10-3

解：记飞经地球时，地球上 $t=0$，飞船上 $t'=0$

在飞船系中，飞船与空间站相遇时，

$$\begin{cases} t' = \gamma\left(t - \beta\frac{x}{c}\right) = t-3 & ① \\ t = \frac{x}{\beta c} & ② \end{cases}$$

解得 $\beta = 0.198$，即 $v = 0.198c$

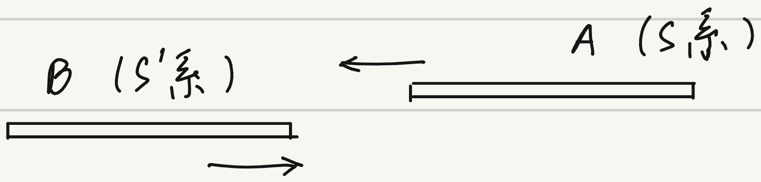
10-6

解：(1)、 $t' = \frac{l_0}{c}$

(2)、 $t_1 = \gamma\left(t' + \beta\frac{-l_0}{c}\right) = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\frac{l_0}{c}$

(3)、 $t_2' = \frac{l_0}{\gamma\beta c} = \sqrt{1-\frac{v^2}{c^2}}\frac{l_0}{v}$

10-9.



解：(1)、B中观测，右端先重合，左端后重合

(2)、记左端重合为事件1，右端重合为事件2，$\Delta t_{21} = \Delta t$

$v\Delta t_{21} = l_0\left(1-\sqrt{1-\beta^2}\right)$ ，$\beta = \frac{v}{c}$.

解得 $v = \frac{2l_0c^2\Delta t}{c^2\Delta t^2 + l_0^2}$

(3)、同时重合

10-13

解：$\beta = \frac{v}{c} = 0.8$，$\gamma = (1-\beta^2)^{-1/2} = \frac{5}{3}$

(1)、$\Delta t_1 = \gamma\Delta t_1' = 50\text{min}$ ，在 12:50 到达空间站

(2)、$\Delta x_1 = v\Delta t_1 = 7.2\times10^{11}\text{ m}$

(3)、$\Delta t_2 = \Delta t_1 + \frac{\Delta x_1}{c} = 90\text{ min}$，在 13:30 收到信号

(4)、$\Delta t_3 = \frac{v\Delta t_2}{c-v} = 360\text{ min}$，$\Delta t_{总} = \Delta t_2 + \Delta t_3 = 450\text{ min}$

$\Delta t_{总}' = \frac{1}{\gamma}\Delta t_{总} = 270\text{ min}$ 在 16:30 收到回答

10-16

解：(1). $\cos\theta' = \dfrac{\beta+\cos\theta}{1+\beta\cos\theta} \Rightarrow \theta' = \arccos\dfrac{\frac{v}{c}+\cos\theta}{1+\frac{v}{c}\cos\theta}$

(2). 由 $\theta' < \frac{\pi}{2}$，得 $0 < \dfrac{\beta+\cos\theta}{1+\beta\cos\theta} \leq 1$

解得 $-\beta < \cos\theta \leq 1$

$0 \leq \theta < \arccos(-\beta)$

当 $v\to c$ 时，$\beta\to 1$，$\arccos(-\beta)\to\pi$，故几乎所有的光都能被看到.

10-20.

解：先算光在厚玻璃中传播时间及距离：

记玻璃左右两面分别为 C，D，则

$\Delta t_{CD}' = \dfrac{nD}{c} \quad \Delta x_{CD}' = D$

得 $\Delta t_{CD} = \gamma\left(\Delta t_{CD}' + \beta\dfrac{\Delta x_{CD}'}{c}\right) = \gamma(n+\beta)\dfrac{D}{c}$

$\Delta x_{CD} = \gamma(\Delta x_{CD} + \beta c\Delta t_{CD}') = \gamma(1+\beta n)D$

故 $\Delta t_{AB} = \Delta t_{CD} + \dfrac{L-\Delta x_{CD}}{c} = \dfrac{L}{c} + \gamma(n-1)(1-\beta)\dfrac{D}{c}$

10-25.

解：(1). $h\nu_1 = \gamma_1(h\nu_0 + \beta_1 h\nu_0) \Rightarrow \nu_1 = \sqrt{\dfrac{1+\beta_1}{1-\beta_1}}\,\nu_0$

同理得 $\nu_2 = \sqrt{\dfrac{1-\beta_1}{1+\beta_1}}\,\nu_0$

由 $\dfrac{\lambda_2}{\lambda_1} = \dfrac{\nu_1}{\nu_2} = \dfrac{1+\beta_1}{1-\beta_1}$

得 $\beta_1 = \frac{1}{11} \Rightarrow V_1 = \frac{1}{11}c$

由 $\nu_2 = \sqrt{\dfrac{1+\beta_2'}{1-\beta_2'}}\,\nu_0$，得 $\dfrac{1+\beta_2'}{1-\beta_2'} = \dfrac{1-\beta_1}{1+\beta_1} \Rightarrow \beta_2' = -\frac{1}{11}$

从而 $\beta_2 = \dfrac{\beta_1-\beta_2'}{1-\beta_1\beta_2'} = \dfrac{11}{61} \Rightarrow V_2 = \dfrac{11}{61}c$ （靠近地球）

(2). 远离地球时，$\beta_2'' = \dfrac{\beta_1+\beta_2}{1+\beta_1\beta_2} = \dfrac{91}{341}$

故 $\nu_2'' = \sqrt{\dfrac{1-\beta_2''}{1+\beta_2''}}\,\nu_0$

$\lambda_2'' = \sqrt{\dfrac{1+\beta_2''}{1-\beta_2''}}\,\lambda_0 = 720\ nm$

10-29

解：(1)、$E_0 = m_0c^2 = \sqrt{E^2-p^2c^2} = 300\,\text{MeV}$

(2)、$p' = \frac{1}{c}\sqrt{E'^2-E_0^2} \approx 500\,\text{MeV}/c$

(3)、$p' = \gamma\left(p - \beta\frac{E}{c}\right)$

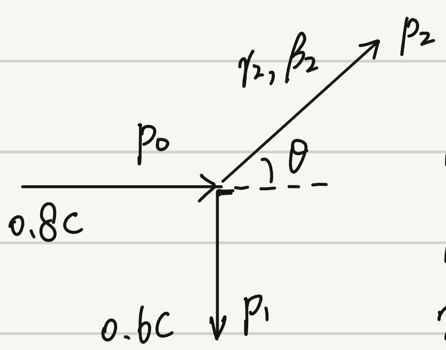
代入 $p'=500\,\text{MeV}/c$，$p=400\,\text{MeV}/c$，$E=500\,\text{MeV}$，得

$\beta = -0.183$

故相对速度 $v=0.183c$，方向与粒子动量方向相反

10-31

解：(1).



$p_1 = \gamma_1 m_0 \beta_1 c = \frac{3}{4}m_0c$ ①

$p_0 = \gamma_0 m_0' \beta_0 c = \frac{4}{3}m_0'c$ ②

$\gamma_0 m_0'c^2 = \gamma_1 m_0c^2 + \gamma_2 m_0c^2$ ③

$(\gamma_2 m_0c^2)^2 = (m_0c^2)^2 + (p_1^2+p_0^2)c^2$ ④

联立得：$\gamma_2 = \frac{205}{36}$，$\beta_2 = 0.984$，$m_0' = \frac{253}{60}m_0$

速率 $v_2 = 0.984c$，方向 $\theta = \arctan\frac{135}{1012} \approx 7.60^\circ$

(2)、由(1)得：$\frac{m_0}{m_0'} = \frac{60}{253} \approx 0.237$

10-33

解：湮灭后，$2h\nu = 2m_ec^2 \Rightarrow h\nu = m_ec^2$

"对心"碰给予速度最大、

$$\begin{cases} h\nu + m_ec^2 = h\nu' + \gamma m_ec^2 & ① \\ \frac{h\nu}{c} = -\frac{h\nu'}{c} + \gamma\beta m_ec & ② \end{cases}$$

联立得：$\beta = 0.8 \Rightarrow v = 0.8c$

10-34

解：利用零动量系：设该系中总能量为 E'，原系总能量 E，动量 p.

$E^2 - p^2c^2 = E'^2$ ①

$E = E_k + 2m_0c^2$ ②

$pc = \sqrt{(E_k+m_0c^2)^2-(m_0c^2)^2}$ ③

解得 $E' = 3843.88\,\text{MeV}$

故 $E_{k\min} = E' - 2m_0c^2 = 1967.88\,\text{MeV}$

10-36

解：分析中间态：速度为 $v=\beta c$ 时，动质量为 m.

由动量守恒：$(m+dm)(v+dv)-mv+(-dm)\left(\dfrac{v-u}{1-\frac{vu}{c^2}}\right)=0$

$$m\,d\beta=-\beta_0\frac{1-\beta^2}{1-\beta\beta_0}dm$$

这里 $\beta=\frac{v}{c}$, $\beta_0=\frac{u}{c}$.

积分，$\displaystyle\int_{m_0'}^{m}\frac{dm}{m}=-\frac{1}{\beta_0}\int_0^{\beta}\frac{1-\beta\beta_0}{1-\beta^2}d\beta$

$$\ln\frac{\gamma m_0}{m_0'}=\frac{1}{2\beta_0}\left[\ln\left|\frac{1-\beta}{1+\beta}\right|-\beta_0\ln(1-\beta^2)\right]$$，这里 $\gamma=\frac{1}{\sqrt{1-\beta^2}}$，$\ln\gamma=-\frac{1}{2}\ln(1-\beta^2)$

$$\frac{m_0}{m_0'}=\left(\frac{c-v}{c+v}\right)^{\frac{c}{2u}}$$

10-38

解：(1). $F_0 d=(\gamma_1-\gamma_2)m_0c^2$

得 $F_0=1\times10^{-9}N$

(2). 由相对论力变换公式，

$$F_0'=\frac{F_0+\frac{vu_x}{c^2}F_0}{1+\frac{vu_x}{c^2}}=F_0=1\times10^{-9}N$$

(3). $F_0t=(\gamma_1\beta_1-\gamma_2\beta_2)m_0c$.

得 $t=1.83\times10^{-9}s$

$F_0't'=(0-\gamma_2'\beta_2')m_0c$，且 $\beta_2'=\dfrac{\beta_2-\beta_1}{1-\beta_2\beta_1}=-\dfrac{3}{5}$

得 $t'=1.5\times10^{-9}s$

10-3 船的固有时 t_0，地球系下 t 船速 $v=\frac{x}{t}$ 有

$$t-t_0=\Delta t=3s, \quad \frac{t}{t_0}=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

解得 $v=\frac{20}{101}c=6\times10^7\ m/s$

10-6 (1) $t'=\frac{l_0}{c}$

(2) $l=l_0\sqrt{1-\frac{v^2}{c^2}}$

$l=(v+c)t_1 \Rightarrow t_1=\frac{l_0}{c}\sqrt{\frac{c-v}{c+v}}$

(3) $t_2=\frac{l}{v}=\frac{l_0}{v}\sqrt{1-\frac{v^2}{c^2}}$

10-9 (1) 先右后左

(2) $l_0\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)=v\cdot\Delta t$

$\Rightarrow v=\frac{2c^2l_0^2\Delta t}{l_0^2+c^2\Delta t^2}$

(3) 显然同时重合

10-13 (1) 地球时 $t=\frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}=50\,min$ 故 12:50

(2) $x_0=vt=7.2\times10^{11}\,m$

(3) $t_{c_1}=\frac{x_0}{c}=40\ min$ 故 13:30

(4) 地面回答时 $x=x_0+vt_{c_1}$

$t_{c_2}=\frac{x}{c-v}$

地面总时间 $t_2=t+t_{c_1}+t_{c_2}$

固有时 $t_0=t_2\sqrt{1-\frac{v^2}{c^2}}=270\ min$ 故 16:30

10-16 (1) 速度变换 $C_x'=\frac{c\cos\theta+v}{1+\frac{v}{c}\cos\theta}$

从而 $\theta'=\arccos\frac{C_y'}{c}=\arccos\frac{\frac{v}{c}+\cos\theta}{1+\frac{v}{c}\cos\theta}$

(3) $\frac{v}{c}\to 1$ 时 $\theta'\to 0$，

临界 $\theta'=\frac{\pi}{2}\Rightarrow\theta=\frac{\pi}{2}+\arcsin\frac{v}{c}\to\pi$

此时几乎所有恒星的光趋向于从正前方照来，故可看到所有恒星

10-20 玻璃内时，$v_c'=\frac{c}{n}$

速度变换．$$v_c=\frac{v+v_c'}{1+\frac{vv_c'}{c^2}}=\frac{v+\frac{c}{n}}{1+\frac{v}{nc}}$$

地面系：$$D'=D\sqrt{1-\frac{v^2}{c^2}}$$

$$t_2=\frac{D'}{v_c-v}$$

剩余距离．$x=L-D'-vt_2$

总时间 $$t=\frac{x}{c}+t_2=\frac{L}{c}+\frac{D}{c}(n-1)\sqrt{\frac{c-v}{c+v}}$$

10-25 (1) 由光源系到S系．$$\lambda_1=\lambda_0\sqrt{\frac{c-v_1}{c+v_1}}=500\,nm$$

$$\lambda_2=\lambda_0\sqrt{\frac{c+v_1}{c-v_1}}=600\,nm$$

得 $v_1=\frac{1}{11}c$

S'系中光源速度为v_1远离 $$v_1=\frac{v_2-v_1}{1-\frac{v_1v_2}{c^2}}$$

得 $v_2=\frac{11}{61}c$

(2) S'系中光源速度 $$v'=\frac{v_1+v_2}{1+\frac{v_1v_2}{c}}=\frac{91}{341}c$$

$$\lambda'=\lambda_0\sqrt{\frac{c+v'}{c-v'}}=720\,nm$$

10-29 (1) 由 $E^2=p^2c^2+E_0^2$

得 $E_0=\sqrt{E^2-p^2c^2}=300\,MeV$

(2) 由 $E'^2=p'^2c^2+E_0^2$

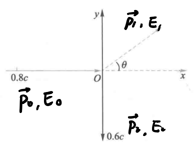
得 $P'=\frac{1}{c}\sqrt{E'^2-E_0^2}\approx 500\,\frac{MeV}{c}$

(3) S''系相对S系沿粒子速度方向的直线运动，速度为S'系在该方向的分速度　可知S'、S''系中粒子动量、能量相同

能量变换　$E' = \dfrac{E - pv}{\sqrt{1-\frac{v^2}{c^2}}}$　　解得　$v = 0.983c$　或　$v = -0.183c$

故在S系中，S'系的速度在粒子运动方向的速度大小为　$0.983c$　或　$-0.183c$

10-31



题 10-31 图

能量守恒：　$E_0 = E_1 + E_2$　　其中　$E_0 = \dfrac{m_0'c^2}{\sqrt{1-0.8^2}}$　　$E_2 = \dfrac{m_0c^2}{\sqrt{1-0.6^2}}$

动量守恒：　$p_0 = p_{1x}$

$0 = p_{1y} - p_2$

$\Rightarrow$　$p_{1x} = \dfrac{m_0' \cdot 0.8c}{\sqrt{1-0.8^2}} = \dfrac{4}{3}m_0'c$

$p_{1y} = \dfrac{m_0 \cdot 0.6c}{\sqrt{1-0.6^2}} = \dfrac{3}{4}m_0c$

动量能量关系：　$E_1^2 = p_1^2c^2 + m_0^2c^4$

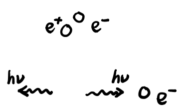
代入得　$\dfrac{m_0}{m_0'} = \dfrac{6}{25} = 0.24$　，　$E_1 = \dfrac{205}{36}m_0c^2$

$E_1 = \dfrac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$　$\Rightarrow$　$v_1 = \sqrt{1-\left(\frac{36}{205}\right)^2}\,c \approx 0.98446\,c$

$\theta = \arctan\dfrac{p_{1y}}{p_{1x}} = \arctan\dfrac{27}{200} = 7.688^\circ$

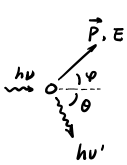
$\dfrac{m_0}{m_0'} = 0.24$

10-33



$2m_ec^2 = 2h\nu$　$\Rightarrow$　$h\nu = m_ec^2$

康普顿散射：

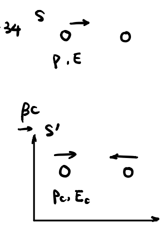


$$\begin{cases} \dfrac{h\nu}{c} = \dfrac{h\nu'}{c}\cos\theta + p\cos\varphi \\ 0 = p\sin\varphi - \dfrac{h\nu'}{c}\sin\theta \\ m_ec^2 + h\nu = E + h\nu' \\ E^2 = p^2c^2 + m_e^2c^4 \end{cases} \Rightarrow \frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_ec^2}(1-\cos\theta)$$

代入　$h\nu = m_ec^2$　得　$E = m_ec^2\left(2 - \dfrac{1}{2-\cos\theta}\right)$

故　$\theta = \pi$　时取最大速度　$E = \dfrac{5}{3}m_ec^2 = \dfrac{m_ec^2}{\sqrt{1-\frac{v^2}{c^2}}}$　得　$v = 0.8c$

10-34



在零动量系中动能最小，$\gamma=\frac{1}{\sqrt{1-\beta^2}}$ 如图

$$p_c\cdot c=\gamma(pc-\beta E)$$

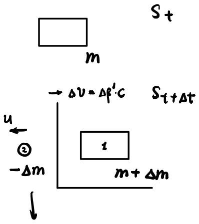
$$-p_c\cdot c=\gamma(0-\beta E_0)\quad\Rightarrow\quad\beta=\sqrt{\frac{E-E_0}{E+E_0}}$$

其中 $E^2=p^2c^2+E_0^2$

故 $E_c=\gamma E_0$

$$E_k=2(E_c-E_0)=2\left(\sqrt{\frac{E+E_0}{2E_0}}-1\right)E_0=2\left(\sqrt{\frac{E_{k_0}}{2E_0}+1}-1\right)E_0=1967.875\,MeV$$

10-36



（注意此处 $-\Delta m$ 为动质量，对应喷出物的总能量）

在 S_t 一系列瞬时惯性系中观察，$\gamma=\frac{1}{\sqrt{1-\Delta\beta'^2}}$

$S_{t+\Delta t}$ 中 $p_2'=-\Delta m(-u)$　$p_1'=0$　$E_2'=-\Delta mc^2$　$E_1'=(m+\Delta m)c^2$

变换到 S_t 中 $p_2c=\gamma(p_2'c+\Delta\beta'E_2')$　$p_1c=\gamma\Delta\beta'E_1'$

有 $p_1+p_2=0\Rightarrow\Delta\beta'=\frac{-u}{c}\cdot\frac{\Delta m}{m}$

最终换回地面系 $\beta+\Delta\beta=\frac{\beta+\Delta\beta'}{1-\beta\Delta\beta'}$

故 $d\beta=\frac{\beta+\Delta\beta'}{1-\beta\Delta\beta'}-\beta=(1-\beta^2)\Delta\beta'$　$(\Delta\beta'\to0)$

从而 $\frac{d\beta}{1-\beta^2}=\frac{-u}{c}\frac{dm}{m}\Rightarrow\frac{1}{2}\ln\frac{1+\beta}{1-\beta}=\frac{-u}{c}\ln\frac{m_0}{m_0'}$　其中 $\beta=\frac{v}{c}$

故 $\frac{m_0}{m_0'}=\left(\frac{c-v}{c+v}\right)^{\frac{c}{2u}}$

10-38 (1) $Fd=\frac{m_0c^2}{\sqrt{1-\frac{v_1^2}{c^2}}}-\frac{m_0c^2}{\sqrt{1-\frac{v_2^2}{c^2}}}\Rightarrow F=\frac{7}{12}\frac{m_0c^2}{d}=1\times10^{-9}\,N$

(2) 粒子运动方向与参考系相对运动方向在同一条直线上，有 $\vec{v}\cdot\vec{F}=vF$

力变换 $F'=\frac{F-\frac{u}{c^2}\vec{v}\cdot\vec{F}}{1-\frac{uv}{c^2}}=F=1\times10^{-9}\,N$

(3) $Ft=\frac{m_0v_1}{\sqrt{1-\frac{v_1^2}{c^2}}}-\frac{m_0v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}$　$F't'=\frac{m_0v_1'}{\sqrt{1-\frac{v_1'^2}{c^2}}}-0$

$\Rightarrow t=1.833\times10^{-9}\,s$

$t=1.5\times10^{-9}\,s$