\cos\theta\right.$

质心运动: $mg\cos\theta - N = ma_n = mR\dot{\theta}^2$

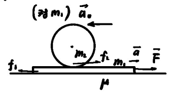
质心系下: $fR = \frac{1}{2}mR^2\cdot\ddot{\theta} = \frac{1}{2}mR^2\cdot\frac{d\dot{\theta}^2}{2d\theta} = \frac{1}{2}mR^2\cdot\frac{2g}{3R}\sin\theta$

得 $N = \frac{1}{3}(7\cos\theta - 4)mg$

$f = \frac{1}{3}mg\sin\theta$

不滑: $\frac{f}{N} = \frac{\sin\theta}{7\cos\theta - 4} \leq \mu = 0.25$ 得 $\theta \leq \arccos\frac{56}{65}$

6-21



(1) m_1系下对m_2: $\begin{cases} f_2R = \frac{1}{2}m_2R^2\cdot\beta_2 & 其中\ \beta_2 = \frac{a_0}{R} \\ f_2 - m_2a = -m_2a_0 \end{cases}$ 得 $a_0 = \frac{2}{3}a$

对系统: $F - f_1 = m_1a + m_2(a - a_0)$ 其中 $f_1 = \mu(m_1 + m_2)g$

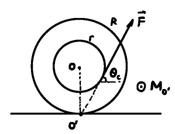
得 $a = \frac{F - \mu(m_1 + m_2)g}{m_1 + \frac{1}{3}m_2}$

(2) 由(1)知 $f_2 = \frac{1}{3}m_2 a = m_2 \cdot \frac{F-\mu(m_1+m_2)g}{3m_1+m_2} \leq \mu m_2$

得 $F \leq 2\mu(2m_1+m_2)g$

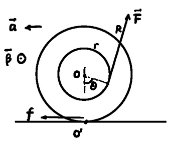
6-27

(1) 对瞬心 O' $(\vec{a}_{o'} \perp \vec{OO'})$ 力矩(向外为正方向) $M_{o'}$ 如图 $\theta_c = \arccos\frac{r}{R}$



(i) 当 $\theta > \arccos\frac{r}{R}$ 时 $M_{o'} > 0$ 向后动

$$M_{o'} = F\cdot(r - R\cos\theta) = I\cdot\beta \quad 其中\ I = J + mR^2,\ \beta = \frac{a}{R}$$



对柱: $f - F\cos\theta = ma$

得 $f = \frac{mRr + J\cos\theta}{J + mR^2}F$ 要求 $f \leq \mu N$, 其中 $N = mg - F\sin\theta$

(ii) 于是 $\mu \geq \frac{(mRr + J\cos\theta)F}{(J+mR^2)\cdot(F - mg\sin\theta)}$

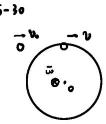
(iii) $a = \frac{F(r - R\cos\theta)R}{J + mR^2}$

(2) 同理 (i) 当 $\theta < \arccos\frac{r}{R}$ 时向前动

(ii) $\mu \geq \frac{(mRr + J\cos\theta)F}{(J+mR^2)\cdot(F - mg\sin\theta)}$

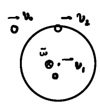
(iii) $a = -\frac{F(r - R\cos\theta)R}{J + mR^2}$

6-30



(i) 有轴: 角动量守恒: $mv_0R = I\omega$, $I = \frac{1}{2}m_0R^2 + mR^2$

$$E_{k1} = \frac{1}{2}I\omega^2 = \frac{m^2v_0^2}{m_0 + 2m}$$



(ii) 无轴 动量守恒: $mv_0 = mv_2 + m_0v_1$

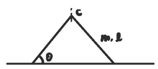
对碰点角动量守恒: $0 = m_0Rv_1 - \frac{1}{2}m_0R^2\omega$

其中 $\omega = \frac{v_2 - v_1}{R}$

$$E_{k2} = \frac{1}{2}mv_2^2 + \frac{1}{2}m_0v_1^2 + \frac{1}{2}\cdot\frac{1}{2}m_0R^2\omega^2$$

$$= \frac{3mv_0^2}{2(m_0+3m)}$$

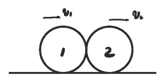
故 $E_{k1}/E_{k2} = \frac{2(m_0+3m)}{3(m_0+2m)}$

6-31 

机械能守恒：$mgl\sin\theta = 2\cdot\frac{1}{2}I\omega^2$ 其中，$\omega=\frac{v_c}{l}$，$I=\frac{1}{3}ml^2$

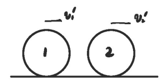
得 $v_c=\sqrt{3gl\sin\theta}$

6-32 (1) 动量守恒，$mv_0 = mv_1+mv_2$



碰后瞬间

完全弹性：$e=\frac{v_2-v_1}{v_0}=1$　　得 $\begin{cases} v_2=v_0 \\ v_1=0 \end{cases}$



稳态.

对地上某点角动量守恒：

球1　$mv_1R+I\omega_0 = mv_1'R+I\omega_1'$　　其中 $I=\frac{2}{5}mR^2$

球2　$mv_2R = mv_2'R+I\omega_2'$

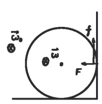
且 $v_1'=R\omega_1'$，$v_2'=R\omega_2'$

得 $v_1'=\frac{2}{7}v_0$，$v_2'=\frac{5}{7}v_0$

(2) 初态动能 $E_0=\frac{1}{2}mv_0^2+\frac{1}{2}I\omega_0^2$

末态动能 $E=\frac{1}{2}mv_1'^2+\frac{1}{2}I\omega_1'^2+\frac{1}{2}mv_2'^2+\frac{1}{2}I\omega_2'^2$

$\Delta E = E_0-E=\frac{2}{7}mv_0^2$

6-33 

(1) 碰撞 Δt 时间，$F\Delta t = mv_0 - m(-v_0)$

若碰后仍转动：$fR\Delta t = I(-\omega)-I(-\omega_0)$　其中 $f=\mu F$，$I=\frac{2}{5}mR^2$，$\omega_0=\frac{v_0}{R}$

得 $\omega R = v_0(1-5\mu)$　要求 $\mu<\frac{1}{5}$

对地上某点角动量守恒：$mv_0R - I\omega = mv'R+I\omega'$　其中 $v'=\omega R$，以离墙为正

得 $v=\frac{3+10\mu}{7}v_0$

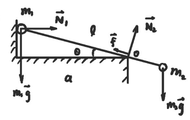
若 $\mu\geq\frac{1}{5}$ 碰后 $\omega=0$　得 $v=\frac{5}{7}v_0$

故 $v=\begin{cases} \frac{3+10\mu}{7}v_0 & \mu<\frac{1}{5} \\ \frac{5}{7}v_0 & \mu\geq\frac{1}{5} \end{cases}$　（如果无此条件，μ 极大时 v 也极大，这是荒谬的。）

(2) 默认为(1)中 $\mu<\frac{1}{5}$ 的情况，$f=\mu F=\frac{2\mu mv_0}{\Delta t}\leq mg$

故 $\mu\leq\frac{g\Delta t}{2v_0}$

6-40



对O点： $m_1 g a - N_1 a \tan\theta - m_2 g (l\cos\theta - a) = 0$

对m_1点， $N_2 \dfrac{a}{\cos\theta} - m_2 g\, l\cos\theta = 0$

竖直 $f\sin\theta + N_2\cos\theta = m_1 g + m_2 g$

要求 $\dfrac{f}{N_2} = \dfrac{m_1+m_2}{m_2}\dfrac{a}{l\sin\theta\cos^2\theta} - \tan\theta \le \mu$

$$N_1 = \frac{m_1 g a - m_2 g l\cos\theta + m_2 g a}{a\tan\theta} \ge 0$$

整理得 $\dfrac{l}{a}\cos\theta \le \dfrac{m_1+m_2}{m_2} \le \dfrac{l}{a}(\mu + \tan\theta)\sin\theta\cos^2\theta$ （当然由 $\dfrac{l}{a}\cos\theta \le \dfrac{l}{a}(\mu+\tan\theta)\sin\theta\cos^2\theta$ 知有必要条件 $\mu \ge \cot\theta$）