第五章作业

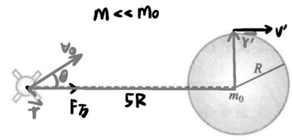
5-6

解：$\because F_万 = \dfrac{Gm_0m}{(5R)^2}$ 指向星球中心，故角动量守恒：

$$5R \cdot v_0 \cdot \sin\theta = R \cdot v' \quad \therefore v' = 5\sin\theta \cdot v_0$$

又$\because$机械能守恒：$\frac{1}{2}mv_0^2 + \left(-\dfrac{Gm_0m}{5R}\right) = \frac{1}{2}mv'^2 + \left(-\dfrac{Gm_0m}{R}\right)$

$$\therefore \sin\theta = \frac{1}{5}\sqrt{1+\frac{8Gm_0}{5Rv_0^2}} \quad \therefore \theta = \arcsin\frac{1}{5}\sqrt{1+\frac{8Gm_0}{5Rv_0^2}}$$



题 5-6 图

5-7

解：对 A、B、沙子整体而言角动量守恒：（以逆时针为正向）

$$r_A \cdot \omega_0 r_A(m_A+m) = r_A \cdot \omega_A r_A(m_A+m') + r_B \cdot \omega_B r_B(m_B+m-m')$$

又$\because m' = m - \lambda t$

又$\because$ A的角动量守恒，故 $\omega_A = \omega_0$

$$\therefore \omega_B = \frac{r_A^2\omega_0(m_A+m) - r_A^2\omega_0(m_A+m-\lambda t)}{r_B^2(m_B+\lambda t)} = \frac{\omega_0 r_A^2(\lambda t)}{r_B^2(m_B+\lambda t)} \quad \left(t \le \frac{m}{\lambda}\right)$$

当 $t > \frac{m}{\lambda}$ 时 $\quad \omega_B = \dfrac{\omega_0 r_A^2 m}{r_B^2(m_B+m)}$

5-10

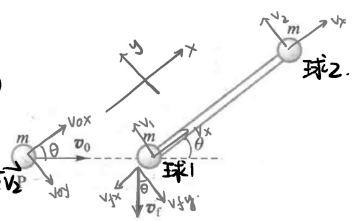
解：(1) 弹性碰撞：$m\vec{v}_{0x} = m\vec{v}_{fx} + 2m\vec{v}_x$

$$m\vec{v}_{0y} = m\vec{v}_{fy} + m\vec{v}_1 + m\vec{v}_2$$

$$\cancel{\tfrac{1}{2}}mv_0^2 = \cancel{\tfrac{1}{2}}mv_f^2 + \cancel{\tfrac{1}{2}}\cdot 2mv_x^2 + \cancel{\tfrac{1}{2}}m(v_1^2+v_2^2)$$

化简：$\frac{\sqrt{2}}{2}\cancel{m}v_0 = -\frac{\sqrt{2}}{2}\cancel{m}v_f + 2\cancel{m}\vec{v}_x \qquad v_0 = -v_f + 2\sqrt{2}\,\vec{v}_x$

$-\frac{\sqrt{2}}{2}\cancel{m}v_0 = -\frac{\sqrt{2}}{2}\cancel{m}v_f + \cancel{m}\vec{v}_1 + \cancel{m}\vec{v}_2 \qquad \therefore -v_0 = -v_f + \sqrt{2}\,\vec{v}_1 + \sqrt{2}\,\vec{v}_2$



题 5-10 图

又$\because$对球2：只受杆的作用力 $\therefore v_2 = 0$

$$\therefore v_f = \frac{v_0 + \sqrt{1+7v_0^2}}{7} \approx 0.55v_0$$

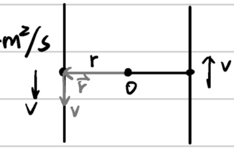
(2) $\quad v_1 = \dfrac{v_f - v_0}{\sqrt{2}} \approx -0.32v_0$

以球2为参考点：$\omega_1 = \dfrac{v_1}{l} = \dfrac{0.32v_0}{l}$

又$\because$刚体转动角速度与基点无关 $\therefore \omega = \dfrac{0.32v_0}{l}$

5-12

解：(1) 抓住绳子前：$L_前 = 2\vec{r} \times m\vec{v} = 2\times5\times60\times6.5 = 3.9\times10^3 kg\cdot m^2/s$



抓住绳子水平方向受力仅有沿绳T，

则角动量守恒：$L_后 = L_前 = 3.9\times10^3 kg\cdot m^2/s$

(2) 角动量守恒：$L = 2\cdot\vec{r'} \times m\vec{v'} = L_前$ $\therefore v' = 13 m/s$

(3) 以某个人为参考系：$T = \mu\frac{(2v')^2}{2r'} = \frac{60\times60}{60+60} \times \frac{4\times169}{5} = 4056N$

(4) $2W = E_{k末} - E_{k初}$

$= \frac{1}{2}\cdot 2m\cdot v'^2 - \frac{1}{2}\cdot 2mv^2$

$W = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2 = 3802.5J$

5-14

解：角动量守恒：$\vec{r}\times m\vec{v_0} = \vec{r'}\times m\vec{v}$

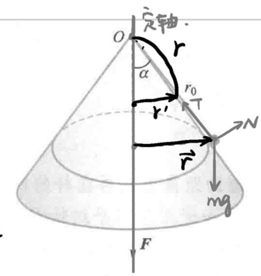
即：$r_0\cdot \cancel{\sin\alpha}\cdot v_0 = r\cdot\cancel{\sin\alpha}\cdot v$ $\therefore v = \frac{r_0 v_0}{r}$

脱离锥面临界条件：$\cancel{m}\frac{v^2}{\sin\alpha r} = \cancel{m}g\tan\alpha$

$\therefore r = 0.5m$. $\therefore v = 1.5m/s$

动能定理：$W - mg(r_0\cos\alpha - r\cos\alpha) = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

$W = 3.6J$



题 5-14 图

5-16

解：(1) 作椭圆运动，运动的周期为：1920-1870 = 50年.

(2) $\because r_A m_A = r_B m_B$ 且 $r_A : r_B = 2:5$ $\therefore m_A : m_B = 5:2$

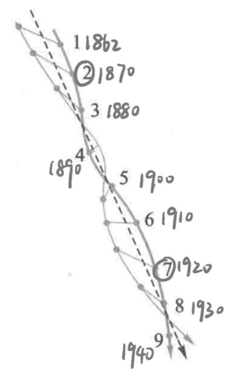
设 $m_A = 5m$. $m_B = 2m$

$\therefore \mu = \frac{m_A\cdot m_B}{m_A+m_B} = \frac{10m^2}{7m} = \frac{10}{7}m$

$\therefore \frac{G5m\cdot 2m}{a^2} = \mu(\frac{2\pi}{T})^2 a = \frac{10m}{7}\cdot\frac{4\pi^2}{250(a)}\cdot a.$

$\frac{Gm_s\cdot \cancel{m_地}}{r^2} = \cancel{m_地}(\frac{2\pi}{1(a)})^2\cdot r$

代入后解得 $m = \frac{1}{7\times250}\cdot\frac{a^3}{r^3}\cdot m_s = \frac{(20.4)^3}{7\times250}m_s \approx 0.485m_s$

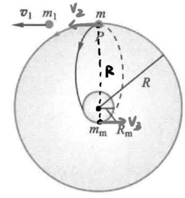


题 5-16 图

$\therefore m_A = 5m \approx 2.43m_s$. $m_B = 2m \approx 0.97m_s$

5-17

解：发射物体前：$\frac{Gm_m \cancel{m}}{R^2} = \frac{\cancel{m} v_0^2}{R}$ $v_0 = \sqrt{\frac{Gm_m}{R}}$



题 5-17 图

发射物体：$m \cdot v_0 = m_1 v_1 + (m-m_1) v_2$.

角动量守恒：$v_2 \cdot R = v_3 \cdot R_m$.

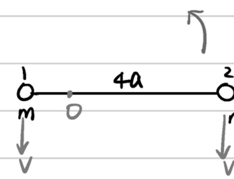
能量守恒：$\frac{1}{2}\cancel{m'} v_2^2 - \frac{Gm_m \cancel{m'}}{R} = \frac{1}{2}\cancel{m'} v_3^2 - \frac{Gm_m \cancel{m'}}{R_m}$

$$v_2 = \sqrt{\frac{2Gm_m R_m}{R(R+R_m)}}$$

$$\therefore v_1 = \frac{mv_0 - (m-m_1)v_2}{m_1} = \frac{m}{m_1}\sqrt{\frac{Gm_m}{R}} - \frac{m-m_1}{m_1}\sqrt{\frac{2Gm_m R_m}{R(R+R_m)}}$$

5-20

解(1) 以O为固定点，

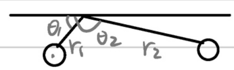


球1、球2角动量守恒：$L_1 = mva$ $L_2 = -3mva$

$\therefore L_1 = mva = mv_{1\perp} \cdot r_1$ $L_2 = -3mva = -mv_{2\perp} r_2$

能量守恒：$\cancel{\frac{1}{2}} 2\cancel{m} v^2 = \cancel{\frac{1}{2}}\cancel{m} v_{1\perp}^2 + \cancel{\frac{1}{2}}\cancel{m} v_{2\perp}^2 + \cancel{\frac{1}{2}} \cdot 2\cancel{m} v_{//}^2$ ①

最大距离时 $v_{//} = 0$



$2v^2 = (\frac{va}{r_1})^2 + (\frac{3va}{4a-r_1})^2$ $\therefore$ 解得 $r_1 \approx 1.653a$

(2) $-ma_{//} = T + m\frac{v_\perp^2}{r_1}$ $\therefore T = -m\cdot\ddot{r}_1 - m\cdot\frac{v^2a^2}{r_1^3}$

由①式：$2v^2 = \frac{v^2a^2}{r_1^2} + \frac{9v^2a^2}{(4a-r_1)^2} + 2\dot{r}_1^2$

$$0 = \frac{-2v^2a^2}{r_1^3}\cdot\cancel{\dot{r}_1} + \frac{18v^2a^2}{(4a-r_1)^3}\cdot\cancel{\dot{r}_1} + 4\cancel{\dot{r}_1}\cdot\ddot{r}_1 \quad \therefore \ddot{r}_1 = \frac{1}{4}\left(\frac{2v^2a^2}{r_1^3} - \frac{18v^2a^2}{(4a-r_1)^3}\right)$$

$$\therefore T = m\frac{v^2a^2}{2r_1^3} + \frac{m9v^2a^2}{2(4a-r_1)^3} + m\frac{v^2a^2}{r_1^3} = \frac{mv^2a^2}{2r_1^3} + \frac{9mv^2a^2}{2(4a-r_1)^3}$$

$\because$ 当 $r_1 \approx 1.46a$ 时 T_{min} $\therefore$ 此时 $T_{min} = 0.4352\frac{mv^2}{a}$

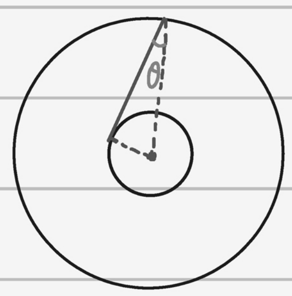
5-6

角动量守恒： $5mRv_0\sin\theta = mRv$ ①

机械能守恒： $\frac{1}{2}mv_0^2 - \frac{Gm_0m}{5R} = \frac{1}{2}mv^2 - \frac{Gm_0m}{R}$ ②

联立①②得： $\theta = \arcsin\left(\frac{1}{5}\sqrt{1+\frac{8Gm_0}{5v_0^2R}}\right)$

5-7



$\sin\theta = \frac{r_A}{r_B}$ 设A筒中沙子质量为 m_s

对A筒： $L_A = (m_A+m_s)r_A^2(-\omega_A)$

对B筒： $L_B = (m_B+m-m_s)r_B^2\omega_B$

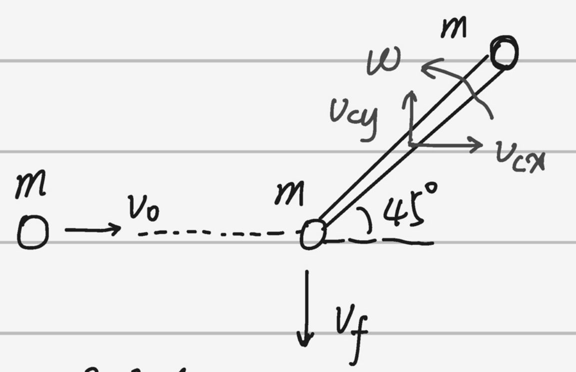
$\frac{dL_A}{dt} = \frac{dm_s}{dt}\cdot r_A^2(-\omega_A) + (m_A+m_s)r_A^2\frac{d(-\omega_A)}{dt} = (-\lambda)\cdot r_A^2(-\omega_A) \Rightarrow \frac{d\omega_A}{dt} = 0.$

得 $\omega_A = \omega_0$ 不变.

由角动量守恒： $L_B = (m-m_s)r_A^2\omega_0$

又由 $m-m_s = \lambda t$，得： $\omega_B = \frac{\lambda t r_A^2}{(m_B+\lambda t)r_B^2}\omega_0 \quad (t \le \frac{m}{\lambda})$. 当 $t > \frac{m}{\lambda}$ 时, $\omega_B = \frac{mr_A^2}{(m_B+m)r_B^2}\omega_0$

5-10.



由动量守恒, $v_{cx} = \frac{1}{2}v_0$, $v_{cy} = \frac{1}{2}v_f$ ①，②

由角动量守恒, $2mv_{cx}\cdot(\frac{\sqrt{2}}{4}l) = 2mv_{cy}\cdot(\frac{\sqrt{2}}{4}l) + \frac{1}{2}ml^2\omega$ ③

由机械能守恒, $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + m(v_{cx}^2+v_{cy}^2) + \frac{1}{4}ml^2\omega^2$ ④

联立① -④ 得：

$$\begin{cases} v_f = \frac{2\sqrt{2}+1}{7}v_0 \approx 0.547v_0 \\ \omega = \frac{3\sqrt{2}-2}{7}\frac{v_0}{l} \approx 0.320\frac{v_0}{l} \end{cases}$$

5-12

(1). $L_0 = 2m\cdot(\frac{1}{2}l)\cdot v_0 = 3900\ J\cdot s$

(2). 相距5m时, $m(\frac{1}{2}l)v_0 = m(\frac{1}{2}l')v_1 \Rightarrow v_1 = 13\ m/s$

(3). 拉力提供向心力. $T = m\frac{v_1^2}{\frac{1}{2}l'} = 4056\ N$

(4). $W = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = 3802.5J$

5-14

角动量守恒（对圆锥轴）：

$$m r_0 \sin\alpha\, v_0 = m r \sin\alpha\, v \quad \Rightarrow \quad v = \frac{r_0}{r} v_0$$

脱离时，支持力 $N=0$

$$mg\sin\alpha = m\frac{v^2}{r\sin\alpha}\cos\alpha \quad \Rightarrow \quad v^2 = gr\sin\alpha\tan\alpha$$

联立得：
$$\begin{cases} r = \sqrt[3]{\dfrac{r_0^2 v_0^2}{g\sin\alpha\tan\alpha}} = 0.5\,m \\ v = \sqrt[3]{g r_0 v_0 \sin\alpha\tan\alpha} = 1.5\,m/s \end{cases}$$

做功：$W = \frac{1}{2}m(v^2 - v_0^2) + mg(r_0 - r)\cos\alpha = 3.6\,J$

5-16

(1)、由图判断，相对作椭圆运动，运动周期约为50年

(2)、设 m_A，m_B 的单位均为 m_s，距离单位为 AU，时间单位为年 (year)，

则该单位制下，$G=1$，以下式子中不含 G，且 $\omega = \frac{1}{T}$ 而非 $\frac{2\pi}{T}$

二体运动动力学方程：$\frac{m_A m_B}{a^2} = \frac{m_A m_B}{m_A + m_B} a\omega^2 \Rightarrow \omega = \sqrt{\frac{m_A + m_B}{a^3}}$

故 $\frac{1}{T} = \sqrt{\frac{m_A + m_B}{a^3}}$ ①

$m_A : m_B = 5:2$ ②

联立得：$m_A = 2.426\ (m_s)$ $m_B = 0.970\ (m_s)$

5-17.

设喷后速度为 v_0，落至月球速度为 v.

角动量守恒：$m' R v_0 = m' R_m v$ ①

能量守恒：$\frac{1}{2}m' v_0^2 - \frac{G m_m m'}{R} = \frac{1}{2}m' v^2 - \frac{G m_m m'}{R_m}$ ②

联立①②得：$v_0 = \sqrt{\frac{2Gm_m R_m}{R(R+R_m)}}$

初始速度：$v_{原} = \sqrt{\frac{Gm_m}{R}}$

动量守恒：$m v_{原} = m_1 v_1 + (m - m_1) v_0$

得 $v_1 = \sqrt{\frac{Gm_m}{R}}\left[\frac{m}{m_1}\left(1 - \sqrt{\frac{2R_m}{R+R_m}}\right) + \sqrt{\frac{2R_m}{R+R_m}}\right]$

5-18

初始状态：$\frac{Gmm'}{r_0^2}=\frac{mh^2}{r_0^3} \Rightarrow h$ 为比角动量，$h=\sqrt{Gm'r_0}$

给予径向冲量 I 后，有 $\dot{r}(0)=\frac{I}{m}=v_{r0}$，为小量.

$\dot{r}=0$ 时，$\frac{mh^2}{2r^2}-\frac{Gm'm}{r}=\frac{mh^2}{2r_0^2}-\frac{Gm'm}{r_0}+\frac{1}{2}mv_{r0}^2$

$\Rightarrow r_{1,2}=\frac{Gm'\pm hv_{r0}}{Gm'-v_{r0}^2 r_0}r_0 \approx r_0\left(1\pm\frac{hI}{Gmm'}\right)$，故有 $r_2-r_0\approx r_0-r_1\approx\frac{I}{m}\sqrt{\frac{r_0^3}{Gm'}}$

对径向运动分析：

$$m\left(\ddot{r}-\frac{h^2}{r^3}\right)=-\frac{Gm'm}{r^2}$$

代入 $r=r_0+\Delta r$ 得：

$$\Delta\ddot{r}-\frac{h^2}{r_0^3}\left(1-\frac{3\Delta r}{r_0}\right)=-\frac{Gm'}{r_0^2}\left(1-\frac{2\Delta r}{r_0}\right)$$

$$\Delta\ddot{r}+\frac{Gm'}{r_0^3}\Delta r=0$$

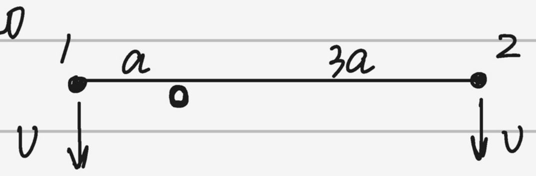
故 Δr 的变化为简谐振动方程，代初始条件得：

$\Delta r=\frac{I}{m}\sqrt{\frac{r_0^3}{Gm'}}\sin\left(\sqrt{\frac{Gm'}{r_0^3}}t\right)$，

而恰有 绕转角速度 $\omega_0=\sqrt{\frac{Gm'}{r_0^3}}$，$\Delta r=\frac{I}{m\omega_0}\sin\omega_0 t$，

所以 r_1、r_2 的取到恰相差 $180°$，故轨迹为偏心圆

5-20



(1)、相碰之后，1、2 的比角动量分别为：

$h_1=av$，$h_2=3av$.

设拉力为T，则有 $\begin{cases}-T=m\left(\ddot{r}_1-\frac{h_1^2}{r_1^3}\right) & ①\\ -T=m\left(-\ddot{r}_1-\frac{h_2^2}{(4a-r_1)^3}\right) & ②\end{cases}$

联立①②得：$\begin{cases}\ddot{r}_1=\frac{1}{2}\left(\frac{h_1^2}{r_1^3}-\frac{h_2^2}{(4a-r_1)^3}\right)\\ T=\frac{1}{2}m\left(\frac{h_1^2}{r_1^3}+\frac{h_2^2}{(4a-r_1)^3}\right)\end{cases}$

由机械能守恒：$\frac{mh_1^2}{2r_1^2}+\frac{mh_2^2}{2(4a-r_1)^2}=2\cdot\frac{1}{2}mv^2$ ③

由③得 $r_1=1.653a$，即为最大值

(2)、由 $\frac{dT}{dt}=\frac{3}{2}m\left(-\frac{h_1^2}{r_1^4}+\frac{h_2^2}{(4a-r_1)^4}\right)\dot{r}_1=0$

解得 $r_1=a$, $1.464a$ 或 $1.653a$,

经验证，$r_1=1.464a$ 时，T 最小.

$T_{min}=0.435\frac{mv^2}{a}$

5-6 m 角动量守恒： $m v_0 \cdot 5R \sin\theta = m v R$ （其中 v 为掠过时速率）

机械能守恒： $\frac{1}{2} m v_0^2 - \frac{G m_0 m}{5R} = \frac{1}{2} m v^2 - \frac{G m_0 m}{R}$

由上述二式得： $\theta = \arcsin\left(\frac{1}{5}\sqrt{1+\frac{8 G m_0}{5 R v_0^2}}\right)$

5-7 t 时刻两桶(带沙)质量为 $m_A + m - \lambda t$ 和 $m_B + \lambda t$

系统对轴角动量守恒： $(m_A + m) r_A^2 \omega_0 = (m_A + m - \lambda t) r_A^2 \omega_A + (m_B + \lambda t) r_B^2 \omega_B$

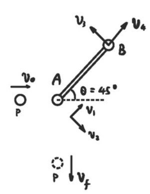
t 时 $\Delta m = \lambda \Delta t$ 砂到 B 瞬间角动量守恒， $(m_B + \lambda t) r_B^2 \omega_B + \Delta m r_A^2 \omega_A = (m_B + \lambda t + \Delta m) r_B^2 (\omega_B + \Delta\omega_B)$

$\Delta t \to 0$ 得 $\frac{d\omega_B}{dt} = \lambda \cdot \frac{\frac{r_A^2}{r_B^2}\omega_A - \omega_B}{m_B + \lambda t}$

结合上式得： $\begin{cases} \omega_A = \omega_0 \\ \omega_B = \frac{r_A^2}{r_B^2} \frac{\lambda t}{m_B + \lambda t} \omega_0 \end{cases} \quad (0 \le t < \frac{m}{\lambda})$

$t \geq \frac{m}{\lambda}$ 时沙漏尽，故综上： $\begin{cases} \omega_A = \omega_0 \\ \omega_B = \begin{cases} \frac{r_A^2}{r_B^2} \frac{\lambda t}{m_B + \lambda t} \omega_0 & (0 \le t < \frac{m}{\lambda}) \\ \frac{r_A^2}{r_B^2} \frac{m}{m_B + m} \omega_0 & (t \geq \frac{m}{\lambda}) \end{cases} \end{cases}$

5-10



如图，系统对 B 点角动量守恒： $m v_0 l \sin\theta = m v_f l \cos\theta + m v_2 l$

对 A 点角动量守恒： $0 = m v_3 \Rightarrow v_3 = 0$

系统机械能守恒： $\frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} m (v_1^2 + v_2^2) + \frac{1}{2} m (v_3^2 + v_4^2)$

沿杆动量守恒： $m v_0 \cos\theta = m v_1 + m v_4 - m v_f \sin\theta$

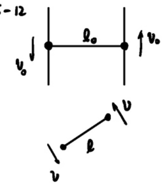
沿杆运动： $v_1 = v_4$

在 B 平动系下看： $\omega = \frac{v_2 + v_3}{l}$

由上述各式得： (1) $v_f = \frac{1 + 2\sqrt{2}}{7} v_0$

(2) $\omega = \frac{3\sqrt{2} - 2}{7} \frac{v_0}{l}$

5-12



(1) 对绳中点角动量 $L = mv_0\frac{l_0}{2} + mv_0\frac{l_0}{2} = mv_0 l_0 = 3900\ kg\cdot m^2\cdot s^{-1}$

(2) 距离为 l 时，由对称性，绳中点不动

对绳中点角动量守恒： $L = mv\cdot\frac{l}{2} + mv\frac{l}{2} = mvl = mv_0 l_0$

得 $v = \frac{l_0}{l}v_0 = 13m/s$

(3) 默认此时两人未拉动绳

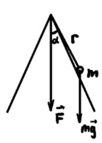
$$T = \frac{mv^2}{l/2} = 4056\ N$$

(4) 记每人作功 W，由动能关系.

$$2\times\frac{1}{2}mv_0^2 + 2W = 2\times\frac{1}{2}mv^2$$

得 $W = \frac{1}{2}mv_0^2\left(\frac{l_0^2}{l^2} - 1\right) = 3802.5\ J$

5-14



m对轴角动量守恒　$mv_0 r_0\sin\alpha = mvr\sin\alpha$

脱离时临界：

$$T\sin\alpha = m\frac{v^2}{r\sin\alpha}$$

$$T\cos\alpha = mg$$

由动能原理： $\frac{1}{2}mv_0^2 + W = mg(r_0 - r)\cos\alpha + \frac{1}{2}mv^2$

得 $$W = \frac{1}{2}m\left[\left(gr_0v_0\frac{\sin^2\alpha}{\cos\alpha}\right)^{\frac{2}{3}} - v_0^2\right] + mg\left[r_0 - \left(\frac{v_0^2r_0^2\cos\alpha}{g\sin^2\alpha}\right)^{\frac{1}{3}}\right]\cos\alpha$$

代入数据得　$W = 3.6\ J$

5-16 (1) 显然为椭圆运动，周期约50年

(2) 由质心距离比知 $\frac{m_A}{m_B} = \frac{5}{2}$ ，约化质量 $\mu = \frac{m_Am_B}{m_A + m_B}$

$\frac{Gm_Am_B}{a^2} = \mu a\left(\frac{2\pi}{T}\right)^2$ 与太阳系地球比较， $\frac{GM_sm_e}{r^2} = m_er\left(\frac{2\pi}{T_e}\right)^2$ （因为 $M_s \gg m_e$）

得 $m_A + m_B = M_s\left(\frac{a}{r}\right)^3\left(\frac{T}{T_e}\right)^{-2}$ 从而 $\begin{cases} m_A \approx 2.4256\ M_s \\ m_B \approx 0.9702\ M_s \end{cases}$

5-17. 记发射前后船速分别为 v_0, v

发射动量守恒： $m v_0 = (m-m_1)v + m_1 v_1$ （如图 $v>0$）

船角动量守恒 (m') $m' v R = m' v' R_m$

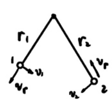
机械能守恒： $\frac{1}{2}m'v^2 - \frac{G m_m m'}{R} = \frac{1}{2}m'v'^2 - \frac{G m_m m'}{R_m}$

初态： $\frac{G m_m m}{R^2} = m\frac{v_0^2}{R}$

由上述各式得 $v_1 = \frac{m}{m_1}\sqrt{\frac{G m_m}{R}} - \left(\frac{m}{m_1}-1\right)\sqrt{2G m_m\left(\frac{1}{R}-\frac{1}{R+R_m}\right)}$

$$= \frac{m}{m_1}\sqrt{\frac{G m_m}{R}}\left[1-\left(1-\frac{m_1}{m}\right)\sqrt{\frac{2R_m}{R+R_m}}\right]$$

5-20



1,2 分别角动量守恒： $\begin{cases} m v a = m v_1 r_1 \\ m v\cdot 3a = m v_2 r_2 \end{cases}$ $(r_1+r_2=4a)$

机械能守恒： $2\times\frac{1}{2}mv^2 = \frac{1}{2}m(v_1^2+v_r^2)+\frac{1}{2}m(v_2^2+v_r^2)$

记绳中张力为T： $\begin{cases} T = m\frac{v_1^2}{r_1} - m\frac{dv_r}{dt} \\ T = m\frac{v_2^2}{r_2} + m\frac{dv_r}{dt} \end{cases}$

由上述各式得 $\left(\frac{dr_1}{dt}\right)^2 = v^2\left[1-\frac{a^2}{2}\left(\frac{1}{r_1^2}+\frac{9}{(4a-r_1)^2}\right)\right]$

令 $\frac{dr_1}{dt}=0$ 不难得 r_1最大时： $r_1 \approx 1.653\,a$

且 $T = \frac{1}{2}mv^2a^2\left[\frac{1}{r_1^3}+\frac{9}{(4a-r_1)^3}\right]$，对 r_1 求导得 $\frac{dT}{dr_1} = \frac{3}{2}ma^2\left[-\frac{1}{r_1^4}+\frac{9}{(4a-r_1)^4}\right]$ 不难得 $r_1 \approx 1.4641\,a$ 时

T 最小且 $T \approx 0.435256\cdot\frac{mv^2}{a}$