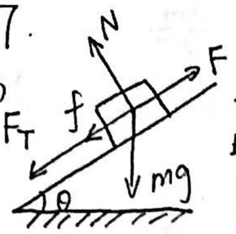
4-7.

(1)



明显地，随着F_T的增大，直至$F=F_T+f+mg\sin\theta$时，物体减速，故f始终做负功 随后静止

且 $f=\mu N=\mu mg\cos\theta$

[系统机械能守恒，开始重力势能为0

$\because F\cdot x-\frac{1}{2}kx^2-mgx\sin\theta=\mu mg\cos\theta\cdot x$

由功能关系 $\therefore 2F-kx-2mg\sin\theta=2\mu mg\cos\theta$

$\therefore x=\dfrac{2(F-mg(\sin\theta+\mu\cos\theta))}{k}$

$\therefore W=Fx=2F\cdot\dfrac{F-mg(\sin\theta+\mu\cos\theta)}{k}$

(2) $F=F_T+f+mg\sin\theta$

$=kx+\mu mg\cos\theta+mg\sin\theta$

$\therefore x=\dfrac{F-mg(\sin\theta+\mu\cos\theta)}{k}$

由动能定理：$\frac{1}{2}mv_m^2-0=Fx-\frac{1}{2}kx^2-mgx\sin\theta-\mu mgx\cos\theta$

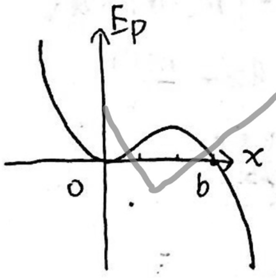
$=x(F-mg(\sin\theta+\mu\cos\theta)-\frac{1}{2}kx)$

$=\frac{1}{2}\cdot\dfrac{[F-mg(\sin\theta+\mu\cos\theta)]^2}{k}$

$\therefore v_m=\dfrac{1}{\sqrt{mk}}[F-mg(\sin\theta+\mu\cos\theta)]$

4-15. (1) $F=-\dfrac{\partial E_p}{\partial x}=ax(3x-2b)$

(2)



(3) 易见平衡位置为$x=0$和$x=\frac{2b}{3}$处

且$\left.\dfrac{\partial^2E_p}{\partial x^2}\right|_{x=0}=2ab>0$，稳定

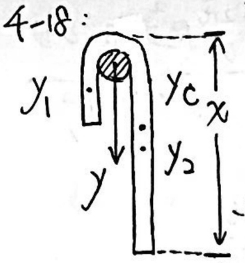
$\left.\dfrac{\partial^2E_p}{\partial x^2}\right|_{x=\frac{2b}{3}}=-2ab<0$，不稳定

(4) 若v_0向x轴负方向，显然不可能达到$-\infty$处，一定会在之后以相同速率v_0向x轴正方向运动；

若v_0向x轴正方向，只需突破势垒即可，即 $\frac{1}{2}mv_0^2>E_{p极大}=\frac{4ab^3}{27}$

$\therefore$ 当 $v_0\in\left[-\sqrt{\frac{8ab^3}{27}},\sqrt{\frac{8ab^3}{27}}\right]$ 时，达不到$+\infty$处

4-18：



设左端重心y_1，右端重心y_2，绳的质心为y_c。

$\therefore y_2=\frac{x}{2}$，$y_1=\frac{2l-x}{2}$

$\therefore y_c\cdot 2l=y_2x+y_1\cdot(2l-x)$

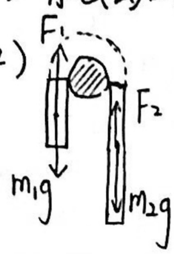
$\therefore y_c=\frac{1}{2}\dfrac{x^2+(2l-x)^2}{2l}=\dfrac{x^2-2xl+2l^2}{2l}$

由动能定理：$\frac{1}{2}mv_x^2=m(y_c-\frac{l}{2})\cdot g$

$\therefore v_x=(x-l)\sqrt{\frac{g}{l}}$

(1) 脱离时，$x=2l$，$v_{末}=\sqrt{gl}$

(2)



设绳加速度为a_x，速度为v_x

$\therefore l\le x\le 2l$时，$a_x=\frac{dv_x}{dt}$，设轴给绳F_1与F_2的力

$\therefore m_1a_x=F_1-m_1g+F_T$

$m_2a_x=m_2g-F_2+F_T$

删 $\therefore F_1+F_2=(m_1+m_2)g+(m_1-m_2)a_x$

$=mg-(\frac{x}{2l}-\frac{2l-x}{2l})\cdot m\cdot\frac{dv_x}{dt}$

$=mg-\frac{x-l}{l}m\cdot\frac{dv_x}{dt}$

由 $V_x=(x-l)\cdot\sqrt{\frac{g}{l}}$，知

$$\frac{dV_x}{dt}=\frac{d(x-l)}{dt}\cdot\sqrt{\frac{g}{l}}+(x-l)\frac{d(\sqrt{\frac{g}{l}})}{dt}$$

$$=\sqrt{\frac{g}{l}}\frac{dx}{dt}[=\sqrt{\frac{g}{l}}\cdot V_x]=\sqrt{\frac{g}{l}}\cdot 2V_x$$

$$=\frac{2(x-l)g}{l}$$

$\because F_1+F_2=mg-mg\cdot\frac{2(x-l)^2}{l^2}$

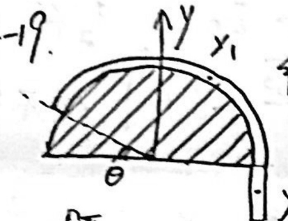
$$=-\frac{2x^2-4xl+l^2}{l^2}mg$$

删

∴ 以y轴为正方向，$F=-\frac{2x^2-4xl+l^2}{l^2}mg$

讨论：明显地，当 $x=\frac{2+\sqrt{2}}{2}l$ 时，$F=0$，即若绳子自由下落至此处，轴钉受力为0，此时绳脱离轴钉，并在之后不再与轴钉接触。

4-19.



当绳端与圆形中心连线与水平面夹角为θ时，

显然 $x=R\theta$

$y_2=-\frac{R\theta}{2}$

$$y_1=\frac{\int_\theta^\pi y\cdot\rho_l\,dl}{\int_\theta^\pi \rho_l\cdot dl}\underset{令\begin{cases}x=R\cos\theta\\ y=R\sin\theta\end{cases}}{=}\frac{\int_\theta^\pi R^2\sin\theta\cdot d\theta}{R(\pi-\theta)}$$

$$=\frac{-R^2\cos\theta\Big|_\theta^\pi}{R(\pi-\theta)}=R\cdot\frac{1+\cos\theta}{\pi-\theta}$$

质心为 y_c，则 $y_c\cdot\pi R=y_2\cdot R\theta+y_1\cdot R(\pi-\theta)$

$\therefore y_c=-\frac{R\theta^2}{2\pi}+\frac{R(1+\cos\theta)}{\pi}$

$\therefore y_{c0}=\frac{2R}{\pi}$

由动能定理：$\frac{1}{2}mV_0^2=mg(-y_c+y_{c0})$

$\therefore \frac{1}{2}mV^2=mgR\cdot\frac{2-2\cos\theta+\theta^2}{2\pi}$

$\therefore V=\sqrt{\frac{gR}{\pi}}\cdot\sqrt{2+\theta^2-2\cos\theta}$

$\because V=\frac{dx}{dt}=\frac{d(R\theta)}{dt}=R\frac{d\theta}{dt}$

$\therefore \frac{d\theta}{dt}=\sqrt{\frac{g}{\pi R}}\sqrt{2+\theta^2-2\cos\theta}$

$\therefore a=\frac{dv}{dt}=\sqrt{\frac{gR}{\pi}}\cdot\frac{2\theta+2\sin\theta}{2\sqrt{2+\theta^2-2\cos\theta}}\cdot\frac{d\theta}{dt}=\frac{g}{\pi}(\theta+\sin\theta)$

(1) 将 $\theta=\pi$ 代入，$V=\sqrt{\frac{gR}{\pi}}\cdot\sqrt{4+\pi^2}$

(2) 将 $\theta=\frac{\pi}{3}$ 代入，$V=\sqrt{\frac{gR}{\pi}}\cdot\sqrt{1+\frac{\pi^2}{9}}$

$$a=\frac{g}{\pi}\cdot\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)$$

4-18.(2). 补充

由变质量问题方程：

$$m\frac{dv}{dt}=F+V_{相对}\cdot\frac{dm}{dt}$$

以左部的绳为主体，$V_{相对}=V_x[-(-V_x)=2V_x]$

$[m_1=\frac{2l-x}{2l}\cdot m]$

$\therefore -m_1\cdot a_x=F_1+m_1g+2V_x\cdot\frac{dm_1}{dt}$

$\frac{dm_1}{dt}=\frac{d\rho_l\cdot l_1}{dt}=\rho_l\cdot\frac{dl_1}{dt}=-\rho_l\cdot V_x=-\frac{mV_x}{2l}$

$\therefore -F_1=m_1g-\frac{mV_x^2}{2l}+m_1a_x$

以右部的绳为主体时，$V_{相对}=-V_x$

$\therefore m_2a_x=F_2+m_2g-2V_x\cdot\frac{dm_2}{dt}$

$\therefore -F_2=m_2g-\frac{mV_x^2}{2l}-m_2a_x$

$\therefore -F_1-F_2=(m_1+m_2)g+(m_1-m_2)a_x-\frac{mV_x^2}{l}$

$\therefore -F=-\frac{2x^2-4xl+l^2}{l^2}$

利用体系动量定理

$mg-F=\frac{\Delta p}{\Delta t}$，记线密度为 ρ_l

$\therefore p(t)=m_2\cdot V_x-m_1\cdot V_x$

$=(x-l)\cdot\sqrt{\frac{g}{l}}\cdot\frac{x-l}{l}\cdot m$

$p(t+\Delta t)=(m_2+\Delta m)\cdot(V_x+\Delta V)-(m_1-\Delta m)(V_x+\Delta V)$

$=p(t)+2\Delta m\cdot V_x+(m_2-m_1)\cdot\Delta V$

其中 $\Delta m=\rho_l\cdot\Delta l=\frac{m}{2l}\cdot\Delta l$

$\therefore mg-F=2\cdot\frac{mV_x\Delta l}{2l\cdot\Delta t}+(m_2-m_1)\cdot\frac{\Delta V}{\Delta t}$

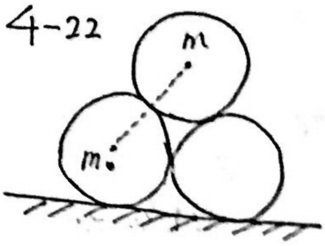
$=\frac{mV_x^2}{l}+(m_2-m_1)\cdot a_x$

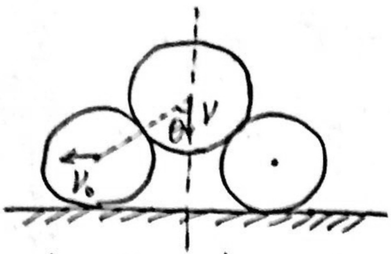
$\therefore F=mg+(m_1-m_2)a_x-\frac{mV_x^2}{l}$

$=-\frac{2x^2-4xl+l^2}{l^2}$

讨论与前相同

4-22





由动能定理：$mg\Delta h=\frac{1}{2}mv^2+2\times\frac{1}{2}m_0v_0^2$

$\Delta h=2R\cdot(\cos30^\circ-\cos\theta)$

$\therefore mgR(\sqrt{3}-2\cos\theta)=\frac{1}{2}mv^2+m_0v_0^2$

由相对运动关系，沿径向速度相等

即 $v\cdot\cos\theta=v_0\cdot\sin\theta\Rightarrow v_0=\frac{v}{\tan\theta}$

$\therefore mgR(\sqrt{3}-2\cos\theta)=v^2\cdot\left(\frac{m}{2}+\frac{m_0}{\tan^2\theta}\right)$

$\therefore v=\sqrt{gR}\left[\frac{\sqrt{3}-2\cos\theta}{\frac{1}{2}+\frac{\cos^2\theta}{\sin^2\theta}\frac{m_0}{m}}\right]^{\frac{1}{2}}$

$=\sqrt{gR}\cdot\frac{2(\sqrt{3}-2\cos\theta)\sin^2\theta}{\sin^2\theta+2\cos^2\theta}$

$=\sqrt{gR}\cdot\frac{2(\sqrt{3}-2\cos\theta)\cdot(1-\cos^2\theta)}{1+\cos^2\theta}$

分离时，两球A、B间无压力，则B受力平衡，以B为参考系（惯性系），则A做圆周运动

$v'=v\cdot\sin\theta+v_0\cdot\cos\theta=\frac{v}{\sin\theta}$

重力在径向的分量提供向心力

$\therefore mg\cos\theta=m\frac{v'^2}{2R}=m\frac{v^2}{2R\sin^2\theta}$

$\therefore 2gR\cos\theta\cdot(1-\cos^2\theta)=v^2=gR\cdot\left(\frac{\sqrt{3}-2\cos\theta}{\frac{1}{2}+\frac{\cos^2\theta}{\sin^2\theta}\frac{m_0}{m}}\right)$

∴代入 $\theta=\arccos\frac{\sqrt{3}}{3}$

$\therefore 2\cdot\frac{\sqrt{3}}{3}\cdot\frac{2}{3}=\frac{\frac{\sqrt{3}}{3}}{\frac{1}{2}+\frac{1}{2}\frac{m_0}{m}}$

$\therefore \frac{m}{m_0}=2$

4-24：系统的总重力与浮力的合力为0，故质心不移动

$\therefore v=g\cdot\frac{t}{2}$

抛出后，由动量守恒：$mv+m_0v_0=0$

$\therefore v_0=\frac{mv}{m_0}=\frac{m}{m_0}\frac{gt}{2}$

$\therefore W=\frac{1}{2}m_0v_0^2+\frac{1}{2}mv^2$

$=\frac{1}{8}mg^2t^2\left(1+\frac{m}{m_0}\right)$

4-29 (1) 最大时：$m_2u=(m_1+m_2)\cdot v_0$

∴机械能守恒：

$\frac{1}{2}m_2u^2=\frac{1}{2}(m_1+m_2)\cdot v_0^2+\frac{1}{2}kx^2$

$\therefore x=\sqrt{\frac{m_1m_2u^2}{(m_1+m_2)k}}$

(2) 不受外力，质心以恒定速度 $v_c=\frac{m_2}{m_1+m_2}\cdot u$ 运动

∴ m_2 在质心系中做简谐振动

易见 $\frac{x_2}{x_1}=\frac{m_1}{m_2}$ ∴ $x_2=\frac{m_1}{m_1+m_2}\cdot l$

∴ $x_2k_c=l\cdot k$ ∴ $k_c=\frac{m_1+m_2}{m_1}\cdot k$

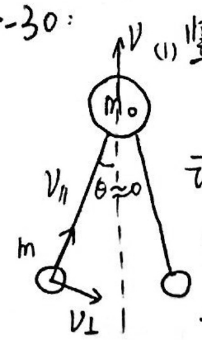
∴ $t=\frac{T}{4}=\frac{1}{4}\cdot2\pi\sqrt{\frac{m_2}{k_c}}=\frac{\pi}{2}\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}$

(3) $x_{\text{架}}=x_c+\Delta x_1=v_c\cdot t-x\cdot\frac{m_2}{m_1+m_2}$

$=\frac{m_2}{m_1+m_2}\cdot\left(u\cdot\frac{\pi}{2}\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}-u\cdot\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}\right)$

$=\frac{m_2u}{m_1+m_2}\cdot\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}\cdot\left(\frac{\pi}{2}-1\right)$

4-30：(1) 竖直方向动量守恒：



$m_0 v_0 = m_0 v + 2m\cdot v_{\parallel}$

动能定理：

$\frac{1}{2} m_0 v_0^2 = \frac{1}{2} m_0 v^2 + 2\times\frac{1}{2} m(v_\parallel^2+v_\perp^2)$

且由沿绳速度相同

$v_\parallel = v$

∴ $v=\frac{m_0}{m_0+2m}\cdot v_0$

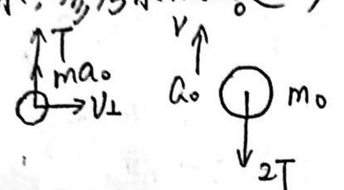
$v_m^2 = v_\parallel^2+v_\perp^2 = \frac{m_0v_0^2-m_0v^2}{2m} = \frac{2(m+m_0)m_0}{(m_0+2m)^2}v_0^2$

∴ $v_m=\sqrt{\frac{2(m+m_0)m_0}{(m_0+2m)^2}}\cdot v_0$

(2) ∴ $v_\perp^2 = v_m^2 - v_\parallel^2 = \frac{m_0}{m+2m}\cdot v_0^2$

以m_0为参考系，参考系以a_0运动

对m来说：



$T+ma_0 = m\frac{v_\perp^2}{a}$

对m_0：$2T=m_0a_0$ ∴ $a_0=\frac{2T}{m_0}$

∴ $T\cdot\frac{m_0+2m}{m_0} = \frac{m}{a}\cdot\frac{m_0}{m_0+2m}\cdot v_0^2$

∴ $T=\frac{m_0^2 m}{(m_0+2m)^2\cdot a}\cdot v_0^2$

(3) 系统不受外力，故质心以恒定速率

$v_c=\frac{m_0}{m_0+2m}\cdot v_0$运动

相碰时，质心与m_0距离为 $\Delta x = a\cdot\frac{2m}{2m+m_0}$

∴ $v_c\cdot T = x-\Delta x$

∴ $\frac{m_0v_0}{m_0+2m}\cdot T = x - a\frac{2m}{m_0+2m}$

∴ $(m_0+2m)x = m_0v_0T+2ma$

4-32 可将飞船-火星质心系看作火心系

故飞船以相对速率u发射

$\frac{1}{2} m_{船} u^2 - \frac{Gmm_{船}}{R} = \frac{1}{2} m_{船}\cdot v_0^2$

脱离火星引力后以 $v'=v_0+v$ 飞出太阳系

故 $\frac{1}{2} m_{船}\cdot(v_0+v)^2 - \frac{Gm_sm_{船}}{r}=0$

∴ $v_0+v=\sqrt{\frac{2Gm_s}{r}}$

而 $m\frac{v^2}{r}=G\frac{m_sm}{r^2}$

∴ $v=\sqrt{\frac{Gm_s}{r}}$

∴ $v_0=(\sqrt{2}-1)v$

∴ $u=\sqrt{\frac{2Gm}{R}+v_0^2}=\sqrt{\frac{2Gm}{R}+(3-2\sqrt{2})\cdot v^2}$

$=11184.3\,m/s$

4-37 代公式

$v_1=\frac{m-em}{m+m}\cdot v_{10}+\frac{m(1+e)}{m+m}\cdot v_{20}$

$=\frac{1}{40}v_{10}+\frac{39}{40}v_{20}$

$v_2=\frac{m-em}{m+m}\cdot v_{20}+\frac{m(1+e)}{m+m}\cdot v_{10}$

$=\frac{1}{40}v_{20}+\frac{39}{40}v_{10}$

∴ $u_1=v_1-v_2=\frac{19}{20}(v_{20}-v_{10})=-\frac{19}{20}u_0$

∴ $\Delta E_{损}=(1-e^2)\frac{1}{2}\frac{m_1m_2}{m_1+m_2}\cdot u^2$

$=\frac{1}{2}\cdot\frac{39}{400}\cdot\frac{m}{2}\cdot u^2=\frac{39}{1600}mu^2$

∴ $\sum\Delta E_{损}=\frac{39}{1600}\cdot m\cdot\left(1+\frac{19}{20}^{2\cdot1}+\left(\frac{19}{20}\right)^{2\cdot2}+\cdots+\left(\frac{19}{20}\right)^{2(n-1)}\right)\cdot v^2$

$\geqslant\frac{1}{5}mv^2$

∴ $\frac{400}{39}\cdot\left(1-\left(\frac{19}{20}\right)^{2n}\right)\geqslant\frac{320}{39}$

∴ $\left(\frac{19}{20}\right)^{2n}\leqslant\frac{1}{5}$ ∴ $n\geqslant16$ 至少撞16次

4-39 设 $m_{He}=4m$，$m_{Li}=7m$，$m_B=11m$

E_0最小时，$v_B=v_n$，$E_0=2mv_0^2$

∴ $4mv_0=(10m+m)\cdot v \Rightarrow v=\frac{4}{11}v_0$

∴ $E'=\frac{1}{2}\cdot11m\cdot v^2=\frac{8}{11}mv_0^2=\frac{4}{11}E_0$

∵ $E_0=E'+Q$ ∴ $E_{0min}=4.4MeV$

∴ $E_{kn}=\frac{1}{2}mv^2=\frac{8}{121}mv_0^2=\frac{4}{121}E_0$

$=\frac{16}{11}\times10^{-1}MeV=0.145MeV$