1-4

(1)、x坐标最大时，$\frac{dx}{dt} = \vec{v}\cdot\vec{i} = 0$

$\vec{v} = \vec{v_0} + \vec{a}t = [(5-t)\vec{i} - t\vec{j}]\ m/s$

由 $\vec{v}\cdot\vec{i} = 0$，得 $t = 5s$

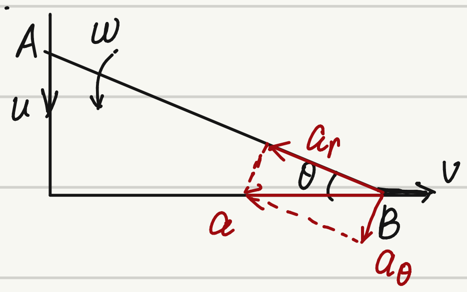
此时 $\vec{v} = -5\ m/s$，速度大小 $5m/s$

(2)、$\vec{x} = \vec{v_0}t + \frac{1}{2}\vec{a}t^2$

$= 25\vec{i} + \frac{25}{2}(-\vec{i}-\vec{j})$

$= \frac{25}{2}\vec{i} - \frac{25}{2}\vec{j}$　　位置为 $(\frac{25}{2}m, -\frac{25}{2}m)$

1-14.



设 B点速度为v，杆转动角速度 ω

沿杆速度相等：$u\sin\theta = v\cos\theta \Rightarrow v = u\tan\theta \Rightarrow \omega = \frac{1}{l}(u\cos\theta + v\sin\theta) = \frac{u}{l\cos\theta}$

对B加速度分析：$\vec{a_B} = \vec{a_A} + \vec{a_{BA}} = \vec{a_{BA}}$　（因为 $\vec{a_A} = 0$）

$\vec{a_{BA}} = \vec{a_r} + \vec{a_\theta}$，如上图所示

由于B一直在水平面运动，故 $\vec{a_B}$（即 $\vec{a_{BA}}$）一定沿水平方向

$a_r = \omega^2 l = \frac{u^2}{l\cos^2\theta}$

故 $a = \frac{a_r}{\cos\theta} = \frac{u^2}{l\cos^3\theta}$

1-18

小球每次落在楼梯上竖直速度 v_y 为：

$v_y = \sqrt{2g(h+l)}$

碰后，$v_y' = ev_y$

由于碰后系统复原，故上升高度为h，$v_y' = \sqrt{2gh}$

故 $2gh = 2ge^2(h+l) \Rightarrow h = \frac{e^2}{1-e^2}l$

每两次碰撞所用时间：

$t = \frac{(1+e)v_y}{g} = \sqrt{\frac{1+e}{1-e}\cdot\frac{2l}{g}}$

水平方向分析：

$v_0 = \frac{l}{t} = \sqrt{\frac{1-e}{2(1+e)}gl}$

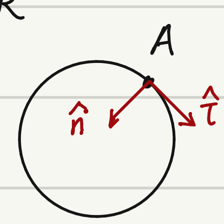
1-24

物体与A速度大小一致，$v=\frac{ds}{dt}=bt$

切向加速度：$a_\tau=\frac{dv}{dt}=b$

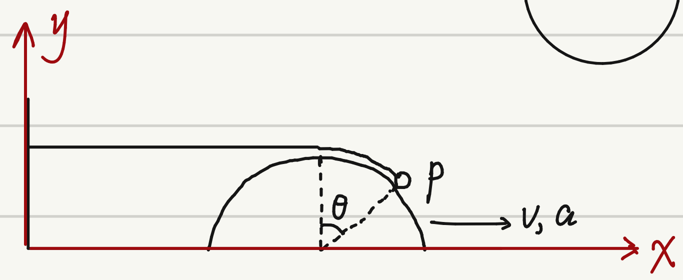
法向加速度：$a_n=\frac{v^2}{R}=\frac{b^2t^2}{R}$

总加速度：如右图所示：



$\vec{a}=b\hat{\tau}+\frac{b^2t^2}{R}\hat{n}$

1-31

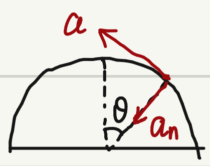


半圆柱相对于墙面速度为v，故墙面与绳子相对半圆柱速度大小也为v

即P相对于半圆柱速度大小为v，方向沿圆柱切向

坐标系如上图所示，则 $\vec{v}_P=v(1-\cos\theta)\vec{i}+v\sin\theta\vec{j}$

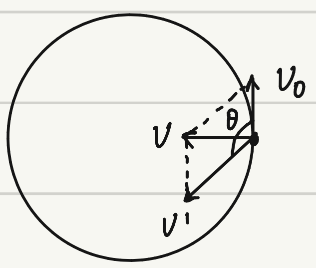
P相对于半圆柱的加速度：



其中 $a_n=\frac{v^2}{R}$

故 $\vec{a}_P=\left[a(1-\cos\theta)-\frac{v^2}{R}\sin\theta\right]\vec{i}+\left(a\sin\theta-\frac{v^2}{R}\cos\theta\right)\vec{j}$

1-33



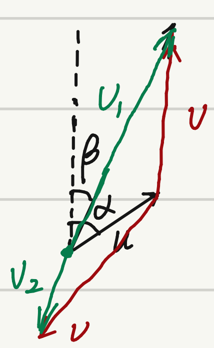
落地时间：$t=\sqrt{\frac{2h}{g}}$

故 $v=\frac{R}{t}=\sqrt{\frac{gR^2}{2h}}$

$$\begin{cases} v_0+v'\cos\theta=0 & ① \\ v=v'\sin\theta & ② \end{cases} \Rightarrow \begin{cases} v'=\sqrt{v_0^2+\frac{gR^2}{2h}} \\ \theta=\arctan\sqrt{\frac{2hv_0^2}{gR^2}}+\frac{\pi}{2} \end{cases}$$

代入数据，$\begin{cases} v'=28\,m/s \\ \theta=\arctan\frac{\sqrt{15}}{15}+\frac{\pi}{2} \end{cases}$

1-35



速度矢量图如左图所示.

去时速度为v_1，回时为v_2

由余弦定理，$\begin{cases} v_1^2+u^2-2v_1u\cos(\beta-\alpha)=v^2 \\ v_2^2+u^2+2v_2u\cos(\beta-\alpha)=v^2 \end{cases}$

得 $\begin{cases} v_1=\sqrt{v^2-u^2\sin^2(\beta-\alpha)}+u\cos(\beta-\alpha) \\ v_2=\sqrt{v^2-u^2\sin^2(\beta-\alpha)}-u\cos(\beta-\alpha) \end{cases}$

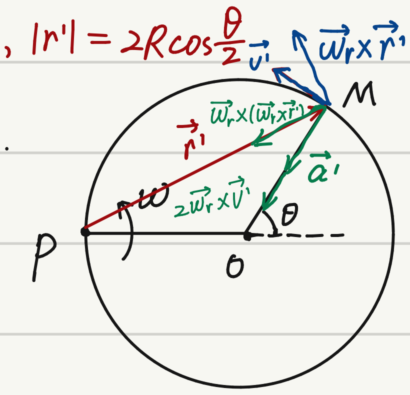
总可飞行时长为 $t=\frac{2R}{v}$.

最远距离R'满足：$\frac{R'}{v_1}+\frac{R'}{v_2}=t$

解得 $R'=\frac{R(v^2-u^2)}{v\sqrt{v^2-u^2\sin^2(\beta-\alpha)}}$，证毕

$|\vec{v}'| = \omega R$, $|r'| = 2R\cos\frac{\theta}{2}$

1-37.



$\vec{v} = \vec{v}' + \vec{\omega}_r \times \vec{r}'$

$\vec{a} = \vec{a}' + \vec{\omega}_r \times (\vec{\omega}_r \times \vec{r}') + 2\vec{\omega}_r \times \vec{v}' + \vec{\beta}_r \times \vec{r}'$ ，这里 $\vec{\omega}_r = \vec{\omega}$.

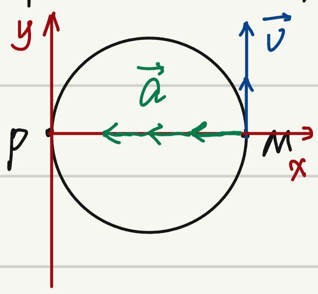
这里，速度：$|\vec{\omega}_r \times \vec{r}'| = 2\omega R\cos\frac{\theta}{2}$　加速度：$|\vec{a}'| = \frac{v'^2}{R} = \omega^2 R$

$|2\vec{\omega}_r \times \vec{v}'| = 2\omega^2 R$.

$|\vec{\omega}_r \times (\vec{\omega}_r \times \vec{r}')| = 2\omega^2 R\cos\frac{\theta}{2}$

当 OM 与 OP 在一条直线上，分 $\theta = 0$ 与 $\theta = \pi$.

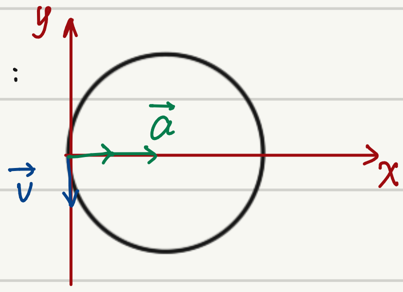
$\theta = 0$:



$\vec{v} = 3\omega R\,\hat{y}$

$\vec{a} = -5\omega^2 R\,\hat{x}$

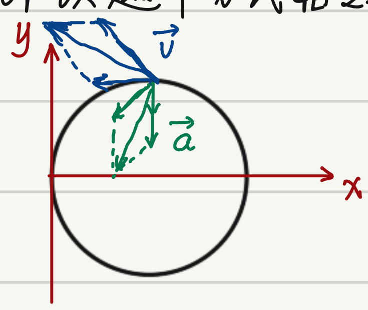
$\theta = \pi$:



$\vec{v} = -\omega R\,\hat{y}$

$\vec{a} = 3\omega^2 R\,\hat{x}$

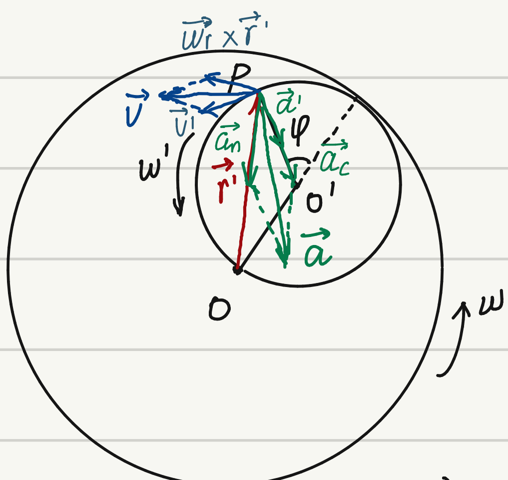
当 OM 与 OP 以题中方式相互垂直时，$\theta = \frac{\pi}{2}$



$\vec{v} = -2\omega R\,\hat{x} + \omega R\,\hat{y}$　　$|\vec{v}| = \sqrt{5}\,\omega R$

$\vec{a} = -\omega^2 R\,\hat{x} - 4\omega^2 R\,\hat{y}$　　$|\vec{a}| = \sqrt{17}\,\omega^2 R$

1-38



$|\vec{r}'| = 2R\cos\frac{\varphi}{2}$　　$|\vec{\omega}_r| = \omega$

$\vec{v} = \vec{v}' + \vec{\omega}_r \times \vec{r}'$.

$$\vec{a} = \vec{a}' + \underbrace{\vec{\omega}_r \times (\vec{\omega}_r \times \vec{r}')}_{\vec{a}_n} + \underbrace{2\vec{\omega}_r \times \vec{v}'}_{\vec{a}_c} + \underbrace{\vec{\beta}_r \times \vec{r}}_{\vec{a}_\beta = 0}$$

$|\vec{v}'| = \omega R$,　$|\vec{\omega}_r \times \vec{r}'| = 2\omega R\cos\frac{\varphi}{2}$

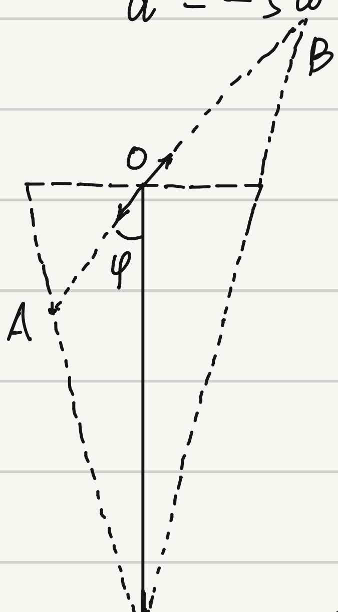
$|\vec{a}'| = \omega^2 R$,　$|\vec{a}_n| = 2\omega^2 R\cos\frac{\varphi}{2}$.　$|\vec{a}_c| = 2\omega^2 R$

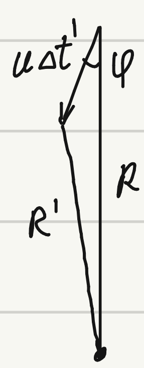
记由 O 指向 P 为矢径方向，单位向量为 $\hat{r}$，横向单位向量为 $\hat{\theta}$.

故　$\vec{v} = -\omega R\sin\frac{\varphi}{2}\,\hat{r} + 3\omega R\cos\frac{\varphi}{2}\,\hat{\theta}$　　$|\vec{v}| = \sqrt{1 + 8\cos^2\frac{\varphi}{2}}\;\omega R$

$\vec{a} = -5\omega^2 R\cos\frac{\varphi}{2}\,\hat{r} - 3\omega^2 R\sin\frac{\varphi}{2}\,\hat{\theta}$　　$|\vec{a}| = \sqrt{9 + 16\cos^2\frac{\varphi}{2}}\;\omega^2 R$

1-39





$R' = \left[(u\Delta t')^2 + R^2 - 2R\cdot u\Delta t'\cos\varphi\right]^{1/2} \approx R - u\Delta t'\cos\varphi$

$\Delta t = \Delta t' + \frac{R'}{c} - \frac{R}{c} = \Delta t'(1 - \beta\cos\varphi)$

故　$v_1 = \frac{u\Delta t'\sin\varphi}{\Delta t} = \frac{\beta\sin\varphi}{1 - \beta\cos\varphi}\,c$

只要满足 $\frac{\beta\sin\varphi}{1 - \beta\cos\varphi} > 1$ ，便可满足 $v_1 > c$.（β 较大，φ 较小时易实现）

对 v_2，同理得　$v_2 = \frac{\beta\sin\varphi}{1 + \beta\cos\varphi}\,c$　$\Longrightarrow$　$\begin{cases} u = \dfrac{\sqrt{(v_1 - v_2)^2 + 4v_1^2 v_2^2/c^2}}{v_1 + v_2}\,c \\ \varphi = \arctan\dfrac{2v_1 v_2}{(v_1 - v_2)c} \end{cases}$

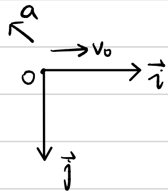
第一章 质点运动学

1-4 (1) $a=(-\vec{i}-\vec{j})\ m/s^2$

∴ 对x轴方向分析: $t_1=\dfrac{v_0}{a_x}=5s$.

y轴方向分析: $v_y=a_y\cdot t_1=-5m/s$.

∴ 此时速度为 $\vec{V}=-5\vec{j}\ m/s$.

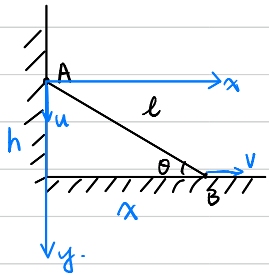


(2) 对x轴方向: $x=\dfrac{v_0}{2}\cdot t_1=\dfrac{25}{2}m$

y轴 : $y=\dfrac{v_y}{2}\cdot t_1=-\dfrac{25}{2}m$.

∴ 此时质点的坐标: $(\dfrac{25}{2}m,-\dfrac{25}{2}m)$

1-14 如图建立坐标系: ∴ $u=\dfrac{-dh}{dt}$



$$\vec{v}=\frac{d\vec{x}}{dt}=\frac{dx}{dt}\cdot\vec{i}=\frac{d\sqrt{l^2-h^2}}{dt}\vec{i}$$

$$=\frac{-h}{\sqrt{l^2-h^2}}\frac{dh}{dt}\vec{i}$$

$$=\frac{h}{x}\cdot u\vec{i}=\tan\theta\cdot u\vec{i}$$

$$\vec{a}=\frac{d\vec{v}}{dt}=\frac{u}{\cos^2\theta}\frac{d\theta}{dt}\vec{i}$$

又∵ $h=l\sin\theta$ 两边求导: $\dfrac{dh}{dt}=\dfrac{dl}{dt}\sin\theta+l\cos\theta\cdot\dfrac{d\theta}{dt}$

$$-u=l\cos\theta\cdot\frac{d\theta}{dt}$$

$$\therefore\ \vec{a}=\frac{u}{\cos^2\theta}\cdot\frac{-u}{l\cos\theta}\cdot\vec{i}=\frac{-u^2}{l\cos^3\theta}\vec{i}$$

1-18 解: 第一级楼梯: $t_1=\sqrt{\dfrac{2(h+l)}{g}}$

$V_1=gt_1=\sqrt{2(h+l)g}$

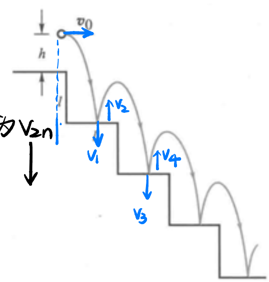
反弹后竖直方向速度: 设第n次与楼梯碰撞后速度为V_{2n}

$$\begin{cases} l=-V_{2n}t_2+\frac{1}{2}gt_2^2 \\ l=v_0t_2 \\ V_{2n+1}=-V_{2n}+gt_2 \\ V_{2n+2}=eV_{2n+1}=V_{2n} \end{cases}$$

∴ $t_2=\sqrt{\dfrac{2(1+e)}{1-e}\dfrac{l}{g}}$

∴ $V_0=\dfrac{l}{t_2}=\sqrt{\dfrac{(1-e)gl}{2(1+e)}}$

$h=\dfrac{e^2}{1-e^2}l$.



题 1-18 图

1-24 ∵轮周与绳子没有相对滑动

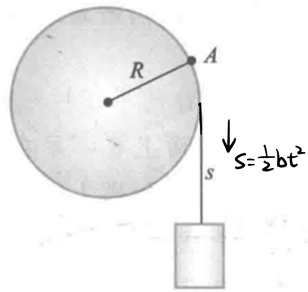
∴绳子向下运动距离 $s = s_A$

∴ $\vec{v_A} = v_A \vec{e_t} = \frac{ds}{dt}\vec{e_t} = \frac{b}{2}\frac{dt^2}{dt}\vec{e_t} = bt\vec{e_t}$

切向加速度： $\vec{a_t} = \frac{dv_A}{dt}\vec{e_t} = \frac{dbt}{dt}\vec{e_t} = b\vec{e_t}$

法向加速度： $\vec{a_n} = \frac{v_A^2}{R}\vec{e_n} = \frac{(bt)^2}{R}\vec{e_n} = \frac{b^2t^2}{R}\vec{e_n}$

总加速度： $\vec{a} = \vec{a_t} + \vec{a_n} = b\vec{e_t} + \frac{b^2t^2}{R}\vec{e_n}$



题 1-24 图

1-31. 小球对圆柱面做圆周运动

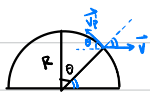
∴ $\vec{v_P} = \vec{v_P'} + \vec{v} = \vec{\omega}\times\vec{r}$

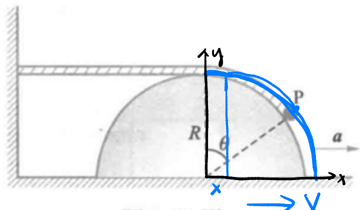
∵ $\vec{v} = \frac{d\vec{x}}{dt} = \frac{dx}{dt}\hat{x}$

又∵ $\vec{v_P'} = \frac{ds}{dt}\vec{e_t} = \frac{dx}{dt}\cdot\vec{e_t} = v\cdot\vec{e_t}$

∴ $\vec{v_P} = (v - v\cdot\cos\theta)\vec{i} + v\cdot\sin\theta\vec{j}$

$= (1-\cos\theta)v\vec{i} + v\sin\theta\vec{j}$





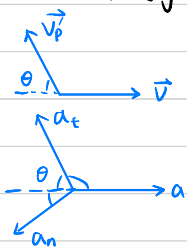
题 1-31 图

∵ $\vec{v_P'} = v\cdot\vec{e_t} = \vec{\omega}\times\vec{r}$ ∴ $\omega = \frac{v}{R} = \frac{-d\theta}{dt}$

∴ $a_x = \frac{dv_x}{dt} = \frac{d(1-\cos\theta)}{dt}v + 1-\cos\theta\frac{dv}{dt} = \sin\theta\frac{d\theta}{dt}\cdot v + (1-\cos\theta)\cdot a = -\sin\theta\cdot\frac{v^2}{R} + (1-\cos\theta)\cdot a$

$a_y = \frac{dv_y}{dt} = \frac{dv}{dt}\sin\theta + v\frac{d\sin\theta}{dt} = \sin\theta\cdot a + \cos\theta\cdot v\cdot\frac{d\theta}{dt} = \sin\theta\, a - \cos\theta\frac{v^2}{R}$

∴ $\vec{a} = \left[-\sin\theta\cdot\frac{v^2}{R} + (1-\cos\theta)\cdot a\right]\vec{i} + \left[\sin\theta\cdot a - \cos\theta\cdot\frac{v^2}{R}\right]\vec{j}$



法2： ∵ $\vec{v_P'} = v\cdot\vec{e_t}$ ∴ $v_P = 2v\sin\frac{\theta}{2}$

$\vec{a_t} = \frac{dv_P'}{dt}\vec{e_t} = \frac{dv}{dt}\vec{e_t} = a\vec{e_t}$

$\vec{a_n} = v\cdot\frac{d\theta}{dt}\vec{e_n} = \frac{v^2}{R}\cdot\vec{e_n}$

∴ $a_{px} = a - a\cos\theta - a_n\sin\theta$

$= a(1-\cos\theta) - \frac{v^2}{R}\sin\theta$

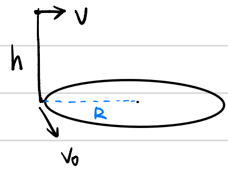
$a_{py} = a\sin\theta - \frac{v^2}{R}\cos\theta$

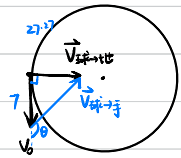
1-33 $\frac{1}{2}gt^2=h$ $t=0.55s$. ∴ $V_{球\to地}=\frac{R}{t}=27.27\ m/s$

∵ $\vec{V}_{球\to地}=\vec{V}_{球\to手}+\vec{V}_0$ ∴ $\vec{V}_{球\to手}=28.15\ m/s$

∵ $\pi-\theta=\arctan\frac{27.27}{7}$

∴ $\theta=\pi-\arctan\frac{27.27}{7}$





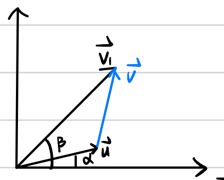
1-35

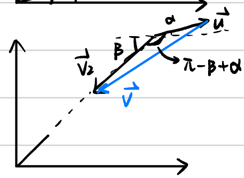
证明: $\cos(\beta-\alpha)=\frac{u^2+v_1^2-v^2}{2uv_1}$

$\cos(\pi-(\beta-\alpha))=-\cos(\beta-\alpha)=\frac{u^2+v_2^2-v^2}{2uv_2}$

$v_1=u\cos(\beta-\alpha)+\sqrt{v^2-u^2\sin^2(\beta-\alpha)}$

$v_2=-u\cos(\beta-\alpha)+\sqrt{v^2-u^2\sin^2(\beta-\alpha)}$





由 $\frac{2R}{v}=\frac{R'}{v_1}+\frac{R'}{v_2}=R'\cdot\frac{v_1+v_2}{v_1v_2}$

$$R'=\frac{2R}{v}\cdot\frac{v^2-u^2\sin^2(\beta-\alpha)-u^2\cos^2(\beta-\alpha)}{2\sqrt{v^2-u^2\sin^2(\beta-\alpha)}}=\frac{R(v^2-u^2)}{v\sqrt{v^2-u^2\sin^2(\beta-\alpha)}}$$

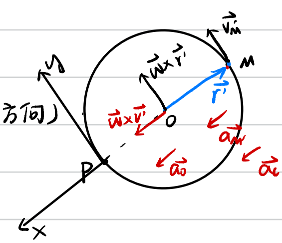
1-37

(1) 情况1: M与P不共点: $\vec{V}_M=\vec{V}_M'+\vec{\omega}\times\vec{r}'+\vec{V}_0$

$=\omega R+\omega R+\omega R=3\omega R$ (y方向)

$\vec{a}_M=\vec{a}_M'+\vec{\omega}\times(\vec{\omega}\times\vec{r}')+2\vec{\omega}\times\vec{V}_M'+\vec{a}_0$

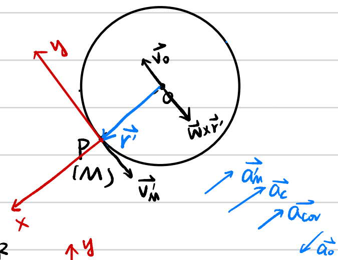
$=\omega^2R+\omega^2R+2\omega^2R+\omega^2R=5\omega^2R$ (x方向)



情况2： $\vec{V}_M = \vec{V}_M' + \vec{w}\times\vec{r}' + \vec{V}_0 = wR$ （$-y$方向）

$\vec{a}_M = \vec{a}_M' + \vec{w}\times(\vec{w}\times\vec{r}') + 2\vec{w}\times\vec{V}_M' + \vec{a}_0$

$= w^2R + w^2R + 2w^2R - w^2R = 3w^2R$ （$-X$方向）

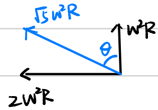


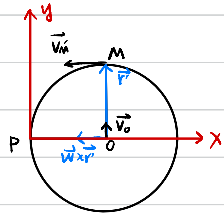
(2) $\vec{V}_M = \vec{V}_M' + \vec{w}\times\vec{r}' + \vec{V}_0 = \sqrt{5}w^2R$

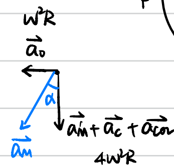
（与y正方向呈arctan2 向左上方）

$\vec{a}_M = \vec{a}_M' + \vec{w}\times(\vec{w}\times\vec{r}') + 2\vec{w}\times\vec{V}_M' + \vec{a}_0$

$= \sqrt{17}w^2R$ （与y负方向呈 $\arctan\frac{1}{4}$ 向左下方）







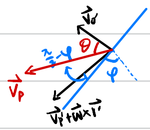
1-38 $\vec{V}_P = \vec{V}_P' + \vec{w}\times\vec{r}' + \vec{V}_0 = 2wR\cdot\sin\varphi$

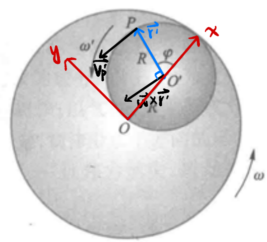
$= -2wR\cdot\sin\varphi\,\vec{i} + (wR + 2wR\cos\varphi)\vec{j}$

$= wR\sqrt{4\sin^2\varphi + 1 + 4\cos^2\varphi + 4\cos\varphi}$

$= wR\sqrt{5 + 4\cos\varphi}$

方向：与y正向夹角为 $\arctan\dfrac{2\sin\varphi}{1+2\cos\varphi}$





题 1-38 图

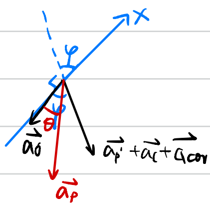
$\vec{a}_P = \vec{a}_P' + \vec{w}\times(\vec{w}\times\vec{r}') + 2\vec{w}\times\vec{V}_P' + \vec{a}_0$

$= -(4w^2R\cos\varphi + w^2R)\vec{i} - 4w^2R\sin\varphi)\vec{j}$

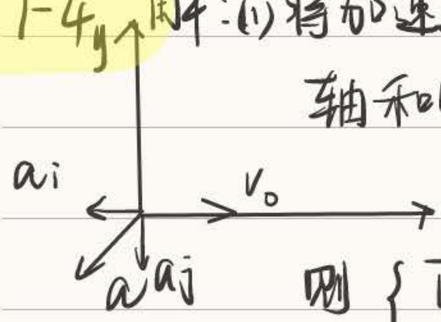
$= w^2R\sqrt{16\cos^2\varphi + 1 + 16\sin^2\varphi + 8\cos\varphi}$

$= w^2R\sqrt{17 + 8\cos\varphi}$

方向：与 X负半轴夹 $\arctan\dfrac{4\sin\varphi}{4\cos\varphi+1}$



1-4 解：(1) 将加速度分解为沿 -x 和 -y 方向，且物体可看作做x轴和y轴两个方向的运动，当 $v_x=0$ 时到达 x 坐标最大值

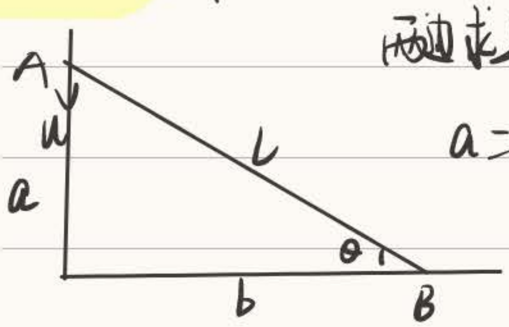


则 $\begin{cases} \vec{v_0}+\vec{a_i}t=0 \\ v_y=\vec{a_j}t \end{cases}$ 解得 $\vec{v_y}=-5\vec{j}$ m/s

(2) $\begin{cases} v_0^2=2a_x \\ y=\frac{1}{2}a_j t^2 \end{cases}$ 解得 $\begin{cases} x=12.5\,m \\ y=-12.5\,m \end{cases}$

则此时质点的位置是 (12.5m, -12.5m).

1-14 解：由题：$a^2+b^2=l^2$



两边求导 $2a\frac{da}{dt}=-2b\frac{db}{dt}$

$a=l\sin\theta$　$b=l\cos\theta$　$\frac{da}{dt}=-u$　$\frac{db}{dt}=v$

得 $v=u\tan\theta$

则加速度为

$$a=\frac{dv}{dt}=\frac{u}{\cos^2\theta}\frac{d\theta}{dt} \quad —①$$

又 $u=-\frac{da}{dt}=\frac{d\,l\sin\theta}{dt}=l\cos\theta\frac{d\theta}{dt}$ —②

⇒ ①②联立得 $a=-\frac{u^2}{l\cos^3\theta}$

故B点速度 $v=u\tan\theta$ 加速度 $a=-\frac{u^2}{l\cos^3\theta}$

1-18 解：$v_y^2=2g(h+l)$ —①

$v_y'=ev_y$ —②

$v_y'^2=2gh'$ —③

若要满足题设可知 $h=h'$ —④

①②③④联立得 $h=\frac{e^2}{1-e^2}l$

$$\begin{cases} t_1=\frac{v_y}{g} \\ t_2=\frac{v_y'}{g} \\ x=v_0(t_1+t_2) \end{cases}$$

若满足题设 则 $x=l$

解得 $v_0=\sqrt{\frac{(1-e)gl}{2(1+e)}}$

故小球抛出高度 $h=\frac{e^2}{1-e^2}l$ 速度 $v_0=\sqrt{\frac{(1-e)gl}{2(1+e)}}$

1-24 解：绳与滑轮之间无相对滑动可知

$v_A=v_绳$

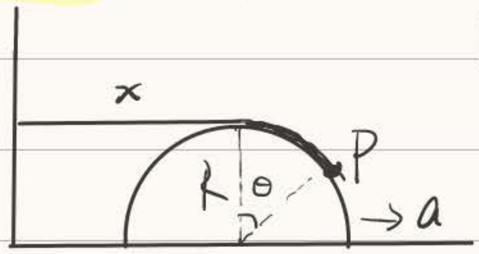
$s=\frac{1}{2}bt^2$ 对t求导得 $v=bt$ 再求得 $a_绳=b$

可知 $v_A=bt$　$a_t=b$　$a_f=\frac{b^2t^2}{R}$

$a=\sqrt{a_t^2+a_f^2}=\sqrt{1+\frac{b^2t^4}{R^2}}\,b$

1-31 解：可知 $x+R\theta=l$ —①

P点坐标 $\begin{cases} x_p=x+R\sin\theta & —② \\ y_p=R\cos\theta. & —③ \end{cases}$



①②③分别对t求导得

$$\frac{dx}{dt}=-R\frac{d\theta}{dt}\Rightarrow\frac{d\theta}{dt}=-\frac{v}{R}$$

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$$v_x = \frac{dx}{dt} + R\cos\theta \frac{d\theta}{dt} = v - v\cos\theta$$

$$v_y = -R\sin\theta \frac{d\theta}{dt} = v\sin\theta$$

$$v_p = \sqrt{v_x^2 + v_y^2} = 2\sin\frac{\theta}{2}\, v$$

$$a_x = \frac{dv_x}{dt} = \frac{dv}{dt} - \cos\theta \frac{dv}{dt} + v\sin\theta \frac{d\theta}{dt} = a - a\cos\theta - \frac{v^2}{R}\sin\theta$$

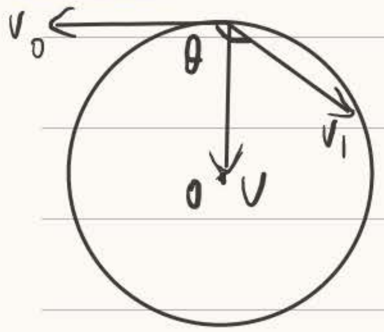
$$a_y = \frac{dv_y}{dt} = \sin\theta \frac{dv}{dt} + v\cos\theta \frac{d\theta}{dt} = a\sin\theta - \frac{v^2\cos\theta}{R}$$

故 $a_p = \sqrt{\left(a - a\cos\theta - \frac{v^2}{R}\sin\theta\right)^2 + \left(a\sin\theta - \frac{v^2}{R}\cos\theta\right)^2}$

综上：小球此时速度为 $v_p = 2\sin\frac{\theta}{2}\, v$

加速度为 $a_p = \sqrt{\left(a - a\cos\theta - \frac{v^2}{R}\sin\theta\right)^2 + \left(a\sin\theta - \frac{v^2}{R}\cos\theta\right)^2}$

1-33



解：可知 v_0 与抛球速度合成 $\vec{v} = \vec{v_0} + \vec{v_1}$，$v$ 应沿着指向圆心 O 方向.

$$\begin{cases} h = \frac{1}{2}gt^2 \\ R = vt \end{cases}$$ 得 $v = 5\sqrt{30}\,m/s$

可知 $\begin{cases} v_1\cos\theta + v_0 = 0 \\ v_1\sin\theta = v \end{cases} \Rightarrow \begin{cases} v_1 = 27.91\,m/s \\ \theta = 104.5^\circ \end{cases}$

即以 27.91 m/s 的速度抛出，速度方向夹角 $\theta = 104.5^\circ$

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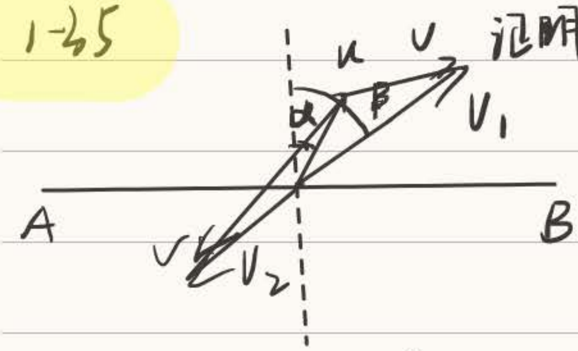
1-35



证明：由题：飞机飞行最长时间为 $t = \frac{2R}{v}$.

由示意图可得

$$v_1 = u\cos(\beta-\alpha) + \sqrt{v^2 - u^2\sin^2(\beta-\alpha)}$$

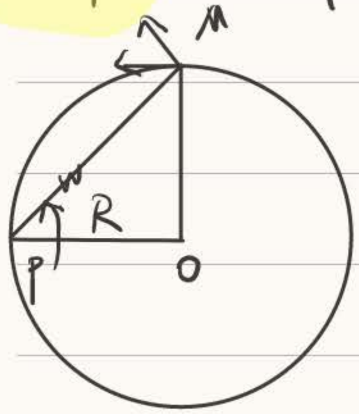
$$v_2 = \sqrt{v^2 - u^2\sin^2(\beta-\alpha)} - u\cos(\beta-\alpha).$$

设飞出最远距离为 R'

则 $t = \frac{R'}{v_1} + \frac{R'}{v_2}$

解得 $R' = \frac{R(v^2-u^2)}{v\sqrt{v^2-u^2\sin^2(\beta-\alpha)}}$

1-37



解：(1) ① 当M与P位于直径两端时

$V_1 = V_{M对圆} + V_{圆对地} = \omega R + 2\omega R = 3\omega R$

$a = a' + \omega \times (\omega \times 2R) + 2\omega \times \omega \times R = 5\omega^2 R$

② 当M与P重合时

$V_2 = V_{M对圆} + V_{圆对地} = \omega R$

$a = a' = \omega^2 R$

(2) $V = V_{M对圆} + V_{圆对地} = \sqrt{5}\,\omega R$

$a = a' + \omega \times (\omega \times \sqrt{2}R) + 2\omega \times \omega \times R = \sqrt{17}\,\omega^2 R.$

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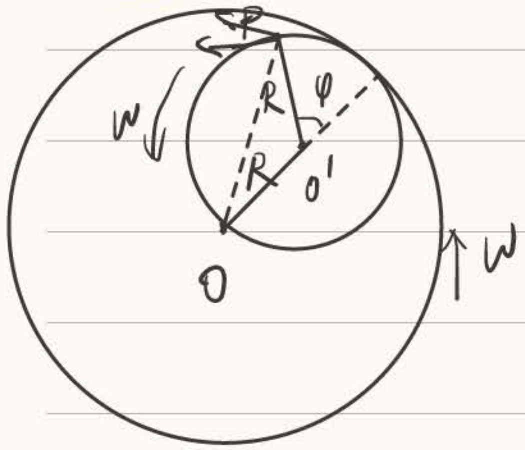
1-38



解: $\vec{v}=\vec{w}\times R+\vec{w}\times R_{op}=\sqrt{5+4\cos\varphi}\,wR$

$$a=\vec{w}\times\vec{w}\times R+\vec{w}\times(\vec{w}\times 2R\cos\frac{\varphi}{2})+2\vec{w}\times R$$

$$=\sqrt{9w^2R^2+4w^2R^2\cos^2\frac{\varphi}{2}+2\times 3wR\times 2wR\cos\frac{\varphi}{2}}$$

$$=\sqrt{17+8\cos\varphi}\,w^2R$$

故P点相对地面速度为$\sqrt{5+4\cos\varphi}\,wR$，加速度为$\sqrt{17+8\cos\varphi}\,w^2R$